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A genetic algorithm for combined topology and shape optimisations

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Abstract

A method to find optimal topology and shape of structures is presented. With the first the optimal distribution of an assigned mass is found using an approach based on homogenisation theory, that seeks in which elements of a meshed domain it is present mass; with the second the discontinuous boundaries are smoothed. The problem of the optimal topology search has an ON/OFF nature and has suggested the employment of genetic algorithms. Thus in this paper a genetic algorithm has been developed, which uses as design variables, in the topology optimisation, the relative densities (with respect to effective material density) 0 or 1 of each element of the structure and, in the shape one, the coordinates of the keypoints of changeable boundaries constituted by curves. In both the steps the aim is that to find the variable sets producing the maximum stiffness of the structure, respecting an upper limit on the employed mass. The structural evaluations are carried out with a FEM commercial code, linked to the algorithm. Some applications have been performed and results compared with solutions reported in literature.

Keywords: Genetic algorithms; Topology optimisation; Shape optimisation

1. Introduction

Optimisation of continuous mechanical structures is much employed in industry, as far as properties, shape and topology are concerned [1]. If topology and shape are fixed by means of different criteria, the optimisation can solve problems of design variables calculation, such as thickness, height, radius of a circular boundary, fibre orientation in composite structures [2,3]. If only the topology is given, then optimisation can find the optimal shape of inward and outward boundaries [4,5]. These can be constituted by curves, defined by the location of some points, which are the design variables; but number, distribution and geometric entities of the boundaries have to be fixed in advance, using often heuristic criteria. However, the topology has a high influence on behaviour of a structure and therefore it is advisable to find it on more rational basis, e.g. with an optimisation procedure which determines the optimal distribution in a certain domain of an assigned limit mass, and consequently of its boundaries.

There has been studies on methods in topology optimisation principally in the last 20 years about. In Refs. [6–9] methods are used determining optimal topology by searching the optimal values of the densities of finite elements, in which a fixed feasible domain is meshed (homogenisation approaches). In other methods elements are removed from design domain or added to this one, depending on stress values and on the basis of rules (e.g. [10]).

Employing methods based on homogenisation approaches, it is opportune to perform a shape optimisation just after the topology one as in Ref. [11], in order to smooth out the rough boundaries obtained in the first step, due to the coincidence of the latter with the discontinuous edges of the elements.

The procedure developed in Ref. [11], which searches the configurations with maximum stiffness and mass below an assigned value and employs the gradient method, has been resumed in this work. The problem of the optimal topology, using the homogenisation approach, is that to seek the elements, in which the domain is meshed, having mass; thus it has an ON/OFF nature, similar to the chromosomal one of the genetic algorithms (GAs) [12,13]. Then for the purpose of this paper, as optimisation technique a genetic algorithm has been developed. This uses as design variables, in the topology optimisation, the relative densities 0 or 1 of

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each element of the structure and, in the shape optimisation, the coordinates of the keypoints of segments of curves and straight lines that constitute the boundaries.

GAs employ a random, yet directed, search for locating the globally optimal solution. Like other methods, e.g. of response surface [2] and gradient [14], GAs can be applied to problems for which it is not possible to have an analytical relationship for the objective function. However they are superior to other techniques [15], as e.g. the gradient descent one, since the search is not biased toward a locally optimal solution. In fact GAs are able not only to improve the solution close to a local optimum, but also to explore a larger extension of the design space. On the other hand, they differ from random sampling algorithms, due to their ability to direct the search toward relatively 'prospective' regions in the search space [15]. For these reasons they can be used in problems for which there are no data on the possible solution; therefore they are useful in several kinds of problems. The advantages are more substantial when the number of variables is very high [11], as in the topology optimisation methods employed in Refs. [6–9,11].

The developed procedure includes also the use of a FE processor for structural computation, it can be easily used in structure design, like has been verified with some examples, and it permits:

- to avoid arbitrary assumptions on the topology of the structures;
- to obtain smoothed boundaries;
- to reduce the risk to find local optima and then to realise significant economies - of material, especially in the topology optimisation due to the binary representation of the variables;
- to get advantage in run-time terms from one bit formulation of the variables in the topology optimisation.

2. Description of the procedure

2.1. Used genetic algorithm

In the GAs each variable is treated as a binary string corresponding to a gene; the variable set constitutes an individual, codified in a structure like the chromosomal one, having the genes one next to the other; more individuals constitute a population. In some cases decimal strings are used instead of binary ones [16,17], with the advantage of having strings with decimal ciphers much similar between them for two values near to one another. The population evolves owing to the modifications performed by the operators of *crossover* [18] (interchange of chromosome segments between mating pairs) and *mutation* (variation of bits). Different strategies can be employed in the GAs [12, 19]; their efficiency can depend on the analysed problem. In this work population size increases increasing number of variables, it is randomly generated inside the feasible domain and keeps constant size in the next generations [20].

