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DISTRIBUTED CLOSED-LOOP EO-STBC FOR A TIME-VARYING RELAY CHANNEL BASED ON KALMAN TRACKING

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ABSTRACT

This paper considers distributed closed-loop extended orthogonal space-time block coding (EO-STBC) for amplify-forward relaying over time-varying channels. In between periodically injected pilot symbols for training, the smooth variation of the fading channel coefficients is exploited by Kalman tracking. We show in this paper that the joint variation of both relay channels still motivates the use of a higher-order auto-regressive model for the a priori prediction step within a decision-feedback system, compared to a first-order standard Kalman model. Simulations results compare these two case and highlight the benefits of the proposed higher-order Kalman filter, which offer joint decoding and tracking.

1. INTRODUCTION

In cooperative communications, the range of communications is extended in the absence of a dedicated infrastructure through the use of relaying nodes. We here consider the case of an amplify and forward (AF) scheme, whereby devices in the relay layer receive from a source and retransmit signals towards the destination. The relay nodes are assumed to be equipped with single antennas, and although they are not linked, can be utilised as a virtual MIMO system to achieve combined diversity and array gain.

Transmit diversity in a distributed environment has been discussed for $N = 2$ using distributed orthogonal STBC (DO-STBC) [5]. For $N = 4$, either full diversity or full rate has to be sacrificed, unless channel state information (CSI) can be exploited akin to closed-loop extended orthogonal STBC [8] in order to achieve maximum diversity and array gain by introducing phase rotation to two of the transmit antennas. The optimum angles are estimated at the destination, and can be fed back to the relay nodes through quantisation and differential encoding [3], exploiting smooth variations of the channel coefficients in Doppler-fading environments.

If the time-varying channel coefficients can be identified and tracked, a joint maximum likelihood (JML) decoding approach proposed in [9] for slow fading channels can be utilised. Based on regular intervals of training, tracking can be accomplished by a Kalman filter in decision-directed mode [6].

In this paper, we formulate the smoothly time-varying channel that is formed by the products of the source–relay and relay–destination links. In order to exploit this smooth variation a higher-order prediction mode akin to [2] is embedded in the Kalman filter. Based on a definition of the system model in Sec. 2, the higher-order model Kalman tracker is introduced in Sc. 3, with results and conclusions presented in Secs. 4 and 5.

In our notation, lower and upper-case bold face variables such as, $\mathbf{h}$ and $\mathbf{H}$ represent vector and matrix quantities respectively. For a matrix $\mathbf{H}$, the transpose is denoted by $\mathbf{H}^{\mathrm{T}}$, the Hermitian by $\mathbf{H}^{\mathrm{H}}$, and the complex conjugate by $\mathbf{H}^{*}$. The statistical expectation operator is given by $\mathcal{E}\{\cdot\}$.

2. SYSTEM MODEL

We consider the AF scheme shown in Fig 1, where a source S transmits to a destination D via a relay layer consisting of $N = 4$ devices R_i, $i \in (1, N)$, which only possess single antennas and are not interconnected. We assume half-duplex mode, where during a first time slot tS transmits to R_i and during the second time slot, the relay to destination link operates in EO-STBC, whereby D provides feedback to relay devices R_1 and R_3.

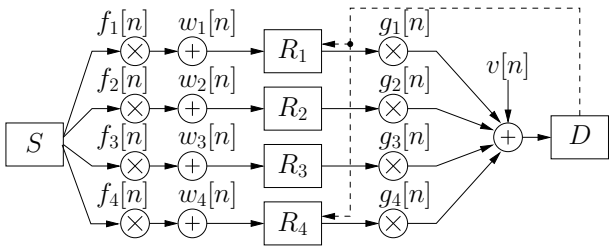

Fig. 1. System model with source S, relay devices R_i, $i = 1 \ldots 4$ and destination D; channels between nodes are characterised by complex fading gains $f_i[n]$ and $g_i[n]$.

