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**Robust Design of a Re-entry Unmanned Space Vehicle by  
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# Robust Design of a Re-entry Unmanned Space Vehicle by Multi-fidelity Evolution Control

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This paper addresses the preliminary robust design of a small-medium scale re-entry unmanned space vehicle. A hybrid optimization technique is proposed that couples an evolutionary multi-objective algorithm with a direct transcription method for optimal control problems. Uncertainties on the aerodynamic forces and vehicle mass are integrated in the design process and the hybrid algorithm searches for geometries that a) minimize the mean value of the maximum heat flux, b) maximize the mean value of the maximum achievable distance, and c) minimize the variance of the maximum heat flux. The evolutionary part handles the system design parameters of the vehicle and the uncertain functions, while the direct transcription method generates optimal control profiles for the re-entry trajectory of each individual of the population. During the optimization process, artificial neural networks are used to approximate the aerodynamic forces required by the direct transcription method. The artificial neural networks are trained and updated by means of a multi-fidelity, evolution control approach.

## Nomenclature

$C_D$	= drag coefficient
$C_E$	= sampling hypersurface
$C_L$	= lift coefficient

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1		
2	$\mathbf{d}$	= design vector
3		
4	$d_{body}$	= thickness of the structure [m]
5		
6	$d_{ll}$	= Euclidean distance between point in $DB_{train}$ evaluated at current fidelity level of the models
7		and the points evaluated with lower level models
8		
9	$d_{min,ll}$	= minimum Euclidean distance to accept points evaluated with lower level models into the
10		database
11		
12	$d_{min,sl}$	= minimum Euclidean distance between the sampled trajectory point and the points of the
13		database evaluated with the same fidelity code, to accept the new point into the database
14		
15	$d_{sl}$	= Euclidean distance between point in $DB_{train}$ and the trajectory points extracted by
16		current solutions
17		
18		
19	$d_{TPS}$	= thickness of the TPS skin covering the vehicle [m]
20		
21	$D$	= drag [N]
22		
23	$DB_{train}$	= matrix of points in the database used to train the approximated model
24		
25	$D_v$	= viscous drag [N]
26		
27	$D_w$	= wave drag [N]
28		
29	$E_q, E_D$	= mean values of the maximum heat flux and the orthodromic distance, respectively
30		
31	$Err$	= error function
32		
33	$g$	= gravity acceleration [ $m/s^2$ ]
34		
35	$h$	= altitude [m]
36		
37	$K_e$	= constant term in the heat flux equation ( $1.742 \cdot 10^{-4}$ )
38		
39	$l$	= nominal length of the wave-rider [m]
40		
41	$L$	= lift [N]
42		
43	$L_C$	= fidelity level of the model
44		
45	$L_{det}$	= deterministic value of the lift [N]
46		
47	$L_{TPS}$	= thickness of the TPS at the nose
48		
49	$L_{unc}$	= value of the lift affected by uncertainties [N]
50		
51	$m$	= total mass of the vehicle [kg]
52		
53	$m_{nose}$	= mass of the TPS at the nose [kg]
54		
55	$m_{skin}$	= mass of the TPS covering the vehicle [kg]
56		
57	$m_{st}, m_{TPS}, m_{pl}$	= structural mass, the mass of the TPS and the mass of the payload
58		(avionics and power system), respectively [kg]
59		
60	$Ma$	= Mach number

1		
2	$n$	= power law exponent for the shape of the waverider
3		
4	$n_k$	= number of kernels
5		
6	$n_{gcyg}$	= defines the generation loop in the generation based evolution control approach
7		
8	$n_{gl}$	= number of generations of the global optimizer, for which the fidelity of the model is
9		increased the upper level
10		
11	$n_{gv}$	= number of generations for which the real model has to be used for the generation based
12		evolution control approach
13		
14	$n_o$	= number of trajectory points to extract from each trajectory
15		
16	$n_p$	= number of collocation points
17		
18	$n_t$	= maximum number of trajectory point to extract for each generation
19		
20	$n_v$	= number of individuals to evaluate by true model in the individual based evolution control
21		approach
22		
23	$N$	= number of trajectory elements
24		
25	$\dot{q}_{conv}$	= convective heat flux [ $W/m^2$ ]
26		
27	$r_f$	= upper bound of the constraints on the final radius [m]
28		
29	$r_{min}$	= lower bound of the constraints on the final radius [m]
30		
31	$r$	= norm of the position vector with respect to the center of the planet [m]
32		
33	$Re$	= Reynolds number
34		
35	$R_E$	= mean radius of the Earth (const. $6.371 \cdot 10^6$ m)
36		
37	$R_n$	= radius of curvature at the edges of the vehicle [m]
38		
39	$S$	= matrix with data of the trajectory
40		
41	$S_n$	= surface of the TPS nose cone [ $m^2$ ]
42		
43	$S_{TPS}$	= TPS surface area except that of the nose [ $m^2$ ]
44		
45	$t$	= time [s]
46		
47	$T$	= thrust [N]
48		
49	$v$	= magnitude of the velocity [m/s]
50		
51	$V_n$	= volume of the nose [ $m^3$ ]
52		
53	$V_\infty$	= free stream velocity module [m/s]
54		
55	$w$	= nominal width of the waverider [m]
56		
57	$x$	= longitudinal coordinate

*Greek symbols*

1		
2	$\alpha$	= angle of attack [deg]
3		
4	$\beta$	= oblique shockwave inclination angle of the waverider
5		
6	$\gamma_v$	= bank angle [deg]
7		
8	$\theta$	= center line wedge angle of the waverider
9		
10	$\theta_p$	= flight path angle [deg]
11		
12	$\lambda$	= longitude [deg]
13		
14	$\rho_{body}$	= density of the vehicle structure [ $kg/m^3$ ]
15	$\rho_{TPS}$	= density of the TPS material
16		
17	$\sigma_q^2, \sigma_D^2$	= variance of the maximum heat flux and orthodromic distance, respectively
18		
19	$\tau_E$	= MOPED coefficient for the generation of new individuals
20		
21	$\phi$	= latitude [deg]
22		
23	$\xi$	= heading angle (azimuth of the velocity) [deg]
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## I. Introduction

The increase in computer performance allows numerical simulation to replace a big portion of experimental tests, and numerical optimization to handle complex multidisciplinary design problems. However, generally only reduced or low-fidelity models are used during the optimization process on sequential machines. Higher fidelity models are used for more detailed investigations of some promising configurations. In order to reduce the duration and cost of the design process and get better solutions from its early stage, it would be desirable to introduce high fidelity models already in the preliminary design phase. Specific techniques are therefore required to efficiently handle computationally expensive, high fidelity models.

Furthermore, in a wide range of problems the system to be designed needs to be optimally controlled during its operational life. In these cases the optimal control component of the design has an impact on the system design component and vice versa. For example, in the case of a re-entry vehicle, the system, i.e. the re-entry vehicle, follows a descent trajectory that needs to be optimally controlled. The outcome of the optimal control problem has an impact on the maximum heat flux that the vehicle has to withstand and ultimately on its design (shape, material, structure, thermal control system). Conversely the system design component has an impact on the solution of the

1 optimal control component because the dynamics changes.  
2

3  
4 Lastly, given the preliminary nature of the computation, the inclusion of uncertainties would  
5 be desirable in order to generate robust solutions and to highlight the sensitiveness of the design to  
6 specific parameters. Hence, the idea is to have an approach capable of 1) integrating system design  
7 and optimal control, 2) handle models with different levels of fidelity, and 3) efficiently incorporate  
8 design uncertainties in the optimization process. Although many authors have addressed each one  
9 of these aspects individually or partially combined, currently there are no examples of integration  
10 of all of them together.  
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18 One decade ago, Tsuchiya and Mori[1, 2] used the integrated design approach, shape plus control  
19 law, for the optimization of launch vehicles. Roshanian and Keshavarz[3] adopted an analogous  
20 integrated approach for the multi-disciplinary design of a sounding rocket, with response surface  
21 method used to approximate the propulsion model. Since the end of 90's, researchers working on the  
22 efficiency of optimization processes have been applying meta-modeling techniques to approximate  
23 the response of expensive models, with or without the use of multiple different models, with different  
24 fidelity levels and costs, during the optimization process[4-9].  
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33 This paper presents a novel approach to the preliminary robust design of complex engineer-  
34 ing systems, **integrating** system design and optimal control into a single optimization process. A  
35 population of individuals evolves multiple system design solutions in parallel using an Estimation  
36 of Distribution Algorithms (EDAs)[10] called MOPED (Multi-Objective Parzen based Estimation  
37 of Distribution) algorithm[11, 12]. At every step of the evolution, an optimal control profile is as-  
38 sociated to each individual of the population (each system design solution). The optimal control  
39 profile is generated by solving an optimal control problem with a **direct transcription method, which**  
40 **transcribes continuous optimal control problems into equivalent non-linear programming problems**  
41 **(NLP) by discretizing both states and controls (for more details on transcription methods for opti-**  
42 **mal control the interested reader can refer to[13]).** Uncertainties in the system design parameters  
43 are propagated through the model to compute mean and variance of the relevant performance  
44 indexes. Mean and variance are then used as objectives for MOPED. Finally, a meta-modeling  
45 technique is used to introduce high fidelity information in the optimization loop while maintaining  
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1 the computational cost at an acceptable level. The balanced mixture of high fidelity, low fidelity  
2 and meta-modeling information is handled through an evolution control process[14].  
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5 The case study considered in this paper is the robust design of a medium scale Unmanned Space  
6 Vehicle (USV). USVs are seen as a test-bed for enabling technologies and as a carrier to deliver and  
7 return experiments to and from low-Earth orbit. They are a potentially interesting solution also  
8 for the exploration of other planets or as long-range reconnaissance vehicles for Earth observation  
9 [15–17].  
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15 In [18] the authors describe a similar approach applied to the design of a small scale USV,  
16 taking into account the availability of last generation thermal protection systems (TPS) based on  
17 ultra-high temperature ceramic materials (UHTC)[19, 20]. Here the integrated design technique  
18 is applied to the design of a larger scale USV, and a different treatment of the optimal control  
19 component is introduced: a dual trajectory optimization loop generates control law solutions that  
20 minimize the maximum heat flux or maximizes the orthodromic distance, during the re-entry phase.  
21 Minimization of maximum heat flux is directly linked to the complexity and costs of the vehicle,  
22 because if the vehicle is able to follow a minimal heat flux descent corridor, then cheaper and lighter  
23 protecting materials, and under-skin structures, could be used. On the other hand, long range  
24 gliding is desirable, because it gives the possibility to robustly select the re-entry point. As in [18],  
25 here the shape of the vehicle is derived from an ideal wave-rider configuration([21–23]), and its  
26 geometry is modified in order to introduce more realistic rounded edges.  
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40 An Artificial Neural Network (ANN) **approximated model** is used as intermediate layer between  
41 the dynamics and aerodynamics of the vehicle such that when the values of drag and lift coefficients  
42 are required to compute the forces acting on the vehicle the ANN is interrogated instead of running  
43 a full CFD simulation. During the optimization process, the aerodynamic database used to generate  
44 the meta-model is updated by a multi-fidelity, evolution control approach: a simplified analytical  
45 model and a computational fluid dynamic (CFD) one are scheduled during the optimization process  
46 in order to efficiently update the database with solutions sampled where optimal solutions are more  
47 likely expected[24–28]. In particular, the low-fidelity model (the analytical one in this case) is used  
48 to generate samples globally over the range of the design parameters, while the high fidelity one will  
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1 be used to locally refine the meta-model in later stages of the optimization.  
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3 The paper starts with a description of the proposed robust multidisciplinary design approach, to-  
4 gether with the meta-modeling integration, the integration of system design and process control and  
5 the treatment of model uncertainties, which are the most innovative aspects of the whole approach.  
6 In the following section the integrated design approach is applied to a low dimensional rocket ascent  
7 problem, then the USV test case is described, with details on the aerodynamic, thermal, mass, and  
8 dynamic models of the vehicle, and some preliminary results are presented. A conclusion section  
9 summarizes the key findings of this work and suggests some directions for future developments.  
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## 18 **II. Robust Multidisciplinary Design Approach**

