Enhancement of the $\nu = 5/2$ Fractional Quantum Hall State in a Small In-Plane Magnetic Field

Guangtong Liu,[1] Chi Zhang,[1,2] D. C. Tsui,[1] Ivan Knez,[2] Aaron Levine,[2] R. R. Du,[2] L. N. Pfeiffer,[1] and K. W. West[1]
[1]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
[2]*Department of Physics and Astronomy, Rice University, Houston, Texas 77251-1892, USA*
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Using a 50-nm-width ultraclean GaAs/AlGaAs quantum well, we have studied the Landau level filling factor $\nu = 5/2$ fractional quantum Hall effect in a perpendicular magnetic field $B \sim 1.7$ T and determined its dependence on tilted magnetic fields. Contrary to all previous results, the 5/2 resistance minimum and the Hall plateau are found to strengthen continuously under an increasing tilt angle $0 < \theta < 25°$ (corresponding to an in-plane magnetic field $0 < B_{\parallel} < 0.8$ T). In the same range of θ, the activation gaps of both the 7/3 and the 8/3 states are found to increase with tilt. The 5/2 state transforms into a compressible Fermi liquid upon tilt angle $\theta > 60°$, and the composite fermion series $[2 + p/(2p \pm 1)$, $p = 1, 2]$ can be identified. Based on our results, we discuss the relevance of a Skyrmion spin texture at $\nu = 5/2$ associated with small Zeeman energy in wide quantum wells, as proposed by Wójs *et al.* [Phys. Rev. Lett. **104**, 086801 (2010)].

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The fractional quantum Hall effect (FQHE) observed at the Landau level (LL) filling factor $\nu = 5/2$ [1–3] has received much attention, since theory suggests that its quasiparticles may obey non-Abelian statistics [4,5]. Here, $\nu = n_e h/eB$, where n_e is the sheet density of 2D electrons and B is the magnetic field. To date, most of the evidence supporting the non-Abelian nature of 5/2 is from exact diagonalization or numerical calculations [6,7] based on modeling electron-electron interaction potentials (including the Coulomb potential), although more recently experimental evidence has accumulated supporting quarter-charged quasiparticles at 5/2 [8–10], or non-Abelian statistics [10]. If the predicted non-Abelian properties can be firmly established both theoretically and experimentally, the braiding of these non-Abelian particles can form the basis for topologically protected quantum computation [11].

The proposed Moore-Read (MR) Pfaffian wave function for 5/2 requires the spin to be at least partially polarized. Recent numerical results indicate that the 5/2 state in a GaAs quantum well system is spin polarized even in the limit of vanishing Zeeman energy [12]. Experimental investigations on spin polarization at 5/2 from electrical transport [13–16] and optical measurements [17–19] have been reported, but these results provide only indirect determination of the spin states. Recent resistively detected nuclear magnetic resonance measurements point to a fully spin-polarized ground state (GS) at 5/2 at a magnetic field around 5 T [20,21].

It is anticipated that increasing the Zeeman energy would help to stabilize the spin-polarized GS in the presence of fluctuations. Therefore, a tilted magnetic field is supposed to enhance the FQHE at 5/2 [12]. However, to date, all the experimental results have contradicted this simple prediction: the 5/2 FQHE is found to be weakened in a tilted field [13–16]. Competition with a striped many-electron phase could be a plausible cause for the complex response of the 5/2 state in a tilted field [22–24]. On the other hand, Wójs *et al.* point to an interesting possibility: based on numerical calculations in wide GaAs/AlGaAs quantum wells (QWs), the MR state could be depolarized by forming a Skyrmion—spin texture or spin topological defect [25,26]. Moreover, Skyrmions are further promoted by disorder [25]. It appears then that, for realistic samples, the GS at 5/2 could experience a spin texture to fully polarized phase transitions as the Zeeman energy is increased. In high n_e samples, Zhang *et al.* [15] reported a quantized 5/2 plateau at 10 T, which is taken as the evidence for a polarized GS. For the opposite limit of small Zeeman energy, we report in this Letter a 5/2 FQHE observed at 1.7 T in a low n_e sample, where the 5/2 plateau and energy gap are found to enhance by a small in-plane magnetic field. Our results render strong experimental support for the notion that the true ground state at 5/2 must be spin-polarized; in realistic materials, the Zeeman energy stabilizes the ground state against fluctuations.

