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AUTONOMOUS SATELLITE CONSTELLATION FOR ENHANCED EARTH COVERAGE USING COUPLED SELECTION EQUATIONS

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This paper presents a novel solution to the problem of autonomous task allocation for a self-organising constellation of small satellites in Earth orbit. The method allows the constellation members to plan manoeuvres to cluster themselves above particular target longitudes on the Earth's surface. This is enabled through the use of Coupled Selection Equations, which represent a dynamical systems approach to combinatorial optimisation problems, and whose solution tends towards a Boolean matrix which describes pairings of the satellites and targets which solves the relevant assignment problems. Satellite manoeuvres are actuated using a simple control law which incorporates the results of the Coupled Selection Equations. Three demonstrations of the efficacy of the method are given in order of increasing complexity - first with an equal number of satellites and targets, then with a surplus of satellites, including agent failure events, and finally with a constellation of two different satellite types. The method is shown to provide efficient solutions, whilst being computationally non-intensive, quick to converge and robust to satellite failures. Proposals to extend the method for on-board processing on a distributed architecture are discussed.

INTRODUCTION

Satellite constellations offer a number of advantages over single-satellite platforms in a variety of space applications. Some missions require multiple satellites by their very nature - with GPS, for example, contact with at least 4 satellites is required from a given position on the Earth's surface to obtain reliable positioning information, and many more satellites are required to extend GPS capabilities for continuous global coverage.[1]

Moreover, there is an increasing trend towards implementing constellations in mission applications which were traditionally performed with single-satellite platforms, such as Earth observation, space science and telecommunications, for example the Disaster Monitoring Constellation (DMC).[2] This is due to the reduced costs and greater coverage associated with the use of a large number of smaller satellites, which are becoming increasingly viable for implementation as advances in miniaturisation and mass production continue. Additionally, multi-satellite missions increase system reliability and robustness, as the failure or success of a constellation mission is not dependent on the longevity and operational efficacy of a single satellite.

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Traditionally, constellation design is driven by the minimisation of the number of satellites to meet the coverage requirements of the mission. Existing constellations, and those proposed for implementation in the near future involve multiple satellites with a fixed relative position, including Teledesic, Iridium, etc.[3] The propellent budget is defined to allow only small station-keeping manoeuvres during the mission, as any other manoeuvring can be expensive for these relatively large spacecraft. As such, much previous work on autonomous constellation control has centred on station-keeping and maintaining a constant relative position between satellites.[3,4,5]

However, as minituarised spacecraft technology advances, including high specific-impulse low-thrust propulsion systems, the notion of massively distributed constellations of small satellites with the ability to reconfigure to service real-time changes in coverage demand can be envisioned.[6,7,8,9] This would enable enhanced coverage of specific regions of the Earth's surface. For example, a constellation of telecommunications satellites could respond to localised peaks in demand, which may be transient and unplanned, or may correspond to specific pre-planned events, for example the Olympics, World Cup, or other large-scale events.

Traditional ground-based approaches for control and station-keeping for such large numbers of distributed spacecraft with high reconfigurability would prove prohibitively complex and expensive. For lower operational costs and increased system flexibility, on-board autonomy is preferred.

Due to the complex nature of the tasks that must be performed by autonomous constellations, it would be difficult to build autonomy into the system by considering every possible outcome comprehensively, and embedding aspects of autonomy to deal with each eventually. Instead, it is desirable to follow the example of nature's emergent systems, by implementing simple behavioural rules which coalesce to produce the desired complex behaviours.

The desired behaviours of an autonomous constellation are vast, and include such things as target detection, reconfiguration and manoeuvring, networking, collision avoidance, failure detection, and a whole host of other tasks. However, one of the first that must be considered is that of task allocation - i.e. in a constellation of many agents, how is it decided which agents are assigned to which tasks? As the number of agents and tasks increases, the number of possible task assignment combinations grows in a combinatorial explosion, making it impossible for an optimal assignment to be found using standard combinatorial optimisation processes.

Hence, this paper presents a novel dynamical systems approach to the problem of task allocation in a micro-satellite constellation in Earth-centered orbits for re-allocating satellite resources to match demand on the Earth's surface. A single ring of satellites at GEO is considered for illustration. The method implements Coupled Selection Equations to solve the linear two-index and axial three-index assignment problems. These represent the problems of assigning tasks to agents from a homogenous constellation of satellites, and from a constellation containing two different satellite types, respectively.

The remainder of the paper is organised as follows. The next section introduces the constellation model - a single circular ring of n spacecraft on GEO. The task allocation problem is then introduced. Coupled Selection Equations are then introduced as a solution to the task allocation problem. Then the manoeuvring strategy for the satellites is given. The results from three demonstration cases of the method are then given. Finally, extensions of the method are discussed and conclusions are made.

CONSTELLATION MODEL

Dynamics

The constellation is modelled as a ring of n satellite members, initially at geostationary altitude, and azimuthally equispaced, as described by Eqs. (1),

$$\left.\begin{aligned} r_{i0} &= r_{\text{GEO}} \\ \theta_{i0} &= \frac{2\pi(i-1)}{n} \end{aligned}\right\} \qquad (i = 1, \cdots, n) \tag{1}$$

Where r_{GEO} is the radius of Geostationary Earth Orbit, $42,164.1$ km, and θ_i is the true anomaly of the i^{th} satellite.

