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The Natural Metaphysics of Computing Anticipatory Systems

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Outline

- Systems theory
 - Pivotal Role of Adjointness
 - Rosen's influence
 - Free and Open Systems
- Composition of Systems for Complexity
 - Godement
 - Cube, Adjunctions
- Anticipation as Structural Ordering

Purpose

- To attempt to show that the natural relationship between category theory and systems provides the basis for a metaphysical approach to anticipation

Systems Theory

- Important for Information Systems
- Challenging Areas
 - pandemics
 - prediction of earthquakes
 - world finance (credit crunch)
 - world energy management policy
 - climate change
- Globalisation
- Freeness and Openness needed

Features of Dynamic Systems

- Natural entities
 - easier to recognise than to define
- Second-order Cybernetics
 - observer is part of the system
 - distinguish between
 - modelling components/components of system itself
- General Information Theory (Klir)
 - handling uncertainty
- Theory of Categories (Rosen)

System Theory

- Basic concepts
 - internal connectivity of components
 - Plato (government institution)
 - Aristotle (literary composition)
 - von Bertalanffy
 - theory of categories (vernacular)
 - to be replaced by an exact system of logico-mathematical laws.

Complexity of System

- System is a model of a whole entity
 - hierarchical structure
 - emergent properties
 - communication
 - control (Checkland)
- Complexity -- openness and freeness
 - self-organisation
 - anticipation (Dubois, Klir)
 - global interoperability

Key Elements in the Definition of a System

system	natural relationship	locality
closed	<i>intra-connectivity</i>	local
open	<i>inter-connectivity</i>	local
self-organised	<i>intra-activity</i>	non-local
free	<i>inter-activity</i>	non-local

