Novel Low-Density Signature Structure for Synchronous DS-CDMA Systems

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***Abstract*— Novel Low-Density Signature (LDS) structure is proposed for synchronous Code Division Multiple Access (CDMA) systems for an uplink communication over AWGN channel. It arranges the system such that the interference pattern being seen by each user at each received sampled chip is different. Furthermore, new near-optimum chip-level iterative multiuser decoder is suggested to exploit the proposed structure. It is shown via computer simulations that, without forward error correction (FEC) coding, the proposed LDS structure could achieve near single-user performance with up to 200% loading condition. As the proposed iterative decoding converges relatively fast, the complexity is kept much more affordable than that of optimum multiuser detection (MUD) with conventional structure.**

I. Introduction

CDMA-based systems are widely known as soft interference-limited systems, so mitigating interference will logically enhance the system performance. Over the last decades, many techniques have been proposed to remove interference and can be classified into several classes: spreading signature optimization, MUD and joint FEC/MUD. Undesirably, they usually come with a complexity increase.

One variant of CDMA systems, i.e. Direct Sequence (DS) CDMA, distinguishes users by spreading their data over unique spreading signatures. In AWGN channel, hence, the only source of interference is the non-orthogonality of those signatures. Let K and N be the number of users and the processing gain, respectively. If $K \leq N$, many orthogonalizing methods can be used to construct the orthogonal sets, e.q. Gram-Schmidt [1]. Nevertheless, when the system is overloaded, i.e. $K > N$, orthogonal sets are impossible to derive. Later, the non-orthogonal sets constructed from Welch-Bound-Equality (WBE), devised in [2], are suggested as the optimum sets that minimize the sum of cross-correlations among users' signatures [3][4][5].

Without channel information at transmitter and the absence of FEC coding, the interference can, thence, be mitigated by using an intensive signal processing by way of MUD. The optimum MUD techniques, e.q. Maximum A Posteriori probability (MAP), yield the best performance in terms of the achievable probability of error [6], yet its computational complexity is intractable for practical implementation. In recent years, numerous proposals that design sub-optimum, low-complexity MUD techniques have been submitted [7]. However, they, generally, fail when the system is overloaded.

In the last decade, with the breakthrough of Turbo Codes [8], the new era of joint FEC/MUD is initiated by employing iterative turbo-style processing. The so-called turbo MUD approximates the highly complex optimum joint scheme by complementing the single-user FEC codes with soft-in-soft-out (SISO) multiuser decoding; the intermediate soft-information from each constituent (multiuser decoder and FEC decoder) are exchanged iteratively to approximate the optimum joint FEC/MUD. Many variants of turbo MUD have been proposed and significance performance improvements have been reported [9][10][11].

In this paper, we come up with a sophisticated structure that enables us to implement turbo-style processing in order to improve the system performance for synchronous DS-CDMA systems. Without FEC coding, we, thus, necessarily need to find sources to enable iterative processing in our algorithm. Most lately, LDPC codes, devised by Gallager in [12], have been attractive due to its capacity-approaching capability and the decoding simplicity by way of message passing algorithm (MPA)[13]. Motivated by these facts, we propose novel Low-Density Signature (LDS) structure that arranges the users' symbol in such a way that the interference seen by each user at each chip is different. Furthermore, we also propose near-optimum chip-level SISO iterative multiuser decoding to complement the proposed LDS structure. With a careful design of the structure of LDS matrix alongside with optimized signatures and BPSK modulation, the new system can approach near single-user performance with upto 200% loading while keeping the complexity more affordable than optimum MUD in conventional CDMA structure.

II. System Model

Consider a synchronous DS-CDMA system with K users and N processing gain. Let $\mathbb{X}_k$ be the modulated symbol constellation alphabet for user k. For simplicity of notation, we assume all users take their symbol from the same constellation alphabet $\mathbb{X}_k = \mathbb{X}, \forall k = 1, \ldots, K$. The symbol x_k is transmitted over AWGN channel synchronously and, thus, is sufficiently detected on symbol-by-symbol basis. The sampled, at chip rate, received symbol can be written as

