Excited state halos in ^{10}Be

Jim Al-Khalili[1] and Koji Arai[2]
[1]*Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom*
[2]*Division of General Education, Nagaoka National College of Technology, 888 Nishikatakai, Nagaoka, Niigata 940-8532, Japan*
(Received 19 July 2005; published 13 September 2006)

The structure of certain bound excited states in ^{10}Be have been shown to have exotic features because of their weak binding and cluster-like configurations. In this article, we investigate $E1$ and $E2$ transitions between these states and compare the findings with recent experimental results. We compare the predictions of two types of structure calculations: a microscopic multicluster model and an *ab initio* no-core shell model. Both predict very similar transition strengths. By considering the relative contributions from the various matrix elements contributing to the transitions arising from the coupling of different ^{9}Be×n configurations in the wave functions making up the states, we conclude that the very weak $B(E1; 2^- \rightarrow 2_1^+)$ can only be understood if the 2^- state (with a separation energy of its predominantly $1s_{1/2}$ neutron of just 0.548 MeV) is a clear halo state. Other nearby states, such as the 2_2^+, do not exhibit a clear halo signature because of the less than clean decoupling into the well-defined ^{9}Be core plus halo neutron.

DOI: 10.1103/PhysRevC.74.034312 PACS number(s): 21.60.Gx, 23.20.−g, 27.20.+n

I. INTRODUCTION

Over the past few decades, a number of neutron-rich nuclei have been discovered to have a ground state structure with an extended spatial distribution due to the long range tail in the wave function of the last one or two valence neutrons, a phenomenon known as the “neutron halo”. The best-known one-neutron halo nucleus is ^{11}Be whose $\frac{1}{2}^+$ ground state is due to the presence of an intruder state from the *sd* shell, the valence neutron occupying the $1s_{1/2}$ orbit rather than the $0p_{1/2}$ as would be expected from a simple shell model description. In this ground state, the neutron is bound by just 0.5 MeV (with the $\frac{1}{2}^-$ excited state 0.32 MeV above it and even closer to the ^{10}Be+n threshold). Other one-neutron halo nuclei include ^{15}C and ^{19}C and there are also two-neutron halo (Borromean) nuclei such as ^{6}He, ^{11}Li, and ^{14}Be. In this article, we investigate the properties of certain excited states of ^{10}Be to see if they qualify as excited state halos.

The field of halo nuclei has generated much excitement and hundreds of papers over the past two decades since their discovery in the 1980s (see the reviews [1–4] for background on the field) after the experimental determination of large $E1$ transitions in ^{11}Be [5] and large interaction cross sections in neutron-rich lithium and helium isotopes [6,7]. The “halo” is essentially a threshold effect arising from the very weak binding of the last one or two valence nucleons (usually neutrons) to, and decoupling from, a well-defined relatively inert core containing all the other nucleons. Textbook quantum mechanics states that a weakly bound nucleon in a short-range nuclear potential can tunnel out to a volume well beyond the range of the potential (assuming in this case that the core is relatively compact). The accepted definition of a halo nucleus is thus that the halo neutrons have more than 50% of their probability density outside the range of the core potential that binds them. In such an open structure it is not surprising that shell model and mean field approaches breakdown and that few-body cluster models of core plus valence particles can account for the most general properties of these nuclei, such as their large matter radii.

In addition to the decoupling of core and valence particles and their small separation energy, the other important criterion for the formation of a halo is that the valence particle must be in a low orbital angular momentum state relative to the core, preferably an s wave, since higher ℓ values give rise to a confining centrifugal barrier. Indeed, it is the additional confining Coulomb barrier that explains why proton halos are not so spatially extended as neutron halos.

The study of excited state halos is difficult because of their very short lifetimes, and one must resort to searching for them as products of reactions or decays. To date, the only clear example of an excited halo state is the proton halo in the $\frac{1}{2}^+$ excited state of ^{17}F. What is surprising is that no analogous neutron halo state has been confirmed despite the view that such states should be rather common in light nuclei.

In this article we first review the evidence, both theoretical and experimental, for the existence of neutron halo states in one or more excited states of ^{10}Be. We then provide further supporting evidence for this by comparing $E1$ and $E2$ transitions between some of these states and show that their strengths can only be understood if we take into account exotic structures, both halo and molecular states.

It is well-known that ^{9}Be has a good $\alpha + \alpha + n$ structure with a ground state three-body separation energy of just 1.6 MeV. However, by adding one extra neutron to make ^{10}Be the binding is increased to a ^{9}Be+n separation energy of 6.8 MeV. This gives the ground and first excited states (0_1^+, 2_1^+) reasonably good shell-model-like structure. However, the more interesting cluster structure returns for the group of four bound excited states of ^{10}Be (2_2^+, 1^-, 0_2^+, and 2^-) concentrated within 0.3 MeV just below the ^{9}Be+n threshold. A number of recent studies (see next section) have suggested that these states have exotic molecular-like (^{6}He+α) structures.

