AAS 13-372

NOVEL ORBITS OF MERCURY AND VENUS ENABLED USING LOW-THRUST PROPULSION

Pamela Anderson,* Malcolm Macdonald,† and Chen-wan L. Yen‡

Exploration of the inner planets of the Solar System is vital to significantly enhance the understanding of the formulation of Earth and other planets. This paper therefore considers the development of novel orbits of both Mercury and Venus to enhance the opportunities for remote sensing. Continuous low-thrust propulsion is used to extend the critical inclination of highly elliptical orbits at each planet, which are shown to require very small acceleration magnitudes. Unlike other bodies in the Solar System, natural sun-synchronous orbits do not exist at Mercury or Venus. This research therefore also uses continuous acceleration to enable both circular and elliptical sun-synchronous orbits, which could significantly simplify the spacecraft thermal environment. Considerably high thrust levels are however required to enable these orbits, which could not be provided by current propulsion systems.

INTRODUCTION

Planetary exploration is vital to gain an insight into the formulation and evolution of not only Earth but of other planets in the Solar System, it can aid the understanding of how life began on Earth, determine whether extra-terrestrial habitable environments exist in the Solar System, enhance quality of life through technology innovation, and reinforce science, technology, engineering, and mathematics (STEM) education.

More specifically, the NASA Vision and Voyages Decadal Survey for 2013 – 2022 has identified three themes for the future development of planetary science, within which the importance of investigating the evolution of the inner planets and their atmospheres is highlighted. Exploration of Venus is emphasized to determine whether the ancient aqueous environment was conducive to early life and investigate if life ever emerged. Furthermore, investigation of the chemistry, geology and climates of the inner planets can lead to better understanding of climate change here on Earth [1]. Thus the importance of exploration of Mercury and Venus is clear, and missions must be developed which are responsive to the identified scientific goals. This work therefore develops novel orbits around Mercury and Venus to enable new and unique investigations.

Similar to Earth, the concentration of mass around the equators of Mercury and Venus mean orbits can be inclined at a 'critical inclination' which allows the apse line to remain stationary over the orbit [2]. These orbits have been used extensively at Earth for high-latitude communications [3] and can offer benefits for remote sensing of other bodies in the Solar System by allowing the spacecraft to spend a large amount of

* pamela.c.anderson@strath.ac.uk, Advanced Space Concepts Laboratory, Mechanical and Aerospace Engineering, University of Strathclyde, Glasgow G4 0LT, Scotland, E.U.
† Malcolm.macdonald.102@strath.ac.uk, Advanced Space Concepts Laboratory, Mechanical and Aerospace Engineering, University of Strathclyde, Glasgow G4 0LT, Scotland, E.U.
‡ chen-wan.l.yen@jpl.nasa.gov, JPL, 4800 Oak Grove Drive Pasadena, CA 91109.

time over a region of interest. The fixed critical inclination can however limit the potential applications of these orbits, thus investigation is conducted into the use of continuous low-thrust propulsion to alter the critical inclination to any inclination required to optimally fulfill the mission objectives. Previous work has considered the extension of the critical inclination of orbits around Earth and Mars [4, 5], and has been shown to offer significant benefits particularly for observation of high-latitude regions of the Earth.

Although critical inclination values can be derived for Mercury and Venus, the reciprocal of flattening of these planets is so low that natural perturbations are of no use for generating sun-synchronous orbits. This paper therefore uses methods previously introduced, [5, 6] to extend existing sun-synchronous orbits at Earth and Mars, to generate sun-synchronous orbits at Mercury and Venus where they otherwise are not possible. This highlights the difference in this research from previous investigations. Enabling sun-synchronous orbits at these bodies could considerably enhance the opportunities for remote sensing and allow significant simplification of the spacecraft thermal environment.

This paper presents the extension of the natural critical inclination values at Mercury and Venus using a general perturbations solution, which is validated using a special perturbations solution. The use of continuous acceleration to force the rotation of ascending node to match the mean rotation of the sun and enable sun-synchronous orbits at these planets is also shown, again using both general and special perturbations solutions.

SPACECRAFT MOTION ABOUT AN OBLATE BODY

As this work will derive new highly elliptical and sun-synchronous orbits around Mercury and Venus, using low-thrust propulsion to counteract the effects of the gravitational fields of each planet, the gravitational potential of oblate bodies is considered [7]

$$U(r,\beta,\lambda)=\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(\frac{R_B}{r}\right)^n\left(C_{n,m}\cos\left(m\lambda\right)+S_{n,m}\sin\left(m\lambda\right)\right)P_{n,m}\sin\beta \tag{1}$$

Where, R_B and μ are the radius and the gravitational parameter of the body under consideration (notation is also defined at the end of this paper). For a body possessing axial symmetry the influence of periodic effects (tesseral and sectorial harmonics) can be neglected for most orbits. The gravitational potential may be written as

$$U(r,\beta)=\frac{\mu}{r}\left[1-\sum_{n=0}^{\infty}J_n\left(\frac{R_B}{r}\right)^n P_n\sin\beta\right] \tag{2}$$

Expanding Eq. (2), the gravitational potential becomes

$$\begin{aligned} U(r,\beta)=\frac{\mu}{r}[1&-J_2\frac{1}{2}\left(\frac{R_B}{r}\right)^2\left(3\sin^2\left(\beta\right)-1\right)\\ &-J_3\frac{1}{2}\left(\frac{R_B}{r}\right)^3\left(5\sin^2\left(\beta\right)-3\right)\sin\beta\\ &-J_4\frac{1}{8}\left(\frac{R_B}{r}\right)^4\left(3-30\sin^2\left(\beta\right)+35\sin^4\left(\beta\right)\right)\\ &-J_5\frac{1}{8}\left(\frac{R_B}{r}\right)^4\left(63\sin^5\left(\beta\right)-70\sin^3\left(\beta\right)+15\sin\left(\beta\right)\right)\\ &-...] \end{aligned} \tag{3}$$

Considering the case where gravitational perturbations to the order of J_4 are considered, and using spherical triangle laws Eq. (3) becomes

$$U(r,\beta)=U_0+U_p=\frac{\mu}{r}-J_2\frac{\mu R_B^2}{2r^3}\left(3\sin^2(i)\sin^2(\theta+\omega)-1\right)$$
$$-J_3\frac{\mu R_B^3}{2r^4}\left(5\sin^3(i)\sin^3(\theta+\omega)-3\sin(i)\sin(\theta+\omega)\right) \tag{4}$$
$$-J_4\frac{\mu R_B^4}{8r^5}\left(3-30\sin^2(i)\sin^2(\theta+\omega)+35\sin^4(i)\sin^4(\theta+\omega)\right)$$

This results in expressions for the perturbing accelerations in the radial, transverse and normal (RTN) directions of

$$R_J=\frac{\mu R_B^2}{8r^7}(-12J_2r^3+15J_4rR_B^2+\sin(i)\sin(\theta+\omega)(-48J_3rR_B+\sin(i)\sin(\theta+\omega)$$
$$(6(6J_2r^2-25J_4R_B^2)+5R_B\sin(i)\sin(\theta+\omega)(16J_3r+35J_4R_B\sin(i)\sin(\theta+\omega))))) \tag{5}$$

$$T_J=\frac{\mu R_B^2}{2r^6}\cos(\theta+\omega)\sin(i)(3J_3rR_B-\sin(i)\sin(\theta+\omega)$$
$$(6J_2r^2-15J_4R_B^2+5R_B\sin(i)\sin(\theta+\omega)(3J_3r+7J_4R_B\sin(i)\sin(\theta+\omega)))) \tag{6}$$

$$N_J=\frac{\mu R_B^2}{2r^6}\cos(i)(3J_3rR_B-\sin(i)\sin(\theta+\omega)$$
$$(6J_2r^2-15J_4R_B^2+5R_B\sin(i)\sin(\theta+\omega)(3J_4r+7J_4R_B\sin(i)\sin(\theta+\omega)))) \tag{7}$$

HIGHLY ELLIPTICAL ORBITS ABOUT VENUS

General Perturbations Solution

As more detailed gravity data is known for Venus, the derivation of orbits around Venus is considered prior to investigation of Mercury. Gravity perturbations at Venus to the order of J_4 are of the same order of magnitude, and thus must be considered in this analysis, the values of J_2, J_3 and J_4 are equal to 4.458E-6, -2.1082E-6, and -2.1471E-6 respectively [14].