19 This paper addresses the integrated design of system and process control under uncertainty. In  
20 particular, the design process aims at the simultaneous optimization of the shape and trajectory  
21 control profiles of an aerospace vehicle. The multidisciplinary design approach proposed in this  
22 paper, integrates an evolutionary multi-objective algorithm with a direct transcription method for  
23 optimal control problems. An outer multi-objective evolutionary optimization procedure manages  
24 the parameters defining the shape of the vehicle and considers as objectives and constraints the  
25 mean and variance of the values of the performance indexes coming from an inner optimization  
26 processes handling the optimal control problem.  
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38 Once a shape is defined by the outer loop, the optimal control problem is solved multiple times,  
39 each time optimizing a different performance index. The individual optimization of each performance  
40 index responds to the need of optimally designing the vehicle for all extreme operational conditions  
41 that could face during its operational lifetime, rather than for an intermediate one.  
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46 In fact, during the operational life, the shape is a fixed characteristic of the vehicle, and cannot  
47 change during the flight (no morphing technology is considered here), while the control law can be  
48 **varied** during the operational phase to optimize the operational performance. As a consequence,  
49 if a given shape has to maximize different and possibly conflicting performance indexes, then it is  
50 necessary to compute the best control law for each performance index, because a single optimization,  
51 combining the performance indexes (e.g., a weighted sum of single objectives), would yield a shape  
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1 that is suboptimal in extreme conditions. Mean and variance of the performance indexes are then  
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3  
4 computed following the procedure described in Sec. II B.

5  
6 The optimization of the optimal control profile makes use of a data-fitted surrogate model to  
7  
8 approximate the dynamic characteristics of the vehicle, such as the aerodynamic forces acting on  
9  
10 the vehicle, as functions of the shape of the vehicle and its operational conditions. The training  
11  
12 and updating of the **approximated model(s)** is optimally managed through a multi-fidelity evolution  
13  
14 control approach. The following sections describe the different aspects of the approach.

### 17 **A. Multi-objective Algorithm**

18  
19 The outer Multi-Objective Optimization (MOO) problem is solved with the Multi-Objective  
20  
21 Parzen based Estimation of Distribution (MOPED) algorithm[11, 12]. MOPED belongs to the class  
22  
23 of Estimation of Distribution Algorithms (EDA)[10]. EDA automatically learn the structure of the  
24  
25 search landscape, by explicitly constructing and evolving a probabilistic model of the search space.  
26  
27 New candidate solutions are then generate by sampling the probabilistic model.

28  
29 MOPED is a multi-objective optimization EDA for continuous problems that uses the Parzen  
30  
31 method[36] to build a probabilistic representation of Pareto solutions, with multivariate dependen-  
32  
33 cies among variables[11, 12]. MOPED implements the general ranking and selection strategies of the  
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35 Non-dominated Sorting Genetic Algorithm II (NSGA-II)[35], while solutions are generated by sam-  
36  
37 pling from the Parzen model instead of using the genetic operators in NSGA-II. The Parzen method  
38  
39 is a non-parametric approach to kernel density estimation and provides an estimator that converges  
40  
41 everywhere to the true Probability Density Function (PDF) in the mean square sense. Should the  
42  
43 true PDF be uniformly continuous, the Parzen estimator can also be made uniformly consistent. In  
44  
45 short, the method allocates  $N_{ind}$  identical kernels (where  $N_{ind}$  is the number of individuals of the  
46  
47 population of candidate solutions), each one centred on a different element of the sample. By means  
48  
49 of the Parzen method, a probabilistic model of the promising search space portion is thus built on  
50  
51 the basis of the information given by  $N_{ind}$  individuals of the current population, and  $\tau_E N_{ind}$  new  
52  
53 individuals ( $\tau_E \geq 1$ ) can then be sampled . The variance associated with each kernel depends on (i)  
54  
55 the distribution of the individuals in the search space and (ii) the fitness value associated with the  
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1 pertinent individual, so as to favor sampling in the neighborhood of the most promising solutions.  
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### 5 **B. Robust Design Optimization Under Uncertainty**

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 7 The objectives and constraints of the outer loop are the statistical moments of the performance  
 8 indexes obtained by solving the inner optimal control problems. In the inner loop, the optimal  
 9 control problems are solved considering deterministic models. Thus the solution of the optimal  
 10 control problems provides the deterministic nominal values of the performance indexes.  
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15 Once a deterministic solution is available, uncertainties on key dynamic characteristics, such as  
 16 the aerodynamic forces or the mass, are introduced by perturbing the nominal value of a generic  
 17 uncertain quantity  $X_{det}$ . When uncertainties are related to operational conditions, uncertain values  
 18  $X_{unc}$  can be obtained as:  
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 23

$$24 \quad X_{unc} = X_{det} + Err C_E X_{det} \quad (1)$$

25  
 26 where  $Err$  is an error function, which depends on the operational conditions, and,  $C_E$  is a sampling  
 27 hyper-surface mapping the set of operational conditions into the interval  $[-1, 1]$ . In the general case,  
 28 the shape  $C_E$  depends on a set of shaping parameters. Later on in this paper  $C_E$  will be defined  
 29 as an interpolating surface whose shape depends on the value of each interpolating node. For each  
 30 nominal solution of the inner loop,  $N_s$  different  $C_E$  surfaces are generated by randomly choosing,  
 31 with uniform distribution in the interval  $[-1, 1]$ ,  $N_s$  sets of values for the shaping parameters and  
 32 then  $N_s$  new trajectories are propagated using the perturbed values  $X_{unc}$  obtained from Eq. (1). For  
 33 each point along one of the  $N_s$  trajectories, the product  $Err C_E$  returns a deterministic value in the  
 34 interval  $[-Err, +Err]$ . For each of the  $N_s$  perturbed trajectories a new value of the performance  
 35 index is computed and used to calculate the mean  $E_J$  and the variance  $\sigma_J^2$ . The two statistical  
 36 moments for each performance index are then returned to the outer loop.  
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### 51 **C. Multi-fidelity Evolution Control**

52 The inner loop needs to evaluate the dynamic models several times both during the solution of  
 53 the optimal control problem and during the computation of the statistical moments.  
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58 In order to make the process computationally affordable, the inner loop interrogates a data-fitted  
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1 surrogate model instead of the true system model. The data-fitted **approximated model** is trained  
2 and updated during the design process by means of a multi-fidelity evolution control technique.  
3

4  
5 The basic idea underneath evolution control (EC) is to manage, throughout the optimization  
6 process, the evaluation of both the true and the surrogated model in a way that reduces the total  
7 computational time but preserving the correctness of the final solution. Due to the necessity to  
8 limit the number of training samples, it is very difficult to construct an initial **approximated model**  
9 that is globally correct. Most likely, the approximation will bring the optimization algorithm to  
10 false optima, i.e. solutions that are optimal for the **approximated model** but are suboptimal for  
11 the true model. Model management or evolution control techniques address this problem and avoid  
12 converging to false optima.  
13

14  
15 Jin et al.[14] in their paper propose two different approaches for the evolution control of the  
16 model: a) individual-based control and b) generation-based control. In the first approach,  $n_v$  indi-  
17 viduals in the current population are chosen and evaluated with the true model at each generation.  
18 In the latter, the whole population is evaluated with the real model, every  $n_{gcyc}$  generations, for  $n_{gv}$   
19 generations, where  $n_{gv} < n_{gcyc}$ . The individuals evaluated with the true model are then introduced  
20 into a data-set that is used to locally improve the surrogated model.  
21

22  
23 The method adopted for this work is a mix of both evolution control strategies. Fig. 1 summa-  
24 rizes the whole optimization process. The MOO algorithm MOPED (on the left of Fig. 1, blocks **0a**  
25 **to 4**) is integrated with an external procedure that monitors the status of the **approximated models**  
26 (box 6 in Fig. 1). At the end of each iteration (generation), the external procedure checks if an  
27 updated version of the approximated model is ready and available. If the approximated model is  
28 updated, then all the individuals in the current population are re-evaluated and re-classified with  
29 the updated model, before the Parzen distribution is updated and sampled. If the approximated  
30 model is not updated, because, for example, a CFD computation is still running, and the difference  
31 between the generation of the previous update and the current generation is  $n_{gcyc}$ , then MOPED  
32 pauses and waits for the new update.  
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The external procedure also manages, in an asynchronous way, the training and updating of the  
**approximated models**. Block 5 in Fig. 1 is detailed in Algorithm 1.

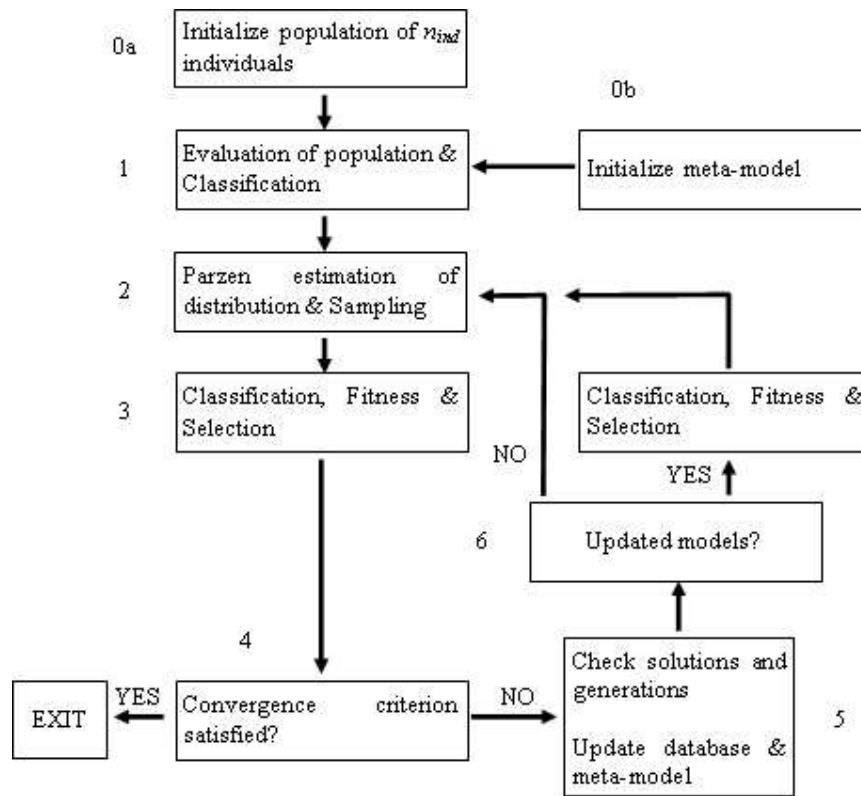


Fig. 1 MOPED with evolution control modification and independent approximated model handler.