The trajectory of each satellite is propagated according to the Gaussian form of the variation of Keplerian elements given in Eq. (2).[10]

$$\begin{aligned}
\dot{a} &= \frac{2a^2 v}{\mu} a_t \\
\dot{e} &= \frac{1}{v}\left[2\left(e + \cos\left(\theta\right)\right) a_t - \frac{r}{a}\sin\left(\theta\right) a_n\right] \\
\dot{i} &= \frac{r\cos\left(u\right)}{h} a_h \\
\dot{\omega} &= \frac{1}{ev}\left[2\sin\left(\theta\right) a_t + \left(2e + \frac{r}{a}\cos\left(\theta\right)\right) a_n\right] - \frac{r\sin\left(u\right)\cos\left(i\right)}{h\sin\left(i\right)} a_h \\
\dot{\Omega} &= \frac{r\sin\left(u\right)}{h\sin\left(i\right)} a_h \\
\dot{M} &= \sqrt{\frac{\mu}{a^3}} - \frac{b}{eav}\left[2\left(1 + \frac{e^2 r}{p}\right)\sin\left(\theta\right) a_t + \frac{r}{a}\cos\left(\theta\right) a_n\right]
\end{aligned} \tag{2}$$

Where a, e, i, ω, Ω, and M are the standard Keplerian elements of semi-major axis, eccentricity, inclination, argument of perigee, right ascension of the ascending node and mean anomaly respectively. In addition, v is the orbital speed, r is the orbital radius, θ is the true anomaly, u is the argument of latitude, p is the semi-parameter, b is the semi-minor axis, h is the specific angular momentum, μ is the standard gravitational parameter of Earth and a_t, a_n and a_h are the control accelerations in the directions tangential, normal and out-of-plane to the satellite velocity vector respectively.

It is assumed that the spacecraft only have the ability to thrust in the tangential direction, that the satellite orbits remain quasi-circular throughout the manoeuvres, and that there will be no out-of-plane motion. Applying these assumptions, the equations of motion simplify to those given in Eq. (3), where the i notation is re-introduced to describe the multiple satellite system, and is understood to run from 1 to n.

$$
\begin{aligned}
\dot{r}_i &= 2\sqrt{\frac{r_i^3}{\mu}}a_{ti} \\
\dot{\theta}_i &= \sqrt{\frac{\mu}{r_i^3}}
\end{aligned} \tag{3}
$$

For numerical propagation a Runge-Kutta method with relative tolerance of 10^{-3} and absolute tolerance of 10^{-6} has been employed.

Two different constellation types are investigated in this paper - one in which the constellation consists of n homogenous satellites, and one in which the constellation consists of n_1 satellites of type 1, and n_2 satellites of type 2. In both cases, the initial spacing and dynamics of each satellite are the same as described above.

THE TASK ALLOCATION PROBLEM

The task allocation problem, in this context, is the problem of deciding which satellites on the constellation should be assigned to the the various targets which arise on the Earth's surface.

In a scenario where the constellation consists of homogenous satellites, the task allocation problem can be described by the linear two-index assignment problem. When the constellation consists of two different types of satellite with different capabilities, and one of each satellite type must visit each target, the problem can be described by the axial three-index assignment problem.[15]

Linear Two-Index Assignment Problem The linear two-index assignment problem is to minimise the total costs, c, associated with the pairings between $N = 1, \ldots, n$ targets and satellites, and is stated mathematically in Eqs. (4) - (6).

$$
\underset{X_{ij}}{\text{minimise}} \qquad c = \sum_{i,j} C_{ij} X_{ij} \tag{4}
$$

$$
\text{subject to} \qquad \sum_{i} X_{ij} = 1, \quad \forall j \in N \tag{5}
$$

$$
\sum_{j} X_{ij} = 1, \quad \forall i \in N \tag{6}
$$

Here, C_{ij} is the cost associated with pairing satellite j to target i, and X_{ij} is a Boolean variable describing status of the pairing according to Eq. (7).

$$
X_{ij} = \begin{cases} 1 & \text{if target } i \text{ is to be visited by satellite } j \\ 0 & \text{otherwise} \end{cases} \tag{7}
$$

The constraints of Eq. (5) - (6) represent the requirement that each target can be visited by only one satellite, and each satellite can only visit one target, respectively. This implies that $\mathbf{X} = [X_{ij}]$ must be a permutation matrix. It should be noted that if it is desired for a target to be visited by multiple satellites, that target should simply be replicated within the cost and Boolean matrices as many times as the number of satellites it requires.

In this work, the classic two-index assignment problem as described above was transformed to the equivalent maximisation problem, where the winnings, W_{ij}, associated with each pairing are to be maximised, and the winnings are related to the costs through the linear transformation given in Eq. (8).

$$W_{ij} = -\gamma \cdot C_{ij} + \delta, \quad \gamma > 0, \quad \gamma, \delta \in \mathbb{R}, \forall (i,j) \in N^2. \tag{8}$$

Axial Three-Index Assignment Problem The axial three-index assignment problem is an extension to the linear two-index assignment problem. In this work, it is used to represent the problem of assigning satellites to targets on a constellation where two different types of satellites exist, and each target must be visited by a satellite of each type. This could represent a constellation of disaggregated spacecraft, where payload, power supply and other subsystems are distributed across separate agents - each satellite type has a different specialisation. If we have $N = 1, \ldots, n$ targets, and $N = 1, \ldots, n$ of satellites of types 1 and 2, then the problem is stated is stated mathematically in Eqs. (9) - (12).

$$\underset{X_{ijk}}{\text{minimise}} \qquad c = \sum_{i,j,k} C_{ijk} X_{ijk} \tag{9}$$

$$\text{subject to} \qquad \sum_{i} X_{ijk} = 1, \quad \forall (j,k) \in N^2 \tag{10}$$

$$\sum_{j} X_{ijk} = 1, \quad \forall (i,k) \in N^2 \tag{11}$$

$$\sum_{k} X_{ijk} = 1, \quad \forall (i,j) \in N^2 \tag{12}$$

Then, the Boolean variable is defined according to Eq. (13).

$$X_{ijk} = \begin{cases} 1 & \text{if target } i \text{ is to be visited by the combination of satellites } j \text{ and } k \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Again, the problem is transformed to a maximisation problem in this work using the transformation in Eq. (8).

