HYBRID FUZZY-BAYESIAN DYNAMIC DECISION SUPPORT TOOL FOR RESOURCE-BASED SCHEDULING OF CONSTRUCTION PROJECTS

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by

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ABSTRACT

This dissertation proposes a flexible and intelligent decision support tool for scheduling and resource allocation of construction projects. A hybrid Fuzzy-Bayesian scheduling network and a new optimization model and solution approach have been developed to assess the combinatory effect of different risk factors on scheduling and optimize the time-cost tradeoff. Developed decision support tool employs interval-valued fuzzy numbers and Bayesian networks to dynamically quantify uncertainty and predict project performance during its make-span. Using interval-valued fuzzy numbers makes the model more flexible and intelligent comparing to conventional fuzzy risk assessment models through incorporating the decision makers` confidence degree. The linguistic assessments of experts regarding the likelihood and severity of increase or decrease in task duration and cost when influenced by different risk factors are used to generate a set of duration and cost prior-probability distributions.

A learning dynamic Bayesian scheduling network is developed to probabilistically combine the prior-probability distributions with initial activity duration estimates and update them as new evidence in form of actual activity data feed into the network. This model also predicts project performance at any point of time during its execution. Optimization model explicitly considers variation of time-cost tradeoff relationship during project execution and complex payment terms to maximize the project net present value (NPV). A sequential solution approach is proposed to combine a procedure for updating time-cost tradeoff data, and mixed-integer linear programming (MILP) methods to obtain optimal project crashing and scheduling solutions that is adaptive to the current project status and crew productivity. Capability of proposed model in quantifying uncertainty at initial phases of project where project performance data are scarce, learning from data and predicting project performance, considering financial aspects of scheduling through optimal resource allocation and providing useful and clear advice to managers are advantages of developed decision support tool over already existing approaches.

APPROVAL PAGE

The faculty listed below, appointed by the Dean of the School of Graduate Studies have examined a dissertation titled "Hybrid Fuzzy-Bayesian Dynamic Decision Support Tool for Resource-Based Scheduling of Construction Projects" presented by Pejman Rezakhani, candidate for Doctor of Philosophy degree and certify that in their opinion is worthy of acceptance.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure Page

LIST OF TABLES

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CHAPTER 1

INTRODUCTION

1.1. Background

Scheduling is the logical sequencing of project tasks to achieve project scope, quality, cost, and duration often in an environment with complex task dependencies, limited resources and uncertain task durations. Two major aspects of construction project scheduling which captured more attention in research are project completion time (make-span) minimization and resource leveling. Construction projects are complex and highly uncertain due to their special characteristics such as uniqueness, variability and ambiguity.

Currently there are three commonly used scheduling methods; two network analysis methods [Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT)] and the S-curve method [e.g. Earned Value Method (EVM)] (Zhang, Du, Sa, Wang, & Wang, 2013) . CPM is widely used but it is purely deterministic and fails to address the inherent uncertainty in task durations. PERT extends CPM by incorporating uncertainty in task durations. However, assumptions of beta distribution for all task durations, independence of tasks, underestimating the mean project duration and neglecting the influence of near-critical paths are PERT`s major shortcomings. EVM only relies on summary project level data and fails in modeling the logical relationship of each task [(Zhang et al., 2013), (Kim & Reinschmidt, 2009)].

These methods require high-quality quantitative data which are rare or incomplete especially at initial project phases and are unable to handle the uncertainty and subjectivity associated with construction activities. In addition, financial aspects of project scheduling and the effect of time value of money are frequently ignored or left as a secondary attention items despite their importance to profitability and even to the existence of the project (Shtub & Etgar, 1997). A dynamic project scheduling approach with the following abilities may prove to be more realistic and can lead to lower cost and scheduling overruns:

- Addressing the inherent risk and uncertainty in construction tasks
- Modeling the logical relationship between related variables
- Capturing actual task data during the project make-span to update risk probabilities and impact
- Optimally allocating resources to maximize the Net Present Value (NPV) of project.

1.2. Research Objectives

This research proposes a flexible and intelligent decision support tool for scheduling and resource allocation of construction projects. The tool uses a hybrid Fuzzy-Bayesian dynamic scheduling model. The power of Fuzzy Set Theory (FST) is utilized in assessing the combinatory effect of different known risk factors on task durations at initial phases of project. The Bayesian Networks (BNs) are used in monitoring and predicting the project performance. The proposed model is different from Fuzzy risk assessment models in literature due to the use of interval-valued fuzzy numbers. Interval-valued fuzzy numbers capture and combine the linguistic evaluations of experts with their confidence degrees regarding their assessment on probability of risk and its impact. A learning dynamic BN model is developed to combine the fuzzy risk assessment results with initial task duration estimates. Initial duration distributions are updated as new evidence in form of actual task data is entered the model. The model predicts project performance at any point of time during project execution. Developed hybrid

Fuzzy-Bayesian model is combined with an algorithm to schedule tasks and to optimally redistribute resources to maximize the net present value of the project.

The developed probabilistic dynamic scheduling model should satisfy following requirements:

- It must estimate the initial task durations by considering the diminishing productivity return when more resources are assigned to a particular task.
- The model must be capable of capturing expert judgments regarding the combinatory effect of different known risk factors on task durations especially at initial phases of project where project performance data are scarce.
- The model must be able to combine expert judgments with initial task duration estimates to come up with a prior (before initiation of project) probability distribution of task durations under risk.
- It must predict and monitor project performance, taking into account the impact of relevant known and unknown risk factors.
- The core model must be simple and applicable without using excessive computational resources. This will enable the modeler to easily replicate it multiple times during project execution.
- It must be able to handle different types of qualitative (i.e. expert judgement) and quantitative (i.e. actual project performance) data during project make-span.
- The model must learn from data, either as a result of observations or as a result of expert judgment entered as evidence.
- The application of such model should be easy and must assist managers in making economically feasible decisions.

1.3. Research Significance

Each construction project is subject to unique risk factors. At initial stages of a project where there is a lack of high-quality quantitative data, statistical methods may not be as efficient as subjective methods in estimating the uncertainty. Fuzzy Set Theory has proven to be an effective method to deal with uncertainty. It quantifies the uncertainty inherent in construction task durations that are affected by a combination of different risk factors. This research extends the current Fuzzy risk assessment methods by incorporating the confidence degree of decision makers in fuzzy computations. The new method, named interval-valued fuzzy numbers, generates a triangular distribution of discrete duration values under the combinatory effect of different risk factors by applying experts` judgments. These distributions are later employed as an input in a Bayesian Network model.

One of the main obstacles of applying BNs in project scheduling is defining the prior probability distributions of task durations under uncertainty. Current approaches do not provide a systematic way to establish prior probability distributions. This research addresses this gap by introducing a hybrid Fuzzy-Bayesian dynamic scheduling model. The model employs interval-valued Fuzzy numbers to elicit subjective estimates from experts and develop them into subjective prior probability distributions for Bayesian analysis. The advantages of proposed model are:

- Quantifying uncertainty at initial phases of project where project performance data are scarce.
- Learning from data and predicting project performance.
- Considering financial aspects of scheduling through NPV optimization.
- Easy application for schedulers and providing useful and clear advice to managers.

1.4. Interdisciplinary Nature of Research

This research contributes to the disciplines of Computing and Mathematics, Operations Research and Civil Engineering. Proposed decision support tool integrates the heavily used Fuzzy Set Theory and more recent Bayesian Networks with optimization methods and solution algorithm and provides a computationally tractable approach to optimize construction project scheduling and resource allocation under an incomplete information process.

1.5. Structure of Dissertation

In next section a comprehensive literature review is presented which addresses the current methodologies in risk identification and assessment under uncertainty, current project scheduling techniques and NPV maximization algorithms. Developed methodology is discussed in chapter 3. Chapters 4 and 5 are dedicated to a bridge project case study and conclusions.

CHAPTER 2

LITERATURE REVIEW

2.1. Risk Identification

2.1.1. Project Risk Classification

Risk is defined as an uncertain future event or condition with an unexpected or unplanned effect on at least one of project objectives (i.e., schedule, cost, quality, etc.). In construction industry, risk may arise from competitive bidding process, weather change, job-site productivity, political situation, market competition etc. Its adverse impacts on project objectives can be mitigated through an effective risk management system. There is a strong relationship between the amount of risk management efforts undertaken in a project and level of the project success PMBok (Guide, 2004).

There are several elements in a project which may affect the risk management plan. Some of these elements are shown in Figure 2.1.

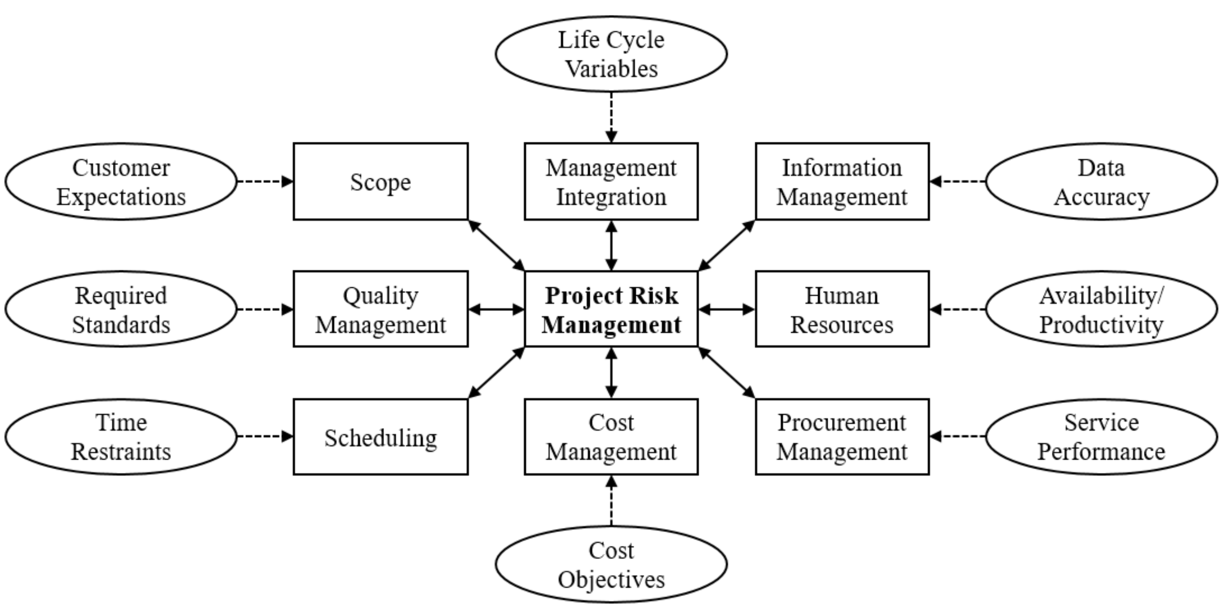


Figure 2.1. Effective elements on project risk management [Source: (Rezakhani, 2012)]

Following is a list of sample actions under each element that may lead to risk events affecting the success of the project:

- Information Management
  - Actions: Lack of communication; Poor complexity management; Lack of adequate consultants.
  - Events: Inaction or wrong action due to incorrect information or communication failure.
- Human Resources
  - Actions: Poor organization; Absence of crew leadership; Conflict management problems.
  - Events: Organizational breakdown.
- Procurement Management
  - Actions: Unforeseen conditions; Using financially unsound contractors.
  - Events: Project delays due to lack of material.
- Cost Management
  - Actions: Estimating errors; Lower than expected productivity; Poor change management.
  - Events: Project over budget.
- Scheduling
  - Actions: Inaccurate time estimating; Poor allocation and management of float; Scope change without due allowance; Act-of-God.
  - Events: Project overdue.
- Quality Management

- Actions: Inadequate quality assurance program; Sub-standard design/material; Accepting the lowest bid price without considering project conditions.
- Events: Low quality project; may result in financial loss.

• Scope

- Actions: Inadequate scope definition; Poor scope management during project make-span.
- Events: Project delays and cost over-runs.

Existing project risk management approaches [Project Risk Analysis and Management (PRAM), Risk Analysis and Management for Projects (RAMP), Risk Management Solutions (RMS), etc.] may be summarized into four phases of identification and classification, assessment, response and monitoring risks. In first phase, risk components which may affect project objectives are defined and their characteristics are documented. Risk assessment step focuses on assessing the effect of identified risks on project objectives (i.e. duration, cost, etc.). There are both qualitative and quantitate methods for risk assessment. Responding to risks is developing strategies to mitigate and reduce the effect of different risk factors on project objectives. Finally, monitoring and reviewing risks helps in implementing a risk response plan, monitoring and keeping track of identified risks and evaluate the effectiveness of the project risk management process (Nieto-Morote & Ruz-Vila, 2011).

PMBok (Guide, 2004) defines risk classification as a "provider of a structure that ensures a comprehensive process of systematically identifying risks to a consistent level of detail and contributes to the effectiveness and quality of the risks process identification." Risk classification is an important step in the risk assessment process, as it attempts to structure the diverse risks that may affect project objectives. Construction risks may be classified in many ways by their types (i.e. natures, magnitudes, etc.), the sources and/or origins, or project phase [(D. F. Cooper & Chapman, 1987; Edwards & Bowen, 1998; Klemetti, 2006; Zhou, Vasconcelos, & Nunes, 2008)].

There are many approaches in literature for construction risk classification. (Perry & Hayes, 1985) published an extensive list of factors assembled from several sources, and classified them in terms of risks retainable by contractors, consultants and clients. However, they were concerned with financial risk rather than hazard. While their main emphasis was to develop a risk management process to avoid cost and time overruns, the adverse effect of risk on other project objectives were neglected. (Abdou, 1996) classified construction risks into three groups: construction finance, construction time, and construction design. The paper addresses these risks by considering the contractual relationships among entities involved in different phases of a project. He criticized the unequal shared responsibility among involved entities due to lack of contractual relationship between the architect/engineer and the contractor. (Shen, 1997) used a survey to identify the project delay risks and effective actions for managing these risks. He categorized delay risks into eight major categories. Among all these factors, "insufficient or incorrect design information" and "abortive works due to poor workmanship" were ranked as the highest and lowest project delay risks.

(Tah & Carr, 2000) proposed a hierarchical risk breakdown structure representation to facilitate risk identification and classification in qualitative risk assessment process. In their approach, project risks are classified into "Internal" and "External" risks. External risks (e.g. economic) are identified as relatively uncontrollable while internal factors (e.g. materials) are known as more controllable risks. (Chapman, 2001) grouped risks into four subsets: environment, industry, client, and project. He also developed a relationship between different risk factors in each sub-category. (Shen, Wu, & Ng, 2001) grouped associated risks with construction joint ventures into six categories as financial, legal, management, market, policy, and technical risks through conducting a survey. For each sub-category risk factor, a risk significant index by multiplying the probability of occurrence by the degree of impact was calculated. This index was used to rank all risks. Chen et. Al (2004) classified risk factors concerning project cost into three groups of resource factors, management factors, and parent factors. Resource factors concern mainly price escalation of material, equipment, and labor. Management factors focus on project`s main cost risk factors. Client`s cash flow situation is described as the main risk factor in this category. Finally, parent factors refer to factors that come from joint venture partners. A case study to investigate risk management practice in a railway construction was also conducted.

(Assaf & Al-Hejji, 2006) defined seventy-three causes of delay categorized into nine groups where the most common cause of delay was identified as "change order". (Dikmen, Birgonul, & Han, 2007) used influence diagrams to define the factors which have influence on project risks. (Zeng, An, & Smith, 2007) categorized risk factors into four major factors of human, site, material, and equipment. (Choi & Mahadevan, 2008) classified critical risks in a project into construction related and act-of-God factors. Construction related class mainly concentrates on risk factors due to incomplete design, improper work, political and financial factors and subcontractors. Act-of-God factors focus on natural hazards like Typhoon, Earthquake, etc.

2.1.2. Project Risk Breakdown

Risks can be represented in a hierarchical structure based on their origin and impact on project objectives [(Tah, Thorpe, & McCaffer, 1993; Wirba, Tah, & Howes, 1996)]. In general, project risks maybe identified as “External”, “Internal” and “Legal” risks (Rezakhani, 2012). The degree of predictability and ability to manage appropriate response varies between them. External risks can be predicted but they are uncontrollable. Internal risks are controllable and mainly are due to technical issues. Majority of legal risks are caused by contractual issues. Figure 2.2. illustrates a general breakdown of mentioned risks.

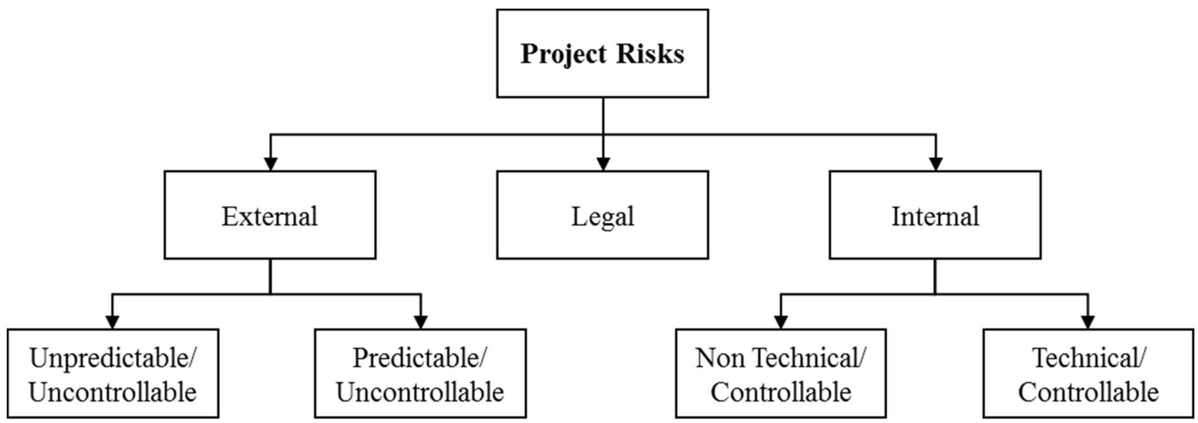


Figure 2.2. General breakdown of project risks [Source: (Rezakhani, 2012)]

External Unpredictable/Uncontrollable risks cannot be predicted or controlled by project manager. They are mainly due to regulatory issues (i.e. environmental issues), natural hazards (i.e. flood), postulated events (e.g. vandalism), indirect effects (e.g. social) and completion problems (e.g. failure to provide financial support to the end of project). *External Predictable/Uncontrollable* risks can be predicted but project manager has no control on them. They mainly arise from market conditions (cost and availability of material and equipment), environmental and social impacts (e.g. environmental pollution), currency changes (e.g. international projects) and inflation (e.g. unpredicted increase in price of material and equipment.)

Internal Non-Technical/Controllable risks are caused by factors inside the project. They are mainly due to managerial issues rather than technical problems. Some of the major ones are caused by management (e.g. loss of project control); schedule (e.g. unforeseen site condition); cost (e.g. underestimating) and cash flow (e.g. interruption in payment to contractors). *Internal Technical/Controllable* risks are the ones which come from technical problems rather than managerial issues. Changes in technology (i.e. complexity introduced as a result of new technology), performance (i.e. productivity) and design inadequacies are some of the factors causing this type of risks. *Legal* risks are mainly related to contractual issues such as misinterpretation and misunderstanding of contract terms.

2.1.3. Hierarchical Risk Breakdown Structure (HRBS)

In order to better understand the effective risks on construction projects, (Rezakhani, 2012) presented a HRBS which classifies risk factors based on predictability and controllability into External (external unpredictable-predictable/ uncontrollable), Operational (external predictable/ uncontrollable), Management (internal non-technical/ controllable), Engineering (internal technical/ controllable) and Financial (internal non- technical/ controllable) and legal. This HRBS is shown in Figure 2.3.

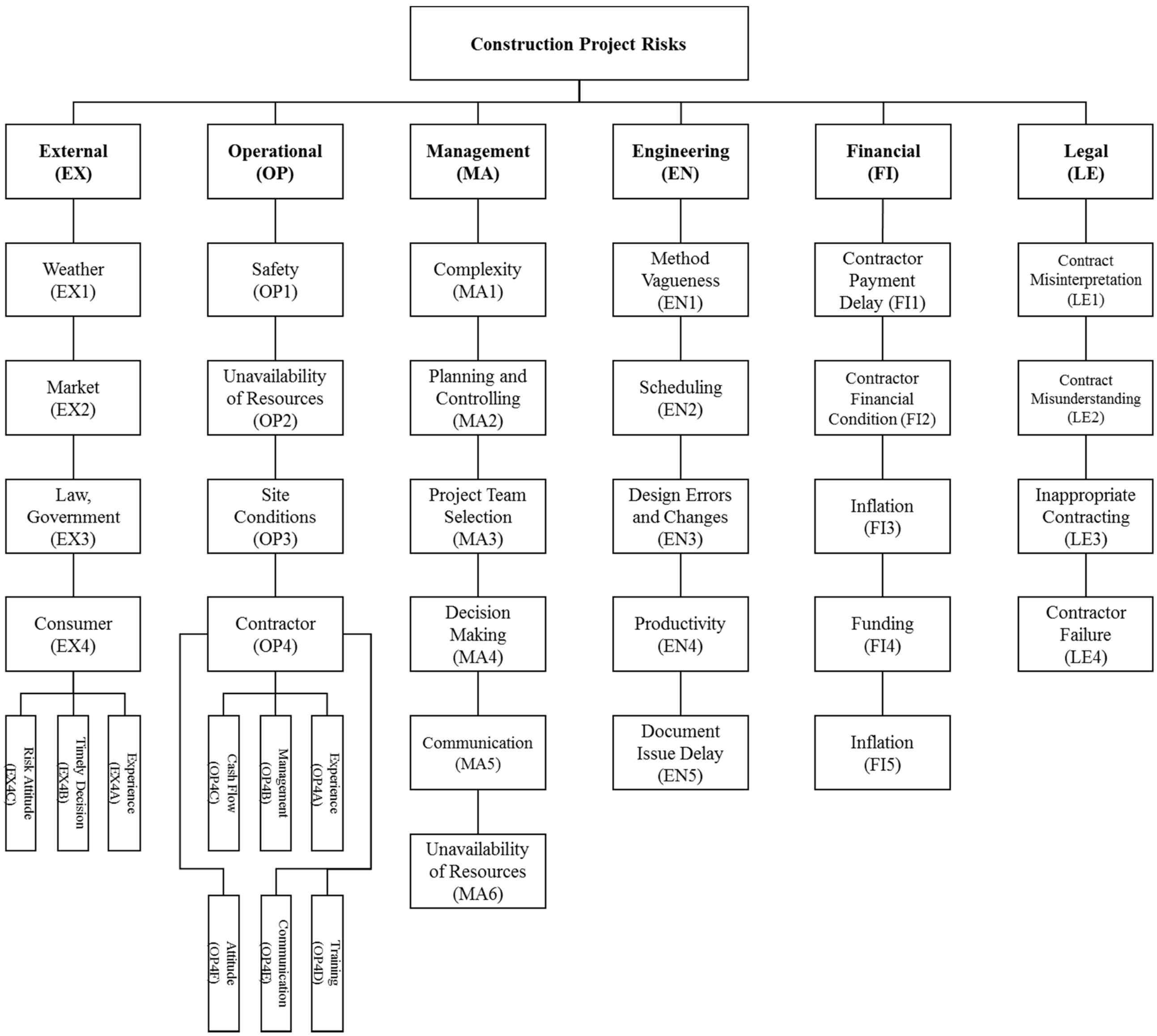


Figure 2.3. Proposed hierarchical risk breakdown structure (Rezakhani, 2012)

2.2. Risk Assessment under Uncertainty

Risk is usually assessed through assessing its probability of occurrence and severity of impact. It was the beginning of the 1980`s when risk management became an independent project management function and research domain. Different approaches have been suggested in literature for assessment of construction project risks, such as Probability Theory (PT), Monte Carlo Simulation (MCS), Analytical Hierarchy Process (AHP) and Fuzzy Set Theory

(FST). In the 1980s PT and MCS were the two main approaches for construction risk assessment. FST was introduced at the end of this period as a possible alternative to handle uncertainty in risk assessment. During 1990s PT and FST captured more attention in risk assessment research. Since 2000 AHP and FST have been mainly used in construction risk assessment domain due to their capability of handling complex and subjective risk assessment problems. However, application of AHP has been limited due to its relative nature of results (Taroun, 2014).

2.2.1. Probability Theory (PT)

Probabilistic analysis is one of the classical models in the risk assessment phase of a risk management plan. Although they have been widely used in construction risk analysis problems they suffer from two major limitations (Mustafa & Al-Bahar, 1991): First, some of these models require detailed quantitative information which may not be available in a construction project; then, they are not able to handle uncertainty and subjectivity inherent in construction projects. (Barnes, 1983) discussed the principles of risk allocation and proposed a risk allocation algorithm. (D. Cooper, MacDonald, & Chapman, 1985) proposed a cost risk analysis method for large projects. Their research focused on reliability of the cost estimate and adequacy of contingency allowance. (P. Dey, Tabucanon, & Ogunlana, 1994) used PT to develop technical and management contingency allowances in order to control uncertainty in projects especially during the planning stage. (Tavares, Ferreira, & Coelho, 1998) defined project risk as a function of the discounted cost and project duration. They assumed lognormal distributions for activity durations where lower bound corresponds to the minimal feasible duration and upper quantiles were assumed to be unbounded. Mode of assumed distribution is defined as expected duration of each activity. Finally, float has been introduced as a managerial tool in selecting the most convenient schedule.

An approach to quantify uncertainty in construction schedules using PT and PERT was introduced by (Mulholland & Christian, 1999). They drew a direct relationship between the variance of project duration and project schedule risk. (Cioffi & Khamooshi, 2009) proposed a method to aggregate multiple risks at a given confidence level and connect them to an appropriate contingency budget. However, simple averaging of probabilities of occurrences to perform the aggregation was one of the shortcomings of the proposed model.

2.2.2. Monte Carlo Simulation (MCS)

One of the earliest attempts in quantifying project cost risks was the work of (Diekmann, 1983). He made a thorough review of available to-date risk quantification methods and concluded that MCS methods are most flexible ones. (Beeston, 1986) utilized MCS to combine risks with assumption of independence between them to define an estimation variance. The project estimate then would be changed with this variance. (Hull, 1990) applied MCS and PERT to perform contract proposal risk assessment from cost, schedule and technical performance perspective. (Al-Bahar & Crandall, 1990) considered the risk management approach from contractors' viewpoint. The influence diagramming and MCS were used to analyze and evaluate project risks. In the same manner, (Huseby & Skogen, 1992) used influence diagramming and MCS to first model dependencies between risks and then assess them.

Using MCS and PERT, (Dawood, 1998) developed a simulation model to consider variations in activity durations and their dependence on risk factors. In his model, MCS generates a random number between (0) and (1) from predefined distributions. These random numbers then would be used as modifiers in calculating each activity duration. (Molenaar, 2005) presented a methodology developed by the Washington State Department of Transportation (WSDOT) for cost risk analysis of highway megaprojects. In his approach, after identifying a set of possible risk events, probability and impact of each event occurrence on activities' duration and cost were assessed. Then, base costs and risk costs were combined into a final range estimate of project costs using MCS. Finally, a sensitivity analysis was performed to identify the most critical risks.

2.2.3. Analytical Hierarchy Process (AHP)

AHP was developed by Thomas Saaty (L. S. Thomas, 1980). It is a multi-criteria decision making tool that captures the decision maker's subjective assessment and quantifies relative priorities of decision alternatives on a ratio scale (Taroun, 2014). Despite its power in capturing subjective assessment, it can only deal with deterministic estimates and may not be a suitable approach for construction projects which usually involve massive uncertainties and subjectivities (Zeng et al., 2007). One of the first approaches in project risk assessment using AHP was proposed by (Mustafa & Al-Bahar, 1991) where they used AHP to assist contractors in risk evaluation of projects which they are bidding. At first step, they structured the risk elements into a hierarchy, then developed their relative weights. At next step, they determined the likelihoods of risk levels by aggregating the relative weights through the hierarchy. Finally, they run a sensitivity analysis to find out the sensitivity of outcomes to change in hierarchy.

In another attempt (Riggs, Brown, & Trueblood, 1994) tried to integrate technical, cost and schedule risks using AHP and a decision tree. They used AHP to assign probabilities to the decision tree. (Zhi, 1995) used risk probability and impact assessment to develop a risk management approach for overseas construction projects. In this approach, AHP was used to assess the impact of risk factors. At first step, project risks were grouped based on their probability and impact level. Then factors in each group were compared pairwise with their upper level ones and were assigned an importance weight for their probability and impact. Multiplying derived probability and impact resulted in factor's degree of risk which then would be used as a ranking factor.

International construction risk assessment was also considered in a paper by (Hastak & Shaked, 2000). They used AHP to determine the relative importance of identified risks in macro (country), market and project level and a probability-impact model to assess these risks. (P. K. Dey, 2001) proposed a quantitative approach to construction risk management using AHP and decision tree analysis. After identifying the possible risk factors affecting time, cost and quality, AHP would be used to assess the likelihood of their occurrence. This paper used Expected Monetary Value (EMV) as a management tool and recommended the best risk response strategy which has the lowest expected cost. AHP was also used by (Dikmen & Birgonul, 2006) in calculation of risk and opportunity ratings of international construction projects. In this paper, risks were classified as project and country specific. AHP was used to compare the probability of occurrence and impact of each level of risk hierarchy. Finally, impact and probability values were multiplied and added up to find the overall risk rating.

(Hsueh, Perng, Yan, & Lee, 2007) proposed an online multi-criterion risk assessment model which utilizes AHP and Utility Theory for risk assessment of construction joint ventures in China. After identifying the risk factors which may affect joint ventures and establish suitable criteria, AHP was used to get the weighting of each criterion. (Zayed, Amer, & Pan, 2008) also used AHP to determine the weight of each risk factor. For this purpose, experts performed a pair-wise comparison between the macro (company) and micro (project) level risk areas and their sub-areas and estimated a relative importance weight.

2.2.4. Fuzzy Set Theory (FST)

FST has been the dominant and key alternative method in assessing construction risk over the last two decades (Baloi & Price, 2003; Taroun, 2014). Its ability in capturing and aggregating the linguistic assessment of experts in early phases of project makes it an appropriate method in assessing construction project risk. FST provides a systematic tool to deal with uncertainty caused by lack of precise information in decision- making process through interpreting the linguistic variables (Zadeh, 1965). Fuzzy systems are best suited for applications where the mathematical models are difficult to derive, incomplete or uncertain information exist and evidence itself is fuzzy in nature (Liu, Yang, Wang, & Sii, 2003).

After (Zadeh, 1965) introduced the concept of fuzzy sets and theory, researchers such as (Kangari & Riggs, 1989), (Paek, Lee, & Ock, 1993), (Tah et al., 1993), (Wirba et al., 1996), (Tah & Carr, 2001), (Cho, Choi, & Kim, 2002), (Choi, Cho, & Seo, 2004), (An, Baker, & Zeng, 2005), (Dikmen et al., 2007), (Zeng et al., 2007), (Wang & Elhag, 2007), (KarimiAzari, Mousavi, Mousavi, & Hosseini, 2011) and (Nieto-Morote & Ruz-Vila, 2011) introduced FST-based risk modeling and analytic methods.

Application of FST in construction risk analysis was first introduced by (Kangari & Riggs, 1989). They presented a risk analysis model which makes use of FST as a risk assessment tool using natural language representation, fuzzy evaluation of risk and linguistic approximation. (Paek et al., 1993) proposed a risk pricing model to assist the contractors decide bid prices and estimate risk contingency using fuzzy numbers obtained from either historical data or expert opinions. (Tah et al., 1993) tried to present contractor`s risk through a new risk breakdown structure called “hierarchical risk-breakdown structure” (HRBS). Linguistic terms and fuzzy calculations were used in proposed approach to determine the contractor`s cost risk. (Tah & Carr, 2001) used HRBS to present a qualitative risk assessment model based on FST. Relationship between the likelihood of occurrence (L), the severity (V), and the effect of each risk factor (E) is defined by IF-THEN rules, i.e., “If L and V, then E.” FST was used to identify and quantify the relationship between the risk sources and the consequences to the project`s performance.

(Choi et al., 2004) presented a fuzzy-based risk assessment model for underground construction projects. Proposed model consisted of four steps as identifying, analyzing, evaluating and managing risks. They used linguistic variables to express subjective judgement. (Dikmen et al., 2007) proposed a fuzzy risk model to assess the risk of cost overruns in international construction projects. Using influence diagrams to model risks, expert judgement to address subjectivity and fuzzy aggregation rules, cost overrun risk rating is assessed. (Zeng et al., 2007) combined FST with the Analytical Hierarchy Process (AHP) method to prioritize risks and derive relative weights for them. However, complexity of AHP calculations, make this approach almost impractical.

(Wang & Elhag, 2007) proposed a fuzzy multi-criteria decision-making approach, which allows decision makers to evaluate multiple fuzzy risk factors using linguistic terms by aggregating the assessments of multiple risk factors. (Zou, Zhang, & Wang, 2007) proposed a methodology which uses expert judgement and fuzzy aggregation to develop a ranking of different risk factors. To solve the inconsistency issue in pair-wise comparison judgements, (Nieto-Morote & Ruz-Vila, 2011) and (KarimiAzari et al., 2011) proposed two different algorithms. A new approach to fuzzy project risk assessment model based on similarity measure of generalized fuzzy numbers was proposed by (Rezakhani, Jang, Lee, & Lee, 2014).

2.3. Current Techniques in Project Scheduling

Current project scheduling techniques can be categorized into network analysis methods [Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT)], simulation and the S-curve method [e.g. Earned Value Method (EVM)] (Zhang et al., 2013). These methods require high-quality quantitative data which are rare or incomplete especially at initial project phases and are unable to handle the uncertainty and subjectivity associated with construction activities. In addition, financial aspects of project scheduling and the effect of time value of money are frequently ignored or left as a secondary attention items despite their importance to profitability and even to the existence of the project (Shtub & Etgar, 1997).

2.3.1. Critical Path Method (CPM)

The CPM is the most known project scheduling technique which has been in use since 1960`s. (Moder, 1988) cited CPM and its calculations as the base for developing more sophisticated scheduling techniques. Project scheduling using CPM includes following steps:

- Define project activities, their sequences and dependency;
- Show activities and their dependency as a network diagram;
- Estimate the discrete activity duration;
- Identify the Critical Path;
- Update scheduling network as the project progresses.

Following are the basic CPM calculations:

$a_j: Activity\ j$

$D_j = Duration\ of\ a_j$

$$ES_j\ (Early\ Start) = Max\ [ES_i + D_i|\ i\ one\ of\ the\ predecessor\ activities] \quad (2\text{-}1)$$

$$EF_j\ (Early\ Finish) = ES_j + D_j \quad (2\text{-}2)$$

$$LF_j\ (Late\ Finish) = Min\ [LF_k - D_k|\ k\ one\ of\ the\ successor\ activities] \quad (2\text{-}3)$$

$$LS_j\ (Late\ Start) = LF_j - D_j \quad (2\text{-}4)$$

$$TF_j\ (Total\ Float) = ES_j - LS_j = LF_j - EF_j \quad (2\text{-}5)$$

By performing the forward (start-finish) and backward (finish-start) passes through the project network, the earliest start (ES) and the earliest finish (EF) time for each activity is calculated. The difference between the latest and earliest finish times of each activity is called total float (TF) which is the allowance in activity duration increase without increasing overall project completion time.

Although nearly 70% of project management professionals use CPM (Pollack-Johnson & Liberatore, 2005) its inability in handling or quantifying uncertainty makes it inappropriate for today's complex projects.

2.3.2. Program Evaluation and Review Technique (PERT)

PERT was introduced in the early 1960`s to incorporate uncertainty in activity duration. PERT uses Beta probability distribution to estimate optimistic, most likely and pessimistic activity times. Expected duration and standard deviation for activity j is calculated as follow:

Expected duration: $\mu_j = (Optimistic + 4 \times Most\ Likely + Pessimistic)/6$ (2-6)

Standard Deviation: $\sigma_j = (Pessimistic - Optimistic)/6$ (2-7)

PERT uses the summation of variances of activities in the critical path to calculate the variance of project completion time. Assuming a normal distribution, probability of project completion date is calculated. However, despite the great success of PERT, studies in 1970s questioned its practicality and theoretical assumptions (MacCrimmon & Ryavec, 1964; Sapolsky). Some of the shortcomings of PERT are as follow:

- Assumption of independence between activities;
- Assumption of Beta distribution for duration of all activities;
- Assumption of only one and unchanged critical path;
- Producing unrealistic and overly optimistic project duration estimates.

2.3.3. Simulation

Monte Carlo Simulation (MCS) became the dominant tool for handling risk and uncertainty in projects in the 1980s. MCS estimates the shortest, most likely and longest activity duration along with shape of duration distribution. Using this distribution function, a random value is generated. After a number of sufficient runs, probability distribution of the possible critical path is generated. This tool has the ability of measuring the correlation between durations of task and project. This feature gives the project managers ability of identifying critical tasks. However, MCS has following drawbacks:

- Assumption of statistical independence between activities which share same risk factors;
- Underestimation of total uncertainty due to neglecting the risk dependence between activities (Van Dorp & Duffey, 1999);
- Inability in addressing subjectivity and uncertainty in activity duration estimates.

2.3.4. S-curves Method

S-curves method (e.g. earned value method) are used to control the overall progress of project. It relies only on summary project-level data and can provide a systematic way of analyzing the actual performance of project (Kim & Reinschmidt, 2009). Although this method has the capability of applying to wide range of project types and sizes but suffers from following shortcomings:

- It cannot demonstrate the logical relationship of project tasks (Zhang et al., 2013);
- It is a deterministic method that is unable to address the inherent uncertainty in project tasks (Kim & Reinschmidt, 2009).

In general, these methods require high-quality quantitative data which are rare or incomplete especially at initial project phases and are unable to handle the uncertainty and subjectivity associated with construction activities. In addition, financial aspects of project scheduling and the effect of time value of money are frequently ignored or left as a secondary attention items despite their importance to profitability and even to the existence of the project (Shtub & Etgar, 1997). A dynamic project scheduling approach with the following abilities may prove to be more realistic and can lead to lower cost and scheduling overruns:

- Addressing the inherent risk and uncertainty in construction tasks
- Modeling the logical relationship between related variables
- Capturing actual task data during the project make-span to update risk probabilities and impact
- Optimally allocating resources to maximize the Net Present Value (NPV) of project.

2.4. Project Dynamic Updating

Construction projects are complex and highly uncertain due to their special characteristics such as uniqueness, variability and ambiguity. Uniqueness arises from the fact that each project has its own characteristics and no similar experience can be applied even for similar projects while trade-off between performance measures and lack of clarity and data are triggers for variability and ambiguity (Khodakarami, Fenton, & Neil, 2007). This uncertainty which causes both the construction productivity and project completion probability vary over time is the source of project risk (Rezakhani, 2012). Purpose of project risk management is handling associated uncertainty in different aspects of project (i.e. schedule, cost, quality, etc.) to improve its performance (Rezakhani et al., 2014). To ensure compliance with schedule, (Zhang et al., 2013) emphasize the need for a dynamic probabilistic evaluation of project performance during its make-span. However, modeling the subjective judgment to deal with the lack of measured project performance data especially at initial phases of project and updating them as new actual performance data become available and measuring the effect of different known and unknown risk factors remain a challenge in project scheduling.