Although the equatorial bulge of Venus is less pronounced than at Earth, the value of the critical inclination of orbits is considered. The value of which is derived using the Gauss form of the Lagrange Planetary Equation for the rate of change of argument of periapsis [9].

$$\frac{d\omega}{d\theta}=\frac{r^2}{\mu e}\left[-R\cos\theta+T\left(1+\frac{r}{p}\right)\sin\theta\right]-\frac{r^3}{\mu p\tan i}\sin(\theta+\omega)N \tag{8}$$

The critical inclination value is therefore derived by substituting Eqs. (5) - (7) into Eq. (8) and integrating analytically over one orbital revolution, which results in the expression for the change in argument of periapsis

$$(\Delta\omega)_{J_4}=\frac{3\pi R_B^2}{512a^4e\left(-1+e^2\right)^4}(e(384a^2\left(-1+e^2\right)^2J_2-135\left(4+5e^2\right)J_4R_B^2+20(32a^2\left(-1+e^2\right)^2J_2$$
$$-\left(52+63e^2\right)J_4R_B^2)\cos(2i)-35\left(28+27e^2\right)J_4R_B^2\cos(4i))$$
$$+10eJ_4R_B^2(-6+5e^2+4\left(-2+7e^2\right)\cos(2i)+7\left(2+9e^2\right)\cos(4i))\cos(2\omega) \tag{9}$$
$$+16a\left(-1+e^2\right)^2J_3R_B(-1-3e^2-4\cos(2i)+5\left(1+7e^2\right)\cos(4i))\csc(i)\sin(\omega)))$$

As higher order gravity terms are included in this expression, the value of the critical inclination will be dependent upon the orbit semi-major axis and eccentricity. For example, a 12 h orbit with a pericenter altitude of 800 km and apocenter altitude of 36,810 km has fixed critical inclinations of 85.3 and 94.7 degrees. It is clear from Eq. (9) that altering the inclination from these critical values will result in a drift in the argument of periapsis due to the effects of the gravity perturbations. Thus, for each value of inclination there exists a constant acceleration magnitude which will negate this drift, and allow free selection of inclination. Low-thrust terms are therefore added to Eqs. (5) - (7) using the argument of periapsis locally optimal control law derived from the variational equation, given in Eq. (8), by consideration of the sine and cosine terms in this equation [10]. The combined gravitational and low-thrust perturbations in each of the RTN directions are thus given by

$$\begin{aligned} R_{J+F_r} = & \frac{\mu R_B^2}{8r^7}(12J_2r^3 + 15J_4rR_B^2 + \sin(i)\sin(\theta+\omega)(-48J_3rR_B + \sin(i)\sin(\theta+\omega)(6(6J_2r^2 - 25J_4R_B^2) \\ & +5R_B\sin(i)\sin(\theta+\omega)(16J_3r + 35J_4R_B\sin(i)\sin(\theta+\omega))))) + F_r\,\mathrm{sgn}\left[\cos(\theta)\right] \end{aligned} \tag{10}$$

$$\begin{aligned} T_{J+F_t} = & \frac{\mu R_B^2}{2r^6}\cos(\theta+\omega)\sin(i)(3J_3rR_B - \sin(i)\sin(\theta+\omega) \\ & (6J_2r^2 - 15J_4R_B^2 + 5R_B\sin(i)\sin(\theta+\omega)(3J_3r + 7J_4R_B\sin(i)\sin(\theta+\omega)))) + F_t\,\mathrm{sgn}\left[\sin(\theta)\right] \end{aligned} \tag{11}$$

$$\begin{aligned} N_{J+F_n} = & \frac{\mu R_B^2\cos(i)}{2r^6}(3J_3rR_B - \sin(i)\sin(\theta+\omega) \\ & (6J_2r^2 - 15J_4R_B^2 + 5R_B\sin(i)\sin(\theta+\omega)(3J_3r + 7J_4R_B\sin(i)\sin(\theta+\omega)))) + F_t\,\mathrm{sgn}\left[\sin(\theta+\omega)\right] \end{aligned} \tag{12}$$

The expression for the rate of change of argument of periapsis with the application of low-thrust propulsion is determined by inserting Eqs. (10) - (12) into Eq. (8) and integrating over one orbital revolution, and the resulting expression given in Eq. (13). Where, the total change in argument of periapsis includes effects from a gravitational parameter, given by Eq. (9), and radial, transverse and normal acceleration components given by Eqs. (14) - (16).

$$\left(\Delta\omega\right)_0^{2\pi} = \left(\Delta\omega\right)_{J_2} + \left(\Delta\omega\right)_{F_r} + \left(\Delta\omega\right)_{F_t} + \left(\Delta\omega\right)_{F_n} \tag{13}$$

$$\begin{aligned} \left(\Delta\omega\right)_{F_r} = & \frac{1}{e\mu}2a^2F_r(-2+2e^2 - 4e\sqrt{-1+e^2}\,\mathrm{Arctanh}\left[\frac{-1+e}{\sqrt{-1+e^2}}\right] \\ & -e\sqrt{-1+e^2}\ln\left[\frac{1-e}{\sqrt{-1+e^2}}\right] + e\sqrt{-1+e^2}\ln\left[\frac{-1+e}{\sqrt{-1+e^2}}\right]) \end{aligned} \tag{14}$$

$$\left(\Delta\omega\right)_{F_t} = -\frac{4a^2\left(-2+e^2\right)F_t}{e\mu} \tag{15}$$

$$\begin{aligned} \left(\Delta\omega\right)_{F_n} = & -\frac{a^2F_n\csc(i)}{\sqrt{-1+e^2}\,\mu}(4\sqrt{-1+e^2} + 2e^2\sqrt{-1+e^2} \\ & -12e\,\mathrm{ArcTanh}\left[\frac{-1+e}{\sqrt{-1+e^2}}\right] - 3e\ln\left[\frac{1-e}{\sqrt{-1+e^2}}\right] + 3e\ln\left[\frac{-1+e}{\sqrt{-1+e^2}}\right]\sin(\omega)) \end{aligned} \tag{16}$$

It should be noted that Eq. (16) is the normal acceleration component given for an argument of periapsis value of 270 degrees, as from Eq. (12) the direction of the normal acceleration is dependent upon the argument of latitude. Previous research has also shown the benefit of including acceleration out of the orbit plane to be minimal when considering the extension of the critical inclination [4], the change in argument of periapsis is therefore made up of a gravitational component and unequal radial and transverse acceleration components which gives the lowest acceleration magnitude to achieve a range of inclinations.

The resulting acceleration magnitudes are shown for a variety of orbit periods between 6 and 24 hours to achieve inclinations between 5 and 175 degrees for a constant pericenter altitude of 800 km, to compensate for the drift in argument of periapsis caused by gravitational perturbations to the order of J_4.

