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A note on the α-quantile option

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Abstract

In this communication, we discuss some properties of a class of path dependent options based on the α-quantiles of Brownian motion. In particular we show that such options are well behaved in relation to standard options and comparatively cheaper than an equivalent class of lookback options.

KEY WORDS: α-quantile of Brownian motions with drift, Dassios-Port-Wendel identity, fixed strike lookback option.

1 Introduction

A new type of path-dependent option which can be interpreted as a modification of lookback options is the α-quantile option, proposed recently by Miura (1992). Its payoff at maturity is defined by the order statistics of the underlying asset price; in particular, this order statistic or, better, the α-percentile point of the stock price for $0 < \alpha < 1$ can be thought of as the level at which the price stays below for $100\alpha\%$ of the time during the option's contract period. The problem of pricing such an option has motivated studies concerning the properties of the distribution function of the α-percentile of the stochastic process driving the stock price, which has been investigated mainly by Akahori (1995), Dassios (1995, 1996), Takács (1996), Yor (1996) and Doney and Yor (1998). Closed formulas in the traditional Black-Scholes framework for the price of this option have been obtained both by Dassios (1995) and Akahori (1995) who also provides an analytical expression for the hedging strategy. However, these formulas are still expressed in integral

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form, which could present serious computational difficulties if they are to be evaluated numerically.

The present paper, other than providing a systematic overview of the results obtained in recent years about the α-quantile option, proposes a numerical method to simulate the option price. Such an approach takes advantage of the Dassios-Port-Wendel identity concerning the α-quantile of a Brownian motion with drift, rather than using numerical integration procedures. The results obtained from this method are then discussed.

2 Pricing α-quantile options

Let $(W_t : t \geq 0)$ be a one-dimensional standard Brownian motion with $W_0 = 0$; let $\sigma \in \mathbb{R}^+$, $\mu \in \mathbb{R}$ and define $X = (X_t : t \geq 0)$ as an arithmetic Brownian motion such that

$$X_t = \mu t + \sigma W_t.$$

The α-quantile of X over the interval $[0, t]$ is defined as follows.

Definition 1 *Let $\Gamma(a, t) = \int_0^t 1_{(X_s \leq a)} ds$ be the occupation time of a Brownian motion with drift, X_t. The α-quantile of X_t is defined as*

$$Q(\alpha, t) = \inf\{x : \Gamma(x, t) > \alpha t\}$$

for all $0 < \alpha < 1$.

From this definition it follows that $Q(\alpha, T) \leq \sup_{0 \leq t \leq T} X_t$ a.s. and $Q(\alpha, T) \geq \inf_{0 \leq t \leq T} X_t$ a.s. More precisely

$$\begin{aligned} \lim_{\alpha \to 0} Q(\alpha, T) &= \inf_{0 \leq t \leq T} X_t \qquad \text{a.s.} \\ \lim_{\alpha \to 1} Q(\alpha, T) &= \sup_{0 \leq t \leq T} X_t \qquad \text{a.s.} \end{aligned}$$

An important result related to the α-quantile concerns its distribution function. Dassios (1995) obtained a useful representation of such a function analogous to a decomposition for random walks due to Wendel (1960) and Port (1963).

Theorem 1 (Dassios-Port-Wendel identity) *Let $0 < \alpha < 1$, then*

$$Q(\alpha, T) \stackrel{d}{=} \sup_{0 \leq t \leq \alpha T} X_t + \inf_{0 \leq t \leq (1-\alpha) T} \tilde{X}_t, \tag{1}$$

where $\stackrel{d}{=}$ means equality in distribution and $\tilde{X}_t$ is an independent copy of the arithmetic Brownian motion.