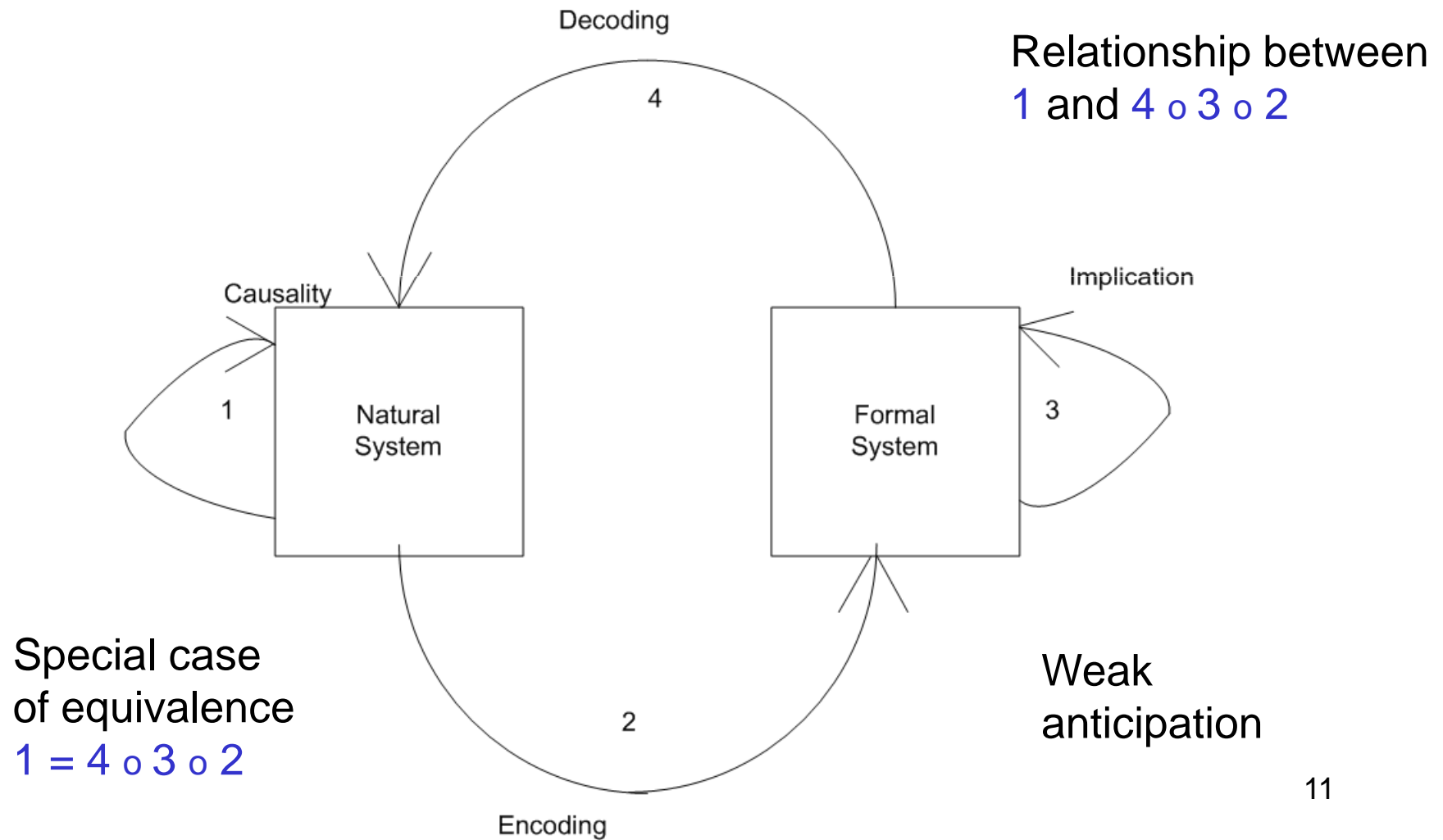
Concept of Openness

- Open
 - defined inductively on open interval -- difficult to formalise
- Dedekind cut
 - section of pre-defined field -- local
- Topology
 - \mathfrak{S} -open
 - system is open to its environment
 - intuitionistic logic
 - Limited by reliance on set theory

Category of Systems

- To make formal
 - intraconnectivity
 - interconnectivity
 - intra-activity
 - Interactivity
- Theory is realisable -- constructive
- Work on process -- Whitehead