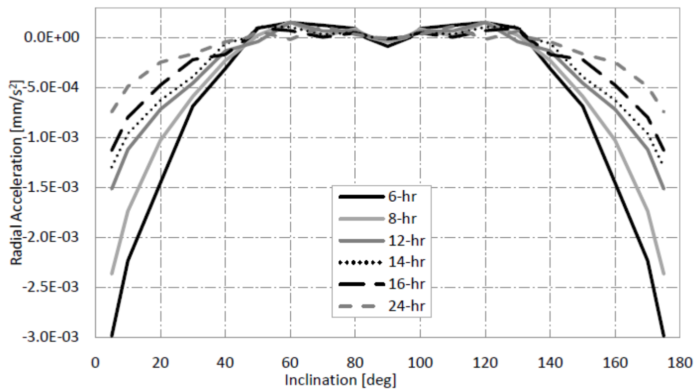


Figure 1 Radial acceleration – Extension of critical inclination at Venus

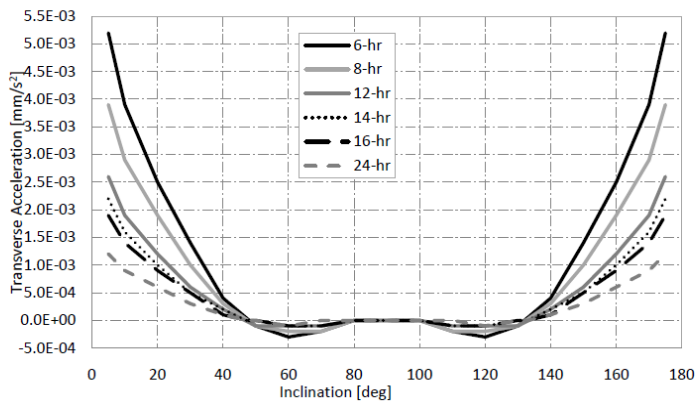


Figure 2 Transverse acceleration - Extension of critical inclination at Venus

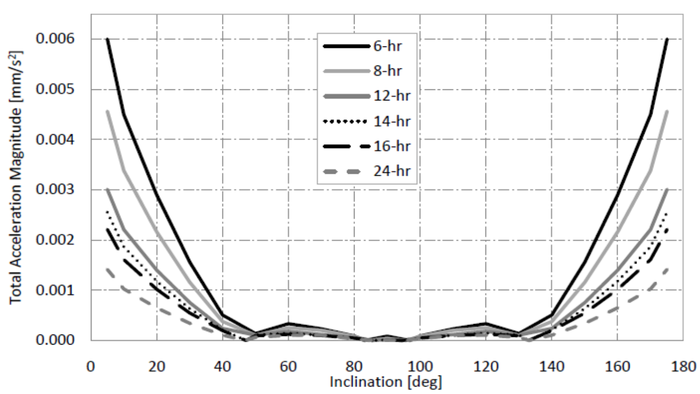


Figure 3 Total required acceleration magnitude for combined thrusting in radial and transverse axes - Extension of critical inclination at Venus

Figure 3 shows curves of the minimum total acceleration made up of unequal radial and transverse components (from Figure 1 and Figure 2) to alter the critical inclination of the orbits to a wide range of possible values. It is shown that as is expected, as the orbit period increases, the total acceleration magnitude decreases. It can also be seen that to enable all of the considered orbits very small acceleration magnitudes are required. For example, considering a 12 h orbit with an inclination of 90 degrees requires a total acceleration magnitude of 1.8E-5 mm/s^2 which, for a 1-ton spacecraft corresponds to a considerably low thrust level of 0.0185 mN.

Special Perturbations Solution

The general perturbations solution can be verified using a special perturbations technique. This model propagates the position of the spacecraft using a set of Modified Equinoctial Elements [12] using an explicit variable step size Runge Kutta (4,5) formula, the Dormand-Prince pair [13]. Results are given for the semi-major axis, eccentricity and argument of pericentre for five revolutions of the spacecraft on a 12 h orbit with orbital elements given in Table 1, in Figure 4. Results are not shown for the remaining orbital elements as these remain unchanged over the orbit. The results show that although the semi-major axis and eccentricity experience some oscillation, the change is negligible over the orbit. The argument of periapsis, however is shown to experience a small drift over the orbit, of around -6.3E-4 deg per orbit which is thought to be acceptable.

Table 1 Orbital parameters – extension of critical inclination at Venus

Orbital Element	Value (units)
Pericenter altitude	800 (km)
Apocenter altitude	36811 (km)
Inclination	90 (deg)
Ascending node	330 (deg)
Argument of periapsis	270 (deg)

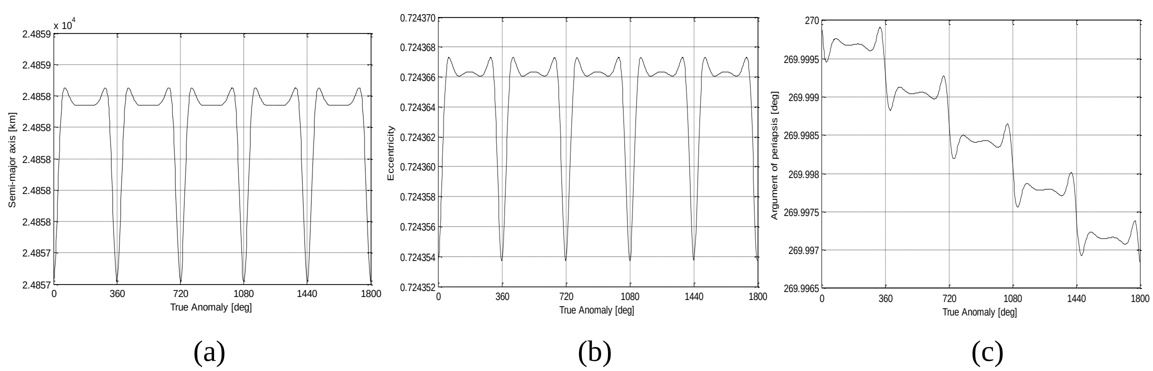


Figure 4 Oscillation of orbital elements over five orbital revolutions around Venus. (a) Semi-major axis (b) Eccentricity (c) Argument of periapsis

SUN-SYNCHRONOUS ORBITS ABOUT VENUS

Circular Sun-Synchronous Orbits - General Perturbations Solution

As the reciprocal of flattening of Venus is so low the rotation of the ascending node cannot naturally match the motion of the mean sun to give sun-synchronous orbits. Thus, continuous acceleration is applied to enable these orbits by forcing a rotation of the ascending node angle to rotate 2π radians in around 225 days. The ascending node angle is described by the Gauss form of the variational equations using classical orbital elements [9].

$$\frac{d\Omega}{d\theta}=\frac{r^3}{\mu p\sin(i)}\sin(\theta+\omega)N \tag{17}$$

The expression for the normal acceleration, given in Eq. (12) is substituted into Eq. (17) and integrated analytically over the orbit to give the change in ascending node angle. In this case, an out-of-plane acceleration is applied which is dependent on the argument of latitude, two solutions exist for an argument of perihelion equal to 0 and 270 degrees, which give the maximum and minimum of the solution. The two resulting expressions for the change in ascending node angle per second are given by

$$\begin{aligned}(\Delta\Omega)_{\omega=0} &= \frac{\sqrt{\mu}}{2\sqrt{a^3}\pi}\Big(\frac{4a^2F_n\cos(\omega)\csc(i)}{\mu}+\frac{1}{(32a^4\left(-1+e^2\right)^4)}3\pi R_B^2(\cos(i)(-32a^2\left(-1+e^2\right)^2J_2\\ &+5\left(2+3e^2\right)J_4R_B^2+35\left(2+3e^2\right)J_4R_B^2\cos(2i))-5e^2J_4R_B^2(5\cos(i)+7\cos(3i))\cos(2\omega)\\ &-2ae\left(-1+e^2\right)J_3R_B(\cos(i)+15\cos(3i))\csc(i)\sin(\omega))\Big)\end{aligned} \tag{18}$$

$$\begin{aligned}(\Delta\Omega)_{\omega=270} &= \frac{\sqrt{\mu}}{2\sqrt{a^3}\pi}\Big(\frac{-1}{\left(\sqrt{-1+e^2}\,\mu\right)}a^2F_n\csc(i)(4\sqrt{-1+e^2}-12e\,\mathrm{ArcTanh}\left[\frac{-1+e}{\sqrt{-1+e^2}}\right]-3e\ln\left[\frac{1-e}{\sqrt{-1+e^2}}\right]\\ &+3e\ln\left[\frac{-1+e}{\sqrt{-1+e^2}}\right])\sin(\omega)+\left(\frac{1}{32a^4\left(-1+e^2\right)^4}\right)3\pi R_B^2(\cos(i)(-32a^2\left(-1+e^2\right)^2J_2\\ &+5\left(2+3e^2\right)^2J_2+5\left(2+3e^2\right)^2J_4R_B^2+35\left(2+3e^2\right)^2J_4R_B^2\cos(2i))\\ &-2ae\left(-1+e^2\right)^2J_3R_B(\cos(i)+15\cos(3i))\csc(i)\sin(\omega))\Big)\end{aligned} \tag{19}$$

The required normal acceleration to enable various circular sun-synchronous orbits around Venus, derived using Eq. (18), is given in Figure 5.