Our samples were modulation-doped symmetrical square $Al_{0.24}Ga_{0.76}As/GaAs/Al_{0.24}Ga_{0.76}As$ QW with a well width of $w = 50$ nm and a spacer width of $d = 220$ nm. The specimens, A and B, were approximately 3 mm × 3 mm square each with eight In/Sn Ohmic contacts diffused around the perimeter. After a brief red light-emitting diode (LED) illumination, the specimen attained a mobility $\mu \sim 1.5 \times 10^7$ cm^2/V s at 300 mK and an electron density $n_e = 1.00 \times 10^{11}$/ cm^2 (sample A) and 1.02×10^{11}/ cm^2 (sample B). The magnetotransport experiments were performed in a top-loading dilution refrigerator with a base temperature of 20 mK and an 18-T superconducting magnet in the National High Magnetic

Field Laboratory (NHMFL). Tilt-field measurements were facilitated by an *in situ* sample rotator, with the total magnetic field B_{total} applied with an angle θ with respect to the sample normal. Standard lock-in technique (7 Hz and 2 nA excitation current) was employed for resistance measurements.

The central findings from our tilted magnetic field experiments in the 50 nm QW were (1) the $5/2$ FQHE was enhanced under a small in-plane magnetic field $B_{\parallel}$, and (2) the $5/2$ FQHE transitioned to a composite fermion state in larger $B_{\parallel}$. The representative data taken from sample A are shown in Fig. 1. Here the R_{xx} and R_{xy} traces were measured at $T \sim 25$ mK. At $\theta = 0°$, we observed a well-defined cusp in R_{xx} at $5/2$ (magnetic field $B \sim 1.66$ T), but no Hall plateau was discernible at this temperature. As can be seen in Fig. 3(a), for sample B at $T \sim 20$ mK, a weak $R_{xx}(5/2)$ minimum and a developing $R_{xy}(5/2)$ plateau could already be observed at $\theta = 0°$. In any case, the $5/2$ FQHE state in this QW at zero tilt angle appears to be rather fragile and the strength of the $R_{xx}(5/2)$ minimum is sensitively dependent on the sample cooling and LED illumination conditions [27].

Both sample A and sample B show quantitatively the same response to a tilted magnetic field. Surprisingly, even a small tilt angle $\theta \sim 10°$ yielding an in-plane magnetic field $B_{\parallel} \sim 0.3$ T dramatically enhanced the $5/2$ FQHE. The $R_{xx}(5/2)$ minimum deepened, along with the development of an $R_{xy}(5/2)$ plateau in a continuous fashion for tilt angles $0 < \theta < 25°$. Above these angles, the trend reverses. Figure 1(b) shows an example for the FQHE at $\theta = 39°$, where a Hall plateau at $R_{xy} = (2/5)(h/e^2)$ is visible, along with a deep minimum in R_{xx}. We also observe that at this tilt angle, the peak around the $R_{xx}(5/2)$ becomes very steep, indicating that the tilt-induced insulating phases (IP) becomes relevant in $\theta \geq 40°$. It is important to note that both the $7/3$ and the $8/3$ FQHE states, which are not discernible at $\theta = 0°$ at this temperature (~ 25 mK), have become remarkably strong in tilted fields, as evidenced by quantized Hall plateaus and steep resistance minima for both $\theta \sim 39°$ and $\theta \sim 60°$.

For a true FQHE state, it is expected that its resistance minimum shows temperature-activated behavior, namely, $R_{xx} \propto \exp(-\Delta/2k_B T)$, where Δ is the gap energy and k_B is the Boltzmann constant. The robustness of the FQHE is measured by Δ. However, a developing FQHE state often shows only a limited range of activation, and therefore Δ is not experimentally accessible. In order to quantitatively assess the strength of the FQHE at $5/2$, $7/3$, and $8/3$ in a tilted field, we plotted the ratio of resistance minimum and the average peak value (see the inset of Fig. 2), $S \equiv 2R_{\min}/(P_1 + P_2)$, against tilt angle. A similar procedure to estimate the $5/2$ strength was used in Refs. [13–15]. The data were taken from sample B at the base temperature $T = 20$ mK. Such plots exhibit systematic patterns for the relevant FQHE states in a wide range of $0° < \theta < 70°$.