Decision support under uncertainty in construction projects may be handled by common approaches in area of artificial intelligence such as Rule-Based expert systems, Neural Networks and Bayesian Networks (BN). Rule-Based expert systems and Neural Networks may not fit the dynamic nature of construction projects because of their inflexibility in accepting new evidence and fixed output information. Also requiring historical data to train neural networks which may not be available due to uniqueness of construction projects and careful analysis of the rule base to determine the effect of each new rule on the others (McCabe, AbouRizk, & Goebel, 1998) make them undesirable as decision support tools in construction projects. On the other hand, BNs are flexible in accepting evidence at any point and dynamically adjust output based on newly entered evidence. They also have the advantage of reasoning in the presence of uncertainty and incomplete data and learning from evidence in order to update their prior beliefs (Hearty, Fenton, Marquez, & Neil, 2009). These preferences make BNs a suitable decision support tool when dealing with uncertainty in construction projects.

2.4.1. Bayesian Network (BN)

Bayesian Network (BN) is a directed acyclic graph consisting of nodes and arcs which uses and exploits Bayes' theorem to find an exact solution and the concepts of conditional probability to express the cause-effect relationships between variables. The nodes represent random variables and arcs are used to show the influential or casual relationships between variables. "Prior" probability distributions are used to define the nodes without parents while nodes with parents are defined with conditional probability distributions in either form of deterministic or standard probability distribution functions (Hearty et al., 2009). All nodes in a BN are conditionally independent which improves the overall computation based on the fact that the probability of either node can be evaluated without consideration of the other (McCabe et al., 1998).

Given prior belief about hypothesis, H, as $P(H)$, the posterior belief about H using evidence, E, by applying Bayes' theorem is calculated as follow:

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)} \qquad (2\text{-}8)$$

When a variable is actually observed, the marginal probability distribution for the observed variable reduces to a probability of 1 for the observed state and zero otherwise. Observed variable updates the conditional probability distribution of its children and through Bayes' theorem (Equation 2-8), the distributions of its parents (Hearty et al., 2009).

Advantages of using BNs include:

- Expressing the complex relationships in a network through conditional probability distributions.

- Facilitating the understanding and modeling of cause and effect relationships using graphical interface.
- Forecasting with small and incomplete data sets.
- Flexibility in accepting subjectively or objectively derived probability distributions.
- Learn from evidence entered into the model to update their beliefs about probable causes.

However, discretizing continues variables which may cause information loss if not done properly is one of the challenges in using the BNs (Uusitalo, 2007). One possible solution to this could be dynamic discretization using Junction Tree (JT) algorithm. (Neil, Tailor, & Marquez, 2007) proposed a dynamic discretization algorithm which uses entropy error as the basis for approximation and JT to perform propagation iteratively on anytime basis when evidence is entered the BN. Another challenge is eliciting and converting experts' knowledge into probability distributions for learning the BN. Combining a powerful expert system tool such as FST into BN may overcome this challenge.

BBNs have been used in construction for different purposes. (McCabe et al., 1998) integrated BBN and computer simulation for improving the construction operation. In this application, the belief network provides diagnostic analysis of the simulated construction performance. Analysis results then would be used to generate alternative actions which may improve the performance. [(Khodakarami & Abdi, 2014) and (Attoh-Okine, 2002)] demonstrated the capabilities of BBNs in modeling the dependencies between cost items in project cost risk analysis and handling the uncertain knowledge about highway construction cost variables through their expressive graphical language. In these approaches, BNs provide a framework for presenting causal relationships and enable probabilistic inference among a set of variables. They also used to perform "what-if" analysis and to predict based on learnt data.

Other applications of BN`s are in modeling the factors influencing falsework installation productivity (Tischer & Kuprenas, 2003) and to analyze construction contract risks (Adams, 2006). In construction project scheduling the application of BBNs is not very significant: (Nasir, McCabe, & Hartono, 2003) used BBN to model the relationship between major risk variables affecting activity durations and determine the upper and lower activity duration limits by suggesting a certain percentage of increase or decrease from most likely value which is assumed to be known. A Monte Carlo Simulation (MCS) then utilizes these limits to incorporate the effect of risks on the project schedule. However, requiring another approach (i.e. MCS) to handle the outputs of proposed BBN and restricting the upper and lower bounds of activity duration to a few pre-defined values are some of the limitations of this model. (Luu, Kim, Tuan, & Ogunlana, 2009) applied BBN to model the cause-and-effect relationships of factors influencing construction delay. Using only two states to assign the variables is one of the shortcomings of this approach. In an attempt to model uncertainty in project scheduling, [(Khodakarami et al., 2007) and (Khodakarami, 2009)] proposed a model which incorporates CPM calculations in BBN.

One of the main challenges of applying BBNs in project scheduling is defining the prior probability distributions of activity durations under uncertainty. Although previously mentioned approaches tried to handle uncertainty inherent in construction schedules but they did not provide a method to establish prior probability distributions. This research addresses this gap by introducing a hybrid Fuzzy-Bayesian Belief Network dynamic scheduling model (F-BBN) which employs FST to elicit subjective estimates from experts and develop them into subjective prior probability distributions for Bayesian analysis. The model is implemented using AgenaRisk toolset due to its user-friendly interface, ability to handle continuous variables and its capability of building dynamic models.

2.5. NPV Maximization

Deterministic Time-Cost Trade-off Problem (DTCTP) may be defined as the process to identify optimum combination of construction tasks to speed up the project while keep the project cost at a reasonable level. DTCTP can find an optimal or near optimal trade-off point between reducing a task's duration and resultant increased cost. There are two different approaches for DTCTP: Unconstrained and constrained resource.

2.5.1. Resource-Unconstrained Scheduling Problem

The vast majority of project scheduling methodologies presented in the literature have been developed with the objective of minimizing the project duration subject to various types of precedence and resource constraints (Willy S. Herroelen, Van Dommelen, & Demeulemeester, 1997). When taken into consideration, there is a decided preference for the maximization of the net present value (NPV) of the project as the more appreciate objective, and this preference increases with the project duration. When significant levels of cash flows are present in the project, in the form of expenses for initiating activities and progress payments for completion of parts of the project, the net present value (NPV) criterion is a more appropriate measure of project performance (Bey, Doersch, & Patterson, 1981).

In recent years, several publications have dealt with the project scheduling problem under the objective of maximizing the net present value (NPV) of the project. The majority of the contributions assume a completely deterministic project setting, in which all relevant problem data, including the various cash flows, are assumed known from the outset. Research efforts have led to optimal procedures for the unconstrained project scheduling problem, where activities are only subject to precedence constraints (Demeulemeester, 1996). In addition, numerous efforts aim at providing optimal or suboptimal solutions to the project scheduling problem under various types of resource constraints (Willy S. Herroelen et al., 1997). Some research works also focus on simultaneous determination of both the amount and timing of payments and stochastic aspects of the scheduling problems.

Much of the research on the NPV project scheduling problem has been concentrated on designing solution approaches for the resource-constrained extension, where the problem is to maximize the NPV of the project subject to precedence, renewable resource, material and capital constraints, time-cost and multi-mode operational issues. Given the complex, combinatorial nature of these problems, optimal approaches have been successful only for small instances. This is due to both the difficulty in representing the problem in mathematical form as well as the difficulty in solving the problem, once formulated (Kolisch & Padman, 2001). Following is the review of literature regarding the resource-unconstrained NPV maximization case.

(Russell, 1970) was the first to introduce the idea of maximizing the net present value of the cash flows in the project. In his model both positive and negative cash flows occur at event completion. The problem was formulated with known durations and precedence relations. He showed that the cost-critical path is quite different from the time-critical path when monetary objectives are considered. Russell's work was extended by (Grinold, 1972) by adding the project deadline in his model. He assumed that a known amount of cash changes hands at each event. He considered the payment scheduling problem for a schedule that maximizes the present value of all transactions.

(Elmaghraby & Herroelen, 1990) Challenged the "no due date" assumption of prior approaches. They argued this assumption may lead to the fact that delaying the project for ever could be the optimal schedule. They proposed a project scheduling algorithm with NPV objective. This algorithm was programmed in C by (Willy S Herroelen & Gallens, 1993). However, (Sepil, 1994) reported on a possible flaw in the algorithm in which the it may not find the optimal solution.

(Sunde & Lichtenberg, 1995) presented a new time-cost trade-off algorithm using successive scheduling to increase the net benefit of a project. This heuristic approach crashes selected activities and balances cost, time and resources at a time. It used a net-present-value representation and the Lichtenberg Quality Picture, as a balancing criterion. (Kazaz & Sepil, 1996) proposed a mixed-integer programming approach with the assumption of occurring cash inflows as progress payments at the end of the month and cash outflows at the completion of activities. They used activity profit curves to maximize the NPV.

(Demeulemeester, 1996) assumed positive and/or negative cash flows are associated with the completion of activities. In proposed approach, positive cash flows are scheduled as early as possible while negative cash flows are scheduled as late as possible within the precedence constraints. His optimal procedure performed a recursive search on partial tree structures. (De Reyck, 1996) extended the (Demeulemeester, 1996) approach to handle the scheduling networks with generalized precedence relations. One of the advantages of proposed approach is allowing time lags between the start and completion of activities.

(Ran Etgar, Shtub, & LeBlanc, 1997) dealt with NPV maximization problem where net cash flow magnitudes are independent of the time of realization. The duration of each activity is known and net cash flow happens at event`s occurrence. (Shtub & Etgar, 1997) developed a branch-and-bound algorithm to maximize project NPV in which payments and cost of resources are time dependent. A comparison between proposed approach and the one presented by (Ran Etgar et al., 1997) showed more efficiency as far as computational time.

Work of (Elmaghraby & Herroelen, 1990) was extended by (R. Etgar & Shtub, 1999), to account for the linear function of cash flows and activities completion times. This linear relationship also appeared in (Vanhoucke, Demeulemeester, & Herroelen, 1999) approach. In former, they presented an exact procedure to maximize the NPV where the activity-based cash flows are linear dependent on the completion times of the corresponding activities. An activity-on-the-node project network with zero-lag finish-start precedence constraints was assumed. They introduce an extension of an exact recursive algorithm which schedules the activities as soon as possible and searches for sets of activities to shift towards the deadline in order to increase the NPV. In latter, an exact algorithm including a recursive search and enumerative procedure was proposed. In this approach precedence constraints and a fixed deadline was considered.

(Vanhoucke, Demeulemeester, & Herroelen, 2003) took the activity profit curves into account. A branch-and-bound algorithm using piecewise linear overestimations was proposed. (Waligóra, 2008) presented a tabu search approach for discrete–continuous project scheduling problem with discounted cash flows. Only positive cash flows which are associated with the execution of each activity are considered. Two common payment models -lump-sum payment and payments at activities' completion times- were considered with the objective of maximization of the NPV of all cash flows of the project. However, considering only the positive cash flows is one of the limitations of this model.

(Nadjafi & Shadrokh, 2008) proposed an exact recursive branch and bound algorithm focusing on reducing the tree size. The algorithm is extended with two bounding rules in order to reduce the size of the branch and bound tree. (Sobel, Szmerekovsky, & Tilson, 2009) presented a project scheduling approach with stochastic activity duration to maximize the expected NPV. Their approach includes randomness in activity durations, costs, and revenues. They presented a NPV maximization algorithm to schedule projects with stochastic activity durations.

(Creemers, Leus, & Lambrecht, 2010) examined project scheduling with NPV objective and exponential activity durations, using a continuous-time Markov decision chain. The cash flows are received or paid at the start of the activity. Comparing to (Sobel et al., 2009) their approach has an improvement both on running times and memory usage. (Wiesemann, Kuhn, & Rustem, 2010) proposed a branch-and-bound algorithm and global optimization to address the maximization of a project's expected NPV. They considered activity-on-node representation with generalized precedence relations scheduling network. The activity durations and cash flows are described by a discrete set of alternative scenarios with associated occurrence probabilities.

2.5.2. Resource-Constrained Scheduling Problem

Based on (Kolisch & Padman, 2001), resources utilized by the activities are classified according to categories, types, and value. The category classification includes resources that are renewable, nonrenewable, partially renewable and doubly constrained. *Renewable resources* are constrained on a period basis only. That is, regardless of the project length, each renewable resource is available for every single period. Examples are machines, equipment, and manpower. *Nonrenewable resources* are limited over the entire planning horizon, with no restrictions within each period. The classic example is the capital budget of a project. *Doubly constrained resources* are limited on a period basis as well as on a planning horizon basis. Budget constraints that limit capital availability for the entire project as well as limiting its consumption over each period are an example of this type of resource. *Partially renewable resources* limit utilization of resources within a subset of the planning horizon. An example is that of a planning horizon of a month with workers whose weekly working time, not the daily time, is limited by the working contract.

The *type* classification further distinguishes each category per the function of the various resources. Each resource type has a *value* associated with it, representing the available amount. Whenever there is at least one category of constrained resources, the resulting Project Scheduling Problem (PSP) is termed as a resource-constrained project scheduling problem (RCPSP). Following is the review of literature regarding the resource-constrained NPV maximization case.

(Doersch & Patterson, 1977) considered the case when progress payments are made upon the completion of certain activities and penalties incurred for late completion. They introduced a zero-one integer programming approach to the NPV maximization problem. This model included a constraint on capital for expenditure such that the available capital increased as progress payments were made. (Smith-Daniels & Smith-Daniels, 1987) extended the (Doersch & Patterson, 1977) model to include material constraints and cost. They demonstrated the importance of materials management factors in planning stage to avoid schedule and cost overruns.

(J. Patterson, Slowinski, Talbot, & Weglarz, 1989; J. H. Patterson, Brian Talbot, Slowinski, & Węglarz, 1990) presented a zero–one programming model and a backtracking algorithm to maximize the NPV of the constrained project scheduling problem. Rather than focusing on maximizing the NPV, they provided a solution methodology to minimize the project duration. Then used this solution to shifting the cash flows to improve the NPV. (Sami M Baroum & Patterson, 1999) proposed a branch and bound procedure for scheduling the activities. In their approach, when activities completed the NPV or the financial reward of resulting cash flows is maximal among all possible schedules. The solution procedure allowed for several potential activity schedules and resources considering several constraints.

(K. K. Yang, Talbot, & Patterson, 1993) presented an integer programming algorithm with the objective of maximizing the NPV of project schedule to the firm. A depth-first branch and bound solution procedure searches over the feasible set of finish or completion times for each of the activities. Then a data set of optimal NPV schedules for projects involving as many as 20-30 activities per project is created. (Icmeli & Erenguc, 1996) developed a branch-and-bound algorithm for the resource constrained project scheduling problems using the minimal delaying alternatives concept originally introduced in (Demeulemeester & Herroelen, 1992) for branching. The procedure used the "minimal delaying alternatives" concept to resolve resource conflicts where payments and activity completion times assumed to have linear relationship.

(Padman, Smith Daniels, & Smith Daniels, 1997) proposed a heuristic non-linear integer programming algorithm to deal with NPV maximization problem in multiple resource constraints. This algorithm suggests scheduling decisions when resource conflicts arise during project scheduling. Proposed algorithm also has the ability of re-optimization of partially completed activities. (Smith Daniels & Aquilano, 1987) proposed a procedure for developing a late-start resource-constrained project schedule using CPM. It was assumed that cash outflows occurred at the beginning of the period and a single project payment was received on completion of the project. In contrast to heuristics that schedule each activity as early as possible, they showed that late-start schedules yield a higher NPV and lower average duration. Also, while the late-start schedule on average was significantly longer than the optimal-duration resource-constrained schedule, no significant difference occurred in the average NPVs of the two scheduling methods.

(Özdamar & Ulusoy, 1996) proposed an iterative scheduling technique which consisted of consecutive forward/backward scheduling passes to smoothing out the project's resource profile. Along with passes, activities shifted in scheduling network to improve both the project duration and NPV. In the assumed cash flow model, activity expenditures occur at their starting times and payment is made on completion of the project. The results demonstrated that under the assumed cash flow model, the iterative scheduling algorithm improved project planning.

(Sami M. Baroum & Patterson, 1996) described the development of a heuristic procedure which activities were assigned weights based upon their cumulative cash flows. In the same manner as Ozdamar and Ulusoy's approach, using multi-pass shifting algorithms, this approach also search locally for improved activity assignments to increase project NPV. (Icmeli & Erenguc, 1994) proposed a tabu search procedure as a heuristic solution technique for Resource Constrained Project Scheduling Problem with Discounted Cash Flows. In this approach, each activity shifts one-time unit from its current completion time, with the restriction that the resulting completion time should not violate earliest and latest completion times for the activity. Comparing to linear programming, tabu search may produce near-optimal solutions with reasonable computational effort.

(Zhu & Padman, 1997) considered projects where cash outflows and inflows occur for project expenditures and payments for completed activities. They provided a heuristic approach to find the optimal solution which proceeded in a single-pass, forward sequence in scheduling network. (Zhu & Padman, 1999) proposed a tabu search metaheuristic procedure for the NPV maximization problem. They designed and conducted several experiments to evaluate both the parameters within tabu search and those that are critical to the project scheduling problem. Tabu Search for Constrained Project Scheduling was tested over 720 small size and medium size projects. The results showed that the impact of tabu search is significant, especially for large projects and can be embedded in decision support environments for constrained project scheduling to guide the generation of better schedules.

(Vanhoucke, Demeulemeester, & Herroelen, 2001) introduced a branch-and-bound algorithm that makes use of extra precedence relations as described by (Icmeli & Erenguc, 1996) to resolve a number of resource conflicts. It also utilizes are cursive search algorithm for the max-NPV problem to compute upper bounds. In their approach, positive cash flows should be scheduled as early as possible while negative cash flows should be scheduled as late as possible within the precedence constraints. Deterministic cash flows are assumed to occur over the duration of activities while progress payments and cash outflows occur at the completion of activities. The promising results indicated that the branch-and-bound procedure is able to optimally solve instances with up to 30 activities and four resource types in a reasonable time limit.

(Schwindt & Zimmermann, 2001) proposed an algorithm which was based on a first-order steepest ascent approach for solving the NPV problem. Temporal constraints between project activities were assumed. A computational performance analysis comparing the proposed algorithm with four procedures from literature has shown that the steepest ascent algorithm outperforms approaches iterating adjacent vertices.

CHAPTER 3

METHODOLOGY

This research aims to address the uncertainty in task durations at initial stages of a project by using FST despite the unavailability of high-quality quantitative data. Once the project is started, BNs will be used to update the subjectively estimated duration of remaining activities by utilizing the actual task data during the project. Project NPV will be maximized by making time-cost trade-off decisions based on the updated duration distributions of remaining activities. Figure 3.1. summarizes the proposed methodology. This will be achieved by following steps:

Before starting the project:

Step 1: Estimating initial task durations considering the diminishing return of crew productivity in case of adding more crew members (discrete alternatives).

Step 2: Grouping project activities based on their crew types and defining the risk factors which may affect their durations.

Step 3: Developing a risk assessment model based on experts' subjective assessments to evaluate the task durations uncertainty when influenced by different risk factors (duration prior-probability distribution).

Project starts:

Step 4: Developing a Bayesian Network model to update the prior probability distributions of task durations based on the actual task data during the project execution (duration posterior-probability distribution).

Step 5: Integrating the developed Bayesian Network in a Mixed-Integer Linear Programming (MILP) optimization algorithm for scheduling tasks and optimally redistributing resources to maximize the NPV of the project. This step will update the scheduling network.

Step 6: Repeat step 5 as new data gets collected.

Project completion.

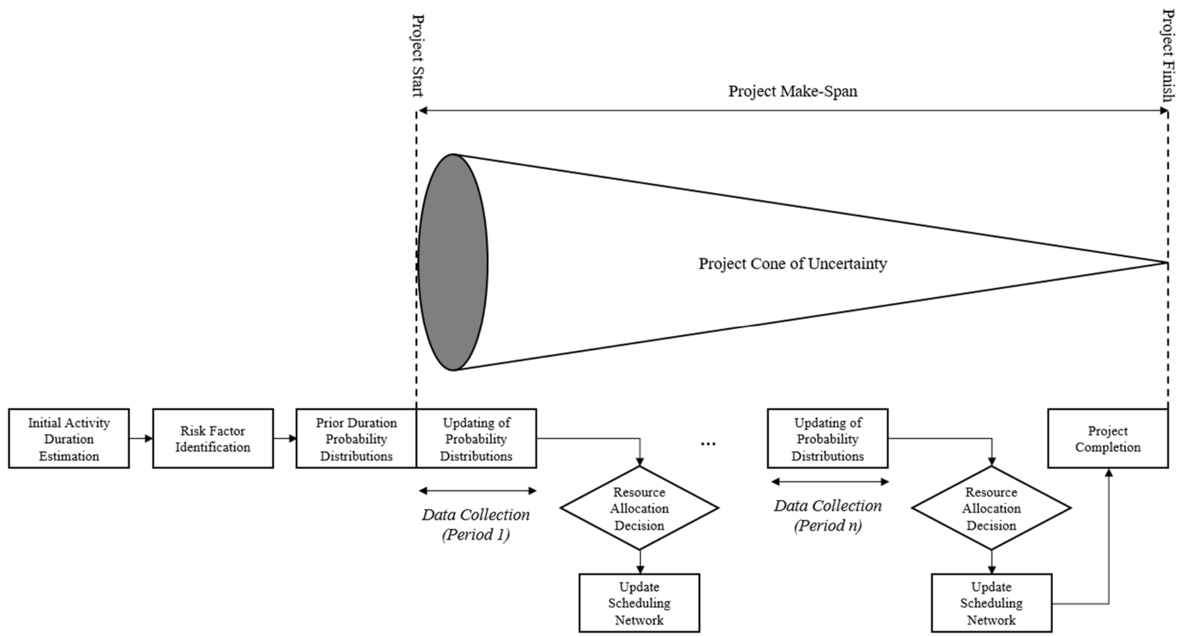


Figure 3.1. Proposed decision support tool model

Proposed model has been divided into two phases: phase I focuses on discrete modeling and optimization while in phase II, probabilistic modeling and optimization are discussed. In phase I, initial discrete task durations are estimated using US average construction cost data and by considering the diminishing productivity return effect. The CPM network is also built based on estimated task durations. Considering the CPM network, different duration and cost crashing options to be used in optimization algorithm are generated. Proposed optimization algorithm optimally allocates resources to unfinished tasks to maximize the NPV of project and finish it within set deadlines.

Phase II starts with identifying possible risk factors from a risk pool by experts. Details of this risk pool are described in section 2.1. of literature review. These risks are called "known" risk factors. Using FST, the combinatory effect of identified known risks on initial discrete task durations is assessed and prior duration probability distributions are generated. In fact, discrete task durations are converted into probability distributions to account for effect of risk and uncertainties. Three values of pessimistic, most likely and optimistic durations maybe defined for completion of each task using percentiles. CPM model in phase I is remodeled as a BN with tasks being nodes and precedence relationship being the arcs. Purpose of this step is to account for "unknown" risks which may happen during the project make-span and dynamic updating based on actual project data. Project data including actual duration and cost of activities are collected in certain time frames and fed into scheduling network. The BN by applying the Bayes theorem updates prior probability distributions and generates posterior probability distributions. This will update the distributions of connected activities in scheduling network who share the same crew. These updated distributions are then used to generate new sets of duration and cost crashing options. Optimization algorithm uses these options to make resource allocation decisions for incomplete activities. As the project progresses, more accurate data become available which results in reducing associated uncertainty. In following each phase is described in details:

3.1. Phase I

3.1.1. Initial Task Duration Estimation

Task initial discrete durations are estimated based on assumed quantity, resources and resource productivity as provided by RS Means 2015 based on US average data. In order to generate duration and cost options to be used in optimization algorithm, different crew configurations are generated. The diminishing return of productivity due to congestion in case of adding more crews is calculated using a regression model proposed by Thomas and Sakarcan (H. R. Thomas & Sakarcan, 1994).

For each task, a Microsoft Excel spreadsheet including specifications such as description, quantity and crew information is generated. RSMeans CostWorks is used to extract crew information such as type based on activity nature, size, daily output per crew, wage, labor, material, equipment and other relative data. A sample spreadsheet is depicted in Figure 3.2.

Activity 7-Make Abutment Forms

ID	*Source*	*Line Number*	*Description*	*Crew*	*Daily Output*	*Labor Hours*	*Quantity*	*Mat/Unit*	*Labor/Unit*	*Equip/Unit*	*Total/Unit*	*Total Cost*
ACT7	CostWorks 2015	31113453050	C.I.P. Concrete Forms	C1	346.00	0.092	1,380.00	$1.52	$3.93	$0.00	$9.39	**$ 12,956**

Number of Crew	1	*Daily Output per Crew*	346.00	*Efficiency*	1.00				*Productivity for Crew C1*		
Crew C1									*Crew Size*	*Labor Hours*	*Daily Output*
ID	*Crew Size*	*Crew and Equipments*	*Wage/hr*	*hr/Day*	*Total Mhrs*	*Duration*	*hr/Unit*	*Cost/Unit*	4	0.092	346.00
C1-C	3	Carpenters	$44.90	8	24	4	0.023	3.114	8	0.147	218.28
C1-L	1	Laborer	$35.45	8	8		0.023	0.820	12	0.143	223.88
Total	4				32	127.63		3.934	16	0.142	224.81

Figure 3.2. Sample activity spreadsheet

Following is a brief description of each cell`s calculations:

Labor Hours is calculated by dividing the *Total Man Hours* for one crew by the multiplication of *Daily Output per Crew* and *Efficiency* factor. Efficiency factor determines the effectiveness of crew and modifies the duration of unfinished activities based on actual project data. For estimation purpose (before the project starts), this factor can be assumed as 1. As the project progresses, different values may be assigned as Efficiency Factor based on actual crew performance to decrease or increase the duration of unfinished activities which share the similar crew.

In this research, productivity is treated as a function of crew size. The diminishing productivity return in case of increasing the crew size is calculated through factor model (H. R. Thomas & Sakarcan, 1994). Mathematically, the factor model is defined as:

$$E_t = I_s + \sum_{i=1}^{m} a_i x_i + \sum_{j=1}^{n} f(y)_j \quad (3\text{-}1)$$

Where E_t = predicted productivity for time period t; I_s = productivity for standard conditions; m = number of condition variables; a_i = coefficient of condition variable i; x_i = indicator of condition variable i (0= not present, 1= present); n = number of submodels; and $f(y)_j$ = mathematical function of sub-model j. Coefficients for the model are selected from (Sanders & Thomas, 1991). In this sense, predicted productivity for time period t as a function of crew size ($E_{t-crew\ size}$) is defined as:

$$E_{t-crew\ size} = I_s + 0.0163(crew\ size) - 0.00156(crew\ size^2) + 0.000046(crew\ size^3) \quad (3\text{-}2)$$

A comparison between nominal and diminishing productivity return for an activity with crew size of four is illustrated in Figure 3.3.

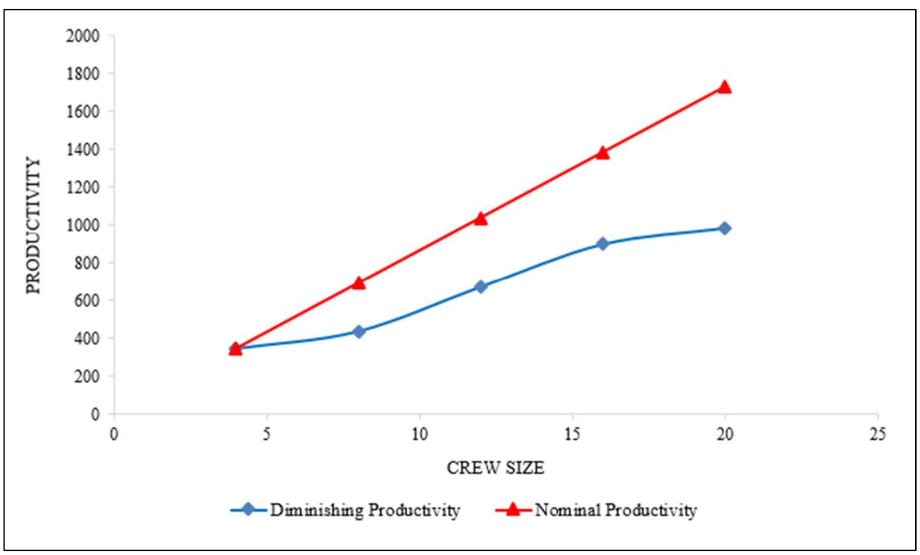


Figure 3.3. Comparing nominal and diminishing productivity return

I_s is defined by labor hours for one crew. As the result, Equation (2) calculates the diminishing labor hours in case of adding more crews. With these new labor hours, activity durations and cost in case of adding more crews are defined as mentioned earlier.

Duration of each activity is calculated by first multiplying the *Labor Hours* by *Quantity* and then dividing the result by *Total Man Hours*. Number of spent hours per unit is defined by dividing the *Labor Hours* by total crew size. Multiplying this value by *Wage* rate and crew size results in labor cost per unit. Same procedure should be followed for equipment cost. Finally, cost of each activity is calculated by totaling the material, labor and equipment unit costs and multiplying it by *Quantity*.

3.1.2. Dynamic Updating

In this model, activities sharing the same crew are related. When actual activity data in form of evidence is entered the model, Efficiency factors of unfinished activities are updated based on average crew performance of related completed activities. Modifying this factor changes the labor hours for each activity. Since the quantity and total man-hours are constant, modifying labor hours results in new activity duration. This change also affects the labor hours per unit of work. Assuming no changes in wage rates and crew size, multiplying the new labor hours by wage rate and crew size results in new labor cost per unit. All other associated costs are updated based on new labor hour values.

3.1.3. Optimization Model and Solution Approach[1]

Crashing options to use in optimization model are generated by assuming four scenarios of "Before Project Start", "Above Cost- Ahead Schedule", "Above Cost- Behind Schedule", and "Below Cost- Behind Schedule". In the "Before Project Start" scenario, which

[1] The optimization model and solution approach in phase I of this research have been developed by Dr. Haitao Li and Liu Yang from College of Business Administration, University of Missouri – St. Louis.

is the base case, the project is in planning stage. Activity durations and costs are estimated but the project has not been started yet. In other scenarios, the project is assumed to be partially completed. Actual duration and cost of finished activities are used to generate different scenarios.

In Above Cost- Ahead Schedule scenario, it is assumed that several activities have been finished sooner with higher cost than the estimation. This results in Efficiency factor of higher than 1 for unfinished related activities and decreases their duration and cost. In Above Cost-Behind Schedule case, not only completed activities have higher than expected cost but also they are behind schedule. These delayed activities change the Efficiency factor to values lower than one indicating possible delays and cost increase in completing the related activities. And finally, in Below Cost- Behind Schedule scenario, it is assumed that even though several activities have higher finish time than what was expected, they are still below the estimated cost. Same as previous scenario, in this case we also have lower than one Efficiency factor and higher predicted cost of completing the activities.

In this section, we first formally describe the addressed Deterministic Time-Cost Trade-off Problem (DTCTP) with varying time-cost relationship and complex payment terms for a construction project. Then details of modeling the payment terms and discounted cash flow are provided. Next, the mathematical formulation of the Mixed Integer Linear Programming (MILP) model is presented, followed by the sequential and adaptive solution framework.

3.1.3.1. Problem Description

A construction project with a total contract value Π must be scheduled to complete within T days. It consists of a set A of activities, and a set E of precedence relationships among the activities. For each pair of arc $(i, j) \in E$, it is required that activity j cannot start until i is finished. Each activity $i \in A$ has a set K^i of modes to be executed. Different modes have tradeoff in time and cost. The duration of activity i executed in mode k is dur_{ik} (in days). Due to variation of crew's productivity and efficiency, the cost of an activity-mode pair may vary over time. Let $\widetilde{cost}_{ik}$ denote the cost of i executed in mode k, which can be updated dynamically over time using data observed during project execution.

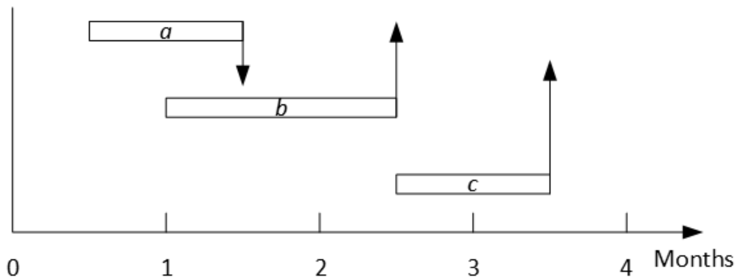
Payment of the construction project is received as monthly installments following mutually agreed terms between the contractor and client. Each month, the contractor calculates the total monthly cost as the summation of direct activity cost and monthly indirect cost based on a daily indirect cost ρ. A ratio ψ is used to markup the monthly cost as the invoice amount. The client agrees to pay no more than a proportion β of the invoice amount every month. The remaining payment will be fulfilled at the completion of the project. The monthly interest rate is r.

The project manager has two decisions to make: (i) project crashing or time-cost tradeoff decision to determine the mode for each activity; (ii) to properly schedule the start and finish time of each activity and cash flows while satisfying all the precedence relationships, payment terms and completion deadline. The objective is to maximize the net present value (NPV) of the project.

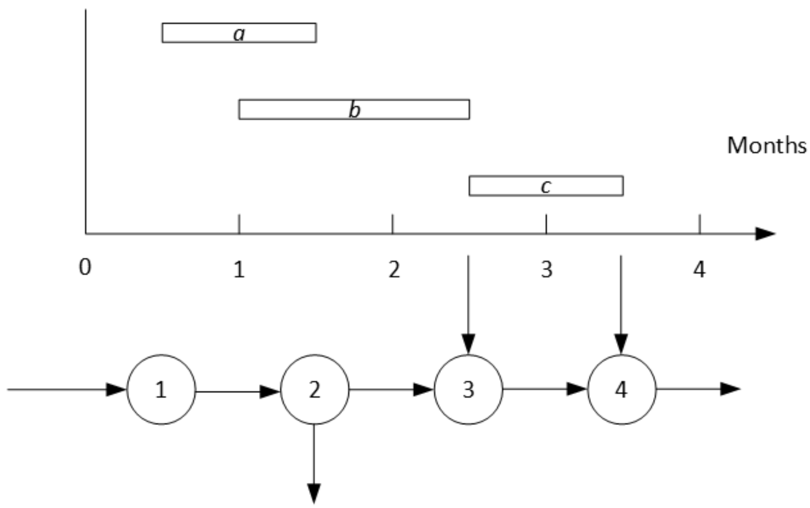
3.1.3.2. Modeling Discounted Cash Flows and Payment Terms

Since payment is made every month, one needs to track the project progress and identify which activities have been completed and should be billed in each month. We present a novel modeling framework that integrates a project network consisting of project activities, and a cash flow network consisting of payment time periods. This framework unifies the project activity progress measured in days and payment interval in months. With proper adjustment, it is flexible enough to cope with arbitrary length of payment interval, e.g., bi-month, quarter, etc. The modeling framework, called integrated project-payment network (IPPN), and its comparison with the traditional modeling approach (Kimms, 2001), can be depicted by Figure 3.4.

In the traditional approach, cash inflow or outflow is assumed to incur at either the start or the end of an activity. As shown in Figure 3.4. (a), Activity a has a cash outflow (down arrow), and both Activity b and c have cash inflow (up arrow) at their finish time. While this approach is straightforward and easy to implement, it is only applicable to the case where payment can be made for each individual activity. Such frequent activity-based payment is rarely the case in real life construction project practice, as it does not consider the realistic payment structure and terms. The integrated project-payment network (IPPN) in Figure 3.4.(b) has two subnetworks: a regular project scheduling subnetwork on top, and a cash flow subnetwork below, with each node representing each payment period of a month, and an arc representing the cash flow of all the activities completed during the same month.



(a) Traditional framework to model project cash flow (Kimms 2001)



(b) Integrated project-payment network

Figure 3.4. Comparing traditional framework with integrated project payment

The two subnetworks are linked through the completion time of each activity. For instance, there is no cash flow in Month 1 as no activity is completed in that month. Month 2 has a cash outflow due to completion of Activity a, and Month 3 has a cash inflow with completion of Activity b. The IPPN is capable of handle construction projects with arbitrary payment periods such as a month, bi-month or quarter.

3.1.3.3. Model Formulation

In this section, we present the complete formulation of the MILP model. In MILP, only some of the variables are constrained to be integers (i.e. whole numbers such as -1, 0, 1, 2, etc.) while in integer programming all of the variables are restricted to be integers. For the convenience of exposition and without loss of generality, a typical monthly payment is assumed in the current formulation.