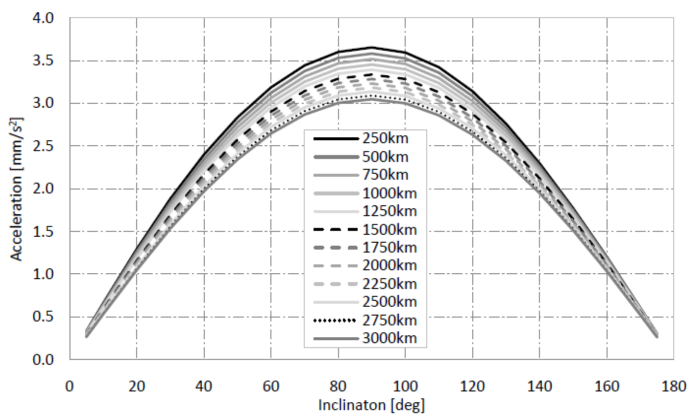


Figure 5 Normal acceleration required for the extension of circular sun-synchronous orbits at Venus, inclination range 5 – 175 degrees

It is shown from Figure 5 that continuous low-thrust propulsion can be used to enable a range of circular sun-synchronous orbits at Venus that would otherwise be infeasible. This being said, it is shown that the acceleration magnitude required to enable circular sun-synchronous orbits is significantly higher than is required to alter the critical inclination. For example, a 1000 km circular orbit inclined at 90 degrees requires 3.45 mm/s^2 of acceleration

Circular Sun-Synchronous Orbits - Special Perturbations Solution

Once again numerical methods are used to validate general perturbations solutions for circular sun-synchronous orbits at Venus, the results are shown for the orbit detailed in Table 2. Figure 6 shows the change in ascending node angle achieved using normal acceleration to achieve the sun-synchronous condition of a 2π radian revolution of the ascending node angle in 225 days, which for the orbit in Table 2 equates to around 0.1 degrees per orbit.

Table 2 Orbital parameters – circular sun-synchronous orbit at Venus

Orbital Element	Value (units)
Pericenter altitude	1,000 (km)
Apocenter altitude	1,000 (km)
Inclination	90 (deg)
Ascending node	330 (deg)

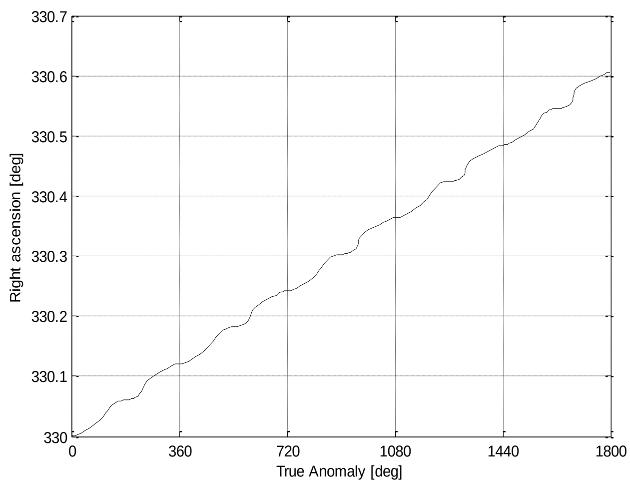


Figure 6 Variation of ascending node angle over five orbital revolutions for a circular orbit around Venus

Elliptical Sun-Synchronous Orbits – General Perturbations Solution

Two conditions are combined to develop elliptical sun-synchronous orbits, which allow both the ascending node angle of the orbit to rotate to match the rotation of the mean sun and allow the argument of periapsis to remain unchanged over the orbit period. This is achieved by using continuous out-of-plane acceleration to force the required rotation of the ascending node (from Eq. (19) for an argument of periapsis value of 270 degrees), and using combined radial and transverse acceleration to compensate for the applied normal acceleration and maintain the zero change in argument of periapsis condition (from Eq. (13))

essential to HEOs. The acceleration required in the radial, transverse and normal directions are shown in Figure 7 - Figure 9, and the total acceleration magnitude is given in Figure 10 for orbits of varying orbital periods and inclinations for an argument of perihelion value of 270 degrees for a constant pericentre altitude of 800 km.

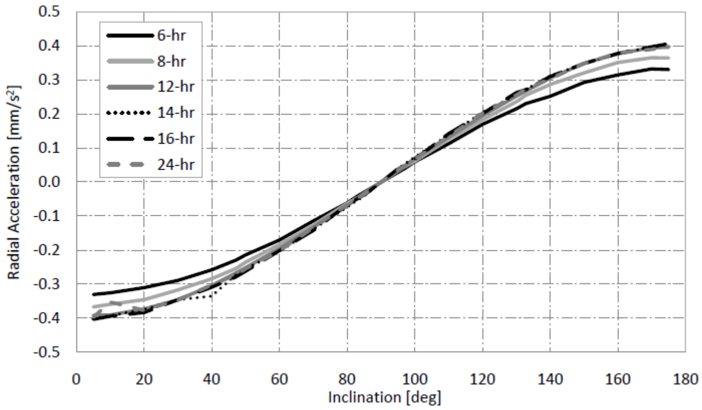


Figure 7 Radial acceleration to achieve sun-synchronous HEOs of varying period and inclination at Venus, ω = 270 degrees

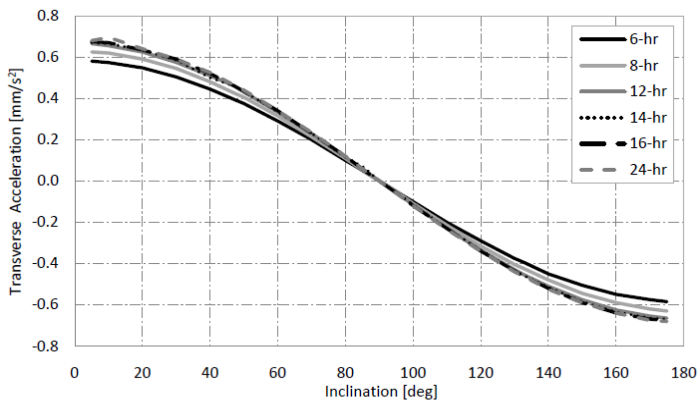


Figure 8 Transverse acceleration to achieve sun-synchronous HEOs of varying period and inclination at Venus, ω = 270 degrees

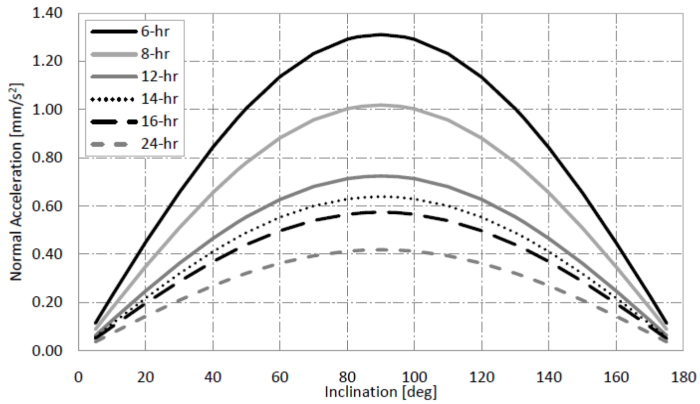


Figure 9 Normal acceleration to achieve sun-synchronous HEOs of varying period and inclination at Venus, ω = 270 degrees