2.1. Transmit Signals

In the first transmission phase, S broadcasts an information vector $\mathbf{s}_n$ with variance σ_s^2, which is received at relay nodes and processed under a constrain of link quality to produce an EO-STBC block. In the second phase, the relay nodes forward the signal to a destination node D. As shown in Fig. 1, the channel links $f_i[n]$ and $g_i[n]$ are spatially independent Rayleigh identically distributed wide-sense stationary (WSS) with Doppler spreads Ω_f and Ω_g and variances σ_f^2 and σ_g^2, respectively. The overall transmitted signal is

$$r[n] = b_2 \mathbf{h}_n^T \mathbf{\Lambda}_n \mathbf{s}_n + w[n] \quad (1)$$

$$w[n] = b_1 \mathbf{g}_n^T \mathbf{\Lambda}_n \mathbf{v}_n + n[n] \quad , \quad (2)$$

where the fixed amplification factor $b_1 = \sqrt{p_2/(p_1+1)}$ is to scale average R_i nodes noise power and $b_2 = b_1\sqrt{p_1}$ is to maintain average signal power. Assuming the total average power is p, therefore $p_1 = p_2 = p/2$, where p_1 is the average power of S and p_2 is the average power of R_i nodes. The aggregate source–relay channels are contained in the channel vector

$$\mathbf{h}_n = \left[\begin{array}{cccc} f_1 g_1 & f_2 g_2 & f_3^* g_3 & f_4^* g_4 \end{array} \right] . \quad (3)$$

During even and odd symbol periods, the transmitted vector with EO-STBC encoding can be written as

$$\mathbf{s}_n = \begin{cases} \frac{1}{2}[s[n], s[n], s[n+1], s[n+1]] \ , & n \quad \text{even} \\ \frac{1}{2}[-s^*[n], -s^*[n], s^*[n-1], s^*[n-1]] & n \quad \text{odd.} \end{cases}$$

The beam steering matrix $\mathbf{\Lambda}_n$ in (1) is diagonal,

$$\mathbf{\Lambda}_n = \text{diag}\left\{ e^{j\vartheta_1[n]},\ 1,\ e^{j\vartheta_2[n]},\ 1 \right\} \ , \quad (4)$$

where $\vartheta_1[n]$and $\vartheta_2[n]$ are acting on the channels $h_1[n]$ and $h_3[n]$ in order to achieve maximum system gain. The optimum values for beam steering angles will be shown in Sec. 2.4.

2.2. Relay Node Processing

The broadcast signal is received as

$$\mathbf{y}_i = \sqrt{p_1}\, f_i \mathbf{s}_n + \mathbf{v}_n \quad . \quad (5)$$

at the relay nodes, where linear processing retransmits the data based on distributed EO-STBC codewords. For complex signal constellations, the processing matrices $\mathbf{A}_i$, $\mathbf{B}_i$ at each relay node are designed such that $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{I}_2$, $\mathbf{A}_3 = \mathbf{A}_4 = \mathbf{0}_2$ and $\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{0}_2$,

$$\mathbf{B}_3 = \mathbf{B}_4 = \left[\begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right] \quad .$$

The forwarded signal therefore is

$$\mathbf{t}_i = b_1(\mathbf{A}_i \mathbf{y}_i + \mathbf{B}_i \mathbf{y}_i) \quad . \quad (6)$$

The channel gains of all the communications links are assumed to be independent Rayleigh fading, giving rise to a covariance matrix

$$\mathbf{R}_h[\tau] = \mathcal{E}\left\{ \mathbf{h}_{n-\tau} \mathbf{h}_n^H \right\} = 4\sigma_f^2 J_0(\Omega_f \tau) \sigma_g^2 J_0(\Omega_g \tau)\, \mathbf{I} \ , \quad (7)$$

where $J_0(\cdot)$ is the zeroth order Bessel function of first kind [2].

