Observations on the Proposed New Mortality Tables Based on the 1991-94 Experience for Male Permanent Assurances

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OBSERVATIONS ON THE PROPOSED NEW MORTALITY TABLES BASED ON THE 1991-94 EXPERIENCE FOR MALE PERMANENT ASSURANCES.

A. E. Renshaw and S. Haberman

1. Introduction

Our objective in this paper is to make some pertinent observations on the proposed new mortality tables based on the 1991-94 UK mortality experience for male lives with permanent life insurance, as presented in CMI Committee (1998). The CMI Committee have followed the historical precedent of graduating the mortality experience independently for each select durational group and for the ultimate group. In order to ensure the desired ordering of the graduation curves and a correspondence of shape and curvature, this approach requires a number of adjustments. We propose and implement a comprehensive method for the modelling of both age and duration effects: based on Renshaw and Haberman (1997). The aim of this method is to graduate by age *and* duration, rather than following the CMI approach of graduating separately by age for each durational group. We would argue that the proposed methodology makes more efficient use of the observational data than the traditional approach because it allows us to infer the shape of the graduated curve at ages and durations (particularly Durations 0 and 1), where the data are sparse, from those age-duration cells were the data are more substantial. The proposed methodology also obviates the need for *ad hoc* adjustments outside of the graduation process.

2. Background

We focus on the construction of a set of select mortality tables and are mindful of the following general considerations

- A set of graduated select duration mortality tables should be ranked with respect to duration for each age.
- Graduated mortality tables should reflect the underlying mortality patterns in the raw data on which they are based.
- Graduated mortality rates should be smooth functions of age and of duration.
- Attention should be given to existing comparable mortality tables.
- Attention should be given to the extrapolation of graduated mortality tables to adjacent ages lying beyond the natural extremities of the data.

It should be emphasised that these considerations are not necessarily exhaustive and that it is possible for certain of them to be in conflict with each other in practice: for example the trade off between adherence to the data and smoothness is well documented in the standard text books. These factors are listed above in what we regard as decreasing order of importance.

The construction of a set of select mortality tables as practised by the CMI Bureau (Mortality Sub-Committee), for the Institute and Faculty of Actuaries, involves the targeting of the force of mortality μ for each duration (individual select and ultimate) separately. This is achieved through the fitting of non-linear parameterised formulae of the type

$$\mu_x = \mathrm{GM}_x(r,s)$$

where the GM function is a parameterised polynomial of degree r-1 in age x plus a second exponentiated parameterised polynomial of degree s-1 in age x. Parameters are estimated by the method of maximum likelihood under an independent Poisson modelling assumption, subject to possible adjustments in the parameter values based on the experience of the CMI Bureau, and making allowance for the considerations listed above. We note that, in this approach, duration is not explicitly modelled. The detailed methodology, including a description of the battery of diagnostic statistical tests which are used to check the adequacy of any adopted fitting formula, is fully documented in Forfar *et al.* (1988). Then, having established the specific nature of such parametric formulae, values of q_x, the probability that a life aged x dies before age x+1, are computed numerically as

$$q_x = 1 - \exp - \int_x^{x+1} \mu_s ds$$

where the integral is evaluated approximately. The resulting values of μ_x and q_x are tabulated as part of the final presentation.

3. The 1979-82 male experience

The CMI Bureau methodology is both well tried and effective. To illustrate this effectiveness, we refer briefly to the AM80 standard select mortality tables, CMI Committee (1990), which are to be updated by the proposed new tables. We recall that the AM80 tables are based on the same $\mu_x = \mathrm{GM}_x(2,2)$ graduation formula, fitted separately for each Duration d = 0, 1, 2+, with some final adjustments at the extreme ages for select Duration d = 0 (at ages below 28) and for the ultimate Duration d = 2+ (at ages above 80). (The AM80(5) tables are ignored for the purpose of this presentation). In particular, we graphically illustrate the goodness-of-fit of these formulae by means of two sets of figures, *viz* Figures 3.1(a,b,c) and Figures 3.2(a,b). In Figures 3.1(a,b,c), we superimpose the values of $\log\mu_x$, based on the relevant mathematical formula, against the background of plotted log{crude mortality rates}, for each duration separately. In Figures 3.2(a,b), we monitor mortality rates for the individual select Durations d = 0, 1 relative to those for the ultimate Duration 2+, matched age for age. This is done by superimposing the values of the differences ($\log\mu_x^d$ - $\log\mu_x^{2+}$) as determined by the relevant formulae, against the background of plotted differences in log{crude mortality rates}. Clearly each of these plots is consistent with the requirements for a satisfactory set of AM80 tables. Full details of the diagnostic statistical tests used are given in CMI Committee (1988).

4. The 1991-94 male experience

For the 1991-94 male experience, the CMI Bureau has proposed separate $\mu_x = \text{GM}_x(2,3)$ formulae for the three Durations $d = 0$, 1 and 2+ without the necessity for final adjustments. We have examined the goodness-of-fit of these proposals by monitoring the same two sets of graphs as above. These are reproduced as Figures 4.1(a,b,c) and Figures 4.2(a,b). Thus, Figures 4.1(a,b,c) depict the graduated values of $\log \mu_x$ against the background of plotted log{crude mortality rates}, for each duration separately, and Figures 4.2(a,b) depict the values of the graduated differences ($\log\mu_x^d$ - $\log\mu_x^{2+}$) against the background of plotted differences in log{crude mortality rates}, for $d = 0$ and 1. We note the systematic lack of fit highlighted by these graphs, a feature which gives cause for concern. However, this is focused at the younger ages rather than at ages where the bulk of the exposed to risk is located.

These proposed graduated values at ages under 25 may be compared with the corresponding AM80 values based on the male 1979-82 experience for permanent assurances: CMI Committee (1990). As the following comparison indicates, mortality, as represented by the graduated values, is appreciably lighter at these young ages in 1991-94 when compared to 1979-82:

q_x - values

		AM 80			*Proposed*	
age x	*Duration 0*	*Duration 1*	*Duration 2+*	*Duration 0*	*Duration 1*	*Duration 2+*
17	0.000755	0.000865	0.000937	0.000427	0.000552	0.000600
18	0.000708	0.000815	0.000886	0.000426	0.000548	0.000594
19	0.000663	0.000768	0.000837	0.000425	0.000544	0.000587
20	0.000622	0.000723	0.000791	0.000425	0.000541	0.000582
21	0.000584	0.000682	0.000747	0.000425	0.000538	0.000577
22	0.000549	0.000644	0.000708	0.000427	0.000535	0.000572
23	0.000518	0.000610	0.000671	0.000429	0.000534	0.000569
24	0.000490	0.000580	0.000639	0.000431	0.000533	0.000567

There is an apparent contradiction between this feature and the reported rising trend in mortality at low ages in the period 1980-90 for males with permanent assurances at durations 5 and over (Renshaw *et al* (1996)). This trend analysis at the youngest ages suggests that the proposed graduation should be revisited.

If the undesired shortcomings revealed in Figures 4.1(a,b,c) and Figures 4.2(a,b) (not present in the previously constructed AM80 standard tables, as noted in Section 3) are to be either eliminated or at the very least minimised, we would argue that it may be necessary to extend current CMI methodology based on the independent fitting of GM(r,s) formulae for the separate durations.

One such possibility is suggested by Renshaw and Haberman (1997), (and is similar in spirit to the approach used by Panjer and Tan (1995) to graduate the 1986-1992 Canadian individual insurance mortality experience). The main ingredients of this alternative can be simply explained by reference to the two sets of Figures 4.1(a,b,c) & 4.2(a,b). Firstly, we make the somewhat obvious comment that the positions of all the data points plotted in these figures are fixed. Then, under current CMI Bureau practice, the continuous curves can be added to Figures 4.1(a,b,c) once the appropriate GM(r,s) models have been fitted, followed by the continuous curves in Figures 4.2(a,b) on differencing the relevant established GM(r,s) formulae. Under the alternative approach proposed by Renshaw and Haberman (1997), only the ultimate experience is graduated in the first instance. This may be done by selecting a suitable GM(r,s) formula, say, thereby leading to the completion of Figure 4.1(c).

Next, we focus on the data plots of Figures 4.2(a,b) and fit continuous curves, denoted by η_x^d, to these, thereby completing these figures. Specifically, we consider the comparison of the mortality experience for select duration d with the corresponding ultimate experience, matched for age, via the statistic

$$Z_x^d = \log\left(\frac{A_x^d}{R_x^d}\right) - \log\left(\frac{A_x^{2+}}{R_x^{2+}}\right)$$

with realised values

$$z_x^d = \log\left(\frac{a_x^d}{r_x^d}\right) - \log\left(\frac{a_x^{2+}}{r_x^{2+}}\right)$$

where A_x^d and R_x^d denote the random numbers of deaths and matching central exposures to the risk of death at age x and select duration d, and a_x^d and r_x^d denote the reported values as observed in the mortality experience under consideration. Our aim is to fit continuous curves η_x^d to the realised values z_x^d and this may be accomplished by the method of weighted least squares using

$$\frac{a_x^d a_x^{2+}}{a_x^d + a_x^{2+}}$$

for weights, where a_x^d and a_x^{2+} denote the actual deaths for select duration d and ultimate duration 2+ respectively, at age x. It then follows that the graduated select and ultimate curves will be related through

$$\mu_x^d = \mu_x^{2+} \exp(\eta_x^d).$$

Further, the first of the bullet points of Section 2 is automatically satisfied provided that we ensure that the fitted η_x^d curves satisfy the inequalities $\eta_x^0 < \eta_x^1 < 0$ for all x.

Finally, these fitted curves translate into multiplicative factors $\exp(\eta_x^d)$ which, when applied to the established graduation under the ultimate experience, generate the graduations for the two select experiences, thereby leading to the completion of Figures 4.1(a&b). Given that parametric formulae have been used to define the η_x^d curves, smoothness is guaranteed and it is possible to convert the resulting μ_xs into q_xs by numerical integration, for each duration - this step conforms with current CMI Bureau practice.

We consider firstly the graduation of the ultimate experience which forms the backbone of any set of select mortality tables. In addition to choosing graduations based on a GM(2,3) formula, we note with much interest that the CMI Bureau also considered the possiblity of using a GM(0,5) formula, before rejecting it on the grounds that the resulting "values of q_x for the GM(0,5) formula rose above those for English Life Table No, 15, Males at low and at high ages, whereas those for GM(2,3) did not." This reason is perhaps not overly convincing given the CMI Bureau's practice of adjusting mortality rates as predicted by GM(r,s) formulae when deemed necessary at extreme ages. Thus, the adjustment to the GM(2,2) formula for ages in excess of 80 years in the case of the ultimate table for the male AM80 experience is a good case in point (CMI Committee (1990)). The rejection of GM(0,5) for the reason given is perhaps all the more surprising in the light of Figure 4.3(d), in which we again superimpose the values of $\log\mu_x$, this time based on the GM(0,5) formula (subject to the minor adjustment for ages less than 19 years described immediately below) against the background of plotted log{crude mortality ratios}, and which may be directly compared with Figure 4.1(c).

