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Ionospheric Studies with the Differential Doppler Technique

by
Fernando de Mendonça

June 1962

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Grant NsG 30-60

RADIOSCIENCE LABORATORY
STANFORD ELECTRONICS LABORATORIES
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ABSTRACT

This report is essentially a review of the measurements related to the ionosphere utilizing the differential Doppler technique.

A brief study of the theory is presented, in which the error incurred in measurements of ionospheric electron content, caused by the assumption of common-path propagation, is shown as a function of the zenith angle of observation for different values of the critical frequency foF2.

Some measurements of electron densities, electron content, horizontal gradients, irregularities, and magnetic-storm effects are discussed. Comments are also made about the method of combining the differential Doppler with the Faraday rotation measurements to establish a necessary constant of integration in the measurements made with satellites.

It is concluded that the differential Doppler technique, with harmonically related frequencies transmitted to or from rockets or satellites, can provide accurate information about the electron density or electron content of the medium, and that the knowledge of these quantities is valuable in understanding the morphology of the ionospheric processes.

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I. INTRODUCTION

In this paper we will be concerned with ionospheric measurements made utilizing the differential Doppler technique.

When a radio wave of frequency f_1, emitted from a moving transmitter, propagates through the ionosphere it will have its Doppler shift slightly altered from the free-space value as a result of dispersion in the medium. This alteration in Doppler shift is very small and can best be detected by comparative measurements on two frequencies f_ℓ and f_h, which were harmonically related at the transmitter, i.e., f_1 and $f_2 = mf_1$.

The frequency beat between f_ℓ and f_h/m, often called differential Doppler, is proportional to the total electron content $\int N\,dh$ if the motion of the vehicle is parallel to the stratification, i.e., along a surface of constant index of refraction. When the vehicle moves along the ray path, the beat frequency will give information about the local electron density. The intermediate case, viz., velocities with components along the ray and along the stratification, will give a lumped result for density and content. Thus, in practice, the best results are obtained with vehicles in trajectories restricted to small zenith angles or satellites in circular orbits.

Normally the frequencies used with the present technique are conveniently chosen to permit approximations with a fair degree of accuracy, as we shall see later.

Initially, we will develop the theory on which the experiment is based and then, following a historical order, we will summarize the measurements made with rockets and finally those with satellites.

II. THEORY

We will begin by considering a medium in which the refractive index is an arbitrary function of position $(\rho, \theta, \emptyset)$, time t, and of the ray slopes $\theta' \equiv \rho \frac{\partial\theta}{\partial\rho}$ and $\emptyset' \equiv \rho \sin\theta \frac{\partial\emptyset}{\partial\rho}$. As we go along, the necessary simplifications, approximations, and special cases will be introduced to avoid undue complications.

The initial restrictions are that one endpoint of the ray is fixed at the observer $(\rho_o, 0, 0)$, and that the variation of the integral of the ray index of refraction along the ray path must vanish satisfying Fermat's principle

$$\delta \int n \, ds = 0. \tag{1}$$

This restriction is equivalent to the statement that we will be confined to the domain of ray optics where we have a slowly varying medium. We are going to apply the ray theory appropriate to an isotropic medium but use refractive indices that are dependent in the presence of the earth's magnetic field.

The phase path length P may be represented by the well known integral of the index of refraction μ; as

$$P = \int_L \mu \, (\rho, \theta, \emptyset, \theta', \emptyset', t) \, ds, \tag{2}$$

where L extends from the observer to the vehicle (ζ, η, ξ), ds is an element of the ray path.

Since

$$ds = \left[1 + (\rho\partial\theta/\partial\rho)^2 + (\rho \sin\theta \, [\partial\emptyset/\partial\rho])^2\right]^{1/2} d\rho \tag{3}$$

or

$$ds = \sec\chi \, d\rho \tag{4}$$

where χ is the angle between the directions of the ray and of the radial coordinate, we can change the variable of integration in (2) and obtain

$$P = \int_{\rho_o}^{\zeta} \mu \, (\rho, \theta, \emptyset, \theta', \emptyset', t) \sec\chi \, d\rho. \tag{5}$$

It is also permissible to carry the derivations to follow in two dimensions, as done by Kelso [Ref. 1], and extend the results to the three-dimensional case. Then

$$P = \int_{\rho_o}^{\zeta} U\,(\rho,\ \theta,\ \theta',\ t)\ d\rho \tag{6}$$

where

$$U = \mu\ (\rho,\ \theta,\ \theta',\ t)\ \sec\chi. \tag{7}$$

In the applications with which we will be concerned, the measurements of instantaneous received Doppler frequency Δf are given by

$$\Delta f = (1/2\pi)\ (d\vartheta/dt) = (f_1/c)\ (dP/dt) \tag{8}$$

where f_1 is the transmitted frequency and c is the free-space velocity of light. Then from (8) and the total derivative of (6) with respect to time we have

$$\Delta f = (f_1/c)\left[(U\ d\zeta/dt)_{\rho=\zeta} + \int_{\rho_o}^{\zeta} (\partial U/\partial t)\ d\rho + \int_{\rho_o}^{\zeta} [(\partial U/\partial\theta)\ (\partial\theta/\partial t) + (\partial U/\partial\theta')\ (\partial\theta'/\partial t)]\ d\rho\right]. \tag{9}$$

Also as shown by Kelso [Ref. 1], one can obtain the fundamental Euler's differential equation of the calculus of variation by applying Fermat's principle to the ray. This will provide, after some algebra, the following simplification to Eq. (9),

$$\Delta f = (f_1/c)\left[\mu_\zeta\ (\hat{r}\cdot\hat{v}) + \int_{\rho_o}^{\zeta} (\partial\mu/\partial t)\ \sec\chi\ d\rho + ([\partial\mu/\partial\theta']\ [\partial\theta'/\partial t]\ \sec\chi)_{\rho=\zeta}\right] \tag{10}$$

where $(\hat{r}\cdot\hat{v})$ is the component of the vehicle velocity along the ray at $\rho = \zeta$, and μ_ζ is the value of the index of refraction at the vehicle.

In the isotropic case, the last term of (10) vanishes and the equation reduces to

$$\Delta f = (f_1/c) \; [\mu_\zeta \; (\hat{r}\cdot\hat{v}) + \int_{\rho_o}^{\zeta} (\partial\mu/\partial t) \sec \chi \; d\rho]. \tag{11}$$

This equation is misleading because it might lead one to assume that the first term in the bracket will always give a contribution to the Doppler shift due to electrons in the neighborhood of the vehicle. That this is not so is easily shown by a simple example. Consider the case of a spherically stratified, time-invariant medium in which the vehicle moves along a surface of constant μ. Snell's law for such a medium is given by

$$\mu\rho \sin \chi = \mu_1 \; \rho_1 \sin \chi_1 = \rho_1 \sin \chi_1 \tag{12}$$

where ρ_1 is the level of $\mu_1 = 1$. From Eqs. (11) and (12) we have

$$\Delta f = (f_1/c) \; \frac{\rho_1 \sin \chi_1}{\rho \sin \chi} \; \hat{r}\cdot\hat{v} \tag{13}$$

But $\hat{r}\cdot\hat{v}$ in this example is equal to $v \sin \chi$; then from Eq. (13)

$$\Delta f = (f_1/c) \; (\rho_1/\rho) \sin \chi_1. \tag{14}$$

This equation shows that the Doppler shift is a measure of electron content, since, as shown by Weekes [Ref. 2], $\sin \chi_1$ is related to $\int N \, dh$. This relationship could have been shown much more easily if we had used Eq. (9) instead of Eq. (11) in the example above. From Eq. (9), with $d\zeta/dt = 0$, and a spherically stratified, time-invariant, isotropic medium we obtain

$$\Delta f = (f_1/c) \int_{\rho_o}^{\zeta} \mu \; (\partial/\partial\theta') \sec \chi \; (\partial\theta'/\partial t) \; d\rho \tag{15}$$

which shows the dependence of Δf on the integration of μ rather than a local value of it.

If one considers now a transmitted higher-frequency f_2 harmonically related to f_1, i.e., $f_2 = m \, f_1$, where m, here assumed to be greater than unity, is the ratio of two integers, then one can establish an expression for the instantaneous received Doppler shift Δf_h by means of Eq. (9); as

$$\Delta f_h = (mf_1/c)\Big([U_2 \, d\zeta/dt]_{\rho=\zeta} + \int_{\rho_o}^{\zeta} [\partial U_2/\partial t] \, d\rho + \int_{\rho_o}^{\zeta} [(\partial U_2/\partial\theta)(\partial\theta/\partial t) + (\partial U_2/\partial\theta')(\partial\theta'/\partial t)] \, d\rho\Big). \qquad (16)$$

In the propagation experiments with rockets as pioneered by Seddon [Refs. 3 and 4] the beat frequency or differential Doppler is obtained by multiplying the lower frequency f_ℓ by m and heterodyning it with the higher frequency f_h. In the work with satellites, some workers have instead divided the higher frequency by m and then combined it with the lower frequency to obtain the beat.

