Polyphase Scrambled Walsh Codes for Zero-Correlation Zone Extension in QS-CDMA

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***Abstract*—Polyphase complex scrambling patterns are designed to extend the zero-correlation zone (ZCZ) of a family of user spreading sequences, called properly scrambled Walsh-Hadamard (PSW) codes, recently proposed for quasi-synchronous (QS) code-division multiple-access systems. The merits of the binary (ZCZ equals 2) and quadri-phase (ZCZ equals 4) PSW codes over the existing QS spreading codes are still retained in the proposed polyphase PSW codes. Scrambling patterns for the case of ZCZ of 8 are derived first. Then, the scrambling pattern design rules are generalized for greater ZCZs and a structured procedure to generate polyphase PSW codes for any ZCZ is developed. Finally, the cross-correlation properties of the proposed spreading codes are evaluated.**

***Index Terms*—Quasi-synchronous CDMA, orthogonal spreading codes, zero-correlation zone (ZCZ), scrambling.**

I. Introduction

A family of user spreading sequences, called properly scrambled Walsh-Hadamard (PSW) codes [1], is proposed recently for quasi-synchronous code division multiple access (QS-CDMA) systems. The work in [1] extends the binary scrambling patterns given in [2] that achieves a zero cross-correlation zone (ZCZ) of 2 to quadri-phase scrambling patterns for a ZCZ of 4; that is, cross-correlation between any two sequences is zero when the offset in time is within ± 3 chips. The major advantage of PSW codes over existing deterministic ZCZ codes is that the effects of long code scrambling such as multiple-access interference randomization and robustness against interception are preserved. Additionally, PSW codes have unique features such as no zero insertion, easy code construction, and flexible code family size.

In this paper, we extend the scheme in [1] to achieve larger ZCZs by designing polyphase scrambling patterns. Although there are existing polyphase codes (e.g., those to extend ZCZ [3] or to maintain flexible code parameters [4]), they are deterministic codes and random scrambling on top of them is not allowed. We first derive the scrambling pattern for ZCZ of 8. The additional constraint to extend ZCZ from 4 to 8 is derived based on the constraint for the case of ZCZ $= 4$. This results in an 8-ary complex scrambling pattern, whose values are like 8-ary phase shift keying (PSK) symbols. From this specific case, we further develop a generalized polyphase PSW code construction procedure for larger ZCZ values, where additional constraints to increase ZCZ from 2^n to 2^{n+1} are generated recursively. The proposed polyphase PSW codes satisfy the theoretical limit of the code size given a ZCZ, and the elements of the scrambling patterns are M-ary PSK symbols with the number of phases M equal to ZCZ.

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II. Polyphase PSW Code Construction

We follow the definition and notations as adopted in [1]:

- N: code length
- $\mathbf{s} = [s_0, s_1, \cdots s_{N-1}]$: length-$N$ scrambling pattern
- $\mathbf{w}^{(i)} = [w_0^{(i)}, w_1^{(i)}, \cdots w_{N-1}^{(i)}]$: the ith row of the $N \times N$ Walsh-Hadamard (WH) matrix
- $\mathbf{c}^{(i)} = \left[s_0 w_0^{(i)}, s_1 w_1^{(i)}, \cdots, s_{N-1} w_{N-1}^{(i)}\right]$: the ith sequence of PSW code set $\mathbf{c}$
- $R_{l,k}(d)$: periodic cross correlation of $\mathbf{c}^{(l)}$ and $\mathbf{c}^{(k)}$ at an offset of d chips, given as $\sum_{n=0}^{N-1} c_n^{(l)} {c_{n\dot{+}d}^{(k)}}^*$, where the subscript $\dot{+}$ denotes modulo-N addition and $(\cdot)^*$ denotes complex conjugate
- ZCZ: zero correlation zone in chips, i.e., the maximum value of T such that $R_{l,k}(d) = 0$, $-T < d < T$.

A. Scrambling Pattern to Achieve ZCZ=8

The scrambling pattern conditions for ZCZ of 4 given in Table 1 of [1] are briefly summarized as follows. First, $s_{0:(N/4-1)}$, which denotes the first quarter part of the binary scrambling pattern, is arbitrarily generated. Then, the second quarter part $s_{N/4:(N/2-1)}$ and the second half part $s_{N/2:(N-1)}$ are generated by the following two steps:

$$s_{N/4:(N/2-1)} = s_{0:(N/4-1)} \odot f\,[a, -a, a, -a, \cdots] \tag{1a}$$

$$s_{N/2:(N-1)} = s_{0:(N/2-1)} \odot [b, -b, b, -b, \cdots] \tag{1b}$$

where $\odot$ denotes element-by-element multiplication, $b = \pm 1$, $a = \left[\pm\sqrt{b}, \pm\sqrt{-b}\right]$, $f = 1$ for the first and 2nd quarter rows of WH matrix or $f = j$ for the third and fourth quarter rows, and $[a, -a, a, -a, \cdots]$ denotes the repeated concatenation of a and $-a$. Since this condition guarantees $R_{l,k}(d) = 0$ when d equals $0, \pm 1, \pm 3, \pm 5, \pm 7, \cdots$, and any integers whose remainder of modulo 4 is 2 (that is, $\pm 2, \pm 6, \pm 10, \cdots$), ZCZ can be extended to 8 if additional constraints are applied on the scrambling patterns to ensure $R_{l,k}(d) = 0$ at $d = \pm 4$. For the case of ZCZ $= 8$, the maximum code size will be $N/8$ by the theoretical bound.

Next, we derive 8-phase scrambling patterns that achieve $R_{l,k}(d) = 0$ with $d = \pm 4$ for $N/8$ rows of the $N \times N$

Walsh-Hadamard (WH) matrix. For simplicity of description, let us call the $N/8$ rows of the $N \times N$ WH matrix a *subset*; thus, the WH matrix has 8 subsets and the vth subset is $\{\mathbf{w}^{((v-1)N/8+1)}, \mathbf{w}^{((v-1)N/8+2)}, \cdots, \mathbf{w}^{(vN/8)}\}$. Consider the scrambling patterns for the 1st subset (v=1). From the WH matrix structure, this subset satisfies the following relation: $w^{(l)}_{mN/8+n} = w^{(l)}_n$ for $m = 1, 2, \cdots, 7$, and $0 \leq n \leq N/8 - 1$. Thus, $w^{(l)}_{mN/8+n} w^{(k)}_{mN/8+n+4} = w^{(l)}_n w^{(k)}_{n+4}$ for $l = 1, \cdots, N/8$ and $k = 1, \cdots, N/8$. Substituting this into Eq. (2) in [1], $R_{l,k}(4)$ is rewritten as $R_{l,k}(4) = 4 \sum_{n=0}^{\frac{N}{8}-1} \left(s_n s^*_{n+4} + s_{\frac{N}{8}+n} s^*_{\frac{N}{8}+n+4} \right) w^{(l)}_n w^{(k)}_{n+4}$, where the identity $s_{m\frac{N}{4}+n} s^*_{m\frac{N}{4}+n+4} = s_n s^*_{n+4}$ for $m = 1, 2, 3$ is used because s_n should satisfy (12), (13) or (14a), (14b) in [1] for $R_{l,k}(\pm 2) = 0$.

To ensure $R_{l,k}(4) = 0$, the scrambling pattern should satisfy

$$s_n s^*_{n+4} + s_{\frac{N}{8}+n} s^*_{\frac{N}{8}+n+4} = 0, \forall n. \tag{2}$$

By applying $s_n s^*_n = 1, \forall n$, (2) is expressed as $\frac{s_n}{s_{\frac{N}{8}+n}} = -\frac{s_{n+4}}{s_{\frac{N}{8}+n+4}}, \forall n$, which is rewritten as

$$\frac{s_i}{s_{\frac{N}{8}+i}} = -\frac{s_{4+i}}{s_{\frac{N}{8}+4+i}} = \frac{s_{8+i}}{s_{\frac{N}{8}+8+i}} \cdots = \frac{s_{\frac{N}{8}+i}}{s_{\frac{N}{4}+i}}, \; i = 0, 1, 2, 3. \tag{3}$$

In further derivations, we consider the case of $b = 1$, $a = [1, j]$ and $f = 1$ in (1a) and (1b). With these parameters, (1a) becomes $s_{\frac{N}{4}:(\frac{N}{2}-1)} = [s_0, js_1, -s_2, -js_3, \cdots]$. Setting $i = 0$ in (3) and substituting $s_{\frac{N}{4}} = s_0$ into the final term lead to $\frac{s_0}{s_{\frac{N}{8}}} = \frac{s_{\frac{N}{8}}}{s_0}$, which results in $\frac{s_0}{s_{\frac{N}{8}}} = \pm 1$. Hence,

$$s_{\frac{N}{8}:4:\left(\frac{N}{4}-4\right)} = \pm\left[s_0, -s_4, s_8, -s_{12}, \cdots, -s_{\frac{N}{8}-4}\right] \tag{4}$$

where $s_{x:y:z}$ denotes $[s_x, s_{x+y}, s_{x+2y}, \cdots, s_z]$.

