

# Northumbria Research Link

Citation: Heather, Michael and Rossiter, Nick (2009) The process semantics of time and space as anticipation. In: CASYS'09: The 9th International Conference on Computing Anticipatory Systems, Symposium 10: Grammatical Cosmos II, 3-8 August 2009, Liège, Belgium .

Published by: UNSPECIFIED

URL:

This version was downloaded from Northumbria Research Link:  
<http://nrl.northumbria.ac.uk/1903/>

Northumbria University has developed Northumbria Research Link (NRL) to enable users to access the University's research output. Copyright © and moral rights for items on NRL are retained by the individual author(s) and/or other copyright owners. Single copies of full items can be reproduced, displayed or performed, and given to third parties in any format or medium for personal research or study, educational, or not-for-profit purposes without prior permission or charge, provided the authors, title and full bibliographic details are given, as well as a hyperlink and/or URL to the original metadata page. The content must not be changed in any way. Full items must not be sold commercially in any format or medium without formal permission of the copyright holder. The full policy is available online: <http://nrl.northumbria.ac.uk/policies.html>

This document may differ from the final, published version of the research and has been made available online in accordance with publisher policies. To read and/or cite from the published version of the research, please visit the publisher's website (a subscription may be required.)

[www.northumbria.ac.uk/nrl](http://www.northumbria.ac.uk/nrl)



# The Process Semantics of Time and Space as Anticipation

Michael Heather

Nick Rossiter

`nick.rossiter@unn.ac.uk`

University of Northumbria

`http://computing.unn.ac.uk/staff/CGNR1/`

# Outline

- Relationships in Category Theory
  - Pivotal Role of Adjointness
  - Intension-Extension
  - Static/Dynamic
  - All a Question of Typing
- Natural Composition of Systems
  - Godement
  - Satisfy Complex Requirements
- Time-Space and other Examples
  - Relevance to Anticipation

# Purpose

- To attempt to show that the natural relationships declared categorically can satisfy those needed in the real world and provide a basis for anticipation

# Relationships Dominate Category Theory

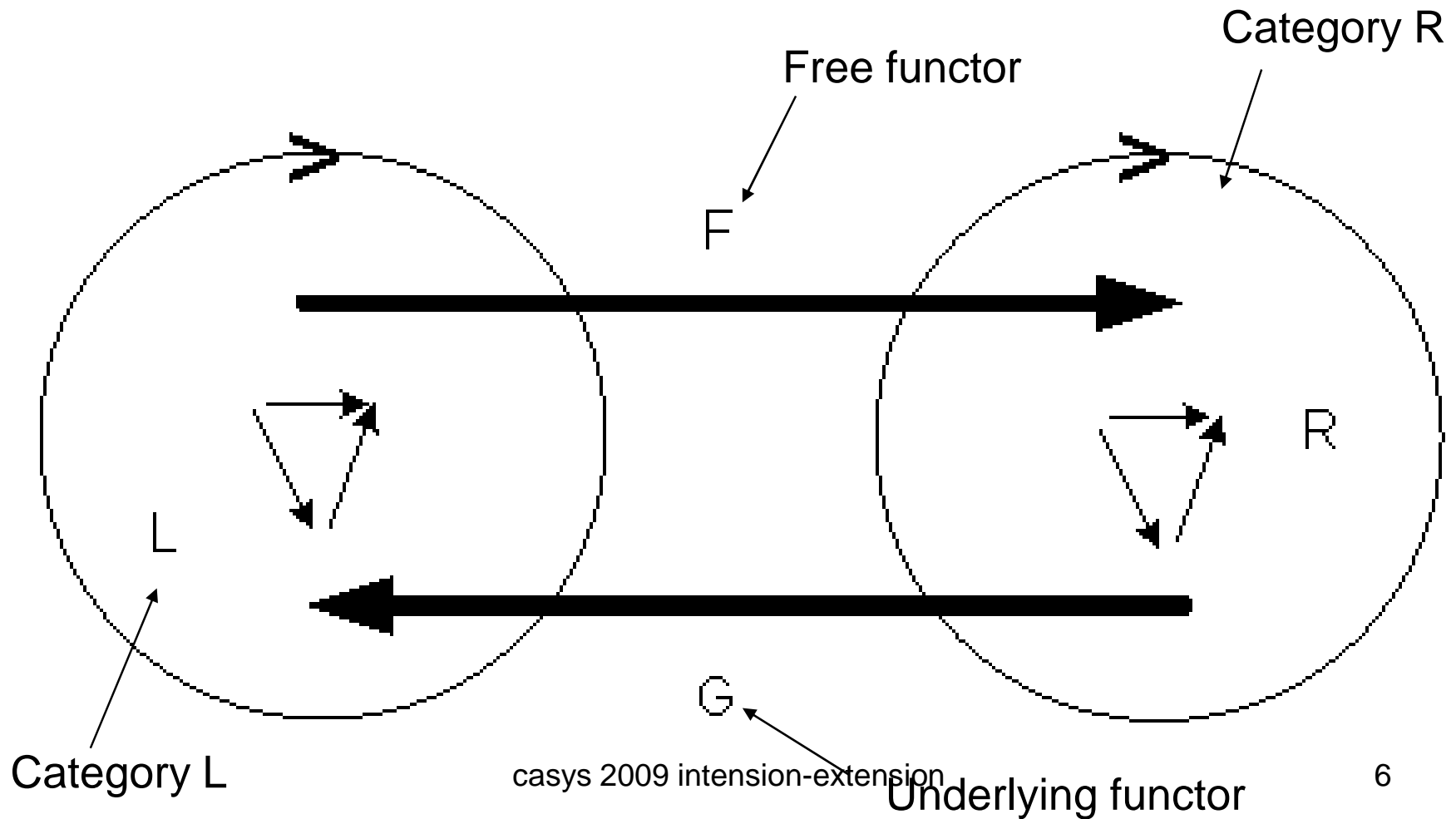
- Categories
  - Cartesian closed – products
  - Locally cartesian closed – pullbacks/comma/slice
  - Intra-category
- Functors
  - Map from one category to another
  - Inter-category
- Natural Transformations
  - Map from one functor to another
  - Inter-functor

