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Multi-Objective Design of Robust Flight Control Systems

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The aim of this work is to demonstrate the capabilities of evolutionary methods in the design of robust controllers for unstable fighter aircraft in the framework of H_∞ control theory. A multi–objective evolutionary algorithm is used to find the controller gains that minimize a weighted combination of the infinite–norm of the sensitivity function (for disturbance attenuation requirements) and complementary sensitivity function (for robust stability requirements). After considering a single operating point for a level flight trim condition of a F-16 fighter aircraft model, two different approaches will then be considered to extend the domain of validity of the control law: 1) the controller is designed for different operating points and gain scheduling is adopted; 2) a single control law is designed for all the considered operating points by multiobjective minimisation. The two approaches will be analyzed and compared in terms of efficacy and required human and computational resources.

Keywords Evolutionary optimization, Robust control, Aircraft control

1 Introduction

In this paper a control synthesis technique in the framework of H_∞ control theory is developed, based on the application of a modern multi–objective evolutionary optimization algorithm to the associated minimization problem. In the last two decades, multiple redundant, full authority, fail/safe operational, fly–by–wire control systems have been brought to a very mature state. As a result, many aircraft, from earlier designs such as the F-16, F-18, and Tornado through the more recent Mirage 2000, European Fighter Aircraft (EFA), Rafale, and advanced demonstrators such as X-29 and X-31, are highly augmented, actively controlled vehicles that possess either a marginal or negative static stability margin without augmentation, for reasons related to improved performances, weight/cost reduction, and/or low observability.

Highly augmented and/or superaugmented aircraft require the synthesis of a control system that artificially provides the required level of stability for satisfactory handling qualities, enhancing pilot capability by properly tailoring the aircraft response to the manoeuvre state. At the same time, modern high performance fighter aircraft are characterized by an extended flight envelope in order to allow the pilot to reach unprecedented maneuvering capabilities at high angles of attack. Such a result can be achieved only if the control system maintains adequate performance in presence of considerable variations of the aircraft response characteristics, avoiding instabilities related to the presence of control surface position and rate saturation limits.

Such a result can be obtained by use of robust controllers. H_∞ control theory was developed in this framework, in order to provide robustness to the closed–loop system to both external disturbance and model uncertainties of known "size". The controller is synthesized by minimizing the infinite norm of the system, determined as the maximum singular value $\bar{\sigma}$ of the tranfer function matrix $\boldsymbol{G}(s)$ for a multi–input/multi–output (MIMO) system, where $\bar{\sigma}$ represents the maximum gain for a (disturbance) signal in the exptecred frequency range. The system will be robust to the worst expected disturbance if $\bar{\sigma}$ is less than 1, in which case all the disturbances will be attenuated by the closed–loop system. The cost of robustness is a certain degree of "conservativeness" of the controller, which may reduce closed–loop performance. For this reason the requirement for robust stability may be accompanied by requirements in the time domain (such as raise time, overshoot, and settling time). These latter requirements can be enforced as inequality constraints to the optimization problem that must solve the minimization problem while pursuing a minimum acceptable level of performance. These acceptable level can be easily derived in aircraft application from requirements for handling qualities.

The synthesis of the controller will be performed by use of an evolutionary optimisation algorithm, motivated by the need for fulfilling different (and possibly competing) requirements in different flight conditions. Highly manoeuvrable aircraft control offers a particularly challenging scenario, where on one side it is unlikely that a single controller synthesized for a given trim operating point performs well over the a sufficiently wide portion of the operating envelope, even by use of robust techniques. In this respect, the classic solution is to use gain scheduled controllers, where the gain are varied as a function of reference parameters for the flight condition (*e.g.* Mach number or dynamic pressure). This classical procedure allows for adapting the system to parameter variation but still requires a certain degree of robustness when the aircraft is flying off–nominal conditions betweem the trim point where the controllers were synthesized. For this reason a gain scheduled controller for an F–16 fighter aircraft reduced short period model will be derived in three different conditions and gain scheduling used for interpolating the gains between the operating points. The F–16 offers a good test–benchmark for the technique as it features most of the characteristics of a modern jet fighter (instability, high-α flight, etc.).

This approach will be compared with the synthesis of a single robust controller derived by enforcing simultaneously the requirements in all the considered operating points. In such a case, a single controller is derived which will be affected by a certain degradation of performance in the nominal operating points. But if the off–nominal behaviour is comparable, the advantage of a simpler, scheduling–free controller may be considerable.