Let μ_1 and μ_2 represent the indices of refraction of the lower and higher frequencies; then, multiplying Eq. (9) by m and subtracting the result from Eq. (16), one obtains an expression for the beat frequency f_b, commonly called differential Doppler:

$$\begin{aligned} f_b &= \Delta f_h - m\Delta f_\ell \\ &= (mf_1/c)\Big[(U_2 - U_1)_{\rho=\zeta} \, d\zeta/dt \\ &\quad + \int_{\rho_o}^{\zeta} (\partial/\partial t)(U_2 - U_1) \, d\rho \\ &\quad + \int_{\rho_o}^{\zeta} [(\partial U_2/\partial\theta_2)(\partial\theta_2/\partial t) - (\partial U_1/\partial\theta_1)(\partial\theta_1/\partial t)] \, d\rho \\ &\quad + \int_{\rho_o}^{\zeta} [(\partial U_2/\partial\theta'_2)(\partial\theta'_2/\partial t) - (\partial U_1/\partial\theta'_1)(\partial\theta'_1/\partial t)] \, d\rho\Big] \end{aligned} \qquad (17)$$

Equation (17) was obtained without any approximations other than restriction to two dimensions. The extension to the three-dimensional case can be accomplished without much difficulty. In the experiment to be studied we wish to determine which of the terms dominates the others in Eq. (17). In the case of rockets in essentially vertical trajectories, the first term is usually much larger than the sum of the three integrals; in the case of a geostationary satellite the second term is the preponderant one; and in the case of common satellites in near-circular orbits the last two integrals are much larger than the sum of the first two terms. Now we shall analyze these cases separately in a more detailed manner.

In rocket experiments, when the zenith angle χ is small, one can assume that the rays of both waves coincide with the straight line of the range R, and that

$$(\partial\theta/\partial t) \approx 0$$

and

$$(\partial\theta'/\partial t) = (\partial/\partial t)\ (\rho\partial\theta/\partial\rho) \approx 0.$$

Thus, from Eq. (17) we obtain

$$f_b \approx (mf_1/c)\ [(\mu_2 - \mu_1)_R\ (dR/dt) + \int_0^R (\partial/\partial t)\ (\mu_2 - \mu_1)\ dR]. \tag{18}$$

The frequencies are conveniently chosen to be much higher than the plasma frequency of the medium given by $f_c = (2K_1N)^{1/2}$, where $K_1 = 40.3$ mks units and N is the electron density. Normally, one can use the quasi-longitudinal approximation [Ref. 5] to the indices of refraction (ordinary and extraordinary) in the absence of collisions, i.e., in the usual notation,

$$\begin{aligned}(\mu_1)_{o,x} &= 1 - K_1N/f_1^2\ (1 \pm [f_L/f_1]) \\ &= 1 -(K_1N/f_1^2)\ (1 \mp [f_L/f_1] + [f_L/f_1]^2 \mp \ldots)\end{aligned} \tag{19}$$

because f_1 is also higher than f_L.

The difference between the indices of refraction of the two waves of frequencies f_1 and mf_1 is easily deduced from (19) as

$$(\mu_2 - \mu_1)_{o,x} = (K_1 N/f_1^2)\ \alpha_{o,x} \tag{20}$$

where $\alpha_{o,x}$ is defined by

$$\alpha_{o,x} = (1 - 1/m^2) \mp (1 - 1/m^3)\ (f_L/f_1) + (1 - 1/m^4)\ (f_L/f_1)^2 \mp \ldots \tag{21}$$

From Eqs. (18) and (20) and the approximations

$$\int_o^R (\partial/\partial t)\ (\alpha_{o,x}\ N)\ dR \approx \int_o^R \alpha_{o,x}\ (\partial N/\partial t)\ dR \approx \bar{\alpha}_{o,x} \int_o^R (\partial N/\partial t)\ dR. \tag{22}$$

We may write the equation used in the rocket experiments

$$(f_b)_{o,x} \approx (mK_1/cf_1) \left[(\alpha_{o,x}\ N)_R\ dR/dt + (\bar{\alpha}_{o,x}) \int_o^R (\partial N/\partial t)\ dR\right]. \tag{23}$$

The integral that appears in Eq. (23) is usually very small in comparison with the first term of the second member, and is therefore treated as a correction term. For rockets which attain heights of only a few hundred kilometers, one can disregard the integral in Eq. (23), while dR/dt is not close to zero, and determine the electron density in the neighborhood of the vehicle (N_R),

$$N_R \approx (f_b)_{o,x}\ cf_1/[mK_1\alpha_{o,x}\ (dR/dt)]_R. \tag{24}$$

Equations (23) and (24) will be discussed further in reviewing the papers about propagation experiments with rockets.

In the case of a geostationary [Ref. 6] or synchronous satellite, one may again use Eq. (17), noting that, with the exception of the second term, all others may be disregarded; then

$$f_b = (mf_1/c) \int_{\rho_o}^{\zeta} (\partial/\partial t)\ (U_2 - U_1)\ d\rho. \tag{25}$$

For sufficiently high frequencies $(f_1 \gg f_c)$, and considering that the resultant beat depends on an integration as shown above, then one can simplify Eq. (25) by assuming a common path for both signals and write

$$f_b = (mf_1/c) \int_{\rho_o}^{\zeta} \sec\chi \, (\partial/\partial t)\, (\mu_2 - \mu_1)\, d\rho. \tag{26}$$

Combining Eqs. (20), (23), and (26) and considering a constant height for the satellite we obtain

$$(f_b)_{o,x} = (mK_1/cf_1)\, \bar{\alpha}_{o,x}\, \overline{\sec\chi}\, (\partial/\partial t) \int_{\rho_o}^{\zeta} N\, d\rho. \tag{27}$$

Thus in this particular case, the ordinary or the extraordinary differential Doppler will provide a measurement of the time rate of change of the total electron content.

Finally, for the case of satellites with orbits of small eccentricity, one may again use Eq. (17), where the first term of the right-hand side, i.e.,

$$(mf_1/c)\, (U_2 - U_1)_{\rho=\zeta}\, d\zeta/dt$$

is a very small contribution to the total of the second member, when the satellite is not near the point of closest approach, because the radial component of velocity is small. If the satellite is well above the level of maximum ionization N_{max}, the difference $(U_2 - U_1)$ is also a small quantity. The second term,

$$(mf_1/c) \int_{\rho_o}^{\zeta} (\partial/\partial t)\, (U_2 - U_1)\, d\rho,$$

is small because the time of observation of the satellite is usually ten minutes or less, in which case one may consider the medium as time-invariant if the observations are not around sunrise or sunset. The third term is related to the horizontal gradients, as can be seen if one considers the high-frequency and common-path approximations with $\mu_1 \approx 1 - K_1 N/f_1^2$. Then

$$(mf_1/c) \int_{\rho_o}^{\zeta} [(\partial U_2/\partial\theta_2)\, (\partial\theta_2/\partial t) - (\partial U_1/\partial\theta_1)\, (\partial\theta_1/\partial t)]\, d\rho$$

$$\approx (mK_1/cf_1)\, (1 - 1/m^2) \int_{\rho_o}^{\zeta} \sec\chi\, (\partial N/\partial\theta)\, (\partial\theta/\partial t)\, d\rho.$$

With these same approximations one can see that the last term of Eq. (17) is proportional to the electron content

$$(mf_1/c) \int_{\rho_o}^{\zeta} [(\partial U_2/\partial\theta'_2)(\partial\theta'_2/\partial t) - (\partial U_1/\partial\theta'_1)(\partial\theta'_1/\partial t) \; d\rho$$

$$\approx (mK_1/cf_1)(1 - 1/m^2) \int_{\rho_o}^{\zeta} N\,[(\partial/\partial\theta')\sec\chi]\,(\partial\theta'/\partial t)\; d\rho.$$

In view of the complexity of Eq. (17) as described in the previous paragraph we shall develop a more suitable approach for the case of satellites in orbits of small eccentricity.