Similarly, substituting $s_{\frac{N}{4}+1} = js_1$, $s_{\frac{N}{4}+2} = -s_2$, $s_{\frac{N}{4}+3} = -js_3$ into (3) for $i = 1, 2, 3$, respectively, results in $s_{\frac{N}{8}+1} = \pm e^{j\pi/4} s_1$, $s_{\frac{N}{8}+2} = \pm e^{j\pi/2} s_2$ and $s_{\frac{N}{8}+3} = \pm e^{j3\pi/4} s_3$. Thus,

$$s_{(\frac{N}{8}+1):4:(\frac{N}{4}-3)} = \pm e^{\frac{j\pi}{4}} \left[s_1, -s_5, s_9, -s_{13}, \cdots, -s_{\frac{N}{8}-3}\right] \tag{5}$$

$$s_{(\frac{N}{8}+2):4:(\frac{N}{4}-2)} = \pm e^{\frac{j\pi}{2}} \left[s_2, -s_6, s_{10}, -s_{14}, \cdots, -s_{\frac{N}{8}-2}\right] \tag{6}$$

$$s_{(\frac{N}{8}+3):4:(\frac{N}{4}-1)} = \pm e^{\frac{j3\pi}{4}} \left[s_3, -s_7, s_{11}, -s_{15}, \cdots, -s_{\frac{N}{8}-1}\right] \tag{7}$$

Eqs. (4)-(7) can be combined as

$$s_{\frac{N}{8}:(\frac{N}{4}-1)} = [\mathbf{p}, -\mathbf{p}, \mathbf{p}, -\mathbf{p}, \cdots] \odot s_{0:(\frac{N}{8}-1)} \tag{8}$$

where the notation $[\mathbf{a}, \mathbf{b}]$ denotes the concatenated sequence of $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{p}$ is expressed as

$$\mathbf{p} = [\pm 1, \pm e^{j\pi/4}, \pm e^{j\pi/2}, \pm e^{j3\pi/4}]. \tag{9}$$

B. Generalization to Larger ZCZ

Eq. (8) shows that the 2nd half of a quarter of $\mathbf{s}$ (i.e., $s_{N/8}, \cdots, s_{N/4-1}$) is obtained by scrambling the first half of the same quarter of $\mathbf{s}$ (i.e., $s_0, \cdots, s_{N/8-1}$) with the sequence formed by repeating $[\mathbf{p}, -\mathbf{p}]$. Let us call $[\mathbf{p}, -\mathbf{p}]$ the unit inner scrambling pattern (UISP). As observed from (9), $\mathbf{p}$ can be obtained by taking the element-by-element square-root of vector $[1, j, -1, -j]$, which is the UISP chosen for generating the 2nd quarter of $\mathbf{s}$ in (1a). Further, the first half of this UISP (i.e., $[1, j]$) is one of the element-by-element square roots of $[1, -1]$, which is the UISP for generating the second half of $\mathbf{s}$ in (1b). This process can be generalized to recursively obtain the UISPs for ZCZs that are a power of 2 (i.e., ZCZ = 2^n) as:

$$\mathbf{p}_i = {}^{\pm}\sqrt{[\mathbf{p}_{i-1}, -\mathbf{p}_{i-1}]}, \quad i = 2, \cdots, n; \quad \mathbf{p}_1 = [\pm 1] \tag{10}$$

where $\sqrt{\mathbf{x}}$ denotes element-by-element square-root of sequence $\mathbf{x}$ and $^{\pm}$ indicates that the polarity of the elements of $\sqrt{\mathbf{x}}$ can be arbitrarily chosen to be 1 or -1. Let $[\mathbf{p}_i, -\mathbf{p}_i]$ be the UISP for generating the second partition of 2^i partitions of $\mathbf{s}$, i.e., $\left[s_{\frac{N}{2^i}}, \cdots, s_{\frac{N}{2^{i-1}}-1}\right]$.

A method to derive the scrambling patterns for the 1st subset (i.e., v=1) has been discussed in Sec. II-A. The same procedure applies in deriving the patterns $\mathbf{p}$ for the remaining subsets ($2 \leq v \leq 8$). Similar to the case of ZCZ = 4, for which f in (1a) is determined based on the selected subset of WH sequences, the WH matrix structure results in two kinds of recursive UISP generation equations: $\mathbf{p}_i = {}^{\pm}\sqrt{[\mathbf{p}_{i-1}, -\mathbf{p}_{i-1}]}$ or $\mathbf{p}_i = j\left({}^{\pm}\sqrt{[\mathbf{p}_{i-1}, -\mathbf{p}_{i-1}]}\right)$, depending on the value of v and i. Therefore, (10) can be further generalized for the vth subset as $\mathbf{p}_i = j^{b_{v,i}}\left({}^{\pm}\sqrt{[\mathbf{p}_{i-1}, -\mathbf{p}_{i-1}]}\right)$, where $b_{v,i}$ equals 0 or 1, and is determined as $b_{v,i} = \text{LSB}$ (least significant bit) of $\lfloor (v-1)2^{i-1}/\text{ZCZ} \rfloor$, by exploiting the structural properties of the WH matrix.

