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Analysis of Receiver Algorithms for LTE SC-FDMA Based Uplink MIMO Systems

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Abstract—This letter derives mathematical expressions for the received signal-to-interference-plus-noise ratio (SINR) of uplink Single Carrier (SC) Frequency Division Multiple Access (FDMA) multiuser MIMO systems. An improved frequency domain receiver algorithm is derived for the studied systems, and is shown to be significantly superior to the conventional linear MMSE based receiver in terms of SINR and bit error rate (BER) performance.

Index Terms—SC-FDMA, multiple-input multiple-output (MIMO), 3GPP LTE uplink.

I. INTRODUCTION

SINGLE Carrier (SC) Frequency Division Multiple Access (FDMA) techniques for uplink transmission have attracted appreciable attention because of its low Peak to Average Power Ratio (PAPR) property compared with competitive Orthogonal FDMA (OFDMA) techniques [1]–[3]. In 3GPP Long Term Evolution (LTE) (also known as Evolved-UMTS Terrestrial Radio Access, E-UTRA or EUTRA), SC-FDMA has been adopted for uplink transmission, whereas the OFDMA signaling format has been exploited for the downlink transmission [4]. The SC-FDMA signal can be obtained using Discrete Fourier Transform (DFT) spread OFDMA, where DFT is applied to convert time domain input data symbols to the frequency domain before feeding them into an OFDMA modulator.

From user capacity point of view, MIMO technique is preferred due to its capacity enhancement ability. For wide band wireless transmission systems, e.g., LTE OFDMA downlink and SC-FDMA uplink [5], [6], to simplify scheduling task, several consecutive subcarriers are usually grouped together for scheduling. A basic scheduling unit is called a Resource Block (RB). The scheduler in a Base Station (BS) may assign single or multiple RBs to a Mobile Station (MS).

Two MIMO schemes for SC-FDMA uplink transmission are being investigated under 3GPP LTE, namely, multi-user MIMO and single user MIMO. For a single user MIMO, the BS only schedules one single user into one RB. For a multi-user MIMO, multiple MSs are allowed to transmit simultaneously on a RB. This paper investigates receiver algorithms for a SC-FDMA based uplink in a multi-user MIMO system.

Frequency domain equalization (FDE) is commonly used for SC-FDMA. This includes frequency domain linear equalization (FD-LE) [7], decision feedback equalization (DFE) [8], and the more recent turbo equalization (TE) [9]. FD-LE is analogous to time domain LE. A zero-forcing (ZF) LE [10] eliminates the intersymbol interference (ISI) completely but introduces degradations in the system’s performance due to noise enhancement. Superior performance can be achieved by using the minimum mean square error (MMSE) criterion [7], which accounts for the additive noise in addition to the ISI. In SC-FDMA, a DFE consists of a frequency-domain feedforward filter and a time-domain feedback filter. It gives better performance than a LE due to its ability to remove past echo ISI. However, a DFE is prone to error propagation problems when incorrect decisions are fed back. Consequently, it suffers from a performance loss for long error bursts. This is especially noticeable when combined with forward error correcting codes [8]. The principle of TE is to improve performance by adding complexity at the receiver through an iterative process, in which feedback information obtained from the decoder is incorporated into the equalizer at the next iteration. The iterative processing allows for reduction of the ISI, multistream interference, and noise by exchanging of extrinsic information between the equalizer and the decoder [9].

The use of FD-LE is a common assumption in the SC-FDMA based LTE uplink systems due to its simplicity, and is also the focus of our work. We show that the conventional frequency domain equalizers are suboptimal for improper signals, for which the performance can be greatly improved by applying widely linear processing to the received signal and utilizing some special properties of improper modulation schemes. The novelties of this paper are the derivation of the received Signal to Interference plus Noise Ratio (SINR) and the proposal of an improved frequency domain receiver algorithm. It will be shown that the proposed receiver algorithm with improper signal modulation schemes significantly outperforms the conventional receiver algorithm.

Notations: we use upper bold-face letters to represent matrices and vectors. The \( (n, k) \) element of a matrix \( \mathbf{A} \) is represented by \( [\mathbf{A}]_{n,k} \) and the \( n \)th element of a vector \( \mathbf{b} \) is denoted by \( [\mathbf{b}]_n \). Superscripts \( (\cdot)^H \) and \( (\cdot)^T \) denote the Hermitian transpose and transpose, respectively, \( (\cdot)^* \) denotes conjugate.

