We note in particular that a full width at half-maximum of ~105 MeV results for the Δ, in good agreement with the on-shell value.

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to evaluate "$t\sigma$" for a given $\Delta t \Delta M$ bin. Here $c = 1500/1750$ μb/event, the sum is over the events in the sample, and the bracketed quantity is evaluated for each event. The integral $\int dt\,dM$ is over that portion of the $\Delta t \Delta M$ bin in question which is included in the physical region of the Chew-Low plot.

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[14]Fits to our data using constant and quadratic terms have also been performed. Although the coefficients of the constant and quadratic terms are consistent with zero in all cases, the constant term carries the same sign in the first five mass bins and the opposite sign in the upper four mass bins, indicating that perhaps we are already observing such an effect.

LEFT-HAND CUTS IN REGGE TRAJECTORIES*

J. S. Ball†
University of Utah, Salt Lake City, Utah

and

F. Zachariasen
California Institute of Technology, Pasadena, California 91109
(Received 9 June 1969)

It is suggested that all Regge trajectories $\alpha(t)$ have a branch point at $t = 0$ and are complex for negative t. A number of consequences and conjectures based on this possibility are discussed.

It is well known that a Regge trajectory $\alpha(t)$ has branch points in t at physical thresholds of all channels to which the trajectory couples. We should like to suggest that every trajectory has a branch point at $t = 0$ as well, and that each trajectory is real only between $t = 0$ and the lowest available threshold.

In potential theory, when two Regge trajectories $\alpha(t)$ and $\alpha_1(t)$ collide [that is, when there is a value t_1 such that $\alpha(t_1) = \alpha_1(t_1)$], then both trajectories develop a square-root branch point at $t = t_1$.[1] Let us suppose this is also true in the relativistic theory. If a trajectory $\alpha(t)$ collides with a set of trajectories $\alpha_i(t)$ at values $t = t_i$ [that is, if $\alpha(t_i) = \alpha_i(t_i)$], then $\alpha(t)$ has a square-root branch point at each t_i, so that $\alpha(t)$ has a set of singularities like $\sum_i C_i(t-t_i)^{1/2}$. A Regge cut is just a continuous superposition of Regge poles; hence if a trajectory $\alpha(t)$ collides with a Regge cut, we may plausibly anticipate that $\alpha(t)$ has a continuous set of singularities of the form

$$\int_{-\infty}^{t_0} C(t')(t-t')^{1/2} dt',$$

where t_0 is the value of t at which $\alpha(t)$ collides with the leading edge of the cut.

Now any Regge trajectory when coupled with the Pomeranchukon $\alpha_P(t)$ generates a Regge cut, and if $\alpha_P(0) = 1$, the trajectory crosses the leading edge of this cut at $t = 0$.[2] Therefore, we expect any Regge trajectory to develop a singularity at $t = 0$ of the form

$$\int_{-\infty}^{0} C(t')(t-t')^{1/2} dt'.$$

If $C(t')$ behaves like any integer power $(t')^n$ near $t' = 0$, then the singularity in $\alpha(t)$ is like $t^{1/2}t^{n+1}$, i.e., a square-root branch point.

It is interesting to note that attempts to calculate a Regge trajectory dynamically, using methods which are sufficiently sophisticated to generate Regge cuts, also seem to yield the result that the trajectory is complex for $t < 0$.[3]

Let us now turn to some of the consequences which may be expected to follow from the existence of a branch point in $\alpha(t)$ at $t = 0$.

Qualitatively, phenomenological Regge-pole analyses of high-energy reactions should be modified to allow the possibility of an imaginary part in $\alpha(t)$ for negative t. However, since the imaginary part arises because of the presence of a Regge cut with which a given Regge pole interferes, it is probably inconsistent simply to allow for an $\mathrm{Im}\,\alpha(t)$ in the analysis without including the effects of the cut as well. Numerically, one might expect that if the cut has an insignificant effect, $\mathrm{Im}\,\alpha(t)$ will also, since in some sense the size of $\mathrm{Im}\,\alpha$ must be proportional to the strength of the cut. Detailed implications for experimental analyses will be discussed elsewhere; here we shall confine ourselves to mentioning the following more qualitative points.

(i) Trajectories cannot have ghosts. A ghost conventionally is assumed to occur if a trajectory passes through a sense value of J at a negative value of t; an example is the A_2 trajectory, which apparently passes through $J = 0$ at $t \approx -0.7$ $(\mathrm{BeV})^2$. However, if the branch point at $t = 0$ exists, the trajectory will be complex for $t < 0$; hence $\mathrm{Im}\,\alpha(t) \neq 0$, and we cannot have $\alpha(t) = J$ for $t < 0$.