We note the structure of the standard form of the GM(0,5) formula

$$\mathrm{GM}_x(0,5) = \exp\sum_{j=0}^{4} \beta_j C_j(t)$$

where $C_j(t)$ denote the customary Chebycheff polynomials of the first type, and $t = (x - 70)/50$. The $C_j(t)$ satisfy the recurrence relation

$$C_{j+1}(t) = 2tC_j(t) - C_{j-1}(t), \quad j = 1, 2, 3, \ldots$$

with $C_0(t) = 1$ and $C_1(t) = t$. Given the deficiency of this GM(0,5) formula at low ages (in comparison with the English Life Table No 15), in the spirit of the CMI Bureau, we have applied the following multiplicative adjustment factor

$$\exp\left\{\left(\frac{t - t_0}{t_1 - t_0}\right)^2 \log r\right\}, \text{ for } t < t_0; \ (t_1 < t_0)$$

for ages 17-19 inclusive, where $t = (x - 70)/50$, $t_0 = (19\text{-}70)/50$, $t_1 = (17\text{-}70)/50$, $r = 0.85$. It is designed so that the value of μ_{17}, as predicted by the GM(0,5) formula, is reduced to 85% of its value, say. It is of academic interest to note that the multiplicative adjustment factor being quadratic in nature preserves the GM(0,5) structure and merely adjusts the values of the first three parameters, β_j (j = 0, 1, 2), within the quartic structure of the formula: we use $\beta_j^!$ for these adjusted parameter values. We have not applied an equivalent adjustment at the other end of the age range, since we are not expressly concerned with the extrapolation of the formula to ages in excess of 92 years. However, it would be a simple matter to make such an adjustment.

We next consider the graduation of the two select experiences. Given the reported difficulties experienced by the CMI Committee (1998) in producing satisfactory graduations for these two cases, especially those concerning "unsatisfactory features in relation to the rates for Duration 2+, especially at the younger ages" based on the use of separate GM(2,2) formulae, we turn to the second stage of our approach which is designed, in part, to identify the extent and nature of such difficulties when they are inherent in the data. Further scrutiny of Figures 4.2(a,b) reveals that if the continuous curves are to be replaced by continuous curves of 'best fit', then such curves, and hence the intrinsic patterns in the two data plots, would appear to cross each other's path, thereby violating the first of the considerations of Section 2 (ie ranking of select mortality curves). Thus, certain underlying inherent features of the data will have to be sacrificed in order to comply with this consideration. The question is, which? One possibility is to fit a pencil of lines to the two combined data sets represented by the equation

$$\eta_x^d = \gamma^d(x - 17); \quad d = 0,\, 1, \quad x \geq 17$$

based on two parameters γ^d and with the focus set at the point η_{17}^d=0. Details of the resulting parameter estimates are as follows

duration	parameter γ^d	standard error	t-statistic
d=0	-0.007109	0.0007359	-9.66
d=1	-0.001549	0.0006158	-2.52

while other aspects of the resulting fit are presented in graphical form in Figure 4.4. In the top two frames of Figure 4.4, we reproduce the data plots of Figure 4.2(a,b) but this time with the fitted lines superimposed. In the bottom frame we display the relative positions of the two fitted lines η_x^d. In focusing on this particular pencil of lines, Figure 4.4 indicates that provided the ultimate graduation is satisfactory over the whole of the age range, then so too will be the resulting graduation based on the straight line for Duration 0. Further, in selecting this particular parameterised form, it is clear that we have favoured Duration 0 at the expense of Duration 1. We investigate this route further.

The formula needed to produce the corresponding select mortality rates μ_x^d, d=0,1, based on these assumptions, then follows *viz*

$$\mu_x^d = \exp(\eta_x^d)\mathrm{GM}_x(0,5) = \exp\{\gamma^d(50t + 53) + \sum_{j=0}^{4} \beta_j^! \, C_j(t)\} \tag{4.1}$$

where $C_j(t)$ denote the customary Chebycheff polynomials of the first type, and $t = (x - 70)/50$. It should be noted that this is still a GM(0,5) formula but with the values of first two parameters adjusted according to select duration. Thus, for select Durations $d = 0,\, 1$

$$\mu_x^d = \mathrm{GM}_x(0,5) = \exp\sum_{j=0}^{4} \beta_j^d \, C_j(t)$$

with $\beta_0^d = \beta_0^! + 53\gamma^d$, $\beta_1^d = 50\gamma^d + \beta_1^!$, $\beta_j^d = \beta_j^!$ otherwise, and for the ultimate duration 2+

$$\mu_x^{2+} = \mathrm{GM}_x(0,5) = \exp\sum_{j=0}^{4} \beta_j^! \, C_j(t).$$

In the case of Duration 1 we refer to this as the *first version*.

While graduations for Duration 0 are well served on the basis of this approach, a visual examination of the top right hand frame in Figure 4.4 indicates that there is scope for possible improvement if the fitted line is replaced by a curve (subject to the constraints $\eta_x^0 < \eta_x^1 < 0$ for all x). One simple and practical possibility, investigated here, is to impose a cubic curve of the form

$$\eta_x^1 = a(x - k_1)(x - k_2)^2$$

with $k_1 = 17$ and $k_2 = 53$, say, in which the value of a is determined by forcing the cubic through the point $\eta_{80}^1 = -0.25$, say. This implies that $\eta_x^1 = 0$ at ages $x = 17$ and 53, based on Figure 4.5, and with $a < 0$, it follows that the cubic η_x^1 has a maximum at 53 and a minimum at 29. This in turn gives rise to the graduation formula

$$\mu_x^1 = \exp(\eta_x^1)\mathrm{GM}_x(0,5) = \exp\{a(50t + 53)(50t + 17)^2 + \sum_{j=0}^{4} \beta_j^1 C_j(t)\} \qquad (4.2)$$

where $a = -5.4434\times10^{-6}$. Once again this is a GM(0,5) formula with, this time, adjustments to the first four β_j^1s. We refer to this as the *second version*.

As a final minor refinement the values of η_x^d at age $x = 17$ are subjected to a minor shift, of amount $(d + 1)z_0/2$, with $z_0 = -0.01$ ($d = 0, 1$), to ensure that the graduations are strictly ordered with respect to duration, at age 17 (as well as the remainder of the age range). Then for completeness, we come full circle by augmenting Figure 4.3(d) with Figures 4.3(a,b,c) in which the values of $\log\mu_x$, based on these formulae, are superimposed against the background of plotted log{crude mortality rates}, for select Duration 0 and both versions of select Duration 1. Figures 4.3(a,b,c,d) bear direct comparison with their respective counterparts in Figures 4.1(a,b,c).

The associated statistical graduation tests are presented in Table 4.1 and corresponding q_x values, determined by numerical integration using Simpson's rule, (with sufficient step lengths to ensure convergence), presented in Table 4.2. These bear direct comparison with their respective counterparts (Table 1.6 & Table 1.15) in CMI Committee (1998).

5. Miscellaneous observations

Every effort has been made to check that our version of the data, kindly supplied by the CMI Bureau for a different purpose on an earlier occasion, matches that used in CMI Committee (1998). Thus, it is possible to check our total deaths and total exposures, for each duration, against the figures quoted in the final column of Table 1.1 of CMI Committee (1998). We report that the matching figures for total deaths are in complete agreement and that the discrepancies between exposure totals are as follows

	duration 0	*duration 1*	*duration 2+*
CMI (1998) Table 1.1 total exposures	837,360.3	835,252.4	15,139,004.8
Total exposures available	837,365	835,259	15,139,034
	(including ages 10-16, which are subsequently not used)		

We also report that our actual deaths for Duration 2+ match, age for age, those quoted in Table 1.17 of CMI Committee (1998), and that there are differences of negligible amounts between matching exposures, within the age range 17 to 91, attributable to rounding errors. We also need to report that our exposures for select Duration 1 terminate at age 89, with the exception of age 100, where the exposure is 0.5.

The extent by which the CMI Committee's proposed GM(2,3) formula understates the mortality rates at low ages, for select Duration 0, is further illustrated by Figure 5.1 in which we have plotted the Pearson residuals (z_x values) against age. Such plots augment the formal statistical test of a graduation, revealing in this case, a feature which is not made apparent by the reported formal statistical tests (although the feature is described in the text of CMI Committee (1998)).

Since only two out of the five parameters are estimated when constructing the GM(2,3) formulae under the CMI Committee (1998) proposals for select durations, a case can be made for increasing the degrees of freedom in the χ^2 tests of their Table 1.6 (d = 0, 1) by three, and for re-computing the matching p-values accordingly. In setting the degrees of freedom in Table 4.1, we have allowed one degree of freedom for each parameter estimated when fitting η_x^d. These issues, which are open to debate, are not crucial at a practical level.

It is informative to augment the reported graduation tests by comparing the values of the total deviations, $\sum_x dev_x$, (coupled with the re-computed chi-square goodness-of-fit p-values, $p(\sum_x z_x^2)$)

Statistic	*Formula*	*d = 0*	*d = 1* (*1st version*)	*d = 1* (*2nd version*)	*d = 2+*
$\sum_x dev_x$	GM(0,5)	-4.70	-22.47	-46.55	0.39
	GM(2,3)	54.78	-2.43		-0.48
$p(\sum_x z_x^2)$	GM(0,5)	0.8958	0.0067	0.0361	0.0085
	GM(2,3)	0.0013	0.0165		0.0023

We note that the proximity of the data plots to the x-axis in the 50s age range, typically Figure 4.3, confirms the data feature, identified in CMI Committee (1998), concerning the closeness of the Duration 1 and Duration 2+ graduations in this age range.

As noted above, the CMI Committee proposals require the fixing of three of the five parameter values prior to estimation for the two select durations. The effects of this device on the standard errors of q_x for the select durations are explained (¶1.8.7 CMI Committee (1998)); however, there is no discussion of which specific features in the data have been used to decide: (a) how many parameters to fix prior to estimation, (b) which parameters to fix, and (c) the levels at which to fix the parameters. We note also that for the case of Duration 1 many combinations of parameters were tried and tested by the CMI Committee. This contrasts with our use of demonstrably recognisable patterns in the data to set the parameters when graduating the two select experiences.

It is possible to estimate the amount of shift to be applied to η_x^d at age 17 through the inclusion of additional free-standing parameters δ^d so that the formula reads

$$\eta_x^d = \delta^d + \gamma^d(x - 17).$$

The estimate for δ^0 is -0.004309 with standard error 0.07851 and so, although negative, is not statistically significant. The estimate for δ^1 is 0.09374 with standard error 0.1091. Since this is positive it violates the ranking criterion, although again it is not statistically significant. Hence, we have imposed the small reported shifts on the data.

It is informative to extend the second stage of our modelling approach to cover the individual 5 year select durations coupled with a 5+ ultimate duration experience. Details of this analysis are displayed in graphical form, Figure 5.2, together with the following parameter estimates for the fitted pencil of lines

$$\eta_x^d = \gamma^d(x - 17)$$

duration	parameter γ^d	standard error	t-statistic
d=0	-0.007272	0.0007420	-9.80
d=1	-0.001690	0.0006210	-2.72
d=2	-0.008899	0.0005968	-1.49
d=3	-0.009795	0.0005742	-1.70
d=4	-0.001834	0.0005778	-3.17

The underlying linear pattern in these data plots for each select duration, with the exception of Duration 1, might be interpreted as supportive evidence for the 1st version GM(0,5) formula discussed in Section 4. It is also of interest to note that the above t-statistics for select durations 2 and 3 are not significantly different from zero, and as such, are supportive of the CMI Committee's decision to combine the experience of durations 2+ and to focus on an ultimate duration of 2+. Note also that the parameter estimates for select Duration 2, 3 and 4 are not well ordered as far as ranking with respect to policy duration is concerned, in any event.

A similar investigation into patterns in differences of log{crude mortality rates} based on individual 5 year select periods relative to ultimate duration 5+ , has been conducted for the historic assured lives experiences from 1990 extending as far back as 1924 for males (and 1975 for females) (Haberman & Renshaw (1996)). A noteworthy feature to emerge from this study is the relative constancy of the established patterns between adjacent quadrennia, at least until the most recent quadrennia. Given also that there is a clear ordering with respect to select Durations 0 and 1 in these patterns up to and including the 1987-90 quadrennium, it is possible that the 1991-94 experience is different from this historical trend: this possibly raises doubts about the choice of this most recent quadrennium's mortality experience on which to base a set of *standard* mortality tables.