Coupled Selection Equations

A selection equation is defined as a dynamical system in which various modes of the system are set in competition, and only the mode with the highest initial value will survive, whilst all others converge to zero in the limit as time tends towards infinity.

Because of this behaviour, the method lends itself well to providing solutions for the task allocation problem, and has been successfully implemented in a variety of contexts involving task allocation for autonomous agents. This includes assigning game strategies to mobile robots playing football, developing self-organising space colonies, and assigning subtasks to robots in the assembly of space station components.[11,12,13]

The most basic of these selection equations is given in Eq. (14).

$$\frac{d\xi_i}{dt} = \xi_i \left(1 - \xi_i^2 - \beta \sum_{i' \neq i} \xi_{i'}^2\right) \quad (14)$$

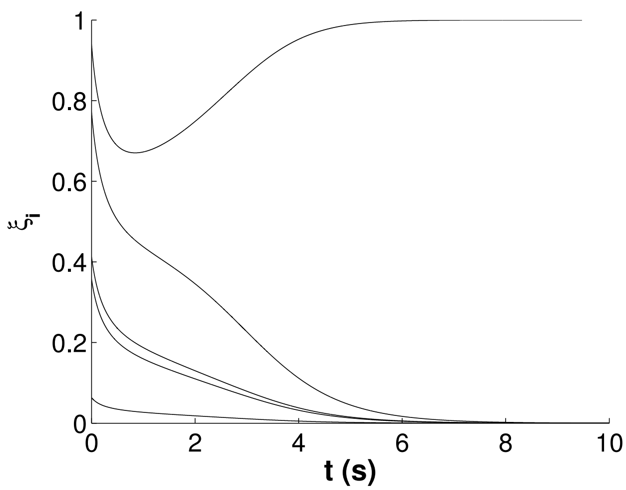


Figure 1: **Typical Time Evolution of Coupled Selection Equations**

It can be shown that the stable equilibrium of Eq. (14) corresponds to a situation in which the ξ_{ij} with the highest initial value converges to 1, while all others converge to 0, subject to the conditions that $\xi_i(0) \geq 0, \forall i$ and $\beta > 1$, where β is a parameter which determines the influence of the modes on each other's time evolution.

A simple demonstration of the time evolution of this system is shown in Figure 1 where 5 modes are initialised.

This equation can be extended for use in the linear two-index assignment problem by numbering the variables with two indices and extending the coupling terms as given by Eq. (15).

$$\frac{d\xi_{ij}}{dt} = \xi_{ij} \left(1 - \xi_{ij}^2 - \beta \left(\sum_{i' \neq i} \xi_{i'j}^2 + \sum_{j' \neq j} \xi_{ij'}^2\right)\right) \quad (15)$$

When the matrix of coupled selection variables, $\mathbf{\Xi} = [\xi_{ij}]$, is initialised with the non-negative winnings from the linear two-index assignment problem, where each winning, W_{ij}, is transformed from the corresponding cost, C_{ij}, using Eq. (8), where $\gamma = (\max \mathbf{C})^{-1}$ and $\delta = 1$ and have been set such that $W_{i,j} \in [0, 1], \forall i, j$, then it can be shown that in the limit as time tends to infinity, the coupled selection variables tend towards a permutation matrix which solves the linear two-index assignment problem, $\hat{\mathbf{X}}$. This process is formalised in Eq. (16).[14] It is also a requirement that $\beta > \frac{1}{2}$ (and, in general $\beta > \frac{1}{D}$ where D is the dimension of the coupled selection variables).

$$\mathbf{C} \mapsto \mathbf{W} \mapsto \mathbf{\Xi}(0) \rightarrow \lim_{t \to \infty} \mathbf{\Xi}(t) = \hat{\mathbf{X}} \quad (16)$$

In this application, the costs are chosen to be equal to the angular distance between the targets and satellites, as given by Eq. (17).

$$C_{ij} = \phi_{ij} = \theta_i - \theta_j, \quad -\pi \leq \phi_{ij} < \pi \tag{17}$$

One further aspect of the two-index coupled selection equations that makes them appropriate for this application is that they will still converge to a Boolean matrix if they are modified to represent the case where there are a surplus of satellites in comparison to targets. In this case, the constraints of Eq. (4) - (6) will not be met, but it will still be the case that each target will be visited by one target only, and that the satellites will be assigned in such a way that Eq. (4) will still be minimised.

The coupled selection equation is extended further for use in the axial three-index assignment problem. Again, another index is added to the variable numbering, and more coupling terms are added as appropriate as given by Eq. (18).

$$\dot{\xi}_{ijk} = \xi_{ijk}\left(1 + (3\beta - 1)\,\xi_{ijk}^2 - \beta\left(\sum_{i'} \xi_{i'jk}^2 + \sum_{j'} \xi_{ij'k}^2 + \sum_{k'} \xi_{ijk'}^2\right)\right) \tag{18}$$

Equation (18) will converge according to Eq. (16) so long as $\beta > \frac{1}{3}$ and $\xi_{ijk}(0) > 0, \forall i, j, k$.

In this case, each cost C_{ijk} is set equal to the sum of the angular distances between satellite i and target k and satellite j and target k.

Although the asymptotically stable solutions of Eqs. (14), (15) and (18) correspond to valid solutions, there may be some unstable equilibria - hence, noise is added to the equations if the maximum rate of change of the equations fall below a set tolerance, in order to prevent stagnation at unsuitable unstable equilibria.