$$y_n = \sum_{k=1}^{K} A_k s_{n,k} x_k + \nu_n \quad (1)$$

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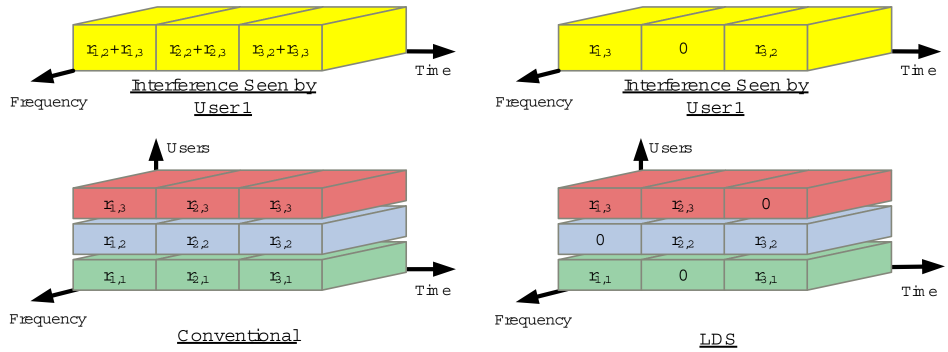

Fig. 1. Interference Pattern of Conventional and LDS Structure of DS-CDMA Systems

where $s_{n,k} \in \mathbb{C}$, $A_k \in \mathbb{R}$ and $\nu_n \sim \mathcal{N}(0, \sigma^2)$ are the n-th element of the normalized complex-valued signature assigned to user k, transmit gain for user k and the additive white noise, respectively, at n-th received chip. Let $\mathbf{s}_k = [s_{1,k}, \ldots, s_{N,k}]^T$ be the normalized spreading signatures with unit energy assigned to user k and $\mathbf{h}_k = A_k \mathbf{s}_k$ be the effective receive signature of user k. By stacking N successive received chip in (1), the received signal at the receiver is given by

$$\begin{aligned} \mathbf{y} &= \sum_{k=1}^{K} \mathbf{h}_k \mathbf{x}_k + \mathbf{v} \\ &= \mathbf{H}\mathbf{x} + \mathbf{v} \end{aligned} \tag{2}$$

where $\mathbf{x} = [\mathbf{x}_1, \ldots, \mathbf{x}_K]^T$ and $\mathbf{v} = [\nu_1, \ldots, \nu_K]^T \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$ denote the symbol vector and noise vector, respectively. The effective signature matrix is represented by $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K]$, where its column $k = 1, \ldots, K$ and its row $n = 1, \ldots, N$ denote the vector of signatures assigned to user k and the vector of the element of signatures for users that contribute their data at received sample chip n, correspondingly.

III. Low-Density Signature Structures

In conventional DS-CDMA systems, each element of $\mathbf{s}_k$ normally takes non-zero values and, hence, it is easy to see from (1) that the transmitted symbols from all users are superimposed at $y_n, n = 1, \ldots, N$.

Instead of optimizing the N-chips signatures as in literatures, we propose a new structure that arranges each user to spread its data virtually over a small number of chips, which is actually the effective processing gain of the user, and then zero-padding process is done such that the total chip is N. Furthermore, a unique interleaver will be employed to each user to scramble their signatures such that each user will see a different interference pattern at each chip it participates. Fig. 1 depicts an example of LDS structure. It is easy to see that the interference being seen by each user is different; at each chip, user sees limited number of interferer(s). Therefore, it is able to prevent a dominant interferer to corrupt all chips of a user, and hence, gives an intrinsic interference diversity to users. Furthermore, LDS structure, in AWGN channel, will also have some advantage in the multiple cell scenarios compared to the conventional structure because of the small effective processing gain as well as the interference diversity. The inter-cell interference can be regarded as an additional source of noise.

Let $d_v < N$ and $d_c < K$ be the maximum number of chips, over which users are allowed to spread their data, and the maximum number of users allowed to interfere to each other at each chip, respectively. The new signature for each user will, then, have a maximum of d_v non-zero values and a minimum of $N - d_v$ zero values. Consequently, at each receive chip, a user will have only maximum of $d_c - 1$ interferers.

In order to make this new structure becomes clearer, we introduce an indicator vector, $\psi_k \in \mathbb{F}_2^N$, for user k. The position of 1's in this vector indicates the position where user k should spread its data and the position of 0's denote the position where user k will not transmit its data. Then, $\Psi_{N \times K} = [\psi_1, \ldots, \psi_K]$ is the indicator matrix for all users that tells the users when to or not to transmit their data.