Consider now the Black-Scholes framework. The non-dividend paying risky asset S has value $S_t = S_0 e^{X_t}$ at time $t \geq 0$ where $S_0 > 0$. Further, the return on the riskless bond is exponential with rate $r > 0$. Then an α-quantile call option with strike price K and underlying asset S has a payoff function at maturity date, T, defined as $\left(S_0 e^{Q(\alpha,T)} - K\right)^+$, where S_0 is the value of the underlying asset at the beginning of the contract and $Q(\alpha, T)$ is the α-quantile of X. Analogously, the payoff of an α-quantile put option of the same type is $\left(K - S_0 e^{Q(\alpha,T)}\right)^+$.

Applying the risk-neutral valuation procedure (Harrison and Pliska, 1981), we can say that the no-arbitrage price at time $t \in [0, T]$ of an α-quantile call option is given by

$$C(S_0, \alpha, T-t) = e^{-r(T-t)} \mathbb{E}\left[\left(S_0 e^{Q(\alpha,T)} - K\right)^+ \mid \mathcal{F}_t\right],$$

where $\{\mathcal{F}_t\}_{t \geq 0}$ is the natural filtration for X and $\mathbb{E}$ denotes the expectation under the risk-neutral probability measure $\mathbb{P}$. The price of the α-quantile put can be defined analogously. Exploiting the convolution property in Theorem 1, Dassios (1995) also obtained the closed formula for the α-quantile call option

$$\begin{aligned} C(S_0, \alpha, T-t) \;=\; & e^{-r(T-t)} \int_K^\infty \mathbb{P}\left[\tilde{Q}(\alpha', T-t) > \ln \frac{z}{S_t} \mid \mathcal{F}_t\right] 1_{\left(\Gamma\left(\ln \frac{z}{S_0}, t\right) > t-(1-\alpha)T\right)} dz \\ & + e^{-r(T-t)} \int_K^\infty 1_{\left(\Gamma\left(\ln \frac{z}{S_0}, t\right) \leq t-(1-\alpha)T\right)} dz, \end{aligned} \tag{2}$$

where

$$\alpha' = \frac{\alpha T - \Gamma\left(\ln \frac{z}{S_0}, t\right)}{T-t}$$

and $\tilde{Q}(.,.)$ is a version of the α-quantile that is independent of $\mathcal{F}_t$.

Given the nature of the α-quantile of a Brownian motion, the α-quantile option can be considered as a "smoothed" version of a more well-known path-dependent option, the fixed strike lookback option. It can be shown quite easily that the α-quantile option is always cheaper than an equivalent lookback option written on the same underlying, with the same strike K, same expiration date T and lookback period $[0, T]$. Specifically, consider the case of a call option. The payoff at maturity of the fixed strike lookback call is $\left(S_0 e^{M_0^T} - K\right)^+$, where $M_0^T = \sup_{0 \leq t \leq T} X_t$. Its price at time t is then

$$L(S_0, T-t) = e^{-r(T-t)} \mathbb{E}\left[\left(S_0 e^{M_0^T} - K\right)^+ \mid \mathcal{F}_t\right].$$

Since $Q(\alpha, T) \leq M_0^T$ *a.s.*, it follows that

$$Y := \left(S_0 e^{Q(\alpha,T)} - K\right)^+ - \left(S_0 e^{M_0^T} - K\right)^+ \leq 0 \quad a.s. \tag{3}$$

Setting $Z = \mathbb{E}[Y \mid \mathcal{F}_t]$, inequality (3)and the definition of conditional expectation imply

$$\mathbb{E}[Z 1_A] = \mathbb{E}[Y 1_A] \leq 0 \quad \forall A \in \mathcal{F}_t.$$

Hence it follows that

$$C(S_0, \alpha, T-t) \leq L(S_0, T-t) \quad a.s. \tag{4}$$

By an analogous argument it is possible to show that also the put option is always less expensive than the fixed strike lookback put. This analytical result is confirmed by the numerical evidence produced in the next section.

