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# An analysis of disability-linked annuities

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**Abstract**

In this paper we investigate a special type of annuity where the annuity is issued to a policyholder who is in reasonable health at the outset; however, if the policyholder subsequently becomes disabled then the annuity payments are increased to a higher level depending on the level of disability.

We analyse different types of disability-linked annuities, both single life and last survivor, in order to examine their main characteristics and assess their suitability as potential new products in the annuity and long term care (LTC) market.

Initially, we use a central set of assumptions - in particular, incorporating the central set of morbidity trend assumptions used by Rickayzen and Walsh (2002) - to calculate various quantities. These include the expected times spent in different states of disability, the probabilities of each level of annuity enhancement eventually being triggered and the single premiums required. We then use pessimistic and optimistic assumptions to replicate the expansion and compression of morbidity theories, respectively, to provide a sensitivity analysis. In addition, we consider the effect of widening the definition of disability used for the product to make it more appealing to consumers. We also examine the underwriting considerations by calculating the theoretical premium which should be charged if a moderately disabled individual were to purchase the product. Finally, we estimate the potential market in the UK for this group of products.

Our main conclusion is that when we compare the disability-linked annuity with the corresponding traditional whole life annuity, the increase in premium involved is relatively modest. The reasons for this are twofold. Firstly, the mortality and morbidity risks act in opposite directions. Therefore, an individual receiving an annuity enhancement due to severe disability will also face shorter life expectancy. Secondly, periods of disability are likely to occur towards the end of life. Therefore, the value of the annuity enhancements is relatively small since they are discounted so heavily. For similar reasons, we find that moving to optimistic or pessimistic morbidity assumptions has relatively little effect on the premiums. Finally, we note that the product is only likely to be affordable to the more affluent sector of the UK elderly population; nevertheless, we estimate that there is a considerable potential market for a product which appears to have many attractive features.

## 1. Introduction

As in many other developed countries around the world, the UK population has been ageing over recent decades, and is expected to continue to do so in the future. For example, the proportion of the population over age 65 is expected to increase between 2000 and 2030 from 16.0% to 23.1% (Karlsson et al, 2005). With an ageing population, an increasing number of people need to consider how they will meet their standard of living requirements in retirement both in good health, and in poor health when they might require long term care (LTC).

In this paper we investigate a special type of annuity where the annuity is issued to a policyholder who is in reasonable health at the outset; however, if the policyholder subsequently becomes disabled then the annuity payments are increased to a higher level depending on the level of disability.

Throughout this paper we will refer to such an annuity as a "disability-linked annuity". Clearly, the definition of "disabled" is important for this product and will be discussed in detail in the body of the paper. Since the highest level of annuity payment will be reached when the individual requires LTC, the annuity is sometimes referred to as being an integrated life annuity and LTC insurance product. For a fuller account of LTC, and of LTC insurance in general, the reader is referred to Booth et al (2005).

This class of annuity business has been considered by Ferri and Olivieri (2000), Warshawsky et al (2002), Murtaugh et al (2001), Spillman et al (2003), Olivieri and Pitacco (2001) and Pitacco (2004). The salient issues arising from these papers are described briefly below.

Warshawsky et al (2002) identified the following positive features of a disability-linked annuity:

- The pooling of the longevity risk (associated with the life annuity) with the morbidity risk (associated with the LTC insurance component) should result in a lower overall risk for the insurer to manage since the two risks are working in opposite directions. In other words, the longer an individual stays healthy and receives the standard life annuity, the lower the present value of the LTC annuity enhancement; whereas, the earlier the individual becomes severely disabled (thereby triggering the LTC annuity enhancement), the shorter the overall term of the annuity since the individual's life expectancy is likely to be compromised by the illness. This is explored in more detail in Section 6.2.

- The annuity enhancement would help to meet the additional care costs associated with severe disability and thereby support any bequest motive for the individual's children.

- The annuity is more flexible than a standard annuity since it increases to help meet LTC costs when required. Indeed, such an annuity could enable the purchaser to continue to live, and receive care, in their own home rather than having to move into an institution such as a residential home. The fact that large unexpected medical expenses can be met, to a certain extent, by the annuity enhancement helps to meet a major concern of retirees that they will have insufficient savings to defray such costs in the future (Panis, 2003)

- From a marketing perspective, the annuity should be attractive to consumers since its two components can be presented in a positive fashion: a life annuity payable whilst the individual is healthy and an enhancement to this annuity payable should the individual suffer very poor health. Stand alone LTC policies have tended not to sell well in the past because, by definition, they force prospective purchasers to dwell on the unsavoury prospect of requiring LTC at some point in the future.

The first advantage listed above suggests that a combined life annuity and LTC product would require less stringent underwriting procedures than those required if the two components had been offered as stand alone products. This is because the opposing longevity and morbidity risks are being pooled together.

Murtaugh et al (2001) explored this point using data taken from the USA. They compared the potential demand for the combined product, after minimum underwriting had removed the applicants unsuitable for this product (which were those people who would immediately be able to claim the disability benefit), with the potential demand for the two stand alone products, after current underwriting procedures had removed the applicants unsuitable for those products. They found that the relaxation in underwriting procedures would enable 98% of 65 year olds to be considered for the integrated product as opposed to 77% for the stand alone products. In addition, they found that the increase in the size of the pool would lead to a reduction in premium of approximately 3.5% for the integrated product compared to the total premium payable in respect of the two stand alone products.

The authors pointed out that, with such a small proportion of people being excluded from cover for underwriting reasons, the idea of developing a mandatory State scheme which offers such an integrated product to every citizen regardless of current health status becomes quite feasible (Spillman et al, 2003).

Ferri and Olivier (2000) considered the risks to which the providers of such an annuity would be exposed. In particular, they analysed the risks presented by demographic changes (ie future trends in mortality and morbidity rates). Olivieri and Pitacco (2001) extended the analysis to assess the minimum solvency reserve required with this type of product, and discussed the ways in which stop-loss reinsurance could be used to reduce the minimum solvency margin. Pitacco (2004) demonstrated the way in which, as the number of policyholders increases, the process risk (ie the risk from random fluctuations within the model) reduces but the systematic risk (ie the longevity risk emanating from fundamental changes in future mortality and morbidity rates) does not.

A type of disability-linked annuity was launched in the early 1990's by an insurance company in the UK. However, this product had to be withdrawn from the market after an objection from HM Revenue & Customs (the UK tax authority formerly known as the Inland Revenue). They argued that such a product could not be treated as a "pension" (with the all important accompanying tax concessions) since it was a change in health status (ie the individual becoming disabled) which caused the annuity to increase.

Other insurance companies have launched similar products in the UK over the last few years which were not offered within the company's pension business. However, without the tax concessions for which a pension product would be eligible, these types of disability-linked annuities have tended to be unattractive to consumers. The take-up rate, therefore, for such annuities has been very low. The tax position, however, appears to have improved as a result of the simplification of the tax regime post A-Day ($6^{th}$ April 2006). This is considered in more detail in Section 3.8.

It is interesting to note that this type of annuity was alluded to in Chapter 12 of the final report prepared by the Wanless Social Care Review team in March 2006 (Wanless Social Care Review, 2006). The report was a result of a wide ranging review of social care arrangements in England prepared for the health and social care think tank, the Kings Fund.

In this paper, we will examine disability-linked annuities by considering, by way of illustration, a particular class of disability-linked annuity where the annuity is increased to a certain level once the annuitant has reached a particular state of disability, and a higher level once the annuitant has reached a more severe state of disability. In other words, there are two levels of annuity enhancement due to onset of disability.

The main objective of the research is to consider the characteristics of disability-linked annuities under a range of assumptions (eg from pessimistic to optimistic morbidity assumptions, and from narrow to wide definitions of disability). In particular, under the different assumptions, we wish to compare the single premium required for the disability-linked annuity with the single premium in respect of the corresponding traditional whole life annuity which is not enhanced upon disability. In addition, we wish to consider the expected times spent receiving each level of annuity enhancement and the probabilities of reaching each of the associated states of disability.

This paper has two main intentions from a practical perspective: firstly, to help inform the debate on annuities which is currently occurring in the UK partly prompted by the Government's consultation paper "Modernising Annuities" (Inland Revenue, 2002); and secondly, to consider whether disability-linked annuities offer a viable alternative to traditional means of obtaining LTC insurance cover.

Our main conclusion is that disability-linked annuities have a number of favourable qualities from both the insurer's and the policyholder's point of view.

The longevity risks and morbidity risks contained within the combined life annuity and LTC product act in opposite directions which should minimise the overall risk to the insurer. In consequence, the insurer can be more flexible over the underwriting requirements and disability definitions used. In addition, the premiums appear to compare very favourably with those offered for traditional life annuities when the additional disability benefits provided with the former are taken into account.

The relatively high probability that at least one level of annuity enhancement will be reached, particularly in the case of females, should make the products marketable. Recent changes in tax regime could have increased the product's appeal to consumers further.

Although by no means everyone will be able to afford the premiums for such products, our research suggests that a significant number could, and would be in sufficiently good enough health to satisfy the eligibility requirements.

This paper is organised as follows. Section 2 describes the single life disability-linked annuities being considered as examples to be explored within the remainder of the paper. Section 3 describes the model and assumptions. Section 4 examines the expected times spent in different states of disability whilst Section 5 considers the associated probabilities of reaching each of these states. Section 6 presents the results using the central set of morbidity assumptions, and Section 7 considers the results if more optimistic or pessimistic assumptions are adopted instead. Section 8 revisits the results if a wider definition of disability is employed. Section 9 investigates the underwriting considerations and Section 10 considers another possible product: a last survivor disability-linked annuity. Section 11 explores the potential demand for the products described in the paper and Section 12 concludes.

## 2. The annuity products under consideration

### *2.1 The level 1/1.5/2.5 annuity*

We carry out our analysis of disability-linked annuities by determining the characteristics of particular examples. We begin by investigating the following level disability-linked annuity:-

The annuity commences at a rate of £10,000 per annum. It increases to £15,000 per annum once the annuitant has become moderately disabled and £25,000 per annum once the annuitant has become severely disabled. It is assumed that the annuity is purchased by single premium by an annuitant who is healthy at outset. The terms “healthy”, “moderately disabled” and “severely disabled” will be discussed in Section 3.5 below. For illustrative purposes, we consider alternative starting ages for males and females of 60, 65, 70, 75 and 80.

For convenience, we assume that the annuity is payable annually in arrears. Therefore, it is the individual’s state of health at each anniversary of the commencement of the policy which determines the level of annuity payable. In practice, such an annuity is more likely to be paid more frequently. However, for the purposes of comparing the single premium required for this annuity with that required for a standard annuity (also assumed to be paid annually in arrears), it is acceptable to adopt this simplified approach.

The transition probabilities described in Section 3.1 below are based on annual transitions so, clearly, it is more convenient to assume that the annuities are paid annually. Separate calculations, not shown in this paper, confirm that assuming that payments are made annually rather than more frequently does not distort the findings. It also needs to be borne in mind that, in theory, at each annuity payment date the health status of the individual needs to be verified in case the level of payment should be changed. Clearly, this would be impractical if payments were made very frequently, say, monthly.