Considering again Eqs. (6), (7), and (8) we may write for the lower frequency,

$$\Delta f_{\ell} = (f_1/c)\,(d/dt) \int_{\rho_o}^{\zeta} U_1 \; d\rho, \tag{28}$$

and also for the higher frequency,

$$\Delta f_h = (mf_1/c)\,(d/dt) \int_{\rho_o}^{\zeta} U_2 \; d\rho. \tag{29}$$

Dividing the Doppler shift of the higher frequency by the factor m, and combining it with the Doppler shift of the lower frequency, one obtains the beat frequency

$$f_b = (\Delta f_h/m) - \Delta f_{\ell} = (f_1/c)\,(d/dt) \int_{\rho_o}^{\zeta} (U_2 - U_1)\; d\rho \tag{30}$$

or

$$f_b = (f_1/c)\,(d/dt) \int_{\rho_o}^{\zeta} (\mu_2 \sec\chi_2 - \mu_1 \sec\chi_1)\; d\rho. \tag{31}$$

Equation (31) can be integrated with respect to time, resulting in

$$\int f_b \; dt + C = (f_1/c) \int_{\rho_o}^{\zeta} (\mu_2 \sec\chi_2 - \mu_1 \sec\chi_1)\; d\rho \tag{32}$$

where C is an unknown constant of integration, and the integral of the first member can be easily obtained by merely counting the cycles of the beat. By adding and subtracting $(\mu_1 \cos \chi_2)$ to the integrand of Eq. (32) and rearranging terms, we obtain:

$$(c/f_1)\left(\int f_b dt + C\right) = \int_{\rho_o}^{\zeta} (\mu_2 - \mu_1) \sec \chi_2 \, d\rho - \int_{\rho_o}^{\zeta} \mu_1 (\sec \chi_1 - \sec \chi_2) \, d\rho \tag{33}$$

or

$$\equiv E_1 - E_2 = E_1 (1 - [E_2/E_1]). \tag{34}$$

Note that the common-path assumption has not been made for the two signals. Normally the higher frequency is such that $\sec \chi_2$ can be calculated along the line of sight or range R. Note also that the last integral of Eq. (33) satisfies the following inequalities

$$E_2 \equiv \int_{\rho_o}^{\zeta} \mu_1 (\sec \chi_1 - \sec \chi_2) \, d\rho < \int_{\rho_o}^{\zeta} (\sec \chi_1 - \sec \chi_2) \, d\rho \tag{35}$$

$$E_2 \leq (S_1 - S_2) < (S_1 - R) \tag{36}$$

where S_1 and S_2 are the geometrical lengths of the rays and R is the range.

Now we want to show that the magnitude of the error

$$e \equiv (E_2/E_1) < (S_1 - R)/E_1 \equiv \Delta$$

incurred by disregarding the integral E_2 in Eq. (34) is very small. For simplicity of computation, assume a spherically symmetric layer with a distribution for N as shown in Fig. 1.

Consider also the high-frequency approximation with $f_1 = 54$ Mc and $m = 6$ (Transit satellites), and a satellite in a circular orbit at the height $h_s = \rho_2 - \rho_o$. For this example we can obtain the values for Δ, parametric in N_{max}, as a function of the zenith angle at the observer χ_o as shown in Fig. 2.

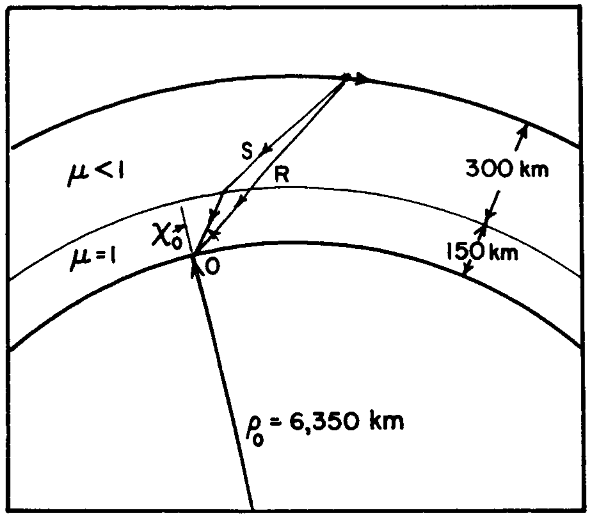

FIG. 1. DISTRIBUTION USED FOR CALCULATION OF $(S_1 - R)/E_1$.

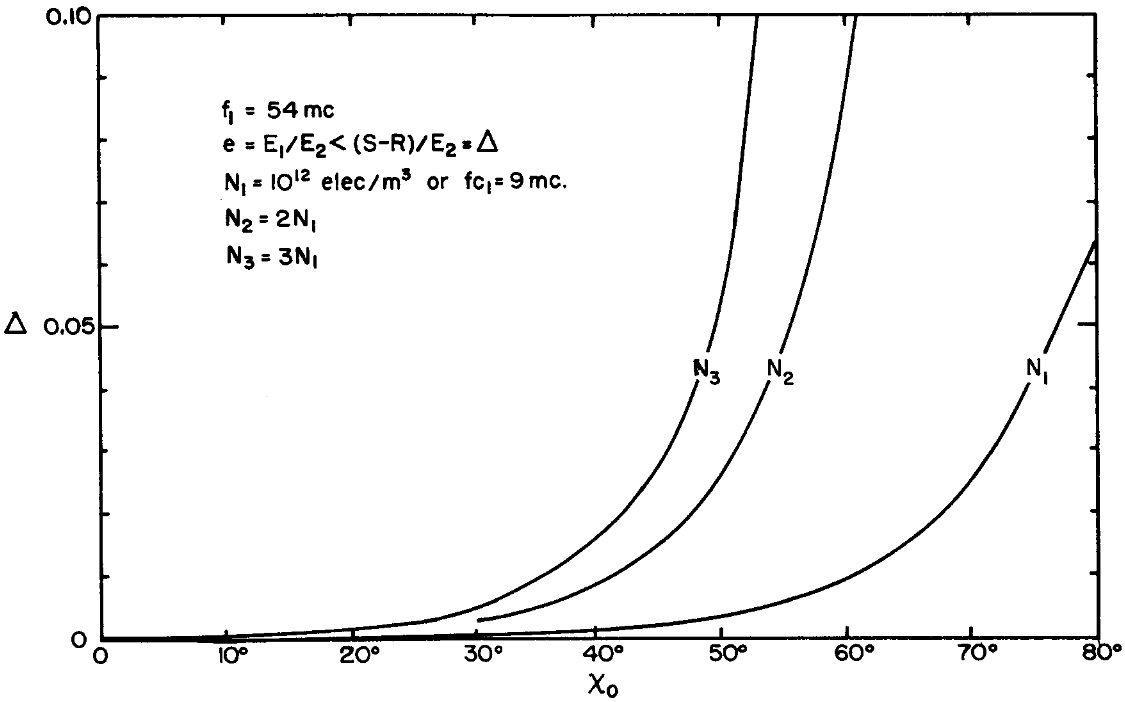

FIG. 2. ERROR Δ AS A FUNCTION OF χ_0, PARAMETRIC IN N_{max}, FOR THE FREQUENCY OF 54 Mc, AND USING THE DISTRIBUTION GIVEN IN FIG. 1.

It can be shown [Ref. 7] that, with a flat-ionosphere approximation, the value of Δ is proportional to f_1^{-2}.

Thus one can establish a boundary in the value of error e which one is willing to tolerate by simply limiting the cone of observation of the satellite to a convenient maximum χ for a given N_{max}.

Equation (34) can be rewritten to read

$$\int_{\rho_o}^{\zeta} (\mu_2 - \mu_1) \sec \chi_2 \, d\rho = [1/(1 - e)] \, (c/f_1) \, (\int f_b \, dt + C). \qquad (37)$$

From Eqs. (20) and (37), we obtain an expression for the total electron content $I(t)$, from the ground to the height of satellite h_s,

$$(K_1 \, \overline{\alpha}_{o,x}/f_1^2) \int_o^{h_s} N \sec \chi_2 \, dh \approx (1 + e) \, (c/f_1) \, (\int [f_b]_{o,x} \, dt + C) \qquad (38)$$

or

$$I(t) \equiv \int_o^{h_s} N \, dh = (f_1 c/K_1) \, (\overline{\cos \chi}/\overline{\alpha}_{o,x}) \, (1 + e) \, [\int (f_b)_{o,x} \, dt + C] \qquad (39)$$

where $\overline{\cos \chi}$ is a weighted average value, usually calculated from the satellite's ephemerides at the level of N_{max}, i.e., $\overline{\cos \chi} = \cos \chi_m$. For "Transit" satellites ($f_1 = 54$ Mc and $m = 6$) with zenith angles smaller than 60 deg, Eq. (39) can be written

$$I(t) \approx (f_1 c/K_1) \cos \chi_m \, (m^2/[m^2 - 1]) \, (\int f_b \, dt + C). \qquad (40)$$

The constant of integration C in Eq. (40) can be established either by means of least-squares best-fit approximations [Ref. 7] or by combining the differential Doppler information with simultaneous Faraday rotation measurements [Ref. 8].

III. SUMMARY OF DIFFERENTIAL DOPPLER MEASUREMENTS WITH THE AID OF ROCKETS

Rocket measurement of electron densities utilizing differential Doppler was started by Seddon [Ref. 3] at the Naval Research Laboratory as early as 1946. Table 1 shows a partial list of some of the experiments performed since then, with the mentioned process.

TABLE 1. SOME DIFFERENTIAL DOPPLER MEASUREMENTS WITH ROCKETS

Year	Rocket	Height (km)	Frequencies (Mc)	References
1946-47	V-2	150	4.27 and 25.62	3, 4
1954	Viking-10	220	7.75 and 46.50	9, 10
1956	Aerobee Hi	220	7.75 and 46.50	11
1958	(Russian)	470	24, 48 and 144	12
1959	Aerobee Hi	220	7.75 and 46.50	13
1961	ARGO D-4	620	12.27 and 73.62	14, 15

Figures 3 and 4 show typical results of electron densities from two flights, the first with an Aerobee Hi [Ref. 11] at a time when sporadic-E was present, and the second [Ref. 14] with a higher-altitude rocket of the type ARGO D-4.

