Addendum to “Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving”

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Addendum to "Analytic and bootstrap estimates of prediction errors in claims reserving"

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Abstract

In England and Verrall (1999), an appropriate residual definition was considered for use in a bootstrap exercise to provide a computationally simple method of obtaining reserve prediction errors for the chain ladder model. However, calculation of the first two moments of the predictive distribution only was considered. In this paper, the method is extended by using a two-stage process: bootstrapping to obtain the estimation error and simulation to obtain the process error. This has the advantage of providing realisations from the whole predictive distribution, rather than just the first two moments.

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1. Introduction

Although stochastic claims reserving methods were introduced about two decades ago, they are still only used by a limited number of practitioners. A number of reasons for this could be suggested, including: a general lack of understanding of the methods, lack of flexibility in the methods, lack of suitable software for ease of use and so on. However, the main reason is probably lack of need for the methods when traditional methods suffice. More recently, a greater interest has been expressed in estimating the downside potential of claims reserves, in addition to the best estimate. For this, it is necessary to be able to estimate the variability of claims reserves, and ideally, to be able to estimate a full predictive distribution from which percentiles (or other measures) of that distribution can be obtained.

To date, the primary focus of stochastic claims reserving methods has been obtaining the reserve root mean square error of prediction (prediction error) in addition to the mean. Essentially, this provides the first two moments of the predictive distribution only. Further assumptions are usually necessary if other statistics are required. For example, in Mack (1993), the distribution of the underlying data is not fully specified, only the first two moments. Although formulae are developed for the reserve prediction errors, further assumptions that the predictive distribution of the reserves is approximately Normal or Lognormal are required in the calculation of confidence intervals.

One area where stochastic claims reserving models are required is dynamic financial analysis (DFA), where cash flows of an insurance enterprise are simulated to help with business planning, capital modelling, and risk profiling generally. One component of risk carried by an insurance operation lies within its outstanding claims reserves, and simulating reserve movements is an ingredient of a full DFA exercise. Methods are therefore required for DFA which enable a predictive distribution of future claim payments to be obtained. Clearly, it is desirable if this can be performed in a consistent manner, such that when the simulated future claim payments are summed to give origin period reserves, or total reserves, the prediction error of those sums matches their analytic equivalents.

This paper augments England and Verrall (1999) and extends the methods to obtain a full predictive distribution of reserve estimates. This is achieved by simulating the process error in addition to using bootstrapping to obtain the estimation error. The procedure is simple to implement, and all calculations can be performed within a spreadsheet; there is no need for sophisticated software.

2. Methodology

We focus on the model described by Renshaw and Verrall (1998), who proposed modelling the incremental claims using an "over-dispersed" Poisson distribution. Adopting the notation of England and Verrall (1999) where the incremental claims for origin year i in development year j are denoted C_{ij}, then

$$E[C_{ij}] = m_{ij} \quad \text{and} \quad Var[C_{ij}] = \phi E[C_{ij}] = \phi m_{ij} \tag{2.1}$$

$$\log(m_{ij}) = \eta_{ij} \tag{2.2}$$

$$\eta_{ij} = c + \alpha_i + \beta_j \qquad \alpha_1 = \beta_1 = 0 \tag{2.3}$$

Equations 2.1, 2.2 and 2.3 define a generalised linear model in which the response is modelled with a logarithmic link function and the variance is proportional to the mean (hence "over-dispersed" Poisson). The parameter ϕ is an unknown scale parameter estimated as part of the fitting procedure. With certain positivity constraints, predicted values and reserve estimates from this model are exactly the same as those from the chain ladder model.