Let ξ_n and ζ_k be the sets of user indices that contribute their data at chip n and the set of chip indices, where user k spread his data over. Then, from the indicator matrix, ξ_n denotes the set of position of 1's in row n and ζ_k represents the set of position of 1's in column k.

The LDS indicator matrix shall be designed with the following requirements:

1) Maximum number of 1's in each column is $d_v < N$.
2) Maximum number of 1's in each row is $d_c < K$.
3) Full Connectivity on the associated Bipartite graph of the LDS Indicator matrix.

The first and second requirement are also required by to design an LDPC matrix. Moreover, for better performance, we impose another constraint as stated in the third requirement so that each user node, in the associated bipartite graph, should be able to reach other user nodes, therefore, fully cooperative processing can be fully achieved. Throughout this paper, however, the Indicator matrix is constructed as an LDPC matrix [14]. In addition, if the LDS indicator matrix has the same number of 1's in each column and has the same number of 1's in each row, then the LDS is said to have regular structure, otherwise irregular.

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With LDS convention, (1) can sufficiently be re-written as

$$\begin{aligned} y_n &= \sum_{k\in\xi_n} h_{n,k}x_k + \nu_n \\ &= \bar{\mathbf{h}}_n^T\mathbf{x}_n + \nu_n \end{aligned} \quad (3)$$

where $\mathbf{x}_n$ and $\bar{\mathbf{h}}_n$[1] denote the symbol vector and the corresponding elements vector of effective receive signatures, respectively, for user $k, \forall k \in \xi_n$ that contributes its symbol at chip n.

Example 1: A LDS DS-CDMA system is designed to serve a maximum of 6 users with the processing gain of 4 chips. Then, one realization of LDS indicator matrix with $d_v = 2$ and $d_c = 3$ is given by

$$\Psi_{4\times 6} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

User 3, for instance, will have to spread its data over $\zeta_3 = [1, 3]$ chips only. And at chip y_1 there are users: $\xi_1 = [1, 3, 4]$ that will interfere to each other.

IV. Iterative Chip-Level Multiuser Decoding

With the basic assumptions that the effective receive signature are available at the receiver, the optimum way to reconstruct $\mathbf{x}$ is by employing MAP detection algorithm on (2). Observing $\mathbf{y}$, the MAP algorithm will estimate $\hat{\mathbf{x}}$ that maximizes the joint *a posteriori* probability (APP)

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}\in\mathbb{X}^K} p(\mathbf{x}|\mathbf{y}). \quad (4)$$

Let x_k be the symbol of interest. Then, the marginal APP of x_k is given by

$$p(x_k|\mathbf{y}) = \sum_{\sim x_k} p(\mathbf{x}|\mathbf{y}) \quad (5)$$

Computing (5) with conventional CDMA structure by *brute-force* requires a complexity of $\mathcal{O}(|\mathbb{X}|^K)$, which is often too prohibitive for practical implementation. Furthermore, having the fact that the symbols are transmitted over a memory-less channel and the assumption that $x_k, k = 1, \ldots, K$ is statistically independent between users and distributed uniformly over $\mathbb{X}$, we can use Bayes' rule to write

$$p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x}) \quad (6)$$

and since noise elements in $\mathbf{v}$ are assumed uncorrelated, we are allowed to factorize

$$p(\mathbf{y}|\mathbf{x}) = \prod_{n=1}^{N} p(y_n|\mathbf{x}). \quad (7)$$

[1]The phase rotation, which is unique for each user, is already included in this vector. This is necessary as suggested for practical CDMA systems

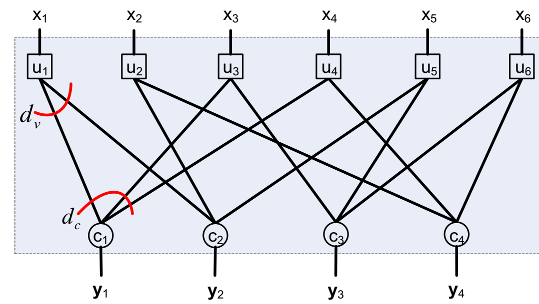

Fig. 2. Factor Graph Representation of LDS in example 1

Moreover, if LDS structure is used, then from (3) we have

$$\begin{aligned} p(y_n|\mathbf{x}) &= p(y_n|\mathbf{x}_n) \\ &= \exp(-\frac{1}{2\sigma^2}|y_n - \bar{\mathbf{h}}_n^T\mathbf{x}_n|^2). \end{aligned} \quad (8)$$

Hence, by using (6), (7), and (8), the resultant complexity of computing (5), with regular LDS, is now reduced to $\mathcal{O}(|\mathbb{X}|^{d_c})$, where d_c is chosen to be much smaller than K.