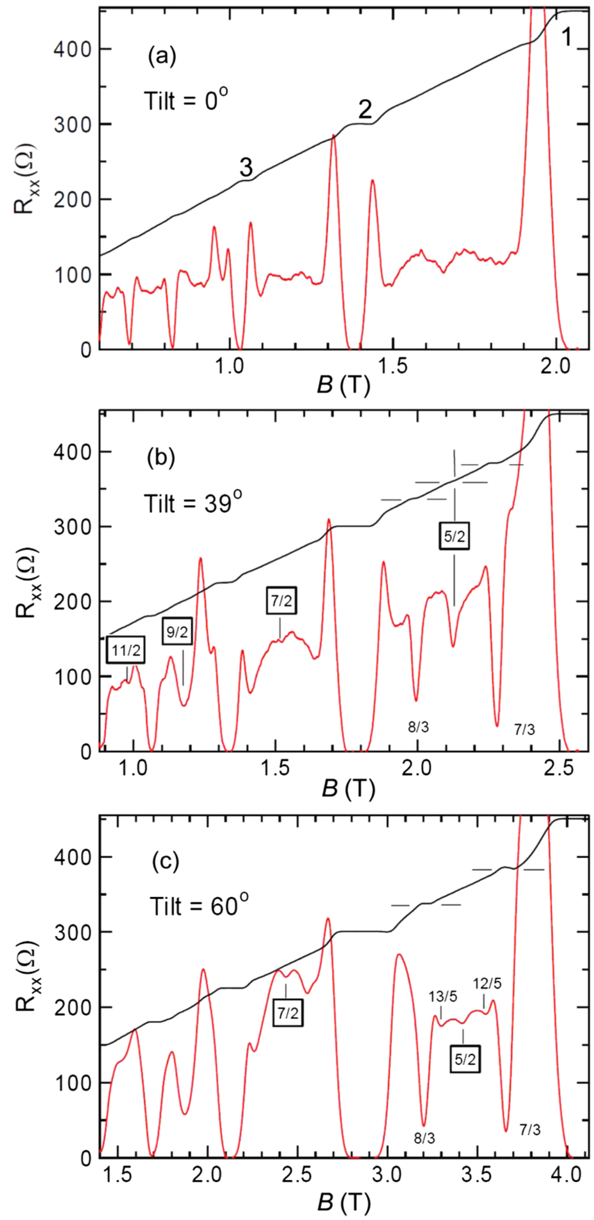

FIG. 1 (color online). Sample A: the magnetoresistance (R_{xx}) and Hall resistance (R_{xy}) traces are shown for representative tilt angles: (a) $\theta = 0°$, $R_{xx}(5/2)$ shows a cusp but no discernible plateau in R_{xy}. No resistance features for the $7/3$ and $8/3$ FQHE were observed; (b) $\theta = 39°$, $R_{xx}(5/2)$ develops into a minimum associated with a visible Hall plateau in R_{xy}. Strong FQHE states at both the $7/3$ and $8/3$ filling factors are observed; (c) $\theta = 60°$, the states around $5/2$ develop into a composite fermion state series at $\nu = 2 + p/(2p \pm 1)$, $p = 1, 2$. All traces were measured at a temperature $\sim$25 mK.

Two types of distinctively different patterns are observed: (1) For $7/3$ and $8/3$, the S values decrease monotonically with the tilt angles, indicating a continuous strengthening of the FQHE at these filling factors. The result for $7/3$ is consistent with that previously reported by Dean *et al.* [13] for a QW having a 40 nm width and an

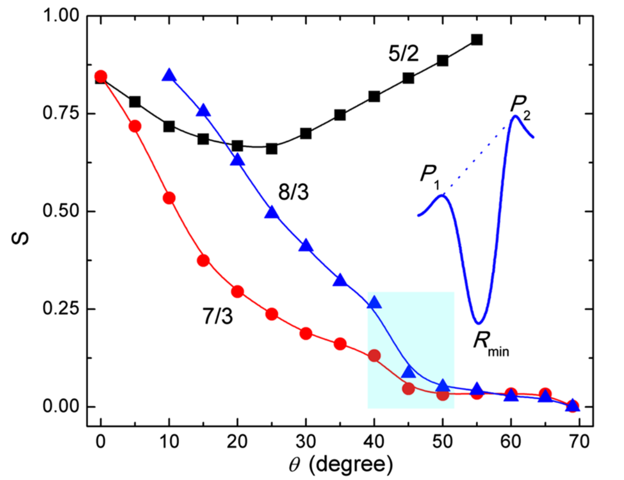

FIG. 2 (color online). Sample B: the evolution of parameter S as a function of tilt angle θ is shown for 5/2, 7/3, and 8/3. The solid curves are a guide to the eye. The schematic in the inset shows the peak-valley structure which defines S (see text). The square area indicates a steep enhancement of the 7/3 and the 8/3 FQHE in the tilt angle range ~40°–50°, as discussed in the text.