II. SYSTEM MODEL

The cellular multiple access system under study has \( n_R \) receive antennas at the BS and a single transmit antenna at
the $i$th user terminal, $i = 1, 2, \cdots, K_T$ where $K_T$ is the total number of users in the system. We consider the multi-user MIMO case with $K (K < K_T)$ users being served at each time slot and $K = n_R$. The system model for a SC-FDMA based MIMO transmitter and receiver is shown in Figs. 1 and 2, respectively.

On the transmitter side, the user data block containing $N$ symbols is firstly transformed by an $N$ point DFT to a frequency domain representation. The outputs are then mapped to $M$ ($M > N$) orthogonal subcarriers followed by an $M$ point Inverse Fast Fourier Transform (IFFT) to convert to a time domain complex signal sequence. There are two approaches to mapping subcarriers among MSs [4]: localized mapping and distributed mapping. The former one is usually referred to as localized FDMA transmission, while the latter one is usually called distributed FDMA transmission scheme. With the localized FDMA transmission scheme, each user's data is transmitted by consecutive subcarriers, while with the distributed FDMA transmission scheme, the user's data is transmitted by distributed subcarriers [4]. Because of the spreading of the information symbol across the entire signal band, the distributed FDMA scheme is more robust against frequency selective fading. Therefore, it can achieve more frequency diversity.

For the localized FDMA transmission, in the presence of frequency selective fading channel, the multiuser diversity and frequency selective diversity can also be achieved if assigning each user to subcarriers with favorable transmission characteristics. In this work, we only consider the localized FDMA transmission. A Circle Prefix (CP) is inserted into the signal sequence before it is passed to the Radio Frequency (RF) module. On the receiver side, the opposite operating procedures are performed after the noisy signals are received by the receive antennas. A MIMO Frequency Domain Equalizer (FDE) is applied to the frequency domain signals after subcarrier demapping, as shown in Fig. 2. For simplicity, we employ a linear Minimum Mean Squared Error (MMSE) receiver, which provides a good tradeoff between the noise enhancement and the multiple stream interference mitigation [11].

In the following, we let $D_{F_M} = I_K \otimes F_M$ and denote by $F_M$ the $M \times M$ Fourier matrix with the element $[F_M]_{m,k} = \exp(-j \frac{2\pi}{M}(m-1)(k-1))$ where $k, m \in \{1, 2, \cdots, M\}$ is the sample number and the frequency tone number, respectively. Here $\otimes$ is the Kronecker product, $I_K$ is the $K$ dimension identity matrix. We denote by $D_{F_M^{-1}}$ the $M \times M$ inverse Fourier matrix, defined as $I_K \otimes F_M^{-1}$, and $F_M^{-1}$ is the $M \times M$ inverse Fourier matrix with the element $[F_M^{-1}]_{m,k} = \frac{1}{M} \exp(j\frac{2\pi}{M}(m-1)(k-1))$. $D_{F_M}$ and $D_{F_M^{-1}}$ are defined in the similar way as $D_{F_M}$ and $D_{F_M^{-1}}$ with the only difference in the matrix size. Furthermore, we let $F_n$ represent the subcarrier mapping matrix of size $M \times N$ and $F_n^{-1}$ is the subcarrier demapping matrix of size $N \times M$.

The received signal after the RF module and removing CP becomes $r = \text{HD}_{F_M}(I_K \otimes F_n)D_{F_M}\tilde{x} + \tilde{w}$, where $\tilde{x} = [\tilde{x}_1, \cdots, \tilde{x}_K]^T \in \mathbb{C}^{KN \times 1}$ is the data sequence of all users, and $\tilde{x}_i \in \mathbb{C}^{N \times 1}$, $i \in \{1, \cdots, K\}$, is the transmitted user data block for the $i$th user; $\tilde{w} \in \mathbb{C}^{MN \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0I \in \mathbb{R}^{MN \times MN}$, i.e., $\tilde{w} \sim \mathcal{CN}(0, N_0I)$; $H$ is an $n_RM \times KM$ channel matrix.