After the description of aircraft model and control system architecture and a brief review of H_∞ control theory in the next Section, the optimization method used is briefly recalled in Section 3. The synthesis of a controller in the neighbourhood of a single trim condition and a comparison between a gain–scheduled controller and a controller synthesized for different competing merit functions is then carried out and presented in Section 4. A Section of conclusions ends the paper.

2 Aircraft dynamic model and control system architecture

2.1 Equations of motion and simplifications

The longitudinal equations of motion of a rigid aircraft are expresse by a set of 4 ordinary differential equations in the form

$$\begin{aligned} \dot{u} &= -qw - g\sin\theta + \left(0.5\rho V^2 S C_x + T\right)/m \\ \dot{w} &= qu + g\cos\theta + 0.5\rho V^2 S C_z/m \\ \dot{q} &= 0.5\rho V^2 S\bar{c} C_m/I_y \;\; ; \;\; \dot{\theta} = q \end{aligned} \tag{1}$$

where the state variables are the velocity components u and w (with $V^2 = u^2+w^2$), the pitch angular velocity q and the pitch angle θ. The control variables are the elevetor deflection δ_E (which acts on the pitch moment aerodynamic coefficient C_m, but it affects the force coefficients C_x and C_z as well) and the throttle setting δ_T, suche the thrust delivered by the engine is $T = T_{\max}(h, M)\delta_T$.

It is possible to linearize the equations of motion in the neighbourhood of a trim condition. By use of a set of stability axes for a level flight condition at velocity V_0, one gets a fourth order linear system in the form

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & V_0 + Z_q & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{bmatrix} X_{\delta_E} & X_{\delta_E} \\ Z_{\delta_E} & 0 \\ M_{\delta_E} & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \delta_E \\ \delta_T \end{pmatrix} \tag{2}$$

The stability derivatives in Eq. 2 depend on the considered flight condition. This means that the response of the aircraft to control action will vary with V_0. In order to deal with a simplified model, it is possible to consider the response to a reduced order short period model, under the assumption that attitude variables (q and $\alpha \approx w/V_0$) respond to control input on a faster time–scale then trajectory ones (namely velocity V and flight–path angle γ, where for longitudinal flight it is $\theta = \alpha + \gamma$), so that V can be considered approximately constant during an attitude manoeuvre. The reduced order model is given by

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{bmatrix} Z_\alpha & 1 + Z_q/V_0 \\ M_\alpha & M_q \end{bmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{bmatrix} Z_{\delta_E} \\ M_{\delta_E} \end{bmatrix} \delta_E \tag{3}$$

Model fidelity is enhanced by including a simple first order actuator model for the respons of elevator deflection to pilot or automatic control inputs:

$$\dot{\delta}_E = \frac{1}{\tau_A}\left(\delta_{E_{\text{com}}} - \delta_E\right) \tag{4}$$

In what follows, an F-16 fighter aircraft model will be considered. The original model, taken from Ref. 1, features a nonlinear aerodynamic model for $-10 \le \alpha \le 45$ deg and $|\beta| \le 30$ deg. Finite differences are used to linearize the aircraft model in the neighbourhood of each trim condition and obtain approximate values for the stability derivatives in Eqs. (2) and (3).

2.2 Longitudinal Stability and Control augmentation system

Figure 1 depicts the structure of a longitudinal stability and control augmentation system (SCAS). The blocks P and A represent the aircraft and elevator actuator dynamics, respectively. The stability augmentation provides increased pitch damping (by q–feedback) and

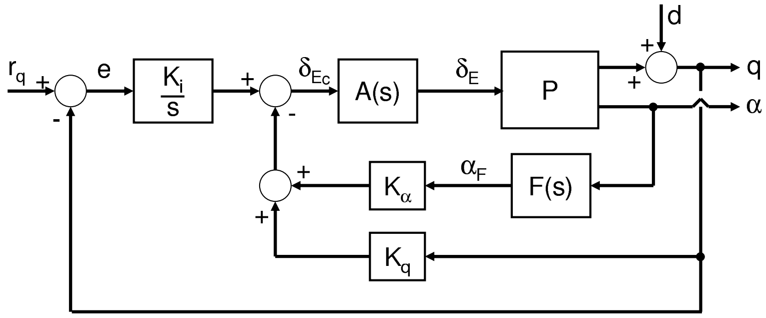

Figure 1: Control system architecture.

artificial static stability (α feedback). In this latter case a filter is included for reducing α sensor noise, with a cut–off frequency of $\tau_F = 10$ rad/s (that is, $F(s) = \tau_F/(s + \tau_F)$).