The disability-linked annuity being considered in this paper is assumed not to be guaranteed for any period (ie the annuity payments cease immediately upon death of the insured life). In practice, it is likely that the annuity would be guaranteed for at least 5 years since, otherwise, a purchaser who dies immediately after the annuity commences would leave no benefits in respect of the product to his/her estate having just paid a substantial premium to the insurer. The reason why we have not allowed for a guaranteed period is that we wish to compare a traditional life annuity with a disability-linked annuity without the results being distorted by the guarantee period.

The annuity amounts quoted throughout this paper are assumed to be gross. However, tax considerations are discussed in Section 3.8.

The £10,000 / £15,000 / £25,000 per annum structure leads us to describe this as a "level 1/ 1.5/ 2.5" annuity. To put these annuity amounts in context, the average annual costs of private residential care and private nursing care in the UK in 2003 were £17,100 and £23,700, respectively (Laing and Buisson, 2006).

### *2.2 The index-linked 1/1.5/2.5 annuity*

The second type of annuity we consider has the same structure as the annuity described in Section 2.1 except that the annuity payments increase in line with price inflation each year. The enhanced levels of £15,000 per annum and £25,000 per annum are also increased in line with price inflation between date of commencement of policy and date when payment at the enhanced rate is due. In other words, this version of the product can be described as follows:

The annuity commences at £10,000 per annum and increases each year in line with the level of price inflation which prevailed over the previous 12 months. Once the annuitant becomes moderately disabled, the annuity is increased by 50% (ie from considering the ratio 15,000: 10,000); and once the annuitant becomes severely disabled, the annuity is further increased by 67% (ie from considering the ratio 25,000: 15,000).

Since this annuity is an index-linked version of the annuity described in section 2.1, we describe this as an "index-linked 1/ 1.5/ 2.5" annuity.

### *2.3 The 1 /1.8 /3 annuities*

For illustrative purposes, we also consider the annuities which correspond to those described in Sections 2.1 and 2.2 that have the 1/ 1.8/ 3 structure. In other words, the level version of this annuity will commence at £10,000 per annum and increase to £18,000 per annum and £30,000 per annum on the annuitant becoming moderately and severely disabled, respectively.

## 3. The model and assumptions

### *3.1 Disability/recovery rates*

Since the annuities described in Section 2 are enhanced once an individual becomes moderately or severely disabled, we need to make assumptions regarding disability and recovery rates when calculating the single premiums required.

We use the disability and recovery rates obtained from the long term care UK population projection model described in Rickayzen and Walsh (2002). The model is a 12 state multiple-state model and is depicted, together with the possible annual transitions, in Figure 1.

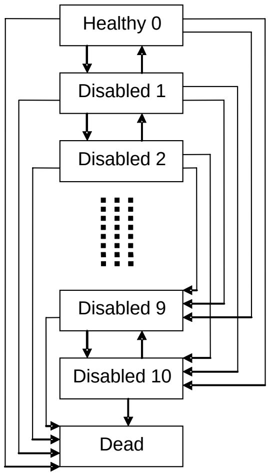


**Figure 1. The multiple state long term care model**

The 10 disability states correspond to those emanating from the OPCS survey of Great Britain which took place in 1985 and 1986. These are described in more detail in Martin et al (1988). State 1 and State 10 are the least, and most, severely disabled states, respectively. State 0 is the healthy state and State 12 is the dead state.

The arrows pointing downwards in Figure 1 indicate that it is assumed that an individual can deteriorate to any other worse state of disability in a year. The arrows pointing upwards show that it is assumed that an individual can only improve by at most one disability category in a year. The allowance made for recovery is considered in more detail in Section 9.

Rickayzen and Walsh (2002) describe how the model was developed to project the number of disabled lives over the next 40 years from the base year of 1996. Importantly, assumptions regarding trends in future disability rates were made in the model which were based on recent trends in healthy life expectancy data. Given the paucity of the data, 16 alternative sets of trend assumptions were considered. As part of the sensitivity analysis carried out in this paper, we will consider the effect of using the most optimistic and most pessimistic sets of those assumptions, as well as the central set of assumptions, in our calculations. These are considered in Section 7.

It should be borne in mind that the disability/recovery rates we are using are based on population data. However, as pointed out by Dullaway and Elliott (1998), we would expect individuals purchasing health insurance products to experience lighter morbidity rates than the general population due to underwriting and the fact that such individuals are likely to come from a relatively high socio-economic grouping. Dullaway and Elliott suggest ways in which general population data can be adjusted to become suitable for insured populations in Chapter 5 of their paper. However, as they acknowledge, this is not only a complicated task but is also fairly arbitrary given the limited amount of data involved.

For the type of disability-linked annuity under consideration in this paper, it is not necessarily the case that the higher the morbidity rates assumed, the higher the single premium required. This is because, whilst higher disability rates will indeed lead to greater numbers of lives receiving enhanced annuities, such disabled lives will also be subject to higher mortality rates (see Section 3.2 below). This would mean that the total period for which the lives receive the annuity from the outset of the policy will be relatively short. The net effect on the single premium of assuming high disability rates coupled with higher mortality rates amongst more severely disabled people could be neutral. This important point is considered in more detail in Section 6.2.

It should be noted that one of the problems encountered by Rickayzen and Walsh in developing their model is that the published report on the OPCS survey (Martin et al, 1988) shows disability prevalence rates for all lives aged over 80 aggregated together. As acknowledged by Rickayzen and Walsh, not being able to split the data into narrower age bands above age 80 may mean that the morbidity rates at the older ages (eg above age 90) may be higher in practice than those assumed in the model. However, the trade-off between mortality rates and morbidity rates mentioned in the previous paragraph, and analysed in Section 6.2, suggests that this should not materially affect the premiums shown in this paper.

### *3.2 Mortality*

We have used the analysis set out in Section 3.2 of Rickayzen and Walsh (2002) to shape the mortality assumptions adopted for this piece of research.

In particular, we assume that the mortality rate at any age has two components: one which applies equally to healthy and disabled people of a given age and sex in a particular year, and one which increases linearly from OPCS disability category 6 to disability category 10, but does not apply at all to people in disability categories lower than 6. The rationale (which is described more fully in Rickayzen & Walsh (2002)) is that people in the lower disability categories are not experiencing life threatening conditions.

For our purposes, the formula used to express the second component for someone aged x in disability category n is:

$$\text{ExtraMort}(x,n) = \frac{0.10}{1+1.1^{(50-x)}} \cdot \frac{\text{Max}(n-5,0)}{5} \qquad (1)$$

The level mortality component has been chosen at each age in such a way as to ensure that the overall mortality at each age (ie when the two mortality components are combined) replicates the IL92 mortality table. For this purpose, we assume that the proportion of lives in each disability category at each age is in accordance with the prevalence rates calculated in Rickayzen & Walsh (2002). Details of the IL92 mortality table can be found in Appendices A6 and A7 of Continuous Mortality Investigation (CMI) Report No. 17 (Continuous Mortality Investigation Bureau, 1999).

The ExtraMort (x,n) component given in equation (1) is half of that used in Rickayzen and Walsh (2002). However, the latter involved general population mortality whereas we are concerned with insured population mortality in this paper. It is therefore appropriate to re-scale the extra mortality component.

The author has investigated using fractions of the ExtraMort (x,n) component adopted in Rickayzen & Walsh (2002) other than 0.5 (in particular, 0.25 and 0.6) and found that the results are relatively insensitive to this fraction. For example, using a fraction of 0.25 rather than 0.5 would have the effect of increasing the premiums involved by approximately 2.5%. Fractions greater than 0.6 were not considered since these would lead, at some ages, to the level mortality rate component at low disability levels needing to be negative in order that the overall mortality rate for that age should accord with the IL92 Table.

### *3.3 Expenses*

We assume that the expenses associated with this product will be as follows:

> 2.5% of the basic part of each annuity payment (ie the part which is payable even while the individual is in good health).
>
> 15% of the annuity enhancement due to the onset of a disability (ie the part of each annuity payment which is in excess of the basic annuity amount).

We assume that the expenses attached to the uplift component will be relatively high since individuals receiving an enhanced level of payment will need to have their health monitored by the provider to ensure that they remain eligible for the enhancement.

### *3.4 Valuation rate of interest*

We use a valuation rate of interest of 7% per annum for the level annuity and 3% per annum for the index-linked annuity. We are, therefore, implicitly assuming a price inflation rate of 3.9% per annum.

### *3.5 Definition of "disabled"*

Since the disability-linked annuity product being investigated pays out different annuity amounts depending on whether the policyholder is "healthy/slightly disabled", "moderately disabled" or "severely disabled", it is important to define these terms carefully.

Before considering these definitions further at this stage, it is helpful to consider when claims are paid in relation to conventional LTC insurance products. In the UK, it is common practice for a claim to be payable if a policyholder is unable to perform a certain number of the six benchmark Activities of Daily Living ("ADLs") or has suffered significant cognitive impairment. The six standard ADLs are: washing oneself, dressing oneself, mobility, toileting, feeding and transferring (from, say, a chair to a bed). It is usual for a full claim to be paid if the individual fails 3 ADLs and, in addition, some policies include provision for a partial payment to be made if the individual fails 2 ADLs. A full claim is always paid if the individual fails the cognitive test regardless of whether or not they have any physical disabilities.

Relating this information to the disability-linked annuity under consideration, it seems reasonable to define "severely disabled" to be when the individual fails 3 or more ADLs and "moderately disabled" to be when he/she fails 2 but not 3 ADLs. However, the data we are using are based on OPCS disability categories rather than ADLs. We must therefore, in turn, relate the OPCS disability categories to ADL failures. This can only be done approximately.

Dullaway and Elliott (1998) suggest that, for both sexes, the number of lives failing 2 ADLs or the cognitive test would, approximately, equate to the number of lives in OPCS categories 7-10. They also suggest that the number of lives failing at least 3 ADLs or the cognitive test would, approximately, include all the lives in categories 9 and 10 and half the lives in category 8.

For our central set of assumptions, we will therefore define the terms "healthy/slightly disabled", "moderately disabled" and "severely disabled" for claims purposes as follows:

"Healthy/slightly disabled" = Lives in OPCS categories 0-6 inclusive.

"Moderately disabled" = Lives in OPCS category 7 and half the lives in OPCS category 8.

"Severely disabled" = Lives in OPCS categories 9 and 10 and half the lives in OPCS category 8.

Since the above definitions of disability are relatively narrow, we also wish to examine the effect of making the disability definitions more generous. After all, there is no reason why the disability-linked annuity should be enhanced only when the individual has failed 2 ADLs. The more generous the definition of disability used, the more attractive such an annuity is likely to be, provided the premium is still affordable to the policyholder. We will investigate this point further in Section 8.

To put the above disability categories in context, a working party of the UK Actuarial Profession assumed that the relationship between OPCS categories and the care needs of individuals would be as summarised in Table 1 (Nuttall et al, 1994).

**Table 1: Care needs according to OPCS category**

| **OPCS categories** | **Care needs** | **Hours per week of care required** |
|---|---|---|
| | | |
| 1-2 | Low | 5 |
| 3-5 | Medium | 15 |
| 6-8 | Regular | 30 |
| 9-10 | Continuous | 45 |

Source: Nuttall et al (1994)

### *3.6 The central set of assumptions*

For the purposes of our calculations we have adopted the central set of trend assumptions that were used in Rickayzen and Walsh (2002) (ie Model C described in Table 14 of that paper).