With the MIMO FDE, the output time domain signal is given by

$$\tilde{z} = D_{F_M^{-1}}A^H(I_K \otimes F_n^{-1})D_{F_M}\tilde{r} = D_{F_M^{-1}}A^H(\text{HD}_{F_M}\tilde{x} + \tilde{w}) = D_{F_M^{-1}}\tilde{z} , \quad (1)$$

where $A$ is a $KN \times KN$ equalization matrix and $H = (I_K \otimes F_n^{-1})D_{F_M}$. $\text{HD}_{F_M}(I_K \otimes F_n) \in \mathbb{C}^{KN \times KN}$; $\tilde{w} \in \mathbb{C}^{MN \times 1}$ is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $N_0I \in \mathbb{R}^{MN \times MN}$, i.e., $\tilde{w} \sim \mathcal{CN}(0, N_0I)$.

In the frequency domain, $z = A^H[Hx + w]$, where $x$ can be expressed as $x = D_{F_M}\tilde{x} = P \cdot D_{F_M}\tilde{s}$, and $\tilde{s} = [\tilde{s}_1, \cdots, \tilde{s}_K]^T$ and $\tilde{s}_i \in \{\mathcal{CN}(0, 1)\}$, $i \in \{1, \cdots, K\}$, is the user data block for the $i$th user, and $E[\tilde{s}_i^H\tilde{s}_i'] = I_N$. The power loading matrix $P \in \mathbb{R}^{KN \times KN}$ is a block diagonal matrix with its $i$th submatrix expressed as $P_i = \text{diag}([\sqrt{p_{i,1}}, \sqrt{p_{i,2}}, \cdots, \sqrt{p_{i,N}}] \in \mathbb{R}^{N \times N}$ and $p_{i,n} (i \in \{1, 2, \cdots, K\})$ is the transmitted power for the $i$th user at the $n$th subcarrier; $\tilde{s} \in \mathbb{C}^{KN \times 1}$ represents the transmitted data symbol vector from different users with $E[\tilde{s}^H\tilde{s}] = I_N$. In the frequency domain, the received signal can be expressed as

$$r = HPs + w = HPD_{F_M}\tilde{s} + w , \quad (2)$$

where $s = D_{F_M}\tilde{s}$ is the transmitted signal in the frequency domain.

We apply the FDE matrix $A$ on $r$ to obtain the equalized signal $z = A^Hr$, where $A$ in the conventional system is derived from the cost function $e = E[\|z - s\|^2] = E[\|A^Hr - s\|^2]$. Minimizing this cost function leads to the optimal matrix of $A$ as

$$A = (HP^H HP + N_0I)^{-1}HP^H . \quad (3)$$
III. IMPROVED FREQUENCY DOMAINE RECEIVER ALGORITHM

In the previous section, we investigated conventional linear MMSE receiver for the SC-FDMA based uplink MIMO system. It is optimum for systems with proper modulations, such as M-QAM and M-PSK (for which $E[s_i^*s_j^*] = 0$). However, for the improper modulation schemes, such as M-ary ASK, OQPSK (for which $E[s_i^*s_j^*] = I_{K\times N} \neq 0$), the conventional solution becomes suboptimum as will become evident later on.

In (2), let us assume $s_i \in \mathbb{C}^{N \times 1}$ is an improper signal vector, satisfying the condition $E[s_i^*s_j^*] = E[s_i^*s_j^*] = I_{K\times N}$. Since $E[s_i^*s_j^*] = D_F E[s_i^*s_j^*]D_F^H = D_F^H = I_{K\times N}, E[s_i^*s_j^*] = D_F E[s_i^*s_j^*]D_F^H = D_F^H = I_{K\times N},$ we can conclude that $s_i$ is also an improper signal vector. In order to utilize the improperness of $s$, we need to apply widely linear processing [12], [13], the principle of which is not only to process $r$, but also its conjugated version $r^*$ in order to derive the filter output, i.e.,

$$z = \zeta r + \eta r^* = \Omega^H y,$$

where $\Omega = \begin{bmatrix} \zeta & \eta \end{bmatrix}^H$ and $y = [r \ r^*]^T$. It is worth noticing that the conventional linear MMSE receiver is a special case of the one expressed by (4), when $\zeta = \Lambda^H$ and $\eta = 0$. The cost function for deriving the new filter is defined by

