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# Spectrum-to-spectrum methods for the generation of elastic floor acceleration spectra

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## Abstract

In seismic codes, the acceleration demand of nonstructural components is commonly expressed in terms of floor response spectra and estimated by means of simple predictive equations. By using the latter, response-history analysis of the structure is not required, being floor spectra calculated directly from the peak ground acceleration expected at the site. The price for this simplicity in the method used for the estimation of floor spectra is the generally poor approximation of the obtained predictions. Codes' equations, in fact, do not explicitly account for important factors influencing floor spectra, such as the contribution of the higher modes of vibration of the structure and the actual value of the nonstructural components' damping ratio. Alternative spectrum-to-spectrum methods for direct generation of floor spectra have been proposed, which include these factors and improve the accuracy of the predictions. Different approaches have been used and several methods developed. Despite large research effort, however, a comparative evaluation of the currently available proposals is still lacking. The objective of this paper is to fill this gap, by reviewing selected proposals representative of practice-oriented spectrum-to-spectrum methods. A case study consisting in a six-story frame is analyzed and predictions obtained with the investigated methods are compared with exact floor spectra derived from time-history analyses of the structure, as well as spectra calculated using the Eurocode 8 equation.

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*Keywords:* nonstructural components; acceleration-sensitive; floor response spectra; seismic demand; uniform hazard

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## 1. Introduction

In the field of earthquake engineering, the study of the seismic performance of nonstructural components is gaining increased research attention. Experiences from earthquakes worldwide keep showing, indeed, that even

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when collapse of a building is prevented, its functionality may be seriously compromised by nonstructural damage. For emergency facilities that are required to remain operational immediately after the earthquake, evaluation of nonstructural damage becomes therefore a primary concern. This is the case, for example, of hospitals, police and fire stations. Past earthquakes have also shown how important components may be for ordinary buildings. Frequently, nonstructural damage contributes for a large portion to the total cost required to repair the building. Besides, severe damage to nonstructural components can significantly increase the risk of fatal injury to the building's occupants. Response parameters used to estimate nonstructural damage differ depending on whether deformation- or acceleration-sensitive components are considered. In the latter case, seismic demand is usually represented through floor spectra, i.e., response spectra in terms of pseudo-acceleration (hereafter simply shortened to acceleration) estimated at the floor levels of the structure where the nonstructural components are attached to.

Floor spectra are determined from the response of single-degree-of-freedom (SDOF) oscillators (representing nonstructural components) to the floor motion of the structure induced by the seismic excitation. Depending on the considered analysis method, they may be explicitly derived from floors' acceleration histories, based on structural response-history analysis, or calculated using a predictive equation from a given input ground motion spectrum. Among the two alternatives the second one is often preferred, especially in seismic codes, because of its simplicity. Its appeal consists in the fact that a response-history analysis of the structure is not required, and only a standard response spectrum, rather than a set of ground motion time-series, is needed to model seismic action at the site. Methods which make use of predictive equations are commonly known in the literature as spectrum-to-spectrum methods, being floor spectra directly calculated from ground spectra. The objective of this paper is both to review these methods and to evaluate their predictive accuracy. Only selected proposals representative of practice-oriented methods that use as input ground spectra expressed in terms of acceleration will be reviewed. The evaluation will be carried out with reference to a case study consisting in a six-story reinforced concrete frame located in Milan. Exact floor spectra derived from a large number of time history analyses will be used to estimate the predictive accuracy of the methods. Improvements in the predictions obtained by using these methods instead of the Eurocode 8 [1] equation will be shown.

## 2. Spectrum-to-spectrum methods

Several spectrum-to-spectrum methods can be found in the literature, which have been developed by employing a range of different approaches from analytical to numerical, deterministic or probabilistic (e.g. [2-11]). Among the large number of methods, however, only those proposed by the following researchers are considered in this study: Yasui et al. [9], Calvi and Sullivan [10], and Lucchini et al. [11] (i.e. the Authors of the present work). By using these methods floor spectra are calculated by means of simple closed-form equations, which require as input the modal properties of the structure and ground spectra expressed in terms of acceleration, representation of the seismic action usually provided by codes. Floor spectra, in particular, are generated by first calculating, through predictive equations, the floor spectrum produced by the ground motion filtered by each mode of vibration of the structure. Modal contributions are then estimated multiplying each spectrum by the participation factor and the shape component, at the floor of interest, of the corresponding mode of vibration<sup>1</sup>. Contributions are finally combined by a simple square root of the sum of the squares (SRSS) combination rule. Note that this framework is identical to that of the conventional modal response spectrum method of analysis used in current practice to calculate structural demand. The methods differ in the proposed equation for predicting the floor spectrum produced by the single mode of vibration of the structure, but are built on the same assumptions, i.e., that the nonstructural component and the supporting structure both behave linearly, and that the dynamic interaction between the two systems can be neglected. This means that the methods can be applied to light components mounted on structures that are expected to respond in their elastic range to the earthquake.