# Adjointness

- Perhaps most important relationship is that of adjointness
- Discovered by Kan
- Elaborated by Lawvere
- Provides a more general type of relationship than equivalence
  - Suited to real-world where relationships are not always so simple

# Inter-relationship between two Categories L, R through Functors F,G

If adjointness holds, we write  $F \dashv G$

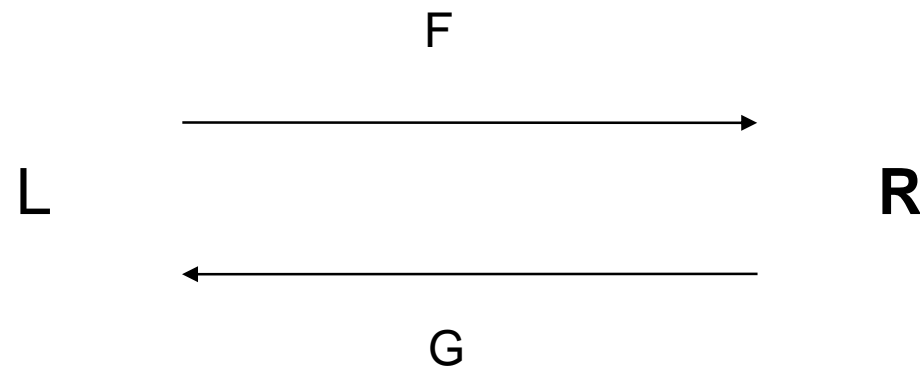


# Features of Adjointness $F \dashv G$

- Free functor (F) provides openness
- Underlying functor (G) enforces rules
- Natural so one (unique) solution
- Special case
  - $GF(L)$  is the same as  $L$       AND
  - $FG(R)$  is the same as  $R$
  - Equivalence relation
- Adjointness in general is a relationship less strict than equivalence
  - $1_L \leq GF$  if and only if  $FG \leq 1_R$



# Example of Adjointness



- If conditions hold, then we can write the adjunction  $F \dashv G$
- The adjunction is represented by a 4-tuple:
  - $\langle F, G, \eta, \varepsilon \rangle$
- $\eta$  and  $\varepsilon$  are unit and counit respectively
  - $\eta : L \rightarrow GF L$ ;  $\varepsilon : FGR \rightarrow R$
  - Measure displacement in mapping on one cycle