$$c^{WL} = E[\|\Omega^H y - s\|^2] = E[(\Omega^H y - s)(y^\Omega - s^\Omega)];$$

$$= \Omega^H C_{yy} \Omega - \Omega^H C_{ys} - C_{sy} \Omega + I_N,$$

where

$$C_{yy} = E\{yz^H\} = E\left\{\begin{bmatrix} r \\ r^* \end{bmatrix} \begin{bmatrix} r^H & r^T \end{bmatrix}\right\},$$

and

$$C_{rr} = E\{rr^H\} = E\{(HPs + w)(s^H P^H H^H + w^H)\} = HP E\{ss^H\} P^H H^H + N_0 I = HP H P^H H^H + N_0 I$$

$$C_{rt} = E\{rr^T\} = E\{(HPs + w)(s^T P^H T^H + w^T)\} = HP E\{ss^T\} P^H T^H T^H,$$

$$C_{ys} = E\{ys^H\} = E\left\{\begin{bmatrix} r \\ r^* \end{bmatrix} s^H \right\} = E\left\{\begin{bmatrix} r^* s^H \end{bmatrix} \right\} = HP E\{ss^H\} P^H E\{ss^T\} = HP D_F D_F^H P^H T^H T^H,$$

$$C_{sy} = E\{ys^T\} = E\{s^T r^H\} = E\{s^T P^H s^H\} = [P^H H^H D_F D_F^H]^*.$$

Differentiating $c^{WL}$ in (5) with respect to $\Omega$ results in $\frac{\partial c}{\partial \Omega} = (C_{yy} \Omega^*) - C_{sy}$, which is set to zero to yield the optimum vector of $\Omega$

$$\Omega = C_{yy}^{-1} C_{sy} = C_{yy}^{-1} C_{ys}$$

$$= \left[\begin{array}{cc} HP & \text{HPD}_F D_F^H P^H T^H T^H \\
H^* P (D_F D_F^H)^* & \text{HPD}_F D_F^H P^H H^H + N_0 I \end{array}\right]^{-1}$$

For the proposed FDE, the augmented autocorrelation matrix $C_{yy}$ and crosscorrelation matrix $C_{ys}$ expressed in (7) which give a complete second order description of the received signal are used for deriving the filter coefficient matrix $\Omega$; whereas for the conventional linear MMSE algorithm, the coefficient matrix $\Lambda$ is calculated using only the autocorrelation of the observation $C_{rr} = E[rr^H]$ and the crosscorrelation $C_{rs} = E[rs^H]$. The pseudo-autocorrelation $C_{rr} = E[rr^T]$ and pseudo-crosscorrelation $C_{rs} = E[rs^T]$ are implicitly assumed to be zero, leading to sub-optimum solutions. For proper signals, the improved FDE converges to the conventional FDE since $E[rr^T] = 0$ and $E[rs^H] = 0$, therefore, $C_{rr} = 0$ and $C_{sy} = \begin{bmatrix} HP \\ 0 \end{bmatrix}$ in (7). In this case, the optimum solution of $\Omega$ boils down to $\lambda$ expressed by (3).

It should be noted that the performance gains achieved by the improved FDE comes at the cost of increased computational complexity. The difference in complexity lies in the computation of the equalization matrix $\Lambda$ for the conventional FDE and the computation of $\Omega$ for the improved FDE as shown in Table I, where we show the number of multiplication ($\times$), division ($\div$), addition ($+$) and subtraction ($-$) operations to calculate the equalization matrices $\Lambda$ and $\Omega$, respectively. However, the complexity increase by the improved scheme is largely compensated by the significant performance improvement. Furthermore, this issue becomes less critical in slow-fading channels for which the equalizer matrices do not need to be updated frequently.