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<sup>1</sup> In the case a spatial model of the structure is used, e.g. for studying the 3D response of a torsional building, the participation factor is calculated for the considered horizontal direction of the earthquake excitation, and the component of the shape vector is the one along the direction where the floor spectra of interest are calculated.

In the following subsections the three methods will be presented in detail. Predictive equations will be reported using the same notation adopted in the works where they were originally presented.

### 2.1. Yasui & others

In the method proposed by Yasui et al. [9], the floor spectral acceleration generated by the  $i$ th mode of vibration of the structure is denoted as  $S_{Ei}$  and is calculated as follows

$$S_{Ei} = \frac{1}{\sqrt{\{1 - (\omega_A/\omega_{Bi})^2\}^2 + 4(h_A + h_{Bi})^2(\omega_A/\omega_{Bi})^2}} \sqrt{\{(\omega_A/\omega_{Bi})^2 S(\omega_{Bi}, h_{Bi})\}^2 + S(\omega_A, h_A)^2} \quad (1)$$

in which  $\omega_A$  and  $h_A$  are the natural circular frequency and the damping ratio, respectively, of the SDOF oscillator representing the nonstructural component;  $\omega_{Bi}$  and  $h_{Bi}$  are the corresponding properties of the mode of vibration of the structure;  $S$  is the input spectral acceleration calculated at both  $(\omega_{Bi}, h_{Bi})$  and  $(\omega_A, h_A)$ .

The equation is derived analytically by using the Duhamel integral for the determination of the time-history response of the oscillator. The latter is expressed as a function of the response of the mode and that of the oscillator to the ground motion excitation. The maxima of these two contributions are combined by means of a SRSS rule and the floor spectral acceleration is finally estimated. The derivation is conducted separately for the case of  $\omega_A = \omega_{Bi}$  ('resonant case') and  $\omega_A \neq \omega_{Bi}$  ('non-resonant case'). Two independent equations are derived and then combined to obtain Equation (1). Note that the equation requires in addition to the dynamic properties of both the component and the structure, the spectral acceleration of the ground motion estimated at two values of the damping ratio, namely,  $h_A$  and  $h_{Bi}$ .

### 2.2. Calvi & Sullivan

In Calvi and Sullivan [10] the predictive equation proposed by Sullivan et al. [12] is used to calculate the floor spectra of the structural modes. The equation is empirical and derived from results of nonlinear time-history analyses run using component-structure systems excited by a large suite of earthquake ground motions. Originally proposed for the general case of nonlinear SDOF supporting systems, the equation takes the following form when applied to predict the floor spectrum of the  $i$ th mode of vibration of the structure

$$\begin{aligned} a_m &= \frac{T}{T_i} [a_{max}(DAF_{max} - 1)] + a_{max} & T < T_i \\ a_m &= a_{max} DAF_{max} & T = T_i \\ a_m &= a_{max} DAF & T > T_i \end{aligned} \quad (2)$$

where  $a_m$  is the maximum acceleration of the supported component with period  $T$ , i.e., the floor spectral ordinate at period  $T$ ;  $a_{max}$  is the maximum acceleration of the supporting system with period  $T_i$ , i.e., the ground spectral acceleration at period  $T_i$ ;  $DAF$  is the dynamic amplification factor expressed as follows as a function of the period ratio  $\beta = T_i/T$  and of the damping ratio of the component  $\xi$

$$DAF = \frac{1}{\sqrt{(1-1/\beta)^2 + \xi}} \quad (3)$$

and  $DAF_{max}$  is the maximum value of the dynamic amplification obtained by substituting  $\beta = 1$  into Equation (3) or directly as

$$DAF_{max} = \frac{1}{\sqrt{\xi}} \quad (4)$$

or using an alternative expression, not reported here, to obtain less conservative estimates of  $DAF_{max}$  in the case of short period structures, that is, when  $T < 0.3$ s.

Contributions of the modes of vibrations to floor spectral accelerations are estimated and then combined to calculate the floor spectra of the structure by using the previously described modal procedure. Up to the mid-height of the structure, however, an adjustment is required. In this case, in order to account for the limited filtering that occurs to the ground motion over the lower levels of the structure, it is proposed that the floor spectra are obtained as the envelopes of the floor spectra calculated according to the modal procedure and the ground response spectrum.