Motivated by the fact that the APP computation is visible in chip-level as in (8), we propose near-optimum chip-level iterative SISO multiuser decoding algorithm that is based on MPA to compute the marginal APP of all users efficiently simultaneously. The proposed iterative algorithm can be fully described by using factor graph $\mathcal{G}(\mathcal{C}, \mathcal{U})$ [15]. The symbol variables x_k and the channel observation function $p(y_n|\mathbf{x}_n)$ are, respectively, denoted by user nodes $u_k \in \mathcal{U}$ and chip nodes $c_n \in \mathcal{C}$. Node u_k is connected to nodes $c_n, \forall n \in \zeta_k$. Figure 2 depicts a factor graph representation for the LDS structure used in example 1.

The proposed decoding algorithm works as follows. Each node will exchange message along the corresponding edge $e_{n,k}$ defined in the factor graph. The message being exchanged is a vector[2] that contains the *extrinsic* 'opinion/reliability' of that node regarding each possible values for the corresponding x_k in $\mathbb{X}$ and, throughout this paper, is represented by, numerically stable, the logarithmic value of its corresponding APP. As explained in [13], the fundamental of MPA is that the message passed one node along the edge cannot depend on the message received from the same edge, therefore, the term *extrinsic* is used.

Without loss of generality and for notation brevity, BPSK is assumed as the modulation scheme, i.e. $\mathbb{X} = \{+1, -1\}$. Let $L_{n\uparrow}(x_k)$ and $L_{n\downarrow}(x_k)$ be the message being passed from node c_n that denotes the message obtained from the channel observation at node c_n with apriori knowledge coming from nodes $u_l, \forall l \in \xi_n \setminus k$ and the message being passed from node u_k that represents the *extrinsic* apriori information obtained from other nodes $c_m, \forall m \in \zeta_k \setminus n$ for observing the channel at node c_n, respectively, along the edge $e_{n,k}$. They are given

[2]For BPSK modulation, the message is sufficiently represented by its Log-Likelihood-Ratio (LLR)of a bit

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by

$$L_{n\uparrow}(x_k) = \log \frac{\sum_{\mathbf{x}_n : x_k = +1} p(y_n|\mathbf{x}_n) \prod_{l \in \xi_n \setminus k} p^{ext,n}(x_l)}{\sum_{\mathbf{x}_n : x_k = -1} p(y_n|\mathbf{x}_n) \prod_{l \in \xi_n \setminus k} p^{ext,n}(x_l)} \tag{9}$$

where $p^{ext,n}(x_l)$ denotes the *extrinsic* apriori information of x_l at node c_n and

$$\begin{aligned} L_{n\downarrow}(x_k) &= \log \frac{p^{ext,n}(x_k = +1)}{p^{ext,n}(x_k = -1)} \\ &= \sum_{m \in \zeta_k \setminus n} L_{m\uparrow}(x_k). \end{aligned} \tag{10}$$

Let a be the value taken from the constellation $\mathbb{X}$, then

$$\begin{aligned} p^{ext,n}(x_k = a) &= \frac{\exp(x_k L_{n\downarrow}(x_k))}{1 + \exp(x_k L_{n\downarrow}(x_k))} \\ &= \lambda_{n,k} \exp\left(\frac{x_k}{2} L_{n\downarrow}(x_k)\right) \end{aligned} \tag{11}$$

where $\lambda_{n,k}$ is chosen so as $p^{ext,n}(x_k = +1) + p^{ext,n}(x_k = -1) = 1$. Given a numerically stable operation $\max^\star(a, b) = \log\big(\exp(a) + \exp(b)\big)$ [16], by substituting (11) into (9) we can write (12) that can be found in the next page.

At the end of iteration, we can extract the final reliability value as follow

$$L(x_k) = \sum_{n \in \zeta_k} L_{n\uparrow}(x_k), \quad k = 1, \ldots, K. \tag{13}$$

Then a hard-decision can be taken from (13) and is given by

$$x_k = \begin{cases} +1; & L(x_k) \geq 0 \\ -1; & L(x_k) < 0 \end{cases}; \quad k = 1, \ldots, K \tag{14}$$

The proposed chip-level iterative joint MAP can be summarized as follows.