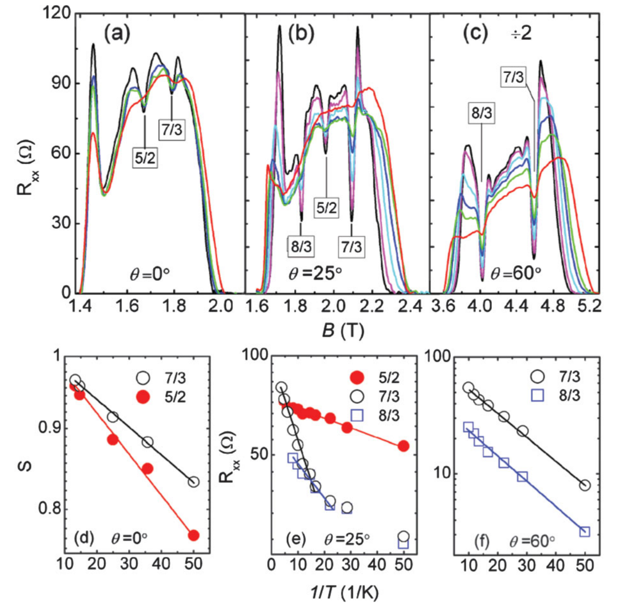

FIG. 3 (color). Sample B: (a)–(c) Temperature-dependent magnetoresistances R_{xx}. The data were taken at temperatures of 20, 28, 40, and 75 mK in (a); 20, 45, 85, 123, 166, and 266 mK in (b); 20, 45, 74, 101, 123, and 208 mK in (c). (d)–(f) Corresponding S (defined in the text) versus $1/T$, or R_{xx} versus $1/T$ at filling factor 5/2, 7/3, and 8/3 are shown, respectively, for tilt angle (d) $\theta = 0°$: $\Delta^S_{5/2} \approx 10$ mK, $\Delta^S_{7/3} \approx 7$ mK; (e) $\theta = 25°$: $\Delta_{5/2} \approx 13$ mK, $\Delta_{7/3} \approx 106$ mK, $\Delta_{8/3} \approx 46$ mK; (f) $\theta = 60°$: $\Delta_{7/3} \approx 88$ mK, $\Delta_{8/3} \approx 103$ mK.

electron density of $n_e \sim 1.6 \times 10^{11}/\text{cm}^2$. The initial 7/3 state ($\theta \sim 0°$) in [13] is proposed to be a Skyrmion, based on the fact that the ratio of the Zeeman energy and the electron interaction energy $\eta \sim 0.01$ is within the range of the critical ratio for Skyrmion formation near $\nu = 1$. (2) For 5/2, S first decreases, reaching a broad minimum at $\theta \sim 25°$, and then increases. This pattern is in sharp contrast with that reported for a 40 nm QW [13], where $S(5/2)$ monotonically increases with tilt angle.

We note that in previous tilt-field experiments, in-plane magnetic fields consistently enhance the IP, which are revealed as resistance spikes around 5/2 [22–24]. Here this effect appears to be weak in the tilt angle range $\theta < 25°$. It is plausible that the competition between the FHQE liquid at 5/2 and the IP around it, which cannot be completely resolved in realistic samples, has led to a complex response of resistance in this filling factor range, and that only in the present samples, the quantum liquid characteristics at 5/2 prevails. On a side note, the precipitous drop of $S(7/3)$ and $S(8/3)$ between $\theta \sim 40°$ and ~50°, as marked by the square area in Fig. 2, is not understood; it could be due to the enhancement of IP in this range, or, a precursor to the transition into a composite fermion (CF) series $[2 + p/(2p \pm 1)]$, with $p = 1, 2$.

Xia *et al.* reported [16] a tilt-induced anisotropic to isotropic phase transition at $\nu = 5/2$, observed in a 40 nm QW. For increasing tilt angles, the authors found that the anisotropic compressible phase can be replaced by isotropic compressible phases reminiscent of the CF fluid at $\nu = 1/2$. In our 50 nm QWs, the electrical subband separation is calculated to be ~5.8 meV [28–30], which indicates that subband mixing in the presence of an in-plane magnetic field is significant. We thus believe that the observed CF series at high tilt angles is a result of subband mixing, confirming the explanation in [16].