Sets

A: set of project activities

E: set of precedence relationships

K^i: set of available modes of activity $i \in A$

M: set of time periods, e.g. months

Parameters

n: number of activities in the project

Γ: number of months in the planning horizon

dur_{ik}: duration (in days) of activity i when taking mode k $\quad i \in A \quad k \in K^i$

$\widetilde{cost}_{ik}$: direct cost of activity i when taking mode k $\quad i \in A \quad k \in K^i$

T: project deadline (in days)

r: monthly interest rate

Π: total contract value

ρ: daily indirect cost ($ per day)

ψ: markup (% of total cost)

β: agreed proportion of monthly payment to total monthly invoice (%)

Decision variables

x_{ikm}=1 if activity i takes option k and finishes in month m; 0 otherwise $\forall i \in A,\ k \in K^i$, $m \in M$

y_m =1 if indirect cost is incurred in month m; 0 otherwise $\quad m \in M$

p_i: duration (in days) of activity i $\quad i \in A$

c_{im}: direct cost of activity i incurred in month m $\quad i \in A \quad m \in M$

SF_i: scheduled finish day of activity i $\quad i \in A$

MIC_m: indirect cost in month m $\quad m \in M$

MC_m: total direct and indirect cost in month m $\quad m \in M$

IR_m: interest paid in month m $\quad m \in M$

IN_m: payment (cash inflow) received in month m $\quad m \in M$

TIN: total cash inflow

π_m= 0 if the project is not finished in month m; remaining payment otherwise $\quad m \in M$

Net_m: net income received in month m $\quad m \in M$

Objective function

Maximize $\frac{\sum_{m \in M} Net_m}{(1+r)^m}$ (3-3)

Constraints

Subject to:

$\sum_{k \in K^i} \sum_{m \in M} x_{ikm} = 1 \quad \forall i \in A$ (3-4)

$p_i = \sum_{k \in K^i} [(\sum_{m \in M} x_{ikm}) \times dur_{ik}] \quad \forall i \in A$ (3-5)

$c_{ik} = \sum_{k \in K^i} (x_{ikm} \times \widetilde{cost}_{im}) \quad \forall i \in A,\ m \in M$ (3-6)

$SF_i - p_i \geq 0 \quad \forall i \in A$ (3-7)

$SF_j - p_j - SF_i \geq 0 \quad \forall (i,j) \in E$ (3-8)

$SF_n \leq T$ (3-9)

$SF_i \leq \sum_{m \in M} (\sum_{k \in K^i} x_{ikm}) \times m \times 30 \quad \forall i \in A$ (3-10)

$SF_i + 29 \geq \sum_{m \in M} (\sum_{k \in K^i} x_{ikm}) \times m \times 30 \quad \forall i \in A$ (3-11)

$SF_n - 30 \times (m-1) \leq y_m \times T \quad \forall m \in M$

(3-12)

$SF_n - 30 \times (m-1) \geq (y_m - 1) \times m * 30 \qquad \forall m \in M \qquad$ (3-13)

$MIC_m \leq 30 \times y_m \times \rho \qquad \forall m \in M \qquad$ (3-14)

$MIC_m \leq (SF_n - 30 \times y_k \times (k-1)) \times \rho \qquad \forall m \in M \qquad$ (3-15)

$\sum_{m \in M} MIC_k = SF_n \times \rho \qquad$ (3-16)

$MC_m = \sum_{i \in A} c_{im} + MIC_m \qquad \forall m \in M \qquad$ (3-17)

$IR_1 = \ MC_1 \times r \qquad$ (3-18)

$IR_m = (\sum_{\tau=1}^{m-1} IR_\tau + \sum_{\tau=1}^{m} MC_\tau - \sum_{\tau=1}^{m-1} IN_\tau - \sum_{\tau=1}^{m-1} \pi_\tau) \times r \qquad \forall m \in \{2..\Gamma\}$

(3-19)

$IN_1 = 0 \qquad$ (3-20)

$IN_m \leq MC_{m-1} \times (1 + \psi) \cdot \beta \qquad \forall m \in \{2..\Gamma\} \qquad$ (3-21)

$TIN = \sum_{m \in M} IN_m \qquad$ (3-22)

$TIN \leq \Pi \qquad$ (3-23)

$\pi_1 = 0 \qquad$ (3-24)

$\pi_m \leq \sum_{k \in K^i} x_{nk(m-1)} \times \Pi \qquad \forall m \in \{2..\Gamma\} \qquad$ (3-25)

$\pi_k \leq \Pi - TIN \qquad \forall m \in M \qquad$ (3-26)

$Net_m = IN_m + \pi_m - MC_m - IR_m \qquad m \in M \qquad$ (3-27)

$x_{ikm}, y_m \in \{0, 1\} \qquad$ (3-28)

$p_i, c_{im}, SF_i, MIC_m, MC_m, IR_m, TIN, IN_m,\ \pi_m, Net_m \geq 0 \qquad$ (3-29)

The objective function (3-3) maximizes the total discounted NPV of the project. Constraint (3-4) assigns exactly one option to an activity, and exactly one month for the activity to finish. That is, no preemption is allowed (once an activity is started, it cannot be interrupted). Constraints (3-5) and (3-6) compute the duration (in days) and cost of each activity, respectively, based on the option chosen. In (3-6), we assume that the direct cost always incur during the month in which the activity is completed. Constraint (3-7) ensures that each activity's starting time is no less than zero. Constraint (3-8) satisfies the precedence relationship between a pair of activities (i, j), i.e. activity j cannot start until activity i is completed. Constraint (3-9) guarantees that the project is completed by the given deadline T (in days). Constraints (3-10) and (3-11) together establish the relationship between the scheduled finish time SF (in days) and the binary decision variable x (in months). For example, if $SF_i = 40$ days, then $\sum_{k\in K^i} x_{ik2}$ and only $\sum_{k\in K^i} x_{ik2}$ would be 1, indicating the activity is completed in month 2. Constraints (3-12) and (3-13) together identify whether the project is active in each month. For example, if the project is expected to be completed in 50 days (SF_n=50 days), then y_1 and y_2 (and only y_1 and y_2) would be 1, indicating the project is active in month 1 and month 2. Constraint (3-14) – (3-16) compute the monthly indirect cost. Specifically, Constraint (3-14) states that a monthly indirect cost is only incurred when the project is active in that month, and together with Constraint (3-16), the monthly indirect cost is calculated as 30-day indirect cost if the month is not the ending month of the project. Constraint (3-15) ensures the correct computation of the indirect cost for the ending month of the project. Constraint (3-16) makes sure that the total indirect costs over all active months equal to the project make-span (in days) multiply by the daily overhead cost rate. Constraint (3-17) computes the total monthly cost as the summation of total monthly direct cost of all activities completed in that month, and the monthly indirect cost. Constraints (3-18) and (3-19) compute the interests paid in Month 1 and the following months, respectively. A company typically has to use bank loan to finance the first month of the project to pay off material costs, equipment costs, and labor costs. In Constraint (3-19) the monthly interest is computed as the cumulative net cash flow in that month multiplied by interest rate. Constraint (3-20) fixes the Month 1 cash inflow to be zero. Constraint (3-21) specifies that the payment (cash inflow) of a month (starting from Month 2) cannot exceed a proportion β of the total cost of the previous month augmented by a markup rate ψ. Constraint (3-22) obtains the total payment received, and Constraint (3-23) makes sure that it does not exceed the contract value. Collectively, Constraints (3-24) – (3-26) identify the remaining of account receivable at the project completion time. Constraint (3-24) forbids the project to complete in Month 1. Constraint (3-25) forces the remaining payment in a month to be zero if the project does not complete in that month. Constraint (3-26) ensures that remaining payment plus the total payment realized does not exceed the total contract value. Constraint (3-27) calculates the net income of each month. Constraints (3-28) and (3-29) specify the domain of binary and continuous decision variables, respectively.

We emphasize two key features of the MILP model. In addition to the typical constraints for modeling project crashing and precedence relationships, i.e. Constraints (3-4) – (3-9), the model also contains new constraints for establishing the link between the two subnetworks in the IPPN, which are essential for proper allocation and scheduling of payments. Furthermore, the parameter $\widetilde{cost}_{im}$ is assumed to vary during project execution, which calls for a new solution approach to be presented below.

3.1.3.4. Solution Framework

The MILP model is NP (Non-Deterministic Polynomial)-hard because it includes the discrete time-cost tradeoff problem as a special case, which is well-known to be NP-hard (De, Dunne, Gosh, & Wells, 1997). The fact that direct cost of an activity when taking an option may vary with the productivity during project execution motivates a sequential solution procedure. The start of each month is a decision point when the activity option (crashing) and scheduling decisions are made, while observing new information about productivity to update the time-cost relationship. As shown in Figure 3.5., an initial resource allocation and scheduling decision is made at the start of the project based on the existing information and data available to the decision-maker. Then external disturbance occurs, which impacts the productivity thus cost of project execution. Such disturbance is observed and utilized to update the time-cost tradeoff data, which is next fed into the MILP model to obtain new allocation and scheduling decisions for the remaining activities. Solutions generated in this way are adaptive to the current state of the project progress and productivity.

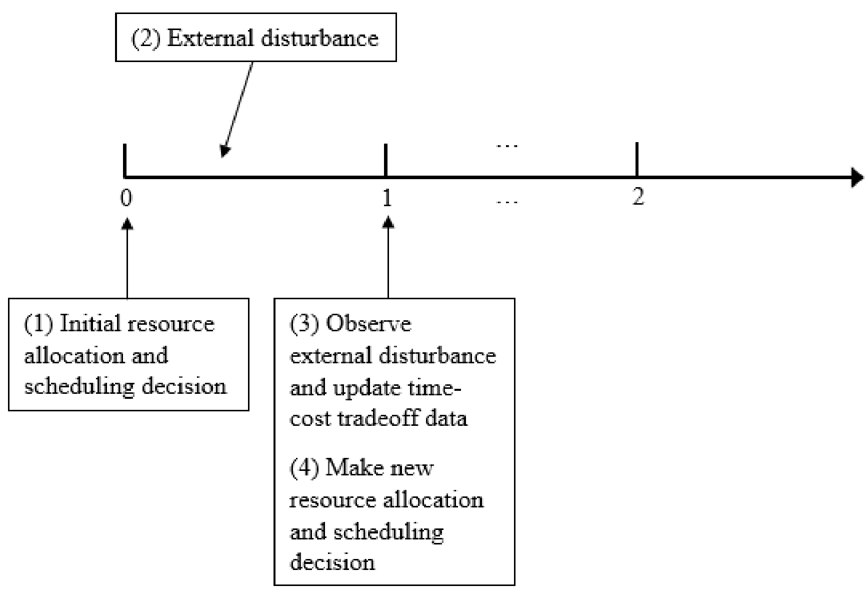


Figure 3.5. Solution framework

In our solution framework, productivity is assumed to be a function of standard productivity and crew size following the factor model of (H. R. Thomas & Sakarcan, 1994). That is:

$$E_t = f(I_s, \delta), \tag{3-30}$$

where E_t is the predicted productivity for time period t, I_s is the productivity in the standard conditions, and δ is the crew size. The functional form $f(\cdot)$ is nonlinear to model the diminishing return of crew size. Every month, information about status of the scheduled (started) activities becomes available and is utilized to update the productivity of utilized resources. Then for every unscheduled activity i that requires the corresponding resource(s), its cost $\widetilde{cost}_{ik}$ when selecting option k is updated using (3-30). At the same time, the finish time SF_i of a completed activity i is fixed at its realized finish time $\overline{SF_i}$, i.e. $SF_i \coloneqq \overline{SF_i}$; and the option decision x_{ikm} of i is fixed at its selected option $\bar{x}_{ikm}$. Next, the MILP model is solved with updated values of $\widetilde{cost}_{ik}$. The obtained solution is implemented for the activities that are eligible to start at the current month. (An activity is *eligible* to start if and only if all its predecessors in the project network have been completed.)

3.2. Phase II

In this phase. the power of Fuzzy Set Theory (FST) is utilized in assessing the combinatory effect of different known risk factors on activity durations at initial phases of project. Bayesian Networks (BNs) are used in monitoring and predicting the project performance. Developed hybrid Fuzzy-Bayesian model is combined with an algorithm to schedule tasks and to optimally redistribute resources to maximize the net present value of the project. The tool will be developed in three phases:

Phase 1: Developing a risk assessment model based on FST to evaluate the probability of increase or decrease in activity durations when influenced by a combination of different risk factors *(duration prior-probability distribution)*.

Phase 2: Developing a BN model to update the prior probability distributions of activity durations by combining them with the actual activity data during the project execution *(duration posterior-probability distribution).*

Phase 3: Integrating the developed hybrid model into an optimization procedure for scheduling tasks and optimally redistributing resources in order to maximize the NPV of the project.

3.2.1. Fuzzy Risk Assessment

This section discusses a flexible and intelligent fuzzy risk assessment model to assess the combinatory effect of different risk factors on task durations. The proposed model is different from fuzzy risk assessment models in literature due to incorporating the confidence degree of decision makers in fuzzy computations by using the interval-valued fuzzy numbers. Interval-valued fuzzy numbers have the capability of capturing and combining the linguistic evaluations of experts with their confidence degrees regarding their assessment on probability of risk and its impact. Employing the interval-valued fuzzy numbers and a discrete fuzzy weighted average algorithm, a set of prior-probability distributions for project activity durations when influenced by a combination of different risk factors are generated. To achieve this, at first experts` linguistic evaluations and their confidence degree regarding the likelihood and severity of the effects of each risk factor on activity durations are translated into appropriate interval-valued fuzzy numbers. An algorithm calculates the fuzzy weighted average of translated values. Two extreme points of calculated fuzzy weighted average and its center of gravity are chosen as “Optimistic”, “Most Likely” and “Pessimistic” activity duration modifiers under combinatory effect of different risk factors. These distributions are later employed as an input in a BN model. This research extends the current fuzzy risk assessment methods by incorporating the confidence degree of decision makers in fuzzy computations and provides a systematic tool to quantify the uncertainty inherent in construction activity durations especially at initial stages of project where project performance data are scarce.

3.2.1.1. Preliminaries

In this section a brief review of interval-valued fuzzy numbers and their arithmetic operations, different linguistic terms applied in proposed model and their corresponding interval-valued fuzzy numbers, fuzzy assessment aggregation, fuzzy weighted average and center of gravity point of a fuzzy number is presented.

Interval-valued fuzzy numbers and their arithmetic operations

Definition 1 (S.-J. Chen & Chen, 2008). An interval-valued fuzzy set A defined in the universe of discourse X is given by

$$A=\left\{\left(x,\left[\mu_A^L(x),\mu_A^U(x)\right]\right)\middle|x\in X\right\},$$

where $0\le\mu_A^L(x)\le\mu_A^U(x)\le 1$ and the membership grade $\mu_A(x)$ of the element x belongs to the interval-valued fuzzy set A, which can be represented by an interval $\mu_A(x)=\left[\mu_A^L(x),\mu_A^U(x)\right]$.

Definition 2 (Hong & Lee, 2002). If an interval-valued fuzzy set A satisfies the following properties:

(1) A is defined in a closed bounded interval,

(2) A is a convex set,

then A is called an interval-valued fuzzy number in the universe of discourse X.

Definition 3 (Chang, Hung, Lin, & Chang, 2006). An interval-valued fuzzy number A on the real line $\Re$ can be represented by the lower and upper bounds (l_A,u_A), mode (m_A), height (w_A) and left-right membership functions ($L-R$) of its lower and upper fuzzy numbers as $A=\left[\left(l_{A^L},m_{A^L},u_{A^L};w_{A^L}\right)_{L-R},\left(l_{A^U},m_{A^U},u_{A^U};w_{A^U}\right)_{L-R}\right]$. The membership function of A which defines the degree of belongingness of elements $x \in \Re$ to A, is denoted as $\mu_A(x) \in [0,1]$ and is defined by the $L(x)$ and $R(x)$. The α-cut, $\alpha \in (0,1]$, of interval-valued fuzzy number A is defined as the ordinary subset $\{x \in \Re \mid \mu_A(x) \geq \alpha\}$ and written as $(A)_\alpha = [a,b]$, where a and b denote the *left* and *right endpoints* of $(A)_\alpha$, respectively.

Assume that there is an interval-valued fuzzy number $A = [A^L, A^U]$ as shown in Figure 3.6. where A^L and A^U are defined as lower and upper fuzzy numbers. Furthermore, the interval-valued fuzzy number A can be represented as $A = [(a_1^L, a_2^L, a_3^L; w_A^L), (a_1^U, a_2^U, a_3^U; w_A^U)]$ where $a_1^L \leq a_2^L \leq a_3^L$, $a_1^U \leq a_2^U \leq a_3^U$, $0 < w_A^L \leq w_A^U \leq 1$ and $A^L \subset A^U$.

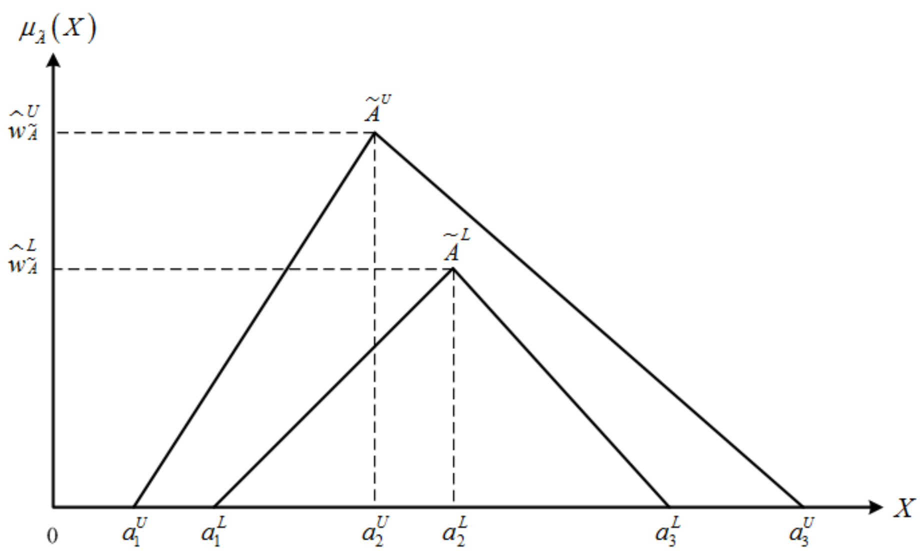


Figure 3.6. Interval-valued fuzzy number *A*

Assume that there are two interval-valued fuzzy numbers A and B, where $A = [(a_1^L, a_2^L, a_3^L; w_A^L), (a_1^U, a_2^U, a_3^U; w_A^U)]$ and $B = [(b_1^L, b_2^L, b_3^L; w_B^L), (b_1^U, b_2^U, b_3^U; w_B^U)]$. The arithmetic operations between the interval- valued fuzzy numbers A and B are as follows (Hong & Lee, 2002):

(1) Interval-valued fuzzy number addition:

$$A \oplus B = \left[\left(a_1^L, a_2^L, a_3^L; w_A^L\right), \left(a_1^U, a_2^U, a_3^U; w_A^U\right)\right] \oplus \left[\left(b_1^L, b_2^L, b_3^L; w_B^L\right), \left(b_1^U, b_2^U, b_3^U; w_B^U\right)\right]$$

$$= \left[\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L; \text{Min}\left(w_A^L, w_B^L\right)\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U; \text{Min}\left(w_A^U, w_B^U\right)\right)\right]$$

(3-31)

Where $a_1^L, a_2^L, a_3^L, a_1^U, a_2^U, a_3^U, b_1^L, b_2^L, b_3^L, b_1^U, b_2^U, b_3^U$ are any real values, $0 < w_A^L \le w_A^U \le 1$ and $0 < w_B^L \le w_B^U \le 1$.

(2) Interval-valued fuzzy number subtraction:

$$A - B = \left[\left(a_1^L, a_2^L, a_3^L; w_A^L\right), \left(a_1^U, a_2^U, a_3^U; w_A^U\right)\right] - \left[\left(b_1^L, b_2^L, b_3^L; w_B^L\right), \left(b_1^U, b_2^U, b_3^U; w_B^U\right)\right]$$

$$= \left[\left(a_1^L - b_3^L, a_2^L - b_2^L, a_3^L - b_1^L; \text{Min}\left(w_A^L, w_B^L\right)\right), \left(a_1^U - b_3^U, a_2^U - b_2^U, a_3^U - b_1^U; \text{Min}\left(w_A^U, w_B^U\right)\right)\right]$$

(3-32)

Where $a_1^L, a_2^L, a_3^L, a_1^U, a_2^U, a_3^U, b_1^L, b_2^L, b_3^L, b_1^U, b_2^U, b_3^U$ are any real values, $0 < w_A^L \le w_A^U \le 1$ and $0 < w_B^L \le w_B^U \le 1$.

(3) Interval-valued fuzzy number multiplication:

$$A \otimes B = \left[\left(a_1^L, a_2^L, a_3^L; w_A^L\right), \left(a_1^U, a_2^U, a_3^U; w_A^U\right)\right] \otimes \left[\left(b_1^L, b_2^L, b_3^L; w_B^L\right), \left(b_1^U, b_2^U, b_3^U; w_B^U\right)\right]$$

$$= \left[\left(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L; \text{Min}\left(w_A^L, w_B^L\right)\right), \left(a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U; \text{Min}\left(w_A^U, w_B^U\right)\right)\right]$$

(3-33)

Where $a_1^L, a_2^L, a_3^L, a_1^U, a_2^U, a_3^U, b_1^L, b_2^L, b_3^L, b_1^U, b_2^U, b_3^U$ are any real values, $0 < w_A^L \le w_A^U \le 1$ and $0 < w_B^L \le w_B^U \le 1$.

(4) Interval-valued fuzzy number division:

$$A/B = \left[\left(a_1^L, a_2^L, a_3^L; w_A^L\right), \left(a_1^U, a_2^U, a_3^U; w_A^U\right)\right] / \left[\left(b_1^L, b_2^L, b_3^L; w_B^L\right), \left(b_1^U, b_2^U, b_3^U; w_B^U\right)\right]$$
$$= \left[\left(a_1^L / b_3^L, a_2^L / b_2^L, a_3^L / b_1^L; \text{Min}\left(w_A^L, w_B^L\right)\right), \left(a_1^U / b_3^U, a_2^U / b_2^U, a_3^U / b_1^U; \text{Min}\left(w_A^U, w_B^U\right)\right)\right]$$

(3-34)

Where $a_1^L, a_2^L, a_3^L, a_1^U, a_2^U, a_3^U, b_1^L, b_2^L, b_3^L, b_1^U, b_2^U, b_3^U$ are all nonzero positive real numbers or all nonzero negative real numbers, $0 < w_A^L \le w_A^U \le 1$ and $0 < w_B^L \le w_B^U \le 1$.

Linguistic terms and their corresponding interval-valued fuzzy numbers

In developed fuzzy risk assessment model, each linguistic term is translated into appropriate interval-valued fuzzy number through a triangular fuzzy membership function. A fuzzy membership function is a curve to map each point in the input space to a degree of membership between 0 and 1. Height (w_A^L) of the lower fuzzy number (A^L) is defined by confidence degree of experts while height (w_B^U) of upper fuzzy number (A^U) is set as 1. In this paper, linguistic terms and their corresponding interval-valued fuzzy numbers are adopted from (S.-M. Chen & Chen, 2009) and shown in Table 3.1 but obviously they can be defined based on project specifications and experts judgments.

Fuzzy assessment aggregation

The fuzzy average operation is used to aggregate experts fuzzy evaluations (Bojadziev & Bojadziev, 2007). Assume *n* experts participate in the assessment process and their linguistic evaluations are translated into interval-valued fuzzy number A_i where $A_i = \left[\left(a_{1i}^L, a_{2i}^L, a_{3i}^L; w_{A_i}^L\right), \left(a_{1i}^U, a_{2i}^U, a_{3i}^U; w_{A_i}^U\right)\right]$ for $i = 1, 2, \ldots, n$. The fuzzy average operation is defined as:

$$A_{average} = \frac{\left(A_1 \oplus A_2 \oplus \cdots \oplus A_n\right)}{n}$$

$$= \left[\frac{\left(\sum_{i=1}^{n} a_{1i}^L, \sum_{i=1}^{n} a_{2i}^L, \sum_{i=1}^{n} a_{3i}^L\right)}{n}; \text{Min}\left(w_{A_i}^L\right), \frac{\left(\sum_{i=1}^{n} a_{1i}^U, \sum_{i=1}^{n} a_{2i}^U, \sum_{i=1}^{n} a_{3i}^U\right)}{n}; \text{Min}\left(w_{A_i}^U\right) \right]$$

(3-35)

Fuzzy weighted average

Weighted average operation ($\overline{W}$) is commonly used in risk and decision analysis to aggregate the evaluated items; "Probability of Failure" (R_i) and "Severity of Loss" (W_i) of each object [(S.-J. Chen & Chen, 2008), (L.-W. Lee & Chen, 2008), (S.-M. Chen & Wang, 2009), (S.-M. Chen & Sanguansat, 2011)]. When the terms R_i and W_i are represented by fuzzy sets or fuzzy numbers, $\overline{W}$ is referred to as a Fuzzy Weighted Average (FWA) (Ngai & Wat, 2005).

Definition 4 (Chang et al., 2006). The FWA can be defined by letting the probability of failure (R_i) and severity of loss (W_i), $i = 1, 2, \ldots, n$, of each object be represented as interval-valued fuzzy numbers $R_1, R_2, \ldots, R_n$ on the universes $X_1, X_2, \ldots, X_n$ with the membership functions $\mu_{R_i}\left(x_i\right)$ and $W_1, W_2, \ldots, W_n$ on the universes $Z_1, Z_2, \ldots, Z_n$ with the membership functions

$\mu_{W_i}(z_i)$. Consider a function f mapping from $X_1 \times X_2 \times \cdots \times X_n \times Z_1 \times Z_2 \times \cdots \times Z_n$ to the universe Y. The FWA ($\overline{W}$) can be defined by:

$$\overline{W} = \frac{W_1 \otimes R_1 \oplus W_2 \otimes R_2 \oplus \cdots \oplus W_n \otimes R_n}{W_1 \oplus W_2 \oplus \cdots \oplus W_n} \tag{3-36}$$

Two exact analytical and discrete algorithm approaches are proposed to compute the FWA [(Baas & Kwakernaak, 1977), (Dong & Wong, 1987), (Liou & Wang, 1992), (H. Q. Yang, Yao, & Jones, 1993), (Guh, Hon, Wang, & Lee, 1996), (D. H. Lee & Park, 1997), (Guu, 2002)]. Compared to the exact analytical approach discrete algorithms are less complicated, more efficient and have dramatically improved the complexity of the FWA computations (Chang et al., 2006).

(Dong & Wong, 1987) applied the vertex method and proposed an algorithm to compute the FWA based on α-cut representation of fuzzy sets, extension principle and interval analysis. Further, (Liou & Wang, 1992) presented an algorithm to improve the complexity of the method developed by (Dong & Wong, 1987). The computational requirements of both (Dong & Wong, 1987) and (Liou & Wang, 1992) methods were improved by the algorithm proposed by (Guh et al., 1996). (D. H. Lee & Park, 1997) and (Guu, 2002) also developed algorithms to reduce the number of comparisons and arithmetic operations of previous approaches. Among all algorithms developed to compute the FWA, the one proposed by (Guh et al., 1996) (*Max-Min Paired Elimination*) is used in applied fuzzy risk assessment model due to lesser calculation complexity. This algorithm is presented in the following section.

Center of gravity point:

The center of gravity point $\left(x^{*}, y^{*}\right)$ of generalized fuzzy number $A=\left[\left(a_1, a_2, a_3; w_A\right)\right]$ (a fuzzy number where $0<w_A\leq 1$ and a_1, a_2 and a_3 are real numbers) based on the concept of geometry is calculated as follows (S.-J. Chen & Chen, 2003):

$$y_A^{*}=\frac{w_A}{2} \tag{3-37}$$

$$x_A^{*}=\frac{y_A^{*}\left(2a_2\right)+\left(a_1+a_3\right)\left(w_A-y_A^{*}\right)}{2w_A} \tag{3-38}$$

3.2.1.2. Methodology

After defining the affecting risk factors on project activity durations, activities are correlated based on their crew type (i.e. concrete work). Experts then evaluate the likelihood of increase or decrease in correlated activity durations when influenced by different risk factors in linguistic terms along with their confidence degree. Three confidence degree levels are considered as High (91%-100%), Medium (81%-90%) and Low (71%-80%). Effect of different confidence degree levels on duration modifiers is tested in case study section. These evaluations are then translated into appropriate interval-valued fuzzy numbers and aggregated to form a triangular distribution for each risk factor whose height is equal to averaged confidence degree of experts. Using the Max-Min Paired Elimination algorithm, fuzzy weighted average of all risk factors is calculated. Two extreme points of calculated fuzzy weighted average and its center of gravity are chosen as “Optimistic”, “Most Likely” and “Pessimistic” activity duration modifiers under combinatory effect of different risk factors, respectively. Using interval-valued fuzzy numbers rather than the classic fuzzy numbers to represent the probability of failure and severity of loss of each sub-component makes the proposed model more flexible and more intelligent than the conventional fuzzy risk analysis methods for handling the risk assessment problems (S.-J. Chen & Chen, 2008). The fuzzy risk assessment model used in this study is presented in following steps:

Step 1: Different risk factors affecting project activity durations are identified and correlated activities are grouped based on their nature (crew members, activity type, etc.). For example, concrete work activities which share the same crew members may be grouped in "Concrete Work" group of activities.

Step 2: Experts express their evaluations of probability of failure (R_i) and severity of loss (W_i) of each risk factor occurrence in linguistic terms along with their confidence degree (w_i). R_i and W_i are defined as the possible activity duration increase under effect of each risk factor i and the effect of delayed activities on project completion time or objectives.

Step 3: Experts` linguistic evaluations are translated into appropriate interval-valued fuzzy numbers using pre-defined values in Table 3.1.

Step 4: Using arithmetic operation (3-31) and fuzzy assessment aggregation (3-35), the interval-valued fuzzy numbers in Step 3 are aggregated and averaged for each risk factor.

Step 5: Based on Definition 3, the Left-Right membership functions of lower and upper fuzzy numbers as well as their respective intervals are calculated. The lower and upper bounds for each interval are $L = \min\{f_L\}$ and $U = \max\{f_U\}$.

Step 6: Applying the Max-Min Paired Elimination algorithm, for different α-cuts, $\alpha \in (0,1]$ the largest and smallest rating coefficients and their matched weighting factors for $\max\{f_U\}$ and $\min\{f_L\}$ are chosen and combined into a new one.

Table 3.1. Linguistic terms and their corresponding interval-valued fuzzy numbers

Linguistic Terms	A^L	$w_A{}^L$	A^U	$w_A{}^B$
Absolutely-Low	(0.0000,0.0000,0.0000)	by Expert	(0.0000,0.0000,0.0000)	1.00
Very-Low	(0.0075,0.0300,0.0525)	by Expert	(0.0000,0.0350,0.0700)	1.00
Low	(0.0875,0.1350,0.1825)	by Expert	(0.0400,0.1350,0.2300)	1.00
Fairly-Low	(0.2325,0.2950,0.3575)	by Expert	(0.1700,0.2950,0.4200)	1.00
Medium	(0.4025,0.4850,0.5675)	by Expert	(0.3200,0.4850,0.6500)	1.00
Fairly-High	(0.6500,0.7200,0.7900)	by Expert	(0.5800,0.7200,0.8600)	1.00
High	(0.7825,0.8450,0.9075)	by Expert	(0.7200,0.8450,0.9700)	1.00
Very-High	(0.9475,0.9700,0.9925)	by Expert	(0.9300,0.9650,1.0000)	1.00
Absolutely-High	(1.0000,1.0000,1.0000)	by Expert	(1.0000,1.0000,1.0000)	1.00

Max-Min Paired Elimination Algorithm to Compute Fuzzy Weighted Average (Guh et al., 1996)

The computational algorithm for each $\alpha_j, j=1,2,\ldots,$m, is summarized as follow and a numerical example is shown in Appendix for clarification:

(1) Find the largest rating coefficient, say $a_1, a_1 \geq a_i, b_1, b_1 \geq b_i$ and the smallest rating coefficient, say $a_n, a_n \leq a_i, b_n, b_n \leq b_i$, for all $i=1,2,\ldots,n$.

(2) For $\min\{f_L\}$, choose c_1 as the corresponding weighting to a_1, choose d_n as the corresponding weighting to a_n.

For $\max\{f_U\}$, choose d_1 as the corresponding weighting to b_1, choose c_n as the corresponding weighting to b_n.

(3) Combine a_1, a_n, and their corresponding weighting c_1, d_n into a new rating coefficient a' and its corresponding weighting w'.

For $\min\{f_L\}$.

$$a' = \frac{a_1 c_1 + a_n d_n}{c_1 + d_n}, \quad w' = c_1 + d_n, \quad c' = d' = w' \tag{3-39}$$

Combine b_1, b_n, and their corresponding weighting d_1, c_n into a new rating coefficient b' and its corresponding weighting w'.

For $\max\{f_U\}$.

$$b' = \frac{b_1 d_1 + b_n c_n}{d_1 + c_n}, \quad w' = d_1 + c_n, \quad c' = d' = w' \tag{3-40}$$

(4) Eliminate a_1, a_n and their corresponding weighting factors c_1, d_n, replace with a' and its corresponding weighting w'. Eliminate b_1, b_n and their corresponding weighting factors d_1, c_n, replace with b' and its corresponding weighting w'. Merge the newly generated criteria and their weighting with the existing ones.

(5) Repeat steps 1 through 4 for $(n-1)$ times, the final $[a', b']$ will be the solution for interval of α_j.

Repeat the above procedure for each α_j.

Step 7: Utilizing Equations (3-37) and (3-38), the lower and upper center of gravity points of computed fuzzy weighted average are defined.

Step 8: Values for α-cut = 0 and center of gravity points are selected and transformed into Type-1 fuzzy numbers as follow and used as the "Optimistic", "Most-Likely", and "Pessimistic" duration modifiers under combinatory effect of different risk factors.

$$\text{Optimistic} = \frac{\left(\min\left\{f_L\right\}_L + \min\left\{f_L\right\}_U\right)}{2} \tag{3-41}$$

$$\text{Most-Likely} = \frac{\left(x_L^* + x_U^*\right)}{2} \tag{3-42}$$

$$\text{Pessimistic} = \frac{\left(\max\left\{f_U\right\}_L + \max\left\{f_U\right\}_U\right)}{2} \tag{3-43}$$

These subjective assessments will be updated using actual activity data during project execution through a learning dynamic Bayesian Network model.

3.2.2. Constructing the Bayesian Network

This dissertation introduces a hybrid Fuzzy-Bayesian Belief Network dynamic scheduling model (F-BBN) which employs FST to elicit subjective estimates from experts and develop them into subjective prior probability distributions for Bayesian analysis. The model is implemented using AgenaRisk toolset due to its user-friendly interface, ability to handle continuous variables and its capability of building dynamic models.

3.2.2.1. Model Scheduling Network as a Bayesian Network

In this BBN model, each activity's duration is modeled by a 'duration block' including five simulation nodes with continues intervals. Purpose of using simulation nodes is to enable the network to perform the dynamic discretization using the algorithm proposed by (Neil et al., 2007). (Fenton & Neil, 2012) suggest to set the number of simulation iterations at 25 in order to provide a reasonable balance between accuracy and efficiency. The number of iterations is the number of times the whole data file will be used to learn the model parameters. Nodes are labeled as 'Initial Estimate', 'Fuzzy Risk', 'Duration', 'Adjusted Duration' and 'Performance Factor'. Figure 3.7. shows the graphic representation of nodes:

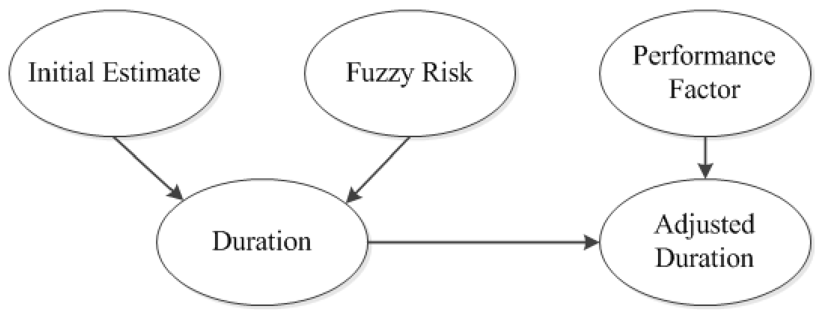


Figure 3.7. Graphic representation of nodes

Description of each node is as follow:

Initial Estimate: This node has uniform distribution with mean and median equal to estimated activity duration. _Activity duration is estimated based on assumed quantity, resources and resource productivity as provided by RS Means 2015 based on US average data.

Fuzzy Risk: This node has triangular distribution which its lower, middle and upper values are determined by proposed fuzzy risk assessment model. Proposed model uses subjective expert assessments to estimate the upper and lower bounds of activity durations based on known risk factors. More details can be found in section 3.2.1.

Duration: This node is defined through probability distributions of its parents (Initial Estimate and Fuzzy Risk nodes). The uniform distribution of Initial Estimate node is modified using the triangular distribution of the Fuzzy Risk node. Mixture of uniform and triangular distributions of Initial Estimate and Fuzzy Risk nodes is performed through Dynamic Discretization. (Fenton & Neil, 2012) provides more information regarding the discretization process.

Performance Factor: This concept was first introduced by (Khodakarami, 2009) and employed for modeling unknown risk factors. It is linked to the Performance Factor nodes of other activities that are affected by the same risk factors. He suggested using truncated normal distribution with mean, variance, lower and upper bounds equal to 0.7, 0.3, 0.5 and 10 which results in an average performance factor of 1 (mean and median are 1.0198 and 0.9557) when no evidence is entered into the model. This subjective probability distribution of Performance Factor will be updated as actual duration data is fed to model.

Adjusted Duration: Is defined through combination of distributions of Duration and Performance Factor nodes by dynamic discretization. Actual activity duration is entered into this node as project progresses and results in updating of its parent node (Performance Factor) based on Bayes' theorem (Equation 2-8). Based on comparison of actual duration with the Duration node, distribution of Performance Factor node gets updated. The updated performance factor is carried to all activities affected by the same risk factor.

3.2.3. Dynamic Updating

To have a better understanding of model behavior when evidence is entered into Adjusted Duration node, one should know the process of building junction tree from BBN and evidence propagation through it. Following is a brief summary of algorithms used for this purpose which is adopted from (Fenton & Neil, 2012).

3.2.3.1. Definitions

Junction tree algorithm: The junction tree algorithm is a general arithmetic framework to calculate the conditional probability of a set of nodes given the observed values. The algorithm decomposes global calculation into local computations using joint probability.

Node parents: Parents are set of nodes which create a child node by use of directed graphs. A conditional distribution for each node given its parents is given as: $P\big(X_i|Parents(X_i)\big)$.

Edges: Edges connect node to form the network. They have two endpoints and may be directed or undirected. Undirected edges are called "lines" and directed edges are called "arcs".

Moral graph: Moral graph is obtained by linking the parents of each node and dropping the directionality of the edges in directed graph.

Clusters: Clusters are subsets of nodes (usually three nodes) of the moral graph which have certain properties.

Complete sub-graph: A complete sub-graph is a form of graph in which all nodes are connected by an edge.

Node weight: Is the number of edges that needed to be added to form a complete sub-graph.

3.2.3.2. Algorithm for Creating a Junction Tree

There are three steps involved in producing the junction tree of a BN:

1. Construct the moral graph (Moralization): In moralization, a directed graph is converted into an undirected graph. This is essential for a uniform treatment of directed and undirected graphs.
2. Triangulate the moral graph (Triangulation): The objective of this step is to identify clusters of the moral graph.

3. Construct the junction tree: In this step clusters obtained in the previous step are connected to form a tree structure.

Step 1: Constructing the Moral Graph

The moral graph is constructed as follow:

1. Define parents of all nodes.
2. Add edges to link parents if an arc does not already connect them.
3. Drop the direction of all arcs.

In developed BBN model, parents of node *Duration* are *Initial Estimate* and *Fuzzy Risk* nodes, parents of node *Adjusted Duration* are *Duration* and *Performance Factor* nodes. All other nodes have no parents. Since nodes *Initial Estimate* and *Fuzzy Risk* are two parents of node *Duration* which are not connected, an edge is added to connect them. Same procedure is followed for nodes *Duration* and *Performance Factor*. After adding edges, direction of all arcs is dropped. Figure 3.8 shows the added edges to *Initial Estimate*, *Fuzzy Risk* and *Duration* nodes. Figure 3.9 illustrates the undirected graph.

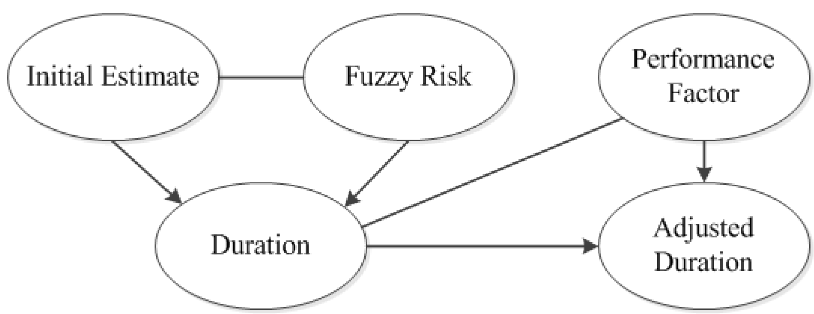


Figure 3.8. Adding edges to link parents of *Duration* and *Adjusted Duration* nodes

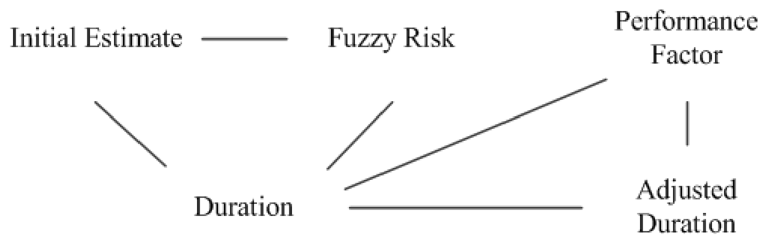


Figure 3.9. Dropping the directions of all arcs

Step 2: Triangulating the Moral Graph

The objective of this step is to identify the clusters. The process involves determining at each step the node with minimum weight and eliminating it. The process is as follow:

1. Determine the node weights.
2. Add the edges to nodes with weight greater than 0 to form a complete sub-graph (set of three nodes connected to each other).
3. Select node with minimum weight to be eliminated. Selected node and its neighbors are defined as cluster.
4. Remove selected node from the graph.
5. Continue until all nodes are eliminated.