IV. PERFORMANCE ANALYSIS

A. SINR expression for conventional FDE

The signal vector detected at the receiver in the time domain can be expressed as

$$\tilde{z} = D_F^{-1} A^H (HD_F^* x + w) = D_F^{-1} A^H (HPs + w).$$

Let $B = A^H HP$, $A$ and $B$ can be expressed as (10) where $A_{ij} \in \mathbb{C}^{N \times N}$ is the equalization matrix between the $j$th transmit and the $i$th receive antenna. $B_{ij}$ is defined similarly. The signal vector detected at the receiver for the $i$th user, $i \in \{1, 2, \cdots, K\}$, in the time domain, can be expressed as

$$\tilde{z}_i = \sum_{j=1, j \neq i}^{K} F_N^{-1} B_{ij} F_N \tilde{s}_j + F_N^{-1} B_{ii} F_N \tilde{s}_i + \sum_{j=1}^{K} F_N^{-1} A_{ii}^H w_j.$$  

The $k$th symbol, $k \in \{1, 2, \cdots, N\}$, of $\tilde{z}_i$ can be expressed as

$$\tilde{z}_i(k) = F_N^{-1}(k,:) B_{ii} F_N(:,k) \tilde{s}_i(k)$$

$$+ \sum_{j=1, j \neq i}^{K} F_N^{-1}(k,:) B_{ij} F_N(:,j) \tilde{s}_j(j)$$

$$+ \sum_{j=1}^{K} F_N^{-1}(k,:) B_{ij} F_N \tilde{s}_j + \sum_{j=1}^{K} F_N^{-1}(k,:) A_{ii}^H w_j.$$  

The first term on the right hand side of (12) represents the desired signal, the second term is the intersymbol interference from the same substream, the third term is the interference
from the other substreams, and the fourth one is the noise. The power of the received desired signal is then
\[ P_d^i(k) = F_N^{-1}(k,:c)B_i,F_N(:,k)F_N(:,k)H_iF_N^{-1}(k,:c)H_i. \]  

(13)
The total power of the received signal can be expressed as,
\[ P_r^i(k) = \sum_{j=1}^{K} F_N^{-1}(k,:c)B_{ij}F_N^{-1}(k,:c)H_i. \]  

(14)
The power of the noise is
\[ P_n^i(k) = N_0 \sum_{j=1}^{K} F_N^{-1}(k,:c)A_{ij}H_iA_{ij}F_N^{-1}(k,:c)H_i. \]  

(15)
The received SINR for the \( k \)th symbol of the \( i \)th user is thus
\[ \gamma_{con}^i(k) = \left[ \frac{P_r^i(k) + P_n^i(k)}{P_d^i(k)} - 1 \right]^{-1}. \]  

(16)

Based on the assumption that the interference signals are completely removed, a simple upper bound on the received SINR can be formulated as \( P_r^i(k)/P_n^i(k) \), where \( P_d^i(k) \) and \( P_n^i(k) \) are given by (13) and (15), respectively.

**B. SINR expression for improved FDE**

With the improved FDE, the frequency domain signal is given by (4) as \( z = \zeta r + \eta \eta \). The corresponding time domain representation is
\[ \tilde{z} = D_{F_N}^{-1}z = D_{F_N}^{-1}(\zeta(HPs + w) + \eta(HPs + w)^\dagger) \]
\[ = D_{F_N}^{-1}(\zeta(HPD_Fs + w) + \eta(HPD_Fs + w)^\dagger). \]  

(17)
Let \( C = \zeta HP \) and \( Q = \eta HP \) and decompose \( C \) and \( Q \) into the block matrices \( C_{ij} \) and \( Q_{ij} \), respectively, in the same way as for decomposing matrix \( A \) (see (10)). The time domain received signal for the \( i \)th user is then
\[ \tilde{z}_i = \sum_{j=1, j \neq i}^{K} F_N^{-1}C_{ij}F_Ns_j + F_N^{-1}C_{ii}F_Ns_i \]
\[ + \sum_{j=1}^{K} F_N^{-1}\zeta_{ij}w_j + \sum_{j=1, j \neq i}^{K} F_N^{-1}Q_{ij}F_Ns_j^* \]
\[ + F_N^{-1}Q_{ii}F_Ns_i^* + \sum_{j=1}^{K} F_N^{-1}\eta_{ij}w_j^*. \]  