A structured procedure to generate the polyphase PSW codes for any ZCZ is thus summarized in Table I. This procedure consists of four steps: parameter input, inner scrambling pattern set generation, outer scrambling pattern generation, and scrambling of WH sequence subset. Line 13 shows that the scrambling pattern is recursively extended via inner scrambling and concatenation until it reaches WH sequence length N. The options in lines 3 and 6, and the first N/ZCZ chips of the scrambling pattern in line 8 can be arbitrarily chosen. This results in nondeterministic scrambling patterns, which preserves randomness in the scrambled WH sequence set, although randomness is reduced compared to the conventional long code scrambling. When used as user spreading sequences in cellular CDMA systems, the proposed polyphase PSW codes can be made random and different for different data symbols and for different cells. This is a feature of the proposed polyphase PSW codes that existing ZCZ codes do not have since random scrambling over them is not allowed [1].

Lines 16 and 17 show that the code size is equal to N/ZCZ, which satisfies the theoretical limit of ZCZ code size with sequence length N (eq. (15) of [5]). Also note that the procedure in Table I includes the scrambling pattern rules for ZCZ $= 4$ given in (1a) and (1b).

III. Cross Correlation of PSW Codes

The cross-correlation properties of the proposed polyphase PSW codes in terms of the following two correlation metrics are assessed in this section:

1) Average periodic cross-correlation power over all sequence pairs, i.e., $\mathbf{E}_{l \neq k}\left[\,|R_{l,k}(d)|^2\,\right]$.

TABLE I
POLYPHASE PSW CODE GENERATION

Parameter input

Input code length N and ZCZ;

Input v, the index of WH sequence subsets, $v \in \{1, 2, \cdots, \text{ZCZ}\}$.

Inner scrambling pattern set $\{\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{\log_2(\text{ZCZ})}\}$ generation

$\mathbf{p}_1 = \pm 1$;

FOR $i = 2$ to $\log_2(\text{ZCZ})$

  $b_{v,i} = \text{LSB of } \lfloor (v-1)2^{i-1}/\text{ZCZ} \rfloor$;

  [a] $\mathbf{p}_i = j^{b_{v,i}} \times \left(\pm\sqrt{[\mathbf{p}_{i-1}, -\mathbf{p}_{i-1}]} \right)$;

END FOR

Outer scrambling pattern generation

Generate first N/ZCZ chips of $\mathbf{s}$, i.e., $[s_0, s_1, \cdots, s_{N/\text{ZCZ}-1}]$ with the random chips $s_i \in \{e^{j\theta}, e^{j2\theta}, \cdots, 1\}$ with $\theta = 2\pi/\text{ZCZ}$;

$\mathbf{r} = s_{0:(N/\text{ZCZ}-1)}$;

FOR $i = 0$ to $\log_2(\text{ZCZ}) - 1$

  $l = \log_2(\text{ZCZ}) - i$;

  [b] $\mathbf{q} = \left[[\mathbf{p}_l, -\mathbf{p}_l], [\mathbf{p}_l, -\mathbf{p}_l], \cdots : \frac{4^i N}{\text{ZCZ}^2} \right]$;

  [c] $\mathbf{r} \Leftarrow [\mathbf{r}, \mathbf{r} \odot \mathbf{q}]$;

END FOR

$\mathbf{s} = \mathbf{r}$;

Scrambling of WH sequence subset

FOR $i = 1$ to N/ZCZ

  [d] $\mathbf{c}^{(i)} = \mathbf{s} \odot \mathbf{w}^{\left((v-1)N/\text{ZCZ}+i\right)}$;

END FOR

(a) $[\mathbf{a}, \mathbf{b}]$ denotes the concatenation of the sequence $\mathbf{a}$ and the sequence $\mathbf{b}$. $\sqrt{\mathbf{x}}$ denotes element-by-element square-root of the sequence $\mathbf{x}$ and $\pm$ indicates that the polarity of the elements of $\sqrt{\mathbf{x}}$ can be arbitrarily chosen to be 1 or -1.
(b) $[\mathbf{a}, \mathbf{a}, \cdots : n]$ denotes n-times concatenation of the sequence $\mathbf{a}$.
(c) $\odot$ denotes element-by-element multiplication.
(d) $\mathbf{w}^{(l)}$ denotes the lth row of the $N \times N$ WH matrix.