3 Monte Carlo simulation

As seen in the previous section, analytic pricing formulas for the α-quantile option are difficult to compute. As we can see from equation (2), in fact, the closed valuation formula of this option requires that all the occupation times $\left\{L\left(\ln \frac{z}{S_0}, t\right) : z \geq K\right\}$ have to be recorded. This can be avoided if the option price is computed for $t = 0$. Nevertheless the option price is still expressed in integral form involving also the distribution function of the α-quantile, which can be obtained through the convolution property of Theorem 1. That means computing another integral expression. On the other hand Theorem 1 and the fact that the distributions of the extremes of the Brownian motion are well known results (see for example Karatzas and Shreve, 1997), provide a straightforward framework to produce the price of the α-quantile option. The idea is to use a Monte Carlo valuation procedure for the price at time 0 of the quantile option, in which the quantile of the Brownian motion is generated directly as the sum of two independent samples of the extremes of X. More precisely, the approach can be described by the following steps:

- for $i = 1, 2, ..., n$ generate independent samples of $M := \sup_{0 \leq t \leq \alpha T} X_t$ and $m := \inf_{0 \leq t \leq (1-\alpha)T} X_t$ by inversion of their distribution functions, through the Newton algorithm;
- let Q_i be the sum of the independent realizations of M and m, then the final payoff, Y_i, of the α-quantile option is computed as $Y_i = \left(S_0 e^{Q_i} - K\right)^+$;

Black-Scholes framework	
Option price	5.7792
Standard error	0.0157
95% confidence interval	$(5.7484, 5.8099)$
Option delta	0.5951
Standard error	0.00167
95% confidence interval	$(0.5918, 0.5984)$
number of iterations: 100,000; time: 8 secs.	

Table 1: Simulation results for the quantile call option in the Black-Scholes framework. Set of parameter: $S_0 = 100$; $K = 100$; $\alpha = 0.5$; $r = 0.05$; $\sigma = 0.2$; $T = 1$.

- let $\hat{C} := e^{-rT} \sum_{i=1}^{n} Y_i/n$; then $\hat{C}$ is a numerical approximation to the option price at time $t = 0$.

Note that in this way we don't need to generate the entire history of the Brownian motion; a procedure that presents problems when choosing an appropriate approximating random walk. Since the α-quantile option is path-dependent, its value is liable to be sensitive to the frequency with which the extremes of the Brownian motion are observed. For a random walk approximation, the "true" extremes of the Brownian motion may not be correctly sampled.

In the same framework, this Monte Carlo procedure can be adapted to approximate also the delta of the α-quantile option at time 0. Let $\partial_i := e^{Q_i} 1_{(S_0 e^{Q_i} > K)}$ be the partial derivative of the sample payoff Y_i with respect to S_0. Then $\Delta = e^{-rT} \sum_{i=1}^{n} \partial_i / n$ is a numerical approximation to the α-quantile option delta at time $t = 0$. To be precise a closed form for the option delta has been derived, as already mentioned, by Akahori (1995). However such a formula suffers of the same problem of the option price, that is it is still expressed in integral form involving also the distribution function of the α-quantile.

The Monte Carlo simulation has been carried out by generating $100,000$ paths. Using a C++ program on a desktop with Pentium(r) III processor and 64,0 MB RAM, the numerical procedure implemented takes 8 seconds to return the option price. The control variate technique is used to reduce the variance of the obtained estimates. The benchmark contract is chosen to be the lookback counterpart; hence the option contract is computed as

$$\hat{C}_q^{CV} = \hat{C}_q + \beta \left(C_L - \hat{C}_L \right),$$

where $\hat{C}_q$ is the price of the quantile call option obtained by the procedure described above, C_L is the exact price of a lookback call contract written on the same underlying, with equal maturity and strike and lookback period $[0,T]$, $\hat{C}_L$ is the estimated value for such a lookback option using a Monte Carlo procedure, and β is a parameter with value other than one. In particular, the choice of β which minimizes the option price variance is $\beta^* = Cov\left(\hat{C}_q, \hat{C}_L\right)/Var\left(\hat{C}_L\right)$. In order to avoid the introduction of a bias in the estimation of the option price, we use a few pilot runs to estimate β^* and then we use this parameter in the main simulation run. The details of the output for the quantile option are presented in Table 1. We choose this version of the control variate technique because β^* allows to reduce significantly the option variance no matter the level of α chosen, that is no matter the degree of correlation between the quantile option and the lookback. In fact, we have to consider that the quantile option loses any similarity with the lookback counterpart for values of α far enough from 1.