On the basis of the efficiency of each individual, evaluated by a fitness, the genetic operator of *selection* chooses (Fig. 1):

- the good individuals, that, based on the principle of 'survival of the fittest', are destined to the generation of a new population, by using both the genetic operators;
- few worse individuals, destined to be modified deeply for the possible random change of all their genes.

Like it is known, the next generations have new characteristics, that can produce a better solution and however can favour the exploration of the feasible domain, reducing the risk of obtaining only local optima, with respect to traditional algorithms. Particularly the mutation on the worse individuals allows to renew the individuals destined to extinction, not dispersing their genetic patrimony, and, at the same time, increasing the diversity in the population and thus favouring the exploration of the design domain.

Run-time is generally high and grows with the number of individual genes. In particular the mutation of the second type allows, with respect to conventional GAs, to limit the number of individuals in the population, since the number of extinct individuals is reduced.

The employed strategy involves also the transfer of the best individual of each population into the next generation without transformations, replacing the worse one. Since for problems with few individuals, the best individual is usually transferred [15], it is believed that the higher the individual number, the higher must be the number of the transferred copies, replacing as many ones extracted randomly, in order to increase the possibility to enhance the population quality

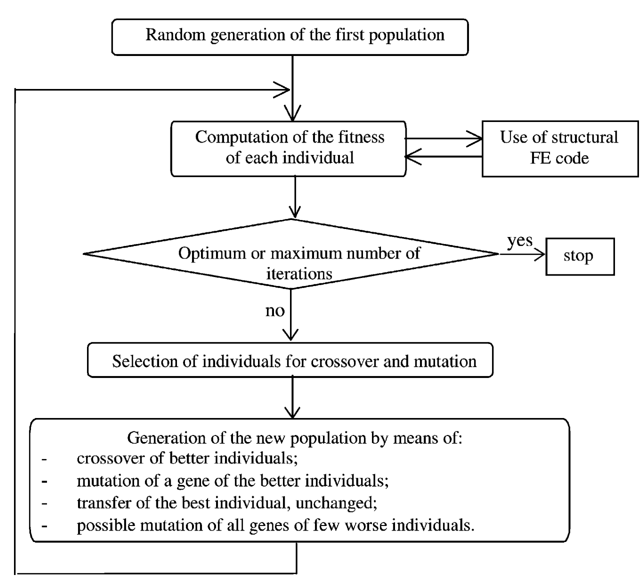

Fig. 1. Flow chart of a genetic algorithm for structural purpose.

and to make the analyses faster; obviously the copy number must not be too high, in order to avoid that the solution tends to get stuck at a local optimum. In the present paper a copy number of 3–7% of the individual number has been transferred.

Naturally, like in other optimisation algorithms, the process is halted when the fitness stops to improve, a prefixed fitness has been achieved or the maximum iteration number has been reached.

The GA developed in this paper is written in the programming language APDL of the ANSYS code [21], by means of which the structural analyses for the calculation of the fitness are executed. The analyses have been performed with a DEC Alpha workstation 500a.

2.1.1. Individual adaptive probabilities of crossover and mutation

The individual selection is controlled statistically by two parameters calculated for each individual—probability of crossover, p_c, and probability of mutation, p_m—and by three coefficients. Two coefficients, r_c and r_m, are randomly drawn for each individual; the individual is selected for the crossover if $r_c < p_c$ and for the mutation if $r_m < p_m$. For the unselected individuals (bad individuals) a coefficient, r'_m, small and constant during all the process, is defined; if $r'_m \geq p_m$ the individual is selected for the possible change of all its genes.

Increasing p_c and p_m increases the probability that the individual is selected for crossover and mutation. Decreasing p_m, increases the probability of changing all individual genes.

In this paper p_c and p_m are adapted to the value of the fitness, f, of each individual; their value increases for the better individuals, according to the relationships:

$$p_c = k_1 \frac{f}{f_{max}}; \qquad p_m = k_2 \frac{f}{f_{max}} \qquad k_1,\ k_2 \leq 1.0 \tag{1}$$

in which f_{max} is the maximum fitness in the population; furthermore it is $0 \leq (r_c, r_m) \leq 1$. It has been imposed $k_1 = 1$ and $k_2 = 0.5$ [15], because crossover must not be prevented and therefore the first coefficient must be high, while the second must be low in order to reduce the probability to destroy good individuals. In one of the studied cases $k_2 = 1$ has been also tested.