Because the equations can be shown to be gradient flows with asymptotically stable equilibria and containing no more complicated attractors than this, a simple first order forward Euler method integrator is sufficient for numerically propagating the equations. This has further advantages for implementation on a highly distributed architecture of small satellites, as it does not require significant on-board processing from each agent in order for them to propagate the equations on-orbit.[15]

Manoeuvre Actuation

Once the Coupled Selection Equations have converged, a simple transformation of the converged matrix can be used to define the destination direction, d, for each satellite. For the two-index assignment problem, this is defined according to Eq. (19).

$$d(\phi_{ij}, \boldsymbol{\Xi}) = \mathbf{N}_{\lambda\sigma}\left(\sum_{i} \xi_{ij}\mathbf{N}_{\lambda'\sigma'}\left(\phi_{ij}\right)\right) \tag{19}$$

Equation (19) is a normal linear combination of the phasing angles between satellite j and all targets i, weighted with the coupled selection variables. This results in the property that the d tends towards the direction of the allocated target - i.e. it is a negative number if the satellite leads the target, and positive if the satellite lags the target. The operator $\mathbf{N}_{\gamma\delta}(\mathbf{y}) = \frac{\mathbf{1}}{|\mathbf{y}|+\mathbf{1}/(\lambda|\mathbf{y}|+\sigma)}$ for $\lambda, \sigma > 0$, and is used to normalise the value of y and avoid singularities at $y = 0$.

Once the direction of rephasing is set, a simple controller can be designed to raise the satellite orbits in order to allow a lagging target to advance towards the satellite or raise them to allow a lagging target to advance towards the satellite. The controller implemented here is given by Eqs. (20) and (21).

$$r_{\text{desired}} = \begin{cases} r_{\text{GEO}} - 1000 \text{ km} & d < 0 \\ r_{\text{GEO}} + 1000 \text{ km} & d > 0 \\ r_{\text{GEO}} & \text{otherwise} \end{cases} \tag{20}$$

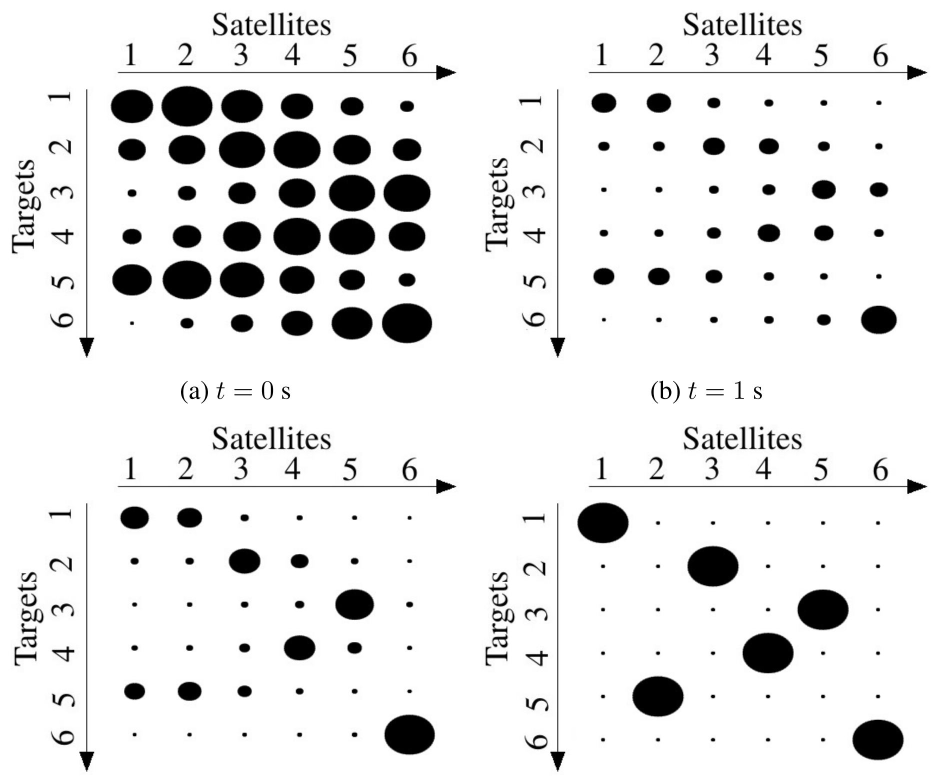
$$a_t = \begin{cases} \kappa & r - r_{\text{desired}} < r_{\text{db}} \\ -\kappa & r - r_{\text{desired}} > r_{\text{db}} \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

Here r_{db} is the dead-band for the satellite radius with a value of 1 km, and κ is a nominal value of the tangential thrust acceleration set according to the capabilities of the on-board propulsion system.

This controller gives a simple thrust-coast-thrust profile, although where minimum time convergence upon the target is desired, a simple controller for continuous thrust can be implemented.[16]

NUMERICAL RESULTS

To demonstrate the efficacy of this method in solving the task allocation problem, three demonstration cases are introduced below, and the results from their numerical simulations are given.



(a) $t = 0$ s

(b) $t = 1$ s

(c) $t = 3$ s

(d) $t = 5$ s

Figure 2: **Time Evolution of Coupled Selection Equation Strengths - Radius of Circle Represents Strength of Pairing Between Target i and Satellite j.**

Test Case #1: Equal Number of Satellites and Targets

In this demonstration, a constellation of six homogenous satellites are initially equi-spaced on geostationary orbit. Six targets are identified on the Earth's surface, and the Coupled Selection Equations are used to deterimine which satellites should be allocated to which targets.

The equations converge quickly, and snapshots of their time evolution are shown in Figure 2.

As can be seen from Figure 2, the equations generally converge as expected - the pairing with the highest initial strength in each row/column tends to persist and grow, whilst all others decay to zero. The only exception to this is in row 1, where the highest initial coupling for target 1 is with satellite 2, yet in the converged state, satellite 1 has been allocated to target 1. This is because targets 1 and 5 both have their strongest initial couplings associated with satellite 2. This results in a column-wise competition between the two elements via the coupling terms of the equation, and since the highest initial value of column 2 lies with target 5, target 5 wins the competition for satellite 2.