(18)
The \( k \)th symbol of \( \tilde{z}_i \) can be expressed as (19). The first term on the right hand side of (19) represents the desired signal, the second term is the intersymbol interferences from the same substream, the third term is the interference from the other substreams, and the fourth one is the noise. The power of the received desired signal is then
\[ P_d^i(k) = F_N^{-1}(k,:)C_{ii}F_N(:,k)F_N(:,k)H_i^iF_N^{-1}(k,:)H_i. \]  

(19)
The total power of the received signal can be expressed as,
\[ P_r^i(k) = \sum_{j=1}^{K} F_N^{-1}(k,:)B_{ij}F_N^{-1}(k,:)H_i^i. \]  

(20)
The power of the noise is
\[ P_n^i(k) = N_0 \sum_{j=1}^{K} F_N^{-1}(k,:)A_{ij}H_i^iA_{ij}F_N^{-1}(k,:)H_i^i. \]  

(21)
The received SINR for the \( k \)th symbol of the \( i \)th user is thus
\[ \gamma_{imp}^i(k) = \left[ \frac{P_r^i(k) + P_n^i(k)}{P_d^i(k)} - 1 \right]^{-1}. \]  

(22)

**TABLE I**

<table>
<thead>
<tr>
<th>Operations</th>
<th>( \times )</th>
<th>( \div )</th>
<th>+</th>
<th>( \cdot )</th>
</tr>
</thead>
<tbody>
<tr>
<td>For ( A )</td>
<td>( 4K^3N^3 + K^2N^2 + 2KN )</td>
<td>( 2K^2N^2 )</td>
<td>( 2K^2N^2 - KN )</td>
<td>( 2K^3N^5 )</td>
</tr>
<tr>
<td>For ( \Omega )</td>
<td>( 23K^3N^3 + K^2N^2 + 3K^2N^2 + 3KN )</td>
<td>( 8K^2N^2 )</td>
<td>( 8K^2N^2 + 2KN^2 - 7KN )</td>
<td>( 16K^3N^5 )</td>
</tr>
</tbody>
</table>

**Note that in [14], an SINR expression for SC-FDMA with a linear MMSE frequency domain receiver was derived for a single antenna case. The analysis derived in this paper is for multiple antennas and can be considered as a generalization of the one derived in [14] for the conventional receiver as well as for the newly proposed receiver for SC-FDMA systems.**
\[ \hat{z}_i(k) = \sum_{j=1, j \neq k} F_N^{-1}(k, :) \left( C_{ii} F_N(\cdot, k) \tilde{s}_i(k) + Q_{ii} F_N(\cdot, k) \tilde{s}_i(k)^* \right) \\
+ \sum_{j=1}^K F_N^{-1}(k, :) \left( C_{ii} F_N(\cdot, j) \tilde{s}_i(j) + Q_{ii} F_N(\cdot, j) \tilde{s}_i(j)^* \right) \\
+ \sum_{j=1, j \neq i} F_N^{-1}(k, :) (C_{ij} F_N \tilde{s}_j + Q_{ij} F_N^* \tilde{s}_j^*) \\
+ \sum_{j=1}^K F_N^{-1}(k, :) (\zeta_{ij} w_j + \eta_{ij} w_j^*). \] (19)

\[ P_n^i(k) = N_0 \sum_{j=1}^K F_N^{-1}(k, :) (\zeta_{ij} \tilde{s}_j^H + \eta_{ij} \eta_{ij}^H) F_N^{-1}(k, :)^H \\
+ \sum_{j=1}^K F_N^{-1}(k, :) (\eta_{ij} E[w_j^* w_j^H] \tilde{s}_j^H + \zeta_{ij} E[w_j w_j^T] \eta_{ij}^H) F_N^{-1}(k, :)^H \\
= N_0 \sum_{j=1}^K F_N^{-1}(k, :) (\zeta_{ij} \tilde{s}_j^H + \eta_{ij} \eta_{ij}^H) F_N^{-1}(k, :)^H. \] (22)

employing improper signals. The derived SINRs are used for performance comparison in this paper. They can also be used for user scheduling. In multiuser transmission systems, the scheduler at the BS can select users with large SINRs for transmission in order to improve system throughput.