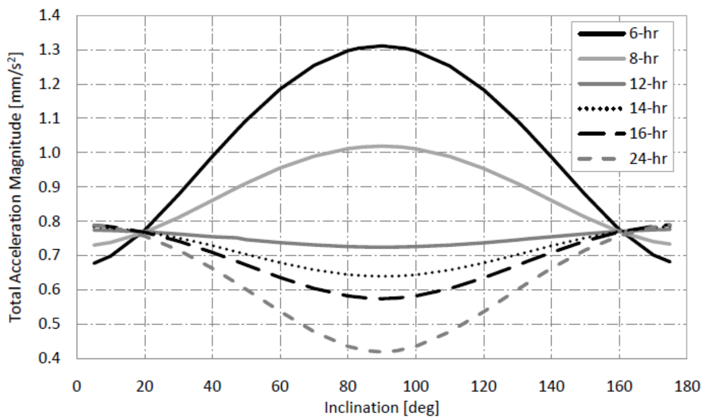


Figure 10 Total acceleration magnitude to achieve sun-synchronous HEOs of varying period and inclination at Venus, ω = 270 degrees

Figure 10 shows the total acceleration magnitude required to enable elliptical sun-synchronous orbits which maintain a constant argument of periapsis over each orbital revolution. For example, considering a 12 h orbit with an inclination of 90 degrees, a total acceleration magnitude of 0.72 mm/s^2 is required, which corresponds to 720 mN of thrust for a 1-ton spacecraft. This is a considerable thrust level which could not be provided by current electric propulsion thrusters. Some development in thrust technology would therefore be required before such mission could become feasible. If the technology was available however, these orbits could potentially enhance the opportunities for remote sensing at Venus as these sun-synchronous orbits do not naturally exist.

Elliptical Sun-Synchronous Orbits - Special Perturbations Solution

Figure 11 gives the variation of orbital elements for a 12 h elliptical sun-synchronous orbit at Venus, with the orbital parameters given in Table 3. The results show that the semi-major axis, eccentricity, and inclination, oscillate during the orbit, but return to the initial value after one revolution and thus verify the analytical solutions. The ascending node angle is also shown to match the analytical results by producing a change of around 0.8 degrees per orbit. However, a small drift of around 1.12E-3 degrees per orbit in the argument of periapsis is displayed. Although a drift is exhibited in this case, the total change in argument of periapsis over one Venus year amounts to less than 0.5 degrees, which is expected to be acceptable. This drift is caused by the oscillation of the semi-major axis, eccentricity, and inclination, which causes a slight drift in the argument of periapsis. The analytical solution makes the assumption that the changes in these elements are zero; the numerical simulation however indicates that in this case the assumption begins to breakdown.

Table 3 Orbital parameters – elliptical sun-synchronous orbit at Venus

Orbital Element	Value (units)
Pericenter altitude	800 (km)
Apocenter altitude	36,811 (km)
Inclination	90 (deg)
Ascending node	330 (deg)
Argument of periapsis	270 (deg)

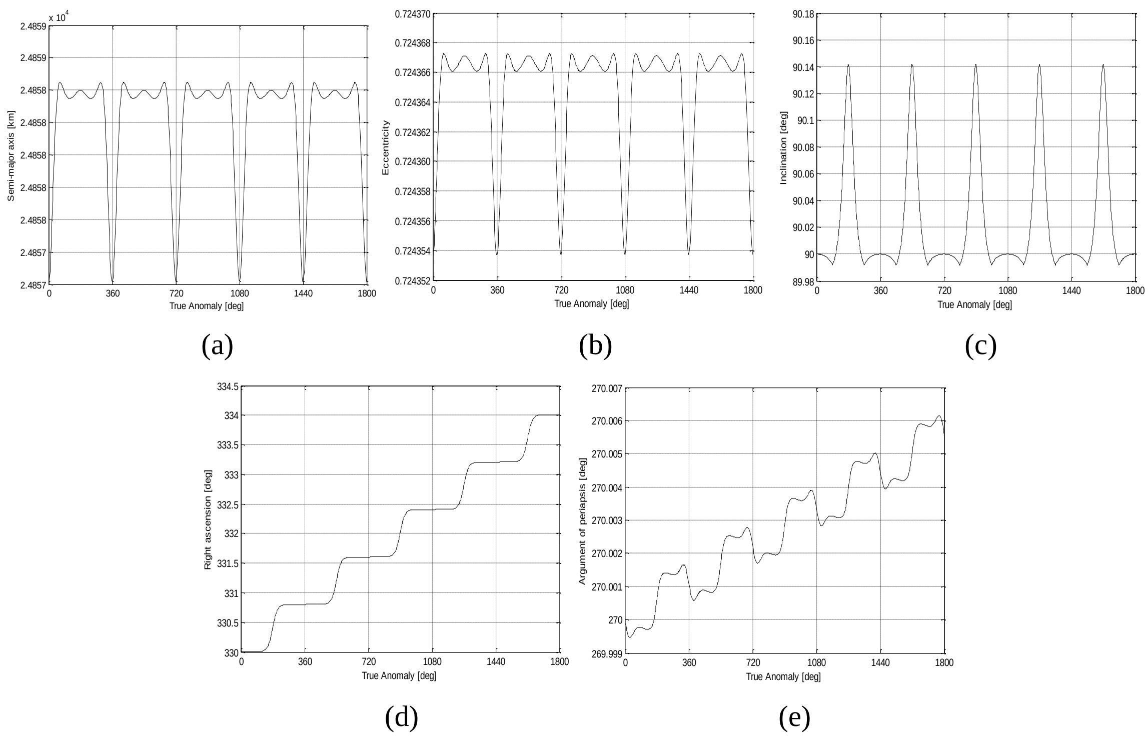


Figure 11 Oscillation of orbital elements over five orbital revolutions around Venus. (a) Semi-major axis (b) Eccentricity (c) Inclination (d) Ascending node angle (e) Argument of periapsis

HIGHLY ELLIPTICAL ORBITS ABOUT MERCURY

General Perturbations Solution

As Mercury is the poorest explored planet of the inner Solar System, detailed gravity field information is not yet available. This paper therefore considers spacecraft motion about Mercury using gravity perturbations to the order of J_2 only, where J_2 is equal to 6E-5 [8]. It is therefore expected that the accuracy of this work could be significantly enhanced by the inclusion of higher order gravity terms when these are made available.

Once again to derive the natural critical inclination values, Eq. (9) is set to zero and solved for the values of the inclination of 63.43 and 116.6 degrees, irrespective of the values of semi-major axis and eccentricity of the orbit. Thus, all Mercury orbits inclined at these values show no rotation of the apsidal line. It is noted that these critical inclination values are the same as at Earth, as in both cases only J_2 perturbations are considered. Mercury is however, more circular than Earth, thus higher order gravity perturbations should be taken into account when made available to significantly enhance the accuracy of these solutions.

To extend the critical inclination using continuous acceleration, Eqs. (10) - (12) are simplified to include perturbations only to the order of J_2, and are substituted into Eq. (8), which is integrated over one orbital revolution. The resulting expression is then solved for the combined radial and transverse accelerations to alter the critical inclination of orbits at Mercury. The resulting acceleration magnitudes are shown for a variety of orbit periods between 6 and 24 hours to achieve inclinations between 5 and 175 degrees for a constant pericenter altitude of 800 km, to compensate for the drift in argument of periapsis caused by gravitational perturbations to the order of J_2.