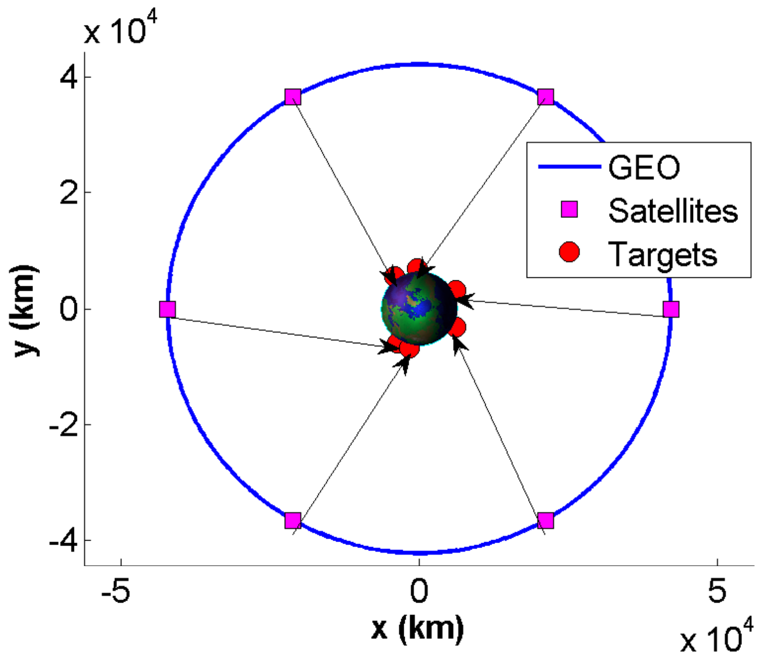
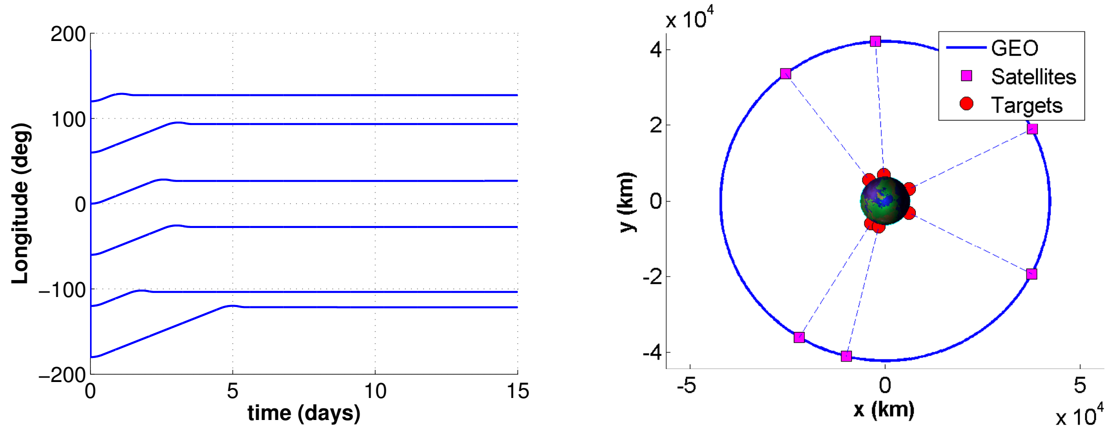


Figure 3: **Satellite Allocations for Test Case #1**

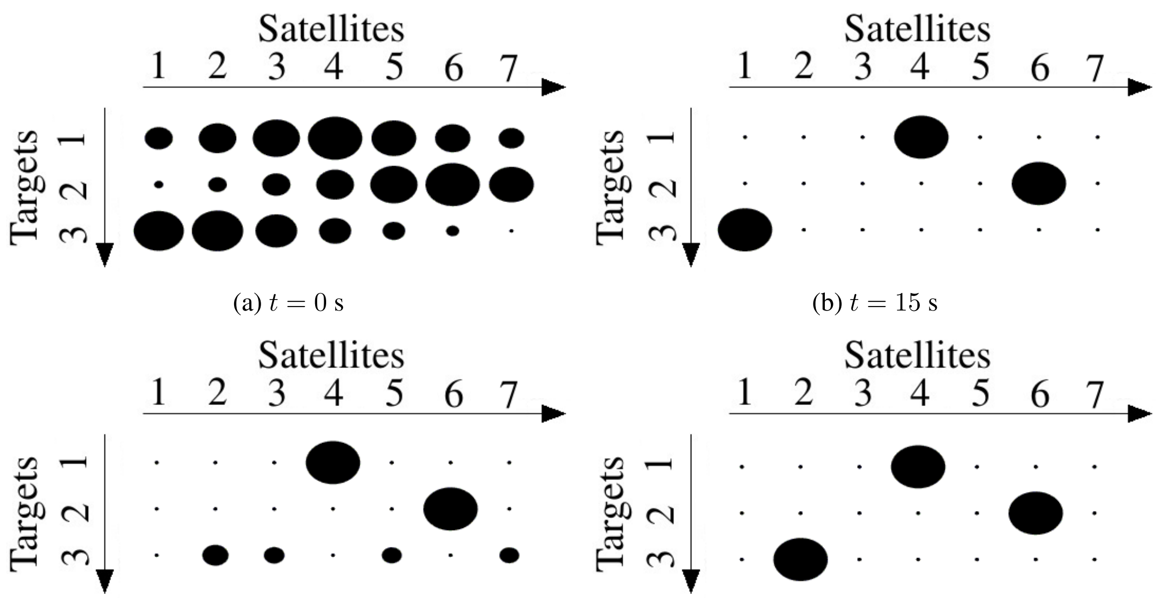
The final on-orbit allocations of this simulation are displayed in Figure 3. The manoeuvre results are shown in Figure 4 - where Figure 4a displays the time histories of the longitudes of the satellites, and Figure 4b shows the final on-orbit positions of the satellites with respect to the targets.