In our duration block, all nodes and their neighbors form a complete sub-graph; hence, their weights are zero. Since all nodes have weight of zero, elimination can start from any of them. Let's choose *Adjusted Duration*; Since *Duration* and *Performance Factor* are already connected and form a complete sub-graph, there is no need to add any more edge. Eliminate node *Adjusted Duration*; the first cluster forms as *Duration-Performance Factor-Adjusted Duration (DPA)* and the new moral graph becomes as shown in Figure 3.10.

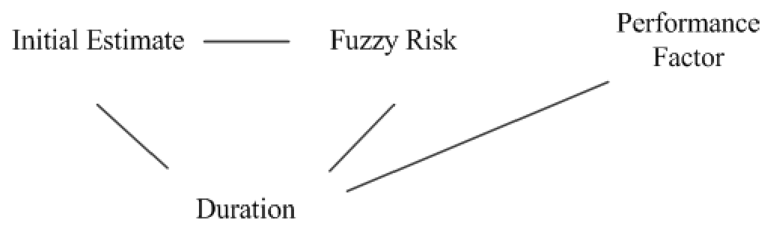


Figure 3.10. Moral graph phase I

The next candidate for elimination is *Duration*. Same process followed and the next cluster is *Initial Estimate-Fuzzy Risk-Duration (IFD)*. Remaining nodes are *Initial Estimate, Fuzzy Risk* and *Performance Factor*. *Initial Estimate* and *Fuzzy Risk* nodes each need one edge (dotted line) to form a complete sub-graph while *Performance Factor* node requires two edges. So, the *Initial Estimate* and *Fuzzy Risk* nodes have node weight equal to one while *Performance Factor* has node weight equal to two.

Figure 3.11. shows the resultant moral graph after eliminating the *Duration* node. Next clusters will be *Initial Estimate-Fuzzy Risk-Performance Factor (IFP), Initial Estimate-Performance Factor (IP)* and *Initial Estimate (I).* List of clusters is represented in Table 3.2.

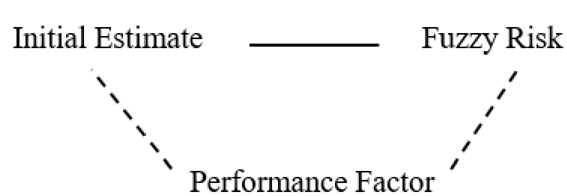


Figure 3.11. Moral graph phase II

Table 3.2. List of clusters

Forming Nodes	Name
Duration-Performance Factor-Adjusted Duration	DPA
Initial Estimate-Fuzzy Risk-Duration	IFD
Initial Estimate-Fuzzy Risk-Performance Factor	IFP
Initial Estimate-Performance Factor	IP
Initial Estimate	I

Step 3: Construct the Junction Tree

The purpose of this step is to connect the clusters obtained in the previous step to form a tree not a graph. Probabilistic inference is much more easier and faster in a junction tree structure. Following is the process for constructing the junction tree:

1. Remove any clusters that are not maximal (subset of another cluster). Usually clusters with less than three nodes are considered as not maximal clusters.
2. The junction tree is formed by inserting edges for the *n* remaining clusters until all clusters are connected by (*n*-1) edges. These edges are also counted as "separators".
3. Separators which connect the highest number of nodes (members) are selected to be included in the junction tree.

Clusters I (Initial Estimate) and IP (Initial Estimate-Performance Factor) are discarded since they are both subsets of IFP (Initial Estimate-Fuzzy Risk-Performance Factor) and not maximal. Remaining clusters are IFP (Initial Estimate-Fuzzy Risk-Performance Factor), IFD (Initial Estimate-Fuzzy Risk-Duration) and DPA (Duration-Performance Factor-Adjusted Duration). By discarding clusters I (Initial Estimate) and IP (Initial Estimate-Performance Factor), the separators which connect the highest number of nodes would be between *Duration* and *Fuzzy Risk* nodes (DF) and *Duration* and *Adjusted Duration* nodes (DA). Resultant junction tree is shown in Figure 3.12.

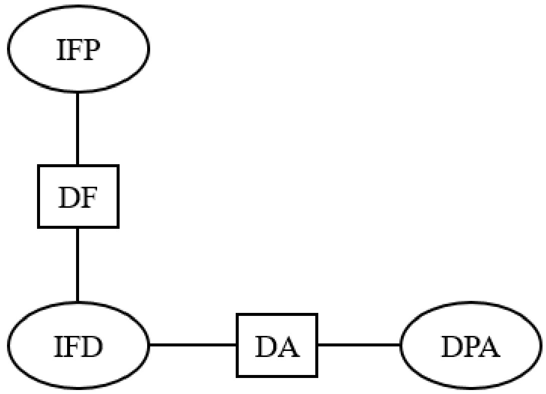


Figure 3.12. Junction tree of duration block

3.2.3.3. Algorithm for evidence propagation in developed junction tree

When evidence in form of task duration is entered in *Adjusted Duration* node, it propagates through developed junction tree shown in Figure 3.12 to update the distribution of *Performance Factor* node. In this section, the propagation process is described. The BN and junction tree for evidence propagation in developed duration block is shown in Figure 3.13 (a) and (b). Note that when an evidence is entered the model, the arc direction changes from *Adjusted Duration* to *Performance Factor* node. For simplicity, nodes are labeled with A, B and C letters.

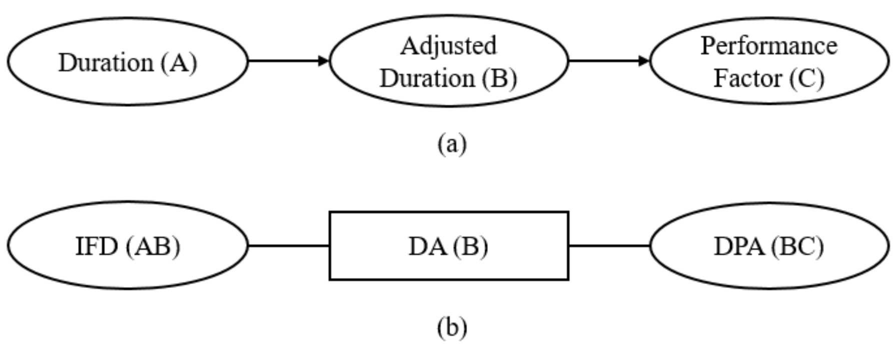


Figure 3.13. (a) BN of duration block for evidence propagation, (b) Resultant junction tree

In the BN shown in Figure 3.13 (a), $P(A, B, C) = P(C|B)P(B|A)P(A)$. *P(A)* is assigned to cluster AB and notation *t(AB)* is used to indicate it. Using the same process, following cluster tables are defined:

$$t(AB) = P(B|A)P(A) \tag{3-44}$$

$$t(BC) = P(C|B) \tag{3-45}$$

In the case of separator DA (B), since the node *Adjusted Duration* (B) has already been assigned to cluster, it can be initiated as 1:

$$t(B) = 1 \tag{3-46}$$

When a task duration is entered in *Adjusted Duration* node (B), an amended table for cluster BC, *t(BC),* is produced which denotes by $t^*(BC)$.

$$t^*(BC) = P(C|B = b_1) \tag{3-47}$$

The conditional change in *Performance Factor* node (C) is calculated through marginalization given the change in *Adjusted Duration* node (B).

$$t^*(C) = \sum_b t^*(BC) \tag{3-48}$$

3.2.4. Optimization Model and Solution Approach

To find the optimum solution approach in probabilistic case, an Excel sheet with built-in Visual Basic (VB) code has been developed. Different task duration and cost options are entered this sheet. These options are generated by combining the discrete task durations with duration modifiers computed by Fuzzy risk assessment. For this purpose, first task duration and cost for three crew sizes are estimated and entered into *Initial Estimate* node in AgenaRisk one at a time for each crew size. A uniform distribution with mean value equal to estimated task duration is assumed for this node. Duration modifiers calculated by Fuzzy risk assessment model are also entered in *Fuzzy Risk* node assuming a triangular distribution with minimum, median and maximum points equal to optimistic, most likely and pessimistic values. For the base case where the project has not been started yet and no actual performance data is available, the performance factor is assumed to be 1. The software combines discrete initial duration values with duration modifiers. It actually increases estimated durations considering the effect of different risk factors. Results are shown in *Duration* node and since the performance factor is 1, same result will be shown in *Adjusted Duration* node. The mean values of distributions in *Adjusted Duration* node are selected as duration options for base case. Using these mean values, corresponding costs would be calculated by considering crew diminishing productivity return, efficiency and other factors.

When the project starts, the scheduling network should be updated using actual performance data. Incorporating actual performance data in scheduling network may produce more realistic duration and cost options. It also has two major benefits: First, it reduces the subjectivity of Fuzzy risk assessment model by using the performance factor modifier; second, can improve the overall project performance prediction. Three different scenarios are assumed for the project and for each scenario, three different duration and cost options for three crew sizes are generated. In each scenario, project is partially completed with different situations (i.e. behind schedule-above cost) and actual performance data is available. These actual data are in form of a discrete value and entered as *evidence* in *Adjusted Duration* node. The AgenaRisk updates the distribution of *Performance Factor* node by propagating the evidence through BN. Updated performance factor is transferred to unfinished related activities and updates the distribution of *Adjusted Duration* node. The mean value of these updated distributions is selected as duration options for each crew configuration. Corresponding costs would be calculated by using these mean values and other related factors such as crew productivity and efficiency.

The VB code finds the Maximum NPV by combining the different duration and cost options. At first, the user should define the predecessor and successor tasks in project's CPM network. The code then reads task durations from the network table, calculates Early Start, Early Finish, Late Start and Late Finish and converts duration from days to whole months using integer division. For each task, the daily cost is calculated by dividing the total cost by duration and adding the daily indirect cost. The solution approach generates a cashflow matrix and calculates the cumulative and total cost, invoice, retained amount, beginning and ending balance, interest and payment by considering the percentages of, markup, retained and monthly interest rate. Finally, the Maximum NPV is calculated and shown. To clarify, the definitions of these terms are as follow:

Cashflow: is the difference between opening balance (available cash at the beginning of month) and closing balance (available cash at the end of month). It is positive if the closing balance is higher than opening balance.

Markup: is the increase to project cost to account for contractor's profit. It is usually represented as percentage of cost.

Retainage: is the amount that is being withheld from total payment to contractor to assure that contractor or subcontractor will satisfy project requirements and complete the project.

Invoice: is the total amount of cost and markup for each month.

Net Present Value (NPV): is the difference between the present value of cash inflows and the present value of cash outflows.

CHAPTER 4

CASE STUDY AND MODEL VALIDATION

4.1. Phase I: Deterministic Case

Proposed decision support model and sequential solution procedure are tested on a simulated bridge project with 38 activities. Table 4.1 lists all the activities with their descriptions, required crew resources and normal duration (in days). For instance, Activity 7 “Make abutment forms” requires Crew C1 and normally takes 4 days. Table 4.2 provides details of each crew with labor and equipment. For example, Crew C14C consists of a carpenter a foreman and a carpenter, and a gas engine vibrator. The activity-on-node (AON) network in Figure 4.1 shows the precedence relationships among activities.

Table 4.1. Activities list of sample bridge project

Activity	Description	Crew ID	Duration (days)
1	Shop drawings: abutment-deck steel	FAB	10
2	Shop drawings: footings steel	FAB	5
3	Move in	N/A	3
4	Deliver piles	N/A	15
5	Shop drawings: girders	FAB	10
6	Deliver footings steel	N/A	7
7	Make abutment forms	C1	4
8	Excavate abutment 1	B10L	4
9	Drive piles abutment 1	B19	3
10	Excavate abutment 2	B10L	1
11	Deliver abutment and deck steel	N/1	15
12	Forms and steel footing 1	C1, ROD	3
13	Drive piles abutment 2	B19	4
14	Pour footing 1	C20	1
15	Strip footing 1	N/A	1
16	Forms and steel abutment 1	C1, ROD	5
17	Forms and steel footing 2	C1, ROD	2
18	Pour abutment 1	C20	1
19	Pour footing 2	C20	1
20	Strip and cure abutment 1	C14C	3

Table 4.1. Activities list of sample bridge project (Continue)

Activity	Description	Crew ID	Duration (days)
21	Strip footing 2	N/A	1
22	Saw abutment 1	B89	14
23	Forms and steel abutment 2	C1, ROD	5
24	Pour abutment 2	C20	1
25	Strip and cure abutment 2	C14C	2
26	Deliver girders	N/A	25
27	Saw abutment 2	B89	7
28	Set girders	C2, ROD	6
29	Forms and steel deck	C1, ROD	5
30	Rub concrete abutment 1	CF	2
31	Pour and cure deck	C20	1
32	Rub concrete abutment 2	CF	4
33	Strip deck forms	C1	4
34	Saw contraction joints	B89	2
35	Painting	B78	1
36	Guardrail	N/A	2
37	Clean up	A5	2
38	Inspection	N/A	1

Table 4.2. Description of crew types

Crew ID	Labor	Equipment
FAB	Fabricator	N/A
C1	Carpenter, Laborer	N/A
B10L	Equipment Operator, Laborer	Dozer (80 HP)
B19	Pile driver foreman, Pile drivers, Crane operator, Oiler operator	Crawler crane (40 Ton), Lead (90' high), Diesel hammer (22k ft-lb)
ROD	Rodman	N/A
C20	Labor foreman, Laborer, Cement finisher, Equipment operator	Gas engine vibrators, Concrete pumps
C14C	Carpenter foreman, Carpenter, Rodman, Laborer	Gas engine vibrators
B89	Equipment operator, Truck driver	Flatbed truck, Concrete saw, Water tank
C2	Carpenter foreman, Carpenter, Laborer	N/A
CF	Cement Finisher	N/A

Table 4.2. Description of crew types (Continue)

Crew ID	Labor	Equipment
B78	Labor foreman, Laborer, Truck driver	Paint striper (40 Gallon), Flatbed truck (3 Ton), Pickup truck (3/4 Ton)
A5	Laborer, Truck driver	Flatbed truck (1.5 Ton)

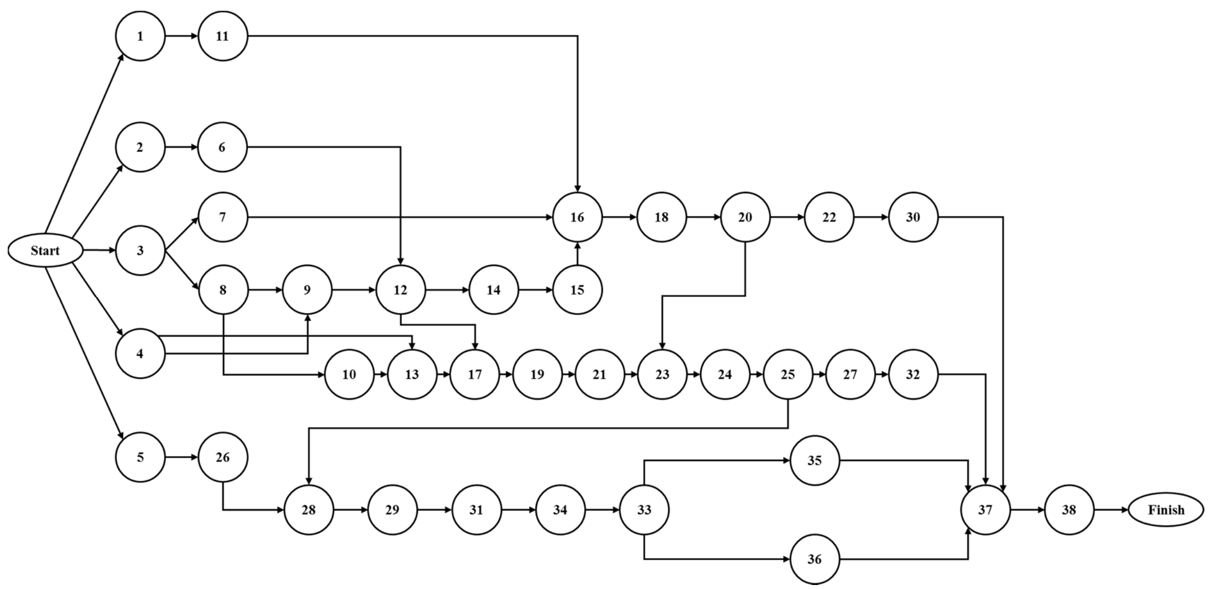


Figure 4.1. The precedence relationships among activities

Detailed activity information including crew requirement, wage and productivity was prepared in the form of Excel spreadsheets. A snapshot of spreadsheet for Activity 7 is shown in Figure 4.2. *Labor Hours* is calculated by dividing the *Total Man Hours* for one crew by the multiplication of *Daily Output per Crew* and *Efficiency* factor. Efficiency factor determines the effectiveness of crew and modifies the duration of unfinished activities based on actual project data. In this example, this factor is set to be one since the project has not started yet. As the project progresses, different values may be assigned to adapt with the crew performance and the current project progress.

Activity 7-Make Abutment Forms

ID	Source	Line Number	Description	Crew	Daily Output	Labor Hours	Quantity	Mat/Unit	Labor/Unit	Equip/Unit	Total/Unit	Total Cost
ACT7	CostWorks 2015	31113453050	C.I.P. Concrete Forms	C1	346.00	0.092	1,380.00	$1.52	$3.93	$0.00	$9.39	**$ 12,956**

Number of Crew	1	Daily Output per Crew	346.00	Efficiency	1.00				Productivity for Crew C1		
Crew C1									Crew Size	Labor Hours	Daily Output
ID	Crew Size	Crew and Equipments	Wage/hr	hr/Day	Total Mhrs	Duration	hr/Unit	Cost/Unit	4	0.092	346.00
C1-C	3	Carpenters	$44.90	8	24	4	0.023	3.114	8	0.147	218.28
C1-L	1	Laborer	$35.45	8	8		0.023	0.820	12	0.143	223.88
Total	4				32	127.63		3.934	16	0.142	224.81

Figure 4.2. A snapshot of spreadsheet for Activity 7

In this case study, productivity is estimated as a nonlinear function of crew size with diminishing returns. Thus, equation (3-1) can be written as:

$$E_t = I_s + 0.0163 \cdot \delta - 0.00156 \cdot \delta^2 + 0.000046 \cdot \delta^3 \quad (4\text{-}1)$$

All the coefficients are taken from (Sanders & Thomas, 1991). *Duration* of each activity is calculated by first multiplying the *Labor Hours* by *Quantity* and then dividing the result by *Total Man Hours*. Number of spent hours per unit is defined by dividing the *Labor Hours* by total crew size. Multiplying this value by *Wage* rate and crew size results in labor cost per unit. Same procedure should be followed for equipment cost. Finally, cost of each activity is calculated by totaling the material, labor and equipment unit costs and multiplying it by *Quantity*. A comparison between nominal and diminishing productivity return for an activity with varying crew size is illustrated in Figure 3.3. While the nominal case assumes constant productivity, the nonlinear function in (Equation 4-1) appropriately captures the decreasing margin of contribution of crew size in real construction project delivery.

Our computational study considers a base case of Before Project Start, when the decision-maker plans for the entire project before it starts. Then three additional scenarios are considered to mimic the situations encountered during project execution: Above Cost – Ahead Schedule, Above Cost – Behind Schedule, and Below Cost – Behind Schedule. In each of the three scenarios, the project is partially completed, and the actual duration and cost of finished activities are used to update the information needed for the updating the new time-cost relationships.

The MILP model[2] can be solved using the branch-and-cut algorithm in integer programming through the state-of-the-art commercially available solver CPLEX 12.1. All computations were executed on a PC with Intel i7 CPU and 8G RAM. It took only seconds to find optimal solution for an instance.

4.1.1. Analysis of the Results

Table 4.3. compares the results of the traditional CPM solution without crashing and optimal solution generated by our optimization approach for four scenarios: the base case where the project has not started and three additional scenarios where the project is partially completed. At the planning stage before the project starts (Scenario-1 in Column 2), the model suggests to complete the project 6 days earlier to achieve a 16% higher NPV than the CPM solution by properly crashing certain activities.

When the project is partially completed above cost and ahead of schedule (Scenario-2 in Column 3), both solutions generate less NPV than planned. However, our solution has 16% higher NPV than the CPM solution, and is able to avoid less loss (4.4% compared with the planned CPM solution, and 17.5% compared with the planned optimal solution).

When the project is partially completed above cost and behind schedule (Scenario-3 in Column 4), both solutions suffer the most loss compared with the planned solutions before the

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[2] Developed by Dr. Haitao Li and Liu Yang from College of Business Administration, University of Missouri – St. Louis.

project starts, and the project make-span will also be prolonged. Also, note that the NPV of our optimization solution generates 19.5% higher NPV than the CPM solution.

When the project is partially completed below cost and behind schedule (Scenario-4 in Column 5), the CPM solution achieves only 0.6% less NPV than the planned schedule before the project starts; while our optimization solution is still able to improve the planned CPM NPV by 2.1%, and is 2.8% higher than the current CPM NPV.

Table 4.3. Results comparison for the base case and three scenarios

	Before Project Start		Above Cost, Ahead Schedule		Above Cost, Behind Schedule		Below Cost, Behind Schedule	
	All-Normal	ILP Model	All-Normal	ILP Model	All-Normal	ILP Model	All-Normal	ILP Model
Discounted Expenses	$424,103	$422,712	$427,127	$425,980	$436,317	$435,092	$423,899	$425,755
Discounted Revenue	$437,736	$438,522	$438,363	$439,019	$437,772	$436,831	$437,446	$436,675
NPV of Profit	$13,634	$15,810	$11,236	$13,039	$1,455	$1,739	$13,547	$13,920
Improvement (model result vs. all-normal)		16%		16%		19.5%		2.8%
Improvement (3 scenarios vs. all-normal before project start)			-17.6%	-4.4%	-89.3%	-87.2%	-0.6%	2.1%

Table 4.3. Results comparison for the base case and three scenarios (Continue)

	Before Project Start		Above Cost, Ahead Schedule		Above Cost, Behind Schedule		Below Cost, Behind Schedule	
	All-Normal	ILP Model	All-Normal	ILP Model	All-Normal	ILP Model	All-Normal	ILP Model
Improvement (3 scenarios vs. ILP before project start)			-28.9%	-17.5%	-90.8%	-89.0%	-14.3%	-12.0%
Project make-span (days)	65	59	63	59	70	69	70	66

In all, when a project is partially completed during execution, our optimization approach can capture the actual productivity and update the corresponding option costs, so that better project crashing and scheduling decisions can be achieved.

4.1.2. Computational Experiments

Additional computational experiments were conducted to examine the impact of problem parameters on the optimal solution. In particular, we would like to understand how the interest rate, indirect cost rate and the size of the contract may affect the optimal project make-span and NPV, as well as how NPV trades off with project make-span.

Experiment I

Sensitivity analysis is conducted to show how optimal project make-span and NPV are influenced by interest rate, indirect cost and contract values. The deadline on project make-span is set to be 90 days. Three parameters are controlled in the experiment. The interest rate varies in the interval of [0, .02] with a .002 increment; indirect cost changes from $182 to $432 in increments of $25; and contract value is set at $460,519 (baseline), and 5% and 10% higher than the baseline. Thus, a total of 363 problem instances are generated and solved by our optimization approach.

Figure 4.3. shows how optimal project make-span vary with interest rate and indirect cost. The optimal make-span decreases as interest rate or indirect cost increases. It becomes inelastic with respect to interest rate after indirect cost reaches $307/day, and an interactive effect between interest rate and indirect cost is also evident. To quantify the relationship, we employ a multiple linear regression model, $y = b_0 + b_1x_1 + b_2x_2 + b_3x_3$, in which dependent variable is the optimal project make-span, and independent variables include interest rate (x_1), indirect cost (x_2) and contract value (x_3). The regression output is summarized in **Error! Reference source not found.**4.4. (row of "A – Incl. 3 factors"). Both interest rate and indirect cost appear to be statistically significant (with negative sign), but the contract value does not. This suggests that a higher interest rate will bring more incentive to crash project activities, such that project make-span can be reduced to save interest cost and indirect cost.

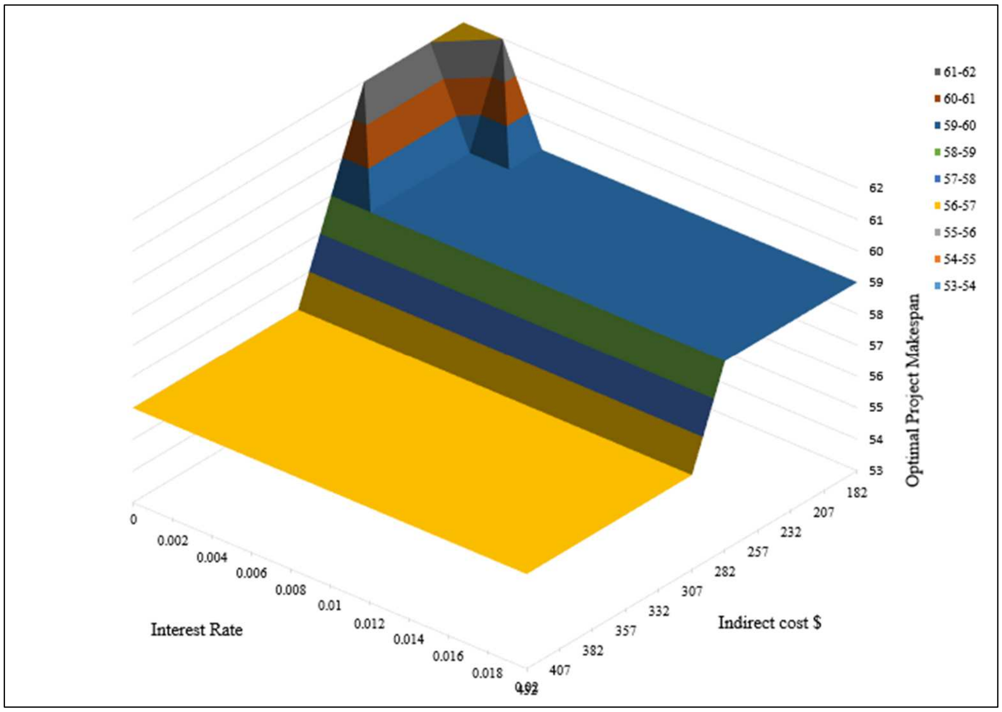

Figure 4.3. How optimal project make-span vary with interest rate and indirect cost

As a second analysis, we remove the insignificant contract value, shown in "B- Incl. 2 factors" of Table 4.4. Adjusted R-square is now slightly improved. We then add an interaction term in our third model, $y = b_0 + b_1x_1 + b_2x_2 + b_4(x_1 \times x_2)$, to capture the interactive effect observed in Figure 4.3. The result is summarized in the last row of Table 4.4 ("C – Incl. interaction"). All three independent variables are statistically significant and adjusted R-square is further improved to 72.28%.

Table 4.4. Regression analysis of optimal project make-span

		b_0	b_1	b_2	b_3	b_4	Adjusted R square
A-Incl.3 factors	Coefficients	63.705	-31.405	-0.018	-0.000001		70.43%
	P-value		<.0001	<.0001	0.84		
B-Incl.2 factors	Coefficients	63.453	-31.405	-0.018			70.51%
	P-value		<.0001	<.0001			
C-Incl. Interaction	Coefficients	64.904	-176.532	-0.023		.0473	72.28%
	P-value		<.0001	<.0001		<.0001	

We now examine the impact of contract value, interest rate and indirect cost rate on optimal NPV in Figure 4.4. Each surface graph represents the dimension of contract value: the bottom one is generated from baseline contract value, the middle has a contract value 5% higher than the baseline, and the top 10% higher. From the graph, all three factors appear to be linearly related to optimal NPV, as interest rate and indirect cost form a rather flat surface and all three surfaces look parallel to each other. To quantify such relationship, we use multiple linear regression: $y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3$, where dependent variable is now the optimal NPV, and independent variables include interest rate (x_1), indirect cost (x_2) and contract value (x_3). We summarize regression result in Table 4.5., which shows that all three factors are statistically significant with p-value close to 0, and this multiple linear regression model provides very good estimate of the maximum profit that can possibly be achieved (adjusted R-square is 99.995%). Note that the contract value variable alone can only explain about 79% of the variation in optimal NPV; but by adding interest rate and indirect cost in the regression model, nearly all the variation can be accounted for. This suggests that the solution obtained from our optimization approach can provide a reliable estimate on the return of a project.

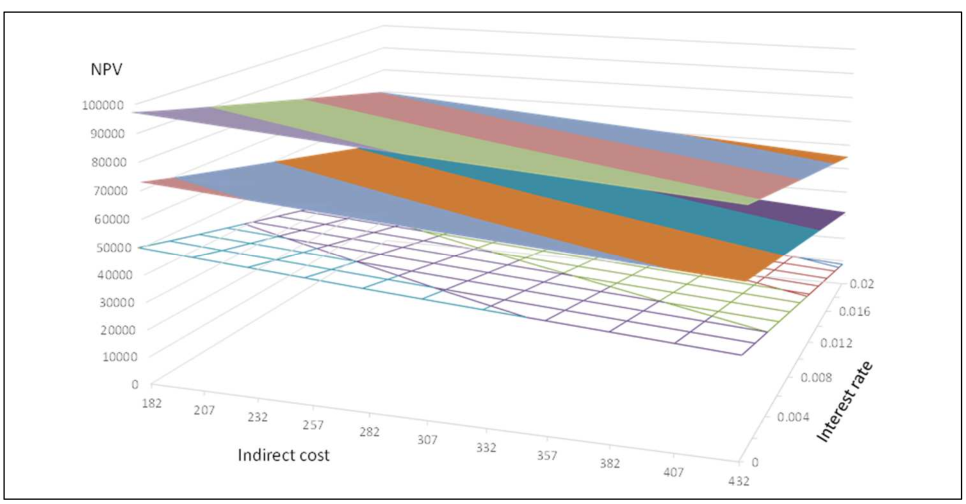

Figure 4.4. The impact of contract value, interest rate and indirect cost rate on optimal NPV

Table 4.5. Regression analysis on optimal NPV

	b_0	b_1	b_2	b_3	Adjusted R square
Coefficients	-395647.247	-1348504.665	-58.097	0.991	99.995%
P-value		close to 0	close to 0	close to 0	

Experiment II

In the second experiment, the relationship between project make-span and NPV is examined. We fix the project make-span in a solution and vary it from 50 days (the minimum possible with all activities crashed at highest cost) to 65 days (the longest value with no crashing), while keeping all other parameters at their baseline values. In Figure 4.5., one observes a nonlinear and concave relationship between the NPV and make-span. The all-crash solution (50 days) is a least profitable one, due to the highest direct cost incurred by crashing all activities with highest cost options. The project return then increases as make-span becomes longer until it reaches 59 days. This reflects the well-known time-cost tradeoff in project crashing. What occurs next is particularly interesting. Although the direct cost continues to decrease with higher make-span, i.e. the time-cost tradeoff still holds, the project return may decrease with longer project make-span. This is caused by significantly more indirect costs and interests paid when the project is unnecessarily prolonged.

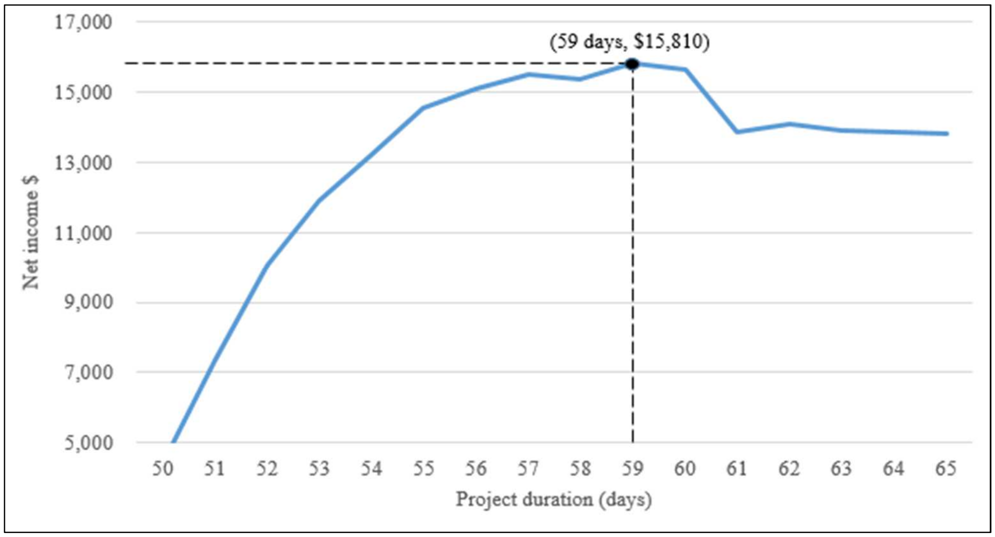

Figure 4.5. Observing a nonlinear and concave relationship between the NPV and make-span

The type of sensitivity analysis in Figure 4.5 will benefit practitioners in the following ways. First, it helps find the optimal project crashing and scheduling decisions, thus optimal NPV and make-span, in an automatic and efficient way. Without an optimization solution approach, the optimal make-span of 59 days will not be intuitive to obtain. Second, when performed before the start of a project, such analysis will help a construction company decide an appropriate project deadline during the negotiation process. For example, when the deadline is very tight, the curve in Figure 4.5 appears to be steep, so that for every one day of extension on make-span, some significantly more NPV can be generated. However, when the deadline approaches the optima (59 days), there appears to be less incentive to negotiate a longer deadline. When it exceeds the optima, having longer make-span will actually reduce the maximum possible project return, thus there is no need to negotiate a longer deadline. Such decision-support offered by our optimization approach will enable the project managers quickly adjust their negotiation strategy and tactics.

4.2. Phase II: Probabilistic Case

To illustrate the application of proposed decision support tool in probabilistic case, the bridge project in phase I is chosen as the case study. List of activities, their descriptions, crew information, initial duration estimate and corresponding precedence network are shown in Tables 4.1., 4.2 and Figure 4.1.

4.2.1. Fuzzy Risk Assessment

Proposed Fuzzy risk assessment model for this case study includes following steps:

Step 1: Four different risk factors relating to scheduling, cost, quality and procurement (Rezakhani, 2012) are considered to affect the project activity durations. Conditions of these risk factors are shown in Table 4.6. Activities 7, 12, 16, 17, 20, 23, 25, 28, 29 and 33 are correlated and share almost same crew members so they may be categorized in one group.

Table 4.6. Risk factors` conditions (Rezakhani, 2012)

Risk Factor	Risk Conditions
1 (Scheduling)	- Errors in estimating time or resource availability. - Poor allocation and management of float. - Scope of work changes without due allowance for time extensions/ acceleration.
2 (Cost)	- Estimating errors, including estimating uncertainty. - Lack of investigation of predictable problems. - Inadequate productivity, cost or change control. - Poor maintenance, security, purchasing, etc.
3 (Quality)	- Poor attitude to quality. - Substandard design/ materials/ workmanship. - Inadequate quality assurance program.
4 (Procurement)	- Unenforceable conditions/ clauses. - Incompetent or financially unsound workers/ contractors. - Adversarial relations. - Inappropriate or unclear contractual assignment of risk.

Step 2: Three experts are assumed to contribute to the risk assessment process. Their linguistic evaluations of probability of failure (R_i) and severity of loss (W_i) of each factor occurrence along with their confidence degree (w_i) are depicted in Table 4.7. It is assumed that all experts are highly confident in their assessment (confidence degrees are in high, 91%-100%, range). To verify the effect of different confidence degree levels on duration modifiers, two other cases where experts' confidence degree levels are in Medium (81%-90%) and Low (71%-80%) range are tested.

Table 4.7. Experts` linguistic evaluations with high level of confidence

Risk Factor	Expert 1		
	R_i	W_i	w_i
Risk Factor 1	Fairly Low	Low	95.00%
Risk Factor 2	Medium	High	94.00%
Risk Factor 3	Fairly Low	Very Low	97.00%
Risk Factor 4	Very Low	Medium	98.00%
	Expert 2		
Risk Factor 1	Medium	Medium	99.00%
Risk Factor 2	Fairly Low	Low	85.00%
Risk Factor 3	Low	Medium	95.00%
Risk Factor 4	Low	Fairly Low	98.00%
	Expert 3		
Risk Factor 1	Low	Medium	93.00%
Risk Factor 2	Fairly Low	Very Low	95.00%
Risk Factor 3	Medium	Fairly Low	94.00%
Risk Factor 4	Fairly Low	Very Low	95.00%

Step 3: Using Table 3.1., experts` evaluations in Step 2 are translated into appropriate interval-valued fuzzy numbers. Resultant fuzzy numbers can be seen in Table 4.8.

Table 4.8. Translating the experts` linguistic evaluations into interval-valued fuzzy numbers

Expert 1				
Risk Factor	R_{1L}	w	R_{1U}	w
Risk Factor 1	(0.2325,0.2950,0.3575)	0.95	(0.1700,0.2950,0.4200)	1.00
Risk Factor 2	(0.4025,0.4850,0.5675)	0.94	(0.3200,0.4850,0.6500)	1.00
Risk Factor 3	(0.2325,0.2950,0.3575)	0.97	(0.1700,0.2950,0.4200)	1.00
Risk Factor 4	(0.0075,0.0300,0.0525)	0.98	(0.0000,0.0350,0.0700)	1.00
	W_{1L}	w	W_{1U}	w
Risk Factor 1	(0.0875,0.1350,0.1825)	0.95	(0.0400,0.1350,0.2300)	1.00
Risk Factor 2	(0.7825,0.8450,0.9075)	0.94	(0.7200,0.8450,0.9700)	1.00
Risk Factor 3	(0.0075,0.0300,0.0525)	0.97	(0.0000,0.0350,0.0700)	1.00
Risk Factor 4	(0.4025,0.4850,0.5675)	0.98	(0.3200,0.4850,0.6500)	1.00
Expert 2				
Risk Factor	R_{2L}	w	R_{2U}	w
Risk Factor 1	(0.4025,0.4850,0.5675)	0.99	(0.3200,0.4850,0.6500)	1.00
Risk Factor 2	(0.2325,0.2950,0.3575)	0.85	(0.1700,0.2950,0.4200)	1.00
Risk Factor 3	(0.0875,0.1350,0.1825)	0.95	(0.0400,0.1350,0.2300)	1.00
Risk Factor 4	(0.0875,0.1350,0.1825)	0.98	(0.0400,0.1350,0.2300)	1.00
	W_{2L}	w	W_{2U}	w
Risk Factor 1	(0.4025,0.4850,0.5675)	0.99	(0.3200,0.4850,0.6500)	1.00
Risk Factor 2	(0.0875,0.1350,0.1825)	0.85	(0.0400,0.1350,0.2300)	1.00
Risk Factor 3	(0.4025,0.4850,0.5675)	0.95	(0.3200,0.4850,0.6500)	1.00
Risk Factor 4	(0.2325,0.2950,0.3575)	0.98	(0.1700,0.2950,0.4200)	1.00
Expert 3				
Risk Factor	R_{3L}	w	R_{3U}	w
Risk Factor 1	(0.0875,0.1350,0.1825)	0.93	(0.0400,0.1350,0.2300)	1.00
Risk Factor 2	(0.2325,0.2950,0.3575)	0.95	(0.1700,0.2950,0.4200)	1.00
Risk Factor 3	(0.4025,0.4850,0.5675)	0.94	(0.3200,0.4850,0.6500)	1.00
Risk Factor 4	(0.2325,0.2950,0.3575)	0.95	(0.1700,0.2950,0.4200)	1.00
	W_{3L}	w	W_{3U}	w
Risk Factor 1	(0.4025,0.4850,0.5675)	0.93	(0.3200,0.4850,0.6500)	1.00
Risk Factor 2	(0.0075,0.0300,0.0525)	0.95	(0.0000,0.0350,0.0700)	1.00
Risk Factor 3	(0.2325,0.2950,0.3575)	0.94	(0.1700,0.2950,0.4200)	1.00
Risk Factor 4	(0.0075,0.0300,0.0525)	0.95	(0.0000,0.0350,0.0700)	1.00

Step 4: Translated fuzzy numbers in Step 3 are aggregated and averaged for each risk factor as Table 4.9.