The trend assumptions incorporated within this set relate to improvements in both overall mortality and disability rates over time. Regarding the latter, we assume that the probability that a healthy life aged x in year t becomes disabled during year t is the same as the probability that a healthy life aged x+1 in year t+10 becomes disabled during year t+10 (eg the probability that a healthy 61 year old becomes disabled in 2018 is the same as the probability that a healthy 60 year old becomes disabled in 2008). Since the underlying assumption is that the probability that a person becomes disabled increases with age, this trend assumption represents an improvement (ie reduction) in disability rates. The one year shift in age every 10 years of time leads to the trend assumption being referred to as “1 in 10” in the aforementioned paper.

In Section 7 we consider the results if more optimistic or more pessimistic trend assumptions are adopted.

### *3.7 Base year for calculations*

Although 1996 was the base year used for the projections described in Rickayzen and Walsh (2002), we have carried out our calculations for this paper assuming that annuities were purchased during 2005. This has been done by allowing for changes in the mortality and morbidity rates between 1996 and 2005 in accordance with the particular trend assumptions adopted.

### *3.8 Tax considerations*

Since the disability-linked annuity product described in this paper is not currently available, it is not possible to be precise about the taxation aspects. However, correspondence with the HM Revenue & Customs suggests that such an annuity could either be offered as a pension product or be treated as a combination of a Purchase Life Annuity (PLA) and a Permanent Health Insurance (PHI) benefit – the latter being more commonly known as Income Protection.

If the annuity were treated as a pension product (which would not have been possible prior to 6$^{th}$ April 2006, but now seems allowable as a result of the simplification changes in tax regime which have occurred with effect from that date) the product could be purchased out of the individual’s pension account. The contributions made by the individual towards this fund would have been made out of pre-tax income. The premium would accumulate within the insurer’s fund without being subject to capital gains or income tax (other than tax on any UK dividends received by the fund) and the annuity payments would be subject to income tax in the hands of the individual. It should be noted that it is unlikely that the insurer would be allowed to reduce any annuity enhancement even if the individual were subsequently to recover to a healthier grouping. This could be a substantial disadvantage to offering the annuity as a pension product since recoveries are expected to occur (see Section 9 ).

If, on the other hand, the annuity were treated as a combination of separate PLA and PHI products then the tax position is very different. The premium would be purchased by the individual out of post-tax savings. Regarding benefit payments, both the capital element of the PLA component of each annuity payment and the whole of the PHI benefit would be payable tax-free whilst the interest element (ie non-capital element) of the PLA component would be subject to income tax.

As mentioned in Section 2.1, for the purposes of our calculations, we assume that all amounts considered in this paper are gross of tax.

## 4. Expected time spent disabled

Based on the assumptions set out in Section 3, we can calculate the expected time that an individual will spend healthy/slightly disabled , moderately disabled or severely disabled in the future.

This has been done as follows:

Let:

$l_x$ = the number of lives aged x in a life table

$\omega$ = the limiting age of the life table

$l_x^h$ = the number of lives aged x in the life table who fall under the "healthy/slightly disabled" definition

$l_x^m$ = the number of lives aged x in the life table who fall under the "moderately disabled" definition

$l_x^s$ = the number of lives aged x in the life table who fall under the "severely disabled" definition

where $l_x = l_x^h + l_x^m + l_x^s$

Then, the complete expectation of life for a person aged x is, as an approximation to the underlying integral, is given in (2):

$$\mathring{e}_x = \frac{1}{l_x}\left\{\left(\sum_{y=x}^{\omega} l_y\right) - \frac{l_x}{2}\right\} \qquad (2)$$

For an individual who is healthy (ie in OPCS category 0) at age x, the expected number of years spent "healthy/slightly disabled", "moderately disabled" and "severely disabled" in the future (depicted by $\mathring{e}_x^h$, $\mathring{e}_x^m$ and $\mathring{e}_x^s$, respectively) are approximately:

$$\mathring{e}_x^h = \frac{1}{l_x}\left\{\left(\sum_{y=x}^{\omega} l_y^h\right) - \frac{l_x}{2}\right\} \qquad (3)$$

$$\overset{\circ}{e}{}_x^m = \frac{1}{l_x}\left\{\sum_{y=x+1}^{\omega} l_y^m\right\} \tag{4}$$

$$\overset{\circ}{e}{}_x^s = \frac{1}{l_x}\left\{\sum_{y=x+1}^{\omega} l_y^s\right\} \tag{5}$$

where x is the starting age of the life table so that $l_x = l_x^h$.

Table 2 shows, for each sex, the expected number of years spent in each disability category for the five different initial ages under consideration. The calculations have been done using the central set of assumptions described in Section 3.6. In each case, the life is assumed to be healthy (ie in State 0) at outset.

**Table 2. Disabled life expectancies (in years) – central assumptions**

| | **Initial Age** | $\overset{\circ}{e}{}_x^h$ | $\overset{\circ}{e}{}_x^m$ | $\overset{\circ}{e}{}_x^s$ | $\overset{\circ}{e}_x$ |
|---|---|---|---|---|---|
| **Males** | 60 | 21.7 | 1.1 | 1.5 | **24.3** |
| | 65 | 17.3 | 1.0 | 1.4 | **19.7** |
| | 70 | 13.4 | 0.9 | 1.3 | **15.6** |
| | 75 | 9.8 | 0.8 | 1.3 | **11.9** |
| | 80 | 7.1 | 0.7 | 1.2 | **9.0** |
| | | | | | |
| **Females** | 60 | 23.6 | 1.6 | 2.2 | **27.4** |
| | 65 | 19.0 | 1.5 | 2.2 | **22.7** |
| | 70 | 14.8 | 1.3 | 2.1 | **18.2** |
| | 75 | 11.2 | 1.1 | 1.9 | **14.2** |
| | 80 | 8.1 | 0.9 | 1.7 | **10.7** |

The following features should be noted:

- Since in each case the life is healthy at outset, the life will, on average, spend most of his/her future remaining life falling under the “healthy/slightly disabled” definition with relatively short periods of time spent moderately or severely disabled. For example, a female who is healthy at age 65 is expected to spend 19.0 years “healthy/slightly disabled”, 1.5 years “moderately disabled” and 2.2 years “severely disabled”.

- Not only are females expected to live longer than males but they are expected to spend more time both moderately and severely disabled than males of the same initial age. This finding accords with observations made by others - for example, Murtaugh et al (2001).

- For both sexes, the proportion of future lifetime expected to be spent severely disabled doubles (approximately) as the initial age moves from age 60 to age 80 (ie from 6.2% to 13.3% in the case of males, and from 8.0% to 15.9% in the case of females).

- For both sexes, the period of time expected to be spent severely disabled is, to a large extent, independent of initial age. The amount of time expected to be spent severely disabled is approximately 1.3 years in the case of males and 2.1 years in the case of females, regardless of initial age. This feature whereby proximity to death is roughly constant and independent of age is consistent with the findings of Himsworth and Goldacre (1999) and Seshamani (2004).

## 5. Claim probabilities

Potential consumers will need to be convinced that there is a strong likelihood that they will eventually be paid an annuity enhancement if they are to consider purchasing a disability-linked annuity. Therefore, in considering the marketing of such a product, it is helpful to consider the probability that an individual taking out such a product will eventually become so disabled that they satisfy the eligibility requirements to receive either of the enhanced levels of annuity.

Table 3 shows these probabilities for individuals who take out such a product at different starting ages, based on the central set of assumptions. In each case (and in all the following tables, other than Tables 24-26), the individual is assumed to be healthy (ie in State 0) at commencement of the annuity.

**Table 3. Claim probabilities – central assumptions**

| | **Age at outset** | **Prob (remains healthy/slightly disabled)** | **Prob (becomes moderately but not severely disabled during remaining life)** | **Prob (becomes severely disabled during remaining life)** |
|---|---|---|---|---|
| **Males** | 60 | 0.627 | 0.123 | 0.250 |
| | 65 | 0.631 | 0.120 | 0.249 |
| | 70 | 0.633 | 0.117 | 0.250 |
| | 75 | 0.624 | 0.117 | 0.259 |
| | 80 | 0.621 | 0.114 | 0.265 |
| | | | | |
| **Females** | 60 | 0.489 | 0.159 | 0.352 |
| | 65 | 0.493 | 0.155 | 0.352 |
| | 70 | 0.501 | 0.149 | 0.350 |
| | 75 | 0.512 | 0.142 | 0.346 |
| | 80 | 0.528 | 0.134 | 0.338 |

It can be seen that the probability of eventually claiming an enhanced annuity is substantially higher for females than for males. This is particularly true when considering the higher level of enhancement and will be reflected in the respective premiums charged in relation to the sexes. It is also interesting to note that the probabilities vary very little with age at purchase.

The relatively high probability that an individual will eventually receive the second level of annuity enhancement, particularly in the case of females, should make such a disability-linked annuity attractive to consumers. A second reason why such an annuity might appeal to females in particular is that wives usually outlive their husbands; there is therefore less opportunity for females to receive informal care provision from their spouse when they require long term care.

## 6. Results using central assumptions

### *6.1 Single premiums required for each type of annuity*

Table 4 and Table 5 set out the single premiums required for different commencement ages using the central set of assumptions for the level and index-linked annuities, respectively. The 1/1.5/2.5 and 1/1.8/3 annuities were described in Section 2 and the 1/1/1 annuity is the standard whole life annuity to which these two annuities are being compared. In all cases, the initial level of annuity paid to the policyholder is £10,000 per annum.

**Table 4. Single premium (in £) required for different types of level annuity – central assumptions**

| | **Age at commencement** | **1/1/1** | **1/1.5/2.5** | **1/1.8/3.0** |
|---|---|---|---|---|
| **Males** | 60 | 120,697 | 128,118 | 131,004 |
| | 65 | 109,418 | 118,082 | 121,429 |
| | 70 | 96,431 | 106,310 | 110,095 |
| | 75 | 82,272 | 94,236 | 98,787 |
| | 80 | 68,507 | 81,600 | 86,539 |
| | | | | |
| **Females** | 60 | 127,943 **(6.0%)** | 138,167 **(7.8%)** | 142,162 **(8.5%)** |
| | 65 | 118,031 | 130,420 | 135,228 |
| | 70 | 105,993 | 120,441 | 126,006 |
| | 75 | 92,255 | 108,296 | 114,420 |
| | 80 | 77,655 **(13.4%)** | 94,421 **(15.7%)** | 100,762 **(16.4%)** |

**Table 5. Single premium (in £) required for different types of index-linked annuity – central assumptions**

| | Age at commencement | 1/1/1 | 1/1.5/2.5 | 1/1.8/3.0 |
|---|---|---|---|---|
| **Males** | 60 | 176,259 | 192,331 | 198,539 |
| | 65 | 152,065 | 168,952 | 175,436 |
| | 70 | 127,495 | 144,925 | 151,571 |
| | 75 | 103,575 | 122,299 | 129,392 |
| | 80 | 82,453 | 101,071 | 108,070 |
| | | | | |
| **Females** | 60 | 192,238 **(9.1%)** | 215,462 **(12.0%)** | 224,449 **(13.1%)** |
| | 65 | 168,833 | 193,773 | 203,372 |
| | 70 | 144,139 | 170,095 | 180,020 |
| | 75 | 119,299 | 145,251 | 155,099 |
| | 80 | 95,687 **(16.1%)** | 120,384 **(19.1%)** | 129,677 **(20.0%)** |

From the size of the premiums, it can be seen that disability-linked annuities are unlikely to be affordable for the less affluent members of the population. The latter would be more concerned with maximising their initial level of income than purchasing an annuity enhancement should they suffer ill health in the future. The potential market for such a product, taking means into account, is considered in more detail in Section 11.