The procedure is supplied with a list of system models ordered by increasing level of fidelity and a scheduling report detailing how and when the different models should be used. Every  $n_{gl}$  generations of the global solver, the fidelity level of the model,  $L_C$ , is increased, till the maximum level,  $L_{Cmax}$ , is reached. Then it extracts for each optimal trajectory the matrix  $\mathbf{S}_{opt} = [Shape\ parameters, Operational\ parameters]$ , containing the geometrical parameters and the operative points along the trajectory. Each row in  $\mathbf{S}_{opt}$  is then compared to the values in  $\mathbf{DB}_{train}$ , which is the matrix of points in the database used to train the data-fitted surrogate. The approximated models are initially trained by using the lower fidelity models (fidelity level  $L_C = 0$ ), then during the evolutionary process new promising points are inserted in the database  $\mathbf{DB}_{train}$  and used to improve the training. Following a predefined schedule, the level of fidelity is progressively increased from one stage of the evolution to another; therefore the training database will contain a mix of values coming from models at different levels of fidelity. The solutions evaluated with the

1 highest fidelity level are inserted in the database if their distance from all the points of the database  
2 evaluated with the same fidelity level,  $d_{sl}$ , is greater than a predefined value. Once the higher fidelity  
3 solutions are in the database, all lower fidelity solutions falling at a distance  $d_{ll} < d_{min, ll}$  from the  
4 higher fidelity solutions are discarded. The surrogate structure is then updated with the new data.  
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10 The data obtained by the higher fidelity models progressively become the main source of updates  
11 for the surrogate model till, near the end of the optimization process, the data obtained by the lower  
12 fidelity models have no influence in the optimal regions of the search space.  
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15  
16 Note that on small scale problems with smooth and differentiable functions a grid search, coupled  
17 with a gradient method, could potentially replace the evolutionary optimization process. The grid  
18 search, however, would scale exponentially with the number of dimensions and the gradient method  
19 would experience difficulties with noisy and discontinuous functions. The proposed robust MDO  
20 approach, on the other hand, is composed of strategies and algorithms that scale polynomially  
21 with the number of dimensions. In particular, the coupled EC and evolutionary multi-objective  
22 optimization algorithm provides an optimal sampling of the true models, and an update of the  
23 surrogates, only in the region of interest, thus avoiding the exponential increase of the number of  
24 samples required for a global update. Furthermore, the use of a bi-level approach is deemed to be  
25 more efficient than the use of a single level approach. In a single level approach the evolutionary  
26 process should handle a large number of parameters and constraints and the gradient should search  
27 a plethora of inhomogeneous parameters. In the proposed bi-level approach, instead, each technique  
28 is applied to the solution of the problem they can handle best.  
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#### 44 **D. Surrogate Model**

45  
46 General principles of evolution control do not depend on any specific approximation technique  
47 but, of course, the approximation approach strongly affects the outcome of any EC strategy. For  
48 the particular application presented in this paper, the approximated model should be able to filter  
49 the noise coming from the CFD models and correctly generalize the dynamic response for a broad  
50 range of shapes and operational conditions. Response surfaces and artificial neural networks were  
51 considered[5, 6], but ANNs have been preferred, because they can be used with minimal knowledge  
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60

---

**Algorithm 1** Evolution control algorithm
 

---

```

1
2
3 1: if mod( $i_g, n_{gl}$ )=0 &  $L_C < L_{Cmax}$  then
4
5 2:    $L_C = L_C + 1$ 
6
7 3: end if
8
9 4: Extract  $n_o$  operative points from each individual of the current population ; Put extracted operative
10 points and shapes into  $S_{opt}$ 
11
12 5: Initialize  $DB_{comp}=\emptyset$  ; Find maximum fidelity level of data in the database  $DB_{train, LCDBmax}$ 
13
14 6: if  $L_C = LCDBmax$  then
15
16 7:   Put elements of  $DB_{train}$  with level =  $L_C$  into database  $DB_{LC}$  and elements of  $DB_{train}$  with level
17   <  $L_C$  into database  $DB_{LL}$ 
18
19 8: end if
20
21 9: if  $L_C > LCDBmax$  then
22
23 10:   Initialize  $DB_{LC} = \emptyset$  ; Put all elements of  $DB_{train}$  into  $DB_{LL}$ 
24
25 11: end if
26
27 12: Initialize  $i_c = 0$ ;  $i_{pop} = 0$ ;  $DB_{LCp} = DB_{LC}$ 
28
29 13: while  $i_c < n_t$  &  $i_{pop} < size(S_{opt}, 1)$  do
30
31 14:    $i_{pop}=i_{pop}+1$  ; compute  $d_{sl,i}$  [min distance of  $S_{opt}(i_{pop}, :)$  from  $DB_{LCp}$ ]
32
33 15:   if  $d_{sl,i} > d_{min,sl}$  then
34
35 16:     Put  $S_{opt}(i_{pop}, :)$  into  $DB_{LCp}$  ; Put  $S_{opt}(i_{pop}, :)$  into  $DB_{Lcomp}$  ;  $i_c = i_c + 1$ 
36
37 17:   end if
38
39 18: end while
40
41 19: if  $L_C > 0$  then
42
43 20:   for  $i=1:size(DB_{LL}, 1)$  do
44
45 21:     compute  $d_{ll,i}$  [min distance of  $DB_{LL}(i, :)$  from  $DB_{LCp}$ ]
46
47 22:     if  $d_{ll,i} > d_{min,ll}$  then
48
49 23:       Put  $DB_{LL}(i, :)$  into  $DB_{LC}$ 
50
51 24:     end if
52
53 25:   end for
54
55 26: end if
56
57 27: Compute function values for operative points and shapes in  $DB_{Lcomp}$  and insert obtained values in
58  $DB_{train}$ 
59
60

```

---

about the structure of the function, which should be approximated.

When dealing with ANNs, usually radial basis ANNs are preferred due to the modest computational effort required to train them [5, 6], but here the generic Multi Layer Perceptron (MLP) ANN with one hidden layer was used, due to the expected better generalization in regions *far* for the training data.

The training process is based on a Bayesian regularization back-propagation[38], which limits any overfitting problem, and the idea is that the computational costs of initial training and online update are negligible if compared to the calls to the high-fidelity model.

### III. Rocket Ascent Case Study

The robust multi-disciplinary design approach, presented in the previous sections, was initially tested on a simple low-dimensional problem with easily verifiable solutions: the design of a vertically ascending single-stage rocket vehicle. The objective of this problem is to find the shape parameters that maximize the expected value of the final altitude and minimize the effect of uncertainties on the aerodynamic forces, given a fixed amount of propellant. The vehicle is considered to be a point mass, whose motion is governed by the following set of dynamic equations ([39], Sec. 8.3):

$$\begin{aligned} \dot{h} &= v \\ \dot{v} &= \frac{T - D}{m} - g \\ \dot{m} &= -\frac{T}{g_0 I_{sp}} \end{aligned} \quad (2)$$

where  $h$  is the altitude,  $v$  is the velocity,  $m$  is the mass of the vehicle,  $T$  is the thrust,  $D$  the atmospheric drag,  $g_0$  and  $g$  the gravity accelerations at sea level and for  $h > 0$ , respectively, and  $I_{sp}$  is the specific impulse. The drag  $D$  is given by the classic expression  $D = 1/2\rho C_D S v^2$ , where  $\rho$  is the atmospheric density,  $C_D$  is a shape drag coefficient, and  $S$  is a characteristic surface (usually the cross section). The simplifying hypothesis adopted here is that the product  $C_D S$  is a known, non-linear function of two shape parameters,  $\mathbf{d} = [d_1, d_2]$ . The product  $C_D S$  is assumed to be uncertain, therefore using Eq. (1), the value of  $C_D S$  are perturbed as follows:

$$(C_D S)_{unc} = (C_D S)_{det} + Err C_E (C_D S)_{det} \quad (3)$$

where  $Err$  is a function of speed and altitude and modeled here as a linear 2D surface, with values varying from 0.1, when speed and altitude are both = 0, to 0.8, when the speed is = 2000 m/s and



1 the altitude is  $2 \cdot 10^5$  m:  
 2  
 3

$$4 \quad Err = 0.000175 v + 1.75 \cdot 10^{-6} h + 0.1 \quad (4)$$

5  
 6 while  $C_E$  is a multidimensional polynomial spline, interpolating a regular grid of nodes. If  $Sp_h(h, \mathbf{s}_h)$   
 7 and  $Sp_v(v, \mathbf{s}_v)$  are the splines for altitude and velocity, respectively, then  $C_E$  is defined as:  
 8  
 9

$$10 \quad C_E = Sp_h(h, \mathbf{s}_h) Sp_v(v, \mathbf{s}_v) \quad (5)$$

11  
 12 where  $\mathbf{s}_h$  and  $\mathbf{s}_v$  are the values of the splines at the interpolating nodes.  $N_{R,h}$  and  $N_{R,v}$  equispaced  
 13 interpolating nodes are taken respectively along the  $h$  and  $v$  axis. Then  $N_s$  values of  $\mathbf{s}_h$  and  $\mathbf{s}_v$  are  
 14 randomly sampled in the interval  $[-1, +1]$  leading to  $N_s$  different shapes of  $C_E$ . For each shape a  
 15 **varied** trajectory is propagated.  
 16  
 17

18 Thus, given a nominal trajectory with an optimal control profile  $T^*$ ,  $N_s$  perturbed trajectories  
 19 are propagated, and the mean  $E_h$  and variance  $\sigma_h^2$  of  $\max_t h$  are computed. The mean and variance  
 20 are then returned as performance indexes to the external loop, which optimizes the shape. The  
 21 external optimization problem is formulated as follows:  
 22  
 23

$$24 \quad \min_{\mathbf{d} \in D} [-E_h, \sigma_h^2] \quad (6)$$

25 The design space for this problem is defined by following bounds on the design parameters:  $d_1 \in$   
 26  $[0, 2\pi]$ , and  $d_2 \in [0, 20]$ .  
 27  
 28

29 For the trajectory optimization problem, the control variable is the thrust  $T$ , and the control  
 30 law is computed as the result of the optimal control problem:  
 31  
 32

$$33 \quad \max_T h \quad (7)$$

34 subject to dynamic Eqs. (2) and terminal conditions:  
 35  
 36

$$37 \quad h(t=0) = h_0$$

$$38 \quad v(t=0) = v_0 \quad (8)$$

$$39 \quad m(t=0) = m_0$$

$$40 \quad m(t=t_f) = m_{min} \quad (9)$$

The solution of this optimal control problem is easily verifiable and can be found in ([39], Sec. 8.3). Here, the problem is transcribed with a Gauss pseudo-spectral method[37] to be consistent with the proposed MDO approach. The trajectory is decomposed in  $N$  elements, each of which has  $n_p$  collocation points. After transcription, the optimal control problem defined by Eq. (7), Eqs. (2) and Eqs. (8, 9) becomes the following nonlinear programming problem:

$$\max_{T_s} h \quad (10)$$

subject to the nonlinear algebraic constraints:

$$C(h_s, v_s, m_s, t_s) = 0 \quad (11)$$

with terminal constraints in Eqs. (8, 9), where  $h_s, v_s, m_s, T_s, t_s$  are the discrete values of states, control, and time values at the nodes of the transcription scheme.