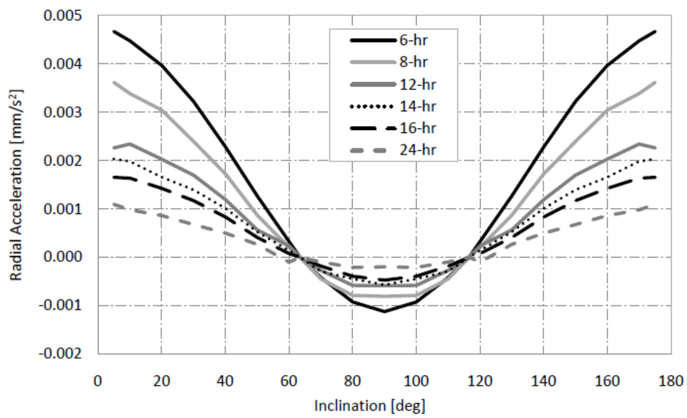


Figure 12 Radial acceleration – Extension of critical inclination at Mercury

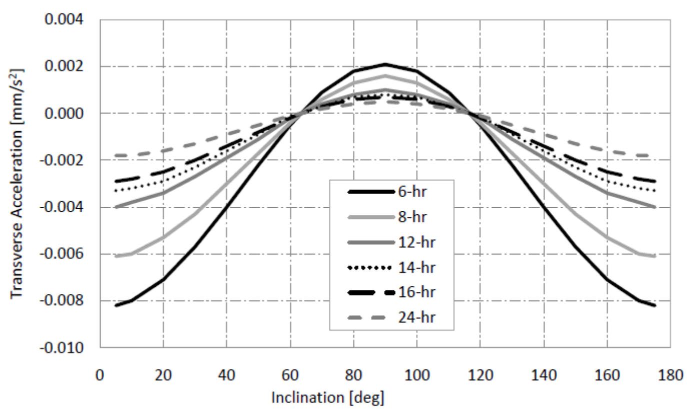


Figure 13 Transverse acceleration – Extension of critical inclination at Mercury

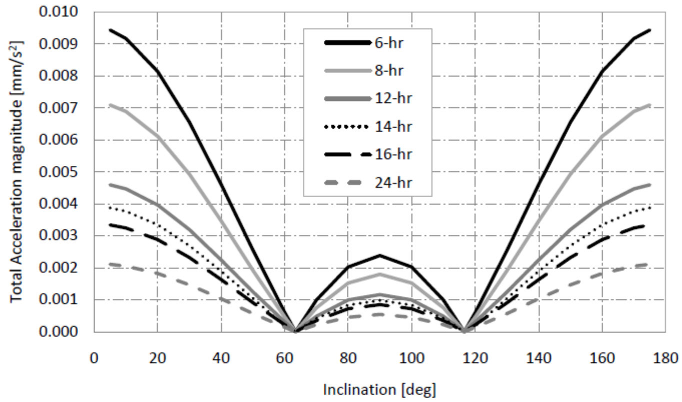


Figure 14 Total required acceleration for combined thrusting in radial and transverse axes - Extension of critical inclination at Mercury

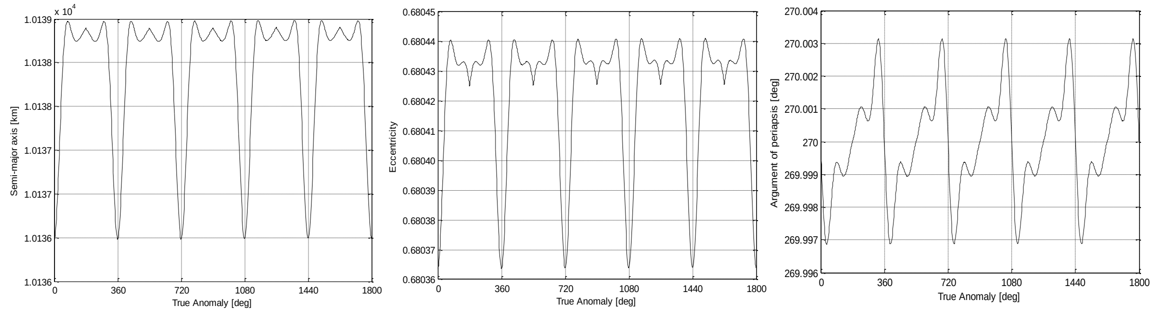
Figure 14 shows that the total acceleration magnitude required to alter the critical inclination of HEOs at Mercury is higher than that required for Venus, however the acceleration required is still within the limits of current thruster technology. For example, a 12 h, 90 degree inclination orbit requires a total acceleration magnitude of 0.0012 mm/s^2. Which equates to 1.2 mN of thrust for a 1-ton spacecraft, which can be provided by the QinitiQ T5 thruster, capable of providing thrust levels between 1 and 20 mN [11].

Special Perturbations Solution

Table 4 gives the orbital parameters for a 12 h highly elliptical orbit at Mercury selected to demonstrate the special perturbations solution, results of which are given in Figure 15. The elements are shown to oscillate over the orbit, but parameters return to the original value after each revolution of the spacecraft. Both the inclination and ascending node angle show no drift over the orbit and so are not included within this paper.

Table 4 Orbital parameters – extension of critical inclination at Mercury

Orbital Element	Value (units)
Pericenter altitude	800 (km)
Apocenter altitude	14,593 (km)
Inclination	90 (deg)
Ascending node	330 (deg)
Argument of periapsis	270 (deg)



(a) **(b)** **(c)**

Figure 15 Oscillation of orbital elements over five orbital revolutions around Mercury. (a) Semi-major axis (b) Eccentricity (c) Argument of periapsis

SUN-SYNCHRONOUS ORBITS ABOUT MERCURY

Circular Sun-Synchronous Orbits - General Perturbations Solution

Similarly to Venus, the reciprocal of flattening of Mercury is so low that natural sun-synchronous orbits do not exist. Low-thrust is therefore firstly considered to enable circular sun-synchronous orbits. Eqs. (8) and (17) are again used to derive novel sun-synchronous orbits around Venus using the normal perturbation given in Eq. (12). Integrating over one orbital revolution gives the following expressions for the change in ascending node angle per second for argument of periapsis values of 0 and 270 degrees respectively. For Venus the ascending node angle is required to rotate 2π radians in around 88 days.

$$(\Delta\Omega)_{\omega=0} = \frac{1}{2\pi}\sqrt{\frac{\mu}{a^3}}\left(-\frac{3\pi J_2 R_B^2\cos(i)}{a^2\left(-1+e^2\right)^2}+\frac{4a^2F_n\cos(\omega)}{\mu\sin(i)}\right) \tag{20}$$

$$(\Delta\Omega)_{\omega=270} = \frac{1}{2\pi}\sqrt{\frac{\mu}{a^3}}\left(\begin{array}{l} -\dfrac{3J_2\pi R_B^2\cos(i)}{a^2\left(-1+e^2\right)^2}-1/ \\ \left(\sqrt{-1+e^2}\,\mu\right)a^2F_n\csc(i) \\ \left(4\sqrt{-1+e^2}+2e^2\sqrt{-1+e^2}-12e\,\mathrm{ArcTanh}\left[\dfrac{-1+e}{\sqrt{-1+e^2}}\right]-3e\ln\left[\dfrac{1-e}{\sqrt{-1+e^2}}\right]+3e\ln\left[\dfrac{-1+e}{\sqrt{-1+e^2}}\right]\right)\sin(\omega)\end{array}\right) \tag{21}$$

The acceleration to achieve circular sun-synchronous orbits directed out of the orbit plane, to achieve various circular sun-synchronous orbits, derived using Eq. (20), is given in Figure 16.