As expected, for each of the annuity products, the single premium is greater for females than males. The percentages shown for females age 60 and 80 represent the increase in single premium required for females as compared with males at those ages. It can be seen that the difference is more pronounced in the case of the 1/1.8/3 annuities than the 1/1/1 annuities. For example, with the index-linked version of the annuities, the premium at age 60 is 9.1% higher for females than males for the 1/1/1 annuity (ie comparing £192,238 with £176,259); whereas the difference is 13.1% for the 1/1.8/3 annuity (ie £224,449 compared with £198,539). This reflects the greater expected time spent disabled by females compared with males, as demonstrated by Table 2. In addition, the percentages increase with initial age. This reflects the fact that life expectancy for females reduces with initial age more gradually than for males
For example, using the figures shown in Table 2, the life expectancy at age 60 is 12.8% greater for females than males (ie 27.4 years compared to 24.3 years) but 18.9% greater at age 80 (ie 10.7 years compared to 9.0 years)

It should be borne in mind that the single premiums shown in these tables will not necessarily be those that would actually be charged by an insurance company. This is because the disability rates used in the calculations have been extracted from general UK population data whereas an insurance company would tend to use lower rates since they would have been derived from insured population data. However, since the purpose of this paper is to calculate the percentage increase in the single premium that would be required (rather than the absolute values of the premium amounts) when comparing each disability-linked annuity with the standard whole life annuity, the latter point should not detract from the analysis. Having said that, we mentioned in Section 3.1 that using higher disability rates than might be appropriate for an insured population would not necessarily lead to higher single premiums being required for the type of disability-linked annuity under consideration in this paper. This point is now considered in more detail in Section 6.2.

### *6.2 The link between disability and mortality rates.*

To illustrate the effect on the single premium of having relatively high morbidity rates and, in consequence, high overall mortality rates, we consider initially a level 0/1.5/2.5 annuity. This is an annuity which is consistent with the 1/1.5/2.5 annuity described in Section 2.1 (ie nothing is paid until the policyholder becomes moderately or severely disabled in which case the annuity is £15,000 or £25,000 per annum, respectively).

In Section 3.6, we referred to the "1 in 10" new disability trend assumption. Let us now consider the single premium required for a female purchasing a 0/1.5/2.5 level annuity at age 60 under the following alternative new disability trend assumptions:-

(1) "1 in 20"

(2) "1 in 10"

(3) "1 in 5"

Assumptions (1), (2) and (3) (which are equivalent to Models B, C and D in Rickayzen and Walsh (2002)) represent progressively lighter disability rate assumptions. For example, the probability that a 60 year old becomes disabled in 2008 is equal to the probability that a 61 year old becomes disabled in 2028 under assumption (1) and in 2013 under assumption (3).

Table 6 shows the corresponding single premiums required under assumptions (1), (2) and (3).

**Table 6. Single premium for female aged 60 purchasing level 0/ 1.5/ 2.5 annuity under alternative disability trend assumptions.**

| **Assumption** | | |
|---|---|---|
| **(1)** | **(2)** | **(3)** |
| £21,339 | £18,915 | £14,671 |

We notice that the single premium required decreases as the disability rate assumption becomes lighter. This is to be expected as nothing is paid whilst the policyholder remains healthy / slightly disabled (which is the majority of her remaining life according to Table 2). Hence, the more likely the policyholder is to remain relatively healthy (which occurs under assumption (3)), the lower the premium she should be charged. Table 6 indicates that the premium decreases by £6,668 from assumption (1) to (3).

Let us now consider the single premiums required under assumptions (1), (2) and (3) for the level 1/ 1.5/ 2.5 annuity (ie the 0/ 1.5/ 2.5 annuity but with £10,000 p.a. paid, in addition, whilst the policyholder is healthy/ slightly disabled). These are set out in Table 7.

**Table 7. Single premiums for female aged 60 purchasing level 1/ 1.5/ 2.5 annuity under alternative disability trend assumptions**.

| **Assumption** | | |
|---|---|---|
| **(1)** | **(2)** | **(3)** |
| £138,981 | £138,167 | £136,747 |

We notice that the premium required decreases by a smaller amount (ie £2,234) than in the 0/ 1.5/ 2.5 case as we move from assumption (1) to (3). This demonstrates that when the disability-linked annuity product includes annuity payments which are made whilst the policyholder is healthy/ slightly disabled, the upward pressure on the single premium of assuming relatively high disability rates (with associated high levels of annuity payments) is ameliorated to some extent by the downward pressure caused by the fact that such lives will spend less time receiving the basic level of annuity.

This point is also illustrated by considering the expected time spent in the three states: “healthy/slightly disabled”, “moderately disabled” and “severely disabled” under assumptions (1) and (3). This information is set out in Table 8 below:

**Table 8: Disabled life expectancies (in years) for female age 60**

| | $\mathring{e}^{h}_{60}$ | $\mathring{e}^{m}_{60}$ | $\mathring{e}^{s}_{60}$ | $\mathring{e}_{60}$ |
|---|---|---|---|---|
| **Assumption (1)** | 22.8 | 1.7 | 2.6 | **27.1** |
| **Assumption (3)** | 25.0 | 1.3 | 1.7 | **28.0** |

It can be seen that, since assumption (1) involves the heavier disability rates, the expected time spent receiving £15,000 p.a. and £25,000 p.a. is greater than under assumption (3) (ie by 1.3 years in total). However, since the model under assumption (3) involves the lighter overall mortality rates (as the disability assumptions are lighter), the expected time spent receiving the basic £10,000 p.a. annuity is much greater than for the model under assumption (1) (ie by 2.2 years).

We can conclude, therefore, that assuming heavier disability rates in our calculations than are likely to be applicable in practice for an insured population will lead to single premiums which are not very different from those actually required for the insured population.

*6.3 Comparison of disability-linked annuities with standard life annuities.*

Table 9 shows the percentage increase in single premium required if a disability-linked level annuity as opposed to a traditional level life annuity is purchased. Table 10 shows the corresponding information for the index-linked annuities.

**Table 9. Percentage increase in single premium required, when comparing a disability-linked level annuity with a standard level annuity - central assumptions.**

| | Age at commencement | 1/1.5/2.5 | 1/1.8/3 |
|---|---|---|---|
| **Males** | 60 | 5.5% | 7.7% |
| | 65 | 7.1% | 9.9% |
| | 70 | 9.2% | 12.8% |
| | 75 | 13.1% | 18.1% |
| | 80 | 17.2% | 23.6% |
| | | | |
| **Females** | 60 | 7.2% | 10.0% |
| | 65 | 9.5% | 13.1% |
| | 70 | 12.3% | 17.0% |
| | 75 | 15.7% | 21.6% |
| | 80 | 19.4% | 26.8% |

**Table 10. Percentage increase in single premium required when comparing a disability-linked index-linked annuity with a standard index-linked annuity – central assumptions.**

| | Age at commencement | 1/1.5/2.5 | 1/1.8/3 |
|---|---|---|---|
| **Males** | 60 | 8.2% | 11.4% |
| | 65 | 10.0% | 13.8% |
| | 70 | 12.3% | 17.0% |
| | 75 | 16.3% | 22.5% |
| | 80 | 20.3% | 27.9% |
| | | | |
| | | | |
| **Females** | 60 | 10.9% | 15.1% |
| | 65 | 13.3% | 18.4% |
| | 70 | 16.2% | 22.4% |
| | 75 | 19.6% | 27.0% |
| | 80 | 23.2% | 31.9% |

The most significant point emerging from these tables is that the increase in the single premium in all cases is relatively small given the substantial extra benefits being provided by the disability-linked annuity if the policyholder were to become disabled. This should make such an annuity product relatively attractive to consumers. For example, being able to upgrade the standard level annuity to the 1/1.5/2.5 level annuity for males in return for an additional 5.5% of the standard single premium could be of great interest to males age 60 wishing to purchase an annuity.

The reason why the increases are so low is that, as Table 2 demonstrates, individuals who are healthy at outset are expected to spend relatively short periods in a state of moderate or severe disability. Furthermore, such periods will tend to be towards the end of a person's life so that the annuity enhancement will be heavily discounted and therefore have a relatively low present value to be added to the standard single premium.

Other observations that can be made about Tables 9 and 10 are as follows: -

- The percentage uplift in premium increases monotonically as the age at commencement increases from age 60 to age 80. This is because, as noted in connection with Table 2, the period of time expected to be spent disabled is, to a large extent, independent of age at commencement. This means that individuals are expected to spend an increasing proportion of future lifetime disabled as the age at commencement increases. The disability-linked annuity therefore becomes more valuable relative to the standard annuity the older the policyholder is at outset.

- The percentage uplift in premium is greater for females than males. This is because females are expected to spend more time disabled than males as illustrated by Table 2.

- The percentage uplift in premium is greater for the index-linked annuity than the level annuity. The reason for this is that index-linking causes the effective rate of interest used to discount the benefits to be reduced and so the enhanced payments (which are paid toward the end of life) become relatively more valuable.

The author has compared the results shown in Tables 9 and 10 with those of Murtaugh et al (2001). The latter considered a 1/ 3/ 4 annuity for an individual age 65, used a rate of interest of 6% pa, assumed mortality and morbidity rates based on US data, included a 10 year guarantee and paid the higher disability enhancement on the failure of 4+ ADLs (rather than 3+). They analysed both the level version of the annuity and an increasing version where the life annuity component increased by 3 % pa and the disability part increased by 5% pa (ie significantly higher increases on the disability component than assumed in this paper). The authors found that the increase in unisex premium required for the level annuity was 11.5% and for the increasing annuity was 23% when compared with standard life annuities. Separate calculations (not shown) which allow for the differences between the two products confirm that the results in this paper are consistent with those of Murtaugh (2001).

## 7. Adoption of pessimistic and optimistic assumptions.

It is generally accepted that life expectancy has been increasing in most countries over the last few decades (see, for example, Dorling et al, 2006). However, there is considerable debate over how the extra years have been spent with regard to state of health. Two main hypotheses, one at each end of the spectrum, have emerged: the compression of morbidity and the expansion of morbidity.

The compression of morbidity hypothesis, espoused by Fries (1980), is that periods of disability are deferred by the extra years of life expectancy. In other words, on average, people spend the extra years in a healthy state and, therefore, a decreasing proportion of their total lifetime in a state of disability. This is an optimistic assumption.