V. ANALYTICAL AND SIMULATION RESULTS

We consider a 3GPP LTE baseline antenna configuration, in which two MSs are grouped together and synchronized to form a virtual MIMO channel between BS and MSs. The channel is assumed to be a six path fading channel and the channel matrix is normalized in such a way that the average channel gain for each transmitted symbol is equal to unity. The fading coefficients for each path are modeled as independent identically distributed (i.i.d) complex Gaussian samples. The block size of the user data is 12, which is also the number of subcarriers in a resource block. The size of the FFT is 256, and the length of the Cyclic Prefix (CP) is 8. The power loss incurred by the insertion of the CP is taken into account in the SNR calculation.

Fig. 3 shows the BER performance comparison between the conventional and the improved receivers for 4ASK and OQPSK systems. The improved receiver scheme significantly outperforms its conventional counterpart, especially at high SNRs. The gap can be over 10 dB. The curves for a QPSK system with the conventional receiver and a BPSK system with the improved receiver are also provided for a comparison. Although the performance of the QPSK system is superior to the 4ASK system with the conventional receiver, however, it is outperformed by the 4ASK system with the improved receiver at high SNRs. The BPSK system yields almost the same performance as the OQPSK system with the improved receiver. The rationale is that the OQPSK signal is basically two BPSK signals for which phase transitions are staggered in time by a half symbol interval. Independent decisions are made on the I and Q data bits and thus the time offset of these two channels has no effect on the decision. The curve for the BPSK system with the conventional receiver is similar to that of the QPSK system with the conventional receiver, and is not shown in the figure. Both BPSK and OQPSK belong to improper modulation, for which the improved FDE scheme can be applied. However, the use of OQPSK is preferred since it is more spectrally efficient than BPSK.

Fig. 4 shows the analytical SINR distribution of the 4ASK systems with the conventional and the improved receiver when the transmitted \( E_b/N_0 \) is equal to 10 dB. In both cases, users have equal transmit power. The curves are obtained by evaluating Eqs. (16) and (23), derived in Section IV. One can see that the SINR distribution of the 4ASK system with the improved receiver is significantly better than the
variable with variance $\sigma^2_e$. Apparently, channel estimation errors deteriorate the performance for both systems. The performance becomes worse as the error variance $\sigma^2_e$ increases. By utilizing the improperness of the signal, the system with the improved FDE is more likely to function in presence of channel estimation errors compared to the system with the conventional FDE.

VI. Conclusion

In this correspondence, we derived an improved frequency domain receiver algorithm for the SC-FDMA based uplink MIMO system with improper signal constellation. Mathematical expressions of the received SINR for the considered MIMO systems have been derived. Both simulation and analytical results reveal that the proposed scheme has a superior BER and SINR performance to the conventional linear MMSE receiver for SC-FDMA MIMO uplink systems.

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4ASK system with the conventional MMSE equalizer. Also shown in the figure are the SINR lower bounds for various systems. It can be seen that the gap between the bound and the analytical SINR for the 4ASK signaling with an improved equalizer is smaller than the one with the conventional MMSE equalizer, which implies that the interference from other users for the studied system can be effectively suppressed with the improved equalizer. Both BER and SINR performance analyzes the use of improper signals in conjunction with the proposed frequency domain receiver algorithm in LTE SC-FDMA based uplink MIMO systems.

The shown results are obtained based on the assumption of perfect channel knowledge at the receiver. The impact of channel estimation errors on the performance of the conventional and the improved equalizers is examined in Fig. 5. The transmitted $E_b/N_0$ is 30 dB and both users have equal transmit power. The imperfect channel estimate can be expressed as $H = H + E$ [15], [16], where $H$ is the original channel matrix and $E$ is the channel estimation error matrix, with each element modeled as a zero-mean complex Gaussian random variable with variance $\sigma^2_e$. Apparently, channel estimation errors deteriorate the performance for both systems. The performance becomes worse as the error variance $\sigma^2_e$ increases. By utilizing the improperness of the signal, the system with the improved FDE is more likely to function in presence of channel estimation errors compared to the system with the conventional FDE.

Fig. 4. SINR distribution for SC-FDMA uplink 2 by 2 MIMO system with conventional MMSE equalizer and the improved FDE equalizer. The users have equal transmit power.

Fig. 5. The impact of channel estimation errors on the performance of the FDEs.