Equation (24) was used in the analyses of the data obtained in the early experiments [Refs. 3, 4, and 9] with the approximation $\mu_2 = 1$, i.e., $\alpha_{o,x} \approx (1 \mp f_L/f_1)$. Equation (23) was used in later experiments with the inclusion of roll correction. Jackson and Bauer [Ref. 14] included a correction for obliquity of the trajectory of the rocket as shown in Fig. 4. They also [Ref. 15] based their analysis on the sum of the ordinary and extraordinary beat frequencies, thus eliminating the roll effects, since the individual ordinary and extraordinary beat frequencies have equal but opposite roll corrections.

A summary of the results obtained in experiments with rockets and satellites has been written by Seddon [Ref. 16], in which he discusses measurements of electron densities, electron-density gradients, sporadic-E and spread-F, some parameters involved in the formation of the ionosphere and presents an extensive bibliography on this subject.

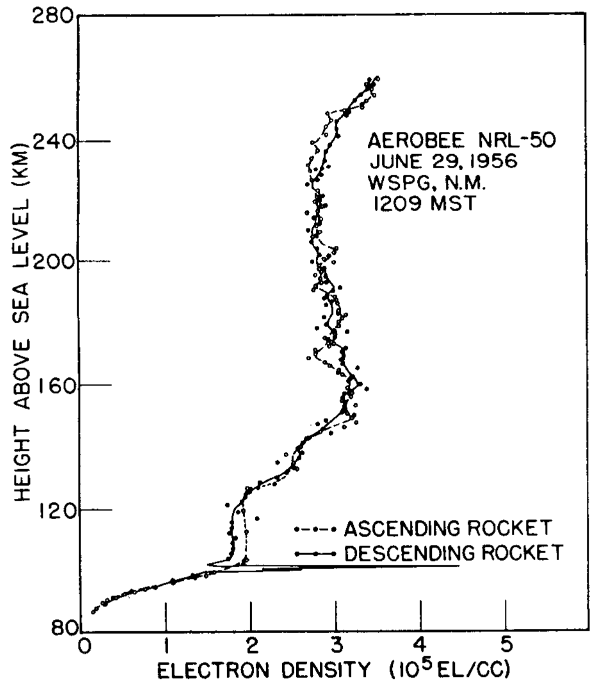

FIG. 3. ASCENT AND DESCENT MEASUREMENTS OF ELECTRON DENSITIES WITH AEROBEE-HI NRL-50, AFTER JACKSON AND SEDDON [Ref. 11]. NOTE THE PRESENCE OF SPORADIC-E AT ABOUT 100 km, AND ALSO THE ABSENCE OF A VALLEY BETWEEN E AND F REGIONS.

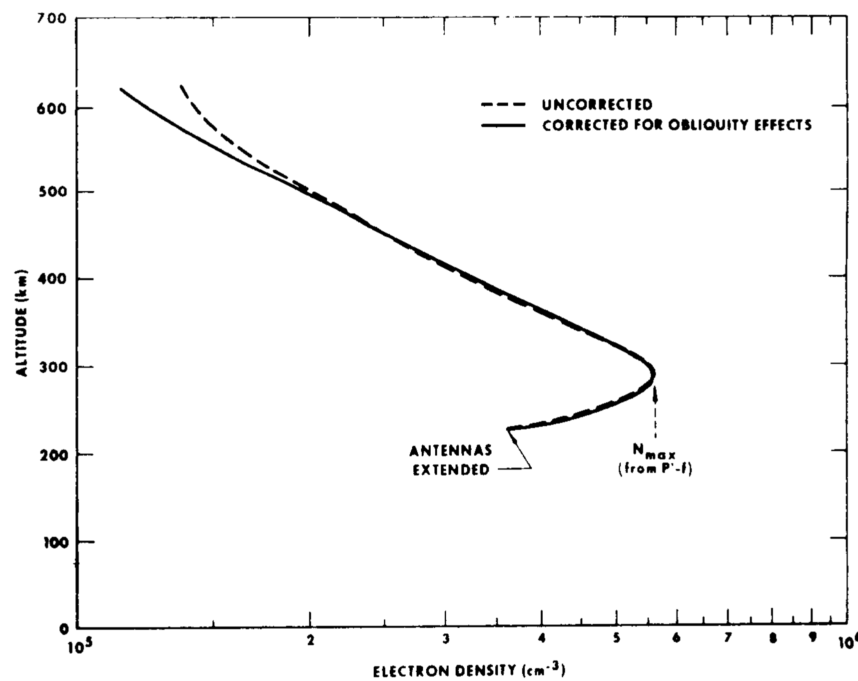

FIG. 4. ELECTRON-DENSITY MEASUREMENTS FROM A FOUR-STAGE RESEARCH ROCKET OF THE TYPE ARGO D-4 FIRED FROM WALLOPS ISLAND, VIRGINIA, 1502 EST, 27 APRIL 1961, AFTER BAUER AND JACKSON [Ref. 14].

Differential Doppler measurements up to one earth radius with rockets are planned for the near future, in which case the variations of the ionosphere below the vehicle must always be taken into consideration in order to arrive at reliable local electron-density data. Corrections for this time variation can be made with the procedure outlined by Bauer and Jackson [Ref. 15]. The potential importance of these high-altitude electron-density measurements in inferring the structural parameters of the earth's outer atmosphere is shown by Bauer in Ref. 17.

Presently, differential Doppler experiments with rockets still are one of the most accurate sources of electron-density profiles of the ionosphere. The best results are obtained in a fairly quiet ionosphere and for that part of the trajectory in which the vehicle has a substantial vertical velocity. Other techniques are required in order that measurements may be made under all ionospheric conditions and utilizing the whole flight path. See, for instance, the simultaneous measurement of electron density made by differential Doppler and RF impedance-probe techniques reported by Kane et al [Ref. 13].

IV. DIFFERENTIAL DOPPLER MEASUREMENTS WITH THE AID OF SATELLITES

It has previously been shown, Eq. (40), that the electron content $I(t)$ can be obtained by integration of the beat frequency. In the experiments with rockets, the constant of integration C in Eq. (40) can easily be determined because either the beat starts from zero below the ionosphere (Fig. 3), implying $C = 0$, or one may use the information about N_{max} from ionosondes for fitting purposes, as shown in Fig. 4.

With satellites, the problem of determining the constant C is not as simple. Ross [Refs. 18 and 19], using the transmissions at 20 and 40 Mc from Sputnik III, obtained C by making use of the geometry of the satellite-observer system, ionosphere height and slopes of the differential Doppler frequency plot. Garriott and Nichol [Ref. 20] calculated C and therefore $I(t)$ using a model ionosphere (spherical symmetry assumed) for best fit between the acquired data and calculated values. Garriott and Bracewell [Ref. 21] also have presented a very simple and straightforward process that can be used in the absence of horizontal gradients.

Another method for obtaining the constant C is the one which combines the simultaneous measurements of Faraday rotation angle and differential Doppler as described in Refs. 8, 22, 23, 24, and 25. This method is derived from the relation that can be established between the Faraday rotation angle Ω and the differential Doppler measurements. This relationship can easily be found by considering the high-frequency approximation for the Faraday rotation angle [Ref. 26],

$$\Omega = (K_2/f_1^2)\ \overline{H}_L \int_0^{h_s} N \sec \chi \, dh \tag{41}$$

where $\overline{H}_L$ is a weighted average of components of the earth's magnetic field along the ray and K_2 is a constant ($2.97\ 10^{-2}$ mks units). Considering, now, Eqs. (38) and (41), one can eliminate the factor

$$\int_0^{h_s} N \sec \chi \, dh$$

and obtain the relation

$$\Omega = (K_2 H_L/K_1\bar{\alpha}) (\int f_b \, dt + C). \tag{42}$$

For two different times, t_a and t_b, one can find C from Eq. (42), i.e.,

$$C = [(K_1\bar{\alpha}/K_2) (\Omega_b - \Omega_a) - (\bar{H}_L \int f_b \, dt)_{t = t_b} + (\bar{H}_L \int f_b \, dt)_{t = t_a}]/(H_{Lb} - H_{La}) \tag{43}$$

where the weighted average values for $\bar{H}_L$ can be computed [Refs. 8 and 27] and the other terms are measured quantities. Note that for all practical purposes, if the antenna systems used are linear [Ref. 28] in the case of high-frequency approximation, we have

$$\bar{\alpha}_{o,x} \approx \bar{\alpha} \approx 1 - 1/m^2. \tag{44}$$

Under these circumstances, the received beat f_b has a value which is approximately an average between the ordinary and the extraordinary beats. Reference 28 presents, in detail, the influence of the presence of the earth's magnetic field in the differential Doppler measurements.