The question we address in this article is whether the 2_2^+ and 2^- states, in particular, have a structure that can be considered molecular-like, halo-like, or some exotic combination of the two: a single halo neutron weakly bound to a molecular-like ^{9}Be core. In the next section we briefly examine experimental evidence for such unusual structures and review several theoretical attempts to model these states. In Sec. III, we lay out the formalism for the $B(E1)$ transition strength between two states, each made up of a sum of different single-particle configurations. Then, in Sec. IV and V, the values obtained are compared with those calculated within both the no-core shell model and a four-body cluster model. Finally, we summarize our results in Sec. VI.

II. EVIDENCE FOR EXOTIC STATES IN ^{10}Be

A number of experimental results have suggested the existence of possible molecular structures in ^{10}Be arising from its $2\alpha + 2n$ clustering [8–10]. In addition, a more recent experiment at TRIUMF [11] provided accurate measurements of the lifetime of the 2^- (6.264 MeV) state following the $\beta n\gamma$ decay of the most famous halo nucleus, ^{11}Li. The study suggested that this state in ^{10}Be can be populated through a rather novel pathway via ^{11}Be* and that if so it would be a strong candidate for halo-like structure involving a weakly bound neutron to a ^{9}Be core. Since the ^{9}Be+n breakup energy is at 6.812 MeV, the 2^- would have a valence neutron separation energy of just 0.548 MeV.

It has been suggested previously [12] that the 2^- state is halo-like, and knockout studies at MSU also back this up. In the one-neutron knockout reaction (^{11}Be,^{10}Be) it was suggested [13] that the 2^- is populated when a deeply bound $p_{3/2}$ neutron is removed from the ^{10}Be core of ^{11}Be, leaving the $1s_{1/2}$ valence (halo) neutron intact. Indeed, what is most interesting is that the 2^- state in ^{10}Be shows remarkable similarity at first glance to the $\frac{1}{2}^+$ ground state of ^{11}Be. Both are dominated by a $1s_{1/2}$ neutron bound to the rest of the nucleus (the core) by about 0.5 MeV.

Few-body structure models assuming ^{10}Be to be a four-body system of $\alpha + \alpha + n + n$ have suggested a molecular-like structure for several of its bound excited states [14–18]. This would suggest that a shell model description is inappropriate. Yet, more microscopic *ab initio* calculations have now reached a high level of sophistication. In particular, the no-core shell model (NCSM) [19,20] has been applied to the study of the lighter Be isotopes, using model spaces up to $9\hbar\Omega$ [21,22]. The many-body NCSM calculation is performed with a version of the shell model code ANTOINE [23]. This approach predicts both the positive and negative parity states in ^{10}Be, in the correct order separately, but the three weakest bound states (1^-, 0_2^+, and 2^-) are too high in excitation energy (and above the ^{9}Be+n threshold). This is possibly due to the model space needing to be even larger, or the need for three-body *NNN* forces. The NCSM predicts the 0_2^+ intruder state to be up at around 9.5 MeV above the ground state. This state is dominated by a $2\hbar\Omega$ excitation of two p-shell neutrons into the $1s_{1/2}$ orbit. The inability of the NCSM to predict this state as bound is most likely related to its problems predicting the $\frac{1}{2}^+$ intruder in ^{11}Be to be its ground state.

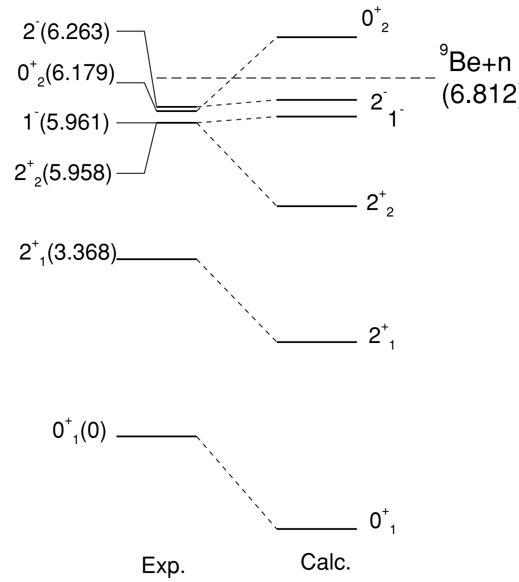

FIG. 1. Energy spectrum for the bound states in ^{10}Be. The experimental levels are from Refs. [27,28] while the theoretical ones are from the microscopic cluster model [24].

One of the most successful models of the structure of excited states of ^{10}Be to date seems to be that based on the microscopic $\alpha + \alpha + n + n$ four-cluster model according to the resonating group method [24]. In this approach, all the nucleons are treated explicitly and two sets of four nucleons (2n+2p) are arranged in 0s clusters (the α particles) whose wave functions are taken to be simple shell model wave functions built up from 0s harmonic oscillator states. For ^{10}Be, the full four-body wave function is then constructed to satisfy the Pauli principle for all the nucleons exactly, is free from any spurious center of mass motion, and has good total angular momentum and parity. Henceforth, when we refer to MCM calculations in this paper we mean the four-body microscopic cluster model of Ref [24].