Early Adjointness from Rosen

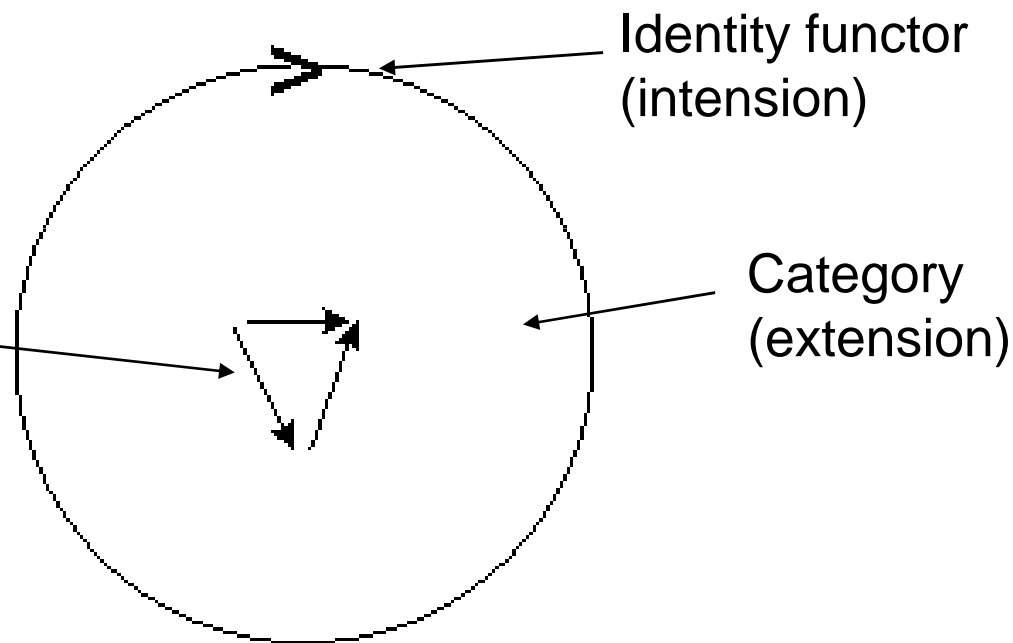


Identity Functor as Intension of Category-System

Cartesian closed
category

Curvilinear
polygon

Intraconnectivity



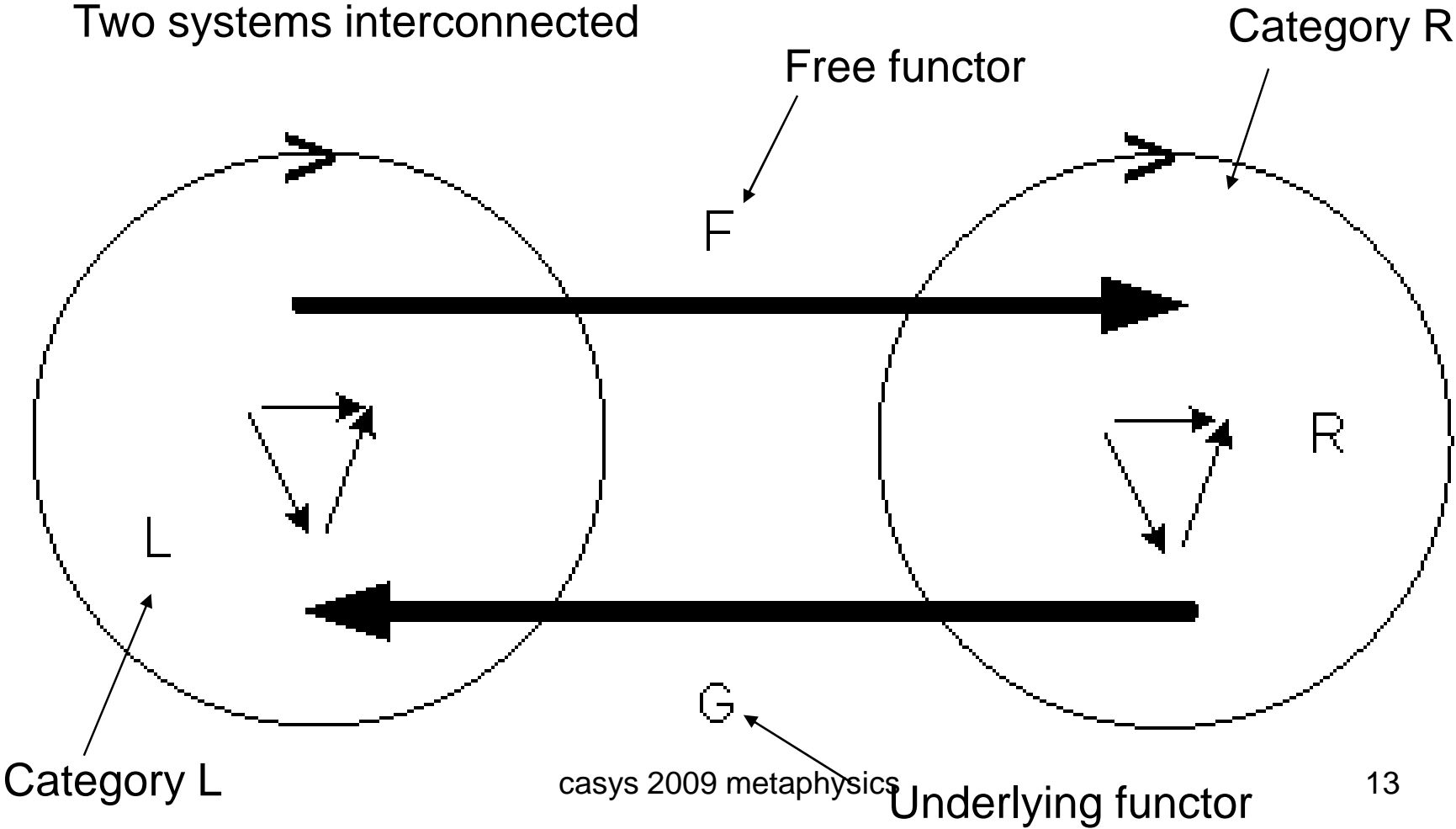
System is one large arrow (process)