Obtaining reserve standard errors (the estimation error component of the prediction error) using the bootstrap procedure involves sampling with replacement from an appropriate set of residuals to obtain a large number of sets of pseudo data. The chain ladder model can be fitted to each set of pseudo data, and reserve estimates obtained. The standard deviation of the set of reserve estimates obtained in this way provides a bootstrap estimate of the standard error (estimation error). The particular residual definition used in England and Verrall (1999) is the unscaled Pearson residual, r_P, defined as

$$r_P = \frac{C-m}{\sqrt{m}}. \tag{2.4}$$

It is unscaled in the sense that it does not include the scale parameter ϕ which is not needed when performing the bootstrap calculations, but is needed when considering the process error. An estimate of the scale parameter consistent with the definition of residuals is the Pearson scale parameter, given by

$$\phi_P = \frac{\sum r_P^2}{n-p} \tag{2.5}$$

where n is the number of data points in the sample, p is the number of parameters estimated and the summation is over the number (n) of residuals. It can be seen that an increased number of parameters used in fitting the model introduces a penalty (*ceteris paribus*).

In England and Verrall (1999), the reserve prediction error is given by

$$PE_{bs}(R) = \sqrt{\phi_P R + \frac{n}{n-p}(SE_{bs}(R))^2} \tag{2.6}$$

where R is an origin year or total reserve, and $SE_{bs}(R)$ is the bootstrap standard error of the reserve estimate. The process variance, $\phi_P R$, is calculated analytically and simply added to the estimation variance (which is suitably scaled to take account of the degrees of freedom).

The purpose of this paper is to describe a methodology which replaces the analytic calculation of the process error with a simulation approach, thereby providing a way of simulating a complete predictive distribution. The methodology proceeds by interrupting the bootstrapping procedure at each iteration and drawing a random observation from the underlying process distribution, conditional on the bootstrap value. To explain the procedure, it is necessary to focus on the triangle of observed data (the past triangle), and the triangle of unknown future payments (the future triangle). The bootstrap procedure involves sampling with replacement from the residuals to obtain a new past triangle of residuals, from which a past triangle of pseudo data is obtained. The chain ladder model is then fitted to the pseudo data and a future triangle of (incremental) payments obtained. For each cell in the future triangle, a random observation is drawn from the underlying process distribution, where the mean is the value obtained from the bootstrap iteration, and the variance is given in equation 2.1, with the scale parameter calculated using equation 2.5. Strictly, a random sample should be drawn from an over-dispersed Poisson distribution (see Section 3). The simulated values in the future triangle can then be summed to provide a realisation of the origin period and total reserves. The process is then repeated a large number of times to provide a predictive distribution. An example showing the computations required can be found in Appendix A. The procedure is performed by completing the following steps:

- Obtain the standard chain ladder development factors from cumulative data
- Obtain cumulative fitted values for the past triangle by backwards recursion as described in Appendix A
- Obtain incremental fitted values for the past triangle by differencing
- Calculate the unscaled Pearson residuals for the past triangle using equation 2.4
- Calculate the Pearson scale parameter, ϕ, using equation 2.5
- Adjust the Pearson residuals using equation 3.1 (see Section 3)
- Begin iterative loop, to be repeated *N* times (*N*=1000, say)
  - Resample the adjusted residuals with replacement, creating a new past triangle of residuals
  - For each cell in the past triangle, solve equation 2.4 for *C*, giving a set of pseudo incremental data for the past triangle
  - Create the associated set of pseudo cumulative data
  - Fit the standard chain ladder model to the pseudo cumulative data
  - Project to form a future triangle of cumulative payments
  - Obtain the corresponding future triangle of incremental payments by differencing, to be used as the mean when simulating from the process distribution
  - For each cell (i,j) in the future triangle, simulate a payment from the process distribution with mean $\tilde{m}_{ij}$ (obtained at the previous step), and variance $\phi\tilde{m}_{ij}$, using equation 2.1 and the value of ϕ calculated previously
  - Sum the simulated payments in the future triangle by origin year and overall to give the origin year and total reserve estimates respectively
  - Store the results, and return to start of iterative loop.

The set of stored results forms the predictive distribution. The mean of the stored results should be compared to the standard chain ladder reserve estimates to check for bias. The standard deviation of the stored results gives an estimate of the prediction error.

It can be seen that essentially the bootstrap procedure provides a distribution of "means" in the future triangle, and the process error is replicated by sampling from the underlying distribution conditional on those means. The result is a simulated predictive distribution of future payments which when summed appropriately provides a predictive distribution of reserve estimates, from which summary statistics can be obtained.