The possible mutation of all the genes is performed on the worse individuals, setting $r'_m = 0.05$, with the aim to change them emphatically. The mutations are effected by extracting a value, s_m, for each gene and comparing it with the following probability for the individual:

$$p_{mm} = k_3 p_m \tag{2}$$

with $k_3 = 1$ in this paper. The genes having $s_m > p_{mm}$ are replaced by new random values inside the feasible domain of each variable. In this way the number of the mutations grows with the worsening of the individual and a higher heterogeneity of individuals can be obtained, favouring the exploration of the domain.

2.2. Topology optimisation

Often the procedure consists in the assignment of a feasible domain, which is meshed in finite elements. Each element is considered to coincide with a cell containing microvoids [6]; by varying the dimensions of the microvoids, the density of the element varies. The optimal topology of the structure is generated searching the microvoid dimension distribution—that is the mass distribution in the domain—which assures the desired requirements. Since the mechanical properties of the element change with the density, it is necessary to know the law of these variations. In literature have been proposed homogenisation based approaches:

some approaches search the above relation vs. dimensions and orientation of the microvoids, that are used as optimisation design variables [6–8];
others assign the relation directly vs. relative density of the elements, that are the design variables [9,22,23].

The last approach, already used in Ref. [11], is based on the assumption that element stiffness grows from 0 to the material one, growing the density from 0 to that of the material. The relation between Young's modulus and density is non-linear [6] and, in linear elastic field, it is assumed, according to Ref. [24]:

$$\frac{E_i}{E} = \left(\frac{\rho_i}{\rho}\right)^{\beta} = (\rho'_i)^{\beta} \tag{3}$$

In this relation ρ and E represent, respectively, effective density and Young's modulus of the material, E_i Young's modulus of the element with average density ρ_i, and relative density ρ'_i. The value of 2 can be assigned to the constant β [22,23].

Aim of the optimisation is to maximise the stiffness, respecting a limit on the usable mass; the stiffness is quantified by the load work, $L(\{\rho'_i\})$, [25], which becomes the objective function, that must be minimised, and which depends on ρ'_i :

$$L(\{\rho'_i\}) = \{\mathbf{u}(\{\rho'_i\})\}^{\mathrm{T}}\{\mathbf{F}_c\} + \int_{\Omega} \{\mathbf{u}(\{\rho'_i\})\}^{\mathrm{T}}\{\mathbf{F}_v\}\mathrm{d}V + \int_{\Gamma} \{u(\{\rho'_i\})\}^{\mathrm{T}}\{\mathbf{F}_s\}\mathrm{d}S \tag{4}$$

where Ω is the feasible domain of the structure, contained by the surface Γ and meshed; $\{\mathbf{F}_c\}$, $\{\mathbf{F}_v\}$ and $\{\mathbf{F}_s\}$ are, respectively, concentrated, body and surface loads; $\{\mathbf{u}\}$ are the nodal displacements; V denotes volume and S surface. The use of finite element method is appropriate because, practically, strains and stresses cannot be calculated in analytical way.

To take into account the limit, M_l, on the feasible material mass, the relative densities of the n finite elements are subject to the constraint of the mass, $M(\{\rho_i^l\})$:

$$M(\{\rho_i^l\}) = \rho \sum_{i=1}^{n} \int_{V_i} \rho_i^l \, dV_i \le M_l \tag{5}$$

It is assumed that discontinuous and jagged contours are allowed, coincident with those of the elements, although they harm the accuracy of the FE solution. Nevertheless these discontinuities can be smoothed performing, successively, a shape optimisation.

It must be observed that the minimisation of Eq. (4) furnishes, generally, densities between a lower value, tending to zero (that is absence of material), and a larger one, tending to effective material density; in this way vast regions of the domain would have intermediate densities, with difficult interpretation from a manufacturing point of view. The drawback can be overcome using a penalty function [8,22,26], with the aim to penalise the L values corresponding to configurations with ρ_i^l different from 0 or 1, like it has been made also in Ref. [11]. In this paper, instead, the use in the GA of the variables 0 or 1 for the element densities simplifies the above procedure, indispensable with other optimisation techniques that require the continuity of the variables in the range 0–1; further speed up of the calculations is obtained just owing to the presence of one bit only for each string.

Effectively, in order to overcome numerical problems with the FE code, the value 0 of relative density has been replaced by 0.01 and Eq. (3) has been used to calculate the E_i values.