1) Initialize the priori information. If no a priori information is available, set $L_{n\downarrow}(x_k) = 0, \forall k, \forall n$
2) Update all chip nodes by using (12)
3) Update all user nodes by using (11)
4) If the number of iteration is met, go to 5) otherwise 2)
5) Calculate the final APP value for x_k with (13)
6) Estimate the transmitted symbol by using (14)

V. Numerical Results

In this section, we show the performance of our proposed LDS structure for synchronous DS-CDMA systems with BPSK modulation for transmission over AWGN channel. Notice that all simulations are done with the same spreading signatures[3] for each of the structures.

The LDS structures under investigation are *regular* LDS, unless otherwise stated. LDS $N \times K$ denotes the LDS structure that supports upto K users by using N processing gain. The structures being used in the simulation are: LDS 96×48 ($d_c = 3$, 50% loading), LDS 16×12 ($d_c = 3$, 75% loading), irregular LDS 12×12 ($d_c = 3$, 100% loading), LDS 12×16 ($d_c = 4$, 133% loading), and LDS 48×96 ($d_c = 6$, 200% loading).

[3]The signatures were found by trial-and-error and the signatures that gave the best performances were chosen.

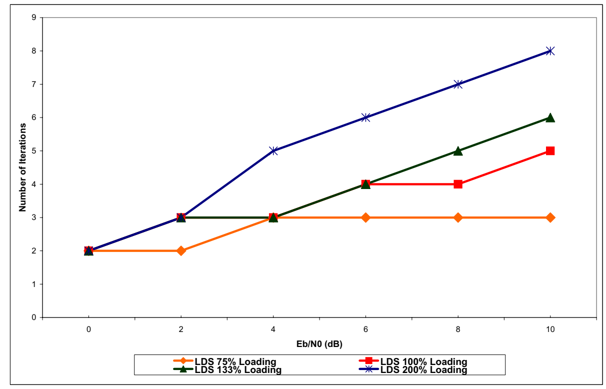

Fig. 3. Convergence Rate of Chip-Level Iterative Multiuser Decoding

Let W be the number of iterations, then the complexity of the proposed iterative decoding algorithm is $\mathcal{O}(W|\mathcal{X}|^{d_c})$. From Fig. 3, we can see that, at SNR lower than 8 dB, most LDS structures converge within 5 iterations, while at higher SNR, a slightly more iterations are needed. The reason behind these results is because of the performance of MAP detection at chip-level itself. The higher the SNR, more reliability can be given to its output and then the MPA can use that information to fine-tune its decoding process.

Moreover, the proposed iterative decoding is now compared with partial Parallel Interference Cancellation (PPIC) [17] with smoothing factor of 0.8 and 5 stages. The purpose is to show that the proposed iterative decoding is necessary to fully exploit the LDS structure. Fig. 4 shows that PPIC cannot use the whole potential of LDS structure even when we employ very light structures: LDS 96×48 or 50% loading and LDS 16×12 or 75% loading only. The proposed decoding is shown to approach single-user performance at overloaded condition. At 200% loading, its performance incurs only approximately 1.2dB loss compared to single-user performance at BER $= 10^{-4}$.

The sensitivity of LDS against near-far effect is shown in Fig. 5. LDS 12×16 is simulated with the following near-far condition: 4 users transmit their data 3dB, 1 user transmit his data 2dB and 9 users transmit their data 1dB above the reference. From Fig. 5(a) depicts the performance of the system from the reference user point of view. It is easy to see that the reference user is helped by the presence of strong user and the strong user degrades his performance because of the presence of weak user as Fig. 5(b) depicts. It is exactly the phenomenon that occurs at optimum multiuser detection [7]. Therefore we can conclude that the optimum MUD with LDS structure is well-approximated by the proposed iterative decoding.