2.3. Lucchini & others

In Lucchini et al. [11] a probabilistic approach is used to derive the predictive equation, which allows the calculation of uniform hazard floor response spectra, namely, floor response spectra whose ordinates are characterized by a given value of the mean annual frequency (MAF) of being exceeded. Given the target MAF of interest  $\lambda$ , the floor spectral acceleration, that is, the response  $S^i$  of a SDOF oscillator to the ground motion filtered by the  $i$ th mode of vibration of the structure, is calculated as follows

$$S^i(\lambda) = \exp \left[ a + \frac{1}{2k_2} \left( -k_1 + \sqrt{\frac{k_1^2}{q} - \frac{4k_2}{q} \ln \frac{\lambda}{k_0 \sqrt{q}}} \right) \right], \quad q = \frac{1}{1+2k_2\sigma^2} \tag{5}$$

which is a function of the five parameters  $a$  and  $\sigma$ ,  $k_0$ ,  $k_1$  and  $k_2$ .

The parameter  $a$  represents, in the log-space, the median dynamic amplification factor of the oscillator’s response with respect to that of the mode of vibration the structure, while  $\sigma$  represents its logarithmic standard deviation (i.e. the record-to-record variability). They depend on the dynamic properties of both the oscillator and the mode of vibration of the structure through the following equations

$$\begin{aligned} a &= a^t r^{n_1} & r \leq 1 \\ a &= a^t + n_2(r^{n_3} - 1) & r > 1 \end{aligned} \tag{6}$$

$$\begin{aligned} \sigma &= \sigma^t [1 - (1 - r)^{n_4}] & r \leq 1 \\ \sigma &= \sigma^t + n_5(r - 1) & r > 1 \end{aligned} \tag{7}$$

in which  $r$  denotes the ratio  $T/T_i$  between the period of vibration of the SDOF oscillator and that of the  $i$ th mode of vibration of the structure. The two coefficients  $a^t$  and  $\sigma^t$  in Equation (6) and (7) represent the (tuning) values at  $r = 1$  of  $a$  and  $\sigma$ , respectively; the coefficients  $n_1$ ,  $n_2$ ,  $n_3$ ,  $n_4$  and  $n_5$  determine the variation of  $a$  and  $\sigma$  for  $r \neq 1$ . Each of these seven coefficients depends on the damping ratio  $\xi$  of the SDOF oscillator and can be calculated through a third-order polynomial of  $\ln(100\xi)$  (refer to [11] for the polynomial coefficients).

The parameters  $k_0$ ,  $k_1$  and  $k_2$  depend on the site seismicity, being the coefficients of the following quadratic expression that relates, in the log-space, the ground spectral acceleration at  $T_i$ ,  $s_a$ , and its corresponding value of the annual exceedance frequency,  $\lambda_{s_a}$

$$\ln \lambda_{s_a} = \ln k_0 - k_1 \ln s_a - k_2 \ln^2 s_a \tag{8}$$

Once three uniform hazard ground spectra are given, three  $\ln s_a - \ln \lambda_{s_a}$  pairs can be extracted and the parameters  $k_0$ ,  $k_1$ ,  $k_2$  simply estimated as the coefficients of a parabola through three points. This means that the ground spectral acceleration  $S_a$  does not directly enter into the predictive equation, as in the case of the two previous proposals, but through the three parameters  $k$ ; in addition, by using the equation of Lucchini et al. [11] three input ground spectra are required to calculate the floor spectra.

The possible case of  $S_a$  representing instead of the spectral acceleration of the arbitrary ground motion component ( $S_{a,arb}$ ) the geometric mean ( $S_{a,gm}$ ) or the maximum spectral acceleration ( $S_{a,max}$ ) of the two horizontal components is also accounted for, by means of a modification of the expressions for  $a$  and  $\sigma$ .  $a$  is calculated through Equation (6) and then modified as follows

$$\begin{aligned} a^{gm} &= a \\ a^{max} &= a^{gm} - 0.1817 \end{aligned} \tag{9}$$

in which the superscripts ‘gm’ and ‘max’ denote the estimate of the parameter when  $S_{a,gm}$  or  $S_{a,max}$  is used instead of  $S_{a,arb}$ . Equation (7) for calculating  $\sigma$  is replaced by

$$\begin{aligned} \sigma^{gm} &= n_4 + n_5 r \\ \sigma^{max} &= \sigma^{gm} + 0.0382 \end{aligned} \tag{10}$$

with the coefficients  $n_4$  and  $n_5$  determined in this case from a first-order polynomial of  $\ln(100\xi)$  instead of a third-order one (refer again to [11] for the polynomial coefficients). Note that while in the common practice seismic hazard is frequently defined in terms of  $S_{a,gm}$ , in some codes (such as the one currently adopted in Italy [13]) the use of  $S_{a,max}$  is preferred.

In order to account for epistemic uncertainty in the modal properties of the supporting structure, an extension of the equations has been recently proposed by the same Authors [14], which is not reported here nor reviewed for the sake of brevity. By using this extended version of the equations, the effect on the floor spectra of possible variations from nominal values of the periods of vibration of the structure can be explicitly estimated.