Table 4.9. Averaged interval-valued fuzzy numbers

Risk Factors	$\overline{R}_L$	w	$\overline{R}_U$	w
Risk Factor 1	(0.2408,0.3050,0.3692)	0.93	(0.1767,0.3050,0.4333)	1.00
Risk Factor 2	(0.2892,0.3583,0.4275)	0.85	(0.2200,0.3583,0.4967)	1.00
Risk Factor 3	(0.2408,0.3050,0.3692)	0.94	(0.1767,0.3050,0.4333)	1.00
Risk Factor 4	(0.1092,0.1533,0.1975)	0.95	(0.0700,0.1550,0.2400)	1.00
	$\overline{W}_L$	w	$\overline{W}_U$	w
Risk Factor 1	(0.2975,0.3683,0.4392)	0.93	(0.2267,0.3683,0.5100)	1.00
Risk Factor 2	(0.2925,0.3367,0.3808)	0.85	(0.2533,0.3383,0.4233)	1.00
Risk Factor 3	(0.2142,0.2700,0.3258)	0.94	(0.1633,0.2717,0.3800)	1.00
Risk Factor 4	(0.2142,0.2700,0.3258)	0.95	(0.1633,0.2717,0.3800)	1.00

Step 5: Utilizing two-dimensional linear equations, the Left-Right membership functions of averaged fuzzy numbers are calculated. Results are shown in Table 4.10.

Table 4.10. Left-Right membership functions

Risk Factor 1			
$\overline{R}_L$		$\overline{R}_U$	
$0.2408 \leq x \leq 0.3050$	$0.3050 \leq x \leq 0.3692$	$0.1767 \leq x \leq 0.3050$	$0.3050 \leq x \leq 0.4333$
Slope: 14.493	Slope: -14.493	Slope: 7.792	Slope: -7.792
Intercept: -3.491	Intercept: 5.351	Intercept: -1.377	Intercept: 3.377
$\overline{W}_L$		$\overline{W}_U$	
$0.2975 \leq x \leq 0.3683$	$0.3683 \leq x \leq 0.4392$	$0.2267 \leq x \leq 0.3683$	$0.3683 \leq x \leq 0.5100$
Slope: 13.129	Slope: -13.129	Slope: 7.059	Slope: -7.059
Intercept: -3.906	Intercept: 5.766	Intercept: -1.600	Intercept: 3.600
Risk Factor 2			
$\overline{R}_L$		$\overline{R}_U$	
$0.2892 \leq x \leq 0.3583$	$0.3583 \leq x \leq 0.4275$	$0.2200 \leq x \leq 0.3583$	$0.3583 \leq x \leq 0.4967$
Slope: 12.289	Slope: -12.289	Slope: 7.229	Slope: -7.229
Intercept: -4.343	Intercept: 6.043	Intercept: -1.792	Intercept: 3.792
$\overline{W}_L$		$\overline{W}_U$	
$0.2925 \leq x \leq 0.3367$	$0.3367 \leq x \leq 0.3808$	$0.2533 \leq x \leq 0.3383$	$0.3383 \leq x \leq 0.4233$
Slope: 19.245	Slope: -19.245	Slope: 11.765	Slope: -11.765
Intercept: -3.570	Intercept: 5.270	Intercept: -1.388	Intercept: 3.388
Risk Factor 3			
$\overline{R}_L$		$\overline{R}_U$	
$0.2408 \leq x \leq 0.3050$	$0.3050 \leq x \leq 0.3692$	$0.1767 \leq x \leq 0.3050$	$0.3050 \leq x \leq 0.4333$
Slope: 14.649	Slope: -14.649	Slope: 7.792	Slope: -7.792
Intercept: -3.480	Intercept: 5.360	Intercept: -1.377	Intercept: 3.377
$\overline{W}_L$		$\overline{W}_U$	
$0.2142 \leq x \leq 0.2700$	$0.2700 \leq x \leq 0.3258$	$0.1633 \leq x \leq 0.2717$	$0.2717 \leq x \leq 0.3800$
Slope: 16.836	Slope: -16.836	Slope: 9.231	Slope: -9.231
Intercept: -2.605	Intercept: 4.485	Intercept: -0.918	Intercept: 2.918
Risk Factor 4			
$\overline{R}_L$		$\overline{R}_U$	
$0.1092 \leq x \leq 0.1533$	$0.1533 \leq x \leq 0.1975$	$0.0700 \leq x \leq 0.1550$	$0.1550 \leq x \leq 0.2400$
Slope: 21.509	Slope: -21.509	Slope: 11.765	Slope: -11.765
Intercept: -1.272	Intercept: 3.172	Intercept: -0.208	Intercept: 2.208
$\overline{W}_L$		$\overline{W}_U$	
$0.2142 \leq x \leq 0.2700$	$0.2700 \leq x \leq 0.3258$	$0.1633 \leq x \leq 0.2717$	$0.2717 \leq x \leq 0.3800$
Slope: 17.015	Slope: -17.015	Slope: 9.231	Slope: -9.231
Intercept: -2.595	Intercept: 4.495	Intercept: -0.918	Intercept: 2.918

Step 6: The Max-Min Paired Elimination algorithm is implemented for $\alpha-\text{cuts}=0.00,0.10,0.20,0.30,0.40,0.50,0.60,0.70,0.80,0.90,0.91,1.00$ to find the $\min\{f_L\}$ and $\max\{f_U\}$ of lower and upper fuzzy numbers. Results for different $\alpha-$cuts and detailed calculations for $\alpha-\text{cut}=0$ are presented in Table 4.11 and Appendix I.

Table 4.11. $\min\{f_L\}$ and $\max\{f_U\}$ for different $\alpha-$cuts

$\alpha-$cut	Lower Fuzzy Number		Upper Fuzzy Number	
	$\min\{f_L\}_L$	$\max\{f_U\}_L$	$\min\{f_L\}_U$	$\max\{f_U\}_U$
0.00	0.210	0.368	0.113	0.436
0.10	0.219	0.360	0.129	0.419
0.20	0.228	0.351	0.145	0.403
0.30	0.236	0.342	0.160	0.386
0.40	0.245	0.334	0.176	0.370
0.50	0.254	0.325	0.192	0.353
0.60	0.262	0.317	0.208	0.337
0.70	0.271	0.308	0.224	0.321
0.80	0.280	0.299	0.240	0.305
0.90	0.288	0.291	0.256	0.288
0.91	0.289	0.289	0.258	0.287
1.00	-	-	0.272	0.272

Step 7: Based on equations (3-37) and (3-38) the lower and upper center of gravity points are (0.289,0.303) and (0.274,0.333).

Step 8: Applying equations (3-39), (3-40) and (3-41) the duration modifiers under combinatory effect of assumed risk factors are computed. In this case, the duration of activities which are under effect of different risk factors may increase optimistically by 16.15%, most-likely by 28.13% and pessimistically by 40.20%. Resultant triangular distribution is shown in Figure 4.6.

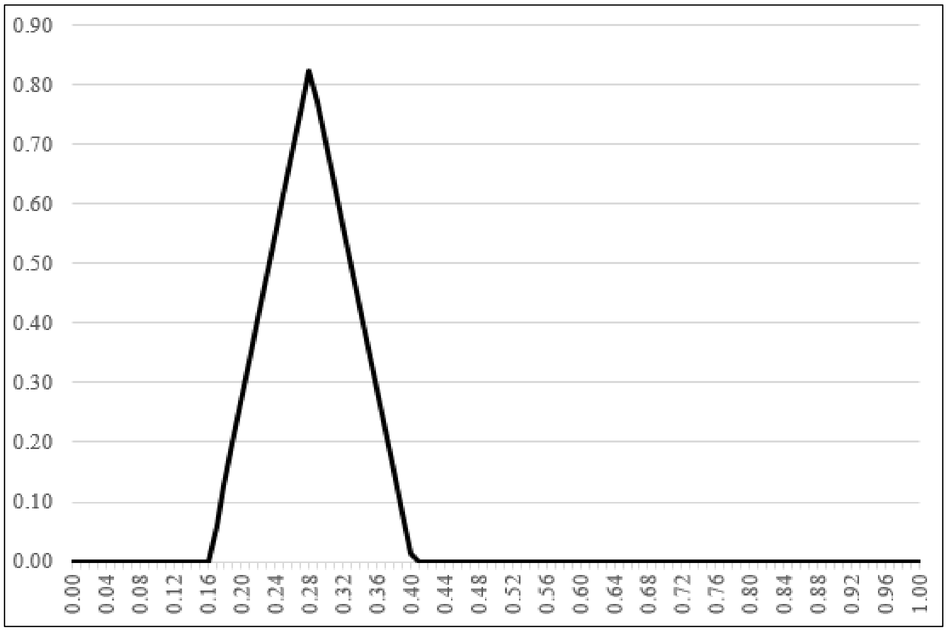

Figure 4.6. Triangular distribution of duration modifiers for high level of confidence

4.2.1.1.Effect of change in experts' confidence level: To explore the effect of change in experts' confidence level on duration modifiers, the calculations are repeated for same evaluation but with different confidence degree levels.

Table 4.12. Experts` linguistic evaluations with medium level of confidence

Risk Factor	Expert 1		
	R_i	W_i	w_i
Risk Factor 1	Fairly Low	Low	85.00%
Risk Factor 2	Medium	High	83.00%
Risk Factor 3	Fairly Low	Very Low	89.00%
Risk Factor 4	Very Low	Medium	87.00%
	Expert 2		
Risk Factor 1	Medium	Medium	88.00%
Risk Factor 2	Fairly Low	Low	82.00%
Risk Factor 3	Low	Medium	87.00%
Risk Factor 4	Low	Fairly Low	86.00%
	Expert 3		
Risk Factor 1	Low	Medium	84.00%
Risk Factor 2	Fairly Low	Very Low	85.00%
Risk Factor 3	Medium	Fairly Low	83.00%
Risk Factor 4	Fairly Low	Very Low	87.00%

Table 4.13. Experts` linguistic evaluations with low level of confidence

Risk Factor	Expert 1		
	R_i	W_i	w_i
Risk Factor 1	Fairly Low	Low	77.00%
Risk Factor 2	Medium	High	75.00%
Risk Factor 3	Fairly Low	Very Low	79.00%
Risk Factor 4	Very Low	Medium	77.00%
	Expert 2		
Risk Factor 1	Medium	Medium	78.00%
Risk Factor 2	Fairly Low	Low	76.00%
Risk Factor 3	Low	Medium	75.00%
Risk Factor 4	Low	Fairly Low	77.00%
	Expert 3		
Risk Factor 1	Low	Medium	74.00%
Risk Factor 2	Fairly Low	Very Low	76.00%
Risk Factor 3	Medium	Fairly Low	75.00%
Risk Factor 4	Fairly Low	Very Low	76.00%

Since there is no change in experts' linguistic evaluations, the mean value and area under computed triangular distribution should stay same for different levels of confidence. As explained earlier, level of confidence in developed model is defined as the height of triangular distribution. When the level of confidence increases, the variation gets smaller to satisfy the constant area requirement while the mean value stays same. Lower level of confidence in fact, means lower height at the peak point of triangular distribution. In this case, the variation should be increased to have the same equivalent area while the mean value does not change. The relationship between different confidence levels and distribution variation is shown in Figure 4.7.

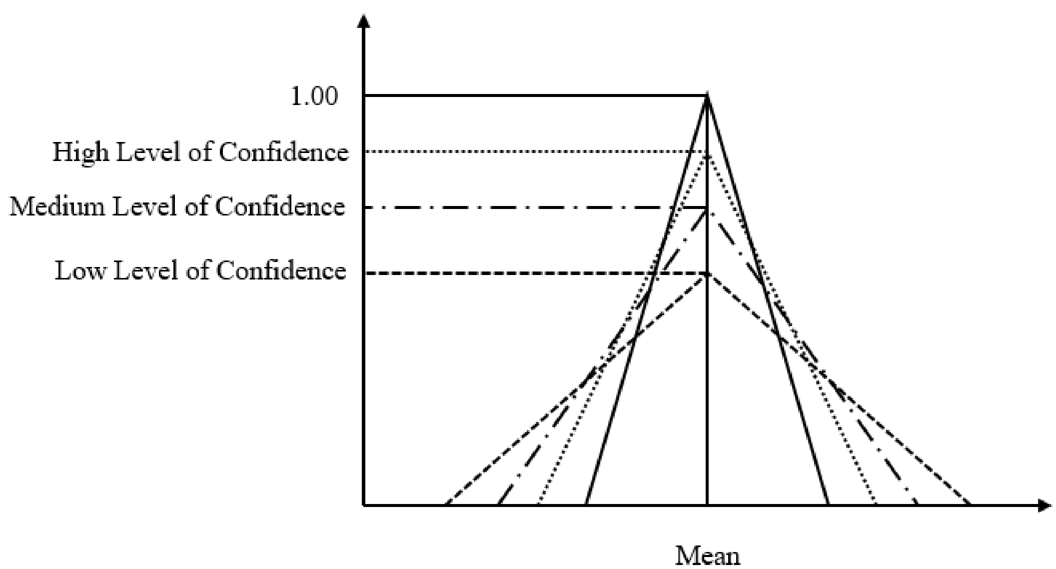

Figure 4.7. Relationship between different confidence levels and distribution variation

For medium level of confidence, the optimistic, most likely and pessimistic values for duration modifiers calculated as 0.151, 0.281 and 0.425. For low level of confidence, the values change to 0.147, 0.281 and 0.465. Figures 4.8 and 4.9 show the resultant triangular distributions based on different levels of confidence. A comparison of duration modifiers for three levels of confidence degree is provided in Table 4.14.

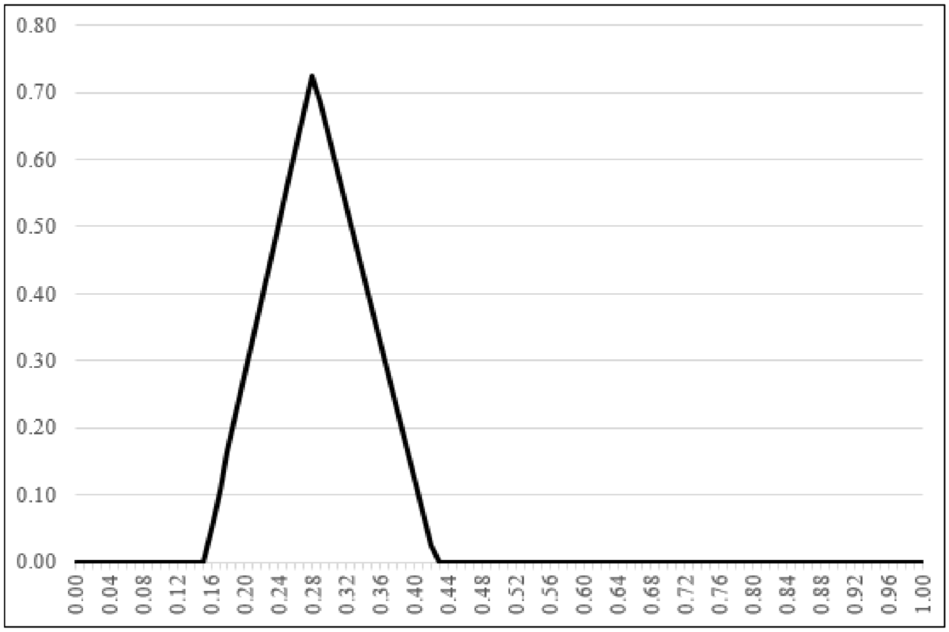

Figure 4.8. Triangular distribution of duration modifiers for medium level of confidence

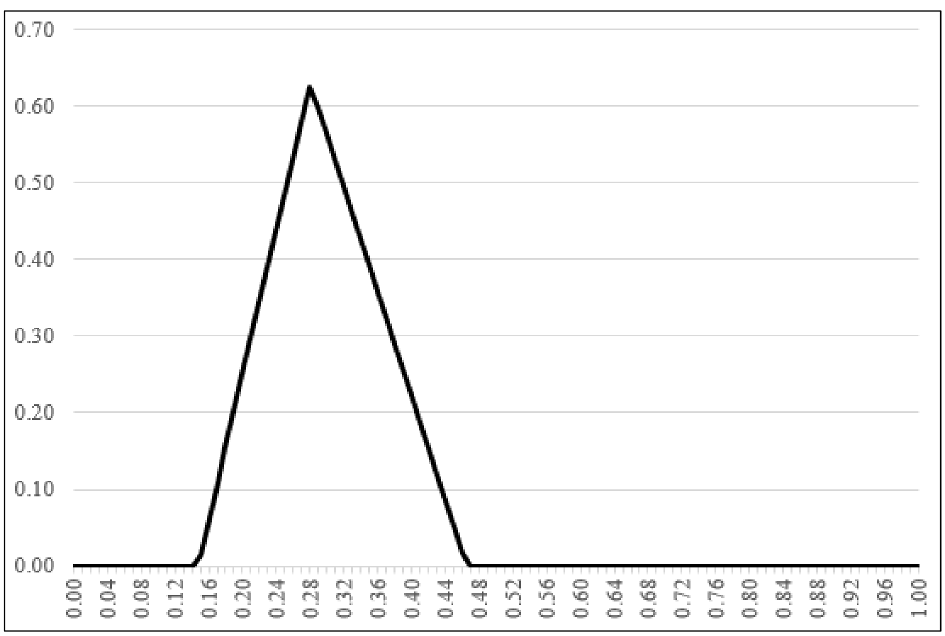

Figure 4.9. Triangular distribution of duration modifiers for low level of confidence

Table 4.14. Comparison of duration modifiers for three levels of confidence degree

Level of Confidence	Duration Modifiers		
	Optimistic	Most Likely	Pessimistic
High	0.162	0.281	0.402
Medium	0.151	0.281	0.425
Low	0.147	0.281	0.465

To verify the effect of changing the confidence levels of experts, the analysis runs again to look at the estimated total project duration and cost for different duration and cost options assuming same predictions but different confidence levels for each expert. Procedure and results are presented in section 5.2.4.

4.2.2. Bayesian Network Modelling

Each task is modeled using five nodes. Each node's distribution and source is defined in Figure 4.10.

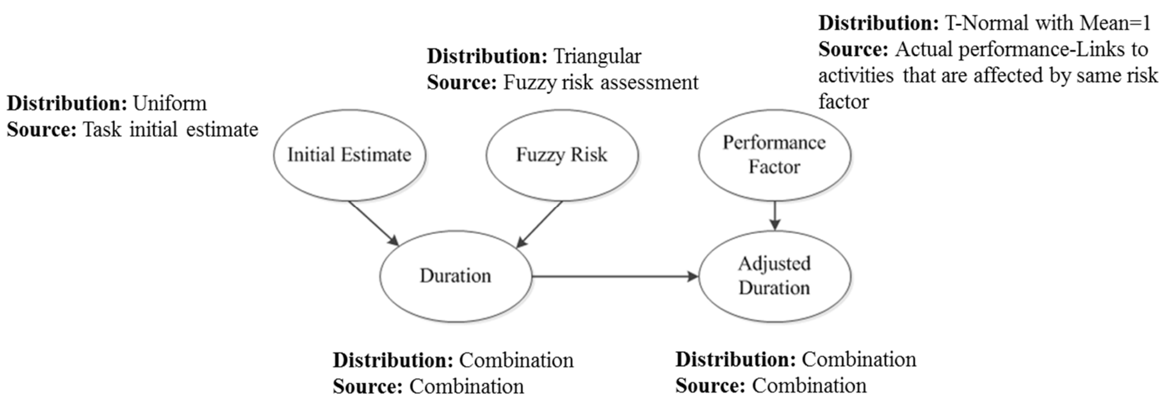

Figure 4.10. Task Bayesian Network modeling

Considering the scheduling network diagram (Figure 4.1.), activities are connected to each other to form the CPM network. The Performance factor node of activities which share the same crew are also connected. A sample scheduling network is shown in Figure 4.11.

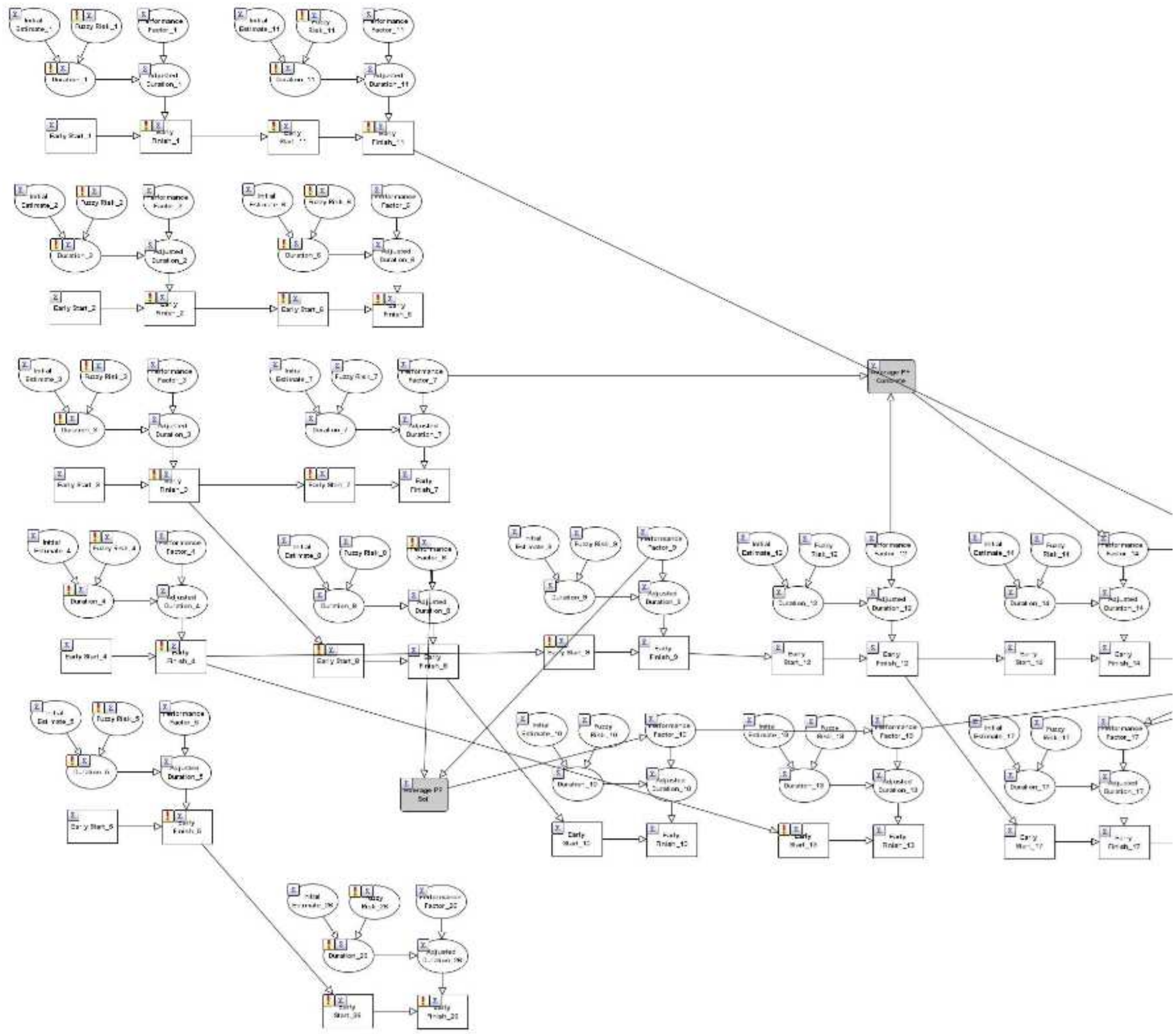

Figure 4.11. Sample scheduling network

To better understand the duration block and performance factor modeling, activity 7 before and after entering the actual duration is explained. The duration block for this activity before entering the evidence (actual duration) is as Figure 4.12.

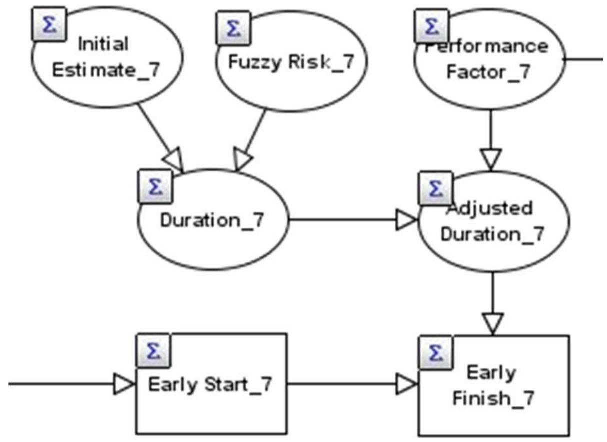

Figure 4.12. Duration block for activity 7 before entering the evidence (actual duration)

In AgenaRisk each node is defined by Node Property Table (NPT) in which user defines the distribution and mathematical functions for each node. For this activity, NPTs of each node are defined as following Figures 4.13 to 4.17.

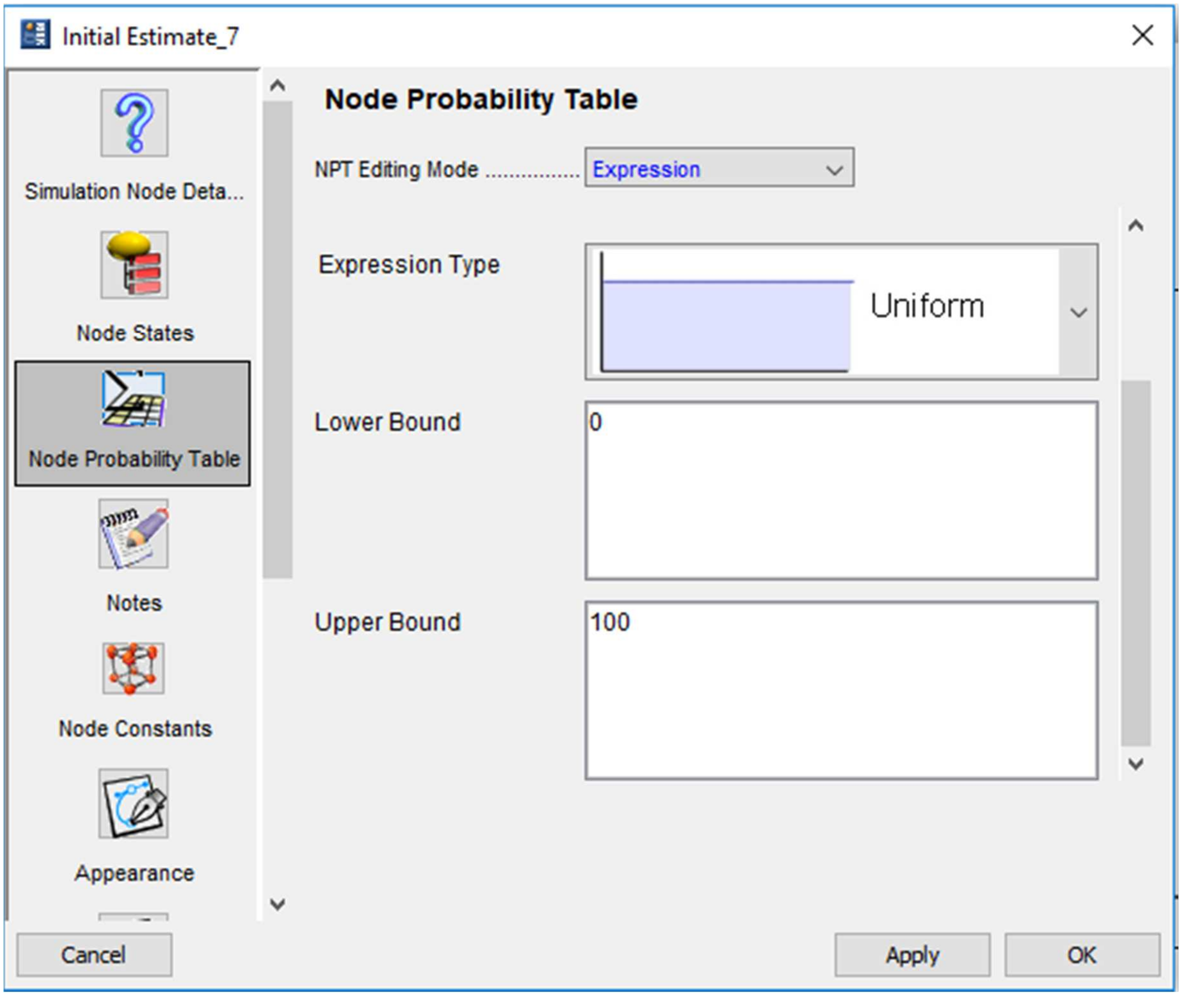

Figure 4.13. NPT of Initial Estimate node

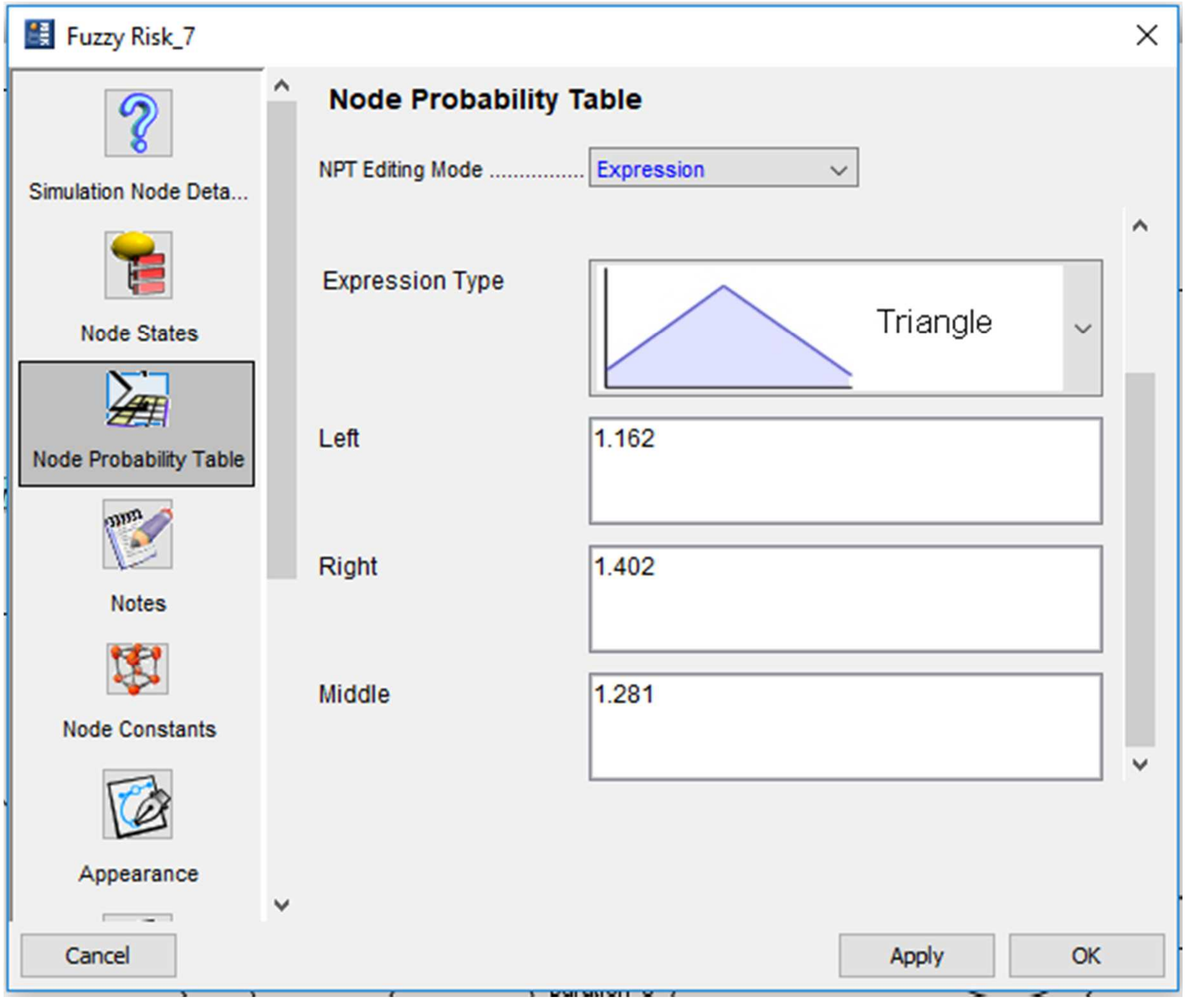

Figure 4.14. NPT of Fuzzy Risk node

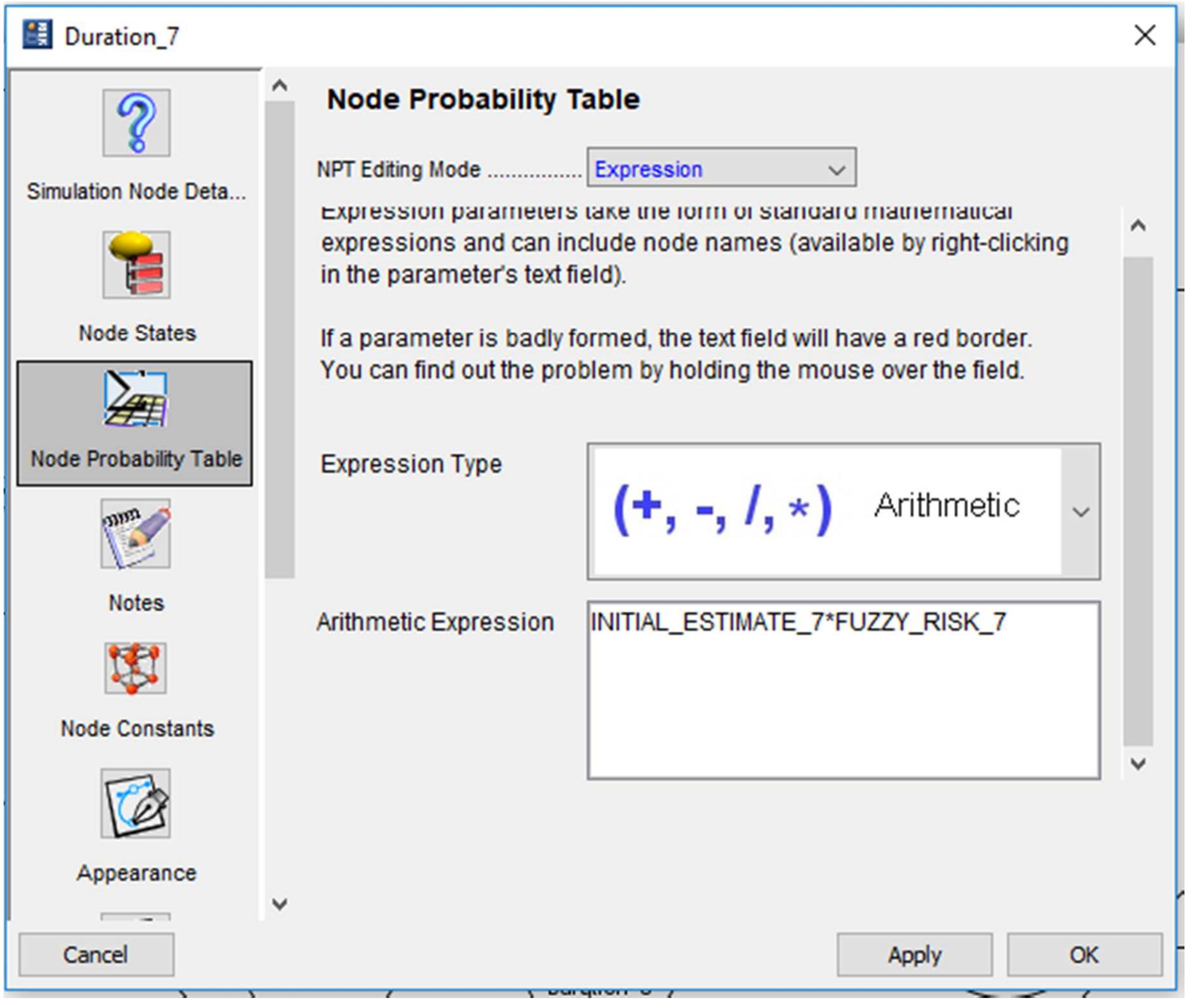

Figure 4.15. NPT of Duration node

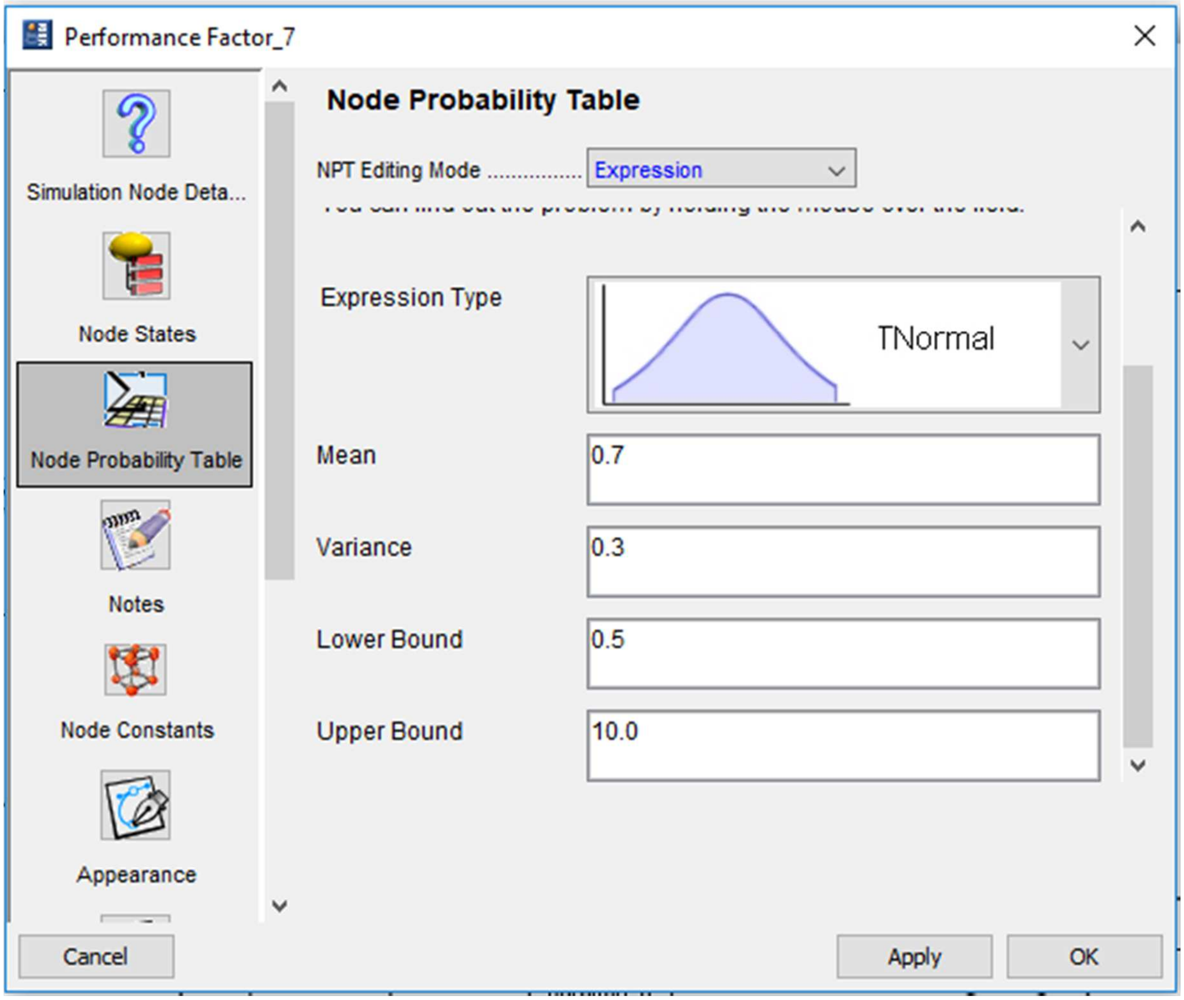

Figure 4.16. NPT of Performance Factor node

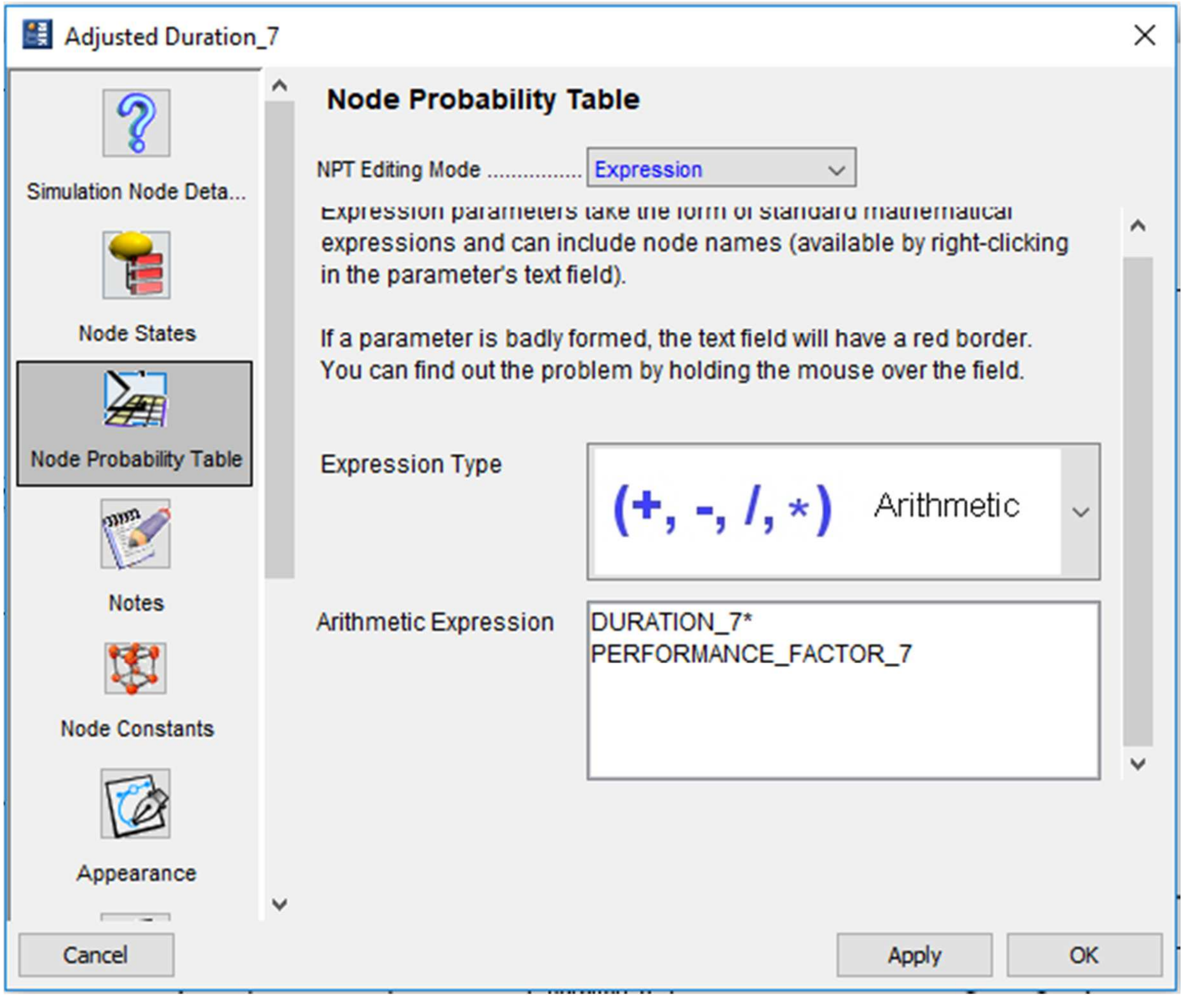

Figure 4.17. NPT of Adjusted Duration node

It is assumed that task 7 is ahead of schedule and has been completed with actual duration of 3 days instead of estimated 4 days. Entering these values in Initial Estimate and Adjusted Duration nodes as shown in Figure 4.18, results in updated distribution of Performance Factor. Completing this task by 1 day sooner than planned, results in its Performance Factor improve by about 3%. Performance Factor before and after entering the evidence (actual duration) are shown in Figures 4.19 and 4.20.