We remark that when CMI Bureau type quadratic adjustments are made at high ages to the ultimate experience, differential adjustments can be introduced for the corresponding select experiences, if required, by making separate adjustments to η_x^d at high ages.

As a final observation about the methodolgy that we have proposed, it should be noted that the weighted least squares methodology with weights dependent on the observed numbers of deaths means that our approach ignores the data for select durations in those age cells for which there is exposure but no recorded deaths. For completeness, we list the contents of the offending data cells in this instance

	Duration 0		*Duration 1*	
age	*deaths*	*exposure*	*deaths*	*exposure*
82	0	13.1		
83	0	9.0		
84			0	10.8
86	0	1.5	0	4.5
87	0	4.5	0	2.0
88	0	1.5	0	5.3
89	0	0.5	0	0.3
100			0	0.5

and note that these represent 0.0036% and 0.0028% of the total exposures, respectively.

6. Conclusions

- Patterns in the raw mortality rates with respect to age, in the 1991-94 male permanent assurances experience, suggest that the underlying mortality rates are not in themselves intrinsically amenable to ranking with respect to policy duration at low ages. Consequently a degree of compromise is necessary when constructing select mortality tables for this experience.
- Under the CMI Committee (1998) proposed new mortality tables, there is evidence of compromise at low ages in both the select Duration 0 tables and the ultimate Duration 2+ table.
- Under the alternative approach developed here, compromise need only be present at low ages in the select Duration 1 table.
- The alternative approach developed here focuses on the relationship between the select duration and ultimate experiences, rather than graduating the select duration experiences independently and then implementing *post hoc* adjustments.

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Table 4.1. Tests of the graduations

Duration	*0*	*1* *1st version*	*1* *2nd version*	*2+*
Formula†	GM(0,5)	GM(0,5)	GM(0,5)	GM(0,5)
Ages used	17-81&84	17-83	17-83	17-91
Parameters:				
β_0	-3.12270	-3.41740	-4.17483	-3.49948
t-statistic				-25.80
β_1	4.41882	4.69685	3.69486	4.77428
t-statistic				20.45
β_2	0.53170	0.53170	-0.06027	0.53170
t-statistic				2.94
β_3	-0.25922	-0.25922	-0.42933	-0.25922
t-statistic				2.91
β_4	0.29501	0.29501	0.29501	0.29501
t-statistic				6.61
Sign test:				
number of +	32	26	24	36
number of -	33	39	41	38
p(pos)	0.500	0.068	0.023	0.454
Runs test:				
number of runs	34	25	27	32
p(runs)	0.600	0.041	0.156	0.100
Kolmogorov-Smirnov test:				
max deviation	0.0141	0.0351	0.0231	0.0044
KS statistic under H_0	0.367	1.049	0.693	0.685
p(KS)	0.999	0.221	0.723	0.736
Serial correlation test:*				
$r1$	0.0077	0.2199	0.1413	0.1929
t-statistic	0.06	1.76	1.13	1.65
$r2$	-0.1659	0.1260	0.0494	-0.1928
t-statistic	-1.317	1.00	0.39	-1.64
$r2$	-0.0409	-0.0383	-0.1272	0.0273
t-statistic	-0.32	-0.30	-1.00	0.23
χ^2 test:				
χ^2	50.22	95.32	86.92	100.1
degrees of freedom	64	64	65	69
$p(\chi^2)$	0.90	0.0067	0.0361	0.0085

† values of the first three parameters are modified for ages under 19 years

* values differ slightly from CMI Bureau values, probably due to precise definition of formulae used

Table 4.2. Values of q_x Males

Age	*Duration 0*	*Duration 1 (1st version)*	*Duration 1 (2nd version)*	*Duration 2+*
17	0.000881	0.000883	0.000886	0.000893
18	0.000846	0.000853	0.000853	0.000864
19	0.000770	0.000781	0.000778	0.000791
20	0.000700	0.000714	0.000709	0.000725
21	0.000644	0.000661	0.000655	0.000672
22	0.000600	0.000619	0.000613	0.000630
23	0.000566	0.000587	0.000581	0.000598
24	0.000539	0.000562	0.000556	0.000574
25	0.000519	0.000544	0.000539	0.000557
26	0.000505	0.000533	0.000527	0.000546
27	0.000496	0.000526	0.000522	0.000540
28	0.000492	0.000525	0.000521	0.000540
29	0.000493	0.000528	0.000525	0.000544
30	0.000497	0.000536	0.000533	0.000553
31	0.000505	0.000548	0.000547	0.000566
32	0.000518	0.000565	0.000565	0.000584
33	0.000535	0.000586	0.000588	0.000607
34	0.000556	0.000613	0.000616	0.000636
35	0.000581	0.000644	0.000650	0.000670
36	0.000612	0.000682	0.000690	0.000710
37	0.000647	0.000726	0.000737	0.000757
38	0.000689	0.000776	0.000791	0.000811
39	0.000737	0.000835	0.000853	0.000873
40	0.000792	0.000902	0.000925	0.000945
41	0.000854	0.000979	0.001007	0.001027
42	0.000926	0.001067	0.001102	0.001121
43	0.001007	0.001167	0.001209	0.001228
44	0.001099	0.001281	0.001331	0.001350
45	0.001203	0.001410	0.001470	0.001488
46	0.001321	0.001557	0.001629	0.001646
47	0.001455	0.001723	0.001808	0.001825
48	0.001605	0.001913	0.002012	0.002028
49	0.001775	0.002127	0.002244	0.002259
50	0.001967	0.002370	0.002506	0.002521
51	0.002183	0.002644	0.002802	0.002817
52	0.002426	0.002955	0.003137	0.003153
53	0.002700	0.003306	0.003516	0.003534
54	0.003007	0.003703	0.003942	0.003964
55	0.003352	0.004151	0.004423	0.004450
56	0.003739	0.004655	0.004962	0.004998
57	0.004172	0.005224	0.005568	0.005617
58	0.004657	0.005863	0.006247	0.006313
59	0.005199	0.006581	0.007006	0.007097
60	0.005803	0.007386	0.007852	0.007978
61	0.006477	0.008288	0.008795	0.008966
62	0.007226	0.009297	0.009842	0.010072
63	0.008058	0.010424	0.011002	0.011310
64	0.008982	0.011681	0.012284	0.012693
65	0.010004	0.013081	0.013698	0.014235
66	0.011133	0.014636	0.015253	0.015950
67	0.012380	0.016361	0.016957	0.017857
68	0.013753	0.018272	0.018820	0.019971
69	0.015263	0.020385	0.020851	0.022313

70	0.016920	0.022717	0.023058	0.024900
71	0.018736	0.025285	0.025450	0.027753
72	0.020722	0.028109	0.028033	0.030896
73	0.022890	0.031208	0.030814	0.034349
74	0.025254	0.034603	0.033799	0.038137
75	0.027826	0.038317	0.036993	0.042286
76	0.030620	0.042372	0.040399	0.046821
77	0.033651	0.046792	0.044021	0.051771
78	0.036935	0.051602	0.047860	0.057163
79	0.040488	0.056830	0.051917	0.063029
80	0.044327	0.062502	0.056192	0.069402
81	0.048470	0.068650	0.060682	0.076314
82	0.052938	0.075305	0.065388	0.083802
83	0.057751	0.082501	0.070304	0.091904
84	0.062934	0.090275	0.075430	0.100662
85	0.068512	0.098665	0.080760	0.110118
86	0.074513	0.107715	0.086292	0.120321
87	0.080967	0.117471	0.092023	0.131322
88	0.087911	0.127983	0.097950	0.143175
89	0.095381	0.139307	0.104073	0.155941
90	0.103422	0.151504	0.110390	0.169685
91		0.164641	0.116903	0.184478
92				0.200395

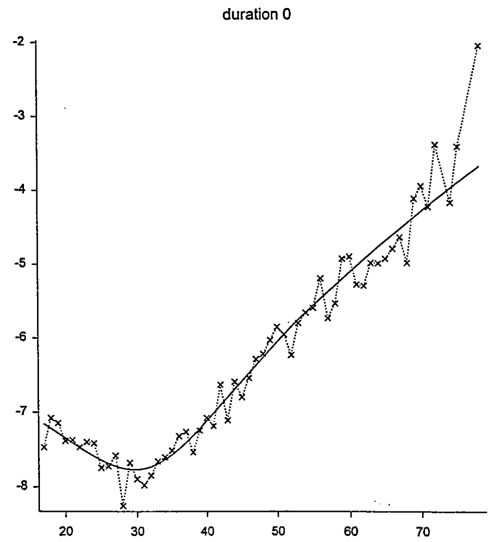


Figure 3.1a AM80 table: $\log \mu_x$ vs age x with log(crude mortality rates)

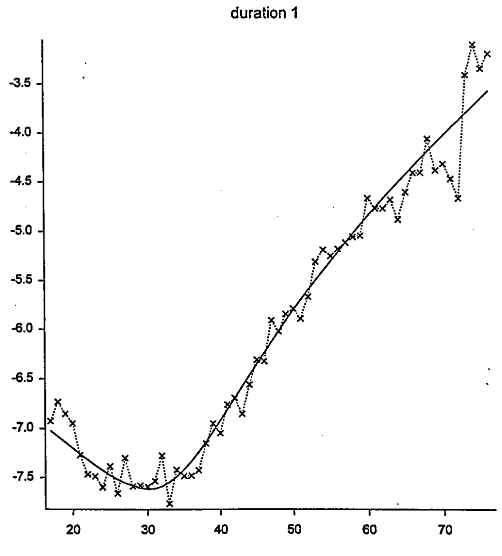


Figure 3.1b AM80 table: $\log \mu_x$ vs age x with log(crude mortality rates)

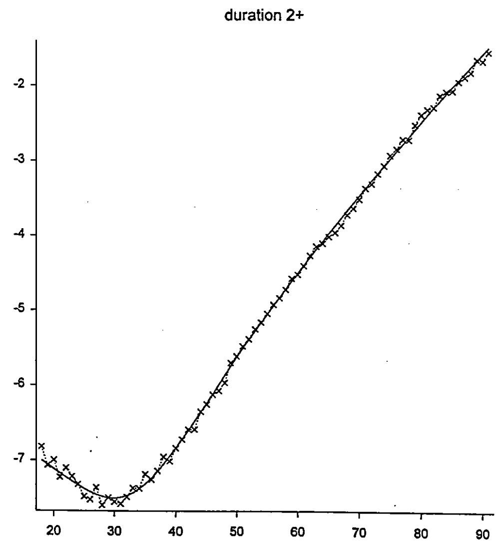


Figure 3.1c AM80 table: $\log \mu_x$ vs age x with log(crude mortality rates)

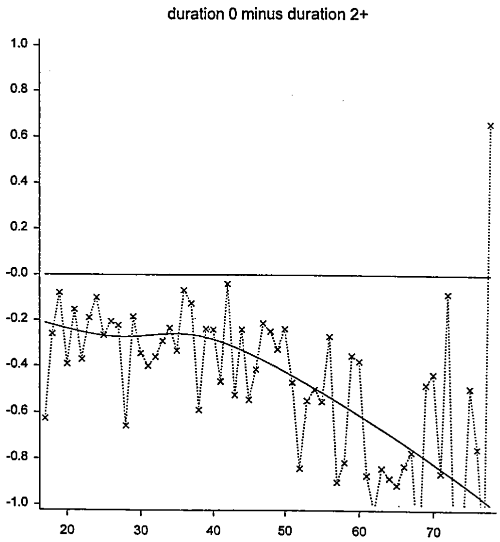


Figure 3.2a AM80 tables: $\log \mu_x^{d=0}$ - $\log \mu_x^{d=2+}$ vs age x with associated data points