Figure 1 shows the energy spectrum for the bound states of ^{10}Be as predicted by the cluster model calculations against the experimental values. The ^{9}Be+n breakup energy is also shown since all states above this threshold are particle unbound.

Table I shows the predicted rms matter, proton, and neutron radii for several of these states from two cluster models: the MCM of [24] and a similar one (also assuming a four-body structure) based on the stochastic variational method (SVM) [15]. It is clear from these values that the 2^- state is much larger in extent, due to the larger neutron distribution, hinting at the existence of a possible neutron halo.

We concern ourselves in this article with the electromagnetic transitions between four of these states: 0_1^+, 2_1^+, 2_2^+, and 2^-. The reason for this is the suggestion, from the recent TRIUMF-ISAC experiment on the β decay of ^{11}Li [11], and subsequent interpretation, that both the 2_2^+ and 2^- states in ^{10}Be are halo-like. It was suggested in that work [11] that these two states are the end products of the β decay of ^{11}Li to the halo-like continuum state in ^{11}Be (8.81 MeV), which has a ^{9}Be+2n cluster structure, followed by the subsequent decay

TABLE I. The rms matter, proton, and neutron radii for several of the bound states in ^{10}Be as predicted by the four-body cluster model based on the resonating group method (RGM) [24]. The values in brackets are those calculated by the stochastic variational method (SVM) of Ref. [15], which also assumes a four-body cluster model.

State in ^{10}Be	rms radius (fm)		
	r_m	r_p	r_n
0_1^+	2.20 (2.28)	2.08 (2.17)	2.28 (2.35)
2_1^+	2.17 (2.41)	2.05 (2.25)	2.25 (2.51)
2_2^+	2.36 (2.58)	2.17 (2.36)	2.49 (2.72)
2^-	2.90 (3.17)	2.32 (2.50)	3.24 (3.50)

of one of the two halo neutrons, leaving the remaining neutron intact in a halo configuration in ^{10}Be. This result has been partially brought into question by another recent experimental result (also from TRIUMF) [25] that claims that the neutron decay pathway from the 8.81-MeV state in ^{11}Be is to the 2^-state and, possibly, the higher (and particle unbound) 3^- state, but not to the 2_2^+ state.

Of interest in this article is the fact that the half-life measurement of the 2^- state of $T_{1/2} = 85 \pm 12$ fs corresponds to the very small experimental value of $B(E1; 2^- \to 2_1^+) = 7.7 \times 10^{-4}$ W.u. Furthermore, surprisingly no γ transition is seen between the 2^- and 2_2^+ states. Of course, because the second 2^+ is an order of magnitude closer in energy to the 2^- than the first one, there is a factor of three orders of magnitude on the lifetime for this $E1$ transition. It is therefore not surprising that the $2^- \to 2_2^+$ transition is not seen because the $2^- \to 2_1^+$ happens much more quickly. But this tells us nothing about any differences in the structure of these states entering into the transition matrix elements.

To understand whether these very small $B(E1)$ values are to be expected we look here more closely at possible cluster structures of these states in ^{10}Be and whether we can understand why these transitions are so weak. In particular, we compare with the very large $B(E1)$ transition between the two bound states in the neighboring ^{11}Be. In all three cases $[B(E1; 2^- \to 2_1^+)$ and $B(E1; 2^- \to 2_2^+)$ in ^{10}Be and $B(E1; \frac{1}{2}^- \to \frac{1}{2}{}_1^+)$ in ^{11}Be], the transition is a pure single-particle one involving the valence neutron between the *sd* and *p* shells. We will see that the ^{9,10}Be $\times$ n configurations that describe these states involve initial and final core states of the same parity, but neutrons in different parity states. Because $E1$ transitions require a change in parity they cannot proceed via a collective excitation within the core and the analysis becomes much simpler.

III. FORMALIZM FOR $E1$ TRANSITIONS

We model the ^{10}Be states as a sum of ^{9}Be $\times$ n configurations and write the wave function in the state J^π as

$$\Psi^{J^\pi M} = \sum_\alpha C_\alpha \left[\Phi_9(I_\alpha^\pi) \times \phi_j\right]_{JM}, \quad (1)$$

where $\Phi_9(I_\alpha^\pi)$ is the wave function of ^{9}Be in state I_α^π and ϕ_j is the ^{9}Be-n cluster wave function. In the four-body cluster model of Ref. [24], the ^{9}Be intrinsic wave function is itself a three-body $(\alpha + \alpha + n)$ cluster one, but we need not concern ourself with its details here since the $E1$ transitions will only involve the neutron-core relative wave functions.