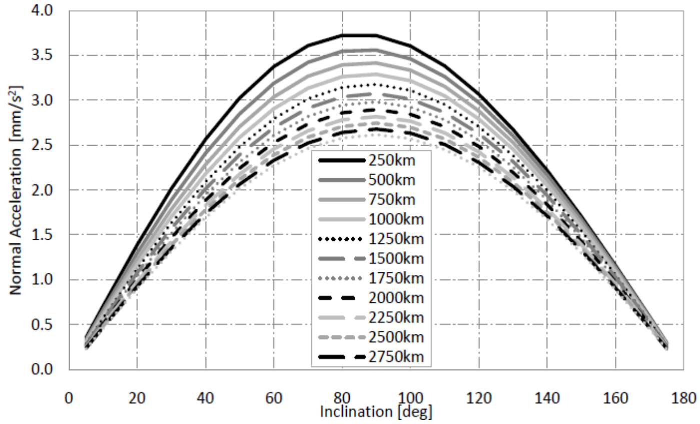


Figure 16 Normal acceleration required for the extension of circular sun-synchronous orbits at Mercury, inclination range 5 – 175 degrees

It is shown that continuous low-thrust propulsion can be used to enable a range of circular sun-synchronous orbits at Mercury that would otherwise be infeasible. This being said, it is shown that the acceleration magnitude required to enable circular sun-synchronous orbits is significantly higher than is required to alter the critical inclination, as was first displayed when considering orbits at Venus. For example, a 1000 km circular orbit inclined at 90 degrees requires 3.29 mm/s^2 of acceleration. Comparing Figure 5 and Figure 16 shows that the acceleration magnitude required to enable circular sun-synchronous orbits is comparable for both Mercury and Venus.

Circular Sun-Synchronous Orbits - Special Perturbations Solution

Numerical methods are once again used to validate general perturbations solutions for circular sun-synchronous orbits, the results are shown for the orbit given in Table 5. Figure 17 shows the change in ascending node angle achieved using normal acceleration to achieve the sun-synchronous condition of a revolution in the ascending node angle of 2π radians in 88 days, which equates to 0.4 degrees per orbit for the orbit in Table 5.

Table 5 Orbital parameters – circular sun-synchronous orbit at Mercury

Orbital Element	Value (units)
Pericenter altitude	1,000 (km)
Apocenter altitude	1,000 (km)
Inclination	90 (deg)
Ascending node	330 (deg)

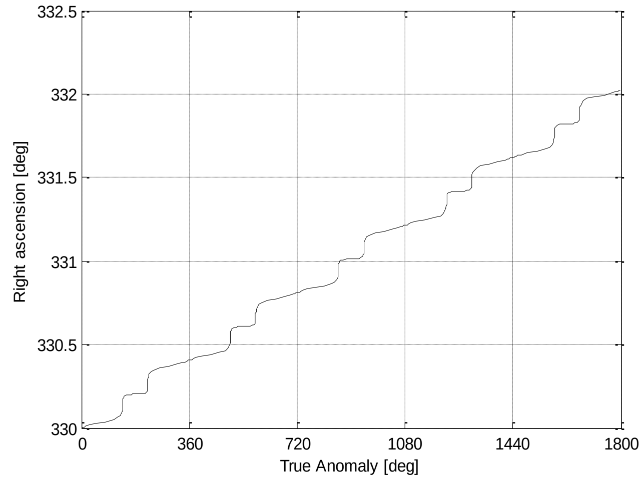


Figure 17 Variation of ascending node angle over five orbital revolutions for a circular orbit around Mercury

Elliptical Sun-Synchronous Orbits - General Perturbations Solution

In a similar manner to Venus, to enable elliptical sun-synchronous orbits around Mercury, two conditions are combined to ensure the rate of change of argument of periapsis is zero over the orbit and the rotation of the ascending node angle matches that of the mean sun. The acceleration required to achieve elliptical, sun-synchronous orbits of varying periods is derived using Eqs. (13) and (21), and the required acceleration for free selection of inclination and altitude, for orbits with constant pericenter altitude of 800 km and argument of periapsis of 270 degrees, is given in Figure 18 - Figure 21.

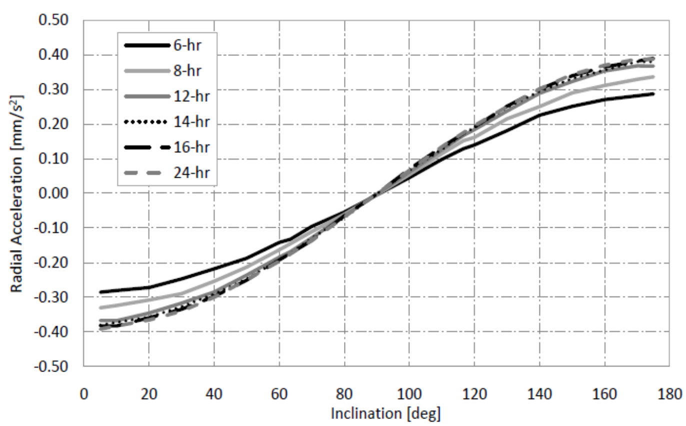


Figure 18 Radial acceleration to achieve sun-synchronous HEOs of varying period and inclination at Mercury, ω = 270 degrees

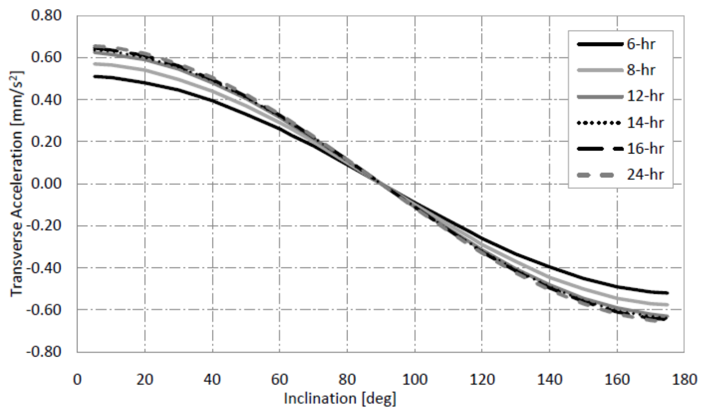


Figure 19 Transverse acceleration to achieve sun-synchronous HEOs of varying period and inclination at Mercury, ω = 270 degrees

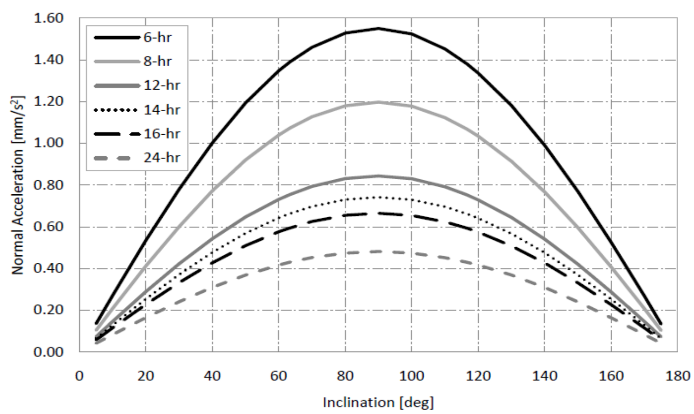


Figure 20 Normal acceleration to achieve sun-synchronous HEOs of varying period and inclination at Mercury, ω = 270 degrees

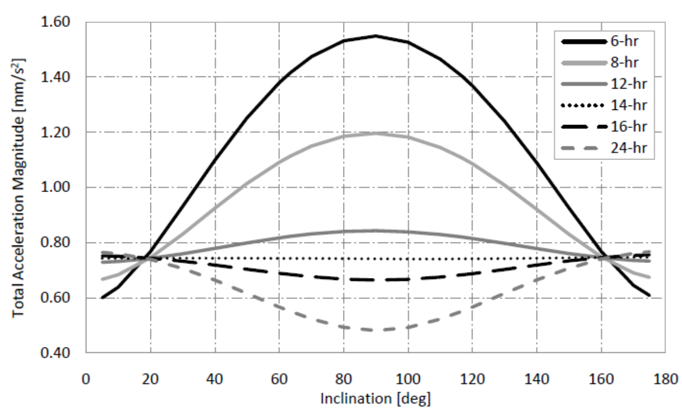


Figure 21 Total acceleration magnitude to achieve sun-synchronous HEOs of varying period and inclination at Mercury, ω = 270 degrees