If $\mathrm{Im}\,\alpha(t)$ happens to be very small when $\mathrm{Re}\,\alpha(t) = J$, for $t < 0$, however, the situation will be analogous to the occurrence of a resonance at positive t, and we might anticipate the cross section for a process in which $\alpha(t)$ can be exchanged to have noticeable structure at large energies and some fixed momentum transfer. [It is, of course, still possible that one of the conventional ghost-eliminating mechanisms, though unnecessary, nevertheless operates and makes the residue function proportional to $\alpha(t)$. In this event no such structure will exist even if $\mathrm{Im}\,\alpha(t)$ is small.] In the case of the A_2 "ghost," no peculiar structure is noticeable in the angular distribution for $\pi^- p \to \eta n$.[5]

(ii) The real part of the trajectory may be an absolutely straight line for $t >$ threshold and $t < 0$, even though the imaginary part of the trajectory is large. For example, suppose that

$$\alpha(t) = A + Bt + C_1[t(t_1 - t)]^{1/2}, \tag{1}$$

where t_1 is the threshold; or, if there are several important thresholds, suppose that

$$\alpha(t) = A + Bt + \sum_{i=1}^{n} C_i[t(t_i - t)]^{1/2}. \tag{2}$$

Then

$$\mathrm{Im}\,\alpha(t) = \sum_{i=1}^{n} C_i[t(t-t_i)]^{1/2}, \quad t > t_n, \quad t < 0,$$
$$= 0, \quad 0 < t < t_1;$$

while

$$\mathrm{Re}\,\alpha(t) = A + Bt, \ t > t_n, \ t < 0;$$
$$= \alpha(t), \ 0 < t < t_1.$$

Thus, no matter how large the C_i, $\mathrm{Re}\,\alpha(t)$ is simply a straight line for t above all thresholds.

In reality, of course, the form in Eqs. (1) or (2) is no doubt too simple. Nevertheless, the true $\mathrm{Im}\,\alpha(t)$ may be just the form given plus a small correction; the small correction to $\mathrm{Im}\,\alpha(t)$ will then produce only a small deviation of $\mathrm{Re}\,\alpha(t)$ from a straight line.

In this connection it may be of value to note that the simple form given for $\alpha(t)$, which says that $\mathrm{Im}\,\alpha(t) \approx Ct$ for $t > t_n$, predicts that the widths Γ_ν, $\nu = 1, 2, 3, \cdots$, of a sequence of Regge recurrences on the trajectory $\alpha(t)$ are related to the masses M_ν of the recurrence by

$$\Gamma_\nu/M_\nu = C/B \text{ independent of } \nu.$$

Experimentally, for the longest sequence of recurrences known, namely $\Delta(\frac{3}{2}^+)$, $\Delta(\frac{7}{2}^+)$, $\Delta(11/2^+)$, $\Delta(15/2^+)$, and $\Delta(19/2^+)$, the ratios Γ/M are 0.10, 0.11, 0.13, 0.14, and 0.13, respectively.[4] For the sequence $N(\frac{3}{2}^-)$, $N(\frac{7}{2}^-)$, $N(11/2^-)$, and $N(15/2^-)$ the ratios are 0.07, 0.13, 0.13, and 0.13, respectively. (Perhaps the deviation from constancy of the lowest member of this sequence is due to the proximity to an important threshold.)

(iii) The Pomeranchuk trajectory $\alpha_P(t)$ can have a real part identically equal to 1 for negative t while satisfying unitarity conditions. Gribov demonstrated many years ago that an asymptotic behavior of the form $T(s,t)\to isf(t)$ as $s\to\infty$ for an elastic-scattering amplitude $T(s,t)$ is imcompatible with t-channel unitarity if it is valid for t > threshold as well as for $t<0$.[5] Gribov's argument has also been translated into the language of partial-wave unitarity in the t channel, where the statement is that a fixed pole at $l=1$ is incompatible with unitarity.[6] Thus the simple behavior

$$d\sigma/dt \to |f(t)|^2 \text{ as } s\to\infty \tag{3}$$

for elastic cross sections, which seems both to be very attractive experimentally[7] and to have considerable intuitive appeal,[8] has always seemed most difficult to accept.

However, the presence of a branch point at $t=0$ in $\alpha_P(t)$ permits us to understand Eq. (3). For example, along the lines of the simple model of Eq. (1), if we write

$$\alpha_P(t) = 1+\textstyle\sum_i C_i[t(t_i-t)]^{1/2},$$

then for $t<0$, $\mathrm{Re}\,\alpha_P(t)=1$ exactly, and evidently this is true for more general forms for $\alpha_P(t)$ as well. Hence the usual Regge form for $T(s,t)$ at large s and negative t becomes

$$T(s,t)\to isf(t)e^{i\,\mathrm{Im}\alpha_P(t)\ln s}. \tag{4}$$

The presence of a nonzero $\mathrm{Im}\,\alpha_P(t)$ produces only a phase in T; thus the cross section does in fact have the form of Eq. (3). Nevertheless, for t > threshold $\mathrm{Im}\,\alpha_P(t)\neq 0$; so that t-channel unitarity conditions are satisfied.

The phase in Eq. (4) can, in principle, be detected experimentally. It produces no effect exactly at $t=0$, of course, since $\mathrm{Im}\,\alpha_P\to 0$ as $t\to 0$ from below, but polarization measurements at finite negative t and high energies could show it up. Qualitatively, the phase should manifest itself by producing t-dependent structure in the polarization, and this structure should (slowly) become more wiggly as the energy increases because of the $\ln s$ factor in the phase. The precise features are difficult to predict because, as mentioned previously, it is inconsistent to include effects from $\mathrm{Im}\,\alpha(t)$ without also including effects of the cut responsible for the existence of $\mathrm{Im}\,\alpha(t)$. A more detailed analysis of these effects will be reported elsewhere.

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