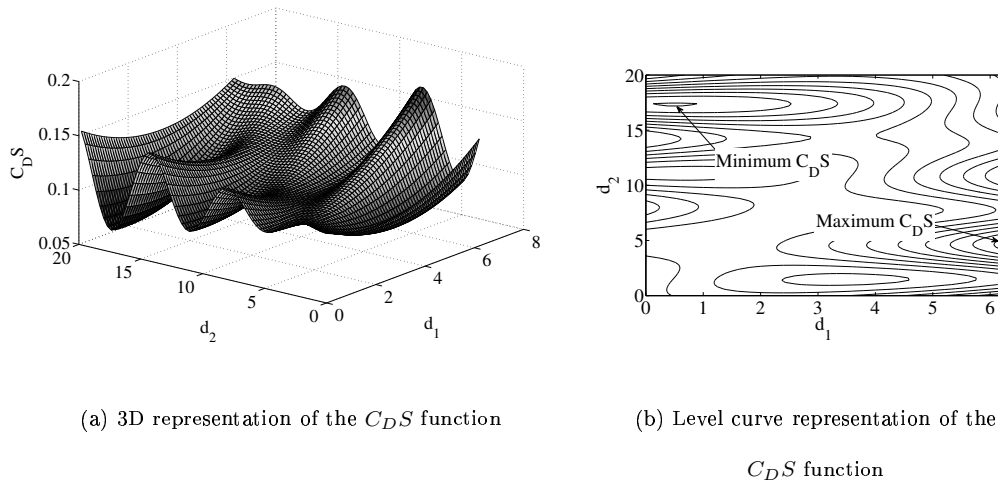
Trajectories are discretized with 7 elements, and 7 nodes for each element. The bounds on the variables of the trajectory optimization are: total time  $T_{tot} \in [0, 1000]$  [s], thrust  $T \in [0, 9 \cdot 10^4]$  [N], altitude  $h \in [0, 2.0 \cdot 10^5]$  [m], velocity  $v \in [0, 2000]$  [m/s], mass  $m \in [150, 700]$  [kg]. The initial conditions (8) are  $x_0 = [h_0, v_0, m_0]^T = [0, 0, 700]^T$ , while the constraints on the final conditions (9) are  $m_f = 150$  kg. The initial guess for the control law of every individual is a constant thrust profile with  $T = 1$  kN. The specific impulse is  $I_{sp} = 100$  s, while the deterministic values of  $C_{DS}$  are given by the function:

$$C_{DS} = \left( \frac{(d_2 - 3d_1) \sin(d_2) + (d_1 - 2)^2 + 11}{60} + 0.5 \right) 0.15 \quad (12)$$

Fig. 2(a) shows a three dimensional view of the  $C_{DS}$  as a function of the design parameters, while Fig. 2(b) shows the level curves with the maximum and minimum values for  $C_{DS}$ .

#### A. Rocket Ascent Results

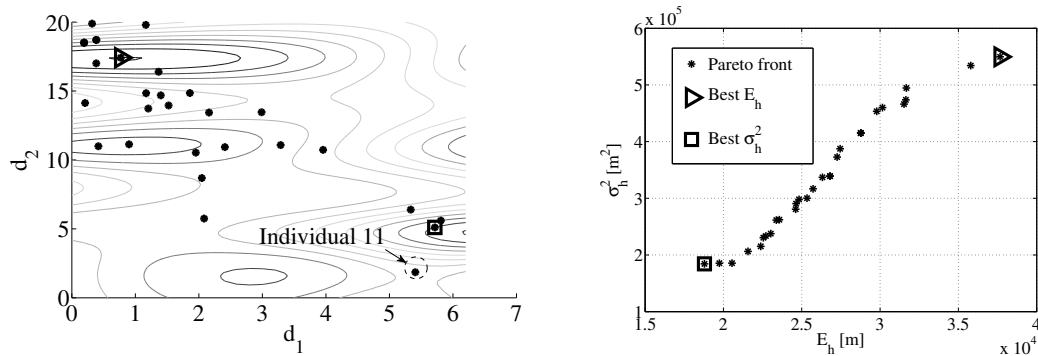
MOPED process was run for 50 generations with a population of 30 individuals. For this simple test case, only one fidelity level has been considered and the characteristic parameters of the evolution control process were set as follows:  $n_t = 10, n_o = 1$  (the approximated coefficient does not



**Fig. 2** Deterministic values of  $C_{DS}$  shape coefficient as function of design variables  $d_1$  and  $d_2$

depend on the operational conditions, but only on the shape),  $n_{gl} = Inf$  (with only 1 fidelity level);  $n_{gcyt} = 3$ ;  $d_{min,sl} = 0.1$  (all the inputs are normalized to  $[-1, 1]$ );  $d_{min,ul} = 0.2$ . Since the coefficient  $C_{DS}$  is considered only function of the shape, the generic row  $j$  of the matrix database used to train the ANN is  $\mathbf{DB}_{train,j} = [d_1, d_2]$ . The initial approximated model of  $C_{DS}$  was built with 30 samples, selected with a randomized Latin Hypercube. For such a small database, the learning procedure for the initialization of the approximating ANN was not able to properly converge to a good representation of the  $C_{DS}$  function. The evolution control added other 29 solutions (for a total of 59 model computations) and brought the approximated model to converge to the correct values of the  $C_{DS}$  function in the regions of interest, enabling the optimizer to converge to the expected optimal solutions. Fig. 3(a) shows the level curves of the approximated model at the last iteration of the EC together with the final population. Fig. 3(b) shows the final approximation to the Pareto front. The structure of the approximated  $C_{DS}$  function in Fig. 3(a) is almost identical to the true one in Fig. 2(b) and the maximum and minimum values are correctly identified. Individual  $Best \sigma_h^2$  is not perfectly located at the position of the maximum of the function, but the achieved approximation is sufficient to make a correct decision on the design solution with best performance and with highest robustness. The relative error between the ANN approximated model and the true  $C_{DS}$  function at the locations of the individuals of the final population is shown in Fig. 4: if the initial ANN is considered, the relative error has an average value of more than 10%, on the other

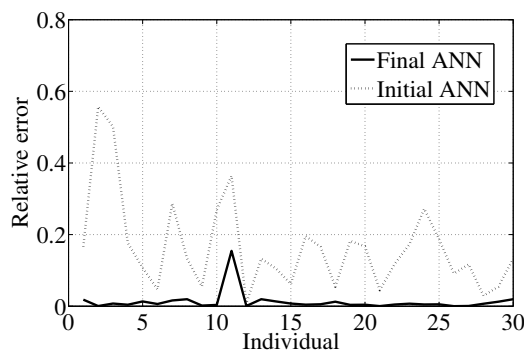
hand, when the final ANN is used, the error is almost always smaller than 3%, which is the required convergence level for the ANN learning process. The higher error displayed by individual 11 means that the value corresponds to a newly founded solution and that the ANN had not time to properly learn the related portion of the search space.



(a) Final population on level curve representation  
for  $C_{DS}$  approximated by ANN

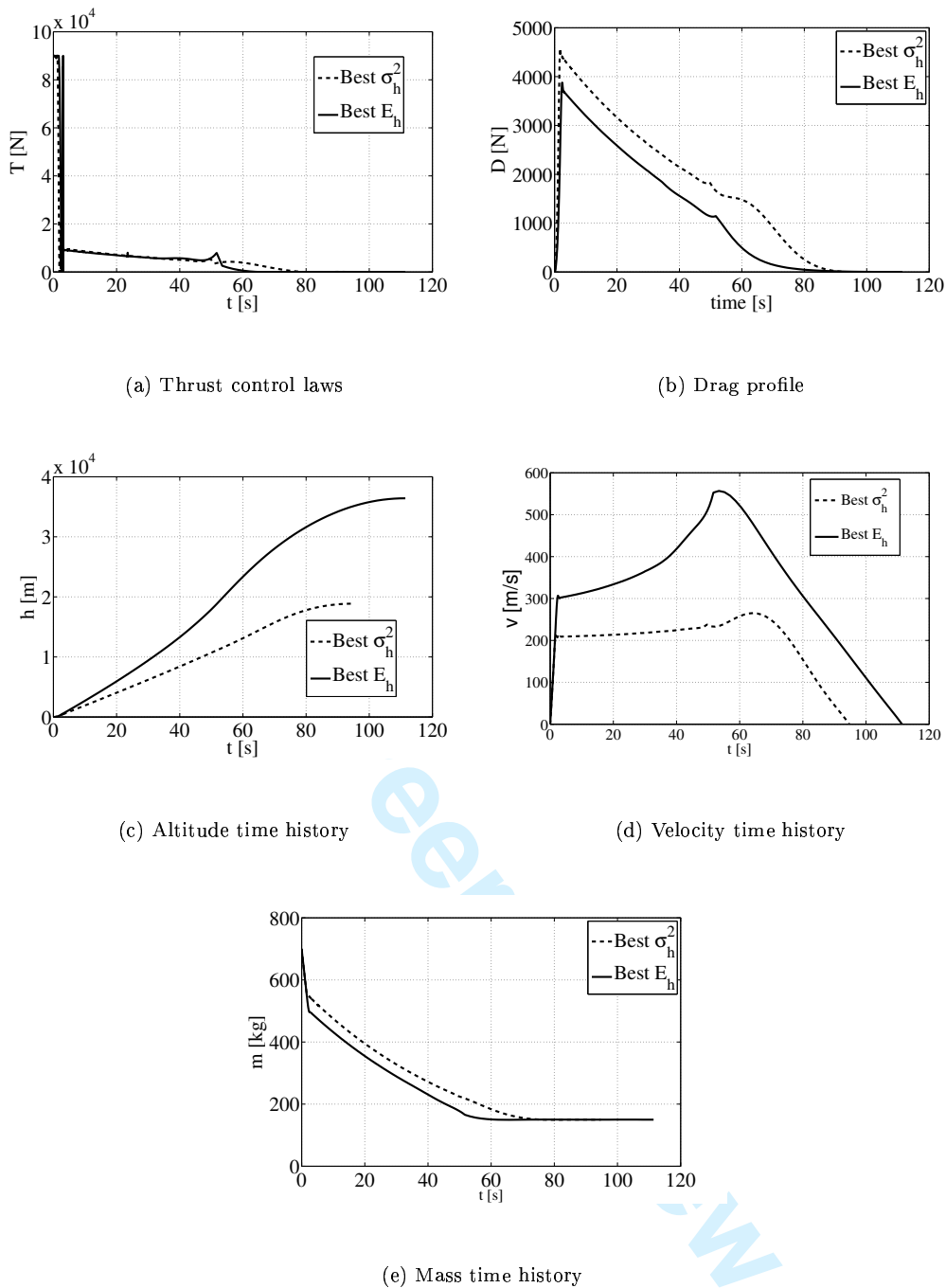
(b) Pareto front approximation

**Fig. 3 Results for the robust design optimization of rocket ascent case**



**Fig. 4 Relative error between true  $C_{DS}$  function and ANN approximation for individual of final population**

Although the deterministic control laws for the **extreme best solutions** (i.e. the one maximizing the expected values of the altitude  $E_h$  and the other minimizing the relative variance  $\sigma_h^2$ ) appear similar as shown in Fig. 5(a), the differences in thrust profile and drag (Fig. 5(b)) allow them to follow different paths in the state space. Note that the obtained control laws well approximate the



**Fig. 5 Results for the robust design optimization of rocket ascent case: best solutions**

analytical solution that can be found in [39]. The solution with smaller  $C_D S$  coefficient can climb higher and faster (see Figs. 5(c) and 5(d)). Due to a higher level of thrust during first few seconds of the climb, the solution with smaller  $C_D S$  also has a faster consumption of mass, making it lighter for most of the trajectory (Fig. 5(e)). Since the uncertainty associated to the  $C_D S$  increases with

1 altitude and velocity (see Eq. (4)), then the best performing individual is also the least robust one.  
2  
3 Vice versa, the least performing solution is less affected by uncertainties on the values of the shape  
4 parameters and thus is the most robust one.  
5  
6  
7  
8