Identity functor is intension

All internal arrows are extension

casys-2009 metaphysics

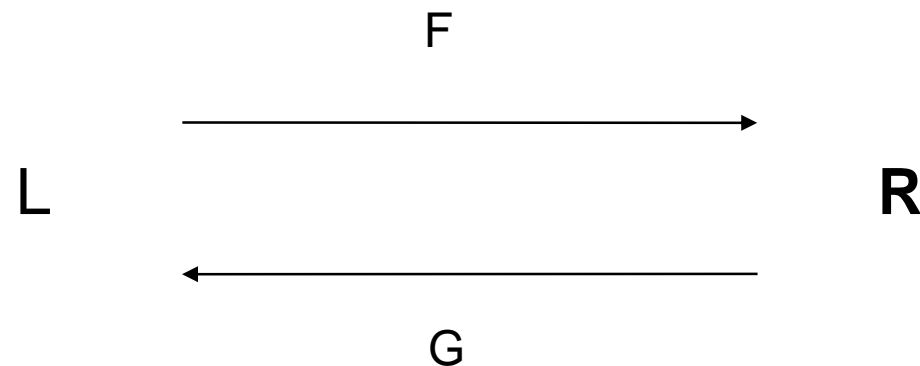
Interconnectivity between two Identity Functors leading to Interactivity between Category-Systems.



Features of Adjointness $F \dashv G$

- Free functor (F) provides openness
- Underlying functor (G) enforces rules
- Natural so one (unique) solution
- Special case
 - $GF(L)$ is the same as L AND
 - $FG(R)$ is the same as R
 - Equivalence relation
- Adjointness in general is a relationship less strict than equivalence
 - $1_L \leq GF$ if and only if $FG \leq 1_R$

Example of Adjointness



- If conditions hold, then we can write $F \dashv G$
- The adjunction is represented by a 4-tuple:
 - $\langle F, G, \eta, \varepsilon \rangle$
- η and ε are unit and counit respectively
 - $\eta : L \rightarrow GF$; $\varepsilon : FG \rightarrow R$
 - Measure displacement in mapping on one cycle
- L, R are categories; F, G are functors

Category-Systems

- Makes formal
 - intraconnectivity identity functor
 - interconnectivity functors
 - intra-activity self-organisation (L and R
are indistinguishable)
 - interactivity adjointness
- Right-hand category-system R
 - free and open category system
 - freedom from free functor F
 - determination by underlying functor G

Anticipation

- Not simply F
 - While this takes process one step forward
 - On its own it lacks context
- Not simply G
 - This appears to take the process backwards
- It's $F \dashv\vdash G$, that is F in the context of G
 - The forward step as limited by G

Composition of Systems for Handling Complexity

- With category-systems
 - Composition is natural
 - Godement calculus
 - Compose all arrows at whatever level they are defined
 - Categories, Functors, Natural transformations, Adjoints
 - Obtain expressions showing equality of paths

The Cube – Composing All Arrows

- Higher dimensional arrows can deconstruct higher dimensional spaces into simple one dimensional paths
- Power of category theory:
 - Abstract (compact) notation can also be represented in an equivalent more detailed notation.
 - Both are robust
 - One more suited to description, the other to implementation.

Abstract Notation

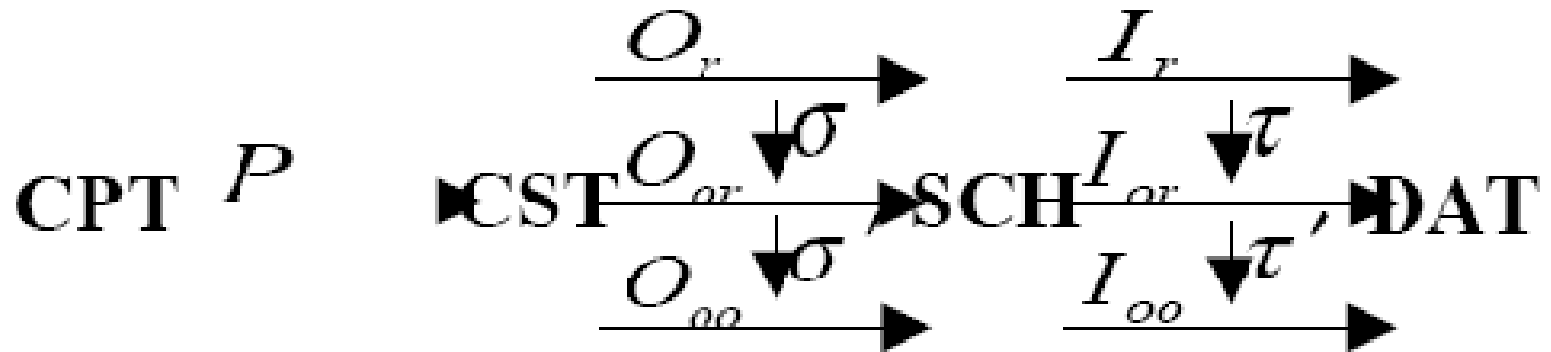
Comparison of four systems (categories):