2.3. Received Signal

During the time intervals n and $n+1$, the received vector can be written as

$$\mathbf{r}_n = \left[\begin{array}{c} r[n] \\ r^*[n+1] \end{array} \right] = b_2 \mathbf{H}_n \mathbf{s}_n + \mathbf{w}_n \quad , \quad (8)$$

based on the equivalent transmit and noise vectors, $\mathbf{s}_n = [s[n], s[n+1]]^T$ and $\mathbf{w}_n = [w[n], w^*[n+1]]^T$, where the latter accounts for both noise propagated through R_i and additive noise at D. The EO-STBC equivalent channel $\mathbf{H}_n$ can be formulated as

$$\mathbf{H}_n = \left[\begin{array}{cc} h_{11}[n] & h_{12}[n] \\ h_{21}[n+1] & h_{22}[n+1] \end{array} \right] . \quad , \quad (9)$$

where the components of $\mathbf{H}_n$,

$$h_{11}[n] = e^{j\vartheta_1[n]} f_1[n] g_1[n] + f_2[n] g_2[n] \quad (10)$$

$$h_{12}[n] = e^{j\vartheta_2[n]} f_3^*[n] g_3[n] + f_4^*[n] g_4[n] \quad (11)$$

$$h_{21}[n+1] = e^{-j\vartheta_2[n+1]} f_3[n+1] g_3^*[n+1] + f_4[n+1] g_4^*[n+1] \quad (12)$$

$$h_{22}[n+1] = -e^{-j\vartheta_1[n+1]} f_1^*[n+1] g_1^*[n+1] - f_2^*[n+1] g_2^*[n+1] \ , \quad (13)$$

are a mixture of dual-hop channel coefficients and rotations due to beam steering.

2.4. Signal Detection & System Gain

Detection is performed over two successive symbols periods, over which in standard space-time block coded systems the channels are assumed to be stationary. This guarantees orthogonality of the equivalent space-time channel matrix $\mathbf{H}_n$, and enables linear decoding according to $\hat{\mathbf{s}}_n = \hat{\mathbf{H}}_n^{\mathrm{H}} \mathbf{r}_n$. In cases, where the matrix is no longer orthogonal, degraded detection performance has motivated approaches such as the joint maximum likelihood method discussed in [9].

For deriving the maximum attainable gain, we assume block stationarity, i.e. $\hat{\mathbf{H}}_n = \mathbf{H}_n$, and $h[n] \approx h[n+1]$. In this case, the maximum achievable gain is

$$\mathbf{G}_n = \mathbf{H}_n^{\mathrm{H}} \mathbf{H}_n = b_2^2 \left[\begin{array}{cc} \alpha+\beta & 0 \\ 0 & \alpha+\beta \end{array} \right] \quad . \quad (14)$$

with

$$\alpha = \sum_{m=1}^{4} |f_m[n]g_m[n]|^2 + \sum_{m=3}^{4} |f_m^*[n]g_m[n]|^2 \quad (15)$$

$$\beta = +\Re\{e^{j\vartheta_1[n]} f_1[n]g_1[n]f_2^*[n]g_2^*[n] + e^{j\vartheta_2[n]} f_3^*[n]g_3[n]f_4[n]g_4^*[n]\} \quad . \quad (16)$$

The maximum factor $\alpha + \max_{\vartheta_1,\vartheta_2} \beta = 4 + \frac{\pi^2}{4}$ can be shown to be attained by setting the beamsteering angles to

$$\vartheta_1[n] = -\angle\{f_1[n]g_1[n]f_2^*[n]g_2^*[n]\} \quad (17)$$

$$\vartheta_2[n] = -\angle\{f_3^*[n]g_3[n]f_4^*[n]g_4[n]\} \quad . \quad (18)$$

3. KALMAN TRACKING

Based on the model and observation equations (1) and (2), this section proposes a higher-order auto-regressive (AR) model for inclusion in a Kalman tracker.