#### 9 **IV. Unmanned Space Re-entry Vehicle Case Study**

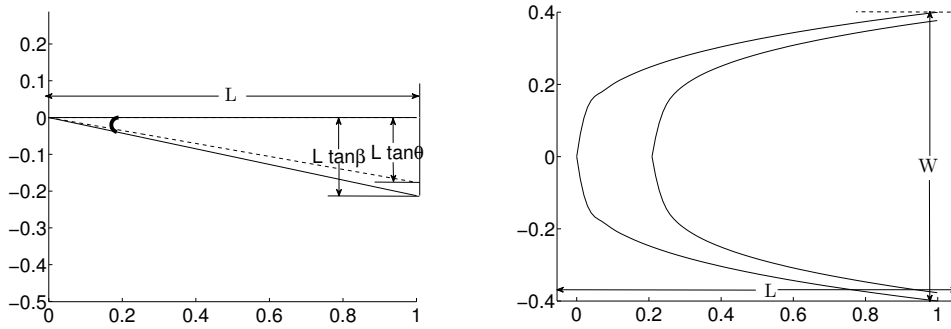
10 The goal of this case study is to design an Unmanned Space Vehicle (USV), for re-entry op-  
11 erations, that minimizes the maximum heat flux along the descent trajectory and maximizes the  
12 orthodromic distance measured on the Earth spherical surface. In the following, the system models  
13 for the design of the USV will be introduced before presenting the application of the proposed robust  
14 MDO process to this design case.  
15  
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21

##### 22 **A. Geometry and Shape Model**

23 The vehicle is a modified version of a waverider with rounded edges. The waverider baseline  
24 geometry is defined by three two-dimensional power-law equations[23]. The planform and the upper  
25 surfaces of the vehicle is parameterized by the length  $l$ , the width,  $w$ , a power law exponent  $n$ ,  
26 the vehicle center line wedge angle,  $\theta$ , and  $\beta$ , which is the oblique shockwave inclination angle[23].  
27 An example can be found in Fig. 6, where the original waverider sharp-edge shape is modified  
28 to introduce a rounded edge with radius of curvature,  $R_n > 0$ . For the example in Fig. 6 the  
29 parameters defining the shape are:  $l = 1.0m$ ,  $w = 0.8m$ ,  $n = 0.3$ ,  $\theta = 10deg$ ,  $\beta = 12deg$ ,  $R_n = 0.02m$ ,  
30 and more details can be found in the cited reference.  
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42

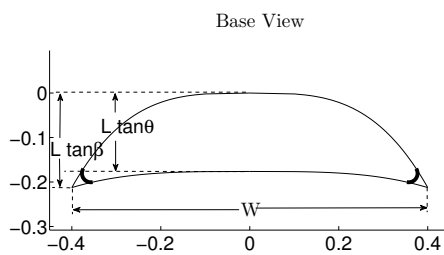
##### 43 **B. Aerodynamic Models**

44 Two different models are used to predict the aerodynamic characteristics of the vehicle. The  
45 former one is a simplified analytical model, which is here applied to the actual rounded-edge vehicle,  
46 although it was originally developed to predict the aerodynamics of the original sharp-edge shape  
47 of the waverider configuration[23]. It gives a very first approximation of the performance at the  
48 early stage of the design process. The latter one is a full high-fidelity CFD model based on a finite  
49 volume integration of Reynolds Averaged Navier-Stokes equations (RANS).  
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60



(a) Side View

(b) Top View



(c) Base View

**Fig. 6 Example of Vehicle Geometry:**  $l = 1.0m$ ,  $w = 0.8m$ ,  $n = 0.3$ ,  $\theta = 10deg$ ,  $\beta = 12deg$ ,  $R_n = 0.02m$

### 1. Analytic Hypersonic Model

The analytic model gives the lift  $L$  and wave drag  $D_w$  as functions of the pressure on the upper, lower and base surfaces,  $P_u$ ,  $P_l$  and  $P_b$ , respectively:

$$\begin{cases} L = S_b(P_b - P_l)\sin\alpha + S_p(P_b - P_l)\cos\alpha \\ D_w = S_b(P_l - P_b)\cos\alpha + S_p(P_l - P_u)\sin\alpha \end{cases} \quad (13)$$

where  $S_p$  and  $S_b$  are the planform area and the area of the base, computed as:

$$\begin{cases} S_p = \frac{wl}{n+1} \\ S_b = S_p \tan(\theta) \end{cases} \quad (14)$$

The pressure on the surfaces can be calculated analytically with the oblique shock theory or Prandtl-Meyer expansion theory.[29] The viscous drag  $D_v$  is given in analytical form by using the

**Table 2 Viscous drag constants for wedge-derived waverider**

Constants	Laminar flow	Turbulent flow
$G_1$	$0.664\sqrt{\rho_e^2 v_e^3 \mu^* C^* / \rho^*}$	$0.037 v_e^{1.8} (\rho^*)^{0.8} (\mu^*)^{0.2}$
$G_2$	0.5	0.8
$F_0$	0.99845	0.99758
$F_1$	-0.57529	-0.80941
$F_2$	0.36737	0.54989
$F_3$	-0.11939	-0.18247

reference temperature method[30] as:

$$D_{v,surf} = G_1 w F(n) \left( \frac{l}{\cos \theta_{surf}} \right)^{G_2} \quad (15)$$

with  $F(n) = F_0 + F_1 n + F_2 n^2 + F_3 n^3$ , and  $\theta_{surf}$  is the inclination angle for the considered surface (upper or lower surface). The adopted constants, which depends on the flow conditions, are given in Table 2.

In Table 2, the Chapman-Rubensin parameter  $C^* \approx (T_*/T_e)^{-\frac{1}{3}}$ ,  $\rho^*$ , and  $\mu^*$  are determined by Eckert's empirical relation and Sutherland viscosity law, while  $\rho_e$ ,  $T_e$  and  $v_e$  are the density, the temperature and the flow velocity at edge of boundary layer in post-shock conditions.

The total drag  $D$  is  $= D_w + D_{v,u} + D_{v,l}$ , where  $D_{v,u}$  and  $D_{v,l}$  are the viscous drag of the upper and lower surface, respectively.

As previously anticipated, this simplified model still considers the sharped shape of the wave rider, and does not take into account the modified one, with the introduction of the rounded edges.

## 2. CFD Model

On the other hand, a commercial code (*Numeca*<sup>®</sup>), solving the Reynolds Averaged Navier-Stokes (RANS) equations, is used to obtain what are considered high fidelity solutions in the entire flight envelope and also to compute initial solutions when the analytical model could not be applied (for supersonic flight regimes).

The computational domain is discretized by a multi-block structured mesh made by 13 blocks



with near  $2.4 \cdot 10^6$  total nodes. For each configuration, the mesh is changed and adapted to the current geometry by internal scripting on the basis of design parameters. Since no out of plane flight conditions are considered, only half of the actual domain is discretized and mirror plane conditions are imposed into the longitudinal plane.

Two different settings are implemented and used during the process: a) Laminar for Reynolds number,  $Re$ ,  $< 0.95 \cdot 10^5$ ; and b) Fully turbulent (no transition model is considered) for  $Re > 1.05 \cdot 10^5$ .

For hypersonic conditions the radiative equilibrium temperature at the nose is imposed on the solid boundaries. Real gases database is enhanced on the basis of reported air data for high temperatures[31].

No solutions are computed for  $0.95 \cdot 10^5 \leq Re \leq 1.05 \cdot 10^5$ , in order to have an aerodynamic database as smooth as possible: since there is no transition model between laminar and turbulent flow, then computations into the transition region could be misleading. The data base will be approximated by a smooth ANN system, and then the ANN itself will provide smooth approximations for the transition region.

### C. TPS and Thermal Model

The thermal protection system (TPS) is assumed to be made of Zirconium Diboride ( $ZrB_2$ ) UHTC, which has following properties: density =  $6000 kg/m^3$ , specific heat =  $628 JKg^{-1}K^{-1}$ , conductivity =  $66 Wm^{-1}K^{-1}$ , and emissivity = 0.8.

We bound the angle of attack to a maximum value of  $20 \text{ deg}$ , hence the highest heat flux is expected to be at the USV nose cap. Thus, the whole nose cone is made of UHTC with length  $L_{TPS}$ . The rest of the vehicle is covered with a thin shell with a constant thickness of  $0.003 \text{ m}$ . [32]

For the design process, the convective heat flux is computed in the simplest way, with the analytical formula[33]:

$$\dot{q}_{conv} = K_e \sqrt{\frac{\rho_\infty}{R_n}} V_\infty^3 \quad (16)$$

where  $K_e = 1.742 \cdot 10^{-4}$  (for the heat flux  $\dot{q}_{conv}$  in  $W/m^2$ ). As previously stated, the minimization of maximum heat flux means that less protecting material could be used, with consequent reduction

1 of building costs.  
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#### 5 D. Mass Model 6

7 The total mass of the vehicle is made of the structural mass  $m_{st}$ , the mass of the TPS  $m_{TPS}$ ,  
8 and the mass of the payload (avionics and power system)  $m_{pl}$ .  
9  
10

$$11 \quad m = m_{TPS} + m_{st} + m_{pl} \quad (17)$$

12 The mass of the payload is here assumed to be 40% of the structural mass, therefore  $m_{pl} =$   
13  $0.4 m_{st}$ . The mass of the TPS is made of the mass of the nose  $m_{nose} = \rho_{TPS} V_n$  plus the mass of  
14 the thin skin covering the rest of the vehicle  $m_{skin}$ , where  $V_n$  is the volume of the nose and  $\rho_{TPS}$   
15 the density of the TPS material. The mass of the TPS skin covering the vehicle, except the nose,  
16 is:  
17  
18  
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24

$$25 \quad m_{skin} = \rho_{TPS} S_{TPS} d_{TPS} \quad (18)$$

26 where  $d_{TPS}$  is the thickness of the TPS, and  $S_{TPS}$  is surface area except that of the nose, which  
27 can be approximated by  $S_{TPS} = 2S_{pE} + S_{bE} - S_n$  ( $S_{pE}$  and  $S_{bE}$  are the total planform surface and  
28 the area of the rear part of the rounded edge waverider, respectively, and  $S_n$  is the surface of the  
29 TPS nose).  
30  
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36

37 The structure of the vehicle is supposed to be made of titanium, with a density of  $4400 \text{ kg/m}^3$ .  
38 The structural mass  $m_{st}$ , can be obtained from:  
39  
40

$$41 \quad m_{st} = \rho_{body} (2S_{pE} + S_{bE}) d_{body} \quad (19)$$

42 where, in this case,  $d_{body} = 0.005 \text{ m}$  is the thickness of the structure of the vehicle, seen as a shell.  
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### E. Dynamic Equations

The vehicle is considered to be a point mass, whose motion is governed by the following set of dynamic equations[34]:

$$\begin{aligned}
 \dot{r} &= v \sin \theta_p \\
 \dot{\lambda} &= \frac{v \cos \theta_p \cos \xi}{r \cos \phi} \\
 \dot{\phi} &= \frac{v \cos \theta_p \sin \xi}{r} \\
 \dot{v} &= -\frac{D(\alpha)}{m} - g \sin \theta_p \\
 \dot{\theta}_p &= \frac{L(\alpha)}{mv} \cos \gamma_v - \left( \frac{g}{v} - \frac{v}{r} \right) \cos \theta_p \\
 \dot{\xi} &= \frac{L(\alpha)}{mv \cos \theta} \sin \gamma_v - \frac{v}{r} \cos \theta_p \cos \xi \tan \phi
 \end{aligned} \tag{20}$$

where  $r$  is the norm of the position vector with respect to the center of the planet,  $\lambda$  is the longitude,  $\phi$  the latitude,  $v$  the magnitude of the velocity,  $\theta_p$  is the flight path angle,  $\xi$  is the heading angle (azimuth of the velocity). No out of plane maneuvers are considered, thus the bank angle  $\gamma_v$  is kept equal to zero during the whole trajectory.