# Uses of Adjointness

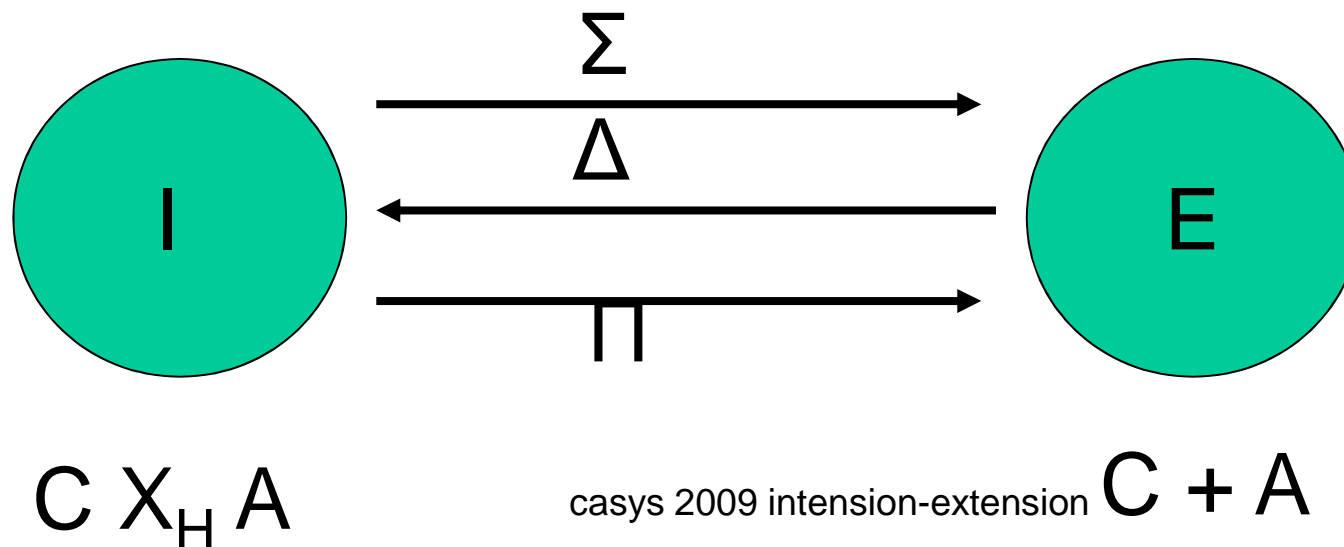
- Representing intension-extension
- Intension-extension is critical for representing information systems
  - Goes back to Port Royal Logic
- Intension is definition of a system
  - The permanent part that does not change
- Extension is time-varying part of a system
  - The time-varying part that is in constant flux

# Example 1

- Banking System
- Intension is definition of structures, rules and procedures that specify how the bank operates; can be a preorder with cycles
- Extension is the data values for the banking operation at a particular time; a partial order

# Consider one relationship

- Customer (C) : account (A) in context of holds (H)
- For Intension (I) - Extension (E) define 2 categories and 3 functors:



# Definitions

- $C \times_H A$  is the relationship  $C \times A$  in the context of  $H$
- $C$  is customer,  $A$  is account,  $H$  is holds
- $C + A$  is all possible values for pairs of  $C, A$ ; by convention written this way but also includes other pools of possible data values (data soup, data types)

# Functors pair 1

- $\Sigma$  selects values that exist for  $C, A$  in  $E$ 
  - Free functor as performing choice
- $\Delta$  takes values that exist for relationship between  $C, A$  in  $E$  and checks conformity with definition in  $I$  for  $C X_H A$ 
  - Underlying functor as checking a rule
- If  $\Sigma$  and  $\Delta$  are adjoint, we write:

$$\Sigma \dashv \Delta$$

# Functors pair 2

- $\Delta$  takes values that exist for relationship between  $C$ ,  $A$  in  $E$  and checks conformity with definition in  $I$  for  $C$   
 $X_H A$ 
  - Because many contexts may exist it is now a free functor as selecting a role (viewpoint is now free)
  - Could have other contexts  $H'$ ,  $H''$  ...
  - Note no use of number (Gödel !)
- $\Pi$  selects values that exist for  $CX_H A$  in  $E$ 
  - Underlying functor as checking that values selected in  $E$  match the type definition
- If both pairs of adjoints hold, we write:

$$\Sigma \dashv \Delta \dashv \Pi$$

# Simultaneity

- In all categorical constructions
  - There is no sequence in the composition
  - The whole structure is evaluated simultaneously (snapped)