3.1. Channel Modelling

As shown in [2], a narrowband time-varying channel can be approximated by an Mth order AR (AR-M) prediction model with coefficient vector $\mathbf{a} = [a_0 \; a_1 \; \cdots \; a_{M-1}]^{\mathrm{T}}$, which linearly combines a past data vector such that

$$h[n] = \mathbf{a}^{\mathrm{H}} \begin{bmatrix} h[n-1] \\ h[n-2] \\ \vdots \\ h[n-M] \end{bmatrix} = \mathbf{a}^{\mathrm{H}}\mathbf{h}_{n-1} + e[n] \quad , \quad (19)$$

where $e[n]$ is the prediction error. Minimisation of this error leads to $\mathcal{E}\{\mathbf{h}_{n-1}\mathbf{e}_n^*\} = 0$, which in our context can be expressed as $\mathcal{E}\{\mathbf{h}_{n-1}h[n]^*\} = \mathcal{E}\{\mathbf{h}_{n-1}\mathbf{h}_{n-1}^{\mathrm{H}}\}\mathbf{a}$. Including temporal correlation in (7) leads to $\mathbf{a} = \mathbf{R}_h\mathbf{p}$ with an appropriately defined cross-correlation vector $\mathbf{p}$. The identical process noise variance σ_e^2 to the mean squared prediction error is

$$\sigma_e^2 = \mathcal{E}\{e[n]e^*[n]\} = \sigma_h^2 - \Re\left\{\mathbf{a}^{\mathrm{H}}\mathbf{P}\right\} + \mathbf{a}^{\mathrm{H}}\mathbf{R_h}\mathbf{a} \quad . \quad (20)$$

Thus, the state-space model incorporated in the Kalman filter with a finite order AR process can be set up for the case of M as

$$\begin{bmatrix} h_{n|n-1} \\ h_{n-1|n-1} \\ \vdots \\ h_{n-M-1|n-M-1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} h_{n-1|n-2} \\ h_{n-2|n-2} \\ \vdots \\ h_{n-M|n-M} \end{bmatrix} + \begin{bmatrix} w[n] \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (21)$$

with the system matrix

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{M-1} & a_M \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}. \quad (22)$$

SNR/[dB]	5	15	25
$M = 1$	0.165	0.134	0.112
$M = 2$	0.016	0.013	0.011
$M = 3$	0.015	0.012	0.010

Table 1. Average MSE for use of different AR-M models in the Kalman tracker.

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3.2. DD-Based Tracking Scheme

A Kalman estimator estimator based on decision-directed (DD) updating akin to [6] is adopted, and extended to the dual-hop relay link and AR-M model. Assuming independence between the channel gains and observation and process noises for different links enable the application of the standard Kalman filter approach [10].

In the DD approach, a periodic insertion of known symbols along with the transmitted data is employed to inhibit KF divergence. Note that the initial channel estimation is the optimal method only in the linear sense. In updating step, a-piriori estimates are performed during two samples intervals which are used to detect coarse symbols. Thereafter, in correction step the predicted state is subsequently refined using the current observation such that the coarse symbols make feedback to produce a posteriori.

The modified channel coefficients require to omit the phase rotation during tracking scheme. It can be noted that the feedback angles can be absorbed either into channel vector or into transmit vector as in equation 1. Therefore, a simple correction can be used with compensation for phase modification. Based on this approach, a low complexity DD-based Kalman estimator can be implemented.

4. SIMULATIONS AND RESULTS

For the simulations below, it is assumed that (i) the initial state $\mathbf{h}_0$, is previously estimated and known and (ii) the compensation for phase modification is based on true phase angles. Simulations are performed over an ensemble of 10^4 randomised channel realisations. We consider that the both dual-hop links are in moderate fading with the normalised Doppler spread $\Omega_f = \Omega_g = \{0.005\pi\}$. The transmission is initially interleaved every $K = \{24\}$ symbol periods by an inserted pilot symbol for channel estimation, incurring 4% loss in bandwidth efficiency.