2.2.1. Calculation of the fitness in the topology optimisation

The fitness of each individual is calculated by using the objective function (Eq. (4)) and taking into account also the constraint of the mass (Eq. (5)). Its expression is:

$$f_{\mathrm{to}}(\{\rho_i^l\}) = \frac{1}{L(\{\rho_i^l\}) + K_{\mathrm{t}}\left(M_l - \rho \sum_{i=1}^{n} \int_{V_i} \rho_i^l \, dV_i\right)^2} \tag{6}$$

in which ρ_i^l is equal to 0 or 1. The relationship is established in order to penalise the individuals with mass much larger or much lower than the limit one. The first are individuals with excessive mass, the latter are individuals using a small part of the available mass. The coefficient K_{t} is a factor, assigned through attempts.

2.3. Shape optimisation

Generally, shape optimisation can be performed with the aim to find the shape of boundaries, for which number, distribution and geometric entities have been fixed heuristically. In this paper it is performed with the aim to smooth the rough boundaries obtained with topology optimisation, simplifying the manufacturability of the structure; otherwise the smoothing would take place with uncertainty, owing to the difficulties to extract by the topology precise information on location and shape of the final boundaries.

In this paper it is established, as in Ref. [11], that the smoothing must be performed by using curve and straight line segments and optimising the locations of their keypoints, with the same objective and constraints of the previous step. The substitution of the rough boundaries with smoothed ones, permits even more accurate and precise FE calculations of the load work and the possibility to represent the more minute details, without increasing substantially the number of variables, like it would be necessary in the topology optimisation to obtain the same refinement. The shape optimisation permits also, in general, to correct an inaccurate evaluation of the optimal topology, due to a possible difficulty of convergence of the optimisation algorithm.

Decimal strings are used in the GA for this optimisation.

2.3.1. Calculation of the fitness in the shape optimisation

In this case each individual is constituted by a set of the keypoint coordinates, $\mathbf{x}_i$. The fitness is calculated by the objective function, $L(\mathbf{x}_i)$, similar to Eq. (4) and includes the penalty term on the mass, like in Eq. (6):

$$f_{\mathrm{so}}(\{\mathbf{x}_i\}) = \frac{1}{L(\{\mathbf{x}_i\}) + K_{\mathrm{s}}\left(M_l - \rho \sum_{i=1}^{n} \int_{V_i} dV_i\right)^2}$$

3. Applications

The procedure has been applied to cases of topology and shape optimisations. Structures, already studied [11,22] using algorithms based on gradient method, have been investigated, in order to compare the results: in both cases the aim was to make maximum the stiffness, under a 25% material usage constraint with respect to the reference domain.

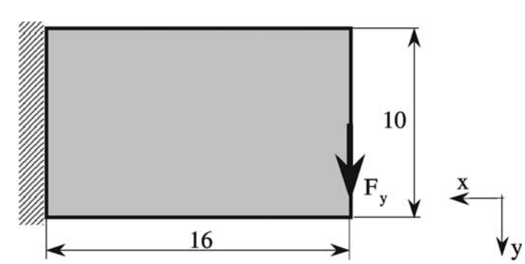

Fig. 2. Feasible domain of a short cantilever.

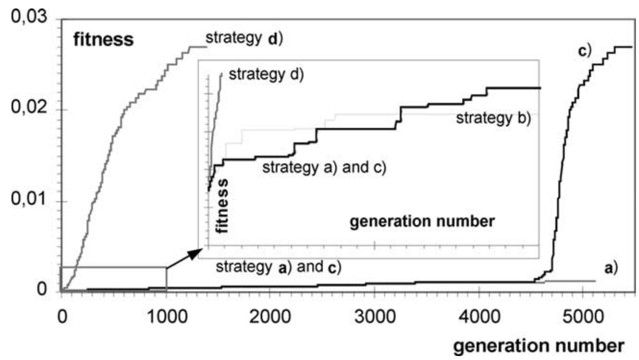

Fig. 3. Maximum fitness value vs. generation number in the topology optimisation of the short cantilever in Fig. 2, varying the strategies. The axes of the zoom cover the ranges 0–1000 (generation number) and 0–0.00045 (fitness).

3.1. Topology optimisation

First of all the topology has been found of a short cantilever and of a plate.

3.1.1. Short cantilever

The structure is loaded at the mid point of an edge by a concentrated force, F_y, and clamped at the opposite edge. Fig. 2 shows the feasible domain and the values of the corresponding mass, M_{dom}, of the usable limit one, M_l, of the material Young's modulus, of the Poisson's ratio, ν, of the thickness, t. Assuming a symmetrical structure, in order to reduce the element number, only half a structure, delimited by the horizontal geometric axis of symmetry, has been modelled and analysed using antisymmetry boundary conditions. The domain has been meshed by means of 32×10 identical square four-noded plane stress elements, as in Refs. [7,11,22], in order to obtain more comparable results. Thus the design variables are the relative densities of the 320 elements. Each generation included 150 individuals.