In Fig. 3 we plot the temperature-dependent magnetoresistance for the tilt angles $\theta \sim 0°$, 25°, and 60°, respectively, measured from sample B. We note that for $\theta \sim 25°$, $R_{xx}(5/2)$ clearly shows $dR_{xx}/dT > 0$, as expected for a developing quantum liquid. This is in contrast to the case of $\theta \sim 0°$, where $dR_{xx}/dT \sim 0$. Such plots demonstrate that, indeed, the 5/2 FQHE is being enhanced by a small in-plane magnetic field component $B_{\parallel} \sim 0.7$ T in this QW. In order to quantitatively access the energy scale of the relevant states, we determined the energy gap Δ^S from $S \propto \exp(-\Delta^S/2k_BT)$, see Fig. 3(d). For the FQHE states at 5/2, 7/3, and 8/3 in tilted fields, we determined the activation gap Δ by fitting the data to an Arrhenius relation, $R_{xx} \propto \exp(-\Delta/2k_BT)$. The values of Δ and Δ^S are presented in the caption of Fig. 3.

In summary, we have observed an enhancement of the low magnetic field FQHE at filling factor 5/2 in a small in-plane magnetic field. This finding is in stark contrast to all previously reported results. In the following paragraphs we discuss briefly the possible relevance of our observation to the phase transition from a partially polarized to a fully spin-polarized ground state at 5/2 driven by Zeeman energy. This regime has not been explored experimentally until this work.

For the 5/2 state in a perpendicular magnetic field B, the ratio between the Zeeman energy $E_Z = g\mu_B B$ and the electron interaction energy $E_C = e^2/\varepsilon l_B$, $\eta \equiv E_Z/E_C$ scales as $\sqrt{B}$, where g is the effective g factor in GaAs, μ_B is the Bohr magneton, ε is the dielectric constant, and $l_B = \sqrt{\hbar/eB}$ is the magnetic length. Wójs *et al.* point out that a wide QW favors the formation of Skyrmions, because the Coulomb repulsion for a spin-singlet pair of electrons softens considerably (relative to a spin-triplet) upon increasing the well width. At a sufficiently low Zeeman energy, it is energetically favorable for charge $e/4$ quasiholes to bind into Skyrmions of charge $e/2$ [25]. In small $B \sim 1.7$ T, $\eta \sim 0.008$, which is comparable to the critical value of ~ 0.01 for forming quasiparticle spin textures, as calculated in [26]. In realistic samples, potential fluctuations are always present and these can further promote Skyrmions by bringing two repelling quasiparticles together. Based on these theoretical considerations, it is plausible that the observed weak plateau at 5/2 and its enhancement in a small tilt angle may be relevant to a Skyrmion spin texture at $\nu = 5/2$ in the limit of small Zeeman energy in wide quantum wells.

In order to further examine the origin of these tilt-field effects, we analyzed quantitatively the behavior of the 5/2 state and the nearby 7/3 and 8/3 states in small tilt angle range $\theta < 25°$. General properties of the Skyrmion states in the FQHE regime have been studied both theoretically (see, e.g., [31–33]) and experimentally (see, e.g., [34,35]). Characteristically, such states prevail in the limit of a small Zeeman energy (originating from a small g factor or a small applied magnetic field). In transport measurements the activation energy at the FQHE filling factor as a function of Zeeman energy can be assessed to reveal the existence of Skyrmions [34,35]. In the Skyrmion state the gap increases faster than a single spin-flip upon increasing the total magnetic field. In the case of "valley Skyrmions," it is found that the interaction-induced quantum Hall gap increases faster than the single particle energy [36].

Estimated from our data in the small tilt angle range, the 7/3 gap increases faster than single spin-flip energy, while for 8/3 and 5/2 the gap openings are better described by a single spin flip. For the lack of experimental g-factor values at these fractions, here we take the electron bare g factor in GaAs, which is a reasonable choice based on the fact that composite fermions carry the same g factor [37,38]. This observation appears to contradict Skyrmion properties, suggesting that a different ground state may be relevant at zero tilt. On the other hand, the Skyrmion model proposed for 5/2 involves fractionally charged quasiparticles, and how Zeeman energy alters the many-body energy gap remains an open question [25,26], which should be addressed in future work. We note that in [38], the energy gap of 8/3 in very high-mobility GaAs/AlGaAs QWs of different electron density (hence different magnetic field) have been carefully measured, and the results can be clearly explained by a Landau fan diagram of composite fermions carrying a spin on the second Landau level. In [39] the 7/2 state in a tilt field has been studied experimentally. At the intermediate tilt angles, the authors observed a strengthening of the 7/2 FQHE and discussed it in the text. Altogether, spin properties of the FQHE states in the second Landau level appear remarkably rich and remain only partially understood.

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