3. Practical Issues

In England and Verrall (1999), an adjustment was made to the bootstrap standard error to take account of the degrees-of-freedom (see equation 2.6 above). This was to enable a comparison to be made with the results obtained analytically. It is desirable to make a similar adjustment here too, but to enable the adjustment to follow through to the predictive distribution automatically, it is suggested that the residuals are adjusted prior to implementing the procedure. That is, replace r_P by r'_P, where

$$r'_P = \sqrt{\frac{n}{n-p}} \times \frac{C-m}{\sqrt{m}}. \tag{3.1}$$

Since the mean of the residuals should be close to zero, this has the effect of inflating the variance while leaving the mean largely unchanged.

To simulate from an over-dispersed Poisson distribution requires a trick. To obtain a realisation from an over-dispersed Poisson distribution with mean m and variance ϕm, sample from a Poisson distribution with mean m/ϕ and multiply by ϕ. This has a few disadvantages. Observations will always be multiples of ϕ, which for large ϕ may not be desirable. Furthermore, for non-integer values of ϕ, observations will be non-integer.

If this feature of the over-dispersed Poisson distribution is considered unacceptable, a pragmatic alternative is to sample from a Gamma distribution parameterised such that the mean is m and the variance is ϕm. The variance remains proportional to the mean, but the simulated values have the advantage of being on a continuous scale and not simply multiples of ϕ. However, this changes the shape of the predictive distribution, while leaving the first two moments unchanged.

The bootstrap procedure can occasionally result in a negative mean from which the process error is simulated. Again, this is undesirable and a number of adjustments can be suggested. A practical approach is to simulate an observation from a distribution with mean $abs(m)$, then subtract $2 \times m$ to retain the appropriate scale.

4. Example

Following the example in England and Verrall (1999) the data from Taylor and Ashe (1983) is used, shown here in incremental form.

357848	766940	610542	482940	527326	574398	146342	139950	227229	67948
352118	884021	933894	1183289	445745	320996	527804	266172	425046	
290507	1001799	926219	1016654	750816	146923	495992	280405		
310608	1108250	776189	1562400	272482	352053	206286			
443160	693190	991983	769488	504851	470639				
396132	937085	847498	805037	705960					
440832	847631	1131398	1063269						
359480	1061648	1443370							
376686	986608								
344014									

The chain ladder reserve estimates together with the means of 1000 simulations of the bootstrap approach of England and Verrall (1999), and the approach adopted here, are shown in Table 1. The results are very close, as expected, with differences due to random variation.

The reserve prediction errors as a percentage of the means are shown in Table 2. The analytic estimates and the bootstrap estimates are taken from England and Verrall (1999). The estimates obtained after simulating the process error as described in this paper are also shown. Again, the results are reassuringly close.

The advantage of the two-stage simulation approach outlined in this paper is the availability of the full predictive distribution. Summary statistics of the predictive distribution of the total reserves (using 1000 simulations) are shown in Table 3. A histogram of the distribution is also shown in Figure 1, together with a smoothed density line.

5. Conclusions

Most papers on the topic of stochastic claims reserving consider measures of variability of claims reserves in addition to a "best estimate". This has the advantage of highlighting precision of the estimates, but falls short of providing the full predictive distribution. The predictive distribution of reserve estimates has been considered by very few authors, although some related papers exist, for example Zehnwirth *et al* (1998), which considers predictive aggregate claims distributions for the collective risk model used in risk theory.

In this paper, one method of obtaining a predictive distribution for the basic chain ladder model is proposed. The method has the advantage of simplicity, and all calculations can be performed in a spreadsheet. The method involves a two-stage procedure: first simulate a forecast mean using the bootstrap, then simulate an observation conditional on the mean.

Recent interest in dynamic financial analysis requires the ability to simulate the full predictive distribution of likely outcomes. Various methods are likely to be proposed, depending on the underlying stochastic model adopted. Whatever method is proposed, it is important that simulated results are consistent with their analytic counterparts. For example, if future payments in individual cells of the future triangle are simulated, it is important that they are simulated in such a way that the variability of their sum is consistent with equivalent results obtained analytically.