Different kinds of strategy have been employed. Fig. 3 shows the evolution of the maximum fitness vs. the generation number, when:

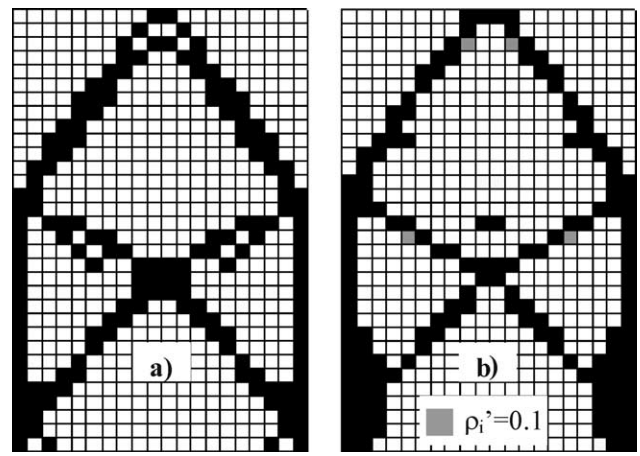

Fig. 5. Optimal topology of the short cantilever obtained in the present paper (a) and in Ref. [11] (b).

Table 1
Results of the topology optimisation found in the present paper and in Ref. [11] for the short cantilever

	Present paper	[11]
M (kg)	314.4×10^{-6}	314.4×10^{-6}
L (N mm)	38.0	137.7
u_y (mm)	0.127	0.459

(a) $k_2 = 0.5$ and only one copy of the best individual transferred to the next generation;
(b) $k_2 = 0.5$ and only one copy $k_2 = 1$ and only one copy of the best individual transferred;
(c) $k_2 = 0.5$ and only one copy of the best individual transferred to the next generation up to about 4500 generations, and 10 copies in the remaining generations until the end;
(d) $k_2 = 0.5$ and 10 copies of the best individual transferred unchanged from the beginning.

It can be observed that at the thousandth generation strategies (a) and (b) lead practically to the same results. Strategies a) and c) show, at least in this case characterised

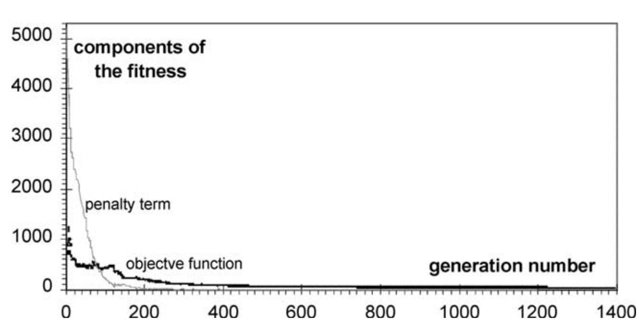

Fig. 4. Values of objective function and penalty term vs. generation number, in the topology optimisation of the short cantilever of Fig. 2, employing the strategy (d).

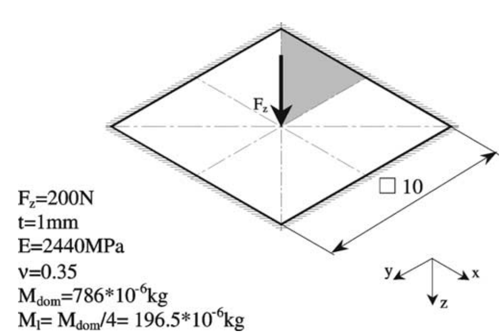

Fig. 6. Feasible domain of a plate clamped in correspondence of the boundary and loaded in the centroid.

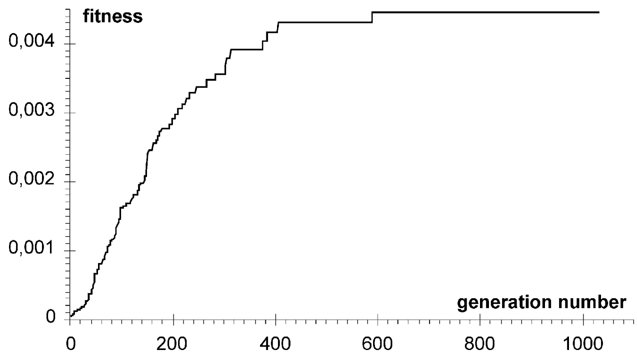

Fig. 7. Objective function vs. generation number in the topology optimisation of the plate of Fig. 6.

by a high individual number, the effectiveness of transferring a copy number higher than 1 to the next generation. In fact the increase to 10 of the copy number beyond the 4500th has increased the convergence rate. Moreover the same fitness, as with (c), has been obtained with the strategy (d), with a much lower generation number, confirming the goodness of the strategy and, in general, of the procedure.