The $E1$ transition strength, from initial state $|J_1^\pi M_1\rangle$ to final state $|J_2^\pi M_2\rangle$ is defined as

$$B\left(E1; J_1^\pi \to J_2^\pi\right) = \sum_{\mu, M_2} |\langle J_2 M_2 | \hat{\mathcal{O}}_{1\mu}(E1) | J_1 M_1\rangle|^2, \quad (2)$$

where the electric transition operator $\hat{\mathcal{O}}_{1\mu}(E1)$ is

$$\hat{\mathcal{O}}_{1\mu}(E1) = \sum_{i=1}^{Z} e r_i Y_{1\mu}(\hat{r}_i). \quad (3)$$

Because the valence particle in our model is a neutron, only the core contributes to electric transitions. One can therefore derive an expression for the transition operator involving a sum of two terms: one involving a collective excitation of the core and the other involving the core recoil as a function of the core-neutron separation vector $\vec{r}$. The latter will be the term that couples the different core-neutron cluster wave functions, $\phi(\vec{r})$, and can be regarded as the single-particle transition with the difference that the initial and final state wave functions that are coupled are relative wave functions between the neutron and the core. For light systems this distinction involves important core recoil effects that are taken into account here. It is then straightforward to show that

$$\hat{\mathcal{O}}_{1\mu}(E1) = \sum_{i=1}^{Z} \left(e x_i Y_{1\mu}(\hat{x}_i) - e\frac{r}{A} Y_{1\mu}(\hat{r})\right)$$
$$= \hat{\mathcal{O}}_{1\mu}^{\text{core}}(E1) - \frac{Z}{A} e\, r\, Y_{1\mu}(\hat{r}). \quad (4)$$

Substituting for the operator into Eq. (2) it is clear that, for the states of interest here, the first term will not contribute to the matrix element because $E1$ transitions require an overall change in parity and for this to be caused by a change in the state of the ^{9}Be core requires the parity of the core-neutron wave function to be unchanged. However, as mentioned earlier, the transitions we consider are all between states involving the valence neutron moving between the *sd* shell and the *p* shell.

Substituting for the initial and final state wave functions, as defined in Eq. (1), into the matrix element in Eq. (2), it is then straightforward to work through the angular momentum algebra to obtained the well-known expression

$$B(E1; J_1^\pi \to J_2^\pi) = \frac{3}{4\pi}\left(\frac{Z}{A}\right)^2 \times \left[\sum_{\alpha,\alpha'} C_\alpha C_{\alpha'} \hat{J}_1 \hat{J}_2 \delta_{I_1, I_2} \left(j_1 \frac{1}{2} 10 | j_2 \frac{1}{2}\right) \times \begin{Bmatrix} j_1 & I_1 & J_1 \\ J_2 & 1 & j_2 \end{Bmatrix} \langle r \rangle \right]^2 \mathrm{e}^2\mathrm{fm}^2, \quad (5)$$

where C_α and $C_{\alpha'}$ are amplitudes associated with the different configurations that make up each of the initial and final states. We will associate these quantities with spectroscopic factors obtained from the four-body RGM model calculations [24]. Note that δ_{I_1,I_2} ensures that only core states with the same angular momentum can be coupled because the transition only takes place because of coupling the different relative core-neutron states (with angular momentum j_1 and j_2. The overlap integral $\langle r \rangle$ is between different relative radial wave functions of the core-neutron system,

$$\langle r \rangle = \int_0^\infty r^3\, dr u_{n_1\ell_1}(r) u_{n_2\ell_2}(r). \quad (6)$$

Note that while the expression in Eq. (5) gives the $B(E1)$ strength in units of e^2fm^2, we quote values in the more traditional Weisskopf units.

The above radial wave functions are calculated assuming Wood-Saxon interactions between the neutron and ^{9}Be core with depths chosen to give the correct neutron separation energy. As was pointed out many years ago [5], the use of harmonic-oscillator wave functions for the neutrons is inappropriate for such weakly bound states and leads to an underestimation of the transition strength because of the importance of the long-range behavior in the overlap intergal that is not picked up unless realistic wave functions are used.

It is clear from the summation in Eq. (5) that the overall transition strength arises from a coherent sum of all allowed matrix elements coupling the different configurations. It is well-known that this can lead to important cancellation effects from matrix elements of opposite signs. A factor missing from this expression, however, is a spatial mismatch factor due to the overlap of different looking ^{9}Be core wave functions. Thus, while the intial and final core states must have the same angular momentum, their spatial distributions can differ because of different clusterization effects. This would lead to one-body transition densities less than unity if different looking core states (with the same quantum numbers) are coupled. This effect has been discussed [26] in the context of the knockout reaction (^{12}Be,^{11}Be) in which the surviving weakly bound or unbound ^{11}Be state following neutron removal differs spatially from the more tightly bound (and hence more compact) initial ^{11}Be state within the bound ^{12}Be projectile.

An alternative qualitative description of this core effect is to say that a deformed, or polarized, core will present to the valence neutron a modified binding potential, which in turn will affect the shape of the radial wave function in either the initial or the final states in the overlap integral of Eq. (6). In particular, if a wave function has a node (such as that involving neutrons in the *sd* shell) then the geometry of the potential will affect the position of the node and possibly have a dramatic cancellation effect in the overlap.