We shall here briefly describe the process used in Ref. 7 for determination of the constant C. Rearranging terms in Eq. (40), we rewrite as

$$(\int f_b \, dt + C) = (K_1\bar{\alpha}/cf_1) \sec \chi_m \, I(t). \tag{45}$$

From the time derivative of Eq. (45) one obtains

$$dI/dt + [\cos \chi_m (d/dt) \sec \chi_m] \, I = (cf_1/K_1\bar{\alpha}) f_b \cos \chi_m. \tag{46}$$

There will be a time $t = t_c$ at which $\sec \chi_m$ assumes a minimum value, then

$$[(d/dt) \sec \chi_m]_{t=t_c} = 0. \qquad (47)$$

Considering Eq. (46) and (47), one can establish an average value of the beat frequency at the time t_c and define a constant K,

$$K \equiv \overline{(dI/dt)}_{t=t_c} = (cf_1/K_1\bar{\alpha})\ (\bar{f}_b \cos \chi_m)_{t=t_c}. \qquad (48)$$

This constant K can be obtained from the differential Doppler data as shown in Fig. 5, where the expression Doppler shift offset [Ref. 21] $\Delta\dot{P}/\lambda$ is equal to $(f_b/\bar{\alpha})$.

We can now, using Eq. (46), define a function F_b which would be the beat frequency if dI/dt were a constant with value K.

$$F_b \equiv (K_1\bar{\alpha}/cf_1) \left[K \sec \chi_m + ([d/dt] \sec \chi_m)\ I\right]$$

or, substituting $I(t)$ from the expression given in Eq. (40),

$$F_b = (K_1\bar{\alpha}/cf_1)\ K \sec \chi_m + \cos \chi_m \quad (\int f_b\, dt + C)\ (d/dt) \sec \chi_m. \qquad (50)$$

Forming the sum of the squares of the differences between the measured beat and F_b, at a properly chosen time interval about t_c, we obtain

$$S = \Sigma(F_b - f_b)^2. \qquad (51)$$

Except for C, all the quantities involved in the right-hand side of Eq. (51) are known. With the aid of a digital computer one can establish a value of C that minimizes S defined above, and this value of C is the sought constant of integration to be used in Eq. (40) for determination of the electron content $I(t)$. Figure 6 shows $I(t)$ obtained with this process [Ref. 7] for a northbound passage of the satellite Transit 2-A, observed from Stanford, California. The point of closest approach (PCA) of the satellite was at 07h 25m 32s (LMT), at a height of $h_s = 1064$ km and $h_s = -10$ m/s.

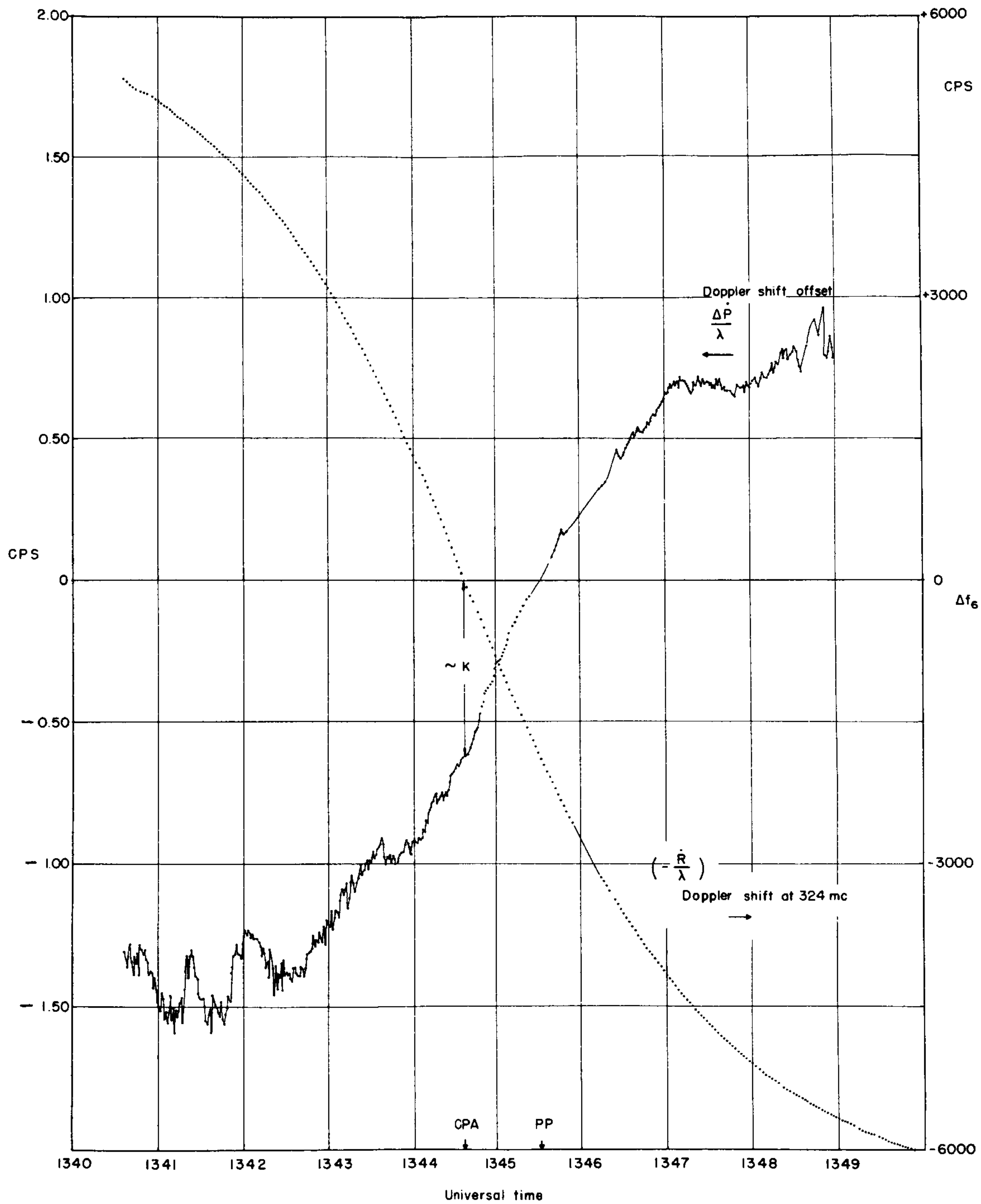

FIG. 5. MORNING PASSAGE OF THE SATELLITE TRANSIT 2-A ON 29 JULY 1960, WHILE CLOSE TO APOGEE ($\dot{h}_s$ = -20 m/s), SHOWING DOPPLER SHIFT OFFSET WHEN IRREGULARITIES IN ELECTRON DISTRIBUTION ARE WEAK. THE DOPPLER SHIFT OF THE 324-Mc CHANNEL IS ALSO SHOWN [Ref. 7].