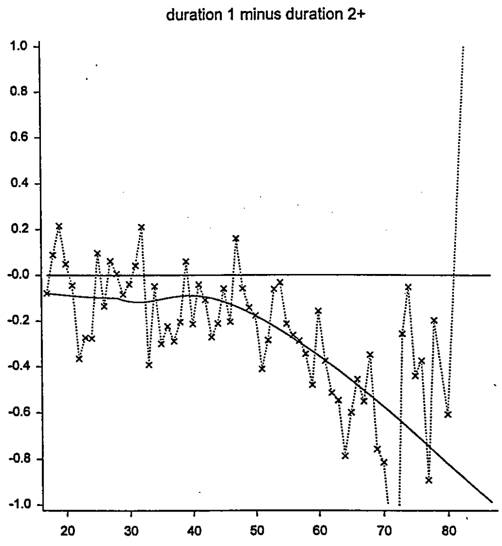


Figure 3.2b AM80 tables: $\log \mu_x^{d=1}$ - $\log \mu_x^{d=2+}$ vs age x with associated data points

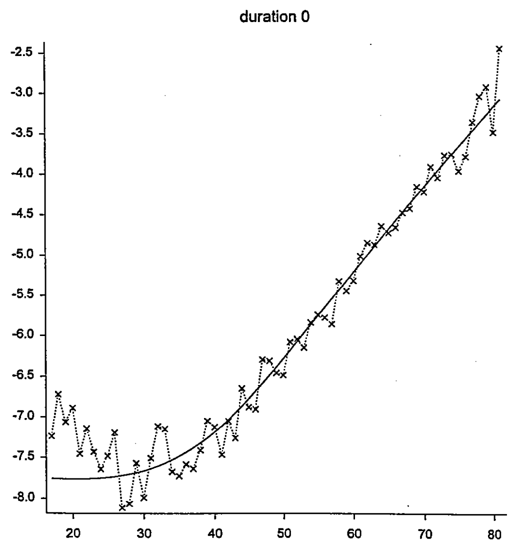


Figure 4.1a GM(2,3) table: $\log \mu_x$ vs age x with log(crude mortality rates)

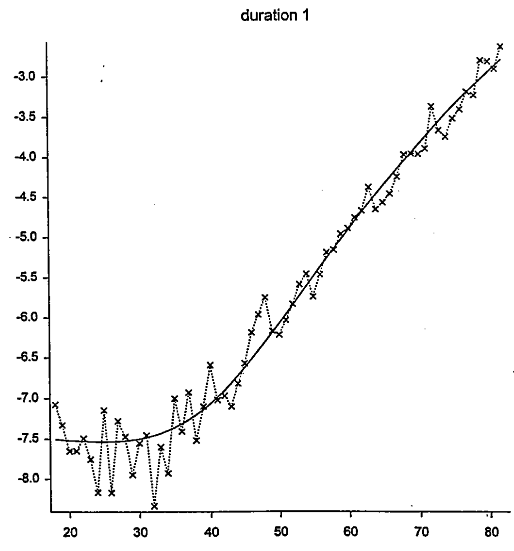


Figure 4.1b GM(2,3) table: $\log \mu_x$ vs age x with log(crude mortality rates)

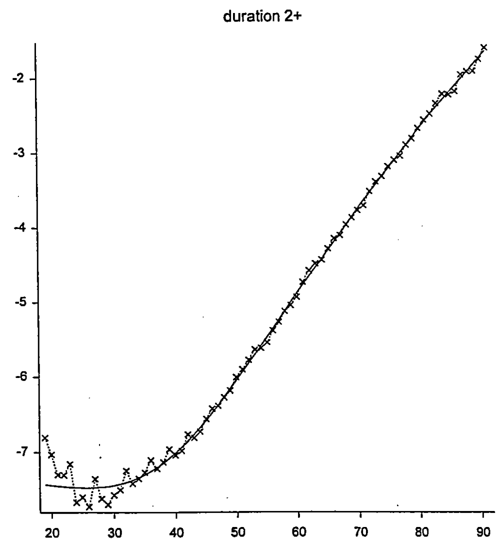


Figure 4.1c GM(2,3) table: $\log \mu_x$ vs age x with log(crude mortality rates)

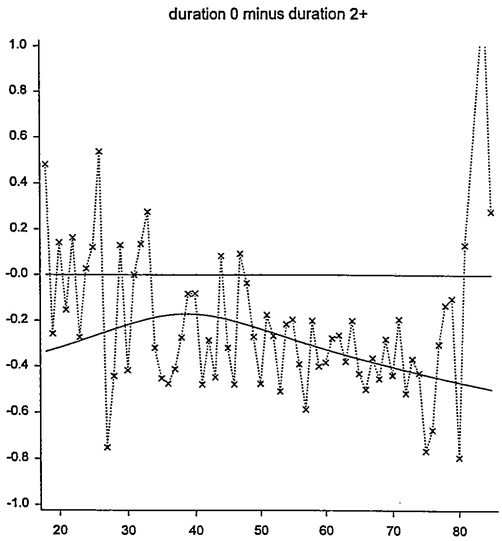


Figure 4.2a GM(2,3) tables: $\log \mu_x^{d=0}$ - $\log \mu_x^{d=2+}$ vs age x with associated data points

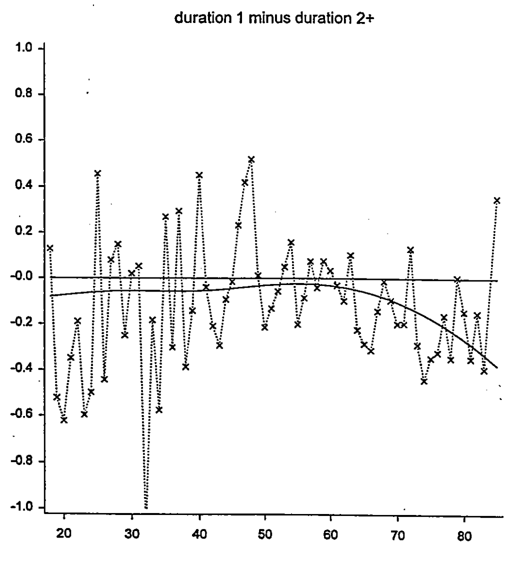


Figure 4.2b GM(2,3) tables: $\log \mu_x^{d=1}$ - $\log \mu_x^{d=2+}$ vs age x with associated data points

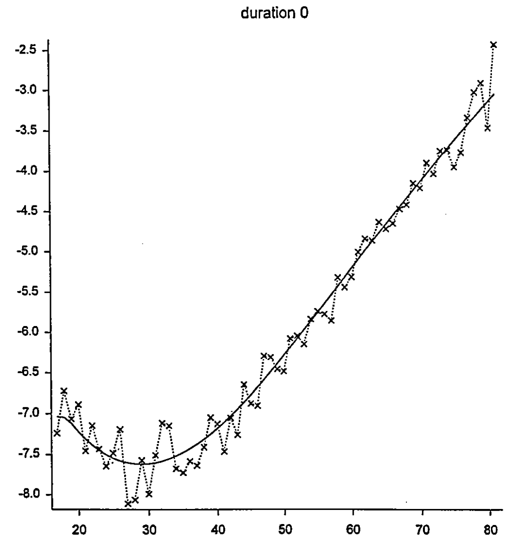


Figure 4.3a GM(0,5) table: $\log \mu_x$ vs age x with log(crude mortality rates)

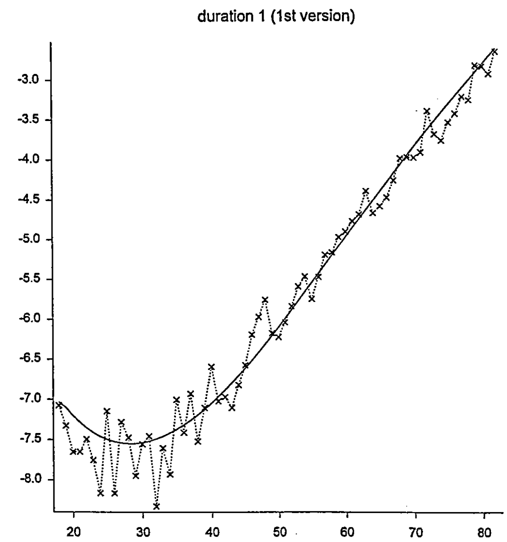


Figure 4.3b GM(0,5) table: $\log \mu_x$ vs age x with log(crude mortality rates)

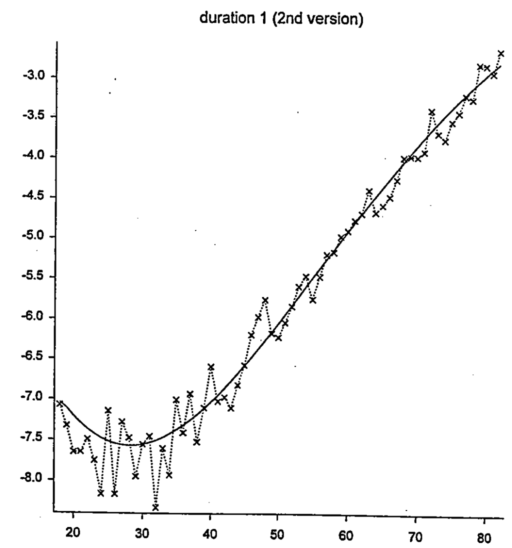


Figure 4.3c GM(0,5) table: $\log \mu_x$ vs age x with log(crude mortality rates)

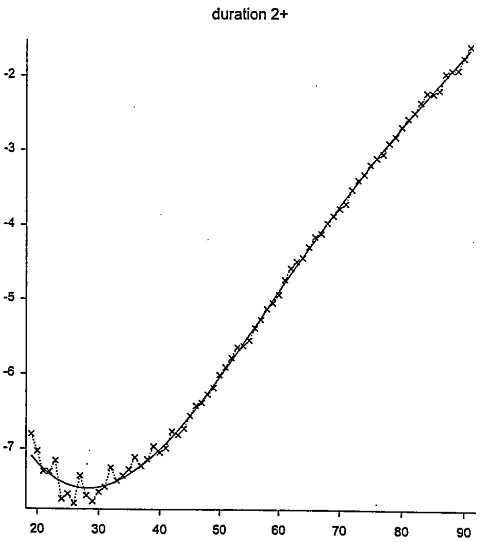


Figure 4.3d GM(0,5) table: $\log \mu_x$ vs age x with log(crude mortality rates)

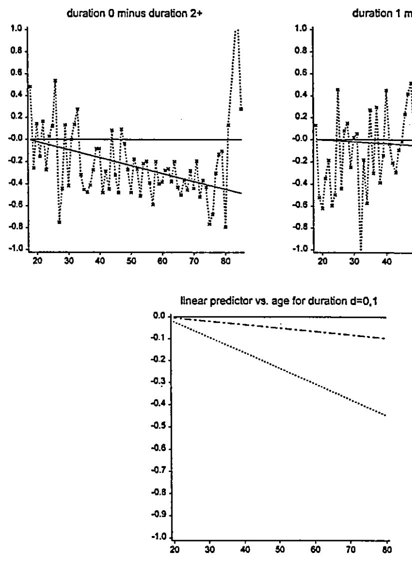


Figure 4.4 Difference plots based on 2+ ultimate duration, fitted pencil of lines