IV. THE TEST CASE: $E1$ TRANSITION IN ^{11}Be

The best-known, and indeed the fastest, $E1$ transition in nuclear physics is that between the first excited state and the ground state of ^{11}Be. One of the most important features of this nucleus is the state inversion that leads to a $\frac{1}{2}^+$ ground state and a $\frac{1}{2}^-$ first excited state. The measured $E1$ transition strength between these two states is 0.3 W.u. To calculate this we take the ground state to be a sum of two terms. The dominant configuration is ^{10}Be$(0^+) \times \nu(s_{1/2})$ with a smaller but quite significant ^{10}Be$(2^+) \times \nu(d_{5/2})$ configuration. If the $\frac{1}{2}^-$ state is then taken to be purely ^{10}Be$(0^+) \times \nu(p_{1/2})$, and since only like-core states contribute, there will be only one term in the sum in Eq. (5) involving the 0^+ ground states of the core. This gives, assuming W-S wave functions for the neutron relative to the core in both initial and final states, a value of $B(E1) =$ 0.86 W.u. It is well known [5] that to reduce this overestimation of the observed strength, we must also take into account excited core configuration in the $\frac{1}{2}^-$ state: ^{10}Be$(2^+) \times \nu(p_{3/2})$. This can couple to the ^{10}Be$(2^+) \times \nu(d_{5/2})$ configuration in the ground state and the two matrix elements lead to a cancellation and a value of $B(E1) = 0.12$ W.u.. Clearly, the precise value can be tweaked by adjusting the amplitudes corresponding to the various configurations. This is not important; what is important is the need for more than one matrix element in a coherent sum. It should be pointed out here that the recent NCSM calculation [22] underestimates the $B(E1)$ strength by over an order of magnitude. The authors attribute this to the unique halo structure of the two bound states. Clearly, the decoupling between core and valence degrees of freedom in these states not only makes the picture simpler but also more accurately reproduces observables such as transition strengths.

V. THE TRANSITIONS IN ^{10}Be

We now turn our attention to the transitions in ^{10}Be and consider first the dominant ^{9}Be×n configurations in the various states of interest as predicted by the MCM calculations of [24]. The 0^+ ground state of ^{10}Be looks almost entirely shell model-like and, in a simple cluster model picture, can be thought of as a $p_{3/2}$ neutron coupled to the $\frac{3}{2}^-$ ground state of ^{9}Be. The first excited state, 2_1^+, is also relatively simple to picture; in this case with the $p_{3/2}$ neutron mainly coupling to the $\frac{5}{2}^-$ first excited state of ^{9}Be. This explains the very strong collective $B(E2)$ transition between these states (see Table II) and the MCM and NCSM predictions are in agreement.

Of the four other bound excited states $(2_2^+, 1^-, 0_2^+, 2^-)$ we focus on the two $(2_2^+, 2^-)$ suggested to be fed by the the decay of the 8.81-MeV state in ^{11}Be as discussed in Ref. [11] and predicted to be possibly halo-like in their structure since they are the remnants of the β decay of ^{11}Li.

TABLE II. A comparison between the measured and calculated values for some $E1$ and $E2$ transition strengths in ^{10}Be. Experimental values are taken from [27] and [11]. Calculated values are from the microscopic cluster model (MCM) and no-core shell model (NCSM) [30].

Transition	MCM	NCSM [30]	Exp.
	(Values are in Weisskopf units)		
$B(E2; 2_1^+ \to 0_1^+)$	4.8	5.4	8.00 (0.80)
$B(E2; 2_2^+ \to 0_1^+)$	0.1	0.17	0.1
$B(E1; 2^- \to 2_1^+)$	0.02	0.022	0.00077
$B(E1; 2^- \to 2_2^+)$	0.006	0.007	Not seen

The 2_2^+ state is dominated by configurations in which a $p_{1/2}$ neutron couples to a ^{9}Be core in either of the $\frac{3}{2}^-$ or $\frac{5}{2}^-$ states. In addition, there is a small mixing from configurations that resemble those in the 2_1^+ state: $^9\mathrm{Be}(3/2^-) \times \nu(p_{3/2})$ and $^9\mathrm{Be}(5/2^-) \times \nu(p_{3/2})$. As for the negative parity 2^- state, this is dominated by the $^9\mathrm{Be}(3/2^-) \times \nu(s_{1/2})$ configuration with small amounts of $^9\mathrm{Be}(5/2^-) \times \nu(s_{1/2})$, $^9\mathrm{Be}(3/2^-) \times \nu(d_{5/2})$, and $^9\mathrm{Be}(5/2^-) \times \nu(d_{5/2})$. It is worth mentioning here that the shell model [29] predicts that both the 2_2^+ and the 2^- states have a strong $\frac{5}{2}^-$ ^{9}Be core component coupled to a $p_{1/2}$ and a $s_{1/2}$ neutron, respectively.