Although the bootstrap/simulation procedure provides prediction errors which are consistent with their analytic counterparts, the predictive distribution produced in this way might have some undesirable properties. For example, for some origin year reserves, the minimum values of the predictive distribution could be negative. A number of other practical and theoretical difficulties exist (such as those outlined in Section 3), and alternative adjustments and improvements are likely to be suggested.

It is not suggested that the method proposed here is the only one which is consistent with the chain ladder model. Other approaches are likely to be suggested which provide a different predictive distribution while also having the same first two moments.

Appendix A – Calculations required

The first seven triangles in this Appendix are similar to those appearing in England and Verrall (1999) except the residuals in Triangle 4 have been adjusted to take account of the degrees-of-freedom, as described in Section 3.

Triangle 1 below shows the cumulative paid claims from the Example, together with the traditional chain ladder development factors.

Triangle 1 – Observed Cumulative Data

357848	1124788	1735330	2218270	2745596	3319994	3466336	3606286	3833515	3901463
352118	1236139	2170033	3353322	3799067	4120063	4647867	4914039	5339085	
290507	1292306	2218525	3235179	3985995	4132918	4628910	4909315		
310608	1418858	2195047	3757447	4029929	4381982	4588268			
443160	1136350	2128333	2897821	3402672	3873311				
396132	1333217	2180715	2985752	3691712					
440832	1288463	2419861	3483130						
359480	1421128	2864498							
376686	1363294								
344014									

Development Factors

3.4906	1.7473	1.4574	1.1739	1.1038	1.0863	1.0539	1.0766	1.0177	1.0000

The first stage is to obtain the cumulative fitted values, given the development factors. The fitted cumulative paid to date equals the actual cumulative paid to date, so we can transfer the final diagonal of the actual cumulative triangle to the fitted cumulative triangle. The remaining cumulative fitted values are obtained backwards by recursively dividing the fitted cumulative value at time t by the development factor at time $t-1$. The results of this operation are shown in Triangle 2. The incremental fitted values, obtained by differencing in the usual way, are shown in Triangle 3.

Triangle 2 – Cumulative Fitted Values

270061	942678	1647172	2400610	2817960	3110531	3378874	3560909	3833515	3901463
376125	1312904	2294081	3343423	3924682	4332157	4705889	4959416	5339085	
372325	1299641	2270905	3309647	3885035	4288393	4658349	4909315		
366724	1280089	2236741	3259856	3826587	4223877	4588268			
336287	1173846	2051100	2989300	3508995	3873311				
353798	1234970	2157903	3144956	3691712					
391842	1367765	2389941	3483130						
469648	1639355	2864498							
390561	1363294								
344014									

Triangle 3 – Incremental Fitted Values

270061	672617	704494	753438	417350	292571	268344	182035	272606	67948
376125	936779	981176	1049342	581260	407474	373732	253527	379669	
372325	927316	971264	1038741	575388	403358	369957	250966		
366724	913365	956652	1023114	566731	397290	364391			
336287	837559	877254	938200	519695	364316				
353798	881172	922933	987053	546756					
391842	975923	1022175	1093189						
469648	1169707	1225143							
390561	972733								
344014									

The unscaled Pearson residuals, shown in Triangle 4, can be obtained using equation 3.1, together with the observed and fitted incremental data.

Triangle 4 – Unscaled Pearson Residuals (adjusted for degrees-of-freedom correction)

208.80	142.16	-138.36	-385.19	210.42	644.02	-291.11	-121.92	-107.42	0.00
-48.38	-67.38	-59.00	161.62	-219.70	-167.45	311.51	31.04	91.03	
-165.74	95.60	-56.50	-26.79	285.86	-499.07	256.12	72.64		
-114.54	252.05	-228.06	659.00	-483.12	-88.71	-323.74			
227.79	-194.98	151.41	-215.29	-25.45	217.73				
87.97	73.62	-97.06	-226.45	266.13					
96.74	-160.52	133.53	-35.37						
-198.70	-123.50	243.69							
-27.44	17.39								
0.00									

A crucial step in performing the bootstrap is resampling the residuals, with replacement. One such sample is shown in Triangle 5. Notice that residuals may appear more than once when resampled with replacement . Care must be taken to ensure that **all** residuals have an equal chance of being selected.