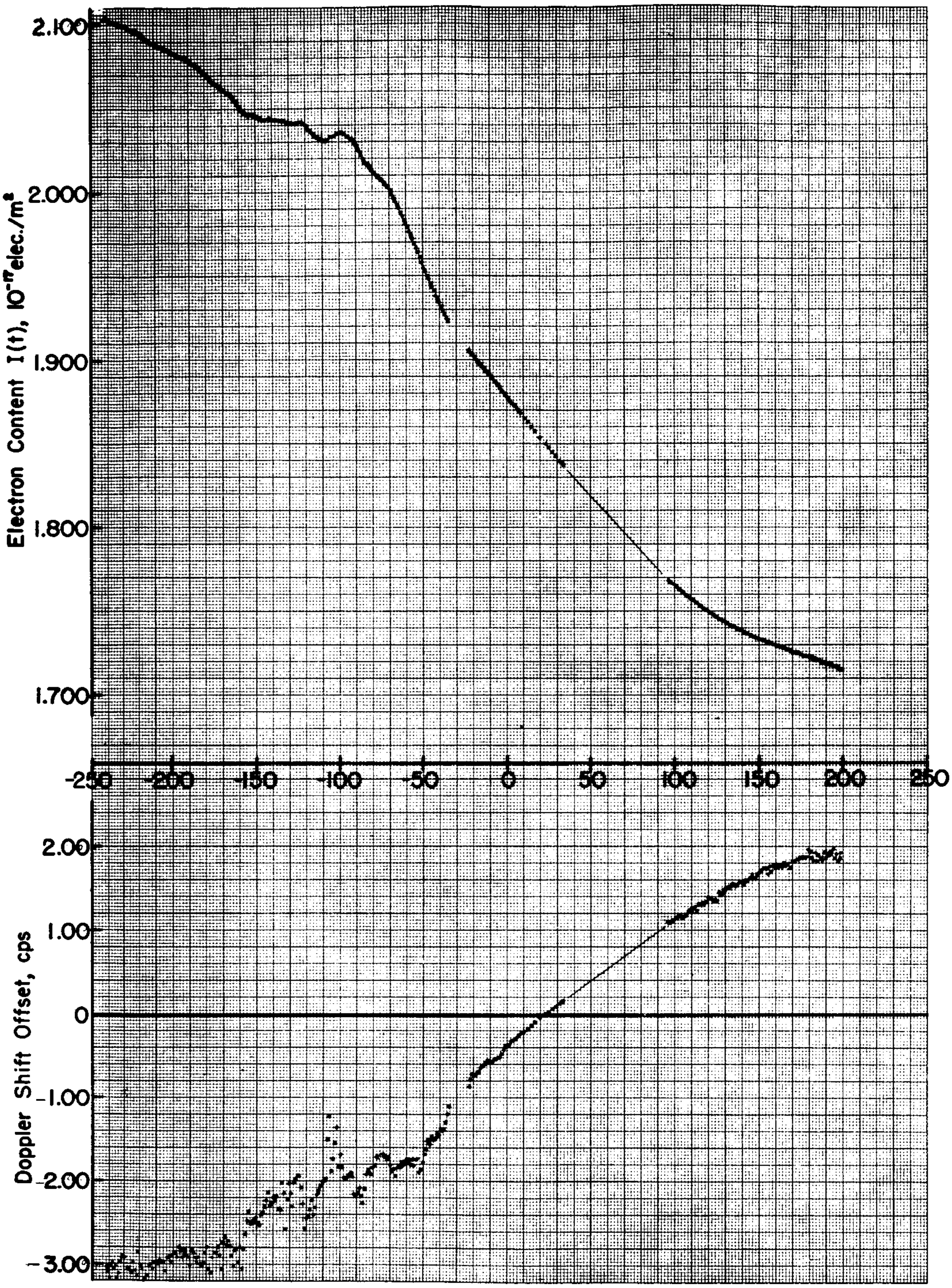

FIG. 6. ELECTRON CONTENT AND DOPPLER SHIFT OFFSET AS A FUNCTION OF TIME (t_i IN SECONDS FROM PCA) FOR A NORTHBOUND PASSAGE OF THE SATELLITE TRANSIT 2-A WITH PCA AT 07h 25m 32s (LMT) ON 23 JULY 1960 [Ref. 7].

V. SOME RESULTS OF THE DIFFERENTIAL DOPPLER MEASUREMENTS WITH SATELLITES

A. SHORT-TIME VARIATIONS IN I(t)

Variations of electron content with time are always observed [Refs. 7, and 20]. As the speed of the satellite is much higher than the drift velocities to be expected, these variations are assumed to be spatial variations. Figure 7 displays the results obtained by Garriott and Nichol [Ref. 20] for a satellite at an average height of 170 km. These E-region variations are a few percent in magnitude and have a horizontal extent of the order of 50 - 200 km. Although the variations in content are quite small, they give rise to large variations of the beat frequency as already shown in Fig. 6 for the case of a much higher satellite. There is some evidence [Ref. 7] that the smaller irregularities are more conspicuous after sunrise than they are in the rest of the day, and also they seem to be aligned with the magnetic-field lines.

B. HORIZONTAL GRADIENTS

The intersection of the ray with a shell at height h_{max} of the maximum electron density is called the ionospheric point, and the projection of this point on the ground is the subionospheric point. When I(t) is combined with the information about the position of the satellite, as a function of time, one can determine the electron content as a function of the local time (LT) and the latitude of the subionospheric point. The average values presented in Fig. 8 were obtained following the procedure above and utilizing data acquired on magnetically quiet days. It is pointed out in Ref. 7 that, for the region under consideration (geomagnetic latitude in the range 37 - 50 deg N), the south-north gradient at midday and the east-west gradient at sunrise or sunset, are of the same order of magnitude, i.e., about 25 percent variation in $\int N\,dh$ for two subionospheric points separated by a distance of 1000 km. Note that there is also, in Fig. 7, a significant horizontal gradient, with an increasing electron content toward the south.

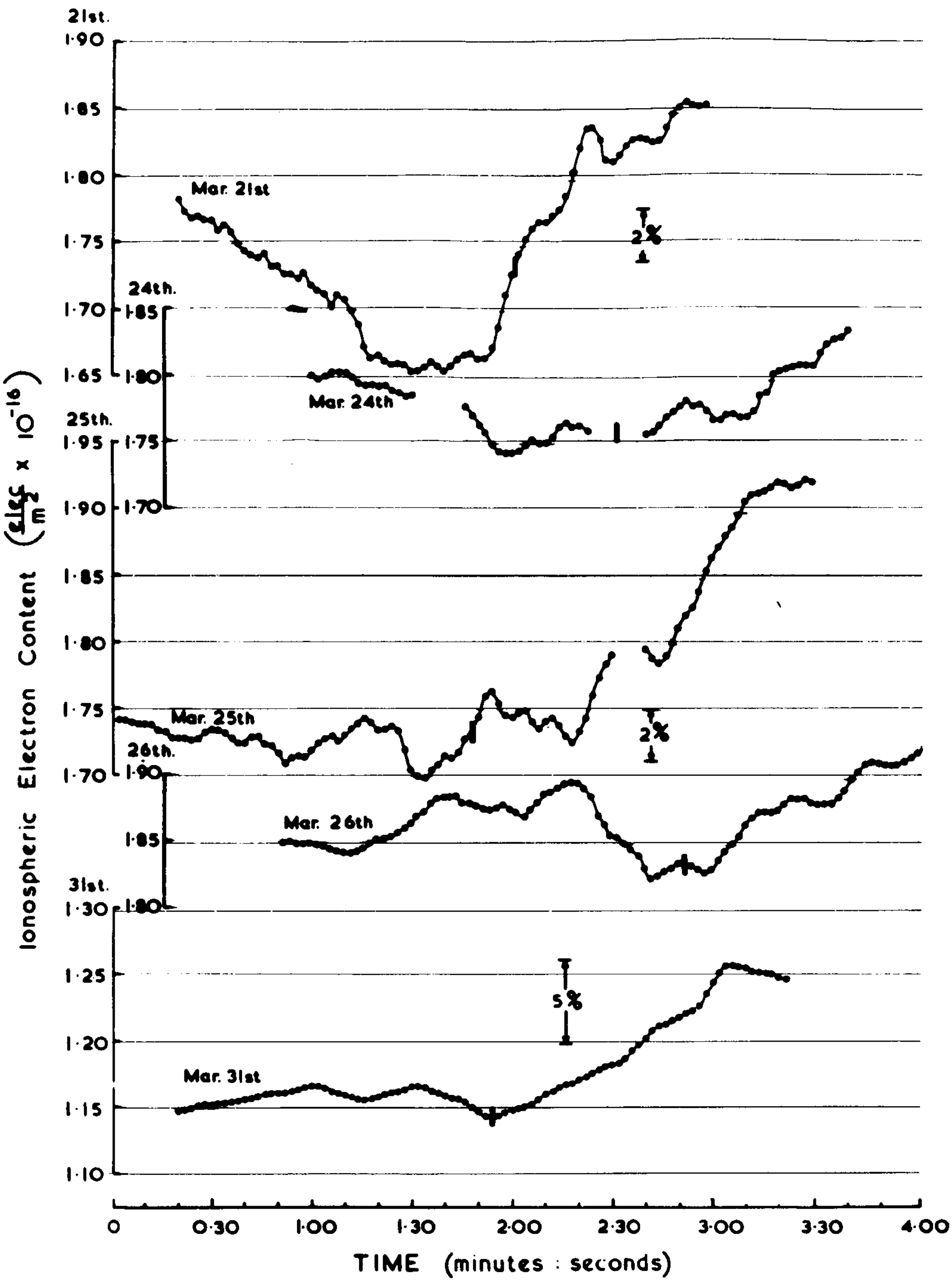

FIG. 7. DEDUCED VARIATIONS OF ELECTRON CONTENT WITH THE POSITION OF THE SATELLITE DURING FIVE LOW PASSAGES OF 1958 δ2, IN MARCH 1960. THE VERTICAL BARS INDICATE THE TIMES OF CLOSEST APPROACH, i.e., FROM TOP TO BOTTOM, 1307, 1141, 1141, 1139, AND 0931 PST [Ref. 20].