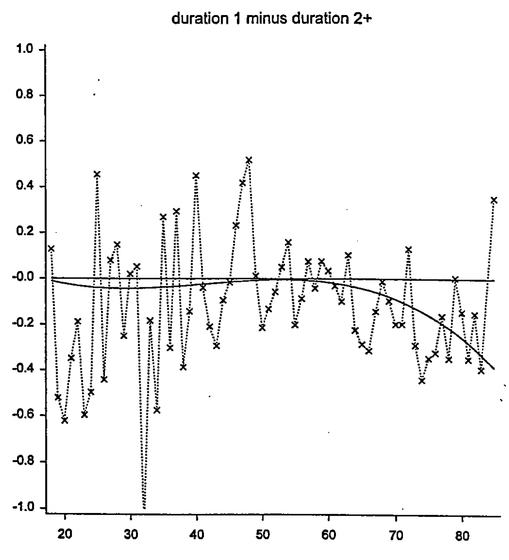


Figure 4.5 Difference plots with fitted cubic

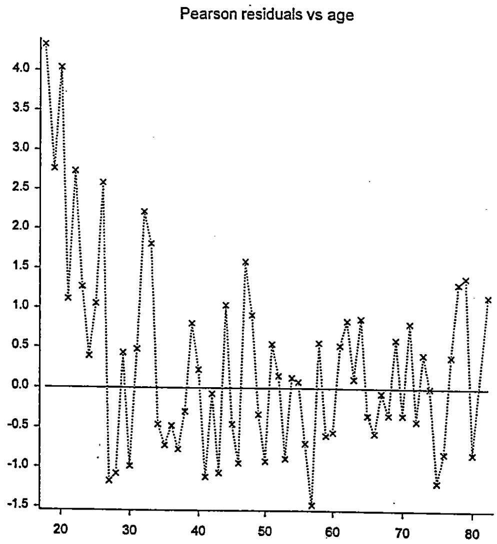


Figure 5.1 GM(2,3) table, $d = 0$: Pearson residuals vs age

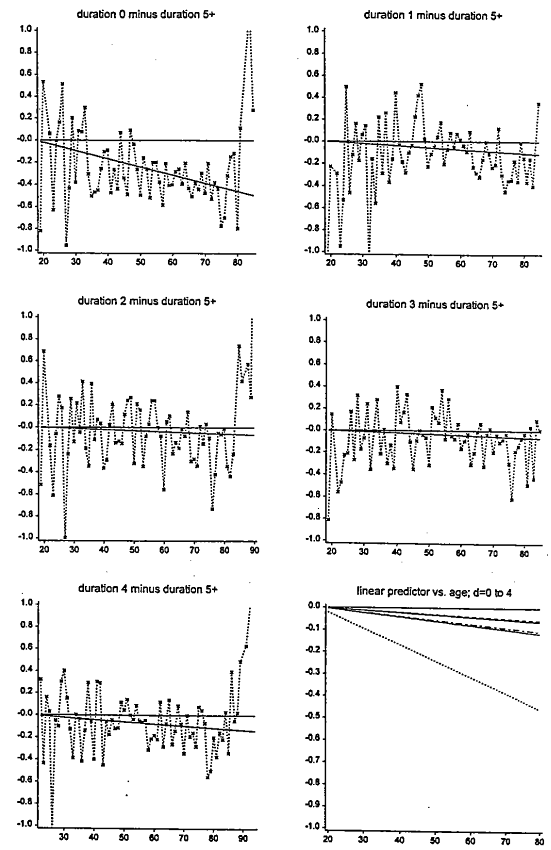


Figure 5.2 Difference plots based on 5+ ultimate duration, fitted pencil of lines

APPENDIX A

Further details of the μ- graduations based on the GM(0,5) described in this paper are presented in this appendix. The details take the form of familiar tabulations of a graduation, for each duration, coupled with the Pearson residual plots (z_x plots) against age x.

Note that the adjusted exposures (and adjusted deaths) for duration 2+ of Table A.3 may be checked against those of Table 1.17 or Table 1.18 in CMI Committee (1998).

Table A.1. Details of graduation for Males Duration 0, $GM_x(0,5)$ formula

Age x	R_x	μ_x	A_x	E_x	Dev_x	$\sqrt{V_x}$	z_x	100A/E
17	8343.8	.000879	6	7.34	-1.34	2.71	-0.49	81.77
18	14125.4	.000873	17	12.34	4.66	3.51	1.33	137.79
19	17526.9	.000811	15	14.21	0.79	3.77	0.21	105.55
20	19630.8	.000732	20	14.38	5.62	3.79	1.48	139.10
21	22483.7	.000670	13	15.07	-2.07	3.88	-0.53	86.28
22	24061.6	.000621	19	14.93	4.07	3.86	1.05	127.22
23	25328.3	.000581	15	14.73	0.27	3.84	0.07	101.81
24	27132.3	.000551	13	14.96	-1.96	3.87	-0.51	86.90
25	28385.1	.000528	16	15.00	1.00	3.87	0.26	106.69
26	29169.6	.000511	22	14.92	7.08	3.86	1.83	147.44
27	29952.1	.000500	9	14.98	-5.98	3.87	-1.55	60.07
28	28604.6	.000494	9	14.13	-5.13	3.76	-1.36	63.72
29	27149.6	.000492	14	13.36	0.64	3.65	0.18	104.83
30	26580.1	.000494	9	13.14	-4.14	3.62	-1.14	68.51
31	25497.9	.000501	14	12.77	1.23	3.57	0.35	109.66
32	24565.6	.000511	20	12.56	7.44	3.54	2.10	159.26
33	22803.3	.000526	18	11.99	6.01	3.46	1.74	150.12
34	21498.3	.000545	10	11.71	-1.71	3.42	-0.50	85.40
35	20349.0	.000568	9	11.56	-2.56	3.40	-0.75	77.88
36	19652.6	.000596	10	11.71	-1.71	3.42	-0.50	85.40
37	18628.5	.000629	9	11.72	-2.72	3.42	-0.79	76.82
38	18129.3	.000667	11	12.10	-1.10	3.48	-0.32	90.91
39	17191.4	.000712	15	12.24	2.76	3.50	0.79	122.54
40	17355.6	.000763	14	13.24	0.75	3.64	0.21	105.68
41	17398.4	.000822	10	14.30	-4.30	3.78	-1.14	69.92
42	17272.0	.000889	15	15.35	-0.35	3.92	-0.09	97.69
43	16989.3	.000965	12	16.40	-4.40	4.05	-1.09	73.18
44	17663.3	.001052	23	18.58	4.42	4.31	1.03	123.82
45	18391.3	.001150	19	21.14	-2.14	4.60	-0.47	89.86
46	17865.7	.001261	18	22.52	-4.52	4.75	-0.95	79.92
47	16211.4	.001386	30	22.47	7.53	4.74	1.59	133.49
48	14813.9	.001528	27	22.64	4.36	4.76	0.92	119.27
49	13963.9	.001688	22	23.58	-1.58	4.86	-0.32	93.31
50	13712.6	.001869	21	25.63	-4.63	5.06	-0.91	81.93
51	13524.4	.002073	31	28.04	2.96	5.29	0.56	110.57
52	12213.6	.002303	29	28.12	0.88	5.30	0.17	103.12
53	11204.6	.002561	24	28.69	-4.69	5.36	-0.88	83.64
54	10598.4	.002852	31	30.22	0.78	5.50	0.14	102.57
55	10537.8	.003178	34	33.49	0.51	5.79	0.09	101.52
56	9299.7	.003544	29	32.96	-3.96	5.74	-0.69	87.98
57	7307.1	.003955	21	28.90	-7.90	5.38	-1.47	72.66
58	5921.1	.004416	29	26.14	2.86	5.11	0.56	110.92
59	5296.1	.004930	23	26.11	-3.11	5.11	-0.61	88.09
60	5673.1	.005505	28	31.23	-3.23	5.59	-0.58	89.65
61	4924.2	.006147	33	30.27	2.73	5.50	0.50	109.02
62	4049.9	.006862	32	27.79	4.21	5.27	0.80	115.15
63	3746.8	.007657	29	28.69	0.31	5.36	0.06	101.08
64	3907.9	.008541	38	33.38	4.62	5.78	0.80	113.86
65	5587.2	.009521	50	53.19	-3.19	7.29	-0.44	94.00
66	4489.0	.010606	43	47.61	-4.61	6.90	-0.67	90.32
67	2969.4	.011806	34	34.95	-0.95	5.91	-0.16	97.28
68	2394.6	.013131	29	31.44	-2.44	5.61	-0.44	92.23
69	2208.3	.014590	35	32.22	2.78	5.68	0.49	108.63
70	2010.9	.016196	30	32.57	-2.57	5.71	-0.45	92.11

71	1523.4	.017961	31	27.36	3.64	5.23	0.70	113.30
72	1179.8	.019896	21	23.47	-2.47	4.84	-0.51	89.46
73	978.4	.022015	23	21.54	1.46	4.64	0.31	106.78
74	881.4	.024332	21	21.45	-0.45	4.63	-0.10	97.92
75	773.8	.026861	15	20.79	-5.79	4.56	-1.27	72.17
76	561.6	.029618	13	16.63	-3.63	4.08	-0.89	78.15
77	423.1	.032621	15	13.80	1.20	3.72	0.32	108.68
78	350.4	.035886	17	12.57	4.43	3.55	1.25	135.20
79	312.4	.039432	17	12.32	4.68	3.51	1.33	138.00
80	223.8	.043281	7	9.69	-2.69	3.11	-0.86	72.27
81	56.9	.047453	5	2.70				
82	13.1	.051975	0	0.68				
83	9.0	.056871	0	0.51				
84	5.5	.062171	2	0.34				
85	7.0	.067908	1	0.48				
86	1.5	.074118	0	0.11				
87	4.5	.080841	0	0.36				
88	1.5	.088124	0	0.13				
89	0.5	.096017	0	0.05				
81-89	99.5		8	5.36	2.63	2.32	1.14	149.11
Totals	833589.9		1344	1348.70	-4.70			

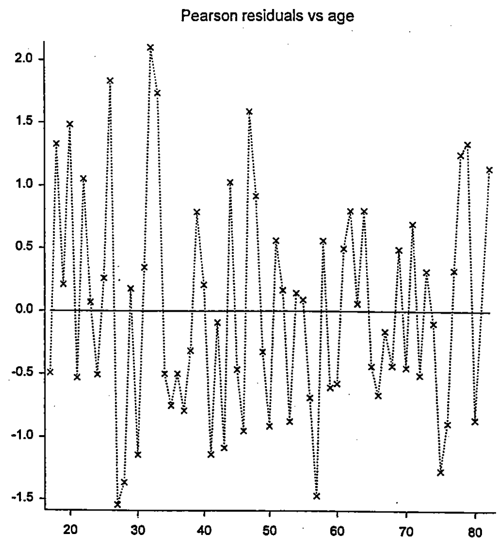


Table A.2(a) Details of graduation for Males Duration 1, (1st. version), $GM_x(0,5)$ formula