By contrast, the expansion of morbidity hypothesis, suggested by Gruenberg (1977), is that increases in life expectancy are due to a decline in accident rates. Consequently, as fewer people die due to accidents, an increasing number die from chronic disease. Hence, on average, people spend an increasing proportion of their lifetime in a state of disability. This is a pessimistic supposition.

An alternative hypothesis, which sits between these two extremes, is known as the dynamic equilibrium hypothesis. Manton (1987) suggested that changes in both mortality and morbidity rates could lead to a conclusion that the extra years of life expectancy are a mixture of years spent in good health and years spent disabled. MacDonald et al (2006) postulated that recent data collected for England and Scotland supported this theory in both those countries. By contrast, Khoman and Weale (2006)
found that, applying an incidence-based rather than prevalence-based approach to calculating healthy life expectancy figures, recent data for the whole of the UK supported the expansion of morbidity hypothesis. This divergence of opinion amply demonstrates the lack of agreement over which theory currently applies.

Karlsson et al (2006) discussed how the sets of assumptions described in Rickayzen and Walsh (2002) relate to the three hypotheses described above. They concluded that Model C of that work reflects the dynamic equilibrium hypothesis, whilst Model A and Model N reflect the expansion and compression theories, respectively.

The assumptions underlying Model C have already been described in Section 3.6 above as they are the central set of assumptions used in this paper. To illustrate the effect of using pessimistic and optimistic trend assumptions in this work, we use the morbidity assumptions underlying Model A and Model N, respectively.

Full details of the assumptions can be found in Rickayzen and Walsh (2002). However, the differences can be summarised as follows: under Model A, we assume no improvement in the morbidity rates over time; under Model N, we assume a “1 in 5” trend assumption as regards improvement in morbidity rates (see Section 3.6) and a slight reduction in the probability that a disabled person becomes even more disabled during the following year.

### *7.1 Results with pessimistic assumptions*

Table 11 shows the time which an individual will expect to spend in each health status over their future lifetime assuming that the pessimistic morbidity assumptions apply, and that they were healthy (ie in State 0) at the initial age.

As expected, with stronger morbidity assumptions, the individual can expect to spend longer in both the moderately and severely disabled states as compared to Table 2. In addition, in every case, the individual's overall life expectancy is reduced since the expected time spent in the relatively healthy state is substantially reduced.

We can note how the proportion of expected time spent disabled (ie in categories "m" or "s") increases as the morbidity rates become higher. For example, for females age 75, the proportion increases from 21.1% to 25.2%.

**Table 11. Disabled life expectancies (in years) – pessimistic assumptions**

| | Initial Age | $\overset{\circ}{e}{}_x^h$ | $\overset{\circ}{e}{}_x^m$ | $\overset{\circ}{e}{}_x^s$ | $\overset{\circ}{e}_x$ |
|---|---|---|---|---|---|
| **Males** | 60 | 20.6 | 1.3 | 2.0 | **23.9** |
| | 65 | 16.4 | 1.1 | 1.8 | **19.3** |
| | 70 | 12.6 | 1.0 | 1.7 | **15.3** |
| | 75 | 9.2 | 0.9 | 1.6 | **11.7** |
| | 80 | 6.7 | 0.8 | 1.4 | **8.9** |
| | | | | | |
| **Females** | 60 | 22.0 | 1.8 | 2.9 | **26.7** |
| | 65 | 17.8 | 1.6 | 2.7 | **22.1** |
| | 70 | 13.8 | 1.5 | 2.5 | **17.8** |
| | 75 | 10.4 | 1.2 | 2.3 | **13.9** |
| | 80 | 7.5 | 1.0 | 2.0 | **10.5** |

Table 12 sets out the probabilities associated with each different claim being made under the pessimistic assumptions.

When compared with Table 3, we note that the probability that the annuity will eventually become enhanced due to the onset of disability increases from approximately 0.37 to 0.44 for males (and approximately 0.50 to 0.58 for females) regardless of initial age. For both sexes, the increase is mainly attributable to the increase in probability that the higher level of enhancement will eventually be paid.

**Table 12. Claim probabilities – pessimistic assumptions**

| | **Age at outset** | **Prob (remains healthy/slightly disabled)** | **Prob (becomes moderately but not severely disabled during remaining life)** | **Prob (becomes severely disabled during remaining life)** |
|---|---|---|---|---|
| **Males** | 60 | 0.536 | 0.141 | 0.323 |
| | 65 | 0.551 | 0.135 | 0.314 |
| | 70 | 0.562 | 0.130 | 0.308 |
| | 75 | 0.563 | 0.128 | 0.309 |
| | 80 | 0.573 | 0.122 | 0.305 |
| | | | | |
| **Females** | 60 | 0.389 | 0.169 | 0.442 |
| | 65 | 0.405 | 0.163 | 0.432 |
| | 70 | 0.424 | 0.156 | 0.420 |
| | 75 | 0.446 | 0.147 | 0.407 |
| | 80 | 0.470 | 0.138 | 0.392 |

Tables 13 and 14 show the single premiums required for the level and index-linked annuities, respectively, under the pessimistic assumptions.

The single premiums are lower for the 1/1/1 annuity than in Tables 4 and 5 where the central assumptions are applied. This is because this annuity is the standard life annuity and, with higher disability rates assumed in Tables 13 and 14, the total life expectancies are reduced (as shown by comparing Table 2 with Table 11). Such annuities are therefore less valuable.

The premiums are higher for the 1/1.5/2.5 and 1/1.8/3.0 types of annuity with the pessimistic assumptions than with the central assumptions. This reflects the fact that the value of the annuity enhancement on disability more than compensates for the reduction in value due to reduced overall life expectancy. However, it is interesting to note that the net effect is that the premiums increase by at most 3.2% (female, age 80, 1/1.8/3.0 index-linked annuity) even though we have moved from the dynamic equilibrium theory to the expansion of morbidity theory.

As with Tables 3 and 4, the percentages shown for females age 60 and 80 represent the increase in single premium required for females as compared with males at those ages. It can be seen that the percentages are all similar to those in Tables 3 and 4, thus demonstrating that moving to the more pessimistic scenario affects premiums for males and females equally.

**Table 13. Single premium required for different types of level annuity – pessimistic assumptions**

| | **Age at commencement** | **1/1/1** | **1/1.5/2.5** | **1/1.8/3.0** |
|---|---|---|---|---|
| **Males** | 60 | 119,912 | 129,566 | 133,291 |
| | 65 | 108,635 | 119,689 | 123,927 |
| | 70 | 95,538 | 108,235 | 113,068 |
| | 75 | 81,467 | 96,193 | 101,765 |
| | 80 | 67,966 | 83,235 | 88,964 |
| | | | | |
| **Females** | 60 | 126,799 **(5.7%)** | 139,856 **(7.9%)** | 144,911 **(8.7%)** |
| | 65 | 116,859 | 132,324 | 138,273 |
| | 70 | 104,871 | 122,509 | 129,245 |
| | 75 | 91,263 | 110,432 | 117,693 |
| | 80 | 76,857 **(13.1%)** | 96,493 **(15.9%)** | 103,867 **(16.8%)** |

**Table 14. Single premium required for different types of index-linked annuity – pessimistic assumptions**

| | **Age at commencement** | **1/1/1** | **1/1.5/2.5** | **1/1.8/3.0** |
|---|---|---|---|---|
| **Males** | 60 | 174,184 | 195,211 | 203,257 |
| | 65 | 150,243 | 171,798 | 180,005 |
| | 70 | 125,740 | 147,828 | 156,192 |
| | 75 | 102,192 | 124,952 | 133,523 |
| | 80 | 81,600 | 103,186 | 111,255 |
| | | | | |
| **Females** | 60 | 189,149 **(8.6%)** | 218,702 **(12.0%)** | 230,014 **(13.2%)** |
| | 65 | 166,083 | 197,037 | 208,830 |
| | 70 | 141,827 | 173,300 | 185,221 |
| | 75 | 117,487 | 148,286 | 159,873 |
| | 80 | 94,377 **(15.7%)** | 123,120 **(19.3%)** | 133,853 **(20.3%)** |

### *7.2 Results with optimistic assumptions*

Table 15 shows the time which an individual will expect to spend in each health status over their future lifetime assuming that the optimistic assumptions apply, and that they were healthy (ie in State 0) at the initial age.

It can be seen that, particularly at the younger initial ages, the time expected to be spent healthy increases as compared to Table 2. As expected, the figures for the average time spent disabled are lower than those for Table 2, particularly for the more severely disabled category. It is also worth noting that, for this category, the average time spent disabled is virtually constant regardless of initial age.

**Table 15. Disabled life expectancies (in years) – optimistic assumptions**

| | Initial Age | $\mathring{e}_x^h$ | $\mathring{e}_x^m$ | $\mathring{e}_x^s$ | $\mathring{e}_x$ |
|---|---|---|---|---|---|
| **Males** | 60 | 22.9 | 0.8 | 1.0 | **24.7** |
| | 65 | 18.3 | 0.8 | 1.0 | **20.1** |
| | 70 | 14.2 | 0.7 | 1.0 | **15.9** |
| | 75 | 10.5 | 0.7 | 1.0 | **12.2** |
| | 80 | 7.5 | 0.6 | 1.0 | **9.1** |
| | | | | | |
| **Females** | 60 | 25.4 | 1.3 | 1.5 | **28.2** |
| | 65 | 20.5 | 1.3 | 1.5 | **23.3** |
| | 70 | 16.0 | 1.2 | 1.5 | **18.7** |
| | 75 | 12.1 | 1.0 | 1.4 | **14.5** |
| | 80 | 8.8 | 0.9 | 1.3 | **11.0** |

Table 16 sets out the claim probabilities under the optimistic assumptions. Compared to Table 3 (when central morbidity rates are assumed), the probability of remaining healthy/ slightly disabled is much higher. Most of this increase is due to the reduction in probability that the individual becomes severely disabled. Indeed, moving from the pessimistic to the optimistic assumptions (ie comparing Tables 12 and 16) has very little impact on the probability that the individual will become moderately, but not severely, disabled. Even under the optimistic assumptions, there is still a fairly high probability that an individual will receive an annuity enhancement (ie approximately 0.28 for males and 0.39 for females).

**Table 16. Claim probabilities – optimistic assumptions**

| | **Age at outset** | **Prob (remains healthy/slightly disabled)** | **Prob (becomes moderately but not severely disabled during remaining life)** | **Prob (becomes severely disabled during remaining life)** |
|---|---|---|---|---|
| **Males** | 60 | 0.737 | 0.101 | 0.162 |
| | 65 | 0.729 | 0.102 | 0.169 |
| | 70 | 0.722 | 0.102 | 0.176 |
| | 75 | 0.705 | 0.105 | 0.190 |
| | 80 | 0.686 | 0.107 | 0.207 |
| | | | | |
| **Females** | 60 | 0.629 | 0.142 | 0.229 |
| | 65 | 0.618 | 0.143 | 0.239 |
| | 70 | 0.611 | 0.141 | 0.248 |
| | 75 | 0.608 | 0.138 | 0.254 |
| | 80 | 0.610 | 0.132 | 0.258 |

Tables 17 and 18 show the single premiums required for the level and index-linked annuities, respectively, under the optimistic assumptions.

The premiums for the 1/1/1 annuities are all higher than under the central assumptions, reflecting higher overall life expectancy under the optimistic assumptions.