It is natural to make the comparison between the 2^- state in ^{10}Be and the $\frac{1}{2}^+$ ground state in ^{11}Be. The difference now is that we can have an $s_{1/2}$ or $d_{5/2}$ neutron each coupling to either $\frac{3}{2}^-$ or $\frac{5}{2}^-$ ^{9}Be core states, whereas in the ground state of ^{11}Be only two configurations, $^{10}\mathrm{Be}(0^+) \times \nu(s_{1/2})$ and $^{10}\mathrm{Be}(2^+) \times \nu(d_{5/2})$, contribute. Nevertheless, if we are to naively associate the 2^- state in ^{10}Be with the $\frac{1}{2}^+$ state in ^{11}Be then we should also expect to see strong $E1$ transitions from this state. This is not the case. In [11], only a very small $B(E1; 2^- \to 2_1^+)$ strength of 0.00077 W.u. was measured and no transition between the 2^- and the 2_2^+ states was seen at all. This is consistent with the calculated MCM values for these strengths of $B(E1; 2^- \to 2_1^+) = 0.02$ W.u. and $B(E1; 2^- \to 2_2^+) = 0.006$ W.u., which are in turn in close agreement with those predicted by the NCSM [30] (see Table II). We address now whether we can understand these calculated $B(E1)$ strengths and what they can tell us about the structure of these states.

We begin by looking first at $B(E1; 2^- \to 2_1^+)$ and consider the simplest scenario in which the 2^- state is described entirely by its dominant $^9\mathrm{Be}(3/2^-) \times \nu(s_{1/2})$ configuration. Because this can only couple to the same core state in 2_1^+, we only have one matrix element in Eq. (5) to consider involving the $^9\mathrm{Be}(3/2^-) \times \nu(p_{3/2})$ configuration in the final state. By following the method used to calculate the $B(E1)$ for ^{11}Be described earlier (using W-S core-neutron wave functions) and assuming only a single-particle transition between an initial state as a pure $3/2^- \times s_{1/2}$ and a final state as a pure $3/2^- \times p_{3/2}$ we arrive at a value of $B(E1; 2^- \to 2_1^+) = 0.12$ W.u., which is 6 times larger than the value calculated in the cluster model and more than three orders of magnitude larger than the experimental value. This suggests that it is important to take into account other configurations in the 2^- state to allow for possible cancellation in the coherent sum of matrix elements in Eq. (5).

First, to do this we must make an assumption about the occupation probabilities of the different $^9\mathrm{Be} \times \mathrm{n}$ configurations (the $C_{\alpha,\alpha'}$s). In Ref. [24] the reduced width amplitudes (spectroscopic amplitudes) are defined as the overlap of the cluster model wave functions for the ^{10}Be states with the various configurations of ^{9}Be+n decay. The (integrated) spectroscopic factors obtained are tabulated in [24], and while not an ideal solution, we take these values for the dominant configurations considered here (Table III). This enables us to take into account in a simple and consistent way the relative importance of the different coupling matrix elements in the $E1$ transition.

TABLE III. The occupation probabilities for the dominant ^{9}Be+n configurations in the ^{10}Be states of interest, defined as renormalized "spectroscopic factors" from [24].

State in ^{10}Be	Configuration $[^9\mathrm{Be}(I^\pi) \times \nu(\ell j)]$	Spectroscopic factors from MCM
0_1^+	$3/2^- \times p_{3/2}$	2.26
2_1^+	$3/2^- \times p_{3/2}$	0.24
	$5/2^- \times p_{3/2}$	1.17
2_2^+	$3/2^- \times p_{3/2}$	0.28
	$3/2^- \times p_{1/2}$	0.54
	$5/2^- \times p_{3/2}$	0.23
	$5/2^- \times p_{1/2}$	0.13
2^-	$3/2^- \times s_{1/2}$	0.70
	$3/2^- \times d_{5/2}$	0.16
	$5/2^- \times s_{1/2}$	0.02
	$5/2^- \times d_{5/2}$	0.10

There are thus four matrix elements that contribute to $B(E1; 2^- \to 2_1^+)$. These are shown in Table IV along with their numerical values to show their cancellation effect. These values are the quantities in square brackets in Eq. (5). The first and fourth of these matrix elements are opposite in sign to the second and third and the full coherent sum gives a value of $B(E1; 2^- \to 2_1^+) = 0.033$ W.u., which is, presumably somewhat fortuitously, in reasonable agreement with the cluster model value of 0.02 W.u. However, the cancellation between the different matrix elements is not as vital here as it was for the $E1$ transition in ^{11}Be. We note in Table III that the $3/2^- \times p_{3/2}$ configuration in the 2_1^+ state has an occupation probability of 0.24. Including this factor but assuming the 2^- state is still a pure $3/2^- \times s_{1/2}$, we obtain a value of 0.028 W.u. Thus we do not necessarily need to include the other three matrix elements that appear when we include the missing configurations in the 2^- because they simply cancel each other out. Of course, by tweaking the relative strengths of the different configurations in the initial and final states, the $B(E1)$ strength can be forced to have any value between a maximum of 0.12 W.u. (when only the first, and largest,

TABLE IV. The values of the different matrix elements contributing to the two $E1$ transitions of interest. The numerical values in column 3 refer to the quantities inside the square brackets in Eq. (5).

