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# LENGTH SCALES AND POSITIVITY OF SOLUTIONS OF A CLASS OF REACTION-DIFFUSION EQUATIONS

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Abstract. In this paper, the sharpest interpolation inequalities are used to find a set of length scales for the solutions of the following class of dissipative partial differential equations

$$u_t = -\alpha_k(-1)^k\nabla^{2k}u + \sum_{j=1}^{k-1}\alpha_j(-1)^j\nabla^{2j}u + \nabla^2(u^m) + u(1-u^{2p}),$$

with periodic boundary conditions on a one-dimensional spatial domain. The equation generalises nonlinear diffusion model for the case when higher-order differential operators are present. Furthermore, we establish the asymptotic positivity as well as the positivity of solutions for all times under certain restrictions on the initial data. The above class of equations reduces for some particular values of the parameters to classical models such as the KPP-Fisher equation which appear in various contexts in population dynamics.

## 1. Introduction.

The problems addressed in this paper concern the analysis of the length scales and the positivity of solutions for a class of nonlinear dissipative partial differential equations (PDEs).

We consider the class of PDEs

$$u_t = -\alpha_k(-1)^k\nabla^{2k}u + \sum_{j=1}^{k-1}\alpha_j(-1)^j\nabla^{2j}u + \nabla^2(u^m) + u(1-u^{2p}), \tag{1.1}$$

in one spatial dimension, where $\alpha_k > 0$, $\alpha_j > 0$, $k, m, p$ positive integers, $k > 1$, $m \geq 1$, $p \geq 1$ and $u = u(x,t)$, $t \geq 0$, $x \in \Omega = [0, L]$ with periodic boundary conditions. A particular case of this equation with $k = 2, m = 0, p = 1$ is known in physical literature as the Swift-Hohenberg equation [18], which is used to describe the Rayleigh-Bénard convection. A similar equation has been used by Pomeau and Manneville [16] in the study of cellular flows just past the onset of instability. For the same values of parameters and positive second term, this is the Extended Fisher-Kolmogorov equation used as a model of phase transitions, onset of spatio-temporal chaos and other phenomena in bistable physical systems [15].

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