The control augmentation system transforms the longitudinal pilot command into a rate command, where the tracked variable is the pitch angular velocity q. In order to provide the system with zero steady–state error an integrator is included in the pitch angular velocity error channel. The resulting open loop dynamics is described by a linear system of ordinary differential equations in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u \ ; \quad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{5}$$

where the state vector is $\boldsymbol{x} = (\alpha, q, \delta_E, \alpha_F, \varepsilon)^T$ (where ε is the integrator variable, such that $\dot{\varepsilon} = r_q - q$), while the only input variable is the pitch velocity reference signal r_q. Provided that the output variables are $\boldsymbol{y} = (\alpha, q, \varepsilon)^T$, the state, control, and output matrices are defined as

$$\boldsymbol{A} = \begin{bmatrix} Z_w & V_0 + Z_q & M_{\delta_E} & 0 & 0 \\ M_w & M_q & M_{\delta_E} & 0 & 0 \\ 0 & 0 & -\tau_A & 0 & 0 \\ 0 & 0 & 0 & -\tau_F & 0 \\ 0 & -\frac{180}{\pi} & 0 & 0 & 0 \end{bmatrix} ; \ \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \tau_A \\ 0 \\ 0 \end{bmatrix} ; \ \boldsymbol{C} = \begin{bmatrix} \frac{180}{\pi} & 0 & 0 & 0 & 0 \\ 0 & \frac{180}{\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

respectively. The optimization algorithm will be exploited in order to find the gains of the stability augmantation system (K_α and K_q) and the integral gain of the control autmentation system (K_i).

2.3 Robust control

Consider the system depicted in Fig. 2, where $\boldsymbol{P}_0(s)$ is the nominal model of a plant with n_i inputs and n_o outputs, $\boldsymbol{C}(s)$ is the controller, $\boldsymbol{r}(s)$ is the reference input signal $\boldsymbol{y}(s)$, $\boldsymbol{d}$ is the noise on the output signal and $\mathbf{n}$ is the noise on the sensors. Given the definition of the output transfer matrix as $\boldsymbol{L}_o = \boldsymbol{P}_0\boldsymbol{C}$, the sensitivity at the output is defined as the transfer matrix $\boldsymbol{y}/\boldsymbol{d}$, that is

$$\boldsymbol{S}_o = (\boldsymbol{I} + \boldsymbol{L}_o)^{-1}, \quad \boldsymbol{y} = \boldsymbol{S}_o\boldsymbol{d} \tag{7}$$

and the complementary sensitivity function at the output is

$$\boldsymbol{T}_o = \boldsymbol{I} - \boldsymbol{S}_o = \boldsymbol{L}_o(\boldsymbol{I} + \boldsymbol{L}_o)^{-1} \tag{8}$$

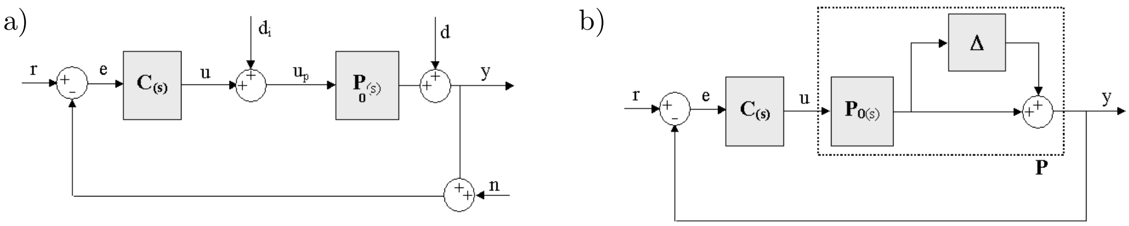

Figure 2: General feedback configuration (a); feedback configuration with multiplicative uncerntainties of the nominal model (b).

From the system represented in Fig. 2, it is easy to derive that

$$\boldsymbol{y} = \boldsymbol{T}_o\boldsymbol{r} - \boldsymbol{T}_o\mathbf{n} + \boldsymbol{S}_{\mathbf{o}}\boldsymbol{P}\boldsymbol{d}_{\mathbf{i}} + \boldsymbol{S}_{\mathbf{o}}\boldsymbol{d} \tag{9}$$

It is thus clear that in order to eliminate or at least reduce the effects of noise on the response of the system, it is necessary to operate on $\boldsymbol{T}_o$ and $\boldsymbol{S}_o$.