By contrast, the premiums for the disability-linked annuities are all lower than under the central assumptions. This reflects the fact that the individuals are less likely to trigger the annuity enhancement which more than offsets the fact that the annuity is payable for longer. It is interesting to note that the net effect is that the premiums decrease by at most 4.2% (female, age 80, 1/1.8/3.0 index-linked annuity) even though we have moved from the dynamic equilibrium theory to the compression of morbidity theory. Indeed, if we move from the expansion of morbidity theory to the compression of morbidity theory (ie comparing Tables 12 and 13 with Tables 17 and 18), we find that the premium only decreases by at most 7.2%.

As with the previous tables of single premiums, the percentages shown for females age 60 and 80 represent the increase in single premium required as compared with males at those ages. Since the percentages are similar to those in the previously tables, it can be concluded that moving to the optimistic scenario affects the premium rates for males and females equally.

**Table 17. Single premium required for different types of level annuity – optimistic assumptions**

| | **Age at commencement** | **1/1/1** | **1/1.5/2.5** | **1/1.8/3.0** |
|---|---|---|---|---|
| **Males** | 60 | 121,376 | 126,774 | 128,901 |
| | 65 | 110,113 | 116,526 | 119,038 |
| | 70 | 97,185 | 104,457 | 107,281 |
| | 75 | 83,152 | 91,965 | 95,358 |
| | 80 | 69,128 | 79,607 | 83,607 |
| | | | | |
| **Females** | 60 | 129,031 **(6.3%)** | 136,395 **(7.6%)** | 139,322 **(8.1%)** |
| | 65 | 119,196 | 128,324 | 131,928 |
| | 70 | 107,159 | 118,064 | 122,337 |
| | 75 | 93,331 | 105,741 | 110,561 |
| | 80 | 78,560 **(13.6%)** | 91,852 **(15.4%)** | 96,965 **(16.0%)** |

**Table 18. Single premium required for different types of index-linked annuity – optimistic assumptions**

| | **Age at commencement** | **1/1/1** | **1/1.5/2.5** | **1/1.8/3.0** |
|---|---|---|---|---|
| **Males** | 60 | 178,104 | 189,387 | 193,819 |
| | 65 | 153,742 | 165,919 | 170,673 |
| | 70 | 129,076 | 141,788 | 146,711 |
| | 75 | 105,127 | 119,034 | 124,376 |
| | 80 | 83,456 | 98,368 | 104,048 |
| | | | | |
| **Females** | 60 | 195,292 **(9.7%)** | 211,619 **(11.7%)** | 218,077 **(12.5%)** |
| | 65 | 171,674 | 189,768 | 196,882 |
| | 70 | 146,629 | 166,049 | 173,628 |
| | 75 | 121,332 | 141,329 | 149,067 |
| | 80 | 97,216 **(16.5%)** | 116,777 **(18.7%)** | 124,275 **(19.4%)** |

## 8. Adopting wider definitions of disability

As suggested in Section 3.5, the wider the definition of “disabled” used by an insurer, the more attractive to a potential consumer the product is likely to be (provided the premium is still affordable). We now consider the impact on the results if we make the definitions of “disabled” more generous.

In this section, we will redefine the three definitions of healthy/disability to be as follows:

“Healthy/slightly disabled” = Lives in OPCS categories 0-4 inclusive.

“Moderately disabled” = Lives in OPCS categories 5-7 .

“Severely disabled” = Lives in OPCS categories 8-10.

These are groupings which have been used by others (see, for example, Mayhew (2001)). We now investigate the results when using these wider definitions of disability as compared to those shown in Sections 4, 5 and 6. We use the central morbidity assumptions throughout this section to be consistent with the earlier sets of results.

Table 19 shows the life expectancy and disabled life expectancy figures which should be compared to Table 2.

As expected, although the total life expectancy figures are the same in the two tables, the expected time spent in the disabled categories is considerably longer with the wider definitions and the expected time spent healthy/slightly disabled is correspondingly shorter. Of the two disabled categories, the largest impact is on the moderately disabled category. For example, females aged 60 would be expected to spend an extra 1.6 years moderately disabled (ie from 1.6 to 3.2 years) but only an extra 0.5 years severely disabled (ie from 2.2 to 2.7

years). As a result, they are expected to spend 2.1 years fewer in a healthy/ slightly disabled state. Overall, they are expected to spend 21.5% of future lifetime in a moderate or severe state of disability, as compared with 13.9% using the narrower definitions of disability. Similar results can be found for the other initial ages, and for males.

**Table 19. Disabled life expectancies (in years) – central assumptions and using wider disabled definition**

| | Initial Age | $\mathring{e}_x^h$ | $\mathring{e}_x^m$ | $\mathring{e}_x^s$ | $\mathring{e}_x$ |
|---|---|---|---|---|---|
| **Males** | 60 | 20.4 | 2.1 | 1.8 | **24.3** |
| | 65 | 16.1 | 1.8 | 1.8 | **19.7** |
| | 70 | 12.3 | 1.6 | 1.7 | **15.6** |
| | 75 | 8.9 | 1.4 | 1.6 | **11.9** |
| | 80 | 6.3 | 1.2 | 1.5 | **9.0** |
| | | | | | |
| **Females** | 60 | 21.5 | 3.2 | 2.7 | **27.4** |
| | 65 | 17.2 | 2.9 | 2.6 | **22.7** |
| | 70 | 13.3 | 2.5 | 2.4 | **18.2** |
| | 75 | 9.9 | 2.1 | 2.2 | **14.2** |
| | 80 | 7.1 | 1.6 | 2.0 | **10.7** |

Table 20 sets out the claim probabilities under the wider definitions of disability. This should be compared with Table 3.

It can be seen that, at all initial ages and for both sexes, the probability that the life remains healthy/slightly disabled reduces by approximately 0.1; the corresponding increase in probability of becoming moderately or severely disabled is fairly evenly split between the two disability categories. The overall effect is that, with the wider disability definitions, the probability of receiving an enhanced annuity increases from approximately 0.50 for females (0.37 for males) to 0.61 for females (0.46 for males). The fact that, with the wider definitions of disability, it becomes more likely than not that females will receive at least one level of annuity enhancement (and almost as likely as not for males) could make the product of great interest to consumers.

**Table 20. Claim probabilities – central assumptions and using wider disabled definition**

| | Age at outset | Prob (remains healthy/slightly disabled) | Prob (becomes moderately but not severely disabled during remaining life) | Prob (becomes severely disabled during remaining life) |
|---|---|---|---|---|
| **Males** | 60 | 0.531 | 0.172 | 0.297 |
| | 65 | 0.537 | 0.167 | 0.296 |
| | 70 | 0.543 | 0.162 | 0.295 |
| | 75 | 0.533 | 0.162 | 0.305 |
| | 80 | 0.532 | 0.158 | 0.310 |
| | | | | |
| **Females** | 60 | 0.375 | 0.219 | 0.406 |
| | 65 | 0.382 | 0.214 | 0.405 |
| | 70 | 0.393 | 0.206 | 0.401 |
| | 75 | 0.409 | 0.197 | 0.394 |
| | 80 | 0.430 | 0.186 | 0.384 |

Table 21 shows the single premiums required with the wider disability definitions for the level version of the product. This should be compared to Table 4.

It can be seen that the premiums for the 1/1/1 products are identical since the annuity payments are not dependent on disability. The numbers in parentheses in the 1/1.5/2.5 and 1/1.8/3.0 columns at sample ages 60 and 80 represent the percentage increases in the premiums compared to Table 4.

The premium increases are relatively modest even when the disability definitions have been widened quite substantially and the most generous (1/1.8/3.0) version of the product is being considered (e.g. the maximum increase is 7.1% for females age 80). The reason for this is the same as the reason why the premium increases for the two disability-linked annuities compared to the 1/1/1 versions are so modest – the annuity enhancements due to disability are expected to commence only towards the end of life (ie are discounted many years into the future) and are not expected to be paid for very long. This suggests that if a disability-linked annuity product were to be launched, the most sensible strategy would be to incorporate a relatively wide definition of disability. In this way, the increase in premium required for widening the definition is relatively low but the appeal of the product should increase considerably since it is very likely that, at some point, at least one level of enhancement would be paid to the policyholder.

The percentage increase in premium as a result of widening the disability definition increases with age. This is because, at older starting ages, individuals spend a greater proportion of their future life in a state of moderate or severe disability. Therefore, the disability definition becomes more significant.

**Table 21. Single premium required for different types of level annuity – central assumptions and using wider disabled definition**

| | **Age at commencement** | **1/1/1** | **1/1.5/2.5** | | **1/1.8/3.0** | |
|---|---|---|---|---|---|---|
| **Males** | 60 | 120,697 | 131,337 | **(2.5%)** | 135,763 | **(3.6%)** |
| | 65 | 109,418 | 121,570 | | 126,565 | |
| | 70 | 96,431 | 109,919 | | 115,380 | |
| | 75 | 82,272 | 98,243 | | 104,625 | |
| | 80 | 68,507 | 85,568 | **(4.8%)** | 92,289 | **(6.6%)** |
| | | | | | | |
| **Females** | 60 | 127,943 | 142,609 | **(3.2%)** | 148,810 | **(4.7%)** |
| | 65 | 118,031 | 135,374 | | 142,613 | |
| | 70 | 105,993 | 125,688 | | 133,790 | |
| | 75 | 92,255 | 113,518 | | 122,128 | |
| | 80 | 77,655 | 99,270 | **(5.1%)** | 107,879 | **(7.1%)** |

Table 22 sets out the corresponding premium information for the index-linked versions of the annuities.

It can be seen that similar comments apply to those made regarding Table 21. In addition, it should be noted that the premium increases in Table 22 are all greater than their counterparts shown in Table 21. The reason for this is that the effect of allowing for index-linked increases to benefits in payment is to use a lower effective rate of interest to discount the payments. Therefore, the annuity enhancements, which are expected to occur towards the end of life, become more valuable when index-linked increases are applied. A similar point was made in Section 6.3 to explain the reason why the percentage increase in the premium for disability-linked annuities compared to the traditional life annuity was higher for index-linked than level annuities.

**Table 22. Single premium required for different types of index-linked annuity – central assumptions and using wider disabled definition**

| | Age at commencement | 1/1/1 | 1/1.5/2.5 | | 1/1.8/3.0 | |
|---|---|---|---|---|---|---|
| **Males** | 60 | 176,259 | 198,801 | **(3.4%)** | 208,064 | **(4.8%)** |
| | 65 | 152,065 | 175,300 | | 184,746 | |
| | 70 | 127,495 | 150,933 | | 160,336 | |
| | 75 | 103,575 | 128,270 | | 138,064 | |
| | 80 | 82,453 | 106,479 | **(5.4%)** | 115,883 | **(7.2%)** |
| | | | | | | |
| **Females** | 60 | 192,238 | 224,581 | **(4.2%)** | 238,028 | **(6.0%)** |
| | 65 | 168,833 | 202,891 | | 216,901 | |
| | 70 | 144,139 | 178,809 | | 192,893 | |
| | 75 | 119,299 | 153,143 | | 166,701 | |
| | 80 | 95,687 | 127,120 | **(5.6%)** | 139,529 | **(7.6%)** |

## 9. Effect of changing initial health status

Until now, it has been assumed that the underwriting being employed by the insurer has been such that every individual accepted for cover is healthy (ie in State 0) at the outset. It is interesting to revisit this assumption and analyse the effect on the premium which should be charged if the individual were, in fact, in State 4 at the outset. For this part, we use mortality assumptions which are consistent with those described in Section 3.2, but applicable to a life in State 4. We also assume throughout this analysis that the central morbidity assumptions apply and that the original (narrow) disability definitions are being used.