(a) **Longitude Time Histories of Satellites** (b) **Satellite Positions $t = 15$ days**

Figure 4: **Manoeuvre Test Case #1**

Test Case #2: Surplus Satellites with Satellite Failure



(a) $t = 0$ s (b) $t = 15$ s

(c) $t = 25$ s (d) $t = 45$ s

Figure 5: **Time Evolution of Coupled Selection Equation Strengths - Radius of Circle Represents Strength of Pairing Between Target i and Satellite Combinations j.**

In this demonstration, a constellation of seven homogenous satellites are initially equi-spaced on geostationary orbit. Three targets are identified on the Earth's surface, and the Coupled Selection Equations are again used to deterimine which satellites should be allocated to which targets. During the process, one of the allocated satellites fails, and the coupled selection equations re-converge to allocate one of the unallocated satellites to the target of the failed satellite. Once again, snapshots of the time evolution of the equations are shown in Figure 5.

The highest initial couplings in each row/column persist and all others decay as expected with satellites 1, 4 and 7 being allocated as shown by Figures 5a-5b. At $t = 15s$, a failure is triggered in satellite 1 and the unallocated satellites begin a sub-competition to fill the position of the failed satellite. This sub-competition converges as expected and satellite 2, which has the strongest coupling with the remaining target compared to the other unallocated satellites, wins the competition and fills in for failed satellite 1, as shown in Figures 5c-5d.

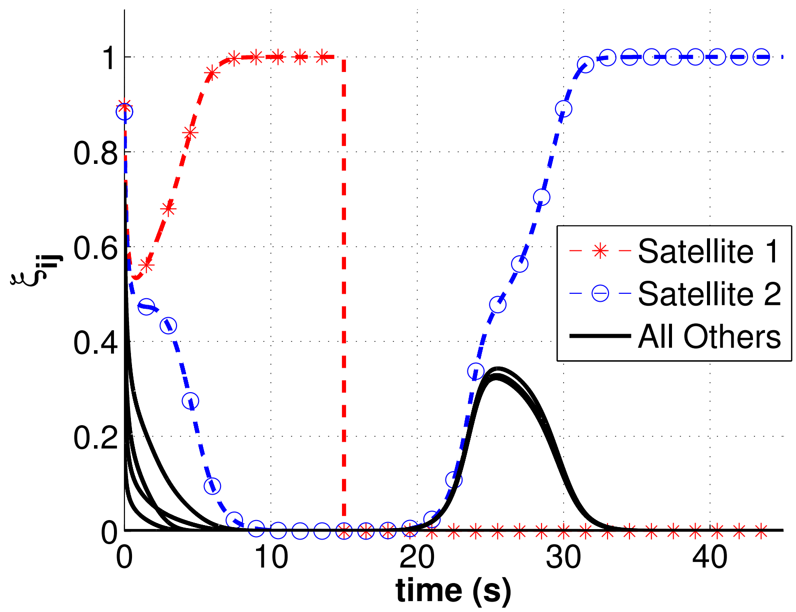
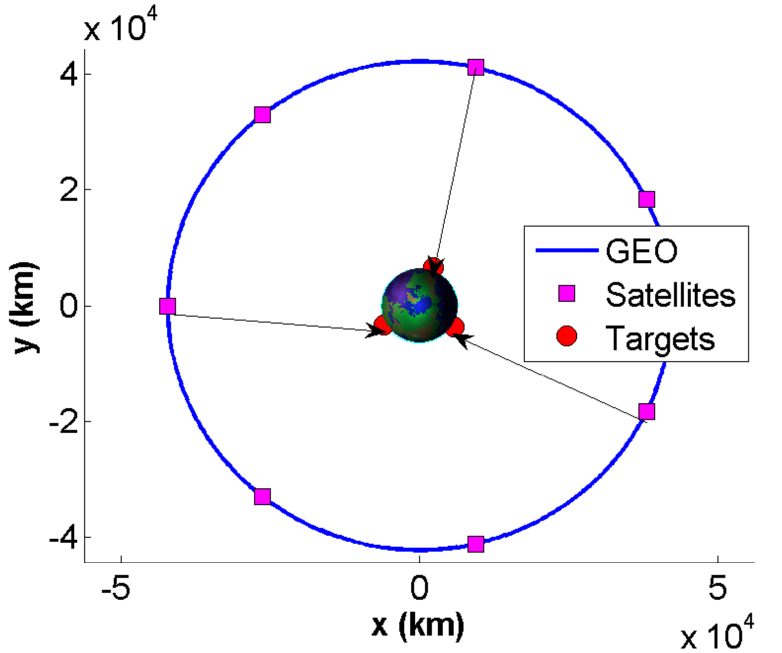


Figure 6: **Coupled Selection Equation Time Histories for Target 3 Test Case #2**