CPT, CST, SCH, DAT

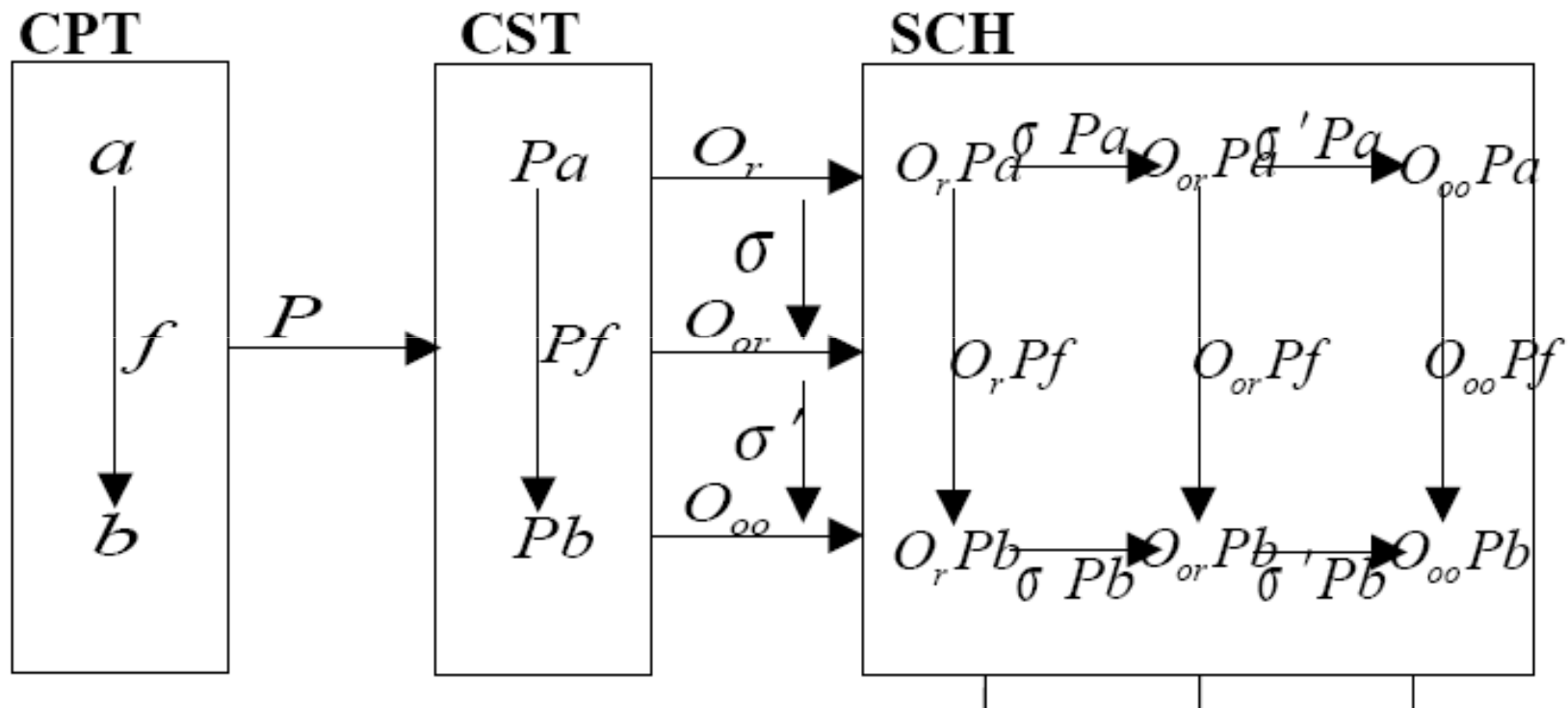
by functors P, O, I

O, I have variants

σ, τ compare variations (natural transformations)