- The I-E relationship is an arrow

$$\Sigma \dashv \Delta \dashv \Pi$$

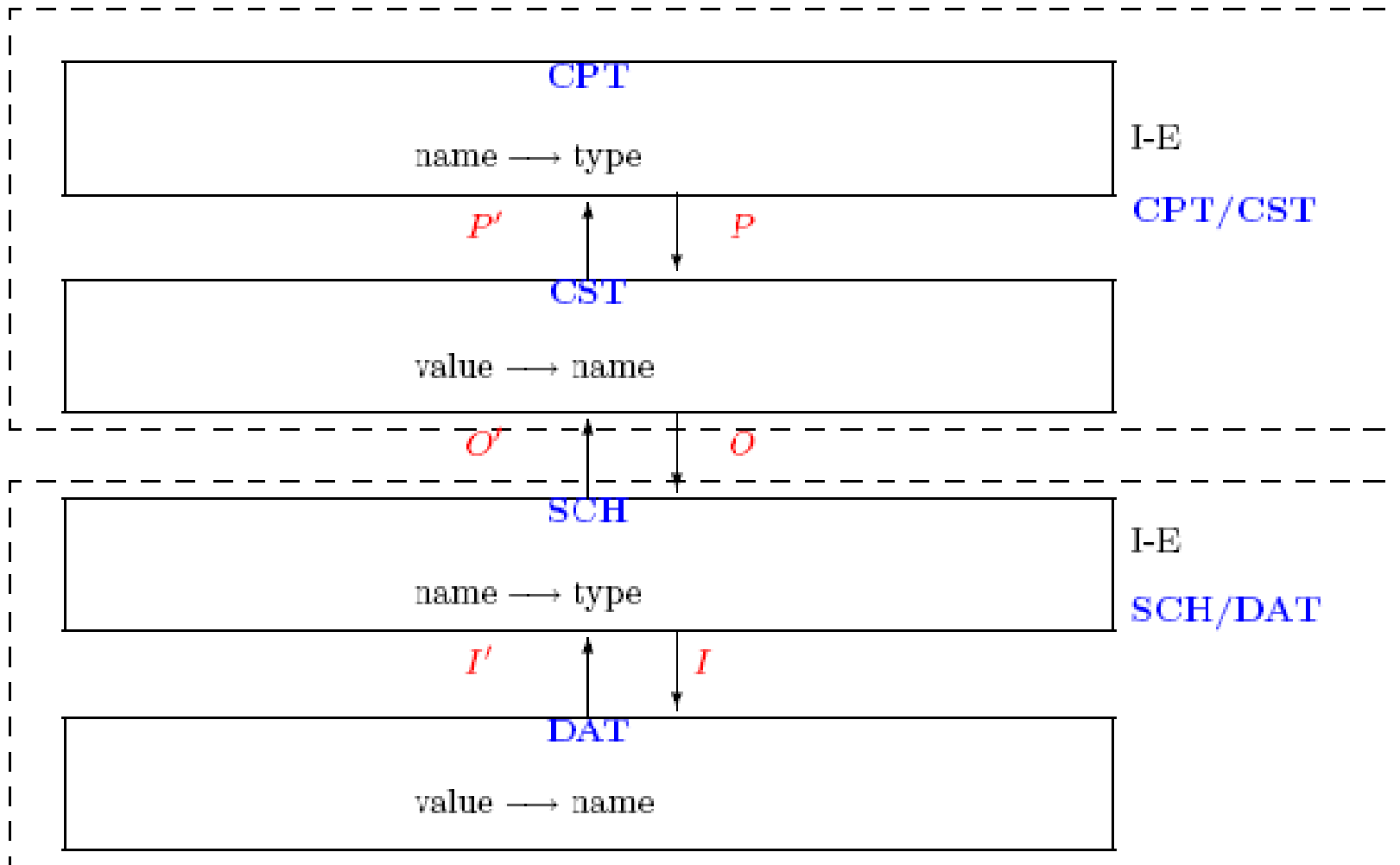
and not any of the categories or functors on their own