In Fig. 4 objective function for strategy (d) and penalty term are plotted vs. generation number. Both tend to minimum values, even if not monotonically especially in the first generations. In particular the minimum value of the penalty term is, practically, equal to zero and involves the use of a mass coincident with the limit M_l. A new population was generated every 15 min about.

Fig. 5(a) shows the meshed feasible domain; the elements with relative density equal to 0 are filled in white, those with $\rho_i^l = 1$ in black, representing the optimal topology. This last is not completely in agreement with those of Refs. [7,22], that is characterised also by few elements with relative densities very different from 0 and 1, and of Ref. [11] (Fig. 5(b)). In particular the displacement, u_y, of the application point of F_y (Table 1) in this work is equal to 0.127, while in Ref. [11] is equal to 0.459, even if the same mass has been practically employed, and in Ref. [22] 0.178 denoting in both cases lower stiffness. The reason for these differences can be, probably, the minor ability of the other methods to explore the design domain. This confirms the need to use more suitable algorithms for finding the global optimum, especially with many variables. The results confirm that jagged boundaries are obtained.

3.1.2. Plate

The procedure has been applied also to the search of the optimal topology of a plate, subject to a load concentrated in the centroid. The plate is clamped in correspondence of the boundary of the feasible domain (Fig. 6). Owing to the symmetries only a 1/8 of the design domain, hatched in figure, has been modelled, with the aim to reduce run-time, defining the suitable constraints along the symmetry sections. The domain has been divided into 190 square four-noded, having the same size as in [11,22], and 20 right-angled triangles three-noded along the bias of the plate, shell elements with 6-degrees of freedom per node. Thus the design variables are the relative densities of the 210 elements. A set of 120 individuals for each generation and the strategy (d) defined in Section 3.1.1, with eight copies of the best individual transferred to the next generation, have been employed. In Fig. 7 the maximum fitness vs. the generation number is shown. It can be observed that, due to the employed strategy, the convergence occurs after a low number of generations, despite the high variable number. Fig. 8(a) shows the optimal topology. Using gradient method the topologies shown in Fig. 8(b) [11] and 8(c) starting from a different initial set, had been obtained. A very different result has been obtained in Ref. [22], which resembles to that of Fig. 8(b), but is characterised by the existence of many elements with relative densities between 0 and 1. Table 2 shows the best values of the displacement, u_z, of the centroid and of the used mass, in this paper and in Ref. [11]. It can be observed that with both the procedures the constraint on the mass is respected.

The results obtained with the procedure of this paper are again better than those obtained with the other methods.

3.2. Shape optimisation

As application the shape optimisation of the plate is performed, whose topology has been found in Section 3.1.2. Shape optimisation requires a lower number of variables as compared to the topology optimisation, and therefore benefits could be expected in terms of run time. But,

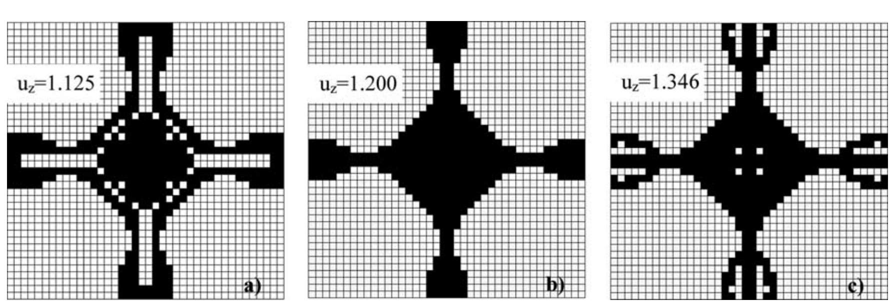

Fig. 8. Optimal topology of the plate of Fig. 6: with genetic algorithm (a) and with gradient method, as in Ref. [11] (b) and with a different starting set of design variables (c).