VI. Conclusions

Novel LDS Structure complemented with near-optimum chip-level Iterative multiuser decoding that is based on MPA is proposed for synchronous DS-CDMA systems for transmission over AWGN channel. The LDS structure has the

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$$
\begin{aligned}
L_{n\uparrow}(x_k) &= \log \frac{\sum_{\mathbf{x}_n : x_k=+1} \exp\left(-\frac{1}{2\sigma^2}|y_n - \bar{\mathbf{h}}_n^T \mathbf{x}_n|^2 + \sum_{l\in\xi_n\backslash k}\left(\frac{x_l}{2}L_{n\downarrow}(x_l)\right)\right)}{\sum_{\mathbf{x}_n : x_k=-1} \exp\left(-\frac{1}{2\sigma^2}|y_n - \bar{\mathbf{h}}_n^T \mathbf{x}_n|^2 + \sum_{l\in\xi_n\backslash k}\left(\frac{x_l}{2}L_{n\downarrow}(x_l)\right)\right)} \\
&= \max_{\mathbf{x}_n : x_k=+1}{}^{\star} \left(-\frac{1}{2\sigma^2}|y_n - \bar{\mathbf{h}}_n^T \mathbf{x}_n|^2 + \sum_{l\in\xi_n\backslash k}\frac{x_l}{2}L_{n\downarrow}(x_l)\right) - \max_{\mathbf{x}_n : x_k=-1}{}^{\star} \left(-\frac{1}{2\sigma^2}|y_n - \bar{\mathbf{h}}_n^T \mathbf{x}_n|^2 + \sum_{l\in\xi_n\backslash k}\frac{x_l}{2}L_{n\downarrow}(x_l)\right) \quad (12)
\end{aligned}
$$

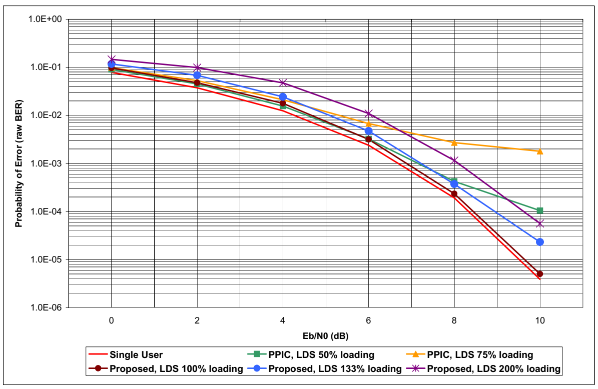
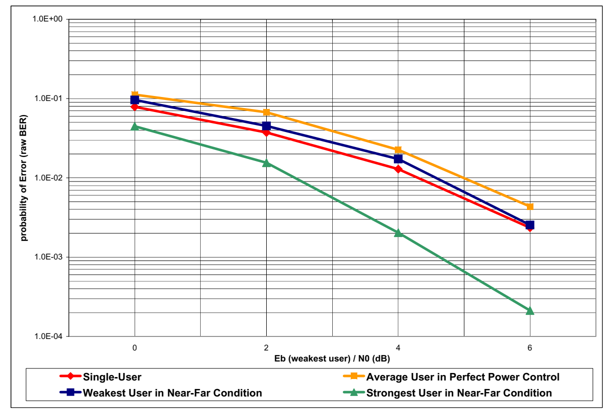
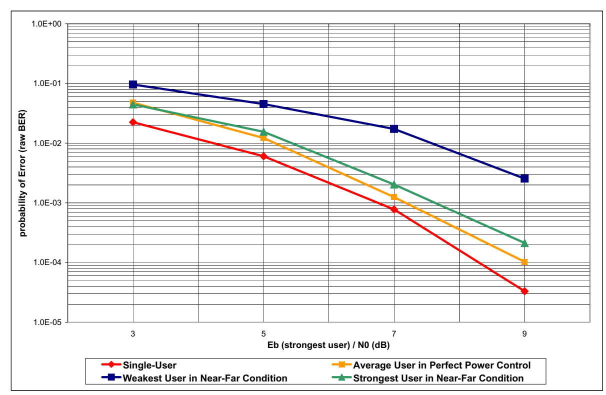


Fig. 4. Performance Comparison Chip-level PPIC and Iterative Joint MAP

interference diversity that prevents a strong interferer to corrupt all chips of a user. Our simulation has suggested that, given the optimized signatures, it could achieve near single user performance with up to 200% loading, while converging relatively fast then keeping the complexity more affordable than MAP algorithm with conventional structure. Furthermore, the proposed system is robust against near-far effect problem and its performance behavior is very similar to optimum MUD.



(a) From Reference User Point of View



(b) From Strongest Interferer Point of View

Fig. 5. Performance of LDS scheme under Near-Far Effect Condition

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