# Can Build up Relations

- Composition of arrows is natural
  - Godement calculus
- As I-E relation is an arrow
- Can compose one I-E relation with another
- Can build up complex levels of types and definitions with flexible meta levels

black - objects



Defining Four Levels of Data Typing with Contravariant  
Functors and Intension-Extension (I-E) Pairs

casys 2009 intension-extension

# Composition of I-E pairs

- Higher I-E pair becomes  $(\Sigma \dashv \Delta \dashv \Pi)'$
- Lower I-E pair becomes  $(\Sigma \dashv \Delta \dashv \Pi)$
- Then top-down is

$$(\Sigma \dashv \Delta \dashv \Pi) \circ (\Sigma \dashv \Delta \dashv \Pi)'$$

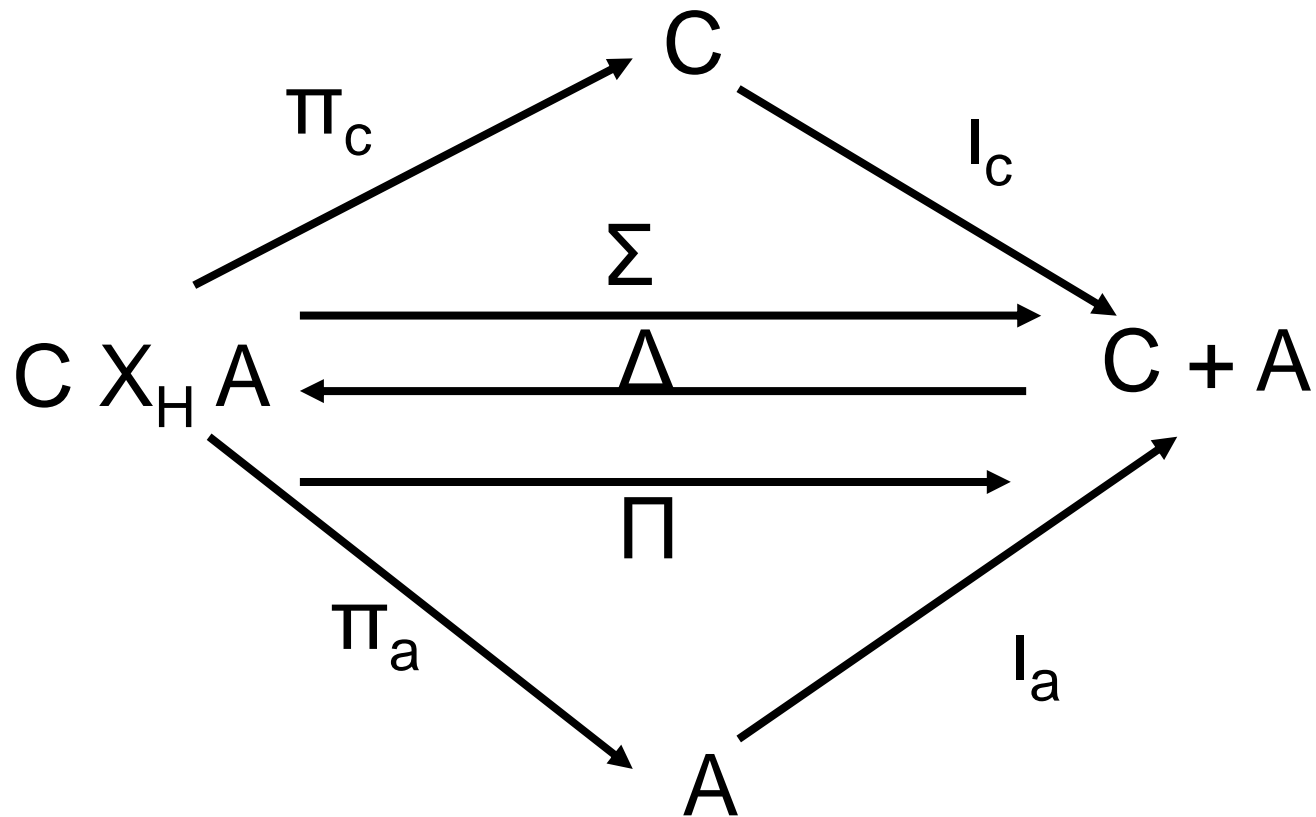
and bottom-up is

$$(\Sigma \dashv \Delta \dashv \Pi)' \circ (\Sigma \dashv \Delta \dashv \Pi)$$

# Form of Underlying Categories

- Might represent data structure (static)