Table 23 shows the disabled life expectancy figures assuming that individuals are in State 4 at commencement. When compared to Table 2 (when lives are assumed to be in State 0), it is perhaps surprising that the figures are not smaller. The reason they are not is that there is an assumption within the morbidity basis that 10% of lives who do not deteriorate in health during a year will recover by one OPCS category during that year. This relatively strong assumption is based on data in both the UK and the USA suggesting that a number of people do recover from disabilities during a year. For the full discussion and justification of this assumption, the reader is referred to Rickayzen & Walsh (2002).

The impact of this assumption is that, with a group of lives starting in State 4, a reasonable number of them will be expected to recover by at least one category during the first few years of the policy.

When compared with Table 2, it can be seen that expected time spent in the two disabled categories increases since the life is starting in a state of disability closer to the moderately disabled grouping (OPCS categories 7- 8.5) than if they were initially in State 0. The overall life expectancy reduces as a result, and the impact of this new assumption is greater on females than males. For example, females aged 60 are expected to spend 0.9 years more either moderately or severely disabled, and with a reduced overall life expectancy of 0.8 years; whereas, males aged 60 are expected to spend only 0.5 years more either moderately disabled or severely disabled, and with a 0.5 year reduction in overall life expectancy.

**Table 23. Disabled life expectancies (in years) – Initial State 4**

| | Initial Age | $\mathring{e}_x^h$ | $\mathring{e}_x^m$ | $\mathring{e}_x^s$ | $\mathring{e}_x$ |
|---|---|---|---|---|---|
| **Males** | 60 | 20.8 | 1.3 | 1.8 | **23.9** |
| | 65 | 16.4 | 1.2 | 1.7 | **19.3** |
| | 70 | 12.5 | 1.1 | 1.7 | **15.3** |
| | 75 | 9.0 | 1.0 | 1.6 | **11.6** |
| | 80 | 6.3 | 0.9 | 1.5 | **8.7** |
| | | | | | |
| **Females** | 60 | 21.9 | 2.0 | 2.7 | **26.6** |
| | 65 | 17.4 | 1.9 | 2.6 | **21.9** |
| | 70 | 13.3 | 1.7 | 2.5 | **17.5** |
| | 75 | 9.7 | 1.5 | 2.4 | **13.6** |
| | 80 | 7.0 | 1.2 | 2.1 | **10.3** |

Tables 24 and 25 show the single premiums that should be charged to a life in State 4 at each initial age for the level and index-linked versions of the annuity, respectively. These should be compared to Tables 4 and 5 which show the premiums that would actually be charged according to the assumptions in Section 3. The figures in parentheses for the two disability-linked annuities show the percentage increase in premium theoretically required, given the individual's actual state of health.

**Table 24. Single premium required for different types of level annuity – Initial State 4, central assumptions**

| | Age at commencement | 1/1/1 | 1/1.5/2.5 | | 1/1.8/3.0 | |
|---|---|---|---|---|---|---|
| **Males** | 60 | 119,570 | 129,538 | **(1.1%)** | 133,429 | **(1.9%)** |
| | 65 | 108,208 | 119,835 | | 124,343 | |
| | 70 | 95,240 | 108,401 | | 113,459 | |
| | 75 | 80,973 | 96,954 | | 103,048 | |
| | 80 | 67,270 | 84,790 | **(3.9%)** | 91,411 | **(5.6%)** |
| | | | | | | |
| **Females** | 60 | 126,063 | 139,998 | **(1.3%)** | 145,470 | **(2.3%)** |
| | 65 | 115,870 | 132,821 | | 139,429 | |
| | 70 | 103,684 | 123,504 | | 131,167 | |
| | 75 | 89,983 | 112,023 | | 120,465 | |
| | 80 | 75,613 | 98,684 | **(4.5%)** | 107,431 | **(6.6%)** |

**Table 25. Single premium required for different types of index-linked annuity – Initial State 4, central assumptions**

| | Age at commencement | 1/1/1 | 1/1.5/2.5 | | 1/1.8/3.0 | |
|---|---|---|---|---|---|---|
| **Males** | 60 | 173,809 | 194,106 | **(0.9%)** | 201,981 | **(1.7%)** |
| | 65 | 149,676 | 171,167 | | 179,455 | |
| | 70 | 125,333 | 147,587 | | 156,101 | |
| | 75 | 101,451 | 125,589 | | 134,759 | |
| | 80 | 80,606 | 104,794 | **(3.7%)** | 113,906 | **(5.4%)** |
| | | | | | | |
| **Females** | 60 | 187,954 | 217,450 | **(0.9%)** | 228,927 | **(2.0%)** |
| | 65 | 164,405 | 196,488 | | 208,898 | |
| | 70 | 139,865 | 173,633 | | 186,599 | |
| | 75 | 115,480 | 149,576 | | 162,559 | |
| | 80 | 92,546 | 125,292 | **(4.1%)** | 137,647 | **(6.1%)** |

The premiums for the 1/1/1 version of the annuity are all lower for Tables 24 and 25 than Tables 4 and 5. This is as expected since this annuity is the standard life annuity and is expected to be paid for a shorter period of time if the individual's initial health status is poor.

As usual in this analysis, the 1/1.5/2.5 and 1/1.8/3.0 versions of the annuity are of more interest. It can be seen that the increases in premium required are of a similar order of magnitude for the level and index-linked annuities. The percentage increases with age since, at older ages, individuals are expected to spend a greater proportion of their future lifetime in a state of disability. The percentage increase is greater for females than males (again, since the proportion of time spent disabled tends to be greater for females than males – see Section 4).

The percentage increase for females purchasing the level 1/1.8/3.0 annuity ranges from 2.3% at age 60 to 6.6% at age 80. These percentages are relatively modest, given the difference between State 0 and State 4 health status, for two reasons: firstly, as explained at the start of this section, the allowance for recovery being made; and secondly, the effect of starting in a state of disability (and therefore increasing the likelihood that enhanced annuity payments will be made) will be mitigated, to some extent, by the fact that the overall life expectancy will be reduced.

The conclusion which can be made following this analysis is that the insurer can afford to be relatively flexible as regards the underwriting requirements. This view is consistent with that of Murtaugh et al (2001) mentioned in the introduction to this paper (Section 1).

## 10. Consideration of last-survivor disability-linked annuities

Karlsson et al (2006) recognised that an individual's requirements for formal long term care depend, inter alia, on whether or not the individual has access to informal care support from a partner. For example, if an individual whose health is poor has a spouse who is willing and healthy enough to provide informal care, then the individual will not need to purchase as much formal care to satisfy his/her care needs.

This observation leads to the concept that individuals might wish to consider purchasing another type of annuity: a disability-linked last-survivor annuity. This is a last survivor annuity where the income level depends on the health statuses of the two individuals involved. It is a natural extension of the disability-linked annuity considered earlier in this paper. Pauly (1990) acknowledges that couples are likely to find LTC insurance products attractive which enable one life to receive costly nursing care without severely depleting the income of the other. This type of annuity should meet that objective.

To analyse the product further, we consider, for illustrative purposes, an annuity having the features described in Table 26.

**Table 26: Last survivor disability-linked annuity payment schedule**

| **Health statuses of the two lives** | **Total amount payable (£ p.a.)** |
|---|---|
| | |
| Both lives in OPCS 0-7 | 10,000 |
| One life in OPCS 0-7,<br>one in OPCS 8-10 | 25,000 |
| Both lives in OPCS 8-10 | 40,000 |
| One life in OPCS 0-7, one dead | 10,000 |
| One life in OPCS 8-10, one dead | 25,000 |

With this product design, it can be seen that a standard last survivor annuity of £10,000 p.a. is payable whilst both lives are in reasonable health. The annuity increases by £15,000 p.a. to £25,000 p.a. whilst one life is in serious ill health (ie OPCS categories 8-10), and by a further £15,000 p.a. to £40,000 p.a. whilst both lives are in serious ill health.

Table 27 sets out the periods of time which lives are expected to spend in different states of disability for different initial ages, assuming that the two lives are the same age and are both healthy (ie in State 0) at the outset. It is assumed that mortality between the two lives is independent. The central set of morbidity assumptions has been adopted.

**Table 27. Expected time (in years) spent by two lives in different states of disability – central assumptions**

| Age of both lives at commencement | Health Statuses of the two lives | | | | | |
|---|---|---|---|---|---|---|
| | Both 0-7 | One 0-7 One 8-10 | Both 8-10 | One 0-7 One dead | One 8-10 One dead | **Total** |
| | | | | | | |
| 60 | 18.437 | 1.991 | 0.154 | 8.365 | 2.240 | **31.187** |
| 65 | 14.267 | 1.799 | 0.156 | 7.742 | 2.237 | **26.201** |
| 70 | 10.626 | 1.577 | 0.156 | 6.893 | 2.198 | **21.450** |
| 75 | 7.539 | 1.389 | 0.158 | 5.848 | 2.120 | **17.054** |
| 80 | 5.217 | 1.154 | 0.151 | 4.707 | 1.964 | **13.193** |

It can be seen, for example, that if both lives are healthy and age 60 at the outset, the expected length of time before the death of the second life is 31.187 years. During this period, 18.437 years are expected to be spent with both lives in OPCS categories 0-7, 8.365 years with one life in OPCS categories 0-7 and the other no longer alive, and 4.385 years with at least one life in poor health.

The expected times shown in the 3$^{rd,}$ 4$^{th}$ and 6$^{th}$ columns are fairly constant. In other words, the times spent with at least one life in OPCS categories 8-10 are reasonably impervious to the age at commencement. For example, the expected time spent in OPCS categories 8-10 by a widow/widower is approximately 2.2 years regardless of the age at commencement.

Table 28 sets out the single premiums required for the level disability-linked last survivor annuity described in Table 26 compared with the level standard last survivor annuity. Table 29 sets out the same information for the index-linked version of the annuity.