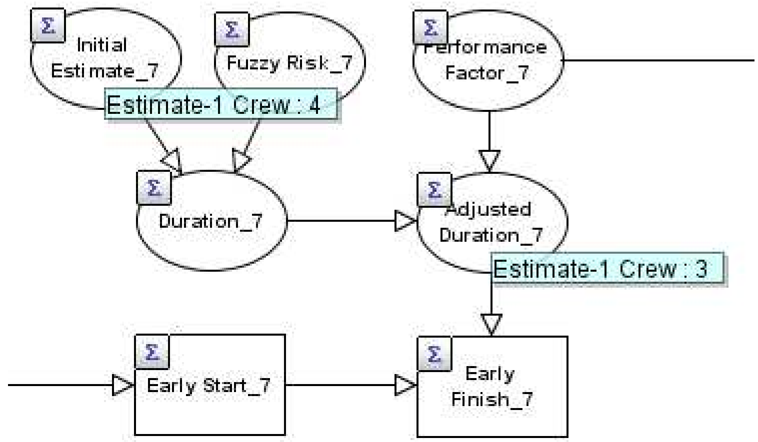

Figure 4.18. Duration block for activity 7 after entering the evidence (actual duration)

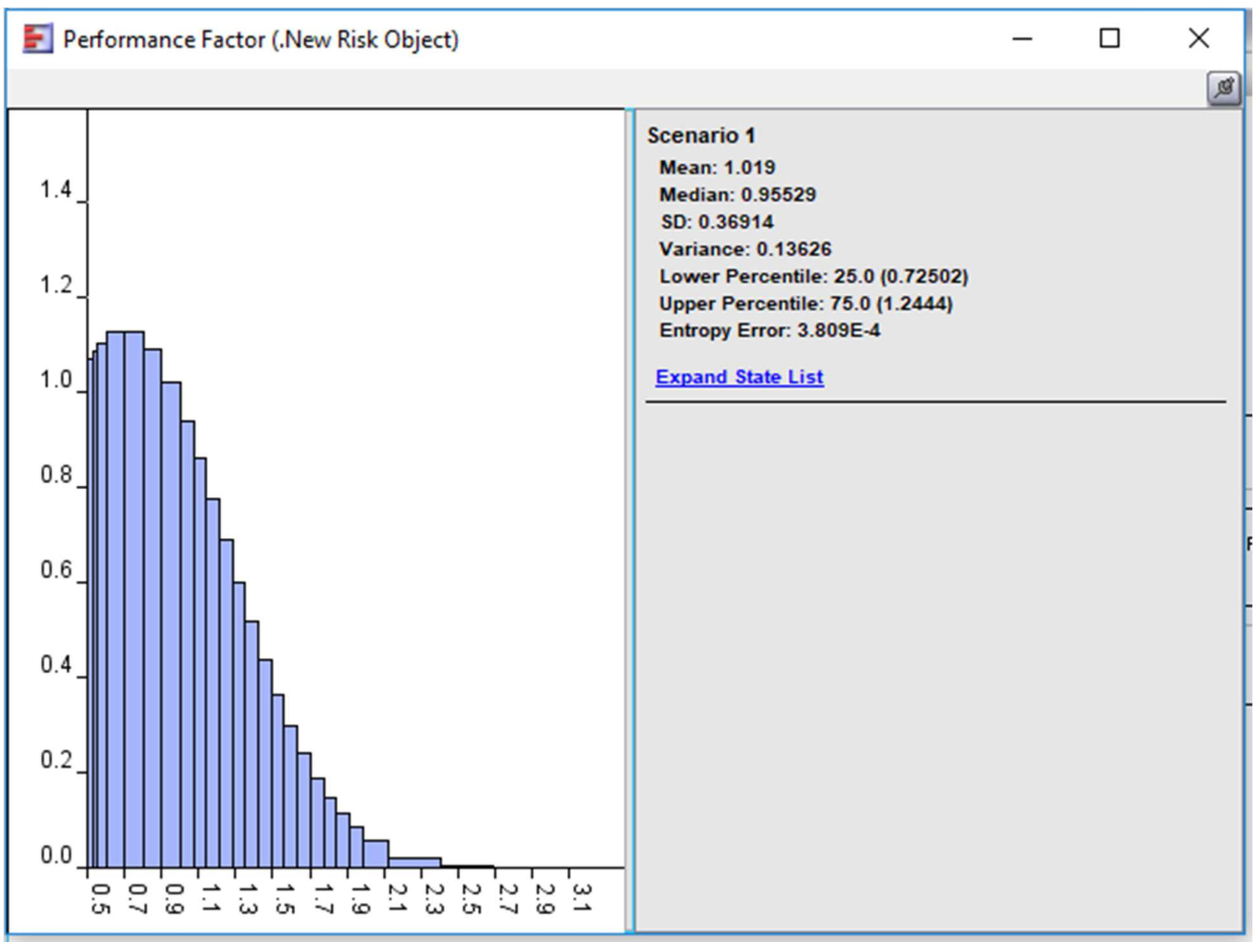

Figure 4.19. Performance Factor of task 7 before entering the evidence (actual duration)

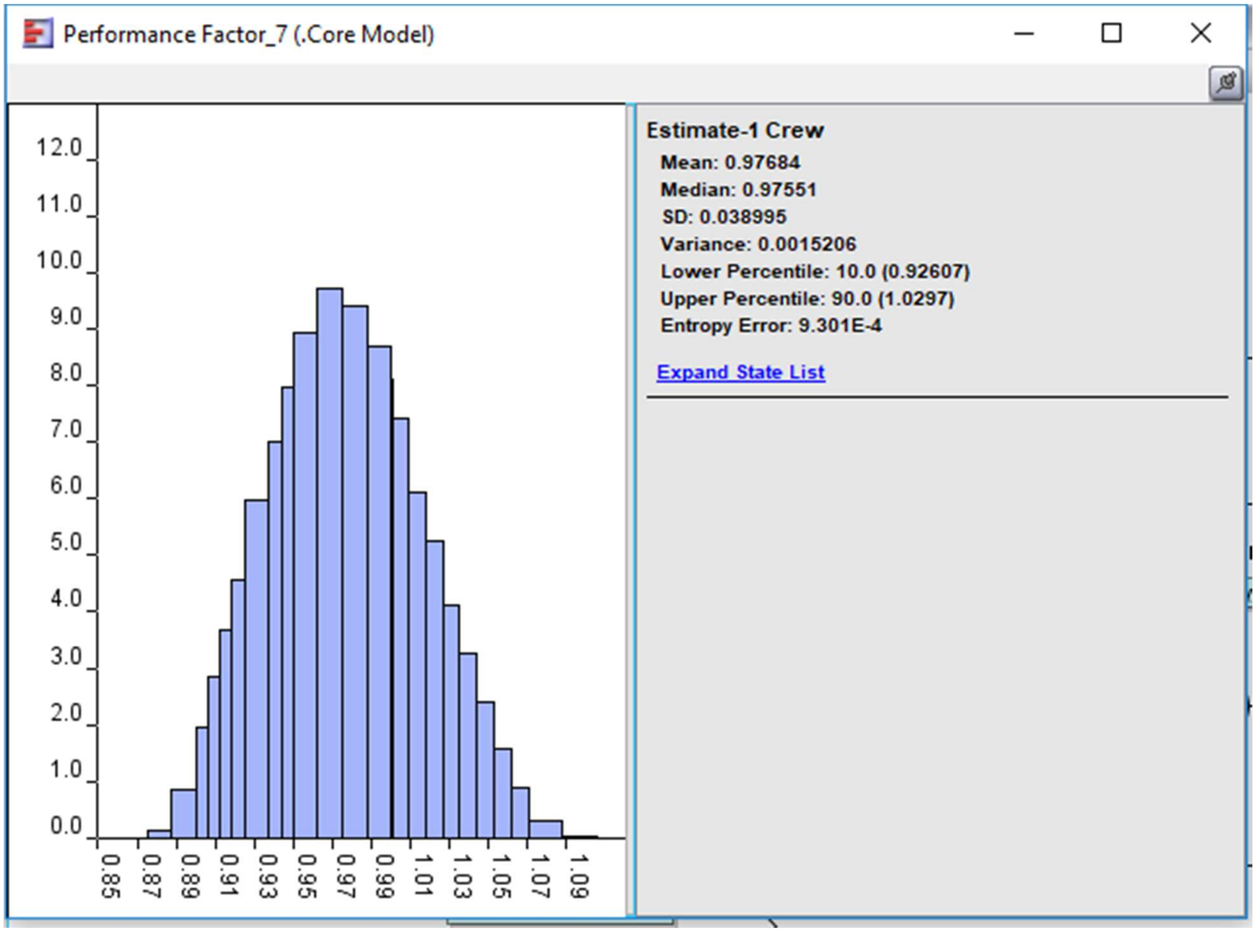

Figure 4.20. Performance Factor of task 7 after entering the evidence (actual duration)

Tasks 7 and 12 are connected since they share the same crew members. Improving the Performance Factor of task 7 directly results in improving the Performance Factor of task 12. The overall performance of project has also an indirect effect on Performance Factor. Since the project is doing well and tasks are finished either on-time or ahead of schedule, the Performance Factor of task 12 improves. Entering the actual duration of task 12 which is 3 days and considering overall project performance, the Performance Factor of task 12 improves by about 22% as shown in Figure 4.21.

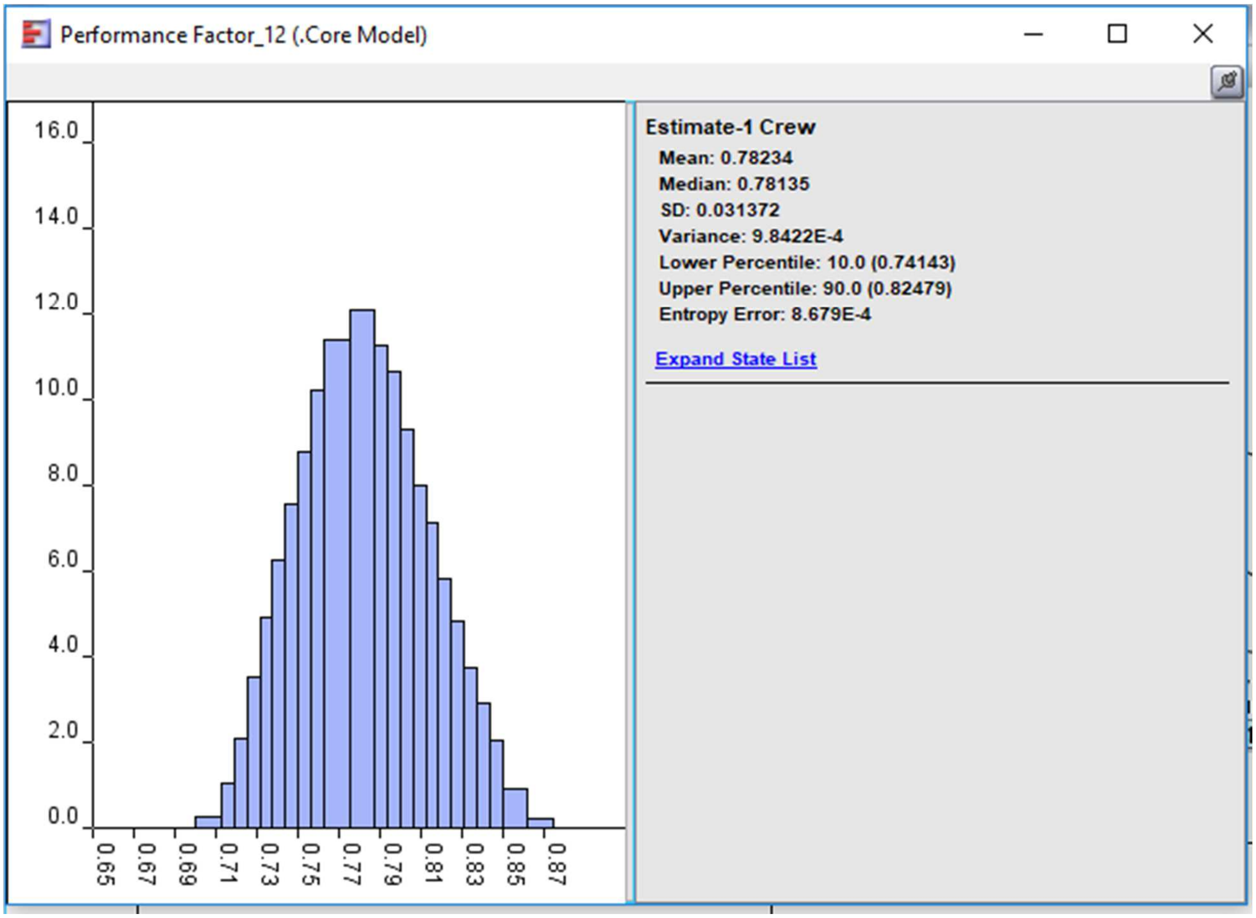

Figure 4.21. Improvement in Performance Factor of task 12

In order to combine and average the Performance Factors, a node called "Average Performance Factor" for each group of tasks is defined. The NPT of this node for tasks 7 and 12 is shown in Figure 4.22.

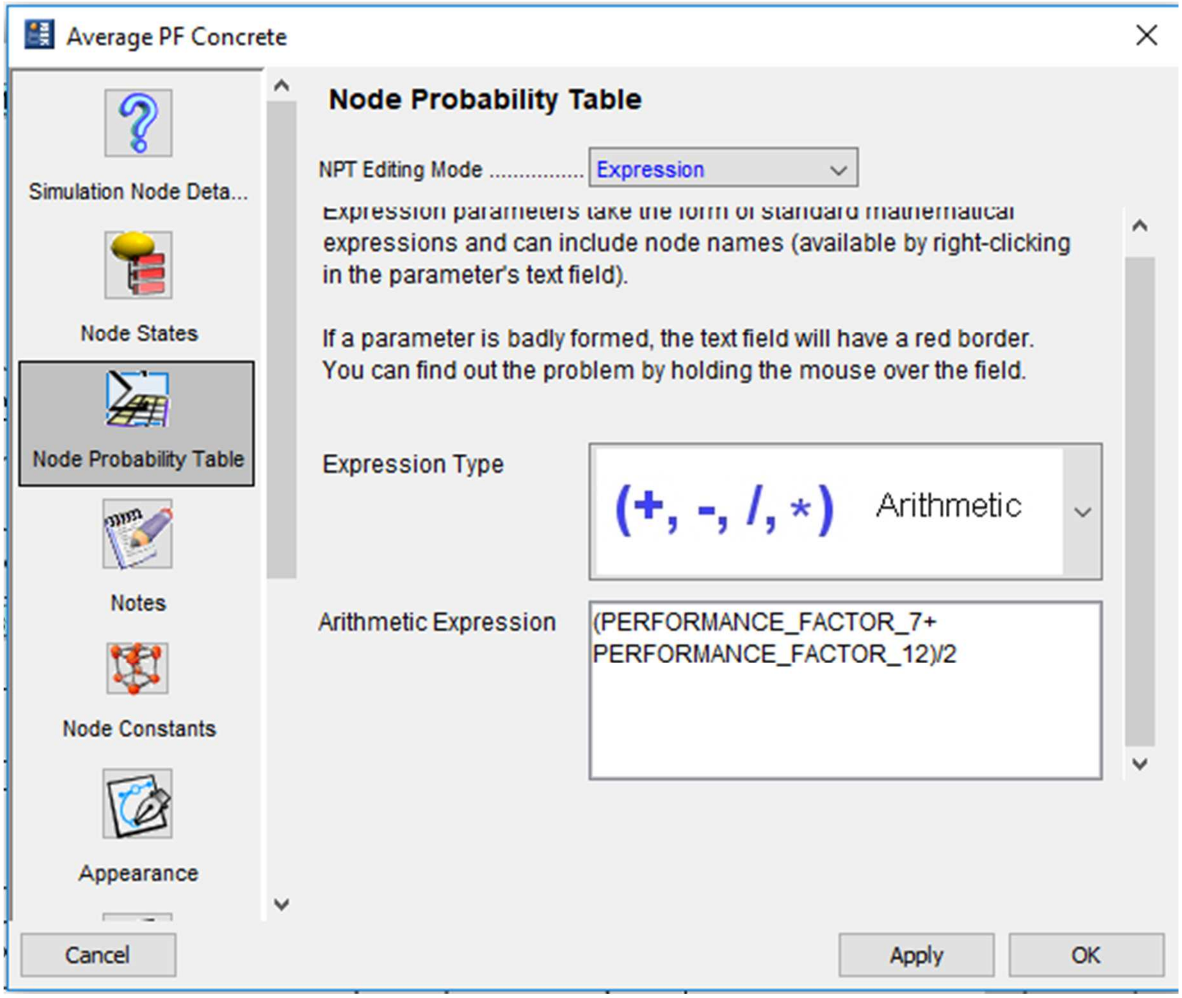

Figure 4.22. NPT of Average Performance Factor node for tasks 7 and 12

In this case study, it is assumed that tasks 7 and 12 are completed. For ahead of schedule case, after entering all evidences (actual durations), the average performance factor for completed tasks of 7 and 12 which are in concrete group of activities is 0.880 as shown in Figure 4.23.

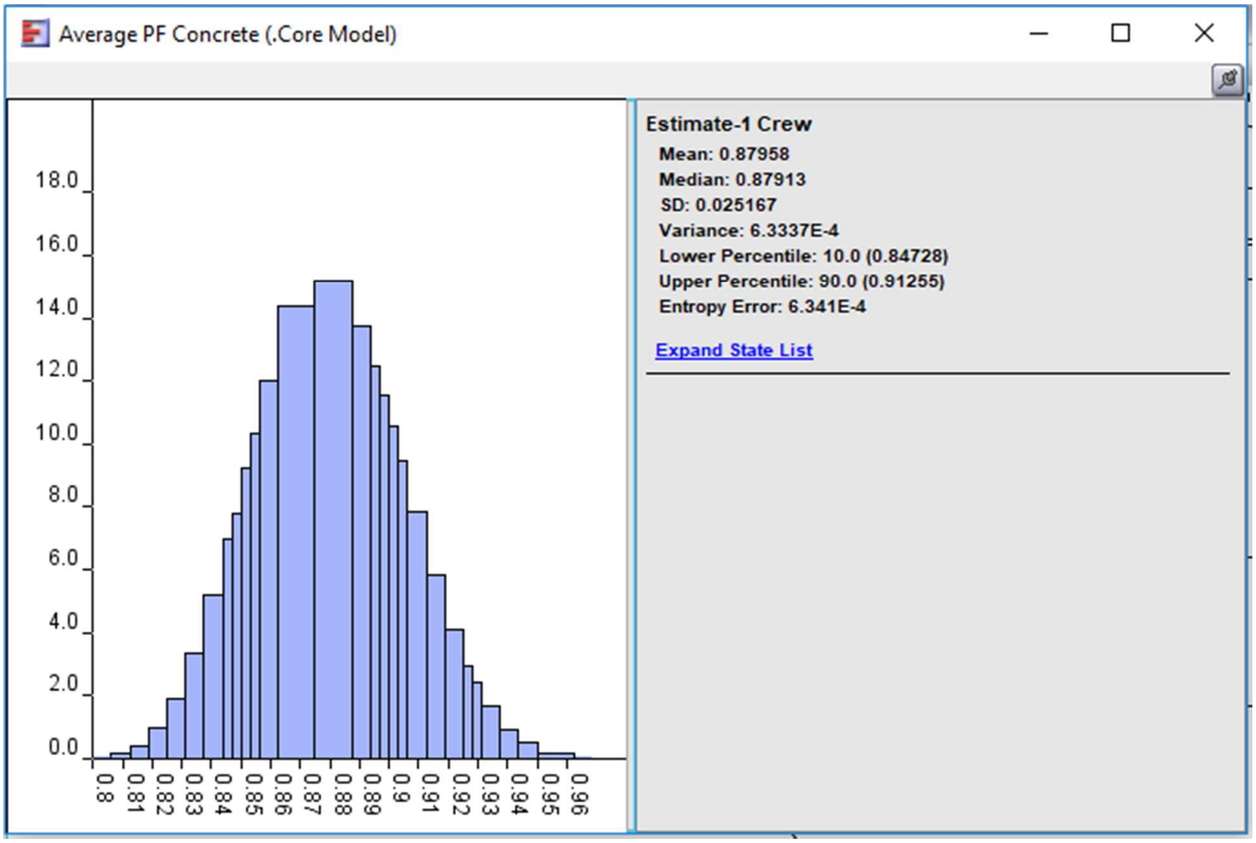

Figure 4.23. Average Performance Factor for tasks 7 and 12 (Concrete Group)

The average performance factor is transferred to next unfinished tasks in related task groups to update their performance factor. In this case, the Performance Factor of task 14 and other related activities are updated from 1 to 0.880. Figure 4.24 illustrates this change.

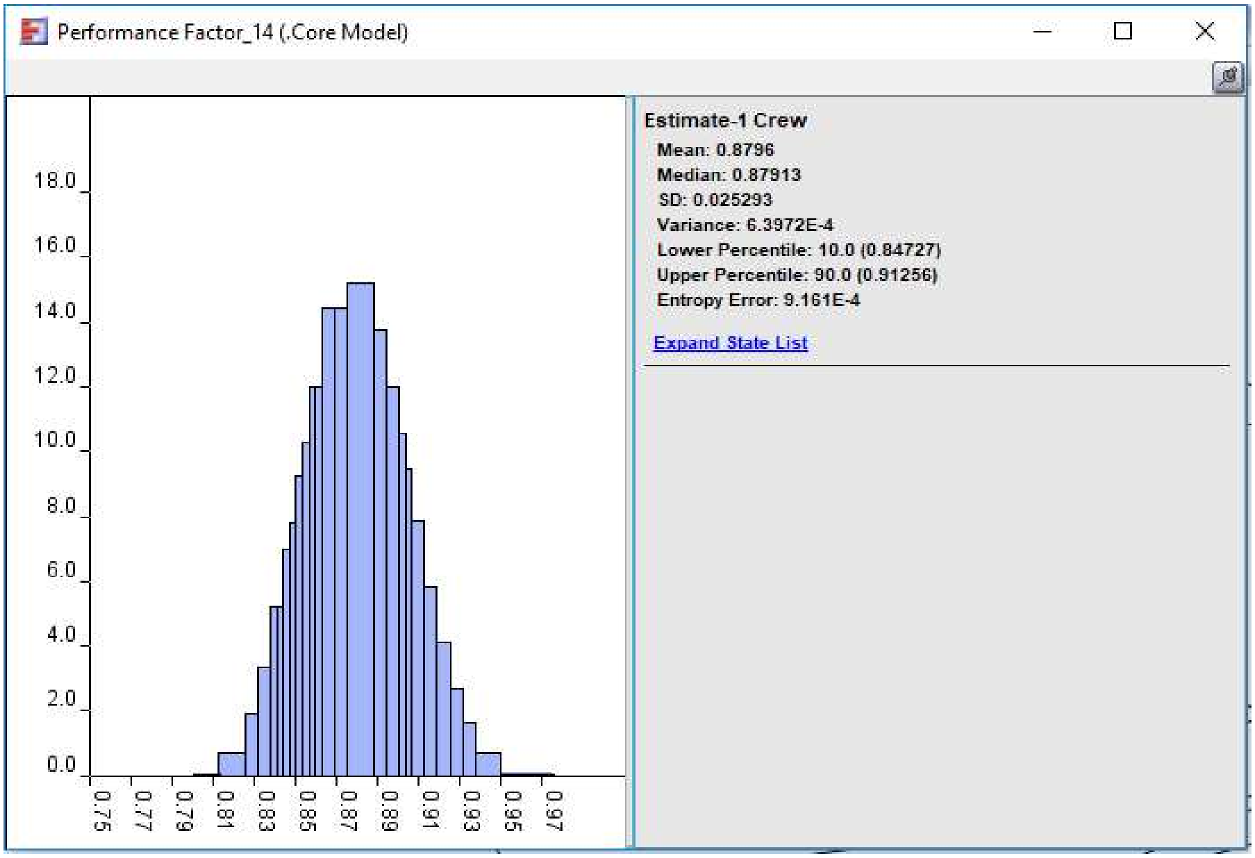

Figure 4.24. Updated Performance Factor of unfinished task 14

With same project performance, developed scheduling model predicts the overall duration of project to be 69 days (mean value 68.696) versus estimated total duration of 75 days. Figure 4.25 depicts the distribution of project completion time for Ahead of Schedule case.

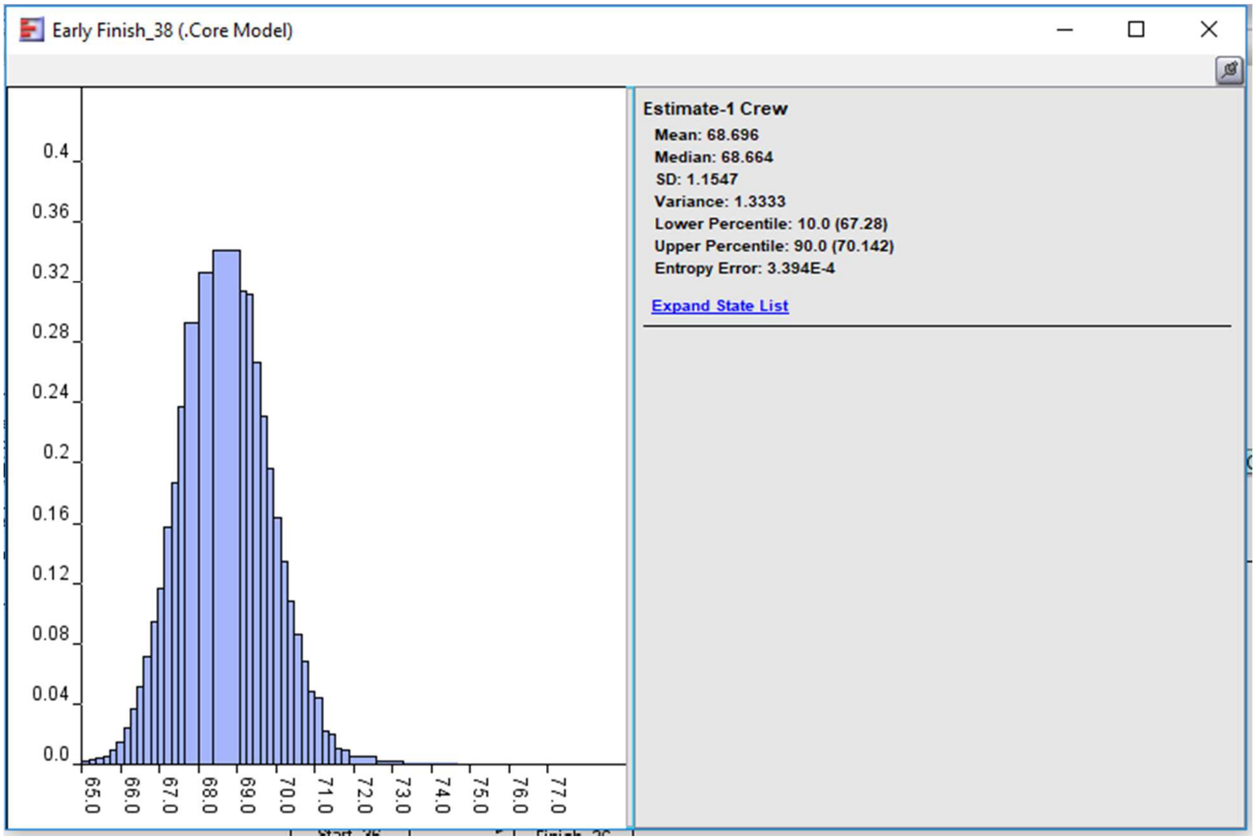

Figure 4.25. Distribution of project completion time for Ahead of Schedule case

For Behind Schedule case, same tasks are assumed to be finished but with actual durations greater than estimation. Table 4.15 shows a comparison between these two scenarios with one crew option:

Table 4.15. Comparison between two scenarios with one crew

Case	Task	Estimate	Actual	Performance Factor	Average P.F.	Predicted Completion	Estimated Completion
Ahead Schedule	7	4	3	0.977	-	-	-
Ahead Schedule	12	4	3	0.782	-	-	-
Ahead Schedule	-	-	-	-	0.880	-	-
Ahead Schedule	14	1	-	0.880	-	-	-
Ahead Schedule	-	-	-	-	-	69	75
Behind Schedule	7	4	7	1.363	-	-	-
Behind Schedule	12	4	5	1.299	-	-	-
Behind Schedule	-	-	-	-	1.331	-	-
Behind Schedule	14	1	-	1.331	-	-	-
Behind Schedule	-	-	-	-	-	87	75

4.2.3. Time-Cost Trade-Off Optimization model

A base case where project has not started yet and no actual task data is available along with three project scenarios of Above Cost-Behind Schedule, Below Cost-Behind Schedule and Above Cost-Ahead Schedule are considered for this case study. For each case, three crashing options of duration and cost are generated by changing the number of crews from one to three. Activities 1 to 12 have been assumed to be finished with 1 crew. The actual duration and cost of these finished activities are entered into the model. Screenshots of these scenarios are shown in Figures 4.26. to 4.28.

Activity	Group	Duration			Cost		
		Option1	Option 2	Option 3	Option 1	Option 2	Option 3
1		10	10	10	$6,720	$6,720	$6,720
2		5	5	5	$3,360	$3,360	$3,360
3		3	3	3	$500	$500	$500
4		15	15	15	$10,000	$10,000	$10,000
5		10	10	10	$6,720	$6,720	$6,720
6		7	7	7	$5,000	$5,000	$5,000
7	Concrete	7	7	7	$20,176	$20,176	$20,176
8	Soil	6	6	6	$7,093	$7,093	$7,093
9	Soil	5	5	5	$65,048	$65,048	$65,048
10	Soil	2	2	2	$1,310	$1,310	$1,310
11		15	15	15	$8,000	$8,000	$8,000
12	Concrete	5	5	5	$13,206	$13,206	$13,206
13	Soil	7	5	5	$88,007	$104,356	$127,071
14	Concrete	2	2	2	$3,650	$3,740	$3,900
15		1	1	1	$500	$500	$500
16	Concrete	9	7	5	$19,879	$24,296	$27,320
17	Concrete	4	4	3	$8,437	$11,165	$11,259
18	Concrete	2	2	2	$7,526	$8,210	$9,500
19	Concrete	2	2	2	$3,560	$4,210	$5,026
20	Concrete	6	4	3	$39,810	$44,647	$53,371
21		1	1	1	$500	$500	$500
22	Soil	23	16	12	$27,227	$36,702	$39,889
23	Concrete	9	7	5	$19,879	$24,296	$27,320
24	Concrete	2	2	2	$7,800	$8,100	$8,620
25	Concrete	4	4	2	$27,338	$40,202	$25,457
26		25	25	25	$15,000	$15,000	$15,000
27	Soil	12	8	7	$13,436	$18,827	$24,818
28	Concrete	11	6	5	$29,176	$30,657	$33,650
29	Concrete	9	6	3	$20,022	$21,235	$18,408
30		2	3	3	$725	$1,760	$2,124
31	Concrete	2	2	2	$11,400	$14,100	$16,520
32		4	6	5	$1,450	$3,521	$4,247
33		4	3	2	$7,227	$10,299	$10,091
34	Soil	4	4	2	$6,759	$14,393	$7,122
35		1	1	1	$1,360	$3,989	$4,290
36		2	2	2	$750	$820	$900
37		2	2	1	$2,597	$2,726	$2,732
38		1	1	1	$500	$500	$500
Total		***87***	***75***	***56***	***$511,647***	***$595,884***	***$627,269***
Estimated		*75*	*64*	*55*	*$433,692*	*$543,199*	*$617,540*
Difference		*12*	*11*	*1*	*$77,955*	*$52,685*	*$9,728*

Figure 4.26. Above Cost-Behind Schedule case

Activity	Group	Duration			Cost		
		Option1	Option 2	Option 3	Option 1	Option 2	Option 3
1		10	10	10	$4,800	$4,800	$4,800
2		5	5	5	$2,400	$2,400	$2,400
3		3	3	3	$500	$500	$500
4		15	15	15	$10,000	$10,000	$10,000
5		10	10	10	$4,800	$4,800	$4,800
6		7	7	7	$5,000	$5,000	$5,000
7	Concrete	7	7	7	$17,620	$17,620	$17,620
8	Soil	6	6	6	$3,500	$3,500	$3,500
9	Soil	5	5	5	$54,237	$54,237	$54,237
10	Soil	2	2	2	$1,112	$1,112	$1,112
11		15	15	15	$8,000	$8,000	$8,000
12	Concrete	5	5	5	$9,230	$9,230	$9,230
13	Soil	7	5	5	$88,007	$104,356	$127,071
14	Concrete	2	2	2	$3,000	$3,251	$2,900
15		1	1	1	$500	$500	$500
16	Concrete	9	7	5	$18,702	$22,497	$25,094
17	Concrete	4	4	3	$6,751	$10,366	$9,981
18	Concrete	2	2	2	$5,230	$7,000	$7,850
19	Concrete	2	2	2	$2,900	$3,500	$3,750
20	Concrete	6	4	3	$36,609	$40,911	$48,670
21		1	1	1	$500	$500	$500
22	Soil	23	16	12	$27,227	$36,702	$39,889
23	Concrete	9	7	5	$18,702	$22,497	$25,094
24	Concrete	2	2	2	$7,200	$7,520	$8,100
25	Concrete	4	4	2	$25,366	$36,808	$23,694
26		25	25	25	$15,000	$15,000	$15,000
27	Soil	12	8	7	$13,436	$18,827	$24,818
28	Concrete	11	6	5	$27,716	$29,037	$31,696
29	Concrete	9	6	3	$18,859	$19,523	$17,900
30		2	3	3	$725	$1,760	$2,124
31	Concrete	2	2	2	$10,825	$13,256	$15,965
32		4	6	5	$110	$2,900	$4,000
33		4	3	2	$6,485	$9,123	$8,944
34	Soil	4	4	2	$6,759	$14,393	$7,000
35		1	1	1	$1,200	$2,291	$3,755
36		2	2	2	$600	$700	$790
37		2	2	1	$2,277	$2,389	$2,394
38		1	1	1	$500	$500	$500
Total		***87***	***75***	***56***	***$466,384***	***$547,304***	***$579,178***
Estimated		*75*	*64*	*55*	*$398,718*	*$456,054*	*$489,955*
Difference		*12*	*11*	*1*	*$67,666*	*$91,250*	*$89,223*

Figure 4.27. Below Cost-Behind Schedule case

Activity	Group	Duration			Cost		
		Option1	Option 2	Option 3	Option 1	Option 2	Option 3
1		10	10	10	$6,720	$6,720	$6,720
2		5	5	5	$3,360	$3,360	$3,360
3		3	3	3	$500	$500	$500
4		15	15	15	$10,000	$10,000	$10,000
5		10	10	10	$6,720	$6,720	$6,720
6		7	7	7	$5,000	$5,000	$5,000
7	Concrete	5	5	5	$19,705	$19,705	$19,705
8	Soil	4	4	4	$6,875	$6,875	$6,875
9	Soil	3	3	3	$68,623	$68,623	$68,623
10	Soil	1	1	1	$1,285	$1,285	$1,285
11		15	15	15	$8,000	$8,000	$8,000
12	Concrete	3	3	3	$14,520	$14,520	$14,520
13	Soil	4	3	3	$77,438	$78,235	$80,458
14	Concrete	2	2	2	$3,650	$3,740	$3,900
15		1	1	1	$500	$500	$500
16	Concrete	6	6	4	$16,665	$25,432	$27,950
17	Concrete	3	3	3	$8,361	$11,018	$14,021
18	Concrete	2	2	2	$6,924	$8,500	$9,024
19	Concrete	2	2	2	$4,012	$4,233	$4,890
20	Concrete	4	3	3	$28,651	$29,998	$35,876
21		1	1	1	$500	$500	$500
22	Soil	14	10	7	$17,031	$20,526	$23,541
23	Concrete	7	6	4	$16,229	$22,014	$28,562
24	Concrete	2	2	2	$8,200	$8,952	$9,025
25	Concrete	3	3	2	$21,596	$40,284	$27,410
26		25	25	25	$15,000	$15,000	$15,000
27	Soil	7	5	4	$9,125	$10,253	$11,202
28	Concrete	7	4	4	$21,520	$25,630	$27,120
29	Concrete	6	4	3	$15,990	$17,260	$19,546
30		2	3	3	$725	$1,760	$2,124
31	Concrete	2	2	2	$13,695	$14,596	$18,967
32		4	6	5	$1,450	$1,860	$1,650
33		4	3	2	$7,227	$10,299	$10,091
34	Soil	2	2	1	$5,100	$6,200	$8,625
35		1	1	1	$1,360	$3,989	$4,290
36		2	2	2	$635	$750	$800
37		2	2	1	$2,597	$2,726	$2,732
38		1	1	1	$500	$500	$500
Total		***69***	***62***	***54***	***$455,989***	***$516,063***	***$539,612***
Estimated		*75*	*64*	*55*	*$398,718*	*$456,054*	*$489,955*
Difference		*-6*	*-2*	*-1*	*$57,271*	*$60,009*	*$49,657*

Figure 4.28. Above Cost-Ahead Schedule case

Duration and cost options for each case are entered into a different Excel sheet which runs an optimization algorithm to find the best combination of task options which maximizes the project NPV.