2) Average data modulated correlation power over all sequence pairs and data symbols within an access window size of W given as:

$$I(W) = \frac{1}{2W+1} \sum_{d=-W}^{W} \mathbf{E}_{l \neq k}\left[|X_{l,k}(d)|^2 \right]. \quad (11)$$

The definition of $X_{l,k}(d)$ is given by Eq. (17) of [1].

Fig. 1 shows $\mathbf{E}_{l \neq k}\left[|R_{l,k}(d)|^2 \right]$ for $N = 256$, $\text{ZCZ} = 16$ and several WH sequence subset indices v's. It is observed that $\mathbf{E}_{l \neq k}\left[|R_{l,k}(d)|^2 \right] \neq 0$ only at $d = \pm 16, \pm 32, \pm 48, \cdots$; thus, $\mathbf{E}_{l \neq k}\left[|R_{l,k}(d)|^2 \right] = 0$ for $-15 \leq d \leq 15$, which verifies that $\text{ZCZ} = 16$. It is also observed that the values of $\mathbf{E}_{l \neq k}\left[|R_{l,k}(d)|^2 \right]$ are identical irrespective of v; this implies that the procedure in Table I achieves the desired ZCZ irrespective of which subset of WH sequence is chosen as the base sequence set.

Fig. 2 plots $I(W)$ expressed in (11) for several values of N and ZCZ in order to assess the multiple access interference when the proposed codes are used as user spreading sequences for quasi-synchronous CDMA. It is observed that interference is not zero even when the access window size W is smaller than ZCZ. This is because the delayed version of other users' sequences after data modulation is not exactly the cyclicly shifted version in the desired sequence period; thus, the correlation values of the data modulated sequences differ from their periodic correlation. However, this is still far smaller than N, which is the average power of cross correlation between two random sequences of length N. Note that $I(W)$ increases drastically when the access timing window W is greater than ZCZ. This is because the large correlation at $d = \pm$ ZCZ is included within the window.

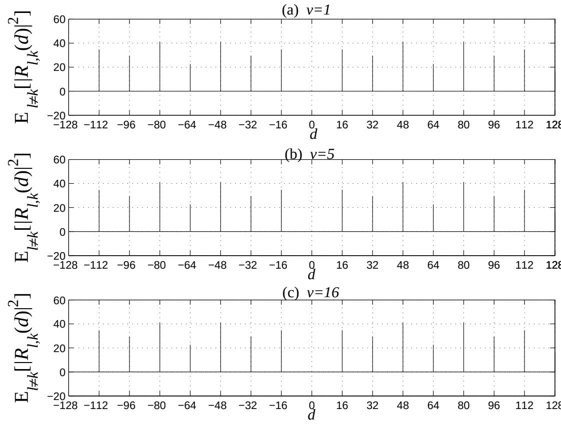

Fig. 1. Average periodic cross correlation power over all sequence pairs ($N = 256, \text{ZCZ} = 16$ and v = 1,5,16).

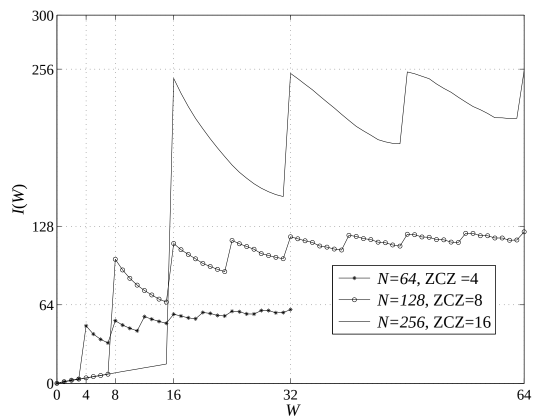

Fig. 2. Average data modulated correlation power over all sequence pairs and data symbols within access window size of W.

IV. CONCLUSIONS

We have proposed polyphase PSW codes for ZCZ extension and developed a systematic code construction procedure. The proposed polyphase PSW codes retain the unique features of existing binary and quadri-phase PSW codes that other existing ZCZ codes do not have. A larger ZCZ allows the polyphase PSW codes be employed in multi-path fading environments, and guarantees other users' signals that arrive either via the direct path or multi-path be removed in the desired user's despreading process.

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