  - Pullbacks/ pushouts
  - Comma/ slice categories
- Might represent process (dynamic)
  - Monads/comonads

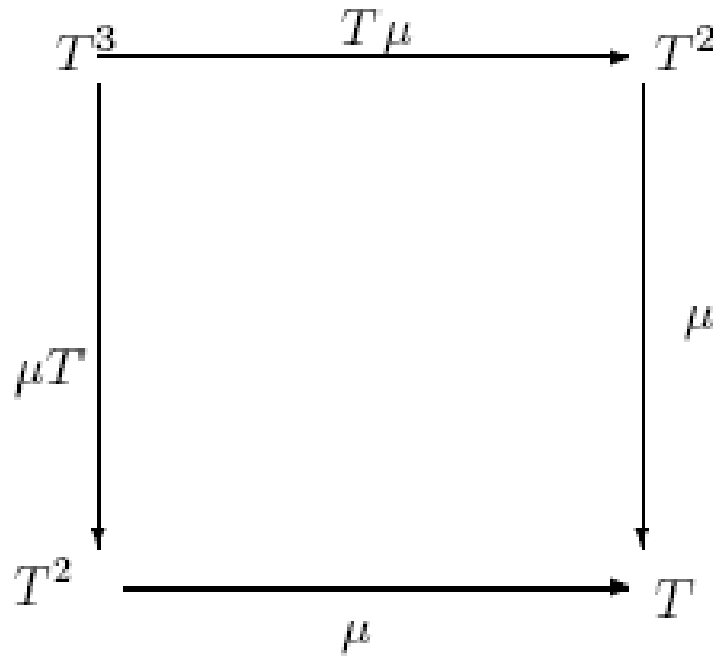
# Pullback



# Pullback versus I-E

- Saying much the same thing
- Pullback useful as a descriptive diagram
- I-E is more useful algebraically as it:
  - Spells out the exact nature of the relationship
  - Is an arrow which can be composed with other categorical arrows

# Monad



Transaction (ACID):

$T$  is one cycle

$T^2$  is two cycles

$T^3$  is three cycles

$T$  is an endofunctor,

can be an adjoint:

$GF$  or  $\Sigma \dashv \Delta \dashv \Pi$ ,

latter as 2 pairs

strictly

# Monad

- Monad is an abstract concept:

$$\text{Monad} = \langle T, \eta, \mu \rangle$$

Where  $T$  is the endofunctor -- endofunctor is functor with same source and target (often an adjoint)