Age x	R_x	μ_x	A_x	E_x	Dev_x	$\sqrt{V_x}$	z_x	100A/E
17	2456.3	.000884	2	2.17				
18	8284.9	.000883	7	7.31				
17-18	10741.2		9	9.48	-0.48	3.08	-0.16	94.90
19	13693.1	.000824	9	11.28	-2.28	3.36	-0.68	79.77
20	16825.9	.000748	8	12.59	-4.59	3.55	-1.29	63.52
21	18908.3	.000689	9	13.02	-4.02	3.61	-1.11	69.12
22	21572.6	.000641	12	13.84	-1.84	3.72	-0.49	86.73
23	23315.1	.000605	10	14.09	-4.09	3.75	-1.09	70.96
24	24668.5	.000576	7	14.21	-7.21	3.77	-1.91	49.25
25	26647.9	.000555	21	14.79	6.21	3.85	1.61	141.96
26	28238.9	.000541	8	15.26	-7.26	3.91	-1.86	52.42
27	29020.7	.000531	20	15.42	4.58	3.93	1.17	129.67
28	29922.8	.000528	17	15.79	1.21	3.97	0.31	107.68
29	28310.1	.000526	10	14.96	-4.96	3.87	-1.28	66.84
30	26712.6	.000534	14	14.26	-0.26	3.78	-0.07	98.15
31	25919.1	.000544	15	14.10	0.90	3.75	0.24	106.39
32	24979.9	.000559	6	13.95	-7.95	3.74	-2.13	43.01
33	24044.1	.000578	12	13.89	-1.89	3.73	-0.51	86.40
34	22209.2	.000602	8	13.36	-5.36	3.66	-1.47	59.87
35	20876.3	.000631	19	13.17	5.83	3.63	1.61	144.28
36	19819.3	.000666	12	13.19	-1.19	3.63	-0.33	90.97
37	19412.7	.000706	19	13.71	5.29	3.70	1.43	138.56
38	18448.1	.000754	10	13.91	-3.91	3.73	-1.05	71.90
39	18225.8	.000809	15	14.74	0.26	3.84	0.07	101.77
40	17479.6	.000872	24	15.24	8.76	3.90	2.24	157.49
41	17940.9	.000944	16	16.94	-0.94	4.12	-0.23	94.46
42	18110.1	.001027	17	18.59	-1.59	4.31	-0.37	91.43
43	18192.8	.001121	15	20.39	-5.39	4.52	-1.19	73.56
44	18310.9	.001228	20	22.49	-2.49	4.74	-0.52	88.94
45	18582.4	.001350	26	25.09	0.91	5.01	0.18	103.63
46	18517.4	.001489	38	27.57	10.43	5.25	1.99	137.84
47	17925.8	.001646	46	29.51	16.49	5.43	3.04	155.89
48	16055.4	.001825	51	29.30	21.70	5.41	4.01	174.08
49	14831.3	.002027	31	30.07	0.93	5.48	0.17	103.10
50	14052.8	.002257	28	31.72	-3.72	5.63	-0.66	88.28
51	13775.6	.002517	33	34.67	-1.67	5.89	-0.28	95.18
52	14002.8	.002811	41	39.37	1.63	6.27	0.26	104.15
53	12817.6	.003144	48	40.30	7.70	6.35	1.21	119.10
54	12001.1	.003521	51	42.25	8.75	6.50	1.35	120.71
55	11212.9	.003945	36	44.24	-8.24	6.65	-1.24	81.37
56	11120.4	.004425	47	49.21	-2.21	7.01	-0.31	95.52
57	9699.9	.004965	54	48.16	5.84	6.94	0.94	112.12
58	7490.9	.005574	43	41.75	1.25	6.46	0.19	102.99
59	6157.5	.006258	43	38.54	4.46	6.21	0.72	111.58
60	5341.8	.007027	40	37.54	2.46	6.13	0.40	106.56
61	5719.6	.007890	49	45.13	3.87	6.72	0.58	108.58
62	4824.1	.008857	45	42.73	2.27	6.54	0.35	105.32
63	3912.2	.009938	49	38.88	10.12	6.24	1.62	126.03
64	3580.0	.011147	34	39.91	-5.91	6.32	-0.93	85.20
65	3765.2	.012495	39	47.05	-8.05	6.86	-1.17	82.90
66	5367.6	.013997	62	75.13	-13.13	8.67	-1.52	82.52
67	4331.1	.015668	62	67.86	-5.86	8.24	-0.71	91.37
68	2817.6	.017523	53	49.37	3.63	7.03	0.52	107.35
69	2308.1	.019580	44	45.19	-1.19	6.72	-0.18	97.36

70	2106.9	.021857	40	46.05	-6.05	6.79	-0.89	86.86
71	1923.6	.024373	39	46.88	-7.88	6.85	-1.15	83.19
72	1498.4	.027149	51	40.68	10.32	6.38	1.62	125.37
73	1061.6	.030208	27	32.07	-5.07	5.66	-0.90	84.19
74	807.9	.033573	19	27.12	-8.12	5.21	-1.56	70.05
75	746.5	.037270	22	27.82	-5.82	5.27	-1.10	79.07
76	698.4	.041325	23	28.86	-5.86	5.37	-1.09	79.69
77	538.9	.045768	22	24.66	-2.66	4.97	-0.54	89.20
78	433.5	.050629	17	21.95	-4.95	4.68	-1.06	77.46
79	363.1	.055943	22	20.31	1.69	4.51	0.37	108.31
80	284.3	.061745	17	17.55	-0.55	4.19	-0.13	96.84
81	202.1	.068076	11	13.76	-2.76	3.71	-0.74	79.95
82	55.6	.074977	4	4.17				
83	15.4	.082498	1	1.27				
84	10.8	.090690	0	0.98				
85	6.5	.099611	1	0.65				
86	4.5	.109325	0	0.49				
87	2.0	.119907	0	0.24				
88	5.3	.131438	0	0.70				
89	0.3	.144010	0	0.04				
81-89	100.4		6	8.54	-2.54	2.92	-0.87	70.28
Totals	833523.3		1771	1793.47	-22.47			

Pearson residuals vs age

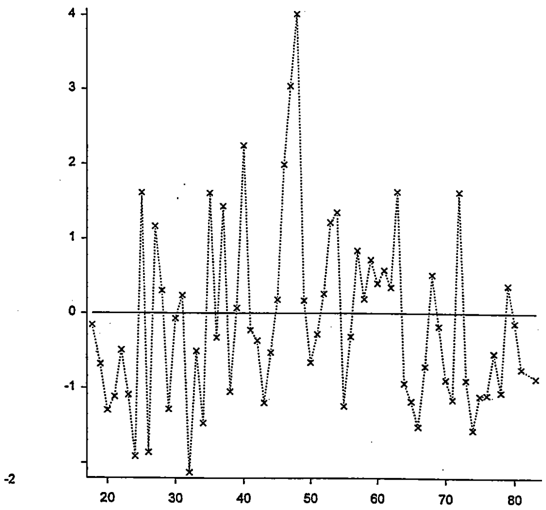


Table A.2(b) Details of graduation for Males Duration 1, (2nd. version), $GM_x(0,5)$ formula

Age x	R_x	μ_x	A_x	E_x	Dev_x	$\sqrt{V_x}$	z_x	100A/E
17	2456.3	.000884	2	2.17				
18	8284.9	.000878	7	7.27				
18-19	10741.2		9	9.44	-0.45	3.07	-0.15	95.27
19	13693.1	.000816	9	11.18	-2.18	3.34	-0.65	80.53
20	16825.9	.000738	8	12.43	-4.43	3.53	-1.26	64.36
21	18908.3	.000678	9	12.81	-3.81	3.58	-1.07	70.24
22	21572.6	.000630	12	13.58	-1.58	3.69	-0.43	88.34
23	23315.1	.000592	10	13.81	-3.81	3.72	-1.03	72.40
24	24668.5	.000564	7	13.91	-6.91	3.73	-1.85	50.31
25	26647.9	.000543	21	14.48	6.52	3.80	1.72	145.08
26	28238.9	.000529	8	14.93	-6.93	3.86	-1.79	53.57
27	29020.7	.000520	20	15.10	4.90	3.89	1.26	132.46
28	29922.8	.000517	17	15.47	1.53	3.93	0.39	109.90
29	28310.1	.000518	10	14.70	-4.68	3.83	-1.22	68.12
30	26712.6	.000525	14	14.02	-0.02	3.74	-0.01	99.86
31	25919.1	.000536	15	13.89	1.11	3.73	0.30	108.02
32	24979.9	.000551	6	13.77	-7.77	3.71	-2.09	43.56
33	24044.1	.000572	12	13.75	-1.75	3.71	-0.47	87.28
34	22209.2	.000597	8	13.28	-5.27	3.64	-1.45	60.30
35	20876.3	.000628	19	13.12	5.88	3.62	1.62	144.83
36	19819.3	.000665	12	13.19	-1.19	3.63	-0.33	91.01
37	19412.7	.000709	19	13.76	5.24	3.71	1.41	138.13
38	18448.1	.000759	10	14.00	-4.00	3.74	-1.07	71.42
39	18225.8	.000817	15	14.90	0.10	3.86	0.03	100.69
40	17479.6	.000884	24	15.46	8.54	3.93	2.17	155.23
41	17940.9	.000962	16	17.25	-1.24	4.15	-0.30	92.74
42	18110.1	.001050	17	19.01	-2.01	4.36	-0.46	89.42
43	18192.8	.001151	15	20.93	-5.93	4.58	-1.30	71.66
44	18310.9	.001265	20	23.17	-3.17	4.81	-0.66	86.32
45	18582.4	.001396	26	25.95	0.05	5.09	0.01	100.21
46	18517.4	.001545	38	28.61	9.39	5.35	1.76	132.81
47	17925.8	.001714	46	30.73	15.27	5.54	2.75	149.69
48	16055.4	.001906	51	30.61	20.39	5.53	3.69	166.62
49	14831.3	.002124	31	31.51	-0.51	5.61	-0.09	98.39
50	14052.8	.002371	28	33.33	-5.33	5.77	-0.92	84.02
51	13775.6	.002651	33	36.52	-3.52	6.04	-0.58	90.36
52	14002.8	.002967	41	41.55	-0.55	6.45	-0.09	98.67
53	12817.6	.003324	48	42.61	5.39	6.53	0.83	112.64
54	12001.1	.003727	51	44.73	6.27	6.69	0.94	114.01
55	11212.9	.004181	36	46.88	-10.88	6.85	-1.59	76.79
56	11120.4	.004691	47	52.17	-5.17	7.22	-0.72	90.09
57	9699.9	.005264	54	51.06	2.94	7.15	0.41	105.75
58	7490.9	.005906	43	44.24	-1.24	6.65	-0.19	97.19
59	6157.5	.006624	43	40.79	2.21	6.39	0.35	105.42
60	5341.8	.007426	40	39.67	0.33	6.30	0.06	100.84
61	5719.6	.008318	49	47.58	1.42	6.90	0.21	102.99
62	4824.1	.009309	45	44.91	0.09	6.70	0.01	100.20
63	3912.2	.010408	49	40.72	8.28	6.38	1.30	120.34
64	3580.0	.011623	34	41.61	-7.61	6.45	-1.18	81.71
65	3765.2	.012963	39	48.81	-9.81	6.99	-1.40	79.91
66	5367.6	.014435	62	77.48	-15.48	8.80	-1.76	80.02
67	4331.1	.016050	62	69.51	-7.51	8.33	-0.90	89.19
68	2817.6	.017815	53	50.20	2.80	7.08	0.40	105.59
69	2308.1	.019738	44	45.56	-1.56	6.75	-0.23	96.58

70	2106.9	.021828	40	45.99	-5.99	6.78	-0.88	87.98
71	1923.6	.024091	39	46.34	-7.34	6.81	-1.08	84.16
72	1498.4	.026534	51	39.76	11.24	6.31	1.78	128.27
73	1061.6	.029163	27	30.96	-3.96	5.56	-0.71	87.21
74	807.9	.031982	19	25.84	-6.84	5.08	-1.35	73.53
75	746.5	.034995	22	26.12	-4.12	5.11	-0.81	84.21
76	698.4	.038204	23	26.68	-3.68	5.17	-0.71	86.20
77	538.9	.041611	22	22.42	-0.42	4.74	-0.09	98.11
78	433.5	.045217	17	19.60	-2.60	4.43	-0.59	87.73
79	363.1	.049019	22	17.80	4.20	4.22	1.00	123.60
80	284.3	.053016	17	15.07	1.93	3.88	0.50	112.79
81	202.1	.057203	11	11.56	-0.56	3.40	-0.16	95.15
82	55.6	.061577	4	3.42				
83	15.4	.066132	1	1.02				
84	10.8	.070861	0	0.76				
85	6.5	.075758	1	0.49				
86	4.5	.080816	0	0.36				
87	2.0	.086026	0	0.17				
88	5.3	.091382	0	0.48				
89	0.3	.0968785	0	0.03				
81-89	100.4		6	6.73	-0.75	2.60	-0.29	88.90
Totals	833523.3		1771	1817.55	-46.55			