### 3. Case study

The three spectrum-to-spectrum methods are evaluated in this section by analyzing a 6-story 3-bay reinforced concrete frame located in Milan, already used in Lucchini et al. [11] as a case study. The structure is characterized by three significant modes of vibration, with periods equal to 1.0s, 0.35s and 0.2s, respectively. A 5% nominal damping ratio is assigned to each mode. Site seismicity is defined through the three uniform hazard ground spectra reported in the left panel of Fig. 1. The spectra are expressed in terms of maximum spectral acceleration, and the hazard level in terms of mean return period,  $T_R$ , equal to  $1/\lambda_{S_a}$ . More details about the modal properties of the structure and the method used to derive the ground spectra can be found in [11].

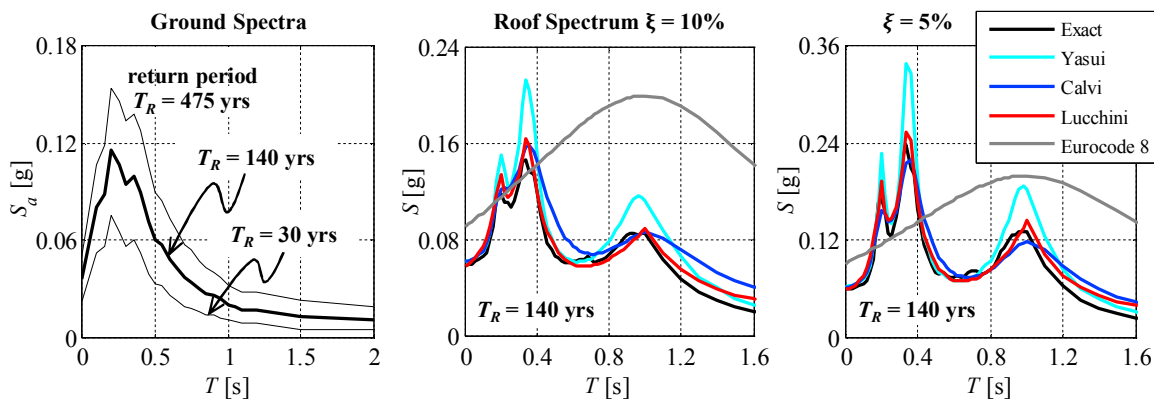


Fig. 1. 5%-damped ground spectra (left), and roof spectra corresponding to a damping ratio equal to 10% (center) and 5% (right), respectively.

The center and right panel of Fig. 1 report the roof spectra of the structure corresponding to a  $T_R$  value of 140 years obtained by assuming a component’s damping ratio equal to 5% and 10%, respectively. The exact roof spectrum in each panel is calculated through a standard probabilistic seismic demand analysis. In particular, the single ordinate of the spectrum, which represents the response of a supported SDOF oscillator, is derived from a demand hazard curve obtained by convolving the response with the seismic hazard; the response is obtained by means of a multiple stripe analysis, run with sets of ground motion records selected to match conditional spectra and shown to be consistent with the uniform hazard spectra used to represent the site seismicity (refer to [11] for more

details). The predictions obtained with the method of Yasui et al. and Calvi and Sullivan use as input the uniform hazard ground spectrum with  $T_R$  equal to 140 years (the same target value considered for the floor spectra). In the case of Yasui et al., the damping correction factor proposed by the Eurocode 8 is used to calculate the ground spectral acceleration corresponding to a damping ratio ( $h_A$ ) equal to 10% (note that the input spectra are 5%-damped).

By using the spectrum-to-spectrum methods rather than the Eurocode 8 equation, the predictions of the floor spectra significantly improve. With the Eurocode 8, the contribution of the higher modes of vibration of the structure is underestimated while it is overestimated that of the fundamental mode. The approximations obtained with Lucchini et al., instead, almost overlap the exact floor spectra. Those obtained with the other two methods match well the exact spectra in the whole period range, being the peak values better predicted by Calvi and Sullivan. It is important to note that the exact floor spectrum depends on the actual ground motions selected for representing site seismicity and used for exciting the structure, but not of course on the definition of  $S_a$  adopted to express the ground spectrum used as input in the spectrum-to-spectrum methods. However, in both Yasui et al. and Calvi and Sullivan the same equations apply independently if the ground spectrum is expressed in term of  $S_{a,arb}$ ,  $S_{a,gm}$  or  $S_{a,max}$ , making the predicted floor spectrum dependent on the used definition for  $S_a$ . On the contrary, in Lucchini et al. this incorrect dependency is avoided by means of Equation (9) and (10). By using Lucchini et al., in addition, epistemic uncertainty in the properties of the structure can be also explicitly accounted for. Results reported in [14] showed that the effect of such uncertainty cannot be neglected consisting in a significant reduction of the peak values of the floor spectra.

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