Transition	Matrix element	Value
$B(E1; 2^- \to 2_1^+)$	$[3/2^- \times s_{1/2}] \to [3/2^- \times p_{3/2}]$	−0.41
	$[3/2^- \times d_{5/2}] \to [3/2^- \times p_{3/2}]$	+0.24
	$[5/2^- \times s_{1/2}] \to [5/2^- \times p_{3/2}]$	+0.19
	$[5/2^- \times d_{5/2}] \to [5/2^- \times p_{3/2}]$	−0.53
$B(E1; 2^- \to 2_2^+)$	$[3/2^- \times s_{1/2}] \to [3/2^- \times p_{1/2}]$	−1.02
	$[3/2^- \times s_{1/2}] \to [3/2^- \times p_{3/2}]$	−0.73
	$[3/2^- \times d_{5/2}] \to [3/2^- \times p_{3/2}]$	+0.55
	$[5/2^- \times s_{1/2}] \to [5/2^- \times p_{1/2}]$	−0.05
	$[5/2^- \times s_{1/2}] \to [5/2^- \times p_{3/2}]$	+0.14
	$[5/2^- \times d_{5/2}] \to [5/2^- \times p_{3/2}]$	−0.26

matrix element is considered) and zero (when there is complete cancellation). Furthermore, we have a similar freedom to force $B(E1)$ to be very small if we assume that the ^{9}Be core remains in its $3/2^-$ ground state in the 2^- state but allowing it to couple to an s- or d-wave neutron (thus leaving us with only the first two matrix elements) or indeed if the valence neutron is purely in s-state and coupling to either a $\frac{3}{2}^-$ or a $\frac{5}{2}^-$ core (the first and third matrix elements). In both these cases, the two surviving matrix elements are of opposite sign and their relative strengths can be adjusted for the appropriate amount of cancellation.

It seems from the previous analysis that we cannot infer too much about the contributions of the different configurations in the 2^- wave function other than to say that its core is probably not too dissimilar to that of the 2_1^+. Of course, we are still left with explaining why the measured strength is over an order of magnitude smaller than the calculated values. But as described earlier, a polarized ^{10}Be core due to molecular configurations will affect the potential felt by the valence neutron and this can have an effect on the overlap integral that comes into the single-particle transition by shifting the position of the node in the initial state radial wave function. However, this is a rather qualitative argument and does not explain why the theoretical values do not pick it up.

If we turn our attention to the $E1$ transition between the 2^- and 2_2^+ states, we find that even more matrix elements can contribute and must be taken into account. However, even so, we arrive at a value of $B(E1; 2^- \to 2_2^+) = 0.25$ W.u., which is far bigger than the MCM and NCSM predictions of 0.006 and 0.007 W.u., respectively. Again, if we consider only the dominant transition $[3/2^- \times s_{1/2}] \to [3/2^- \times p_{1/2}]$, then we arrive at a similarly large value. Even if we allow configurations involving the two different core states of $\frac{3}{2}^-$ and $\frac{5}{2}^-$ while assuming the transition takes place because of the $s_{1/2}$ neutron coupling to either $p_{1/2}$ or $p_{3/2}$ we get nowhere near the correct amount of cancelation, nor when we include those configurations in the 2^- involving the neutron in a $d_{5/2}$ state.

Thus we see that, for the $E1$ transition between these two nearby states (2^- and 2_2^+), not only is it necessary to include many configurations in each state but their occupation probabilities must also be such that they give just the right balance for the matrix elements to cancel each other completely! Using the occupation probabilities predicted by the MCM calculations we do not get this cancellation. And yet the MCM predicts a very small $B(E1)$. A possible interpretation is that the strength of the transition to the 2_1^+ state is, while small, preferred over the one to the 2_2^+ state because the ^{9}Be cores of the dominant configurations in the 2^- and 2_1^+ states look more alike. That is, the 2^- state has a core that looks similar in its spatial structure to that of the 2_1^+ state. However, the 2_2^+ state has a core that is more molecular and extended and therefore has little overlap with the core in the initial 2^- state.

The previous argument is supported if we consider the $E2$ transitions between each of the two 2^+ states and the ground state. We have already mentioned the very large $B(E2; 2_1^+ \to 0^+) = 8$ W.u. due to a collective transition in the core. This compares with a value of $B(E2; 2_2^+ \to 0^+) = 0.1$ W.u., measured experimentally [11] and confirmed by the cluster model prediction. The transition operator in this case is [comparing with Eq. (4)]

$$\hat{O}_{2\mu}(E2) = \hat{O}_{2\mu}^{\text{core}}(E2) + \frac{Z}{A^2} e\, r^2\, Y_{2\mu}(\hat{r}), \tag{7}$$

and we cannot now disregard the first term. The states that are connected now involve configurations in which both the core and the valence parities are unchanged. We find that the observed $B(E2; 2_2^+ \to 0^+)$ strength can be reproduced without including any contribution from the first term in Eq. (7). If the core in the 2_2^+ state looks like that in the 2_1^+ state, then we should expect there to be a large collective core excitation, which would add to the small single-particle transition to give an overall large $B(E2)$, which is not the case. Again, we are forced to conclude that the ^{9}Be core in the second 2^+ state is more molecular in structure that in the first 2^+ state.