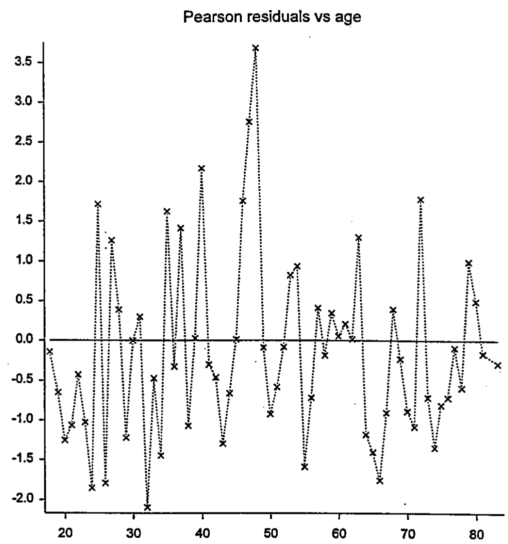
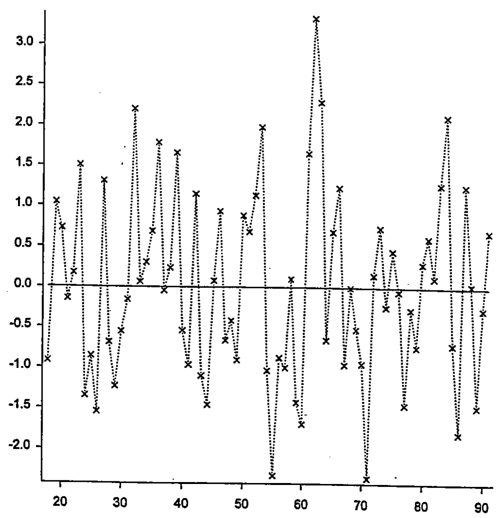


Table A.3 Details of graduation for Males Duration 2+, $GM_x(0,5)$ formula
(Exposed to risk and actual deaths divided by variance ratios)

Age x	R'_x	μ_x	A'_x	E_x	Dev_x	$\sqrt{V_x}$	z_x	100A/E
17	1670	.000888	0.00	1.48				
18	4035	.000889	3.00	3.58				
17-19	5705		3.00	5.48	-2.07	2.25	-0.92	59.19
19	11733	.000831	13.00	9.75	3.25	3.12	1.04	133.38
20	23730	.000756	21.00	17.93	3.07	4.24	0.72	117.10
21	32234	.000696	21.74	22.45	-0.71	4.74	-0.15	96.85
22	40182	.000650	26.98	26.10	0.88	5.11	0.17	103.38
23	50791	.000613	39.53	31.14	8.39	5.58	1.50	126.95
24	75382	.000585	35.19	44.12	-8.94	6.64	-1.35	79.74
25	98036	.000565	49.00	55.38	-6.38	7.44	-0.86	88.48
26	108178	.000551	47.66	59.59	-11.92	7.72	-1.54	79.99
27	104283	.000543	66.41	56.57	9.84	7.52	1.31	117.39
28	106316	.000539	52.11	57.34	-5.23	7.57	-0.69	90.88
29	106131	.000541	48.10	57.43	-9.33	7.58	-1.23	83.76
30	153251	.000548	78.81	83.91	-5.10	9.16	-0.56	93.92
31	169103	.000559	92.92	94.47	-1.55	9.72	-0.16	98.36
32	147738	.000575	105.15	84.87	20.28	9.21	2.20	123.89
33	151442	.000595	90.71	90.12	0.59	9.49	0.06	100.66
34	156258	.000621	100.00	97.00	3.00	9.85	0.30	103.09
35	163409	.000652	113.61	106.53	7.08	10.32	0.69	106.64
36	130215	.000689	106.60	89.70	16.90	9.47	1.78	118.84
37	194052	.000732	141.55	142.10	-0.55	11.92	-0.05	99.62
38	204437	.000783	163.01	160.01	3.00	12.65	0.24	101.88
39	199799	.000841	189.51	168.02	21.48	12.96	1.66	112.79
40	253116	.000908	221.58	229.82	-8.24	15.16	-0.54	96.42
41	282886	.000985	262.50	278.57	-16.07	16.69	-0.96	94.23
42	202255	.001073	233.81	216.93	16.88	14.73	1.15	107.78
43	304640	.001173	336.54	357.27	-20.73	18.90	-1.10	94.20
44	356134	.001287	427.15	458.34	-31.19	21.41	-1.46	93.20
45	341281	.001417	485.29	483.60	1.69	21.99	0.08	100.35
46	363340	.001565	590.80	568.60	22.19	23.85	0.93	103.90
47	370712	.001733	625.62	642.48	-16.85	25.35	-0.66	97.38
48	317516	.001924	600.55	610.94	-10.39	24.72	-0.42	98.30
49	313606	.002141	648.00	671.44	-23.44	25.91	-0.90	96.51
50	305688	.002387	753.53	729.73	23.80	27.01	0.88	103.26
51	283631	.002666	775.00	756.26	18.74	27.50	0.68	102.48
52	277850	.002983	861.31	828.76	32.55	28.79	1.13	103.93
53	265052	.003341	944.51	885.58	58.93	29.76	1.98	106.65
54	283110	.003747	1027.16	1060.76	-33.60	32.57	-1.03	96.83
55	283033	.004206	1110.06	1190.31	-80.25	34.50	-2.33	93.26
56	256730	.004724	1182.35	1212.78	-30.43	34.83	-0.87	97.49
57	264496	.005309	1366.87	1404.25	-37.37	37.47	-1.00	97.34
58	244728	.005969	1464.67	1460.80	3.87	38.22	0.10	100.27
59	244349	.006713	1582.50	1640.18	-57.68	40.50	-1.42	96.48
60	228136	.007549	1652.03	1722.17	-70.14	41.50	-1.69	95.93
61	191875	.008489	1695.54	1628.78	66.76	40.36	1.65	104.10
62	192946	.009544	1984.35	1841.40	142.95	42.91	3.33	107.76
63	171383	.010726	1936.36	1838.21	98.16	42.87	2.29	105.34
64	166608	.012049	1978.01	2007.42	-29.40	44.80	-0.66	98.54
65	105857	.013527	1457.81	1431.94	25.88	37.84	0.68	101.81
66	60318	.015177	952.59	915.42	37.17	30.26	1.23	104.06
67	58525	.017014	965.55	995.77	-30.22	31.56	-0.96	96.97
68	48882	.019058	931.50	931.60	-0.11	30.52	-0.00	99.99

69	47506	.021328	996.55	1013.22	-16.67	31.83	-0.52	98.36
70	41549	.023845	961.02	990.75	-29.73	31.48	-0.94	97.00
71	39094	.026632	965.22	1041.13	-75.91	32.27	-2.35	92.71
72	35851	.029711	1070.18	1065.18	4.99	32.64	0.15	100.47
73	26458	.033110	897.74	876.04	21.70	29.60	0.73	102.48
74	26427	.036856	966.37	973.98	-7.61	31.21	-0.24	99.22
75	18956	.040977	789.31	776.76	12.55	27.87	0.45	101.62
76	17736	.045506	805.60	807.09	-1.49	28.41	-0.05	99.82
77	16941	.050476	812.70	855.09	-42.40	29.24	-1.45	95.04
78	16884	.055924	935.83	944.23	-8.40	30.73	-0.27	99.11
79	15011	.061889	906.50	929.03	-22.53	30.48	-0.74	97.58
80	12906	.068414	891.67	882.98	8.69	29.71	0.29	100.98
81	11584	.075546	893.10	875.09	18.02	29.58	0.61	102.06
82	10082	.083334	843.59	840.18	3.41	28.99	0.12	100.41
83	7843	.091835	754.20	720.22	33.98	26.84	1.27	104.74
84	6927	.101110	756.35	700.40	55.95	26.46	2.11	107.99
85	5831	.111228	630.51	648.61	-18.10	25.47	-0.71	97.21
86	4635	.122265	523.73	566.70	-42.97	23.81	-1.81	92.42
87	3748	.134307	531.36	503.34	28.01	22.44	1.25	105.57
88	2567	.147451	379.10	378.54	0.56	19.46	0.03	100.15
89	2038	.161805	302.98	329.70	-26.71	18.16	-1.47	91.90
90	1660	.177495	289.92	294.57	-4.65	17.16	-0.27	98.42
91	1145	.194665	233.10	222.84	10.26	14.93	0.69	104.61
Totals	9844468		47795.80	47795.40	0.39			

Pearson residuals vs age



APPENDIX B

Further details of the μ-graduations based on the proposed GM(2,3) formulae, described in CMI Committee (1998), for select durations 0 and 1 are presented in this appendix. These augment Table 1.18 in CMI Committee (1998) for ultimate duration 2+ based on a similar formula. The details take the form of familiar tabulations of a graduation, for each select duration, coupled with the Pearson residual plots (z_x plots) against age x.

Note in particular that the three outlier at adjacent ages 46, 47 and 48 in the data for Duration 1, where the actual deaths are substantially higher than those expected for this model, (as identified in ¶1.8.9 CMI Committee (1998)), are much in evidence.