Triangle 5 – Example set of resampled residuals

31.04	142.16	96.74	243.69	-165.74	-483.12	-114.54	142.16	96.74	0.00
-385.19	-215.29	-228.06	227.79	-27.44	151.41	-198.70	31.04	142.16	
-114.54	133.53	208.80	311.51	-59.00	-88.71	72.64	210.42		
644.02	0.00	217.73	-198.70	-228.06	-138.36	644.02			
227.79	-88.71	243.69	311.51	133.53	-56.50				
-88.71	161.62	227.79	161.62	659.00					
-323.74	95.60	95.60	73.62						
256.12	133.53	-26.79							
-291.11	227.79								
-323.74									

Using the resampled residuals in Triangle 5, together with the original incremental fitted values in Triangle 3, a bootstrap data sample can be calculated by solving equation 2.4. The bootstrap sample associated with the resampled residuals in Triangle 5 is shown in Triangle 6. The associated cumulative sample is shown in Triangle 7, together with development factors obtained by applying the standard chain ladder to the bootstrap data. The bootstrap reserve estimate is obtained from the development factors and cumulative bootstrap sample in the usual way.

Triangle 6 – Incremental Bootstrap data sample

286193	789203	785688	964966	310280	31251	209011	242686	323113	67948
139894	728404	755277	1282688	560338	504122	252260	269157	467262	
302436	1055902	1177041	1356228	530633	347018	414137	356376		
756727	913365	1169610	822131	395047	310083	753151			
468386	756374	1105502	1239931	615957	330217				
301033	1032889	1141774	1147627	1034042					
189191	1070369	1118833	1170167						
645170	1314124	1195494							
208635	1197400								
154134									

Triangle 7 – Cumulative Bootstrap data sample together with development factors

286193	1075396	1861085	2826050	3136330	3167581	3376592	3619278	3942391	4010339
139894	868299	1623575	2906263	3466601	3970723	4222983	4492140	4959402	
302436	1358338	2535379	3891607	4422241	4769259	5183396	5539772		
756727	1670092	2839702	3661833	4056880	4366963	5120115			
468386	1224760	2330262	3570192	4186149	4516365				
301033	1333922	2475696	3623322	4657364					
189191	1259560	2378394	3548560						
645170	1959294	3154788							
208635	1406035								
154134									

Resampled Development Factors

3.686	1.786	1.498	1.168	1.079	1.100	1.068	1.097	1.017

The forecast cumulative values are obtained by applying the appropriate development factors from Triangle 7 to the latest diagonal of cumulative payments in Triangle 7, and are shown in Triangle 8. The associated incremental values are shown in Triangle 9.

Triangle 8 – Forecast cumulative payments

									4010339
								4959402	5044878
							5539772	6079567	6184349
						5120115	5467873	6000661	6104084
					4516365	4968309	5305757	5822748	5923105
				4657364	5025417	5528301	5903783	6479046	6590714
			3548560	4145720	4473340	4920978	5255211	5767277	5866677
		3154788	4724649	5519724	5955926	6551923	6996930	7678709	7811053
	1406035	2511177	3760769	4393639	4740852	5215259	5569479	6112168	6217513
154134	568160	1014732	1519676	1775410	1915714	2107415	2250551	2469844	2512413

Triangle 9 – Forecast incremental payments

								85476
							539794	104783
						347758	532788	103423
					451944	337447	516992	100356
				368053	502884	375482	575263	111668
			597160	327620	447638	334233	512067	99400
		1569861	795074	436202	595998	445006	681779	132344
	1105142	1249592	632871	347212	474408	354220	542689	105345
414026	446573	504943	255734	140304	191702	143136	219293	42568

Finally, for each cell in the future triangle, an observation is drawn at random from the process distribution given in equation 2.1, where the means are taken from Triangle 9 and the scale parameter is 52,601, calculated using equation 2.5. In this example, the Gamma distribution has been used with the appropriate means and variances (as discussed in Section 3), with the random observations shown in Triangle 10. This final step is the crucial part in incorporating the process error, having incorporated the estimation error in the bootstrap procedure.