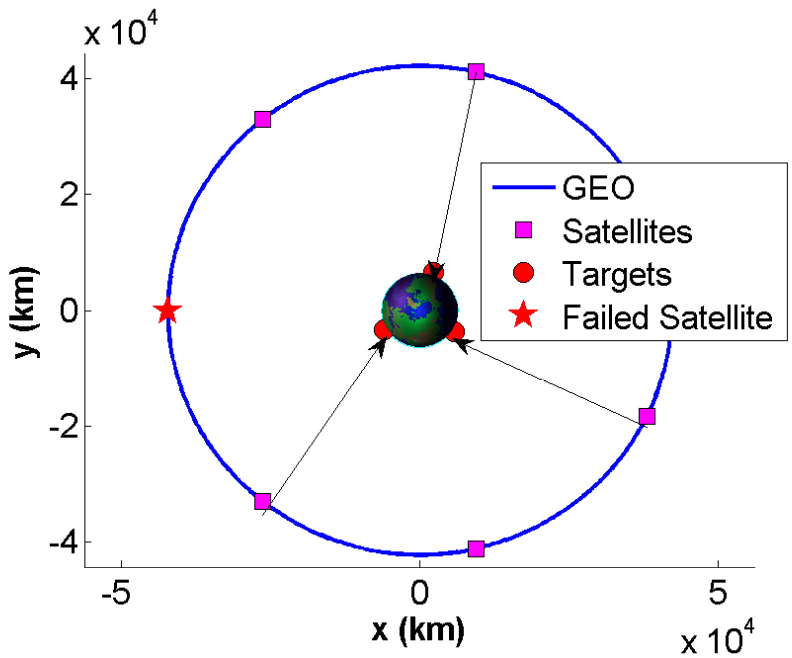
To provide more detail on the time evolution of the equations in this scenario, the coupled selection time histories for the pairings associated with target 3 are shown in Figure 6.

Figures 5 and 6 demonstrate how robust the coupled selection equations are to agent failures, allowing quick adaptations to be made to the allocations in response to unforeseeable events.

The on-orbit allocations from before and after the failure of satellite 1 are shown in Figures 7a and 7b respectively. Manoeuvre results are again shown in Figure 8 - where Figure 8a displays the time histories of the longitudes of the satellites, and Figure 8b shows the final on-orbit positions of the satellites with respect to the targets.

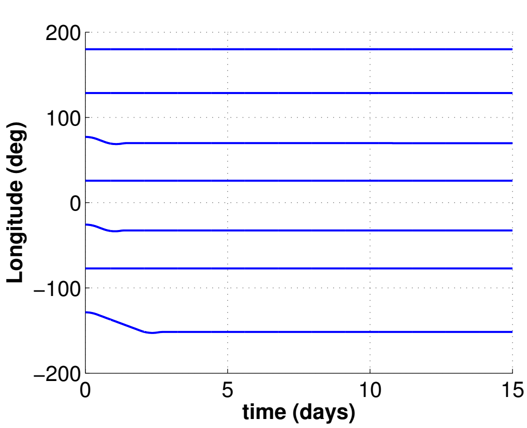


(a) $t = 15$ s **Before Satellite 1 Failure**

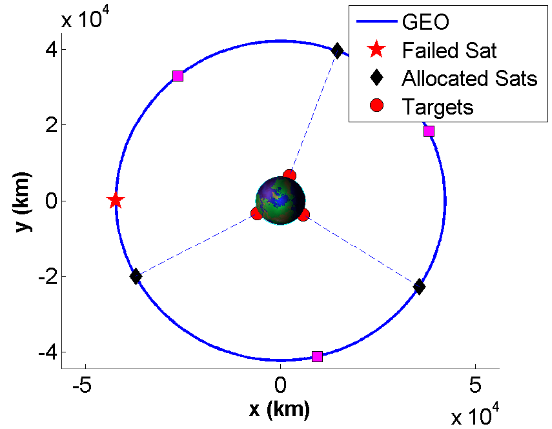


(b) $t = 45$ s **After Satellite 1 Failure**

Figure 7: Satellite Allocations for Test Case #2



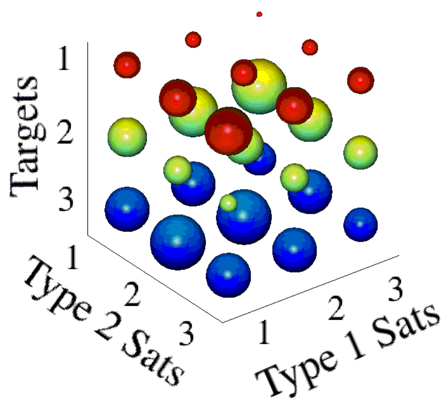
(a) **Longitude Time Histories of Satellites**



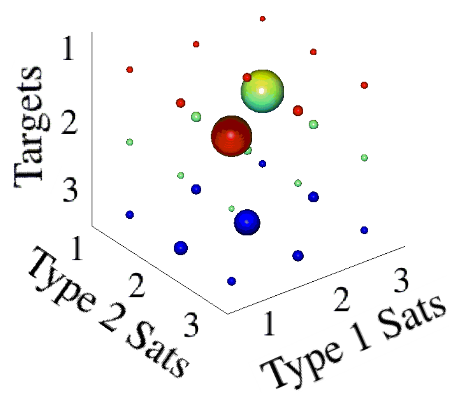
(b) **Satellite Positions $t = 15$days**

Figure 8: **Manoeuvre Test Case #2**

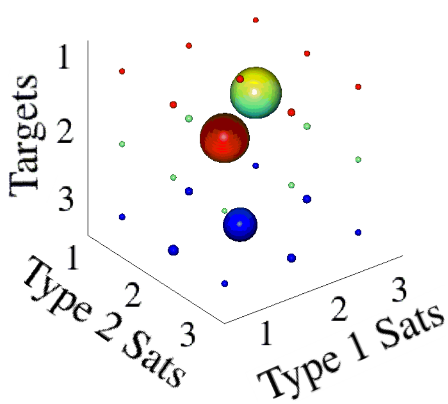
Test Case #3: Multiple Satellite Types



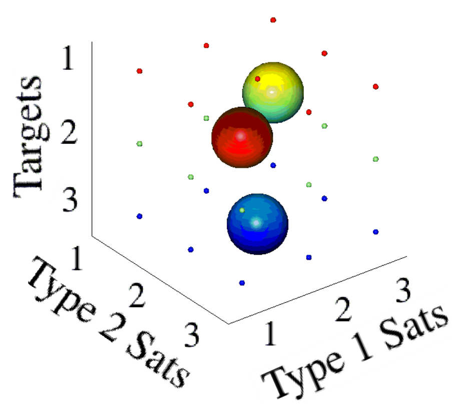
(a) t = 0s



(b) t = 2.6s



(c) t = 3.2s



(d) t = 8s

Figure 9: **Time Evolution of Coupled Selection Equation Strengths - Radius of Sphere Represents Strength of Pairing Between Target k and Satellite Combinations i and j.**