### F. USV Optimization Set-Up

As for the previous test case, the outer multi-objective evolutionary procedure manages the parameters defining the shape of the vehicle and considers as objectives and constraints the mean and variance of the values of performance indexes of inner optimal control processes. On the other hand, now the optimal control problem is solved twice, first to minimize the maximum heat flux,  $\max_t \dot{q}$ , and then to maximize the orthodromic distance,  $Dist$ . Mean and variance of these two performance indexes are then computed following the procedure described in Sec. II B. The optimal control sub-problem, considering deterministic models, gives deterministic values of the performance indexes,  $\max_t \dot{q}$  and  $Dist$ , as a function of the optimal  $\alpha$  profile. However, a number of elements can be considered uncertain, such as the aerodynamic forces and the mass. Therefore, one can associate to the nominal value of lift  $L_{det}$  and drag  $D_{det}$ , the uncertain quantities:

$$\begin{aligned}
 L_{unc} &= L_{det} + Err(\alpha, v, h) C_E(\alpha, v, h) L_{det} \\
 D_{unc} &= D_{det} + Err(\alpha, v, h) C_E(\alpha, v, h) D_{det}
 \end{aligned} \tag{21}$$

where the error function  $Err$  depends on the angle of attack, the speed and the altitude  $h$ , and the

sampling function  $C_E$  maps a triplet of values of angles of attack, speed and altitude into the interval  $[-1, 1]$ . As it was done in Sec. III, the uncertainties in the aerodynamic model are assumed to be increasing with angle of attack, speed, and altitude, thus  $Err$  is modeled as a linear 3D surface, with values that vary from 0.2, when angle of attack, speed and altitude are = 0, to 0.8, when the incidence is = 20 deg, the speed is = 8000 m/s and the altitude is  $10^5$  m:

$$Err = 0.01 \alpha + 2.5 \cdot 10^{-5} v + 2 \cdot 10^{-6} h + 0.2 \quad (22)$$

The  $C_E$  functions is again built as a combination of splines with equidistant nodes. In this case their shape is given by:

$$C_E = Sp_\alpha(\alpha, \mathbf{s}_\alpha) Sp_h(h, \mathbf{s}_h) Sp_v(v, \mathbf{s}_v) \quad (23)$$

where  $Sp_\alpha(\alpha, \mathbf{s}_\alpha)$ ,  $Sp_h(h, \mathbf{s}_h)$  and  $Sp_v(v, \mathbf{s}_v)$  are the splines for angle of attack, altitude and velocity, respectively. The total mass is considered uncertain as well, and it is assumed that  $m$  can uniformly vary in the range  $\pm 0.1$  of the reference value. Then, given a nominal trajectory with an optimal control profile  $\alpha^*$ ,  $N_s$  varied trajectories are propagated, and the mean and the variance of  $\max_t \dot{q}$  and  $Dist$  are computed from the  $N_s$  values of  $\max_t \dot{q}$  and  $Dist$ , one for each varied trajectory.

### G. Shape Optimization

If one calls  $E_{q,1}$  and  $\sigma_{q,1}^2$  the mean value and variance of maximum heat flux, and  $E_{D,1}$  and  $\sigma_{D,1}^2$  the mean value and variance of the orthodromic distance after the first control optimization (considering the maximum heat flux as objective to minimize), and  $E_{q,2}$ ,  $E_{D,2}$ ,  $\sigma_{q,2}^2$ ,  $\sigma_{D,2}^2$  the corresponding values after the second control optimization (considering the orthodromic distance as objective to maximize), then the external process is set as:

$$\min_{\mathbf{d} \in D} [E_{q,1}, -E_{D,2}, \sigma_{q,1}^2 + \sigma_{q,2}^2] \quad (24)$$

subject to the following constraints:

$$\begin{aligned} E_{q,1} &\leq \bar{E}_q \\ (\sigma_{q,1}^2 + \sigma_{q,2}^2) &\leq \bar{\sigma}_q^2 \end{aligned} \quad (25)$$

1 while the design vector  $\mathbf{d}$  is defined as  $\mathbf{d} = [l, w, n, \theta, R_n, \Delta\dot{q}]$  (the meaning of  $\Delta\dot{q}$  is explained in  
 2 the next section). Note that, as previously mentioned, the external optimizer needs to work on  
 3 objectives representing the best performance of the shape, for both different cases.  
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7 The design space for problem(24,25) is defined by the following bounds on the design parameters:  
 8 the nominal length  $l \in [2.9, 4.2][m]$ , the nominal width  $w \in [1.0, 2.0][m]$ , the exponent  $n \in [0.2, 0.7]$ ,  
 9 the angle  $\theta \in [7, 11][deg]$ , the radius of the nose  $R_n \in [0.01, 0.04][m]$ , the constraint on the maximum  
 10 heat flux for the second control law optimization  $\Delta\dot{q} \in [20, 70][W/cm^2]$ . The angle  $\beta$ , as defined in  
 11 Sec. IV A, is kept fixed to  $12deg$ . Moreover, here  $L_{TPS}$  is not considered as a design parameter,  
 12 and its value is set as 10% of the effective length of the vehicle. The constraints on the external  
 13 process are:  $\bar{E}_q = 180$  and  $\bar{\sigma}_q^2 = 1000$ .  
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## 24 H. Trajectory Optimization

25 For the trajectory optimization problem, the control variable is the angle of attack  $\alpha$ . Two  
 26 control laws are computed as the result of the two optimal control subproblems:  
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$$33 \min_{\alpha} \left( \max_t \dot{q} \right) \text{ and } \max_{\alpha} Dist \quad (26)$$

34 both subject to dynamic equations (20) and terminal conditions:  
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$$37 \begin{aligned} 38 r(t=0) &= r_0 \\ 39 \lambda(t=0) &= \lambda_0 \\ 40 \phi(t=0) &= \phi_0 \\ 41 v(t=0) &= v_0 \\ 42 \theta_p(t=0) &= \theta_0 \\ 43 \xi(t=0) &= \xi_0 \end{aligned} \quad (27)$$

$$51 \begin{aligned} 52 r(t=t_f) &\leq r_f \\ 53 r(t=t_f) &\geq r_{min} \\ 54 v(t=t_f) &\leq v_f \\ 55 v(t=t_f) &\geq v_{min} \end{aligned} \quad (28)$$

As for rocket ascent case, these problems are transcribed with a Gauss pseudo-spectral method, and after transcription, the optimal control problems defined by Eqs. (20) and (26, 27, 28) become the following general nonlinear programming problem:

$$\min_{\alpha_s} \left( \max_t \dot{q} \right) \text{ and } \max_{\alpha_s} Dist \quad (29)$$

subject to the nonlinear algebraic constraints:

$$C(r_s, \lambda_s, v_s, \xi_s, \theta_s, \alpha_s, t_s) = 0 \quad (30)$$

and the terminal constraints (27, 28), where  $r_s, \lambda_s, \phi_s, v_s, \xi_s, \theta_s, \alpha_s, t_s$  are the discrete values of the time, states and control values at the nodes of the transcription scheme. In order to obtain feasible solutions that can be integrated, an inequality constraint on the variation of the control law is also imposed: for each node the slope of the control law  $\leq \Delta\alpha$ .

Note that the second optimization problem takes into account an additional constraint on the maximum heat flux, which should be  $\leq (\dot{q}_{max,1} + \Delta\dot{q})$ , where  $\dot{q}_{max,1}$  is the deterministic value of the maximum heat flux resulting from the previous control law optimization, and  $\Delta\dot{q}$  is a variable of the problem, managed by the external evolutionary process. The re-entry time is free and no other terminal conditions are imposed as there is no specific requirement on the landing point.

The trajectories are discretized with 6 elements, and 12 nodes are considered for the first 3 elements (from starting point to half of the trajectory path), where maximum values of the heat flux and major trajectory oscillations are expected, while 5 nodes are considered for the other 3 elements. The bounds on the variables of the trajectory optimization are: total time  $T_{tot} \in [500, 6500]$  [s], angle of attack  $\alpha \in [0, 20]$  [deg], radius  $r \in [6.380 \cdot 10^6, 6.480 \cdot 10^6]$  [m], longitude  $\lambda \in [-180, 21]$  [deg], latitude  $\phi \in [-90, 68]$  [deg], speed  $v \in [100, 10^4]$  [m/s], flight path angle  $\theta_p \in [-80, 10]$  [deg], heading angle  $\xi \in [-225, -90]$  [deg].

The initial conditions (27) are  $x_0 = [R_E + 10^5, 21, 68, 7700, -0.3, -145]^T$ , where  $R_E$  is the mean radius of the Earth, while the constraints on the final conditions are  $r_f = R_E + 50000$  m,  $r_{min} = R_E + 15000$  m,  $v_f = 1100$  m/s,  $v_{min} = 900$  m/s, while the slope of the control law is  $\Delta\alpha = 1/10$  deg/s. The initial guess of the control law for every individual had 18 deg incidence at time  $t = 0$ , linearly decreasing, with decrements  $d\alpha = 1/1000$  deg/s, till the last point of the trajectory

1 obtained by direct integration satisfies the constraints on the required final velocity and altitude.  
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3 Note that, when the optimal control process does not converge, the initial guess of the control law  
4 is considered.  
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## 9 I. USV Results

10 The MOO process was run for 60 generations with a population of 60 individuals. It should be  
11 noted that number of generations and individuals was chosen as a compromise between the expected  
12 amount of function evaluations to converge and the available computational time, and due to the  
13 stochastic nature of the search there could be no proof on the convergence to the global Pareto  
14 set. The initial **approximated models** were built with 1000 samples, selected with a randomized  
15 Latin Hypercube, coming from 920 analytic model computations and additional 80 supersonic CFD  
16 computations to have an extended range of validity, without the need to excessively extrapolate.  
17 The computation of the first database required nearly 1200 hours of computational time, distributed  
18 on a cluster of 20 linux64 processors (near 3 days of effective time). The computations of the CFD  
19 solver were stopped when convergence was obtained on the aerodynamic forces.  
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32 The characteristic parameters of the evolution control process were set as follows:  $n_t = n_o = 20$ ,  
33  $n_{gl} = 10$ , with only 1 switch;  $n_{gyc} = 5$ ;  $d_{min,sl} = 0.3$  (all the inputs are normalized to  $[-1, 1]$ );  
34  $d_{min,ll} = 0.8$ . Since in this case the aerodynamic characteristics of the vehicle are functions of  
35 both shape and operational conditions, the generic row  $j$  of the database used to train the ANN is  
36  $DB_{train,j} = [l, w, n, \theta, R_n, v, h, \alpha]$ .  
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42 At level 0, which is considered up to generation 10, CFD computations were used to verify only  
43 supersonic points, while at level 1, which is considered from generation 11, CFD computations were  
44 used to verify the trajectory points for the whole trajectory.  
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48 During the computation, until generation 50, solutions obtained with the CFD model increased  
49 up to 250, all allocated in the promising region of the search space. On the other hand, analytical  
50 solutions used to build the ANN **approximated models** decreased to nearly 500. From generation  
51 50 to 60 no more new verified values are added to the ANN database.  
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56 The approximation of the Pareto front at the end of the optimization process is shown in Fig. 7.  
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As expected, the front is sparse and irregular, due to the nature of the objective functions, which are extremely noisy because they are the outcome of the Monte-Carlo simulation ( $N_s = 300$ , then each individual required 600 re-propagations). Moreover, the speed and accuracy of convergence of the trajectory optimization loop is quite sensitive to shape parameters and initial conditions. A different population size could improve the quality of the front although a trade-off between exhaustiveness of the search and computational resources is required.