Moreover, apart from external noises affecting the signals, the system may be characterized by other kind of uncertainties. Usually, the nominal model P_0, due to simplifying assumptions and/or linearization, does not correspond to the actual plant. Taking into account a multiplicative uncertainty on the plant model (Fig. **??**), brings to the following expression for the output:

$$\boldsymbol{y} = \frac{\boldsymbol{T}_o + \Delta\boldsymbol{T}_o}{\boldsymbol{I} + \Delta\boldsymbol{T}_o}\boldsymbol{r} \tag{10}$$

In order to reduce the effect of the uncertainty it is necessary to tailor the complementary sensitivity function of the uncertainty itself, ΔT_o.

The main idea behind H_∞ control theory and the design process derived in this framework is to find the values of the controller parameters by minimizing appropriately the infinite norm of the weighted sensitivity and complementary sensitivity functions. In order to achieve this result, the following functions need to be minimized:

$$\|\boldsymbol{W}_1(s)\boldsymbol{S}_o(s)\| = \min \ ; \quad \|\boldsymbol{W}_3(s)\boldsymbol{T}_o(s)\| = \min \tag{11}$$

that is, the effect of noises on the output (Eq. 11 and that of uncertainties of the nominal model $\boldsymbol{P}_0$ is reduced.

Since the H_∞ norm of a system $\boldsymbol{G}(s)$ is

$$\|\boldsymbol{G}\|_\infty = \sup_\omega \bar{\sigma}(\boldsymbol{G}(j\omega)) \tag{12}$$

where $\bar{\sigma}(\cdot)$ is the maximum singular value, this kind of norm provides the worse gain for a sinuisoidal input for a determined frequency, corresponding to the worse energetic gain of the system. The use of weighted functions allows to deal with different kind of signals, when MIMO systems are considered. Moreover, and more important, weights allow to focus the optimization process only within prescribed frequency ranges. As an example, in order to reduce low frequency noise a weight function with high gains at low frequency will be used, that is, it will be

$$\|\boldsymbol{W}_g(s)\boldsymbol{G}(s)\|_\infty < 1 \ ; \quad \|\boldsymbol{G}(s)\|_\infty < \frac{1}{\boldsymbol{W}_g(s)} \tag{13}$$

3 The multi–objective optimization algorithm EDMONDO

Table 1: Trim conditions

	V [ft/s]	h [ft]	Q [psf]
T1	500	0	297
T2	600	3 000	391
T3	700	6 000	486
T4	800	9 000	579
T5	900	12 000	666

Presentatione dell'algoritmo di ottimizzazione
For the first approach, the optimization process is aimed to minimize the function resulting from the sum of the sensitivity and complementary sentitivity function, each appropriately weighted, for three different trim conditions. The optimal gains are then interpolated and tested by means of system linearized for intermediate trim conditions. The objective function is

$$F = |\boldsymbol{W}_1(s)\boldsymbol{S}(s)\|_\infty + |\boldsymbol{W}_3(s)\boldsymbol{T}(s)\|_\infty \tag{14}$$

where $\boldsymbol{W}_1$ is imposed so that the action on the sensitivity function is enfasized in the low frequency zone, where the main disturbance, which can affect the aircraft performance, belong to, while $\boldsymbol{W}_3$ is modeled on the basis of assumed uncertainties on the nominal model of the plant. The weight functions are

$$W_1 = \frac{1+100s}{100s+1}; \quad W_3 = \frac{100+10s}{s+1000} \tag{15}$$

Moreover, the constraints on peek time t_p, settling time t_s and overshoot M_p are set for each trim condition as follows

$$t_p \leq 3[sec]; \quad t_s \leq 4[sec]; \quad M_p \leq 0.2 \tag{16}$$

The 3-dimensional search domain is bounded by $\boldsymbol{lb} = (-30, -30, -30)^T$ and $\boldsymbol{ub} = (0, 0, 0)^T$.

The second approach provides a three points optimization process, which takes care of the 3 objective functions and the 9 constraints at the same time and, again, a test of the system performance for intermediate trim conditions.

4 Results and discussion

Five trim conditions for the F–16 aircraft model were considered (Tab. 1). Trim condition 1, 3 and 5 (T1, T2, and T3) were considered for controller gain synthesis while conditions 2 and 4 (T2 and T4) were used for simulation of the closed–loop behaviour in off–nominal conditions. Controller 1 (C1) is based upon gain scheduling with respect to dynamic pressure, while the second ontroller (C2) employs a fixed set of gains as outlined in the previous section.