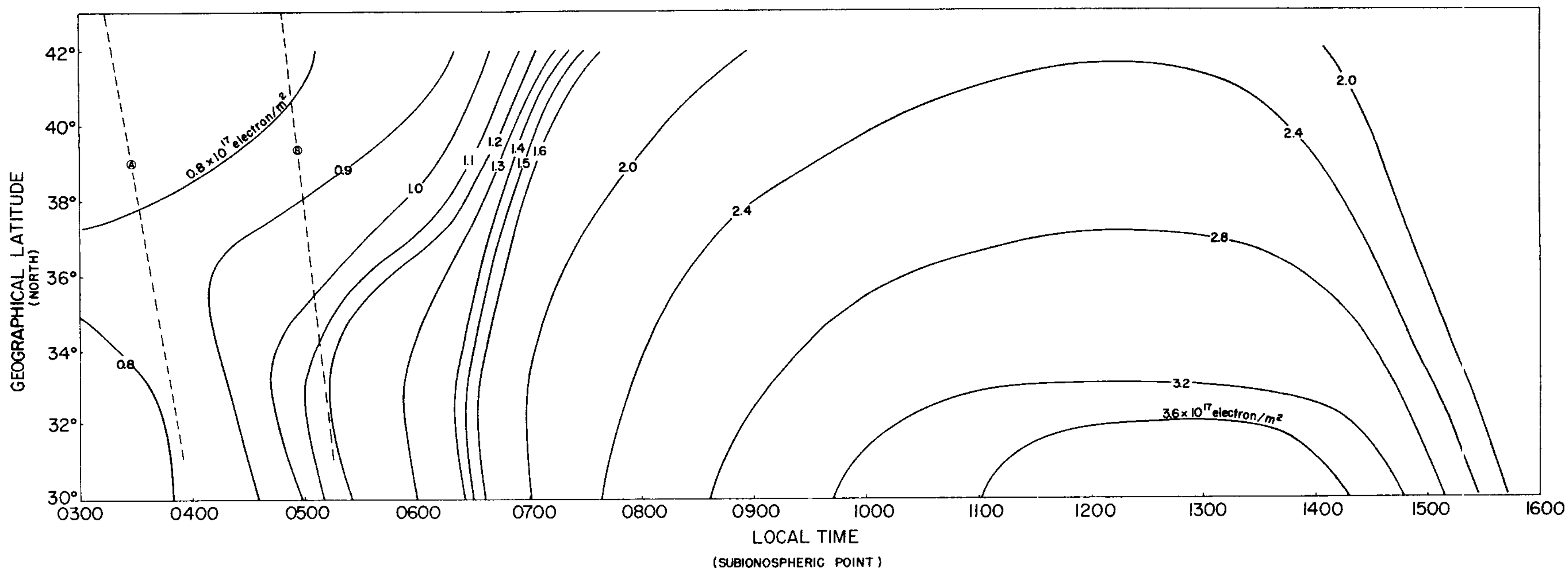

FIG. 8. SMOOTHED CURVES OF CONSTANT $\int_0^{1000} N\,dh$ DERIVED FROM MEASUREMENTS ON QUIET DAYS DURING JULY AND AUGUST 1960 WITH THE SATELLITE TRANSIT 2-A. CURVE (A) REPRESENTS SUNRISE AT SEA LEVEL, AND CURVE (B) SUNRISE AT $h = 400$ km FOR 29 JULY 1960 [Ref. 7].

C. DIURNAL AND DAY-TO-DAY VARIATIONS

Day-to-day variations in electron content, in magnetically quiet periods, have been observed by many workers. Ross [Ref. 18] reported variations which may be as large as $\pm$ 20 percent from the mean and remarked upon the fidelity with which they were reproduced in the lower-ionosphere data. de Mendonça [Ref. 7] observed variations of about $\pm$ 15 percent from the mean, most of the time, and also observed that there were a few cases in which $I(t)$ varied as much as $\pm$ 50 percent from the average on magnetically quiet days.

Figure 9 shows a portion of the diurnal variation of electron content for 3 closely spaced latitudes [Ref. 7], as a function of the local time of the subionospheric point. The northbound passages of the satellite were at an average height of 1000 km, and the southbound passages were between 660 and 750 km. The difference in electron content caused by this difference in height (660 to 1000 km) is small in relation to the total. In the case of an α-Chapman layer, with a scale height of 100 km, the difference in content for the two given heights would be about 10 percent of the total content. Even with this correction applied in Fig. 9, the midday electron content in October would be higher than in midday August, apparently due to seasonal effects.

D. MAGNETIC DISTURBANCES AND ELECTRON-CONTENT VARIATIONS

A tentative result has been obtained by Ross [Ref. 18], namely that for summer months, the electron content varies in an inverse manner with the planetary magnetic index K_p, as observed from a geomagnetic latitude of approximately 53 deg N.

Figures 10 and 11 show measurements made in the geomagnetic latitude range of 39 - 47 deg N (32 - 41 deg N geographic), during disturbed conditions. The results shown in Fig. 10 display a marked increase in electron content from the mean for quiet days, for the lower latitudes. The results shown in Fig. 11 present decreases in content for the entire range of the latitudes surveyed. However, an increase in the north-south horizontal gradients is common to both cases.

Combining the value of $I(t)$ at the point of closest approach of the satellite, i.e., $I(0)$, and the critical frequency foF2 obtained at the same time and place, one can establish the equivalent column height $I(0)/N_{max}$. This quantity tends to larger values for increasing geomagnetic index. This tendency implies that decreases in N_{max} are accompanied by increases in the layer thickness.

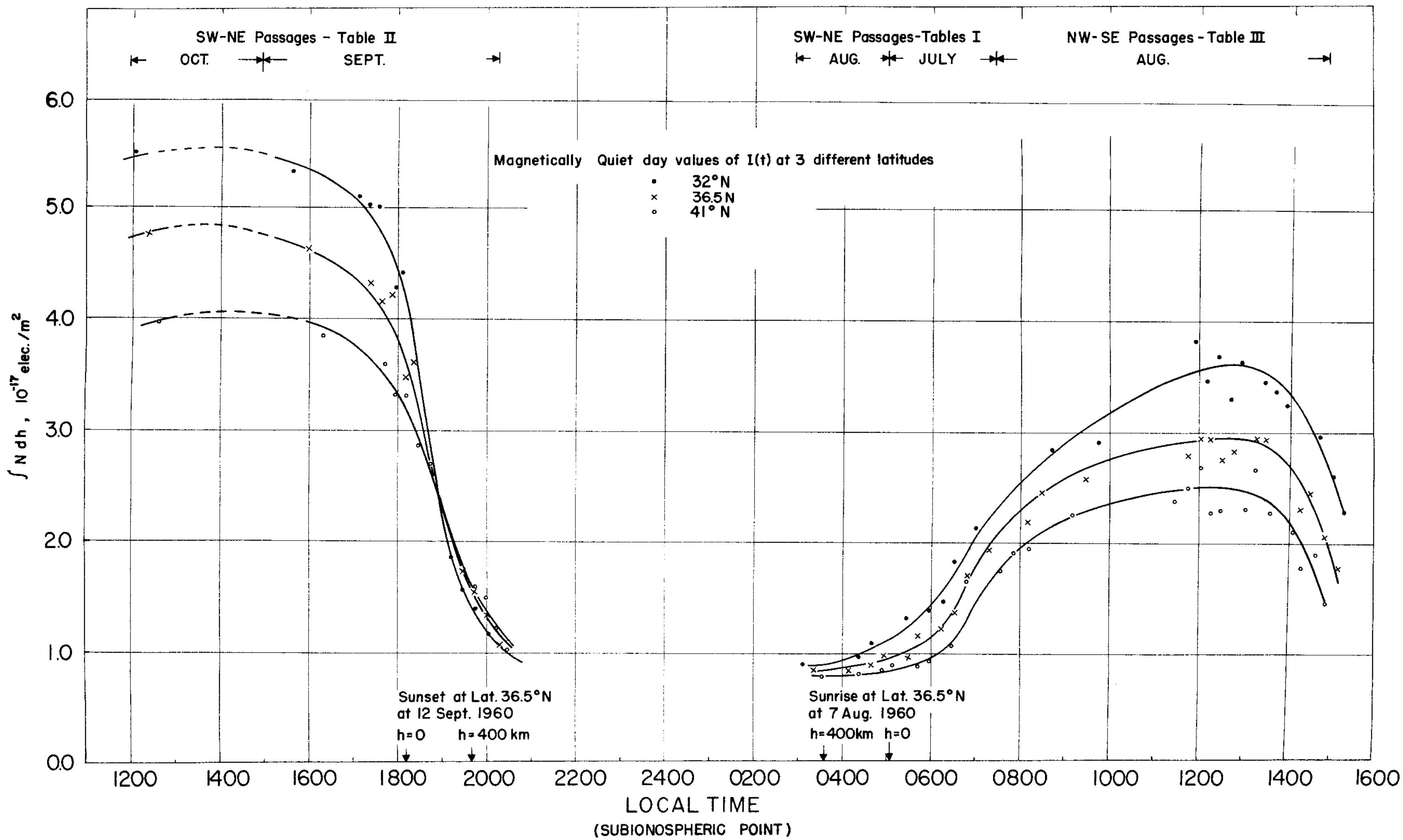

FIG. 9. QUIET-DAY VALUES OF $\int N\,dh$ AS FUNCTION OF LOCAL TIME OF THE SUBIONOSPHERIC POINT AVERAGED FOR THREE LATITUDES, CORRESPONDING TO BEGINNING, MIDDLE, AND END OF THE SATELLITE PASSAGE [Ref. 7].