In this demonstration, a constellation of 6 satellites - 3 of type 1 and 3 of type 2 - are initially equi-spaced in geostationary orbit. 3 targets are identified on the Earth's surface, and the Coupled Selection Equations are used to determine which two satellites of each type are allocated to which targets. The equations again converge quickly, and snapshots of their time evolution are given in Figure 9. Here, the selection variables converge in pairs - i.e. the top (red) layer represents the possible pairs of satellites of both types that can visit target 1, the green layers represents the same for target 2, etc. Only one variable in each layer should persist after convergence.

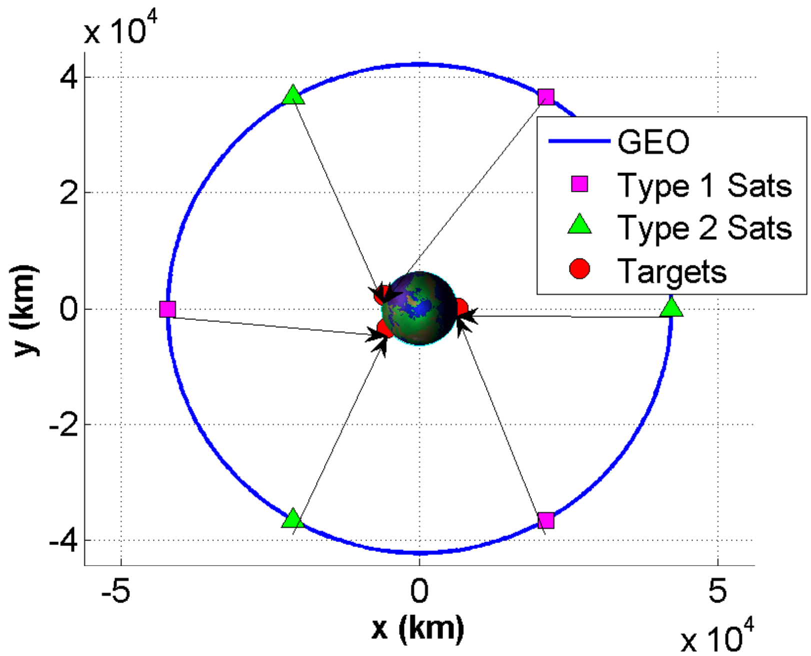
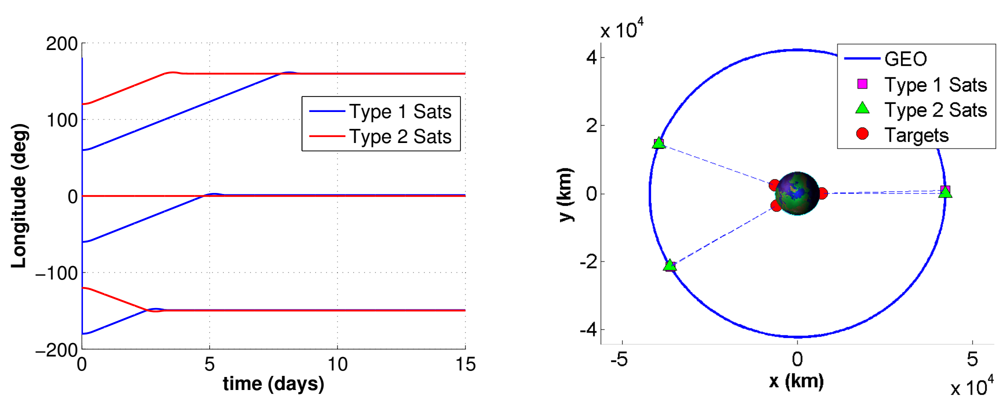


Figure 10: **Satellite Allocations for Test Case #3**

The on-orbit allocations from this demonstration case are given in Figure 10, and manoeuvre results are given in Figure 11 - where Figure 11a displays the time histories of the satellite longitudes, and Figure 11b shows the final on-orbit positions of the satellites with respect to the targets.



(a) **Longitude Time Histories of Satellites** (b) **Satellite Positions** $t = 15$**days**

Figure 11: **Manoeuvre Test Case #3**

CONCLUSIONS AND FUTURE DEVELOPMENTS

This paper has demonstrated a novel solution to the problem of autonomous task allocation on a self-organising constellation of small satellites. Coupled Selection Equations have been shown to converge to solutions which solve the linear two-index assignment problem and axial three-index assignment problem, providing the optimal pairings between satellite members and targets on the Earth's surface.

Demonstrations of the efficacy of the method have been given with various levels of complexity. Through these demonstrations, the method is shown provide these solutions quickly, without much computational effort, and is also shown to be robust to satellite failures.

However, the work presented here has implemented a model of the Coupled Selection Equations which is propagated on a centralised architecture. In order for the autonomy to be embedded within the constellation, the equations must be reformulated to work on a distributed architecture where each agent propagates its own state using information from its neighbours. The reformulated equations must be simple enough for agents with limited processing capabilities to propagate them, and must still converge to a viable solution in the absence of information about the state of all other members of the constellation or where time delays in communications are present. This work provides a strong basis for these future developements for embedded autonomy.

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