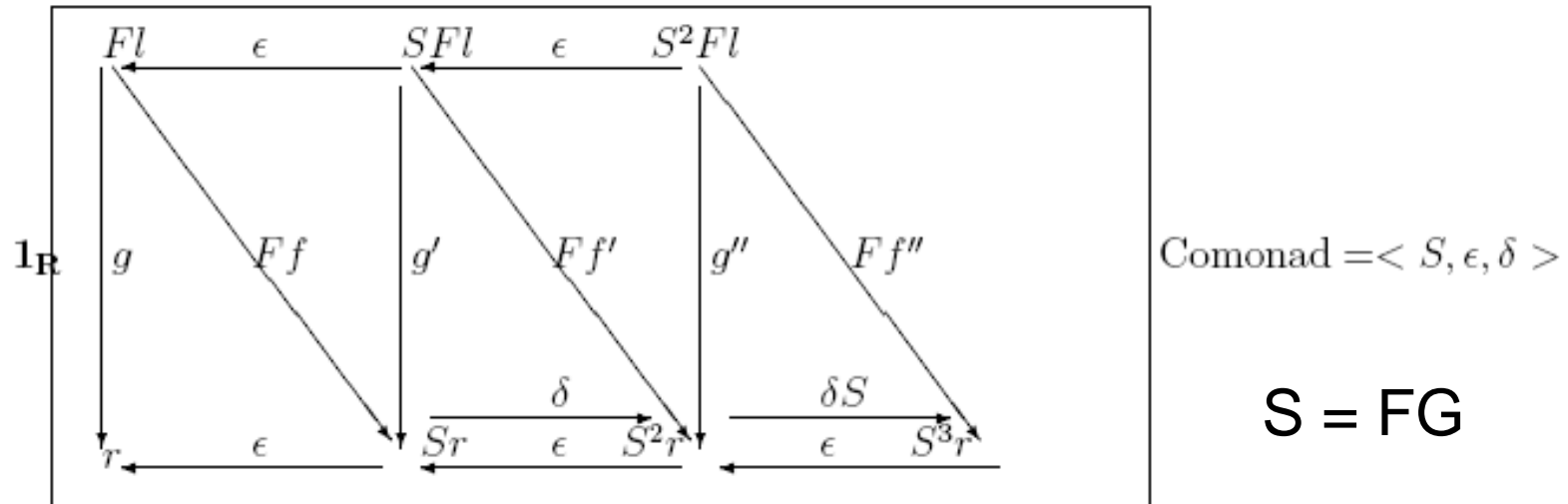
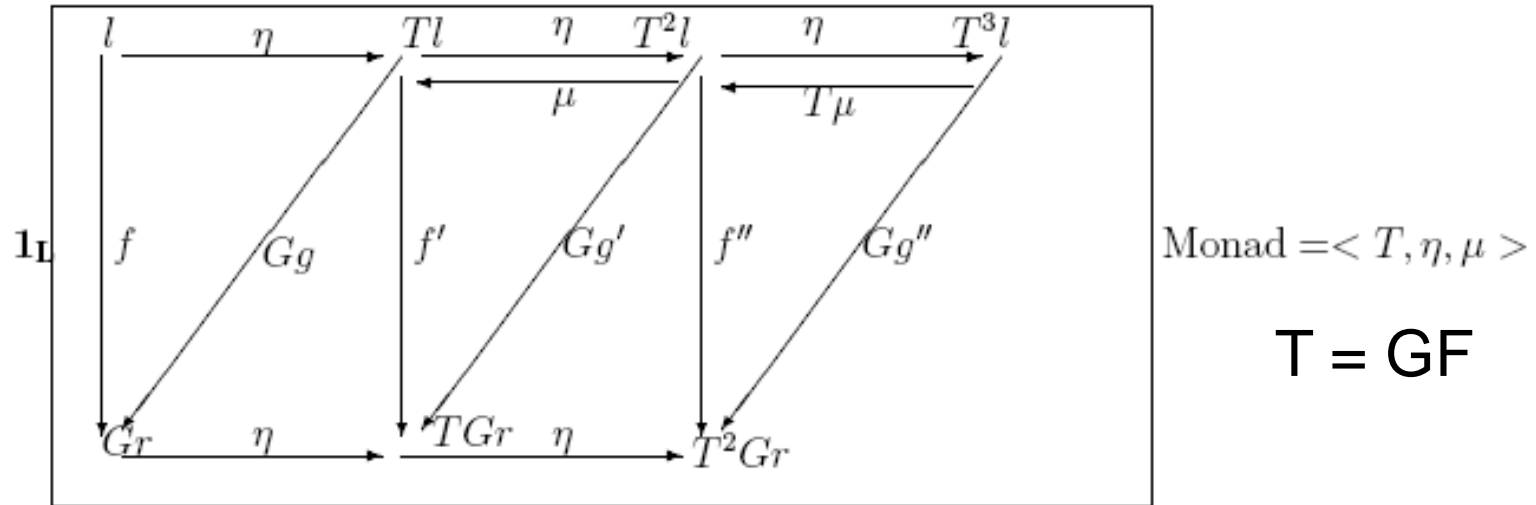
$\eta$  is the unit of adjunction: change in  $L$  on one cycle