4.2.3.1. Base case: Base case is the case where project has not been started yet and no actual performance data is available. Initial task duration estimates are entered in *Initial Estimate* node in AgenaRisk along with duration modifiers computed by Fuzzy risk assessment model. All performance factors are assumed to be 1 and no evidence is entered in *Adjusted Duration* node. The mean values of task durations are taken as duration options. Corresponding cost values are calculated by considering different affecting factors such as crew productivity, efficiency and other factors. The procedure to generate crashing options in this case may summarized as follow:

To generate options in base case (where the project has not been started yet and no actual performance data is available), the initial task duration values for 1, 2 and 3 crew members is estimated by following Excel sheet (Figure 4.29) and entered in Initial Estimate node as evidence.

Activities	Activity Description	Activity Type	Quantity	Crew Information				Resources			Duration	Cost
				Crew ID	Crew #	Daily Output/Crew	Efficiency	labor	material	equipment		
1	Shop drawings, abutment, and deck steel	Shop drawings, abutment, and deck steel	200.00	Fab.	1	20.00	1.00	Fabricator			10	$4,800.00
2	Shop drawings, foot steel	Shop drawings, foot steel	100.00	Fab.	1	20.00	1.00	Fabricator			5	$2,400.00
3	Move in	Move in	Fixed Price		Fixed Price	Fixed Price	Fixed Price	Fixed Price 1			3	$500.00
4	Deliver Piles	Deliver Piles	Fixed Price		Fixed Price	Fixed Price	Fixed Price	Fixed Price 2			15	$10,000.00
5	Shop drawings, girders	Shop drawings, girders	200.00	Fab.	1	20.00	1.00	Fabricator			10	$4,800.00
6	Deliver footer steel	Deliver footer steel	Fixed Price		Fixed Price	Fixed Price	Fixed Price	Fixed Price 3			7	$5,000.00
7	Make abut. Forms	C.I.P. concrete forms, pile cap, square or rectangular, plywood, 2 use, includes	1380.00	C1	1	213.37	1.00	Carpenters Laborer	plywood, 2 use,		7	$17,620.17
8	Exc. Abut. No. 1	Excavating, bulk, dozer, open site, bank measure, common earth, 80 H.P. dozer, 50' haul	1600.00	B10L	1	268.00	1.00	Equip. Oper. (med) Laborer		Dozer, 80 H.P.	6	$6,017.91
9	Drive piles abut. No. 1	Steel piles, "H" Sections, 50' long, HP12 x 53, excludes mobilization or demobilization	1700.00	B19	1	377.60	1.00	Pile Driver Foreman Pile Drivers Equip. Oper. (crane) Equip. Oper. Oiler	Steel piles, "H" Sections, 50' long, HP12 x 53	Crawler Crane, 40 Ton Lead, 90' high Hammer, Diesel, 22k ft-lb	5	$65,048.41
10	Exc. Abut. No. 2	Excavating, bulk, dozer, open site, bank measure, common earth, 80 H.P. dozer, 50' haul	300.00	B10L	1	272.00	1.00	Equip. Oper. (med) Laborer		Dozer, 80 H.P.	2	$1,111.76
11	Deliver abut. And deck steel	Deliver abut. And deck steel	Fixed Price		Fixed Price	Fixed Price	Fixed Price	Fixed Price 4			15	$8,000.00
		C.I.P. concrete forms, pile						Carpenters	plywood, 2			

Figure 4.29. Excel sheet to estimate task duration and cost

For example, Activity 7 is estimated to be finished in 7 days with 1 crew, 5 days with 2 crews and 4 days with 3 crews.

These values are entered in Initial Estimate node as following:

1- Estimated duration for 1 crew is entered in Initial Estimate node as observation (Figure 4.30)

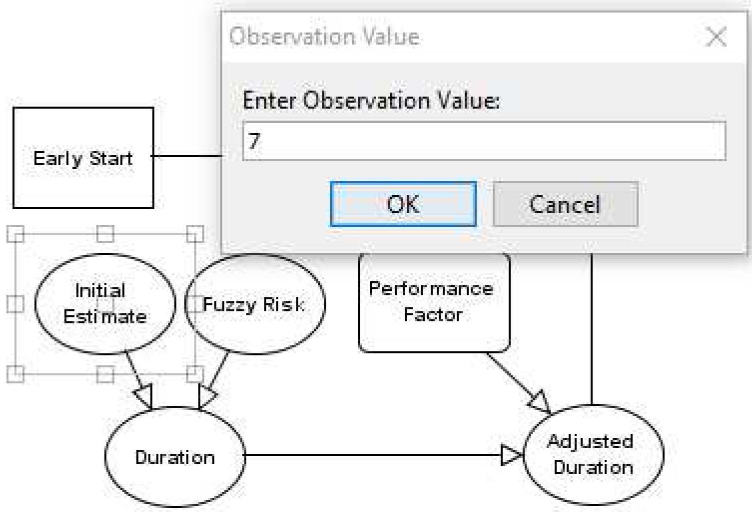

Figure 4.30. Entering estimated duration in Initial Estimate node for 1 crew (Activity 7)

2- Same procedure is followed for all other nodes.

3- Calculation is run with these new observations.

4- The mean value of distributions calculated in Adjusted Duration node is taken as task duration options for 1 crew.

5- The values in Initial Estimate node is changed based on estimated duration for 2 crews (Figure 4.31).

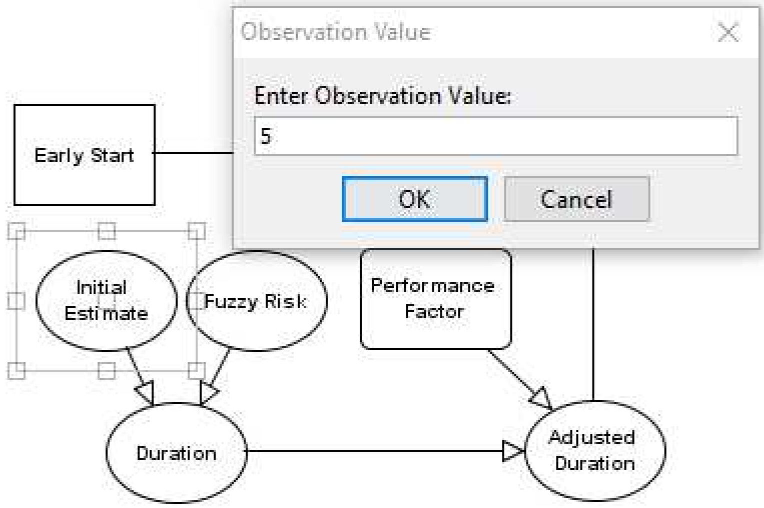

Figure 4.31. Entering estimated duration in Initial Estimate node for 2 crews (Activity 7)

6- Same procedure is followed for all other nodes.

7- Calculation runs again with these new observations.

8- The mean value of distributions calculated in Adjusted Duration node is taken as task duration options for 2 crews.

9- Same procedure is followed to calculated task duration options for 3 crews.

Analysis is run first to find the Internal Rate of Return (IRR) with assuming only one crew option for each activity; then it will be run with all three options to find the best crew combination. IRR values will be compared to see the ability of optimization algorithm in improving the IRR by using different crew options even before the project starts. These values are shown in Table 4.16. In the first two columns, the crew numbers and IRR value for each case is shown. For 1, 2 and 3 crews the calculated IRR value is before running the optimization solution while in combination case, the optimum IRR value is shown. Columns three compares the optimum IRR with the IRR of each crew number.

Table 4.16. IRR values for different crew numbers of base case

Crew Number	IRR Value (%)	Increase (%)
1	17.976	4.76
2	16.374	15.01
3	15.467	21.76
Combination	18.832	-

Results show that presented optimization solution is capable of increasing project's IRR by combining different duration and cost options even before the project starts. The solution provides 4.76% increase in IRR value comparing to using only one crew for the project. These values are 15.01% and 21.76% for using only two and three crews. A comparison between IRR values is shown in Figure 4.32. Chosen options by optimization algorithm along with corresponding duration values are shown in Table 4.17. The scheduling network is run again with these duration values. Results show that with optimum selected options, project is estimated to be finished most likely in about 70 days with optimistic and pessimistic finish times of 68 and 72 days. Project duration distribution is shown in Figure 4.33.

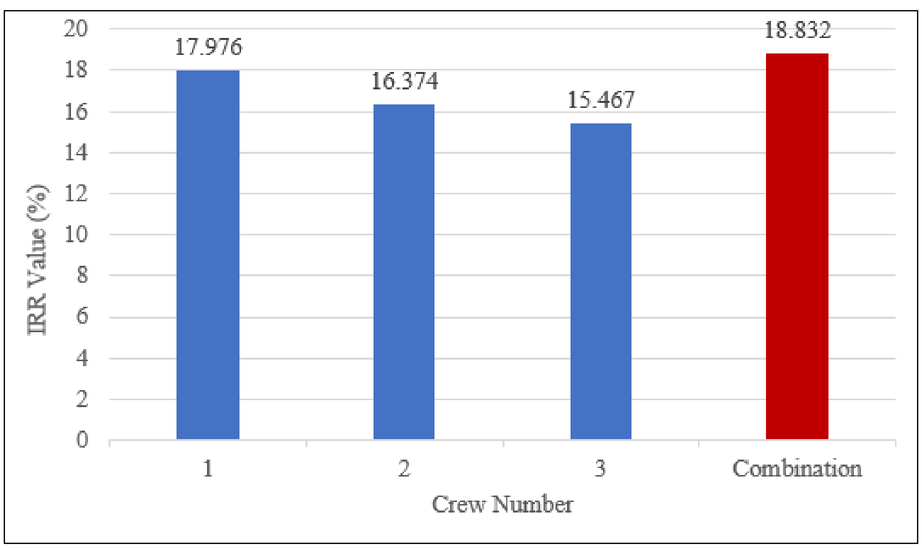

Figure 4.32. Comparing IRR values for different crew numbers in base case

Table 4.17. Chosen options and corresponding duration values for base case

Task	Chosen Option	Corresponding Duration (Days)
1	1	10
2	1	5
3	1	3
4	1	15
5	1	10
6	1	7
7	1	6
8	1	5
9	1	4
10	1	2
11	1	15
12	1	4
13	1	5
14	1	2
15	1	1
16	2	6
17	2	3
18	1	2
19	1	2
20	1	4
21	1	1
22	2	13
23	1	7
24	1	2
25	1	3
26	1	25
27	1	9
28	2	4
29	1	7
30	1	2
31	1	2
32	1	4
33	1	4
34	1	3
35	1	1
36	1	2
37	1	2
38	1	1

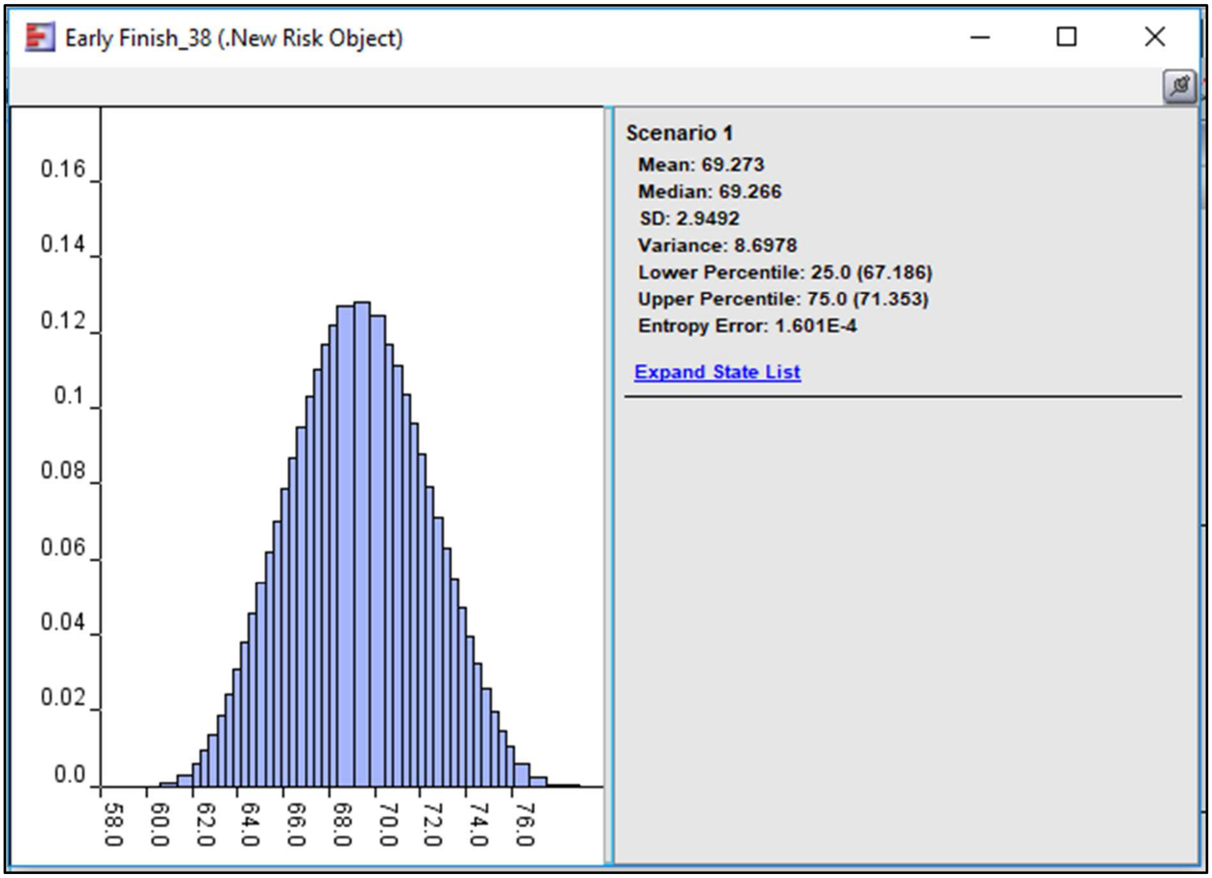

Figure 4.33. Project duration distribution for Base case with selected options

4.2.3.2. Above Cost-Behind Schedule: This is a common case in construction projects. In this case, tasks are finished with actual duration and cost higher than estimated. Options in this case are generated by entering actual task durations as *evidence* in *Adjusted Duration* node. Since the actual duration is higher than estimated, the performance factor would be higher than 1. This results in higher duration values comparing to other cases. The mean value of distributions in *Adjusted Duration* node is taken as task duration option. Different options are generated by changing the initial estimate values based on crew numbers. The procedure to generate duration and cost crashing options is described as follow:

1. For completed tasks, the actual duration is entered as observation in Adjusted Duration node as follow (Figure 4.34). (The project scenario is Above Cost-Behind Schedule)

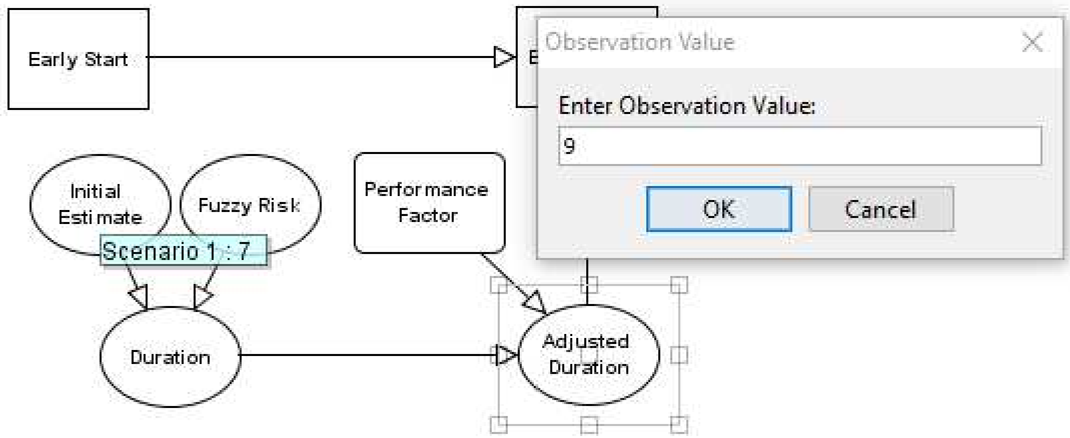

Figure 4.34. Entering actual duration in Adjusted Duration node for 1 crew

2. Entering observation in Adjusted Duration node updates the performance factor.
3. Calculation runs with these new observations.
4. The mean value of distributions calculated in Adjusted Duration node is taken as task duration options for 1 crew.
5. The values in Adjusted Duration node is changed based on actual duration for 2 crews (Figure 4.35).

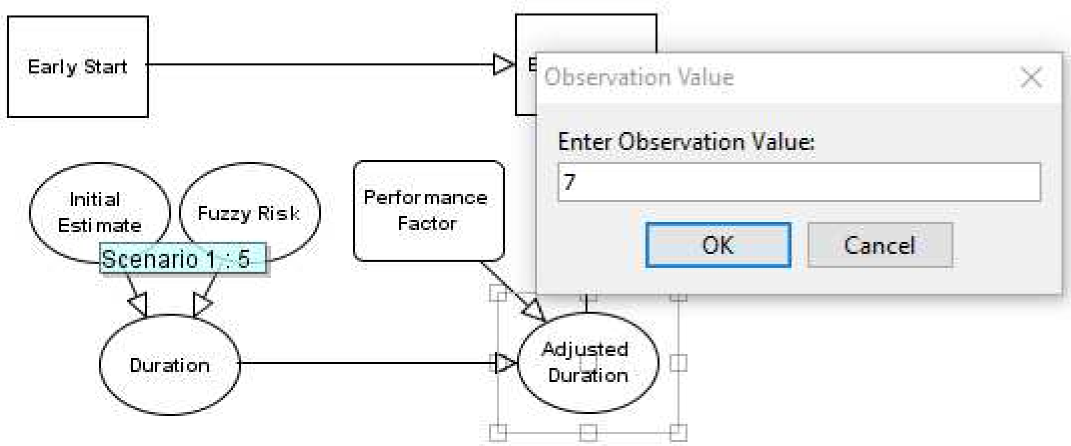

Figure 4.35. Entering actual duration in Adjusted Duration node for 2 crews

6. Calculation runs with these new observations and the mean value of distributions calculated in Adjusted Duration node is taken as task duration options for 2 crews.
7. Same procedure is followed to calculated task duration options for 3 crews.

To find the efficiency of developed optimization model, first the IRR value for three different crew numbers is calculated. The optimization algorithm is then run to find the optimum combination of options. The IRR value of optimum solution is then compared with IRR value of each individual crew number to verify the efficiency of the solution. These values are shown in Table 4.18.

Table 4.18. IRR values for different crew numbers of Above Cost-Behind Schedule scenario

Crew Number	IRR Value (%)	Increase (%)
1	8.725	4.19
2	7.738	17.48
3	6.998	29.90
Combination	9.091	-

Results show that presented optimization solution is capable of increasing project's IRR when it is above the estimated cost and behind the planned schedule by combining different duration and cost options. The solution provides 4.19%, 17.48% and 29.90% increase in IRR value comparing to using only one, two and three crews for the project. A comparison between these IRR values is shown in Figure 4.36. Chosen options by optimization algorithm along with corresponding duration values are shown in Table 4.19. New duration values selected by the optimization algorithm is entered in the scheduling network. Results show that with optimum selected options, project is estimated to be finished most likely in about 74 days with optimistic and pessimistic finish times of 71 and 76 days.

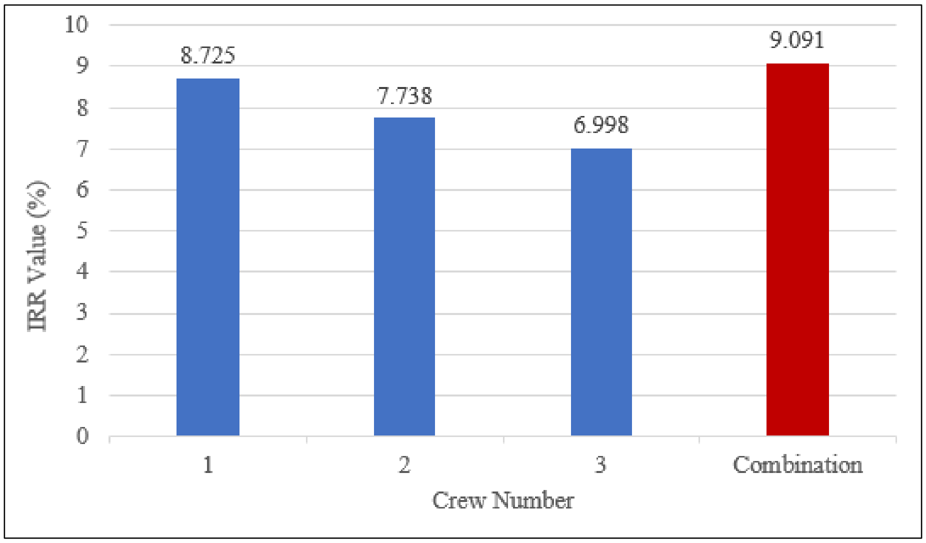

Figure 4.36. Comparing IRR values for different crew numbers in Above Cost-Behind Schedule scenario

Table 4.19. Chosen options and corresponding duration values for Above Cost-Behind Schedule scenario

Task	Chosen Option	Corresponding Duration (Days)
1	1	10
2	1	5
3	1	3
4	1	15
5	1	10
6	1	7
7	1	6
8	1	5
9	1	5
10	1	2
11	1	15
12	1	5
13	1	7
14	1	2
15	1	1
16	1	9
17	1	4

Table 4.19. Chosen options and corresponding duration values for Above Cost-Behind Schedule scenario (Continue)

Task	Chosen Option	Corresponding Duration (Days)
18	1	2
19	1	2
20	1	6
21	1	1
22	1	23
23	1	9
24	1	2
25	1	4
26	1	25
27	1	12
28	1	11
29	1	9
30	1	2
31	1	2
32	2	6
33	3	2
34	1	4
35	1	1
36	1	2
37	1	2
38	1	1

To find the overall project duration distribution, selected optimum options by VB code is entered in AgenaRisk following these steps:

1- The values in Initial Estimate node for completed tasks is cleared. Instead, the actual task duration is entered as observation in Adjusted Duration node. Entering the observation (actual task data) in Adjusted Duration node, updates the distributions in Duration and Performance Factor nodes, so there is no need to enter the initial estimated task duration values for each crew configuration in Initial Estimate node. See Figure 4.37 for completed task 7 which has an actual duration of 9 days.

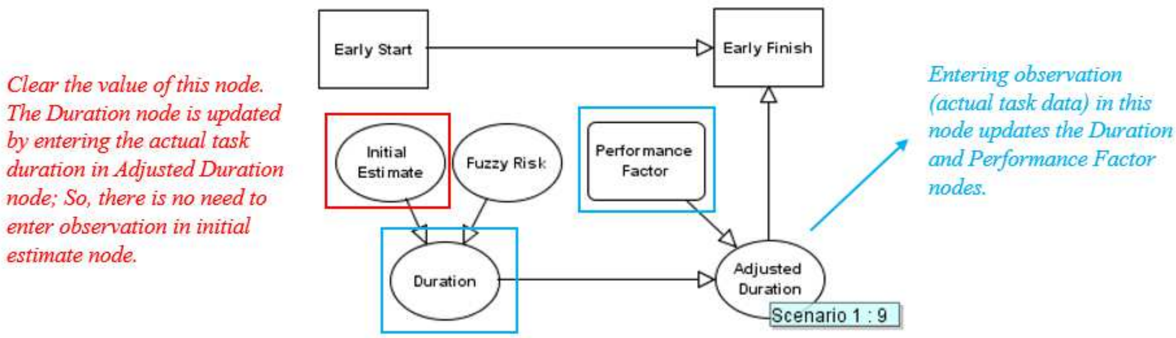

Figure 4.37. Entering actual task data in Adjusted Duration node for completed tasks

2- For incomplete tasks where we have options generated by VB code, the options are entered in following way:

Assume for task 16 the VB code chose option 2 which has corresponding duration value of 7 days. This value is entered in Initial Estimate node as follow (Figure 4.38).

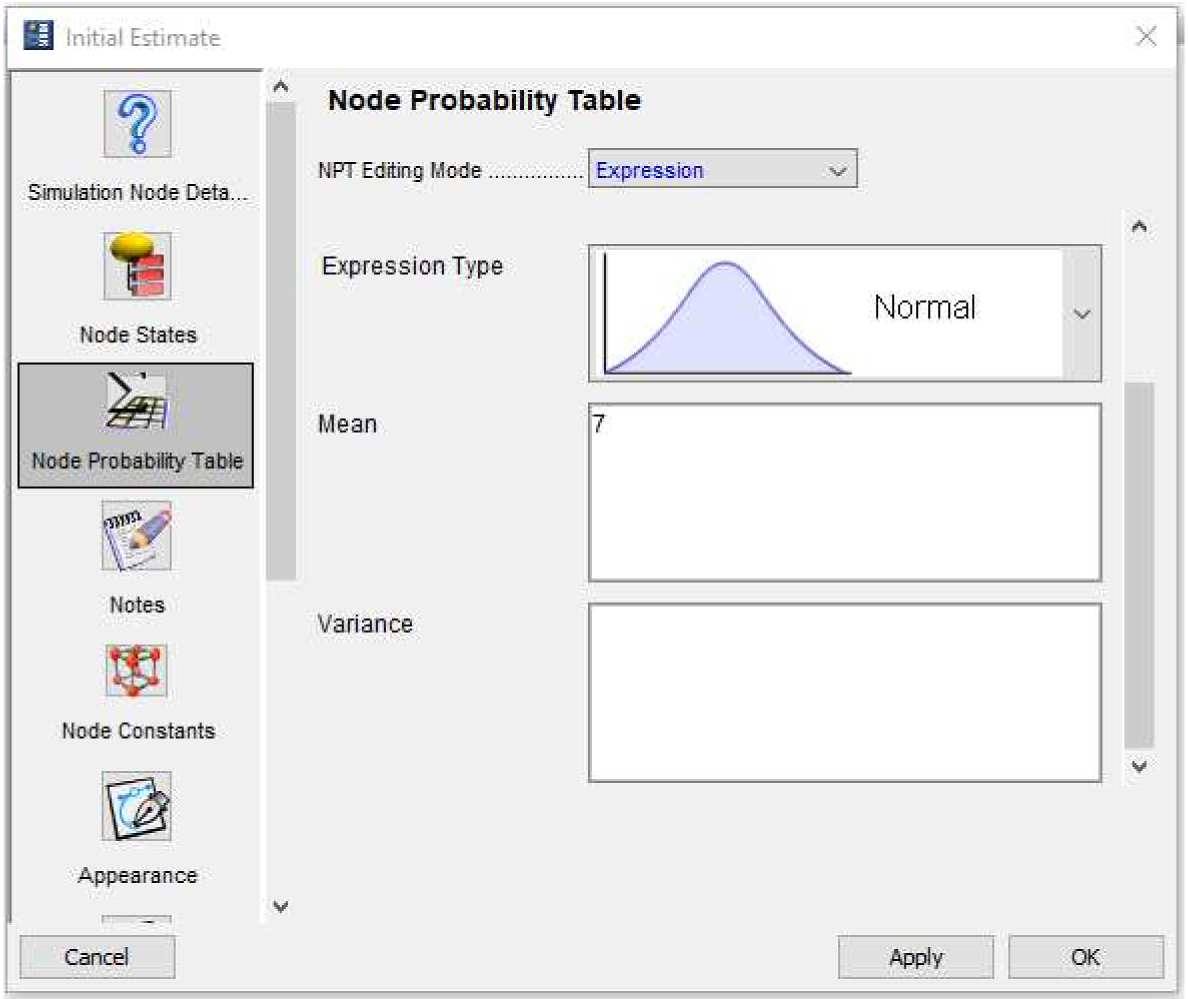

Figure 4.38. Entering task duration option in Initial Estimate node for incomplete tasks

The task is assumed to have an initial normal distribution with mean value equal to duration option chosen by VB code. For this task, it is assumed that the VB Code chose option 2 which has corresponding duration value of 7 days (mean=7).

This node is a simulation node which runs a Monte Carlo simulation to generate task duration distribution.

After entering all options, the analysis runs to generate project total duration distribution. The distribution in Early Finish node of last task is chosen as project duration distribution. This node is a simulation node which defined as Early Start + Adjusted Duration which basically takes the distribution of tasks and generates overall project distribution. Project duration distribution is shown in Figure 4.39.

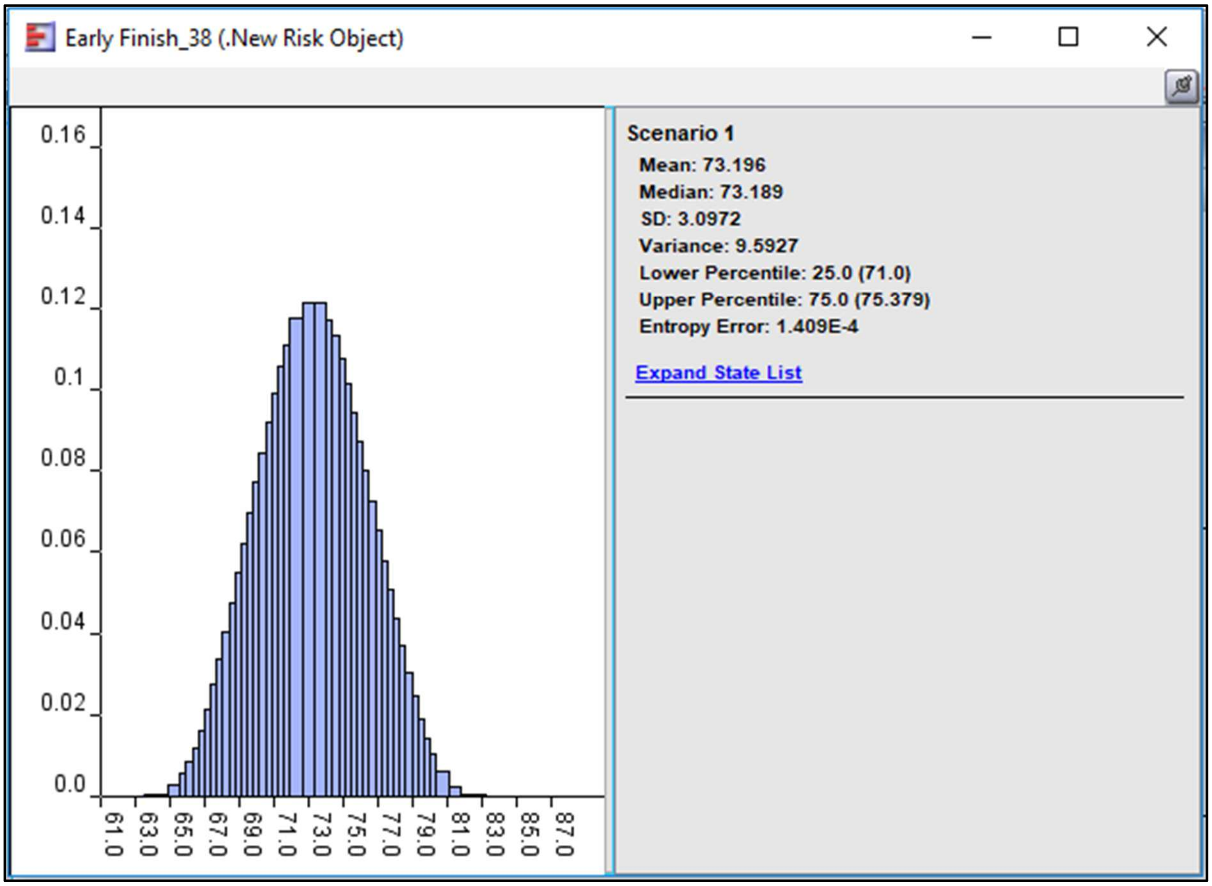

Figure 4.39. Project duration distribution for Above Cost-Behind Schedule scenario with selected options

4.2.3.3. Below Cost-Behind Schedule: This case may happen when the actual cost of finished activities is lower than estimated cost. This may happen due to errors in estimating the cost or decrease in wage and equipment cost. The project is behind the schedule and actions should be taken to get it back on schedule. Same as previous scenario, options are generated by entering the actual task durations as *evidence* in *Adjusted Duration* node. Due to poor performance of crews, the performance factor would be higher than 1. The mean value of distributions in *Adjusted Duration* node is taken as task duration options. Different options are generated by changing the initial estimate values based on crew numbers. Same procedure as described in section 4.2.3.2. is followed to generate duration and cost crashing options as well as overall project duration distribution.

After calculating the IRR value for three different crew numbers, the optimization algorithm is run to find the optimum combination of options. The IRR value of optimum solution is then compared with IRR value of each individual crew number to verify the efficiency of the solution. These values are shown in Table 4.20.

Table 4.20. IRR values for different crew numbers of Below Cost-Behind Schedule scenario

Crew Number	IRR Value (%)	Increase (%)
1	8.737	5.13
2	7.853	16.97
3	7.057	30.17
Combination	9.186	-

This solution provides 5.13%, 16.97% and 30.17% increase in IRR value comparing to using only one, two and three crews when the project is below cost but behind scheduled plan. A comparison between these values is depicted in Figure 4.40. Options which have been chosen by optimization algorithm along with their corresponding durations are shown in Table 4.21. After entering new duration values, the scheduling network predicts the project to be finished most likely in about 72 days with optimistic and pessimistic finish times of 70 and 75 days. Project duration distribution is shown in Figure 4.41.

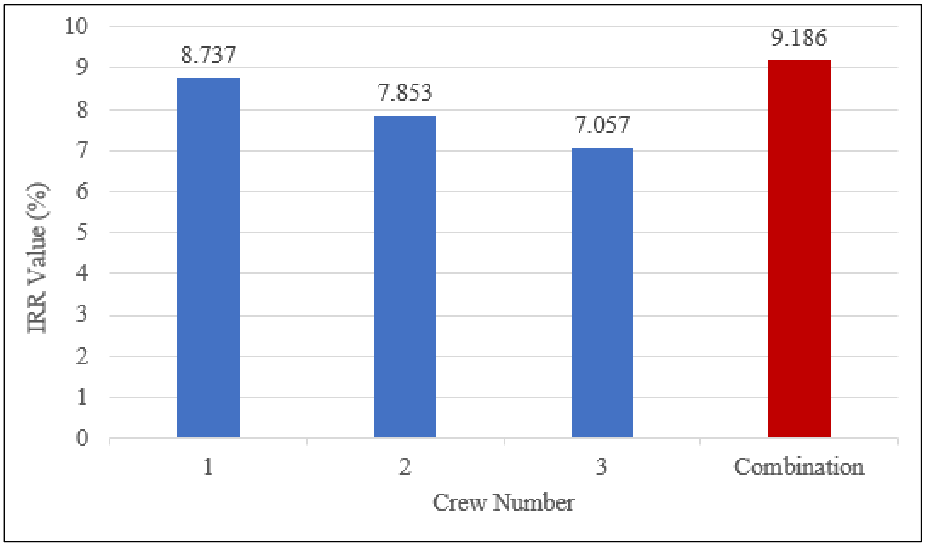

Figure 4.40. Comparing IRR values for different crew numbers in Below Cost-Behind Schedule scenario

Table 4.21. Chosen options and corresponding duration values for Below Cost-Behind Schedule scenario

Task	Chosen Option	Corresponding Duration (Days)
1	1	10
2	1	5
3	1	3
4	1	15
5	1	10
6	1	7
7	1	7
8	1	6
9	1	5
10	1	2
11	1	15
12	1	5
13	1	7
14	1	2
15	1	1
16	1	9
17	1	4

Table 4.21. Chosen options and corresponding duration values for Below Cost-Behind Schedule scenario (Continue)

Task	Chosen Option	Corresponding Duration (Days)
18	1	2
19	1	2
20	2	6
21	1	1
22	1	23
23	1	9
24	1	2
25	1	4
26	1	25
27	2	8
28	1	11
29	2	6
30	1	2
31	1	2
32	1	4
33	1	4
34	1	4
35	1	1
36	1	2
37	1	2
38	1	1

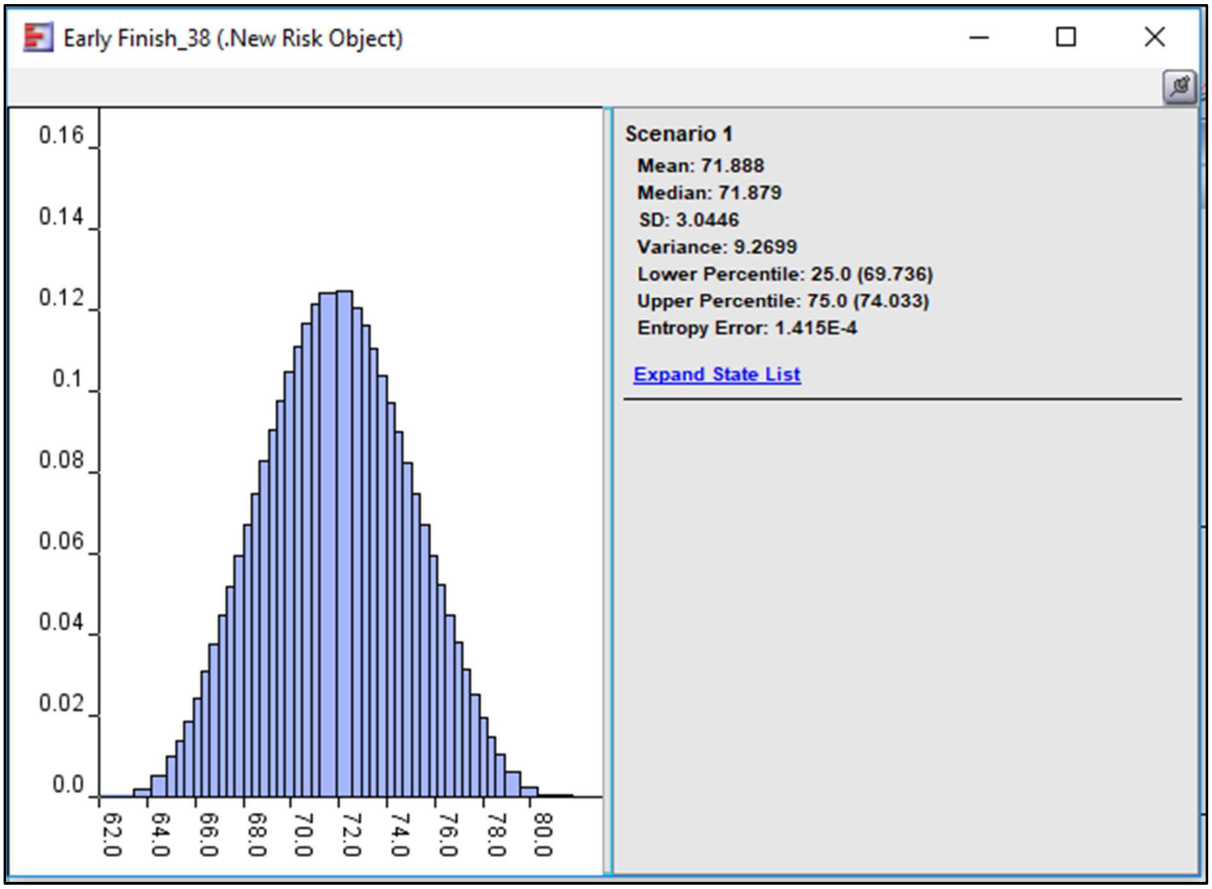

Figure 4.41. Project duration distribution for Below Cost-Behind Schedule scenario with selected options

4.2.3.4. Above Cost-Ahead Schedule: In this case tasks are finished faster than what estimated but with higher cost. Duration and cost options are generated by entering actual task durations as *evidence* in *Adjusted Duration* node. In this scenario, the performance of crew is better than expected so the performance factor would be lower than 1. The mean value of distributions in *Adjusted Duration* node is taken as task duration option. Different options are generated by changing the initial estimate values based on crew numbers. Same procedure as described in section 4.2.3.2. is followed to generate duration and cost crashing options as well as overall project duration distribution.