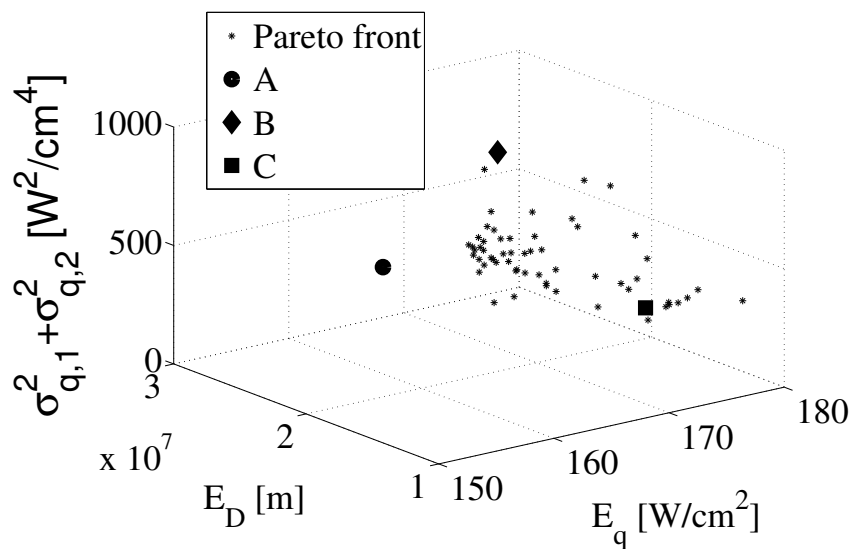


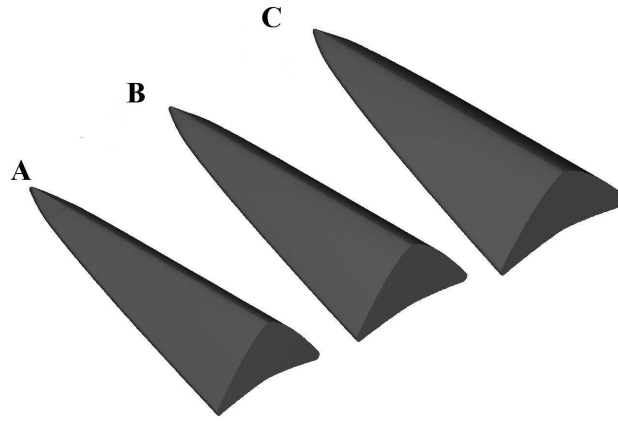
Fig. 7 Approximation of the Pareto front at the end of the optimization process

In Fig. 8 individuals A, B, and C minimize the mean value of the maximum heat flux, maximize the mean value of the distance, and minimize the sum of the variances, respectively. The design parameters for solutions A, B and C are:

1. Solution A:  $l = 2.93998$ ;  $w = 1.19734$ ;  $n = 0.6977$ ;  $\theta = 9.9983$ ;  $R_n = 0.03532$ ;  $\Delta\dot{q} = 41.9184$
2. Solution B:  $l = 2.98669$ ;  $w = 1.28207$ ;  $n = 0.6871$ ;  $\theta = 10.4961$ ;  $R_n = 0.03719$ ;  $\Delta\dot{q} = 62.6298$
3. Solution C:  $l = 3.12287$ ;  $w = 1.49396$ ;  $n = 0.6999$ ;  $\theta = 9.7599$ ;  $R_n = 0.03913$ ;  $\Delta\dot{q} = 33.8409$

The optimization is mainly driven by the heat flux and its variance. In order to limit the heat flux at the nose, the algorithm searches for solutions with a relatively large radius of the nose, and small dimensions. Since the total mass is strictly related to the size, small vehicles are also light weighted and can glide at higher altitude avoiding the very critical part of the flight envelope. A





**Fig. 8 Optimal solutions: individual A minimizes the mean value of the maximum heat flux, individual B maximizes the mean value of the achievable distance, while individual C minimizes the sum of the variances.**

reduction in size, however, could reduce the lift and drag which are required to remain high to glide and dissipate energy over an extended period of time. The result is that short vehicles, that do not sacrifice wingspan to reduce weight, provide acceptable performance and good robustness against uncertainties on the heat flux. All final solutions appear belonging to the same shape niche, but have quite different masses, which are determined by geometrical characteristics. Likely the convergence to the shape niche is due to the stronger influence of the vehicle mass on the optimization of the trajectory.

Fig. 9 shows the nominal, deterministic trajectories of the three selected individuals, when the control law is optimized taking into account the maximum heat flux as performance index. The shapes of the individuals are very similar (meaning similar aerodynamic performance), but the smaller size, meaning a smaller mass as well, allows individual A to follow a higher re-entry path in the critical part of the trajectory, limiting the heat loads (Fig. 10) (the masses of individuals A, B, and C are 191, 212, 248 kg, respectively).

Fig. 11, where the angle of attack is plotted against time, shows small differences in the profile of the angle of attack (less than three degrees in the initial part of the descent). These differences together with the differences in the system design produce a relevant effect on the trajectory and

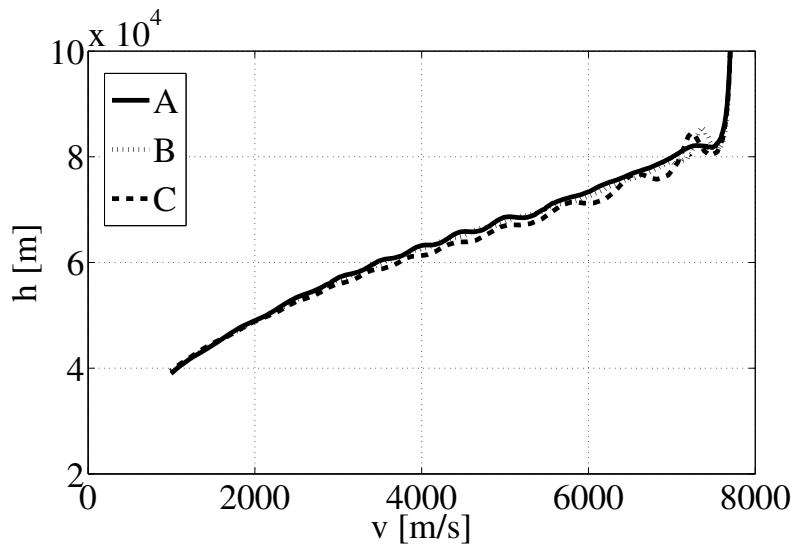


Fig. 9 Trajectories of the selected optimal solutions.

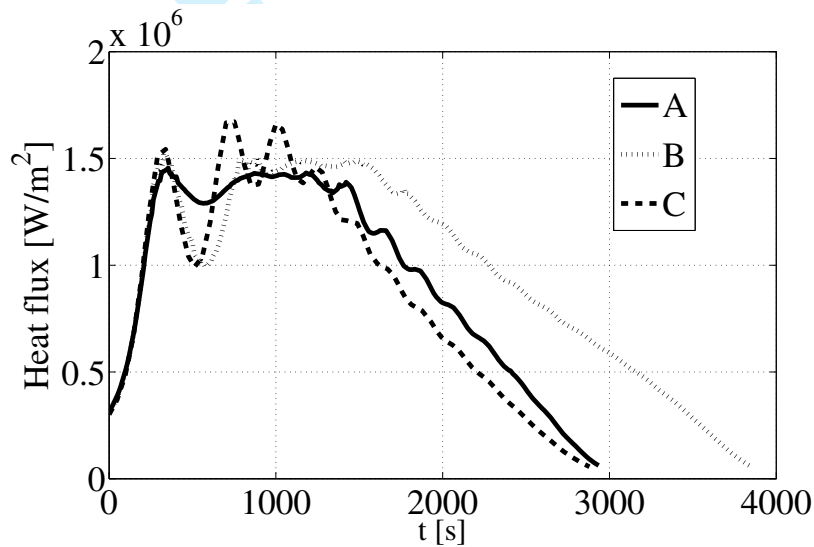


Fig. 10 Convective heat flux of the selected optimal solutions.

on the heat flux.

In order to appreciate the difference between the outcome of the robust optimization process and non-robust solution, a second bi-objective optimization, which only considers the deterministic values of the maximum heat flux and the orthodromic distance as objectives, has been carried out. The deterministic optimization has been carried out with the same bounds, and same MOPED settings of the robust one. The approximated Pareto front is shown in Fig. 12, and compared to

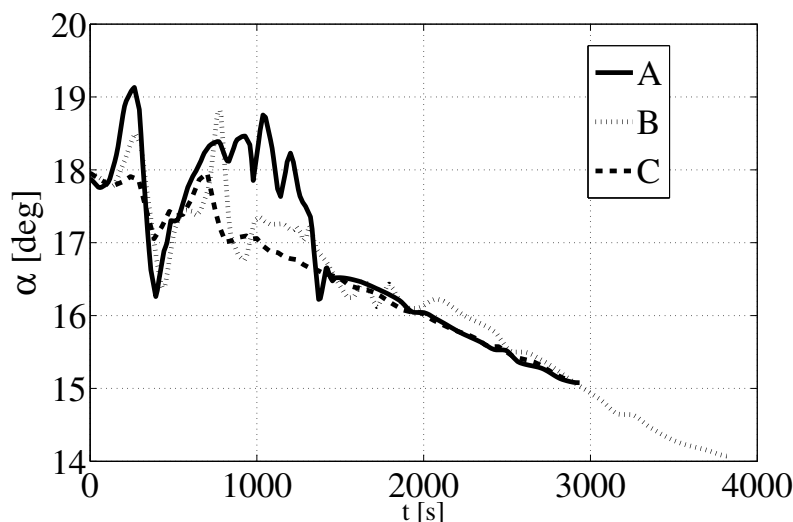


Fig. 11 Angle of attack control law.

a 2D projection of the robust one. As expected the process is able to find solutions with smaller (better) values of the maximum heat flux (left part of the figure), which, in the robust case, are not Pareto optimal because of the third objective function. It has to be noted, however, that this particular run is not capturing the part of the front, which should be expected above individuals A and B. This partial coverage is due to the stochastic nature of the evolutionary process and the noise in the model, even if in the case of deterministic process, the noise is only associated to the convergence of the control law optimization process, since MC simulation is not required. Again, due to the stochastic nature of the optimization process, a rigorous comparison between deterministic and robust solution would require multiple runs to capture the statistical variability of the result. Such an analysis is extremely computationally expensive and out of the scope of this paper.

## V. Conclusions

This paper described a novel evolutionary approach to integrate system design and optimal control into a single optimization process. The procedure implements a combination of a global, population-based solver with a direct transcription method for optimal control problems.