Triangle 10 - Forecast incremental payments (including process error)

								51625
							523596	51561
						449818	303542	210615
					509740	224982	579429	72698
				437494	745695	266993	617457	14791
			779073	336479	437257	224980	353175	45208
		1308427	1295911	433980	915332	229602	802984	116554
	878549	1733080	643355	350541	319135	479250	161422	87312
391039	404381	388433	175582	212715	101511	136093	89687	88928

The associated reserve estimates for this simulation are obtained by summing the values in Triangle 10, as shown below.

Simulated Reserve estimates (1 simulation)

$i = 2$	51625
$i = 3$	575157
$i = 4$	963975
$i = 5$	1386848
$i = 6$	2082430
$i = 7$	2176172
$i = 8$	5102789
$i = 9$	4652644
$i = 10$	1988369
Total	18980009

The process is completed by repeating N times, where N is large (e.g. $N = 1000$), each time creating a new bootstrap sample and new simulated reserve estimates. The simulated prediction errors of the reserve estimates are simply the standard deviations of the N simulated reserve estimates.

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Table 1 : Estimated Reserves (000's)

	Chain Ladder	Bootstrap Mean (E&V 1999)	Bootstrap/Simulation Mean
i=2	95	96	94
i=3	470	474	475
i=4	710	712	719
i=5	985	996	996
i=6	1419	1425	1422
i=7	2178	2171	2164
i=8	3920	3903	3943
i=9	4279	4268	4246
i=10	4626	4645	4629
Total	18681	18690	18688

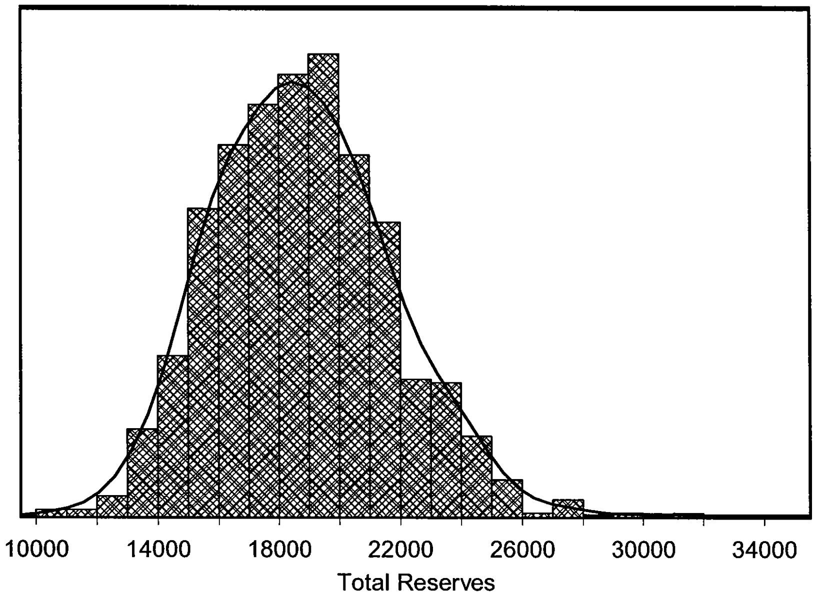
Table 2 : Prediction Errors as % of Reserve Estimate

	Poisson GLM Analytic	Bootstrap (E&V 1999)	Bootstrap/ Simulation
i=2	116	117	117
i=3	46	46	47
i=4	37	36	37
i=5	31	31	31
i=6	26	26	27
i=7	23	23	23
i=8	20	20	21
i=9	24	24	25
i=10	43	43	44
Total	16	16	16

Table 3: Sample Statistics from Predictive Aggregate Distribution of Total Reserves

Number of Observations	1,000
Mean	18,688
Standard Deviation	2,956
Coefficient of Variation	0.158
Skewness	0.350
Kurtosis	0.229
50th Percentile	18,532
75th Percentile	20,640
90th Percentile	22,620
95th Percentile	23,827
99th Percentile	25,967

Figure 1. Predictive Aggregate Distribution of Total Reserves



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