We have not considered here the structure of the other two excited states in ^{10}Be: 1^- and 0_2^+, for which there is strong theoretical evidence of molecular structure. We can conclude tentatively, however, that the 2^- state does indeed appear to be halo-like in its structure because of the dominance of the configurations involving a valence neutron in an $s_{1/2}$ orbit. But, just as was found in the $\frac{1}{2}^+$ ground state of ^{11}Be, there is likely to be an important contribution from configurations in which the neutron is in a less spatially extended (due to the centrifugal barrier) $d_{5/2}$ orbit. There are also likely to be core polarization effects that will affect the value of the overlap integral. The 2_2^+ on the other hand does not lend itself to such a simple halo structure as its single-particle configurations involve p-wave neutrons only. It is more likely to be a molecular cluster structure involving an extended $(\alpha + \alpha + n)$ core coupled to a p-wave neutron.

VI. SUMMARY

We considered the possible halo-like structure of the 2^- state in ^{10}Be that sits just 548 keV below the ^{9}Be+n threshold by examining the only observed electromagnetic transition from this state, a pure $B(E1)$ to the 2_1^+ state. We deduce that, if this state is dominated by a $3/2^- \times s_{1/2}$ configuration, then this configuration alone can explain the calculated weak $E1$ transition. Because the measured transition is even smaller than that predicted by either the microscopic cluster model, or the no-core shell model we conclude that the 2^- state is likely to have a ^{9}Be core that is polarized because of the effects of other, more molecular-like, configurations but that is nevertheless well decoupled from the predominantly s-wave valence neutron. This decoupling of the core and valence degrees of freedom, along with the weak binding, is what defines a halo state. The 2^- state is thus our clearest candidate to date of an excited state neutron halo.

ACKNOWLEDGMENTS

The authors thank B. A. Brown, P. Navrátil, F. Sarazin, and I.J. Thompson for useful discussions, helpful comments, and carrying out calculations.

[1] B. Jonson, Phys. Rep. **389**, 1 (2004).
[2] J. S. Al-Khalili, Lect. Notes Phys. **651**, 77 (2004).
[3] I. Tanihata, J. Phys. G **22**, 157 (1996).
[4] B. Jonson, P. G. Hansen, A. S. Jensen, and B. Jonson, Annu. Rev. Nucl. Part. Sci. **45**, 591 (1995).
[5] D. J. Millener, J. W. Olness, E. K. Warburton, and S. S. Hanna, Phys. Rev. C **28**, 497 (1983).
[6] I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, K. Obayashi, and N. Takahashi, Phys. Rev. Lett. **55**, 2676 (1985).
[7] I. Tanihata *et al*., Phys. Lett. **B160**, 380 (1985).
[8] N. Soić *et al*., Europhys. Lett. **34**, 7 (1996).
[9] M. Milin *et al*., Europhys. Lett. **48**, 616 (1999).
[10] M. Freer *et al*., Phys. Rev. C **63**, 034301 (2001).
[11] F. Sarazin *et al*., Phys. Rev. C **70**, 031302(R) (2004).
[12] F. Negoita *et al*., Phys. Rev. C **59**, 2082 (1999).
[13] T. Aumann *et al*., Phys. Rev. Lett. **84**, 35 (2000).
[14] W. von Oertzen, Z. Phys. A **357**, 355 (1997).
[15] Y. Ogawa *et al*., Nucl. Phys. **A673**, 122 (2000).
[16] N. Itagaki and S. Okabe, Phys. Rev. C **61**, 044306 (2000); N. Itagaki, S. Hirose, T. Otsuka, S. Okabe, and K. Ikeda, *ibid.* **65**, 044302 (2002).
[17] Y. Kanada-En'yo and H. Horiuchi, Phys. Rev. C **66**, 024305 (2002).
[18] P. Descouvemont, Nucl. Phys. **A699**, 463 (2002).
[19] P. Navrátil, J. P. Vary, and B. R. Barrett, Phys. Rev. Lett. **84**, 5728 (2000).
[20] P. Navrátil, J. P. Vary, and B. R. Barrett, Phys. Rev. C **62**, 054311 (2000).
[21] E. Caurier, P. Navrátil, W. E. Ormand, and J. P. Vary, Phys. Rev. C **66**, 024314 (2002).
[22] C. Forssén, P. Navrátil, W. E. Ormand, and E. Caurier, Phys. Rev. C **71**, 044312 (2005).
[23] E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves, J. Retamosa, and A. P. Zuker, Phys. Rev. C **59**, 2033 (1999).
[24] K. Arai, Phys. Rev. C **69**, 014309 (2004).
[25] Y. Hirayama *et al*., Phys. Lett. **B611**, 239 (2005).
[26] P. G. Hansen and J. A. Tostevin, Annu. Rev. Nucl. Part. Sci. **53**, 219 (2003).
[27] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).
[28] N. Curtis, D. D. Caussyn, N. R. Fletcher, F. Marechal, N. Fay, and D. Robson, Phys. Rev. C **64**, 044604 (2001).
[29] B. A. Brown (private communication).
[30] P. Navrátil (private communication).