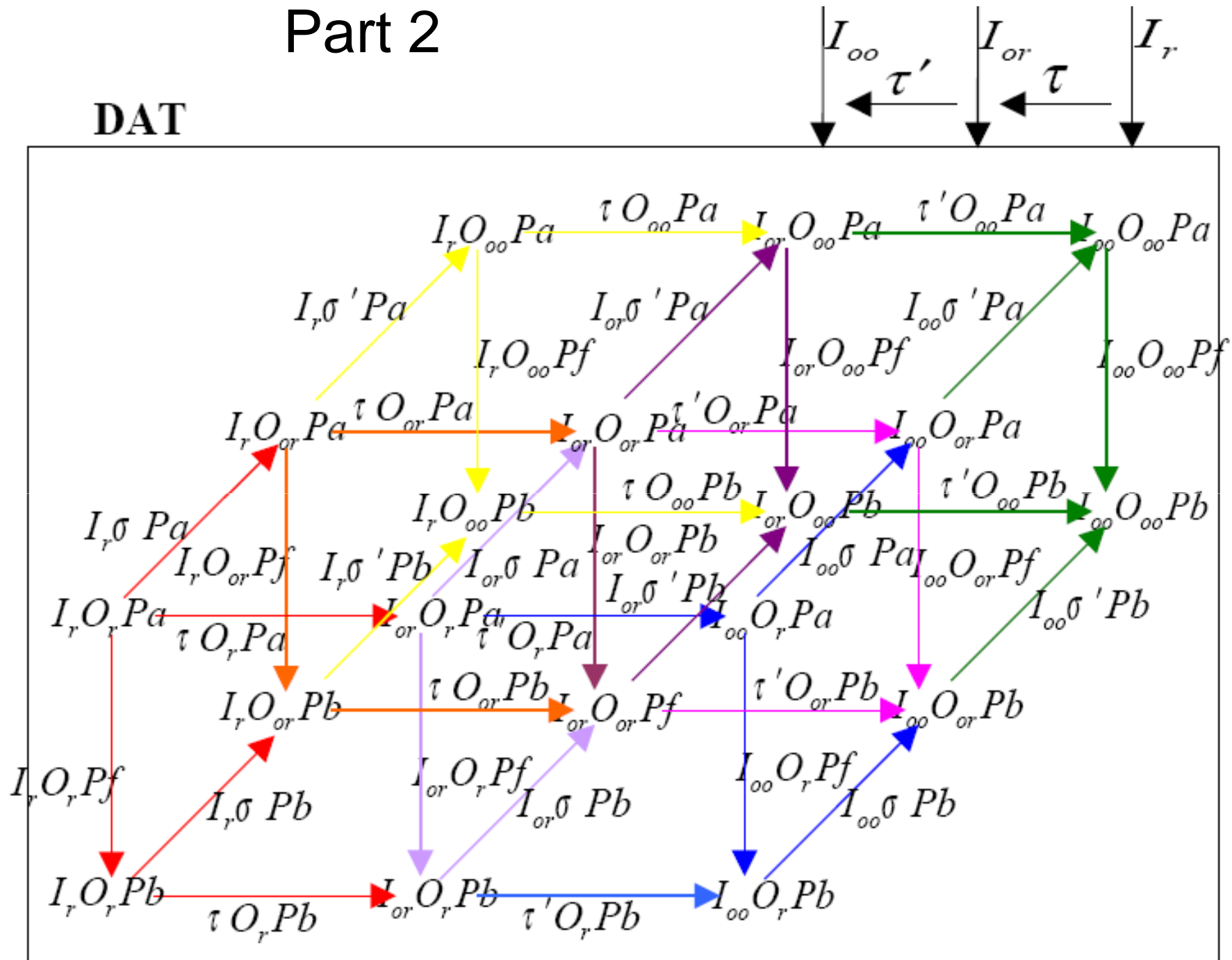


Detailed notation – The Cube – Part 1

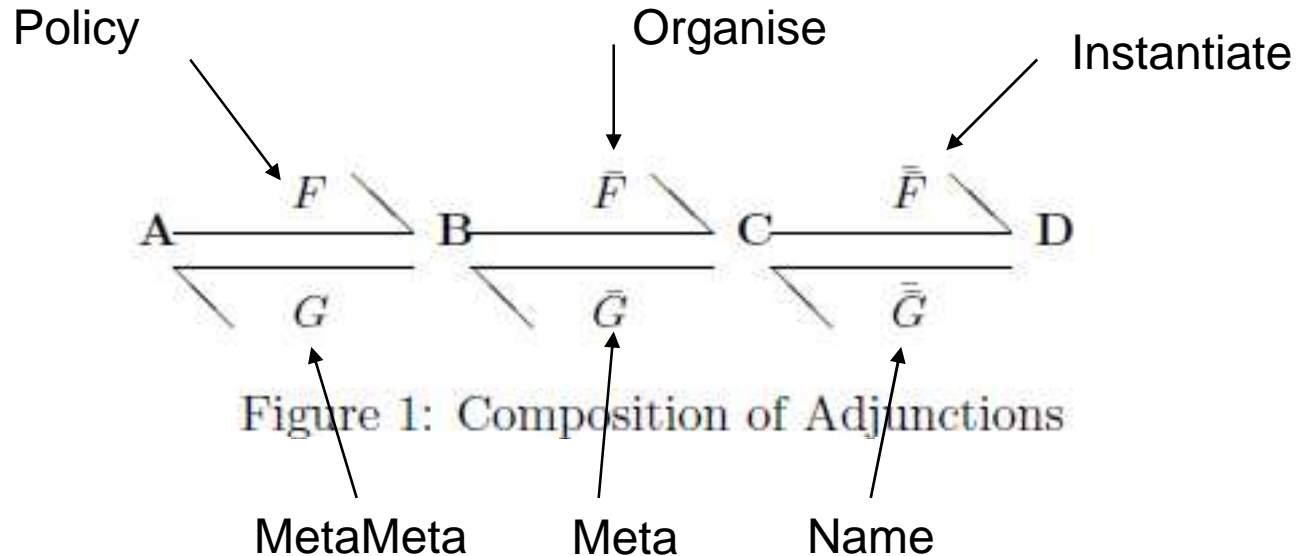


Part 2

DAT



Composing Adjunctions



A is category for Concepts
 B is category for Constructs
 C is category for Schema
 D is category for Data

Example for data structures

Adjunctions compose naturally
 F-|G is one of 6 adjunctions (if they hold)

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Six Possible Adjunctions

Simple

$$F \dashv G$$

$$\overline{F} - \mid \overline{G}$$

$$\overline{\overline{F}} - \mid \overline{\overline{G}}$$

Pairs

$$\overline{F} F - \mid G \overline{G}$$

$$\overline{\overline{F}} \overline{F} - \mid \overline{G} \overline{\overline{G}}$$

Triples

$$\overline{\overline{F}} \overline{F} F - \mid G \overline{G} \overline{\overline{G}}$$

Simple Adjunctions

Not composed

We can define these in more detail with their units and counits of adjunction as follows:

$$\langle F, G, \eta_a, \epsilon_b \rangle: \mathbf{A} \longrightarrow \mathbf{B} \quad (1)$$