Tab. 1 shows the average MSE per channel of D-EO-STBC with Kalman tracking based on different AR orders M, with $M = 1, 2, 3$. It can be seen that $M = 2, 3$ have a significant advantage over the standard first order moded with $M = 1$. In addition, it can be noted that $M = 3$, although it incurs a higher computational complexity than the case of

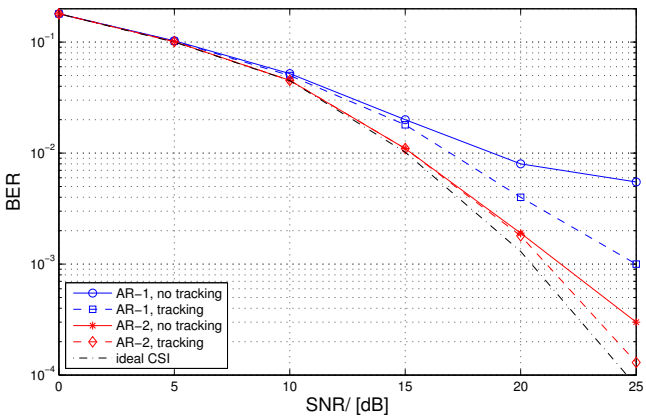

Fig. 2. BER performance of EO-STBC system with Kalman-based channel tracking based on an AR-M with $M = 1, 2$ compared to perfect CSI.

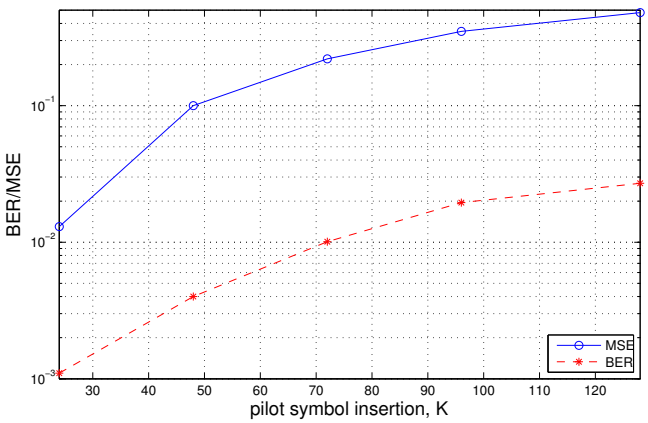

Fig. 3. Average MSE and BER performance at 20dB SNR for D EO-STBC system with Kalman-based tracking based on different tracking periods K.

$M = 2$, does not offer a significant improvement over the latter.

Fig. 2 shows the BER performance, whereby AR-M systems of first and second order are compared. In between a pilot injection after every $K = 24$th time slot, the Kalman filter is either operated in a prediction mode only (labelled as "no tracking") or with a correction step based on DD updating ("tracking"). The It can be seen that the AR-2 performance is very close to the one where the receiver has perfect knowledge of the channel state information (CSI).

We now study the impact of the pilt insertion period K on the performance of the AR-2 system. Fig. 3 shows the BER and MSE performances for pilots interleaved after every $K = \{24, 48, 72, 96, 120\}$ symbol periods. It can be seen that K significantly affects the performance. For the moderate Doppler spread selected here, the simulation indicates than an average BER of 10^{-3} can be maintained for $K = 24$.

5. CONCLUSIONS

We have considered distributed EO-STBC for a two-hop AF relay channel, whereby maximum diverity and array gain can be attained by feedback of appropriate beamsteering angles to the relay layer. The estimation of these angles is based on tracking the aggregate channel at the destination. This is accomplished by employing a Kalman tracker which includes a higher-order prediction model, that can suitably exploit the smoothly time-varying characteritic of the equivalent channel due to Rayleigh fading of the source–relay and relay–destination channels.

6. REFERENCES

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