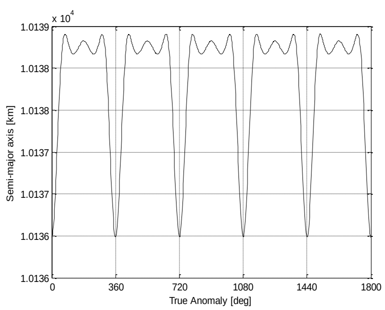
Examination of Figure 21 illustrates the increase in acceleration required to enable elliptical sun-synchronous orbits over simply an extension of the critical inclination. As was the case with circular orbits, the required acceleration magnitude is again comparable to that required at Venus. For example, considering a 12 h orbit with an inclination of 90 degrees, a total acceleration magnitude of 0.84 mm/s^2, corresponding to 840 mN of thrust for a 1-ton spacecraft. This is a considerable thrust level which could not be provided by current electric propulsion thrusters.

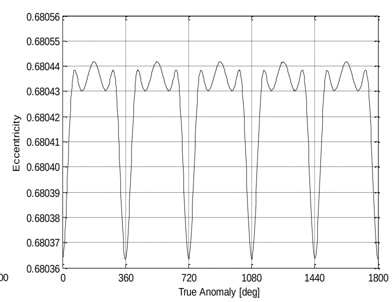
Elliptical Sun-Synchronous Orbits - Special Perturbations Solution

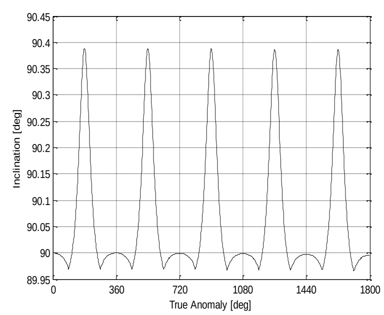
Figure 22 gives the variation in orbital elements for a 12 h elliptical sun-synchronous orbit with the orbital parameters given in Table 6. The required change in the ascending node angle of 2 degrees per orbit is shown to be achieved, with negligible changes in semi-major axis, eccentricity, and inclination over each orbital revolution. As with the elliptical sun-synchronous orbit at Venus, a small drift in argument of periapsis is shown, which in this case is around 0.01 degrees per orbit, equating to less than 2 degrees per Mercury year. This is again expected to be a reasonable cost.

Table 6 Orbital parameters – elliptical sun-synchronous orbit at Mercury

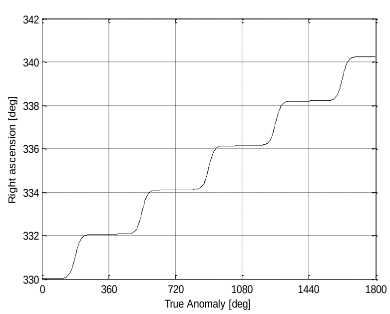
Orbital Element	Value (units)
Pericenter altitude	800 (km)
Apocenter altitude	14,593 (km)
Inclination	90 (deg)
Ascending node	330 (deg)
Argument of periapsis	270 (deg)

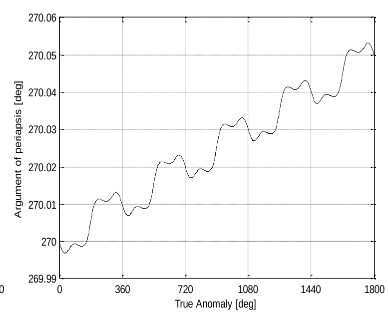






(a) (b) (c)





(d) (e)

Figure 22 Oscillation of orbital elements over five orbital revolutions. (a) Semi-major axis (b) Eccentricity (c) Inclination (d) Ascending node angle (e) Argument of periapsis

CONCLUSION

Continuous low-thrust propulsion has been considered for the extension of orbits at the critical inclination at both Mercury and Venus to enhance the opportunities for remote sensing of these bodies, in some cases these orbits can be enabled using existing electric propulsion systems.

At Mercury and Venus, natural sun-synchronous orbits do not exist; this paper therefore also considers the use of continuous acceleration to enable both circular and elliptical sun-synchronous orbits, which could significantly simplify the spacecraft thermal environment. These orbits do however require significant thrust magnitudes which cannot be provided by current low-thrust propulsion systems.

NOTATION

a	semi-major axis, km
$C_{n,m}$	harmonic coefficients of body potential
e	eccentricity
F_n	low-thrust normal perturbation scalar
F_r	low-thrust radial perturbation scalar
F_t	low-thrust transverse perturbation scalar
i	inclination, degrees
J_n	gravitational perturbations
N	normal perturbation acceleration, mm/s^2
p	semi-parameter, km
$P_{n,m}$	associated Legendre polynomials
r	orbital radius
R	radial perturbation acceleration, mm/s^2
R_B	mean radius of body under consideration, km
$S_{n,m}$	harmonic coefficients of body potential
T	transverse perturbation acceleration, mm/s^2
U	potential
U_0	point-mass gravitational potential
U_p	perturbing component of body potential
β	declination of spacecraft, degrees

θ true anomaly, degrees

λ geographical longitude, degrees

μ gravitational parameter of body under consideration, km^3/s^2

ω argument of periapsis, degrees

Ω ascending node angle, degrees

REFERENCES

[1] Squyres S. Vision and voyages for planetary science in the decade 2013-2022. 2011

[2] Wertz JR. *Mission geometry: Orbit and constellation design and management, the space technology library*. El Segundo, California: Microcosm Press; 2001.

[3] Kidder SQ, Vonder Haar TH. Notes and correspondance - on the use of molniya orbits for meteorological observation of middle and high latitudes. *Journal of Atmospheric and Oceanic Technology*. 1990;7:517-522

[4] Anderson P, Macdonald M. Extension of highly elliptical earth orbits using continuous low-thrust propulsion. *Journal Guidance Control and Dynamics*. 2011

[5] Anderson P, Macdonald M, Yen CW. Extension of martian orbits using continuous low-thrust propulsion. *23rd International Symposium on Spaceflight Dynamics*. 2012

[6] Anderson P, Macdonald M. Sun-synchronous highly elliptical orbits using low-thrust propulsion. *Journal of Guidance, Control and Dynamics*. 2012

[7] Bate RR, Mueller DD, White JE. *Fundamentals of astrodynamics*. Dover Publications, Inc; 1971.

[8] Anderson DJ, Colombo G, Esposito PB, Lau EL, Trager GB. The mass, gravity field and ephemeris of mercury. *Icarus*. 1987;71:337 - 349

[9] Fortescue P, Stark J, Swinerd G. *Spacecraft systems engineering*. 2003.

[10] Macdonald M, McInnes CR. Analytical control laws for planet-centered solar sailing. *Journal of Guidance, Control and Dynamics*. 2005;28:1038-1048

[11] Edwards CH, Wallace NC, Tato C, Put CV. The t5 ion propulsion assembly for drag compensation on goce. *Second International GOCE User Workshop "GOCE, The Geoid and Oceanography"*. 2004

[12] Walker MJH, Ireland B, Owens J. A set of modified equinoctial orbital elements. *Celestial Mechanics* 1985;36:409-419

[13] Dormand JR, Prince PJ. A family of embedded runge-kutta formulae. *Journal of Computational and Applied Mathematics and Statistics*. 1980;6:19-26

[14] Liu X, Baoyin H, Ma X. Extension of the critical inclination. *Astrophysics and Space Science*. 2011;334:115 - 124