η_a is the unit of adjunction $1_a \longrightarrow GFa$ and ϵ_b is the counit of adjunction $FGb \longrightarrow 1_b$

$$\langle \bar{F}, \bar{G}, \bar{\eta}_b, \bar{\epsilon}_c \rangle: \mathbf{B} \longrightarrow \mathbf{C} \quad (2)$$

$\bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{F}b$ and $\bar{\epsilon}_c$ is the counit of adjunction $\bar{F}\bar{G}c \longrightarrow 1_c$

$$\langle \bar{\bar{F}}, \bar{\bar{G}}, \bar{\bar{\eta}}_c, \bar{\bar{\epsilon}}_d \rangle: \mathbf{C} \longrightarrow \mathbf{D} \quad (3)$$

$\bar{\bar{\eta}}_c$ is the unit of adjunction $1_c \longrightarrow \bar{\bar{G}}\bar{\bar{F}}c$ and $\bar{\bar{\epsilon}}_d$ is the counit of adjunction $\bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$

Composition of Adjunction Pairs

$$\langle \bar{F}F, G\bar{G}, G\bar{\eta}_a F \bullet \eta_a, \bar{\epsilon}_c \bullet \bar{F}\epsilon_c \bar{G} \rangle : \mathbf{A} \longrightarrow \mathbf{C} \quad (4)$$