Table 2
Results of the topology optimisation found in the present paper and in Ref. [11] for the plate of Fig. 6

	Present paper	[11]
M (kg)	196.5×10^{-6} kg	196.5×10^{-6} kg
L (Nmm)	225	240
u_z (mm)	1.125	1.200

while the topology optimisation has required only the variables 0 and 1, the variables of the shape optimisation are real numbers, and consequently more combinations of variable sets and an increasing of the population are required; thus can be reduced the above benefits, even if the feasible range for each variable is fixed around the border lines between the regions with ρ_i^j equal to 0 and 1.

3.2.1. Plate

The topology of Fig. 9 has been adopted, considering the result of Fig. 8(a) and an easy manufacturability. It is characterised by rectilinear slots with parallel sides and holes. A slot is defined by the coordinates of the centres of the extreme circular arcs, by the distance between them and by their common radius; a hole is defined by the centre coordinates and by the radius; these quantities are design variables. Owing to the symmetries a 1/8 of the plate has been modelled. Straight boundaries are assumed along the axis parallel to x, the diagonal one and along the boundary coincident with that of the domain. The remaining boundary is modelled by means of four B-spline curves, each defined by three points, whose coordinates are also design variables; even the orthogonality of the tangent to the adjacent straight boundaries has been imposed. In this way the design variables are 24. The mesh has been obtained by means of

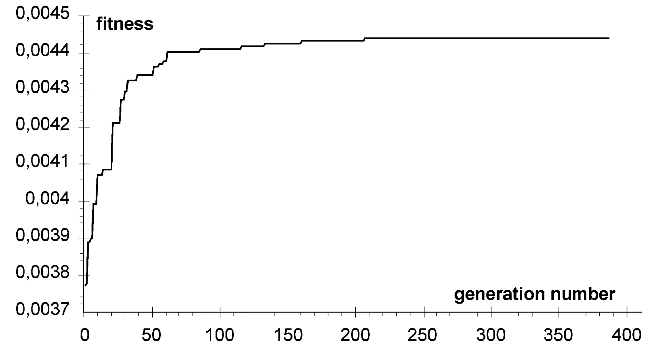

Fig. 10. Maximum fitness value vs. generation number in the shape optimisation of the plate of Fig. 9.

three and four nodes shell elements, using an automatic mesh generator. The mesh refinement has been fixed with the aim to reach an acceptable accuracy of the results, without incurring in excessive run time. A set of 120 individuals for each generation and the strategy (d) of Section 3.1.1, with four copies of the best individual transferred to the next generation, have been employed. In Fig. 10 the maximum fitness value vs. generation number is shown. As said above, considering the low number of variables the convergence is proportionally slower than in the topology optimisation. With respect to the model definition, there are no holes nor inclined slots (Fig. 11). The optimum value of u_z is slightly lower than that found in the first step, while the mass is practically the same. The differences between the results of the two phases of the optimisation are due to the different approximations obtained with the two kinds of elaborations. With respect to the first optimisation in shape optimisation the boundaries are smoothed and therefore give better results, confirming

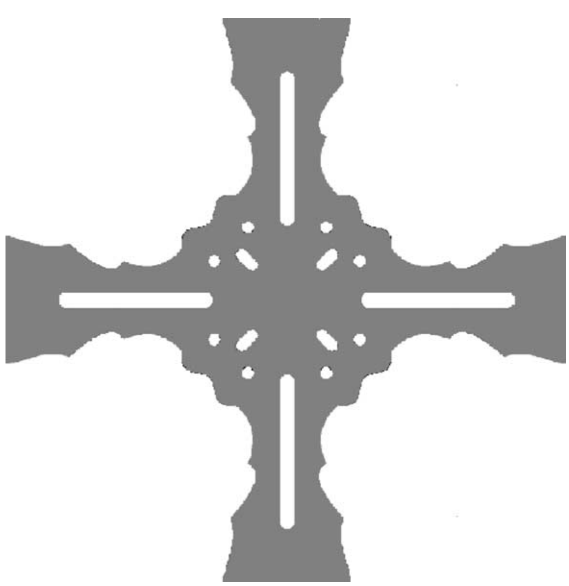

Fig. 9. Definition of the boundaries in the shape optimisation of the plate of Fig. 6, starting from the optimal topology of Fig. 8.