4 Simulated prices

Throughout all the following analysis, unless otherwise stated, the basic parameter set is

$$S_0 = 100; \quad K = 100; \quad \alpha = 0.5; \quad r = 0.05; \quad \sigma = 0.2; \quad T = 1.$$

The α-quantile call price for different values of the initial stock price, with all the other parameters left unchanged, is given in Table 2. In the same table we report also the price of a fixed strike lookback call[1] written on the same underlying and computed for the same values of the parameters K, r, σ and T. As expected, the 0.5-quantile option is cheaper than the lookback option. In order to observe empirically the convergence property of the α-quantile option to the lookback option discussed in the previous section (see equation (4)), for the particular case of $t = 0$, in Figure 1 we plot the prices of both the α-quantile call and put against the stock price for different values of the parameter α. Also, we add to the plot the price function of the equivalent lookback. The dominating value of the lookback as discussed in section 2 is clear. The same kind of analysis is extended to the delta of the options considered and the results are given in Figure 1.b and 1.d. In general, we can say that the slope of the delta when it is considered a function of the stock price represents the so-called gamma. Gamma provides informations about the frequency with which a possible portfolio containing such an option has to be rebalanced in order to maintain a perfect hedge. Hence, it is reasonable

[1] The price in this case in computed through its closed formula (Conze and Viswanathan, 1991).

Stock price	0.5-quantile option	Lookback call
90	1.6588 (0.0093)	9.457
95	3.2695 (0.0125)	13.865
100	5.7501 (0.0157)	19.1676
105	9.0895 (0.0191)	24.8822
110	13.0213 (0.0226)	30.5967
115	17.4030 (0.0256)	36.3112
120	22.0456 (0.0279)	42.0257

Table 2: The α-quantile call price and the lookback call. The numbers in parentheses correspond to the standard errors of the Monte Carlo simulations.

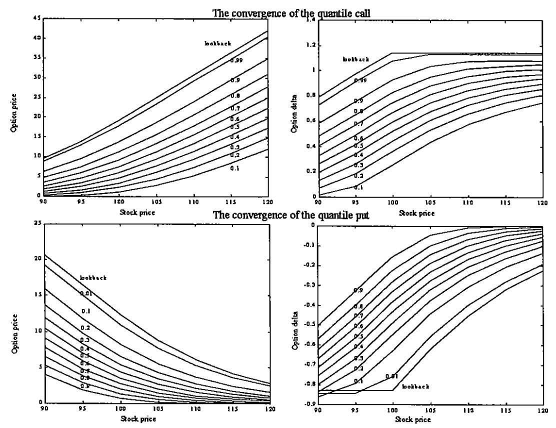

Figure 1: The "convergence" of the α-quantile option to the fixed strike lookback.