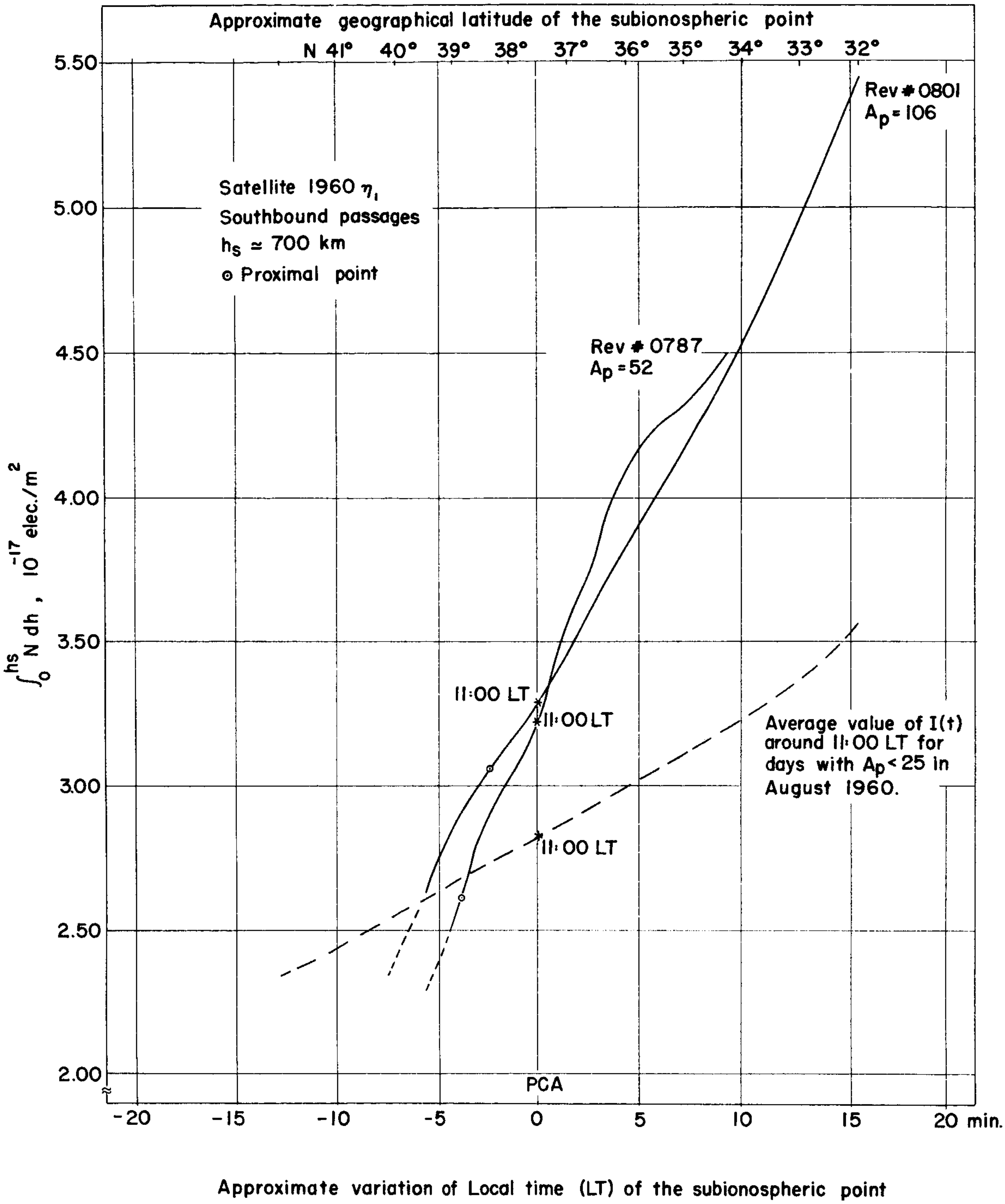

FIG. 10. EFFECTS OF THE MAGNETIC STORM OF 16 AUGUST 1960 ON THE ELECTRON CONTENT $\int N\,dh$ MEASURED IN A SOUTHBOUND PASSAGE OF THE SATELLITE 1960 η_1.

Observe the substantial increase in the north-south horizontal gradient caused by a "positive disturbance" for the lower latitudes. The dashed line represents the average I(t) around noon for days with Ap < 25 in Aug. 1960.

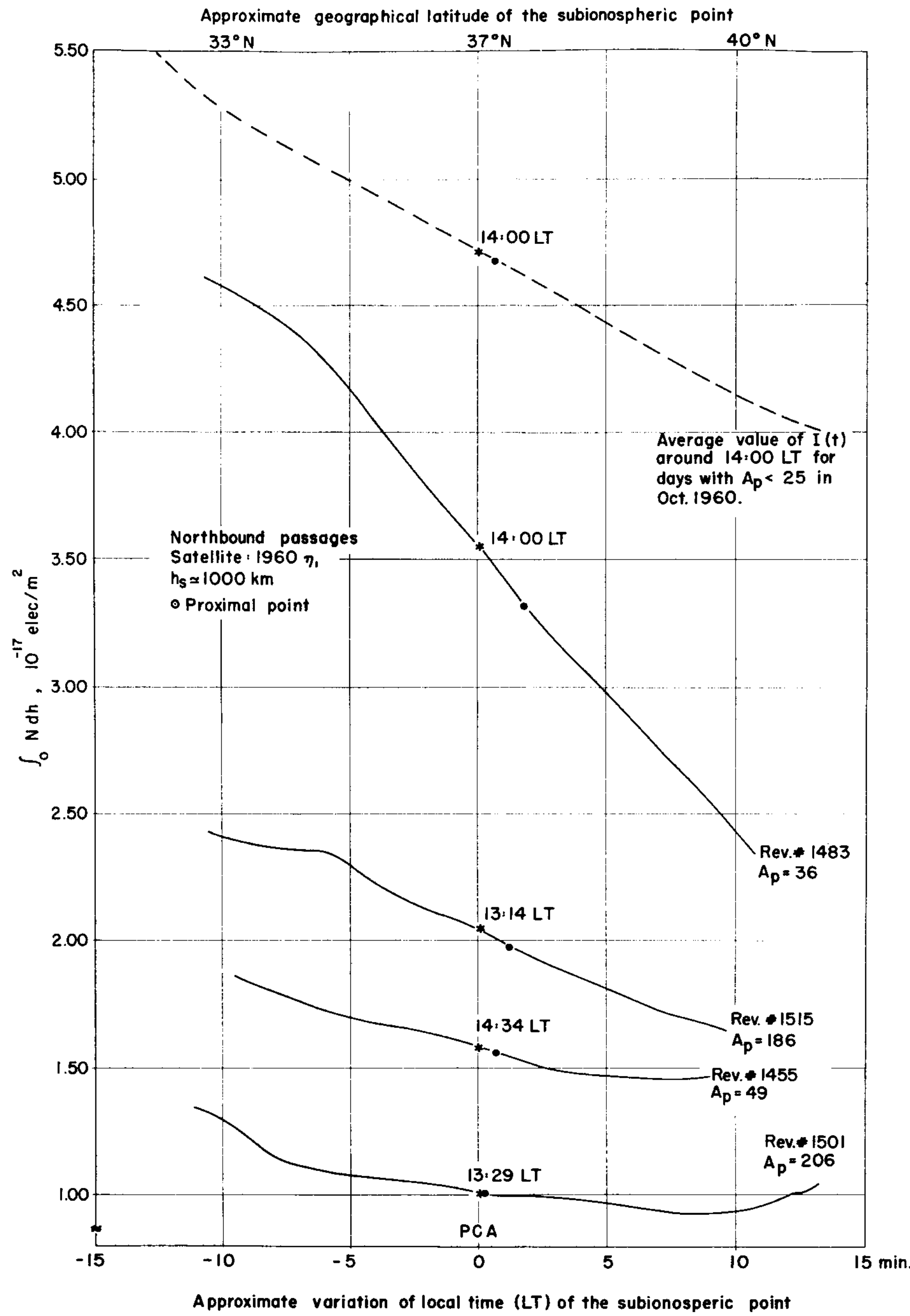

FIG. 11. EFFECTS OF THE MAGNETIC STORM OF 6 OCTOBER 1960 ON THE ELECTRON CONTENT MEASURED IN A NORTH-BOUND PASSAGE OF THE SATELLITE 1960 η_1. OBSERVE THE INCREASE IN NORTH-SOUTH HORIZONTAL GRADIENT AND ALSO THE GREATER DEPRESSION OF I(t) FOR HIGHER MAGNETIC ACTIVITY (A_p).

VI. CONCLUSIONS

The differential Doppler technique, with harmonically related frequencies transmitted to or from rockets or satellites, can provide accurate information about electron density or electron content of the medium. The knowledge of these quantities is valuable for understanding the morphology of the ionospheric processes.

The quiet and the disturbed ionosphere are both greatly dependent on latitude; in view of this fact, since satellite measurements can be be made all over the world for long periods of time, they have obvious advantages over other types of ionospheric measurements that are made at specific locations.

Prospects for the near future are worldwide differential Doppler measurements using the forthcoming Ionosphere Beacon Satellite in a circular polar orbit, the Eccentric Orbiting Geophysical Observatory, satellites of the Transit series, Geostationary Satellite, etc., which will transmit two or more harmonically related radiowaves.

This propagation technique may also find application in the future exploration of the ionosphere of the other planets by means of space probes.

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