**Table 28. Single premium (in £) required for level last survivor annuity – central assumptions**

| **Age of both lives at commencement** | **Standard** | **Disability-linked** | **% increase in premium** |
|---|---|---|---|
| | | | |
| 60 | 136,107 | 153,194 | 12.6 |
| 65 | 128,087 | 148,567 | 16.0 |
| 70 | 117,731 | 141,526 | 20.2 |
| 75 | 105,055 | 132,610 | 26.2 |
| 80 | 90,828 | 120,361 | 32.5 |

**Table 29. Single premium (in £) required for index-linked last survivor annuity – central assumptions**

| Age of both lives at commencement | Standard | Disability-linked | % increase in premium |
|---|---|---|---|
| | | | |
| 60 | 211,064 | 249,393 | 18.2 |
| 65 | 188,907 | 229,858 | 21.7 |
| 70 | 164,724 | 207,389 | 25.9 |
| 75 | 139,334 | 183,475 | 31.7 |
| 80 | 114,417 | 157,399 | 37.6 |

The first thing to note is that the percentage increase in standard premium required to support the disability-linked annuity is substantially higher than in the case of the single life disability-linked annuity considered earlier. Leaving aside the fact that the benefits offered by the two annuities are not directly comparable, it is to be expected that the percentage increase for last survivor disability-linked annuities will tend to be relatively high. This is because, with two lives involved, there is an increased likelihood that an annuity enhancement will eventually be made since it only requires one of the lives to become disabled for this to occur. Notwithstanding this level of percentage increase in premium required, last survivor disability-linked annuities could still be attractive to consumers since they do allow for the health status of each partner in considering future long term care provision.

It can be seen that the percentage increase in the standard premium required for the disability-linked annuity increases substantially with age at commencement. This reflects the fact that the proportion of time spent receiving the basic level of annuity (ie when neither life is in OPCS categories 8-10) decreases with age. For example, Table 27 shows that the proportion of expected time spent with neither life severely disabled decreases with age at commencement from 86% at age 60 to 75% at age 80.

## 11. Affordability of products

It must be recognised that the premiums required for the products described in this paper represent significant lump sums. It is appropriate, therefore, to consider the extent to which such premiums would be affordable to the target group: the older members of the UK population.

Using the information contained within the English Longitudinal Study of Ageing (ELSA) regarding wealth and disability, it is possible to estimate the wealth distribution of the UK population who are healthy and over the age of 65. For information about the first two waves of the ELSA dataset, see Marmot et al (2006).

Table 30 shows, for each asset band (with the value of the house excluded in the first set of columns and included in the second set), the estimated numbers of people over age 65 who report that they have no ADL impairments,. The information is shown separately for single people and couples. The percentages shown are the estimated proportions of the UK population over age 65 within each asset bracket which these numbers represent. In other words, we estimate that 80% of the UK population over age 65 with assets (including housing wealth) of over £1 million have no ADL impairments.

**Table 30. Estimated UK population over age 65 who have no ADL impairments in different wealth brackets (in 000s)**

| **Assets (£000s)** | **Excluding housing** | | | | **Including housing** | | | |
|---|---|---|---|---|---|---|---|---|
| | Single | Married | Total | % of population | Single | Married | Total | % of population |
| 0-50 | 2,071 | 2,209 | 4,280 | 70 | 1,084 | 673 | 1,757 | 66 |
| 51-200 | 442 | 861 | 1,303 | 81 | 1,026 | 1,411 | 2,437 | 74 |
| 201-400 | 87 | 233 | 320 | 78 | 392 | 883 | 1,275 | 79 |
| 401-600 | 21 | 93 | 114 | 83 | 80 | 274 | 354 | 77 |
| 601-800 | 5 | 37 | 42 | 82 | 30 | 106 | 136 | 77 |
| 801-1000 | 2 | 13 | 15 | 83 | 10 | 55 | 65 | 80 |
| >1000 | 7 | 26 | 33 | 94 | 13 | 70 | 83 | 80 |
| **Total** | **2,635** | **3,472** | **6,107** | **73** | **2,635** | **3,472** | **6,107** | **73** |

It should be noted that the percentage generally increases with the level of assets held. This pattern is consistent with the well known link between socio-economic status and health - see, for example, Fuchs (2004). It is interesting to observe that such a high proportion of older people are in good health, particularly at the wealthier end of the spectrum.

It can be seen that, for example, we estimate that 524,000 people over age 65 have assets in excess of £200,000 if housing wealth is excluded, and this number increases to 1,913,000 if housing wealth is included. When assessing potential demand for the disability-linked annuity, the latter is particularly relevant since there is a tendency for pensioners to down size their property once their children have left home (ie release some equity in property by moving to a smaller and cheaper house). These figures, together with the fact that such a high proportion of the elderly appear to be in good enough health to be eligible to purchase a disability-linked annuity, suggests that the potential demand for such a product could be reasonable.

Similar comments can be made about the potential demand for the type of last survivor product discussed in Section 10 by considering the information contained for couples within Table 30. It should be noted that these figures represent the estimated number of individuals who are married and have no ADL impairments - the figures do not represent the number of people who are married where both people have no ADL impairment (as would be required to purchase the last survivor product described earlier). Nevertheless, in view of the high percentage of the population who are in good health and the relatively high proportion of people who are married (as demonstrated in Table 30), we can again conclude that there would be a number of people with the means and necessary health status to purchase such a last survivor product.

## 12. Conclusions

In this paper, we have analysed different types of disability-linked annuities, both single life and last survivor, in order to examine their main characteristics and assess their suitability as potential new products in the annuity and LTC market. In view of the ageing of the UK population over recent decades, and the expectation that this will continue in the future, it is important that innovative insurance products should be developed which might enable individuals to meet their future LTC requirements.

Some types of disability-linked annuity products have been issued in the UK in the past; however, they could not be issued in a tax efficient way and were therefore unattractive to consumers.

The simplification in tax regime that has occurred in the UK post-6th April 2006 suggests that the type of product being described in this paper could be issued more tax efficiently. Correspondence with HR Customs and Revenue suggests that it could either be issued as a pensions product (in which case it could be purchased out of a consumer's pensions account, which has been built up out of contributions from pre-tax income) or it could be issued as a combination of a Purchase Life Annuity and a PHI benefit (in which case a substantial proportion of the benefits could be paid free of income tax). Clearly, the precise tax position would need to be ascertained before such a product could be issued.

We would endorse the views of Warshawsky et al (2002) that such products possess the following attractive qualities:

- The fact that the longevity risk (associated with the life annuity component) and the morbidity risk (associated with the LTC component) operate in opposite directions helps to minimise the overall risk of adverse selection. This is turn enables the underwriting requirements to be relaxed to some extent, and helps to reduce the size of the premiums.

- The annuity enhancement upon onset of disability will help to defray the associated care costs. This should offer reassurance to the policyholder that the care costs can be met if and when they are required, and should support any bequest motive in respect of the policyholder's beneficiaries. Furthermore, the enhancement might enable the policyholder to remain living in their home to receive care, rather than having to move into a residential care home.

- As a way of obtaining some LTC insurance cover, a disability-linked annuity is likely to be viewed in a more positive light than traditional stand alone LTC insurance since the individual knows that they will receive a benefit whilst they are healthy, with an increase in the benefit occurring should their health deteriorate markedly. In other words, a change in emphasis occurs with this type of product.

Initially, we used a central set of assumptions - in particular, incorporating the central set of morbidity trend assumptions used by Rickayzen and Walsh (2002) - to calculate various quantities. These included the expected times spent in different states of disability, the probabilities of each level of annuity enhancement being eventually triggered and the single premiums required.

We found that, at all initial ages, individuals are expected to spend the majority of their lives in the healthy/slightly disabled state, with a short period of more serious disability towards the end of life. As a result, the increase in premium required for a disability-linked annuity as compared to a standard life annuity without enhancement on disability is relatively modest. The period of time spent severely disabled tends to be impervious to the initial age at around 1.4 years for males and 2 years for females.

We found that the premiums are higher for females than males since, not only are females expected to live longer than males, but they tend to spend more time in moderate and severe states of disability.

The probabilities that an annuity enhancement would eventually be paid were relatively high (eg approximately 0.37 for males and 0.50 for females) which should make such a product attractive to consumers, particularly females.

We then considered the results if pessimistic assumptions consistent with the expansion of morbidity were used. We found that the probabilities of the enhancement eventually being paid increased significantly to approximately 0.44 for males and 0.58 for females. However, the effect on the single premium was relatively small. This is because, although there is upward pressure on the premium from the increased likelihood that an individual will become disabled, the overall term of the annuity is shortened due to the consequent reduced life expectancy.

Similar arguments applied when optimistic assumptions consistent with the compression of morbidity were used. The probabilities reduced significantly to approximately 0.28 and 0.39 for males and females, respectively; however, the premiums did not decrease substantially since the expected term expands due to increased life expectancy.

We considered the case if wider, more generous, definitions of disability were adopted than those originally assumed which had been incorporated as a proxy to current LTC claim trigger points.

The product should become more attractive to consumers since it becomes more likely that at least one level of annuity enhancement will be paid, and the individual would be expected to spend longer receiving each level of enhancement under these wider definitions. For example, for females aged 60, the probability that an annuity enhancement will be paid increases from 0.51 to 0.62, and the times spent moderately and severely disabled are expected to increase by 1.6 years and 0.5 years, respectively.

We found, however, that the knock-on effect on the single premium was modest. This is, again, due to the fact that the periods of time spent disabled are relatively short and concentrated towards the end of life (and therefore heavily discounted).

These results suggest that if a disability-linked annuity product were to be launched, the most sensible strategy would be to incorporate a relatively wide definition of disability. In this way, the resulting increase in premium required for widening the definition is relatively small but the appeal of the product increases considerably since it is very likely that, at some point, at least one level of enhancement would be paid to the policyholder.

In order to analyse the importance of the underwriting function, we considered the effect on the premium theoretically required if the individual was, at commencement, already slightly disabled. We found that the implied percentage increase in premium was surprisingly low for two reasons. Firstly, an allowance for recovery is made in the assumptions; and secondly, the effect of starting in a state of disability (and therefore increasing the likelihood that enhanced annuity payments will eventually be made) will be mitigated, to some extent, by the fact that the overall life expectancy is reduced.

Since a key factor for individuals in deciding their formal LTC needs is the extent to which they can rely on informal care from a spouse, we analysed a last survivor type of disability-linked annuity.

We found that the expected time spent with at least one life being severely disabled was almost constant, regardless of the initial ages of the lives. We compared the premiums for this type of product with those for the corresponding standard last survivor annuity and found that the percentage difference increases substantially with age at commencement. This reflects the fact that the proportion of time spent receiving the basic level of annuity (ie when neither life is severely disabled) decreases with age.

Finally, we examined the potential market for disability-linked annuity products. Our analysis suggests that a high proportion of the older UK population are in sufficiently good health to be eligible to take out such a policy. This is particularly true amongst the wealthier citizens (ie the group most able to afford the premiums).

Clearly, the premiums will be large since the product provides a much higher level of benefits than a more traditional life annuity, and by no means everyone will be able to afford such premiums. However, we found that the premiums appear to be affordable for a reasonably large number of people, particularly if individuals are willing to trade down their property to release sufficient equity.

In conclusion, the disability-linked annuity seems to be worthy of attention by insurers as a product providing a certain level of LTC insurance cover, whilst not having the same negative connotations as traditional stand alone LTC insurance appears to have amongst consumers.

The fact that the longevity risks and morbidity risks contained within the product work in opposite directions should make the overall risk more controllable. This facet has other positive effects: the underwriting requirements need not be so stringent, the disability definitions are not so critical and the single premiums appear to compare favourably with their traditional life annuity counterparts when the additional LTC benefits being offered are taken into account.

The fact that the probability that an annuity enhancement will eventually be paid is relatively high, particularly in the case of females, should make the product marketable. In addition, the product appears to have benefited from recent changes in tax regime and should therefore be even more appealing to consumers.

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