$G\bar{\eta}_a F \bullet \eta_a$ is the unit of adjunction $1_a \longrightarrow G\bar{G}\bar{F}F a$ and $\bar{\epsilon}_c \bullet \bar{F}\epsilon_c \bar{G}$ is the counit of adjunction $\bar{F}F G\bar{G}c \longrightarrow 1_c$

The unit of adjunction is a composition of $\eta_a : 1_a \longrightarrow GF a$ with $G\bar{\eta}_a F : GF a \longrightarrow G\bar{G}\bar{F}F a$

The counit of adjunction is a composition of $\bar{F}\epsilon_c \bar{G} : \bar{F}F G\bar{G}c \longrightarrow \bar{F}\bar{G}c$ with $\bar{\epsilon}_c : \bar{F}\bar{G}c \longrightarrow 1_c$

We have retained the symbol \bullet indicating vertical composition as distinct from normal horizontal composition indicated by the symbol \circ [13].

$\bar{G}\bar{\eta}_b \bar{F} \bullet \bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{G}\bar{F}\bar{F}B$ and $\bar{\epsilon}_d \bullet \bar{F}\bar{\epsilon}_d \bar{G}$ is the counit of adjunction $\bar{F}\bar{F}\bar{G}\bar{G}d \longrightarrow 1_d$

The unit of adjunction is a composition of $\bar{\eta}_b : 1_b \longrightarrow \bar{G}\bar{F}b$ with $\bar{G}\bar{\eta}_b \bar{F} : \bar{G}\bar{F}b \longrightarrow \bar{G}\bar{G}\bar{F}\bar{F}b$

The counit of adjunction is a composition of $\bar{F}\bar{\epsilon}_d \bar{G} : \bar{F}\bar{F}\bar{G}\bar{G}d \longrightarrow \bar{F}\bar{G}d$ with $\bar{\epsilon}_d : \bar{F}\bar{G}d \longrightarrow 1_d$.

Composition of Adjunctions Triples

$$\langle \bar{F}\bar{F}F, G\bar{G}\bar{G}, G\bar{G}\bar{\eta}_a\bar{F}F \bullet G\bar{\eta}_aF \bullet \eta_a, \bar{\epsilon}_d \bullet \bar{F}\bar{\epsilon}_d\bar{G} \bullet \bar{F}\bar{F}\epsilon_d\bar{G}\bar{G} \rangle: \mathbf{A} \longrightarrow \mathbf{D} \quad (6)$$

The unit of adjunction is a composition of:

$$\eta_a : 1_a \longrightarrow GFa \text{ with } G\bar{\eta}_aF : GFa \longrightarrow G\bar{G}\bar{F}Fa \text{ with } G\bar{G}\bar{\eta}_a\bar{F}F : G\bar{G}\bar{F}Fa \longrightarrow G\bar{G}\bar{G}\bar{F}\bar{F}Fa$$

The counit of adjunction is a composition of:

$$\bar{F}\bar{F}\epsilon_d\bar{G}\bar{G} : \bar{F}\bar{F}F\bar{G}\bar{G}d \longrightarrow \bar{F}\bar{F}\bar{G}\bar{G}d \text{ with } \bar{F}\bar{\epsilon}_d\bar{G} : \bar{F}\bar{F}\bar{G}\bar{G}d \longrightarrow \bar{F}\bar{G}d \text{ with } \bar{\epsilon}_d : \bar{F}\bar{G}d \longrightarrow 1_d$$

General Solution

- In general, adjointness gives a logical ordering:
 - iff the operation of an environment C on a subobject A has a solution subobject B then
 - A implies B in the environment of C .
 - This can be represented as the adjunction

$$C \times A \rightarrow B \dashv C \rightarrow B^A$$

Metaphysical Ordering

- This adjunction is the natural metaphysical ordering which constitutes **anticipation**
- Thus causation (left adjoint) and Heyting inference (right adjoint) are both stationary forms of the predicate of anticipatory systems
 - these dominate the two mainstream applications of AI and databases
- In AI the left adjoint is a relevance connection in context and the corresponding right adjoint is cognition
- For data warehousing, data mining, the semantic web, etc, a query in context is left adjoint and the resultant retrieval right adjoint

Acknowledgements

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