$\mu$  is the multiplication: change between  $T^2$  and  $T$  (on 2<sup>nd</sup> cycle looking back)

But can express in more detail:



# Adjointness between Monad/Comonad



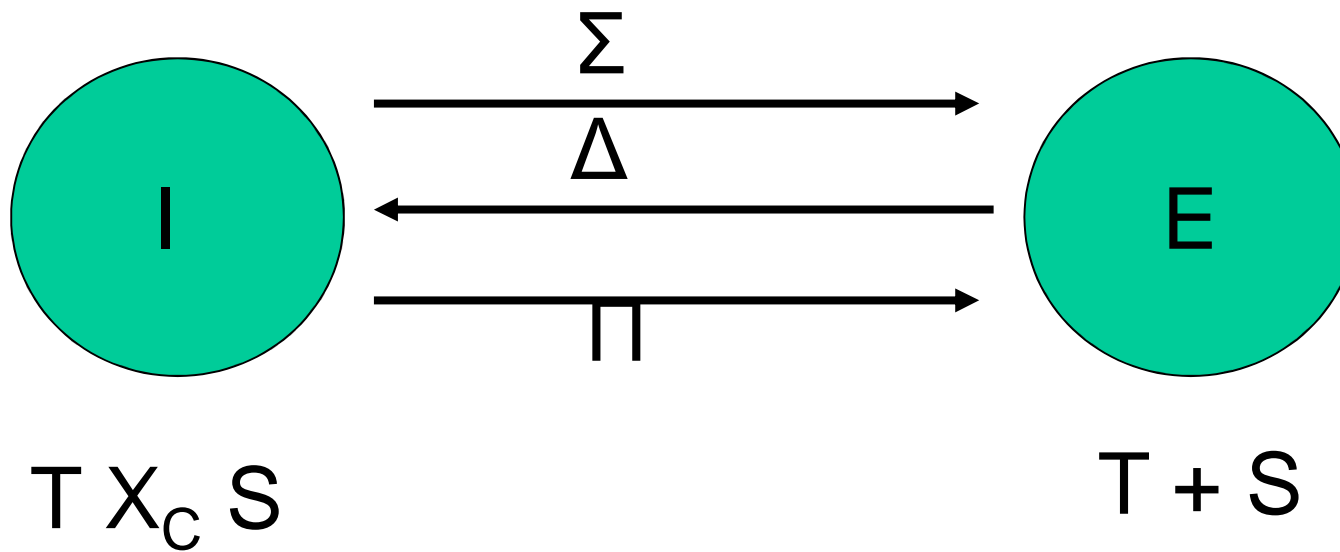
# Comonad

- Comonad is dual of Monad
- Comonad =  $\langle S, \varepsilon, \delta \rangle$
- Where  $S$  is the endofunctor (often an adjoint)
- $\varepsilon$  is the counit of adjunction, measuring change in  $R$  on one cycle
- $\delta$  is the comultiplication, measuring change between  $S$  and  $S^2$  (looking forward)

# Simultaneity

- Monad/comonad are not handled in a sequential fashion
- Cycles are simultaneous
- Structure satisfying the rules is snapped

# Time/Space



# Time/Space as I-E

- $T \& S$  is in the intension
- $T \parallel S$  is in the extension
- $\Sigma \dashv \Delta \dashv \Pi$  gives the relationship for a particular context  $C$  between:

$$T \& S \text{ and } T \parallel S$$

where  $T \& S$  is the invariant intension (I) and  $T \parallel S$  is the time-varying extension (E)

Relativistic:  $\eta$ ,  $\varepsilon$  are significant; classical: maybe not so.

# Anticipation 1

- So is anticipation the comultiplication
  - $\delta: S \rightarrow S^2$
  - taking the comonad forward one cycle
- This is a tempting conclusion
- But it is not so simple
- Anticipation is not one arrow on its own
- Need to consider the full context

# Anticipation 2

- Could better be viewed as looking forward:

$$\delta: S \rightarrow S^2 (FG \rightarrow FGFG)$$

in the context of the monad/comonad adjointness, in particular of the arrow looking back:

$$\mu: T^2 \rightarrow T (GFGF \rightarrow GF)$$