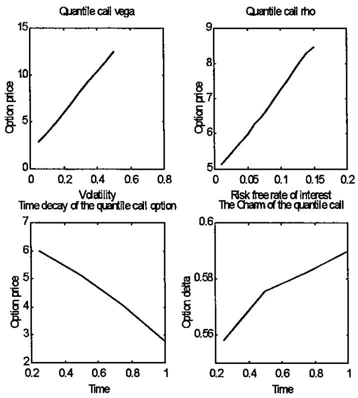

Figure 2: The α-quantile option greeks

to expect that, when the stock price is in the neighborhood of the strike price, the delta is highly sensitive to changes in the underlying asset price, therefore the portfolio is likely to be rebalanced very frequently. Let us now consider the case of the call option. As the stock price becomes very large, delta becomes less sensitive because the option is expected to expire in-the-money. This is the behaviour we can observe in Figure 1.b. For the case of the lookback option this behaviour is very marked. If the initial value of the stock is equal to the strike price, the maximum of the stock itself cannot be less than the strike price. Therefore, above this "critical value" there is no additional risk to hedge and the delta remains constant. The pattern for the α-quantile option, instead, is more "smoothed", but as α approaches the unity we see that the pattern of the delta becomes more and more similar to the one of the lookback, due again to the convergence property of the α-quantile option to the lookback option. Analogous considerations hold also for the put case with the (obvious) difference that the delta becomes less sensitive when the stock price becomes smaller with respect to the strike price.

A general study of comparative statics concerning all the main parameters on which the option value depends, reveals that the α-quantile option presents patterns lined up with the behaviour of other Euro-type options. For example, in Figure 2 it is shown the sensitivity of the α-quantile call option to changes in the stock volatility. As we can observe, the option price is positively correlated with σ, which is again a common feature of all options. In fact, the volatility represents a measure of the uncertainty of the stock price future movements. Since a call option has limited downside risk in the event of stock price falls, meanwhile the put option has limited downside risk in the event of price increases, the value of both options tends to increase when the volatility increases as well. Analogous results can be also obtained for the put option.

5 Conclusions

In this note we have presented properties and features of a new financial instrument, the α-quantile option, introduced first by Miura (1992), which is at the moment only a "theoretical" object since it is not yet traded in the market. The Dassios-Port-Wendel identity has been employed to simulate initial prices and Greeks of the α-quantile. In view of the analytical structure of this option the simulations not surprisingly show consistent characteristics to other Euro-type option contracts. The problem remain how to deal numerically with the mid-contract value of the option. In fact, for this case pricing formulas and numerical approximations cannot avoid the set of occupation times needed to define the α-quantile itself and which makes this process not Markovian.

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References

[1] Akahori J. (1995): Some formulae for a new type of path-dependent option, *The Annals of Applied Probability*, 5, 383-388.

[2] Conze, A. and Viswanathan (1991): Path dependent options: the case of lookback options, *The Journal of Finance*, 46, 1893-1907.

[3] Dassios A. (1995): The distribution of the quantile of a Brownian motion with drift and the pricing of related path-dependent options, *The Annals of Applied Probability*, 5, 389-398.

[4] Dassios A. (1996): Sample quantiles of stochastic processes with stationary and independent increments, *The Annals of Applied Probability*, 6, 1041-1043.

[5] Doney R. A. and M. Yor (1995): On a formula of Takács for Brownian motion with drift, *Journal of Applied Probability*, 35, 272-280.

[6] Harrison M. and S. R. Pliska (1981): Martingales and Stochastic Integrals in the Theory of Continuous Trading, *Stochastic Processes and their Applications*, 215-260.

[7] Karatzas I., Shreve S. E., *Brownian Motion and Stochastic Calculus*, Springer, 1997.

[8] Miura R. (1992): A note on lookback options based on order statistics, *Hitotsubashi Journal of Commerce and Management*, 27, 15-28.

[9] Port S. C. (1963): An Elementary Probability Approach to Fluctuation Theory, *Journal of Mathematical Analysis and Applications,* 6, 109-151.

[10] Takács L. (1996): On a generalization of the arc-sine law, *The Annals of Applied Probability*, 6, 1035-1040.

[11] Wendel J. G. (1960): Order Statistics of Partial Sums, *Annals of Mathematical Statistics*, Vol. 31, 1034-1044.

[12] Yor M. (1995): The distribution of Brownian quantiles, *Journal of Applied Probability*, 32, 405-416.

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