Figure 3 shows the results obtained from a simulation of the closed–loop response to a step input on the input channel r_q for C1 (left) and C2 (right) in three different trim conditions. It should be noted how, in all the considered cases, the response of the tracked variable is satisfactory, and it is only marginally affected by the variation of the trim condition. At the same time, the off–nominal response in T2 lies in between those for the design

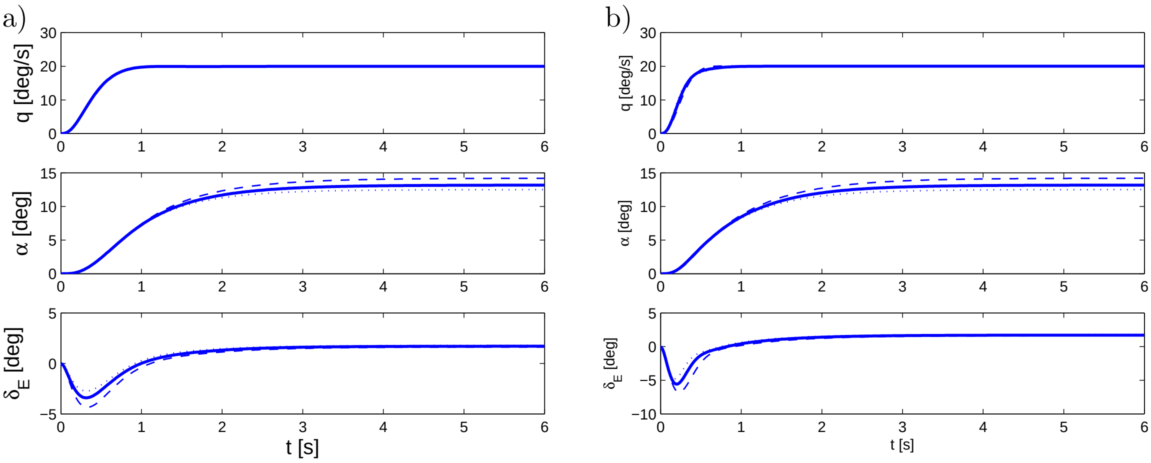

Figure 3: Step responses of scheduled (a) and global (b) controllers in T1 (dashed line), T2 (thick line), and T3 (dotted line).

points (T1 and T3), thus proving that both the gain scheduling and the global approaches provides the required degree of robustness with respect to model parameters variations. Note that similar results are also obtained when considering T3 and T5 as reference trim conditions for the controller gain synthesis and T4 as the off–nominal condition, cases not reported in the figures for the sake of conciseness.

Surprisingly enough, the global controller appears to better exploit the available control power in all the considered situations: the response on the q–channel is faster, yet perfectly damped, with no or marginal overshoot, with a faster variation of the control variable, δ_E, which, nonetheless remains compatible with saturation and rate–saturation constraints. In this respect, the expected result was that the global synthesis approach should provide a more conservative controller, as a compromise between different operating points.

It is not easy to explain the outcome of the analysis performed. One trivial explanation is that the optimization process for single operating points simply did not succeed in finding a global optimum and was stopped for a locally optimal solution, but it is unlikely that such a situation occurred for all the three considered cases. As a consequence, a more likely explanation is that, in some not yet fully understood way, the optimal control problem for a single operating point penalizes more heavily controllers that may evolve subsequently into more aggressive (optimal) ones.

At the same time, it should be underlined that the global controller appears to be unable to satisfy constraints on robustness to variation of system parameters in the third operating point used for controller synthesis (T5). As a matter of fact, the controller is robust to reasonable variations of system parameters, as demonstrated by the reported simulations. Nonetheless this property is not proved from the mathematical standpoint for one of the considered operating points. Apparently, only by reducing considerably the rise time, it is possible to achieve the desired level of robustness in all the considered operating points, thus significantly affecting overall system performance.

4 Conclusions and future work

In this paper an evolutionary optimization technique was demonstrated as a means for control gain synthesis in the framework of H_∞ control problems. Two different techniques were presented: the first one is based on solving three optimization problems at different operating points, by use of gain scheduling for checking control performance in off–nominal conditions. In the second framework, a single set of gains was searched for, which satisfies control constraints and performance in the same set of operating points. Satisfactory results were obtained in both cases, although the second one provided a more aggressive controller on one side, at the expenses of some lack of robustness.

The research will now focus on improving the search of an optimal solution for both techniques (more aggressive controllers in the first case, robust in the whole considered flight envelope for second one). Moreover, a more demanding scenario will also be considered, where simulations are performed by using the fully nonlinear six-degrees-of-freedom model, in order to assess more convincingly the robustness of the control system to both parameter variations and unmodeled dynamics.

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