The proposed approach is applied to two different test cases: the robust design of a single-stage vertically ascending rocket and the robust design of a small scale USV for re-entry operations. In

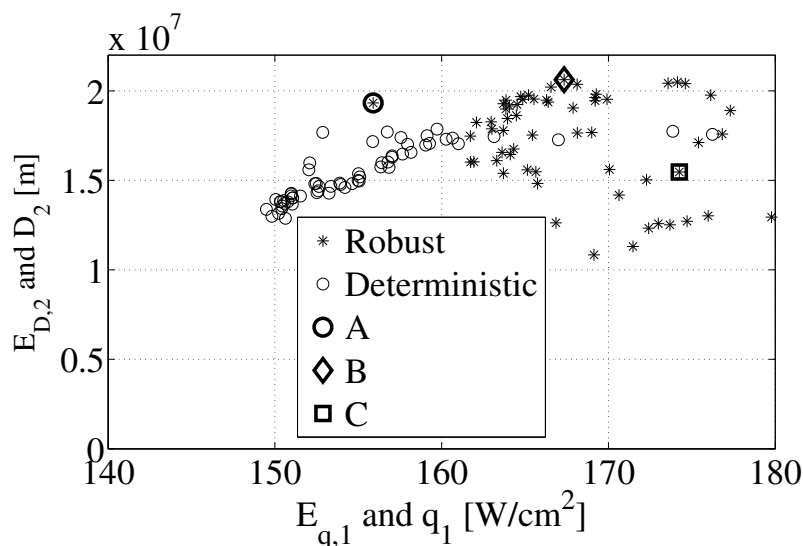


Fig. 12 Comparison between robust and deterministic optimization processes.

both cases, the optimal control solver is interfaced with an ANN, approximating the aerodynamic forces as a function of the geometric parameters and operational conditions. The paper showed that the training and updating of the ANN, by the proposed form of evolution control, converges to a correct representation of the true models in the regions of interest. In the rocket ascent case, the surrogate model could be verified to correctly represent the true model with a final relative error below 3%. On the same case, the MDO process was able to correctly identify the best performing and most robust design solutions under uncertainty. The use of a surrogate allowed for an efficient integration uncertainties on the dynamic models (in particular the aerodynamic forces). For the USV test case, the evolution control procedure was extended to manage and integrates models at different levels of fidelity. Even if the USV model presented in this paper is not fully comprehensive of all the aspects defining the system and phenomena occurring during the re-entry phase, nonetheless it has the main characteristics of a real, complex case, such as: the system and optimal control components are integrated in a multilevel design optimization process, both system and optimal control design processes are computationally expensive and very noisy.

In the future, the approach will be tested on more realistic re-entry problems and generalized to solve analogous problems in other engineering fields, e.g. the design of chemical plants and the design of high performance cars for predefined circuits, also with higher number of shape design

1 variables, at the limits of the meta-modeling capabilities.

2  
3 The current work is mainly focused on the improvement of the aero-thermodynamic model to  
4 insert also unsteady computations into the process. These additional computations are likely to  
5 be needed only for few points of the flight envelope to better assess the errors, and consequent  
6 uncertainties, when lower fidelity models are adopted. The uncertainty modeling is currently one of  
7 the most critical aspects of the system and needs to be improved including knowledge coming from  
8 the expected error of the approximated model and convergence level of the CFD results.  
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10 Other important aspects to consider for future improvements are the techniques for model  
11 approximation and the convergence of the internal optimal control solver, which are crucial for the  
12 correct convergence of the evolutionary process towards optimal shapes.  
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#### References

- [1] Tsuchiya, T., "An Integrated Optimization for Conceptual Design of Spaceplane," *22nd Congress of International Council of the Aeronautical Sciences*, Harrogate, UK, 28 August - 1st September 2000.
- [2] Tsuchiya, T. and Mori, T., "Multidisciplinary Design Optimization to Future Space Transportation Vehicles," *AIAA/AAAF 11th International Space Planes and Hypersonic Systems and Technologies Conference*, Orleans, France, 29 September - 4 October 2002.
- [3] J.Roshanian and Z.Keshavarz, "Effect of Variable Selection on Multidisciplinary Design Optimization: a Flight Vehicle Example," *Chinese Journal of Aeronautics*, Vol. 20, No. 1, 2006, pp. 86–96.  
doi10.1016/S1000-9361(07)60012-0.
- [4] Alexandrov, N., Lewis, R., Gumbert, C., Green, L., and Newman, P., "Approximation and model management in aerodynamic optimization with variable-fidelity models," *Journal of Aircraft*, Vol. 38, No. 6, 2001, pp. 1093–1101.  
doi10.2514/2.2877.
- [5] Shyy, W., Papila, N., Vaidyanathan, R., and Tucker, K., "Global design optimization for aerodynamics and rocket propulsion components," *Progress in Aerospace Sciences*, Vol. 37, 2001, pp. 59–118.
- [6] Giannakoglou, K., "Design of optimal aerodynamic shapes using stochastic optimization methods and computational intelligence," *Progress in Aerospace Sciences*, Vol. 38, No. 1, 2002, pp. 43–76.  
doi: 10.1016/S0376-0421(01)00019-7.
- [7] Ong, Y., Nair, P., and Keane, A., "Evolutionary Optimization of Computationally Expensive Problems via Surrogate Modelling," *AIAA Journal*, Vol. 41, No. 4, 2003, pp. 687–696.

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54  
55  
56  
57  
58  
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60
- doi: 10.2514/2.1999.
- [8] Keane, A. and Nair, P., *Computational Approaches for Aerospace Design: The Pursuit of Excellence*, John-Wiley and Sons, 2005.
- [9] Forrester, A., Sobester, A., and Keane, A., *Engineering Design via Surrogate Modelling: A Practical Guide*, John-Wiley and Sons, 2008.
- [10] Lozano, J. A., Larranaga, P., and Inza, I., *Towards a New Evolutionary Computation: Advances on Estimation of Distribution Algorithms (Studies in Fuzziness and Soft Computing)*, Springer, February 2006.
- [11] Costa, M. and Minisci, E., "MOPED: a Multi-Objective Parzen-based Estimation of Distribution algorithm," *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, edited by C. Fonseca, P. Fleming, E. Zitzler, K. Deb, and L. Thiele, Vol. 2632 of *LNCS*, Springer, Faro, Portugal, 08-11 April 2003, pp. 282–294.
- [12] Avanzini, G., Biamonti, D., and Minisci, E., "Minimum-Fuel/Minimum-Time Maneuvers of Formation Flying Satellites," *Astrodynamics Specialist Conference*, AAS 03-654, Big Sky, Montana, 03-07 August 2003.
- [13] Betts, J. T., *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming, Second Edition*, SIAM, 2010.
- [14] Jin, Y., Olhofer, M., and Sendhoff, B., "A Framework for Evolutionary Optimization with Approximate Fitness Functions," *IEEE Transactions on Evolutionary Computation*, Vol. 6, No. 5, 2002, pp. 481–494. doi: 10.1109/TEVC.2002.800884.
- [15] Russo, G. and P.P. De Matteis, "PRORA-USV: Two Flight Mission Exploring Transonic Conditions," *Proceedings of the 15th AIAA International Space Planes and Hypersonic Systems and Technologies Conference*, AIAA 2008-2660, Dayton, Ohio, USA, 2008.
- [16] Savino, R. and Serpico, D. P. M., "Numerical and Experimental Investigation of PRORA USV Subsonic and Transonic Aerodynamics," *Journal of Spacecraft and Rockets*, Vol. 43, No. 3, 2006, pp. 575–584. doi: 10.2514/1.12352.
- [17] Russo, G., "USV Program Status 2009," *Proceedings of the 16th AIAA/DLR/DGLR International Space Planes and Hypersonic Systems and Technologies Conference*, AIAA-2009-7269, Wahington, DC, 2009.
- [18] Minisci, E., Liqiang, H., and Vasile, M., "Multidisciplinary Design of a micro-USV for Re-entry Operations," *AIAA Guidance, Navigation, and Control Conference*, AIAA 2010-7968, Toronto, Ontario Canada, 02-05 August 2010.
- [19] Viviani, A. and Pezzella, G., "Heat Transfer Analysis for a Winged Reentry Flight Test Bed," *Interna-*

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- tional Journal of Engineering*, Vol. 3, No. 3, 2009, pp. 330–345.
- [20] Squire, T. and Marschall, J., “Material property requirements for analysis and design of UHTC components in hypersonic applications,” *Journal of European Ceramic Society*, Vol. 30, No. 1, 2010, pp. 2239–2251.  
doi: 10.1016/j.jeurceramsoc.2010.01.026.
- [21] Strohmeyer, D., Eggers, T., and Haupt, M., “Waverider Aerodynamics and Preliminary Design for Two-Stage-to-Orbit Missions, Part 1,” *Journal of Spacecraft and Rockets*, Vol. 35, No. 4, 1998, pp. 450–458.  
doi: 10.2514/2.3375.
- [22] Heinze, W. and Bardenhagen, A., “Waverider Aerodynamics and Preliminary Design for Two-Stage-to-Orbit Missions, Part 2,” *Journal of Spacecraft and Rockets*, Vol. 35, No. 4, 1998, pp. 459–466.  
doi: 10.2514/2.3376.
- [23] Starkey, R. P. and Lewis, M. J., “Analytical Off-Design Lift-to-Drag-Ratio Analysis for Hypersonic Waveriders,” *Journal of Spacecraft and Rockets*, Vol. 37, No. 5, 2000, pp. 684–691.  
doi: 10.2514/2.3618.
- [24] Robinson, T., Eldred, M., Willcox, K., and Haimes, R., “Surrogate-Based Optimization Using Multifidelity Models with Variable Parameterization and Corrected Space Mapping,” *AIAA Journal*, Vol. 46, No. 11, 2008, pp. 2814–2822.  
doi: 10.2514/1.36043.
- [25] Singh, G. and Grandhi, R., “Mixed-Variable Optimization Strategy Employing Multifidelity Simulation and Surrogate Models,” *AIAA Journal*, Vol. 48, No. 1, 2010, pp. 215–223.  
doi: 10.2514/1.43469.
- [26] Doherty, J. J., Dean, S. R. H., Ellsmore, P., and Eldridge, A., “A Multi-Fidelity Approach for Supporting Early Aircraft Design Decisions,” *Collaborative Product and Service Life Cycle Management for a Sustainable World*, edited by R. Curran, S. Y. Chou, and A. Trappey, Springer London, London, 2008, pp. 267–279.
- [27] Choi, S., Alonso, J., Kroo, I., and Wintzer, M., “Multifidelity Design Optimization of Low-Boom Supersonic Jets,” *Journal of Aircraft*, Vol. 45, No. 1, 2008, pp. 106–118.  
doi: 10.2514/1.28948.
- [28] Choi, S., Alonso, J., and Kroo, I., “Two-Level Multifidelity Design Optimization Studies for Supersonic Jets,” *Journal of Aircraft*, Vol. 46, No. 3, 2009, pp. 776–790.  
doi: 10.2514/1.34362.
- [29] Chapman, C., *High speed flow*, Cambridge University Press, 2000.

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- [30] White, F., *Viscous Fluid Flow*, McGraw-Hill, New York, 1974.
- [31] Aupoix, B. and Cousteix, J., "Real Gas Effects in Two- and Three-Dimensional Hypersonic, Laminar Boundary Layers," *Computational Methods in Hypersonic Aerodynamics*, edited by T. Murthy, Kluwer Academic Publishers, 1991, pp. 293–340.
- [32] Savino, R., De Stefano Fumo, M., Paterna, D., and Serpico, M., "Aerothermodynamic study of UHTC-based thermal protection systems," *Aerospace Science and Technology*, Vol. 9, 2005, pp. 151–160. doi: 10.1016/j.ast.2004.12.003.
- [33] Anderson, J., *Hypersonic and High-Temperature Gas Dynamics (Second Edition)*, AIAA, 2006.
- [34] Hankey, W. L., *Re-entry Aerodynamics (AIAA Education Series)*, AIAA, 1988.
- [35] Deb, K., Pratap, A., Agarwal, S., and Meyerivan, T., "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II," *IEEE Transaction on Evolutionary Computation*, Vol. 6, No. 2, april 2002, pp. 182–197.
- [36] Fukunaga, K., *Introduction to statistical pattern recognition*, Academic Press, 1972.
- [37] Fornberg, B., *A Practical Guide to Pseudospectral Methods*, Cambridge University Press, 1998.
- [38] MacKay, D., "Bayesian interpolation," *Neural Computation*, Vol. 4, No. 3, 1992, pp. 415–447.
- [39] Bryson, A.E., Ho, Y.C., *Applied optimal control: optimization, estimation, and control*, Taylor & Francis, 1975.