After finding the IRR value for different crew numbers, the optimization algorithm is run to find the optimum combination of duration and cost options. These values along with comparison are shown in Table 4.22.

Table 4.22. IRR values for different crew numbers of Above Cost-Ahead Schedule scenario

Crew Number	IRR Value (%)	Increase (%)
1	11.455	7.32
2	10.356	18.71
3	9.873	24.52
Combination	12.294	-

The optimization approach is capable of increasing project's IRR by 7.32%, 18.71% and 24.52% for one, two and three crews by combining different duration and cost options. A comparison between these values is shown in Figure 4.42. In Table 4.23 chosen options and their corresponding duration values is depicted. Running the scheduling network with selected optimum options results in an estimation of finishing the project most likely in about 68 days with optimistic and pessimistic finish times of 66 and 71 days. Project duration distribution is shown in Figure 4.43.

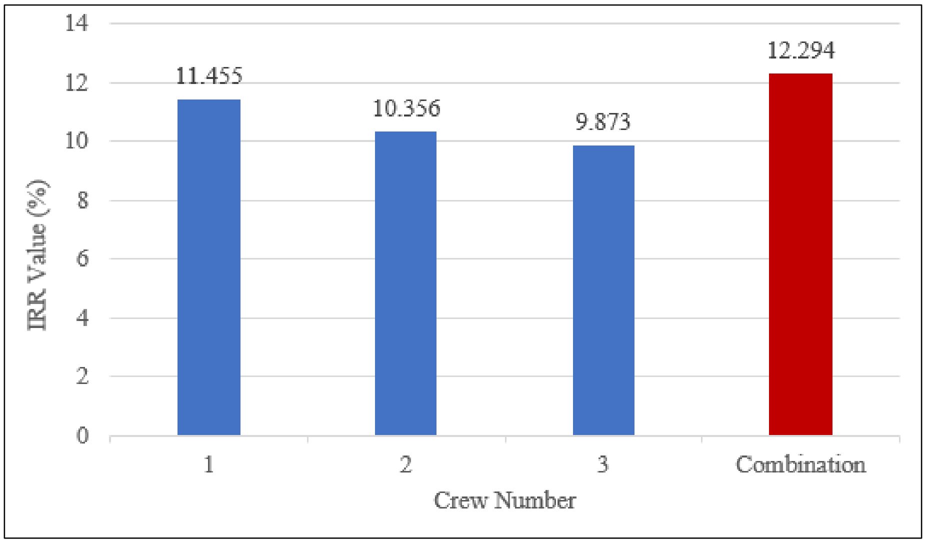

Figure 4.42. Comparing IRR values for different crew numbers in Above Cost-Ahead Schedule scenario

Table 4.23. Chosen options and corresponding duration values for Above Cost-Ahead Schedule scenario

Task	Chosen Option	Corresponding Duration (Days)
1	1	10
2	1	5
3	1	3
4	1	15
5	1	10
6	1	7
7	1	5
8	1	4
9	1	3
10	1	1
11	1	15
12	1	3
13	2	3
14	1	2
15	1	1
16	3	4
17	1	3

Table 4.23. Chosen options and corresponding duration values for Above Cost-Ahead Schedule scenario (Continue)

Task	Chosen Option	Corresponding Duration (Days)
18	1	2
19	1	2
20	1	4
21	1	1
22	1	14
23	1	7
24	1	2
25	1	3
26	1	25
27	1	7
28	2	7
29	1	6
30	1	2
31	1	2
32	1	4
33	1	4
34	1	2
35	1	1
36	1	2
37	1	2
38	1	1

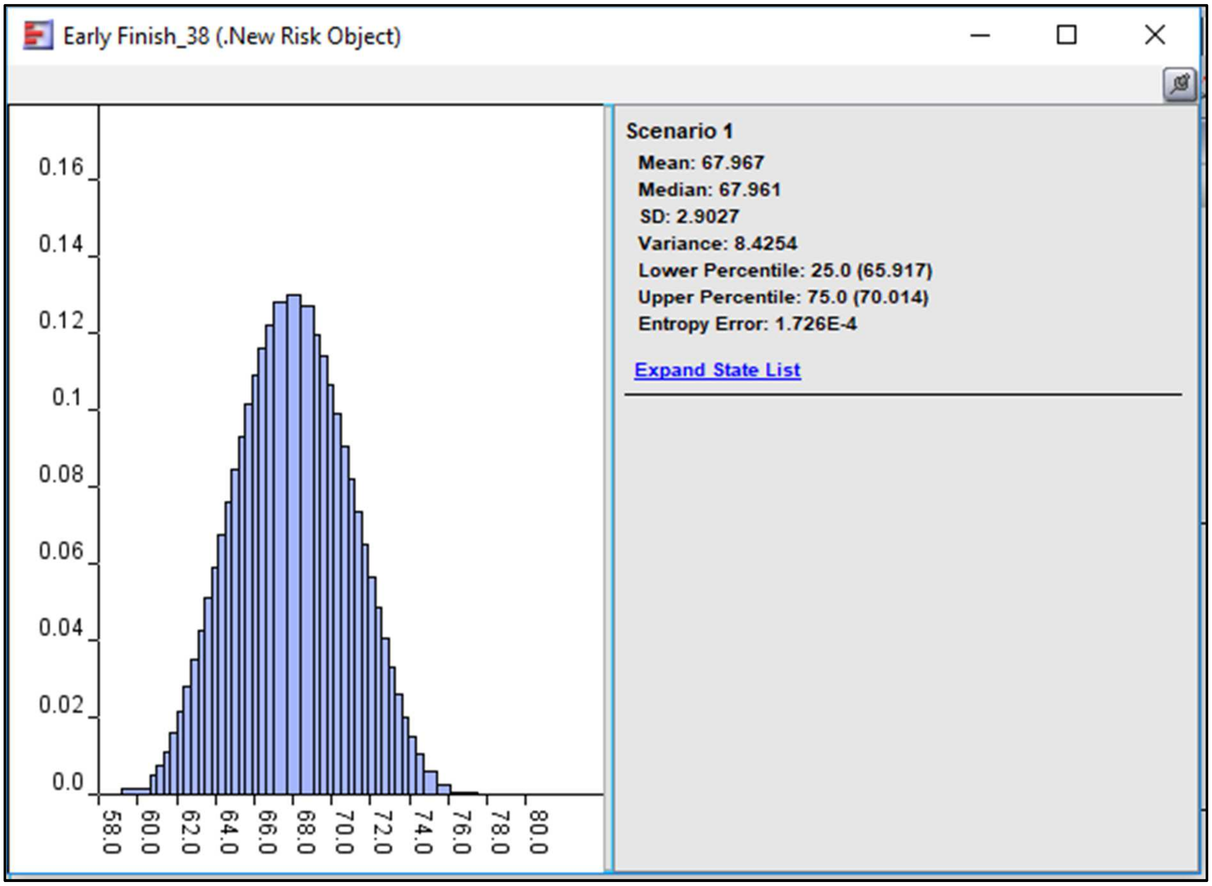

Figure 4.43. Project duration distribution for Above Cost-Ahead Schedule scenario with selected options

4.2.4. Examining the effect of different experts' confidence level on project duration and cost

As mentioned in section 4.2.2.1, decreasing the experts' level of confidence without changing their linguistic evaluations will result in duration modifiers with same mean value but higher variations. The higher variation causes longer estimated project duration and higher cost. To verify this effect, the values of *Fuzzy Risk* node is changed based on duration modifiers for medium and low level of confidence in Table 4.14. New values for this node are shown in Figures 4.44 and 4.45.

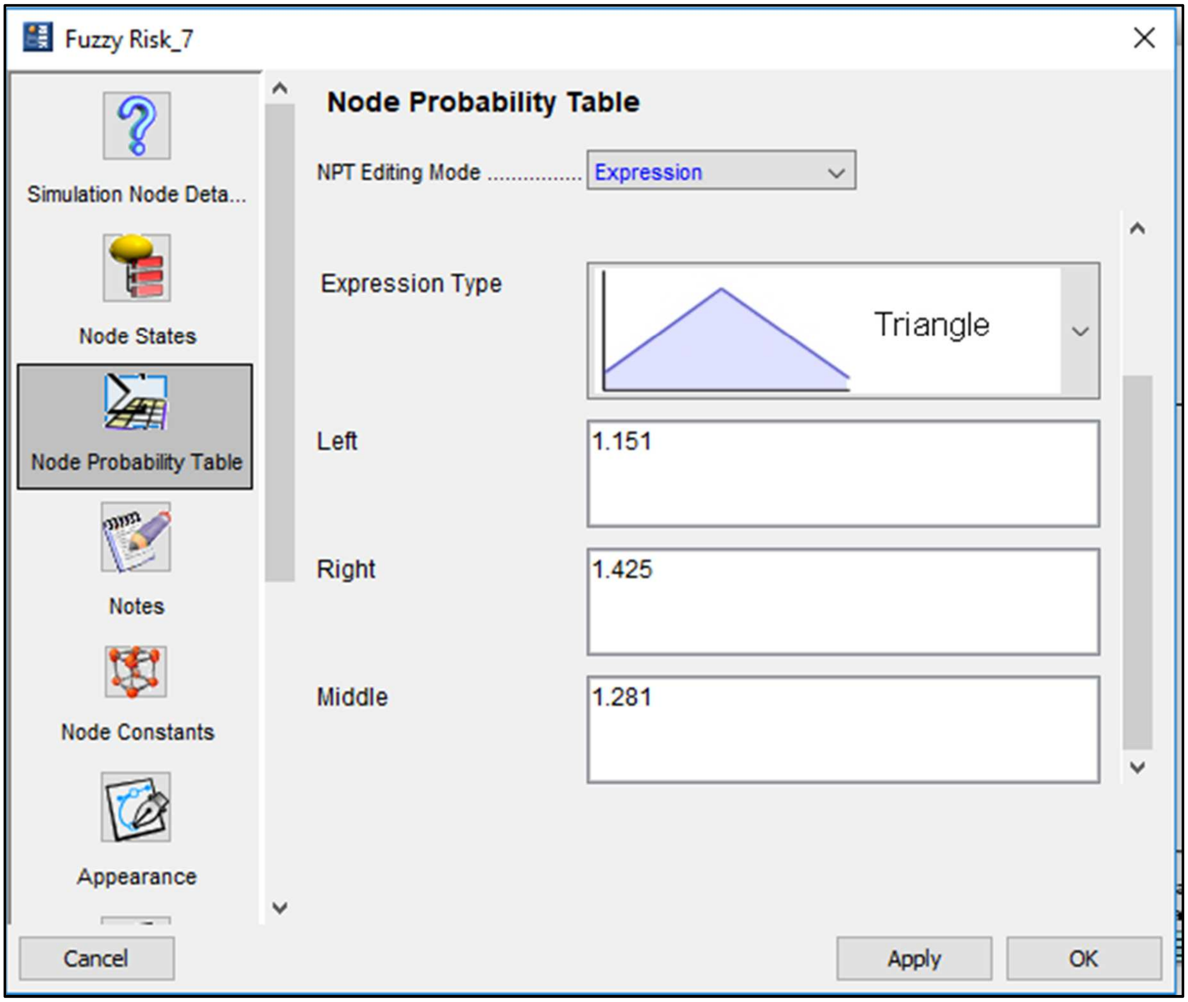

Figure 4.44. Changing the values of Fuzzy Risk node based in medium level of confidence

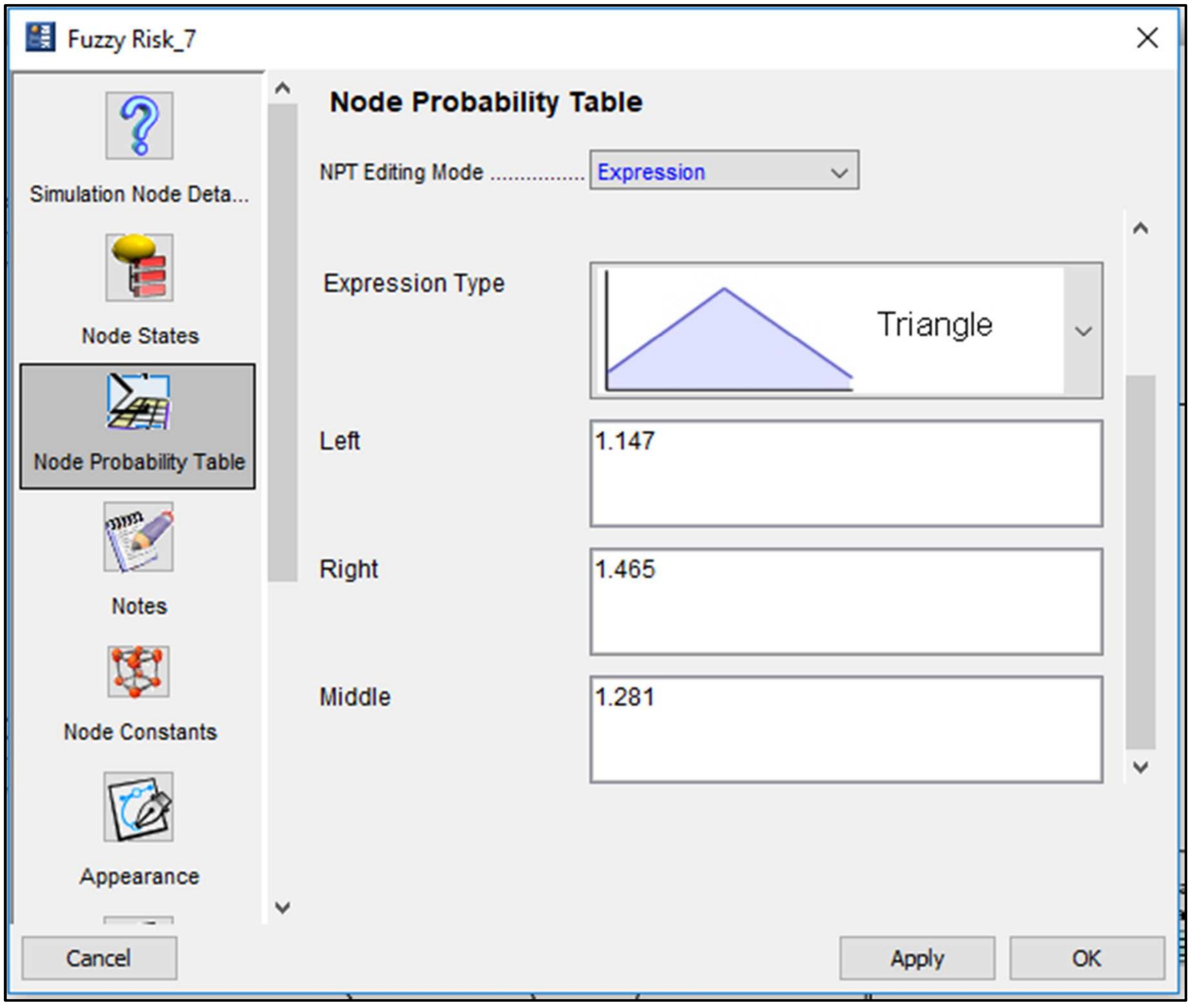

Figure 4.45. Changing the values of Fuzzy Risk node based in low level of confidence

The analysis runs again for both cases and results are compared to project duration and cost of base case with one crew where confidence levels were assumed to be high. Project duration distribution for base case with high, medium and low level of confidence using only one crew and cumulative cost are shown in Figures 4.45, 4.46, 4.47 and Table 4.24. New task durations are used to calculate the cost. For medium level of confidence, the project duration and cost have been increased by 5.33% and 6.92% while this increase is 9.35% and 10.08% for low level of confidence.

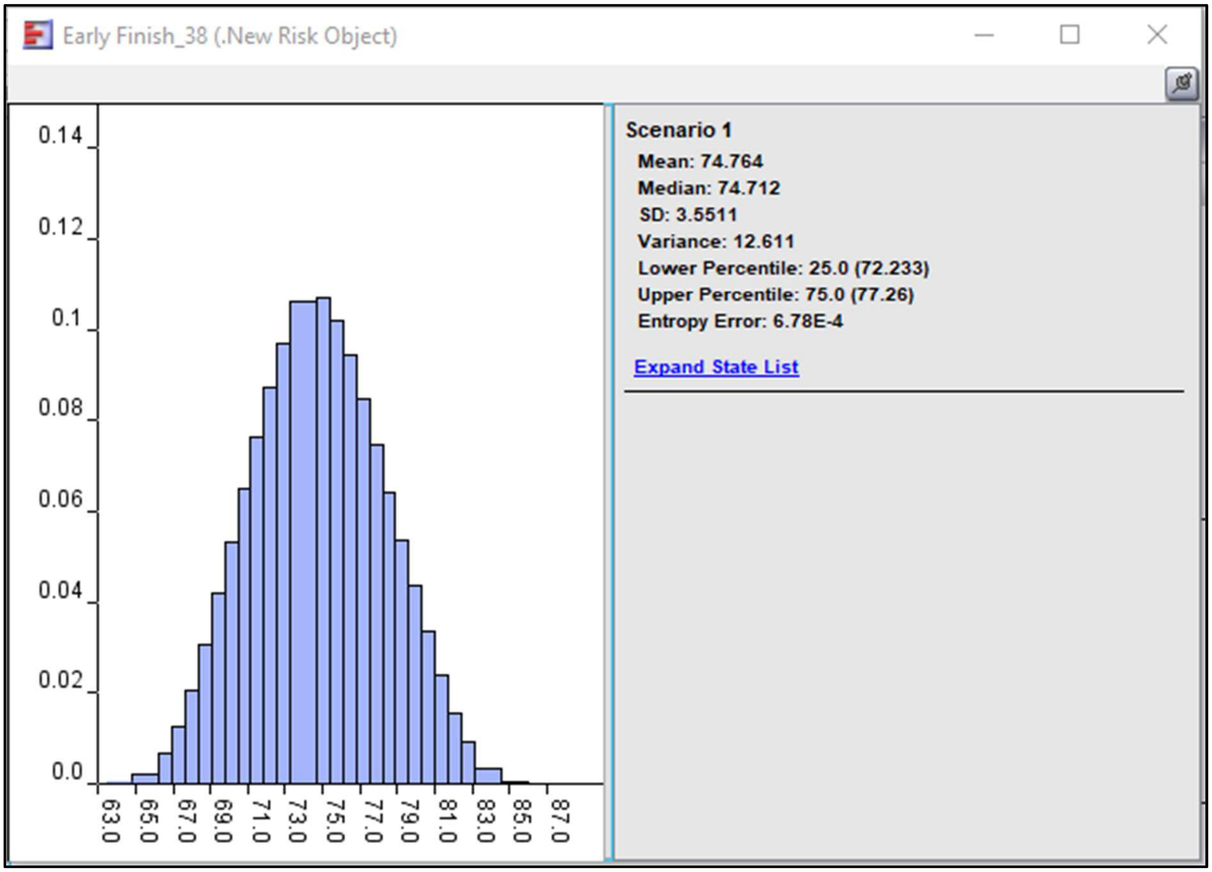

Figure 4.45. Project duration distribution for high level of confidence in base case

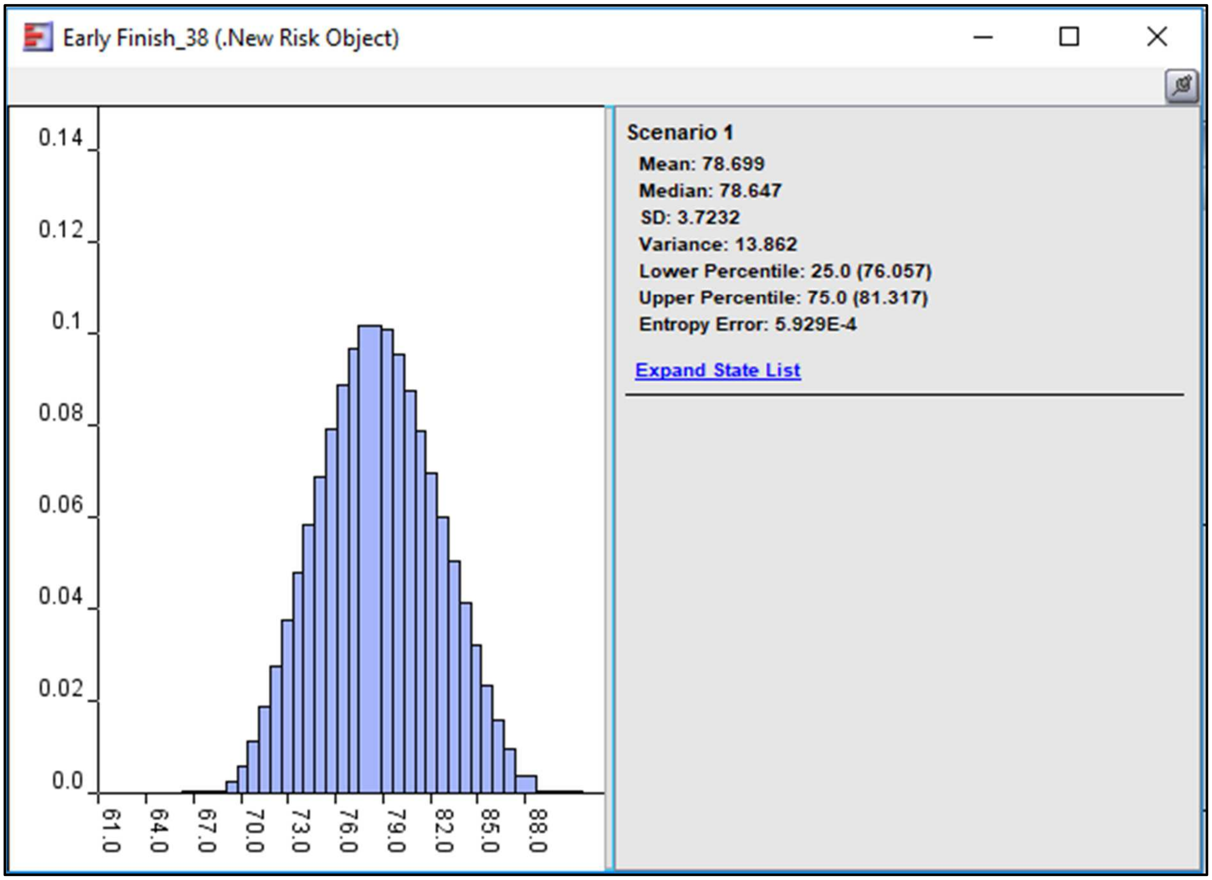

Figure 4.46. Project duration distribution for medium level of confidence in base case

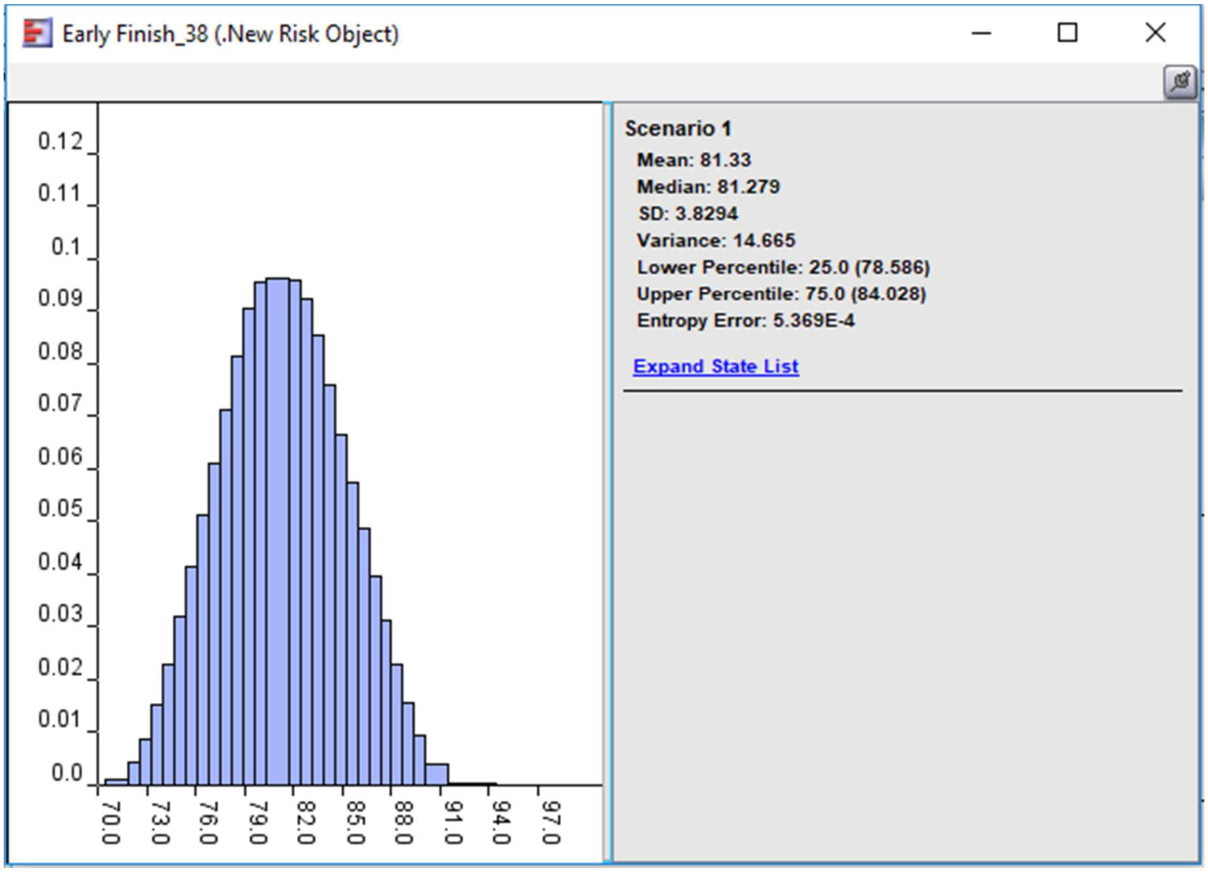

Figure 4.47. Project duration distribution for low level of confidence in base case

Table 4.24. Project duration and cost for high, medium and low levels of confidence

Level of confidence	Duration (Days)	Cost
High	75	$433,692
Medium	79	$463,703
Low	82	$477,408

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

Each construction project is subject to unique risk factors. At initial stages of a project where there is lack of high-quality quantitative data, statistical methods may not be as efficient as subjective methods in estimating the uncertainty. In this case, Fuzzy Set Theory has proven to be an effective method to deal with uncertainty. In this research, a flexible and intelligent fuzzy risk assessment model to quantify the uncertainty in task durations that are affected by a combination of different risk factors is developed. After capturing the experts` linguistic evaluations and their confidence degree regarding the likelihood and severity of affecting each risk factor on task durations, the tool suggests three duration modifiers to generate the optimistic, most likely and pessimistic cases for each task duration. These values can be used to form the triangular distribution of task duration under combinatory effect of different risk factors. This research extends the current fuzzy risk assessment methods by incorporating the confidence degree of decision makers in fuzzy computations which makes the risk assessment model more flexible and intelligent.

The experts` subjective assessments are updated by using the actual task data during project execution through a learning dynamic Bayesian Network model. For this purpose, scheduling network is modeled as a Bayesian Network. For each task, a duration block to model the initial duration estimate, affecting risks, crew performance and start and finish times is developed. For completed tasks, actual task data entered the model and provide basis to estimate crew performance and predict project performance. The model can generate different options by varying the crew numbers for different project scenarios. These options are then used by an optimization algorithm for optimum resource-based scheduling.

Developed model covers both deterministic and probabilistic analysis cases as phase I and phase II. In phase I, an optimization algorithm and solution approach based on MILP model is presented. Developed model considers the variation of crew productivity during project execution, and the complex payment terms. It maximizes the project NPV considering a number of financial factors like direct cost, indirect cost, interest cost and received payments. A novel sequential and adaptive solution procedure is developed to combine a statistical model for updating the time-cost tradeoff. This model uses the discrete duration and cost values as crashing options. In phase II, an optimization model is using VB code in Excel has been developed. In this model, the mean values of distributions calculated by Bayesian scheduling network is used to generate crashing options for different crew numbers. This model and solution approach, combines different crashing options to find the optimum duration and cost options which maximizes the project NPV.

Developed deterministic (phase I) and probabilistic (phase II) models and solution approaches have been examined on a simple bridge project simulation with 38 activities. Four different scenarios including one base case where the project has not been started and three additional scenarios where the project is partially completed are assumed to generate crashing options. Corresponding crashing duration and cost data are dynamically updated during project execution based on observe project performance. Results show that for real project case, this resource-based scheduling model generates significantly higher NPV and IRR values than the traditional CPM solution in both phases. In phase I, for base case the NPV has been increased by about 14% while this increase for Above Cost-Ahead Schedule, Above Cost-Behind Schedule, Below Cost-Behind Schedule is about 18%, 12% and 10%. In phase II, the increase in IRR value using only one crew for base case and Above Cost-Ahead Schedule, Above Cost-

Behind Schedule, Below Cost-Behind Schedule scenarios is about 4.76%, 4.19%, 5.13% and 7.32%. Additional computational experiments were conducted to examine the impact of problem parameters on the optimal solutions. Computational results have validated the solution obtained by proposed optimization approach.

One of the contributions of this research is incorporating the confidence level of experts in decision making. It is assumed that increasing the experts' confidence level will result in more accurate duration and cost estimation. To verify this assumption, Fuzzy calculations to obtain the duration modifiers were performed for three levels of confidence. In high level of confidence (90%-100%) the project is estimated to finish sooner with less cost while in medium level of confidence (80%-90%) and low level of confidence (70%-80%) the project duration is estimated to be increased by 5.33% and 9.35%. This increase for cost is 6.92% and 10.08%. This increase is due to higher variation in duration modifiers when reducing the level of confidence. In this case, the model assumes higher risk values to account for lower level of confidence which leads to longer task duration and cost.

APPENDIX

DETAILED FUZZY WEIGHTED AVERAGE CALCULATIONS FOR $\alpha - \text{cut} = 0$

BASED ON THE MAX-MIN PAIRED ELIMINATION ALGORITHM

Left-Right membership functions defined in Table 8 can be rewritten in terms of α-cuts. Given $\alpha - \text{cut} = 0$, the probability of failure (R_i) and severity of loss (W_i) for each risk factor are calculated as shown in Matrices 1 and 2.

$$\begin{bmatrix} R_{1L} & W_{1L} \\ R_{2L} & W_{2L} \\ R_{3L} & W_{3L} \\ R_{4L} & W_{4L} \end{bmatrix} = \begin{bmatrix} (0.241, 0.369) & (0.298, 0.439) \\ (0.353, 0.492) & (0.186, 0.274) \\ (0.238, 0.366) & (0.155, 0.266) \\ (0.059, 0.147) & (0.153, 0.264) \end{bmatrix} \qquad \text{Matrix 1}$$

$$\begin{bmatrix} R_{1U} & W_{1U} \\ R_{2U} & W_{2U} \\ R_{3U} & W_{3U} \\ R_{4U} & W_{4U} \end{bmatrix} = \begin{bmatrix} (0.177, 0.433) & (0.227, 0.510) \\ (0.248, 0.525) & (0.118, 0.288) \\ (0.177, 0.433) & (0.099, 0.316) \\ (0.018, 0.188) & (0.099, 0.316) \end{bmatrix} \qquad \text{Matrix 2}$$

Lower Fuzzy Number

Lower Bound: find the $\min\{f_L\}_L$

Loop 1.

(1) Choose the smallest ($\downarrow \min$) and the largest ($\uparrow \max$) criteria coefficients from Matrix 1.

Smallest ($\downarrow \min$) = 0.059, largest ($\uparrow \max$) = 0.353

(2) Choose c_1 which matches a_1 and d_n which matches a_n.

$$a_i = (0.059, 0.238, 0.241, 0.353)$$

$$[c_i, d_i] = [(0.153, 0.264), (0.155, 0.266), (0.298, 0.439), (0.186, 0.274)]$$

$$c_1 = 0.264, d_4 = 0.186$$

(3) Calculate a' and w'.

$$a' = \frac{0.353 \times 0.186 + 0.059 \times 0.264}{0.186 + 0.264} = 0.181$$

$$c' = d' = w' = 0.450$$

(4) First delete the coefficients 0.059 and 0.353 and their corresponding weighting factors (0.153,0.264) and (0.186,0.274); then replace with $a' = 0.181$ and $c' = d' = 0.450$.

Loop 2.

(1) Smallest ($\downarrow$ min) = 0.181, largest ($\uparrow$ max) = 0.241

(2) $a_i = (0.181, 0.238, 0.241)$

$$[c_i, d_i] = [(0.450, 0.450), (0.155, 0.266), (0.298, 0.439)]$$

$$c_1 = 0.450, d_3 = 0.298$$

(3) Calculate a' and w'.

$$a' = \frac{0.241 \times 0.298 + 0.181 \times 0.450}{0.298 + 0.450} = 0.205$$

$$c' = d' = w' = 0.747$$

(4) First delete the coefficients 0.181 and 0.241 and their corresponding weighting factors (0.450,0.450) and (0.298,0.439), then replace with $a' = 0.205$ and $c' = d' = 0.747$.

Loop 3.

(1) Smallest ($\downarrow \min$) = 0.205, largest ($\uparrow \max$) = 0.238

(2) $a_i = (0.205, 0.238)$

$[c_i, d_i] = [(0.747, 0.747), (0.155, 0.266)]$

$c_1 = 0.747, d_2 = 0.155$

(3) Calculate a' and w'.

$$a' = \frac{0.238 \times 0.155 + 0.205 \times 0.747}{0.155 + 0.747} = 0.210$$

$c' = d' = w' = 0.902$

(4) First delete the coefficients 0.205 and 0.238 and their corresponding weighting factors (0.747,0.747) and (0.155,0.266), then replace with $a' = 0.210$ and $c' = d' = 0.902$.

Since $a' = 0.210$ is the only coefficient from the two loops, thus the final solution for the lower bound is $\min\{f_L\}_L = 0.210$.

Upper Bound: find the $\max\{f_U\}_L$

Loop 1.

(1) Choose the smallest ($\downarrow \min$) and the largest ($\uparrow \max$) criteria coefficients from Matrix 1.

Smallest ($\downarrow \min$) = 0.147, largest ($\uparrow \max$) = 0.492

(2) Choose d_1 which matches b_1 and c_n which matches b_n.

$b_i = (0.147, 0.366, 0.369, 0.492)$

$$[c_i, d_i] = [(0.153, 0.264), (0.155, 0.266), (0.298, 0.439), (0.186, 0.274)]$$

$$d_1 = 0.153, c_4 = 0.274$$

(3) Calculate b' and w'.

$$b' = \frac{0.492 \times 0.274 + 0.147 \times 0.153}{0.274 + 0.153} = 0.369$$

$$c' = d' = w' = 0.426$$

(4) First delete the coefficients 0.147 and 0.492 and their corresponding weighting factors (0.153,0.264) and (0.186,0.274); then replace with $b' = 0.369$ and $c' = d' = 0.426$.

Loop 2.

(1) Smallest ($\downarrow \min$) = 0.366, largest ($\uparrow \max$) = 0.369

(2) $b_i = (0.366, 0.369, 0.369)$

$$[c_i, d_i] = [(0.155, 0.266), (0.426, 0.426), (0.298, 0.439)]$$

$$d_1 = 0.155, c_3 = 0.439$$

(3) Calculate b' and w'.

$$b' = \frac{0.369 \times 0.439 + 0.366 \times 0.155}{0.439 + 0.155} = 0.368$$

$$c' = d' = w' = 0.594$$

(4) First delete the coefficients 0.366 and 0.369 and their corresponding weighting factors (0.155,0.266) and (0.298,0.439), then replace with $b' = 0.368$ and $c' = d' = 0.594$.

Loop 3.

(1) Smallest ($\downarrow \min$) = 0.368, largest ($\uparrow \max$) = 0.369

(2) $b_i = (0.368, 0.369)$

$[c_i, d_i] = [(0.594, 0.594), (0.426, 0.426)]$

$d_1 = 0.594, c_2 = 0.426$

(3) Calculate b' and w'.

$$b' = \frac{0.369 \times 0.426 + 0.368 \times 0.594}{0.426 + 0.594} = 0.368$$

$c' = d' = w' = 1.020$

(4) First delete the coefficients 0.368 and 0.369 and their corresponding weighting factors (0.594,0.594) and (0.426,0.426), then replace with $b' = 0.368$ and $c' = d' = 1.020$.

Since $b' = 0.368$ is the only coefficient from the two loops, thus the final solution for the upper bound is $\max\{f_U\}_L = 0.368$.

Upper Fuzzy Number

Calculations of fuzzy weighted average using Max-Min Paired Elimination algorithm for upper fuzzy number is implemented in the same manner by replacing the values of Matrix 1 by Matrix 2.

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