Table B.1 Details of graduation for Males Duration 0, $GM_x(2,3)$ formula

Age x	R_x	μ_x	A_x	E_x	Dev_x	$\sqrt{V_x}$	z_x	100A/E
17	8343.8	.000428	6	3.57				
18	14125.4	.000427	17	6.02				
17-18	22469.2		23	9.59	13.41	3.10	4.33	239.74
19	17526.9	.000426	15	7.46	7.54	2.73	2.76	201.11
20	19630.8	.000425	20	8.35	11.65	2.89	4.03	239.66
21	22483.7	.000425	13	9.56	3.44	3.09	1.11	135.97
22	24061.6	.000426	19	10.25	8.75	3.20	2.73	195.35
23	25328.3	.000428	15	10.83	4.17	3.29	1.27	138.52
24	27132.3	.000430	13	11.66	1.34	3.42	0.39	111.45
25	28385.1	.000433	16	12.30	3.70	3.51	1.06	130.10
26	29169.6	.000438	22	12.77	9.23	3.57	2.58	172.31
27	29952.1	.000443	9	13.28	-4.28	3.64	-1.17	67.76
28	28604.6	.000451	9	12.89	-3.89	3.59	-1.08	69.82
29	27149.6	.000460	14	12.48	1.52	3.53	0.43	112.22
30	26580.1	.000470	9	12.50	-3.50	3.54	-0.99	72.01
31	25497.9	.000483	14	12.32	1.68	3.51	0.48	113.64
32	24565.6	.000499	20	12.25	7.75	3.50	2.22	163.32
33	22803.3	.000517	18	11.78	6.22	3.43	1.81	152.79
34	21498.3	.000538	10	11.56	-1.56	3.40	-0.46	86.47
35	20349.0	.000563	9	11.45	-2.45	3.38	-0.72	78.59
36	19652.6	.000592	10	11.63	-1.63	3.41	-0.48	85.99
37	18628.5	.000625	9	11.65	-2.65	3.41	-0.78	77.27
38	18129.3	.000664	11	12.04	-1.04	3.47	-0.30	91.38
39	17191.4	.000708	15	12.18	2.82	3.49	0.81	123.15
40	17355.6	.000760	14	13.18	0.82	3.63	0.22	106.19
41	17398.4	.000818	10	14.24	-4.24	3.77	-1.12	70.25
42	17272.0	.000885	15	15.29	-0.29	3.91	-0.07	98.12
43	16989.3	.000961	12	16.33	-4.33	4.04	-1.07	73.47
44	17663.3	.001048	23	18.52	4.48	4.30	1.04	124.22
45	18391.3	.001147	19	21.09	-2.09	4.59	-0.46	90.08
46	17865.7	.001259	18	22.49	-4.49	4.74	-0.95	80.04
47	16211.4	.001386	30	22.46	7.54	4.74	1.59	133.56
48	14813.9	.001529	27	22.65	4.35	4.76	0.91	119.22
49	13963.9	.001691	22	23.61	-1.61	4.86	-0.33	93.20
50	13712.6	.001873	21	25.68	-4.68	5.07	-0.92	81.77
51	13524.4	.002078	31	28.11	2.90	5.30	0.55	110.30
52	12213.6	.002309	29	28.20	0.80	5.31	0.15	102.84
53	11204.6	.002568	24	28.78	-4.78	5.36	-0.89	83.41
54	10598.4	.002859	31	30.30	0.70	5.50	0.13	102.31
55	10537.8	.003185	34	33.56	0.44	5.79	0.08	101.32
56	9299.7	.003549	29	33.00	-4.00	5.74	-0.70	87.87
57	7307.1	.003956	21	28.91	-7.91	5.38	-1.47	72.64
58	5921.1	.004411	29	26.12	2.88	5.11	0.56	111.04
59	5296.1	.004918	23	26.05	-3.05	5.10	-0.60	88.31
60	5673.1	.005482	28	31.10	-3.10	5.58	-0.56	90.02
61	4924.2	.006111	33	30.09	2.91	5.49	0.53	109.67
62	4049.9	.006809	32	27.57	4.43	5.25	0.84	116.05
63	3746.8	.007384	29	28.41	0.59	5.33	0.11	102.06
64	3907.9	.008443	38	32.99	5.01	5.74	0.87	115.17
65	5587.2	.009395	50	52.49	-2.49	7.25	-0.34	95.26
66	4489.0	.010448	43	46.90	-3.90	6.85	-0.57	91.68
67	2969.4	.011612	34	34.38	-0.38	5.86	-0.06	98.91
68	2394.6	.012897	29	30.88	-1.88	5.56	-0.34	93.90
69	2208.3	.014314	35	31.61	3.39	5.62	0.60	110.72

70	2010.9	.015876	30	31.92	-1.92	5.65	-0.34	93.97
71	1523.4	.017594	31	26.80	4.20	5.18	0.81	115.66
72	1179.8	.019483	21	22.99	-1.99	4.79	-0.41	91.36
73	978.4	.021558	23	21.09	1.91	4.59	0.42	109.05
74	881.4	.023834	21	21.01	-0.01	4.58	-0.00	99.97
75	773.8	.026328	15	20.37	-5.37	4.51	-1.19	73.63
76	561.6	.029058	13	16.32	-3.32	4.04	-0.82	79.66
77	423.1	.032043	15	13.56	1.44	3.68	0.39	110.64
78	350.4	.035304	17	12.37	4.63	3.52	1.32	137.42
79	312.4	.038862	17	12.14	4.86	3.48	1.39	140.03
80	223.8	.042741	7	9.57	-2.57	3.09	-0.83	73.18
81	56.9	.046964	5	2.67				
82	13.1	.051558	0	0.68				
83	9.0	.056549	0	0.51				
84	5.5	.061966	2	0.34				
85	7.0	.067838	1	0.47				
86	1.5	.074199	0	0.11				
87	4.5	.081080	0	0.36				
88	1.5	.088516	0	0.13				
89	0.5	.096543	0	0.05				
81-89	99.5		8	5.32	2.67	2.31	1.16	150.11
Totals	833589.9		1344	1289.22	54.78			

Pearson residuals vs age

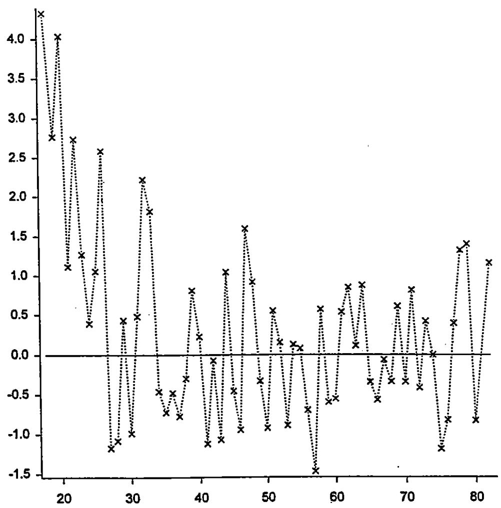
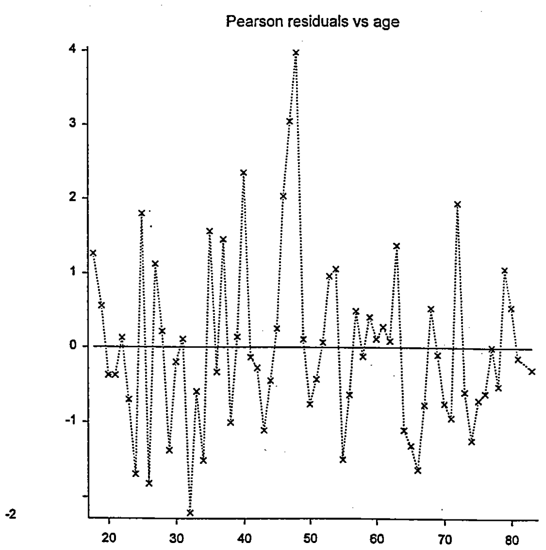


Table B.2 Details of graduation for Males Duration 1, $GM_x(2,3)$ formula

Age x	R_x	μ_x	A_x	E_x	Dev_x	$\sqrt{V_x}$	z_x	100A/E
17	2456.3	.000555	2	1.36				
18	8284.9	.000550	7	4.56				
17-18	10741.2		9	5.92	3.08	2.43	1.26	151.98
19	13693.1	.000546	9	7.48	1.52	2.73	0.56	120.33
20	16825.9	.000542	8	9.13	-1.13	3.02	-0.37	87.64
21	18908.3	.000539	9	10.18	-1.20	3.19	-0.37	88.26
22	21572.6	.000537	12	11.58	0.42	3.40	0.12	103.65
23	23315.1	.000535	10	12.47	-2.47	3.53	-0.70	80.22
24	24668.5	.000534	7	13.16	-6.16	3.63	-1.70	53.19
25	26647.9	.000533	21	14.21	6.79	3.77	1.80	147.79
26	28238.9	.000534	8	15.08	-7.08	3.88	-1.82	53.04
27	29020.7	.000536	20	15.56	4.44	3.94	1.13	128.52
28	29922.8	.000540	17	16.16	0.84	4.02	0.21	105.23
29	28310.1	.000545	10	15.44	-5.44	3.93	-1.38	64.77
30	26712.6	.000553	14	14.77	-0.77	3.84	-0.20	94.78
31	25919.1	.000563	15	14.59	0.41	3.82	0.11	102.79
32	24979.9	.000576	6	14.39	-8.39	3.79	-2.21	41.71
33	24044.1	.000592	12	14.24	-2.24	3.77	-0.59	84.29
34	22209.2	.000612	8	13.59	-5.59	3.69	-1.52	58.85
35	20876.3	.000637	19	13.29	5.71	3.65	1.57	142.96
36	19819.3	.000666	12	13.20	-1.20	3.63	-0.33	90.88
37	19412.7	.000702	19	13.62	5.38	3.69	1.46	139.46
38	18448.1	.000744	10	13.73	-3.73	3.71	-1.01	72.84
39	18225.8	.000794	15	14.48	0.52	3.80	0.14	103.62
40	17479.6	.000853	24	14.92	9.08	3.86	2.35	160.90
41	17940.9	.000923	16	16.55	-0.55	4.07	-0.14	96.66
42	18110.1	.001003	17	18.17	-1.77	4.26	-0.28	93.55
43	18192.8	.001097	15	19.97	-4.97	4.47	-1.11	75.13
44	18310.9	.001206	20	22.09	-2.09	4.70	-0.44	90.54
45	18582.4	.001332	26	24.76	1.24	4.98	0.25	105.02
46	18517.4	.001477	38	27.35	10.65	5.23	2.04	138.92
47	17925.8	.001644	46	29.46	16.54	5.43	3.05	156.14
48	16055.4	.001834	51	29.45	21.55	5.43	3.97	173.20
49	14831.3	.002051	31	30.42	0.58	5.52	0.10	101.89
50	14052.8	.002299	28	32.31	-4.31	5.68	-0.76	86.67
51	13775.6	.002580	33	35.54	-2.54	5.96	-0.43	92.84
52	14002.8	.002899	41	40.59	0.41	6.37	0.06	101.01
53	12817.6	.003259	48	41.77	6.23	6.46	0.96	114.91
54	12001.1	.003665	51	43.98	7.02	6.63	1.06	115.96
55	11212.9	.004121	36	46.21	-10.21	6.80	-1.50	77.90
56	11120.4	.004634	47	51.53	-4.53	7.18	-0.63	91.21
57	9699.9	.005208	54	50.51	3.49	7.11	0.49	106.90
58	7490.9	.005849	43	43.81	-0.81	6.62	-0.12	98.15
59	6157.5	.006563	43	40.41	2.59	6.36	0.41	106.41
60	5341.8	.007357	40	39.30	0.70	6.27	0.11	101.78
61	5719.6	.008238	49	47.12	1.88	6.86	0.27	103.99
62	4824.1	.009213	45	44.45	0.55	6.67	0.08	101.25
63	3912.2	.010291	49	40.26	8.74	6.34	1.38	121.71
64	3580.0	.011477	34	41.09	-7.09	6.41	-1.11	82.75
65	3765.2	.012782	39	48.13	-9.13	6.94	-1.32	81.04
66	5367.6	.014213	62	76.29	-14.29	8.73	-1.64	81.27
67	4331.1	.015779	62	68.34	-6.34	8.27	-0.77	90.72
68	2817.6	.017489	53	49.28	3.72	7.02	0.53	107.55
69	2308.1	.019352	44	44.67	-0.67	6.68	-0.10	98.51

70	2106.9	.021377	40	45.04	-5.04	6.71	-0.75	88.81
71	1923.6	.023573	39	45.34	-6.34	6.73	-0.94	86.01
72	1498.4	.025948	51	38.88	12.12	6.24	1.94	131.17
73	1061.6	.028512	27	30.27	-3.27	5.50	-0.59	89.20
74	807.9	.031273	19	25.27	-6.27	5.03	-1.25	75.20
75	746.5	.034239	22	25.56	-3.56	5.06	-0.70	86.07
76	698.4	.037418	23	26.13	-3.13	5.11	-0.61	88.01
77	538.9	.040816	22	22.00	0.00	4.69	0.00	100.02
78	433.5	.044441	17	19.27	-2.27	4.39	-0.52	88.24
79	363.1	.048298	22	17.54	4.46	4.19	1.07	125.45
80	284.3	.052392	17	14.90	2.10	3.86	0.55	114.13
81	202.1	.056726	11	11.46	-0.46	3.39	-0.14	95.95
82	55.6	.061304	4	3.41				
83	15.4	.066126	1	1.01				
84	10.8	.071193	0	0.77				
85	6.5	.076504	1	0.50				
86	4.5	.082054	0	0.37				
87	2.0	.087841	0	0.18				
88	5.3	.093857	0	0.50				
89	0.3	.100094	0	0.03				
82-89	100.4		6	6.77	-0.77	2.60	-0.29	88.69
Totals	833523.3		1771	1773.43	-2.43			



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