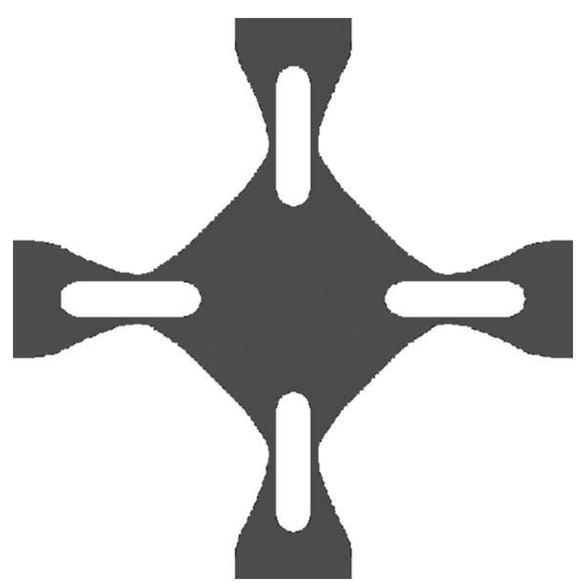

Fig. 11. Configuration of the plate of Fig. 6 after topology and shape optimisations.

Table 3
Results of the shape optimisation found in the present paper and in Ref. [11] for the plate of Fig. 6

	Present paper	[11]
M (kg)	196.5×10^{-6} kg	196.5×10^{-6} kg
L (Nmm)	224	228
u_z (mm)	1.122	1.14

the need to refine the boundaries, using even a higher number of elements.

As regard the solution of Ref. [11], a slightly higher stiffness is achieved (Table 3).

The procedure developed in this paper is easily applicable to the optimisation of ribs, which can be modelled using shell elements, overcoming the drawback to model them only through basis plate thickness variation, as in Refs. [27,28]. This problem is not dealt with by many commercial codes and has been experimented already in Ref. [11], employing the gradient method.

4. Conclusions

In the topology optimisation are often employed approaches based on homogenisation. These permit to overcome some arbitrariness necessary using, e.g. rule-based procedures, and confer to the problem an ON/OFF nature, which has suggested to the authors the use of GAs with one bit strings. Genetic algorithms can reduce the risk of obtaining local minima or maxima; they are suitable for topology and shape optimisation, where a local optimum can involve solutions very different from the global ones, and consequently a possible excess of employed material and reduction of performances; the drawback is particularly serious just in the topological optimisation inspired by the homogenisation approach, because the required high number of variables favours the determination of local minima. The one bit variables permit benefits in run-time terms, above all just when a high number of variables is used.

In this paper the following basic strategy has been employed:

to favour the action of the genetic operators of crossover and mutation in the better individuals, by using an adaptive strategy;
to transfer unchanged the best individual of each generation to the following one.

Furthermore the following strategy is added, in order to avoid large population size, recommended by the high number of variables, but which would require high analysis time for the convergence:

to substitute the worst individual with the best one;
to transfer unchanged to the next generation a number of copies of the best individual, dependent on the number of individuals, replacing as many ones extracted randomly (it is advisable not to transfer a number of copies larger than 10% of the individuals, in order to avoid premature convergence to a local optimum);
to renew the worst individuals, without destining them to extinction, but modifying them through the possible random change of all their genes.

The results show the superiority of the developed algorithm, at least in the topology optimisation, with respect to gradient methods. They show, furthermore, the advantages in refining with a shape optimisation the boundaries found by topology optimisation.

References

[1] Olhoff N, Taylor JE. On structural optimisation. J Appl Mech 1983; 50:1139–51.
[2] Cappello F, Mancuso A. Ottimizzazione della disposizione delle fibre della carena di un'imbarcazione in composito. Proceedings of II Seminario Italo-Español, Progettazione e fattibilità dei prodotti industriali, Vico Equense (Napoli); June 1998.
[3] Cappello F, Celestino A, Luparello S. Progettazione ottimizzata dell'alettone di un'auto da competizione mediante un algoritmo genetico. Proceedings of II Seminario Italo-Español, Progettazione e fattibilità dei prodotti industriali, Vico Equense (Napoli); June 1998.
[4] Cappello F, Ceresia A. Ottimizzazione di forma di componenti strutturali, Il Progettista Industriale; May 1996.
[5] Wieghardt K, Hartmann D, Leimbach KR. Interactive shape optimization of continuum structures. Engng Struct 1997;19(4): 325–31.
[6] Bendsoe MP, Kikuchi N. Generating optimal topologies in structural design using a homogenization method. Comp Meth Appl Mech Engng 1988;71:197–224.
[7] Bendsoe MP. Optimis. of struct. topology, shape, and material. Berlin: Springer; 1995.
[8] Rozvany GIN. Aims, scope, basic concepts and methods of topology optimisation. Topology optimisation in structural mechanics, New York: Springer; 1997.
[9] Rozvany GIN, Olhoff N, Cheng K, Taylor JE. On the solid plate paradox in structural optimisation. J Struct Mech 1982;10.
[10] Rodrigues J, Seireg A. Optimizing the shapes of structures via a ruled-based computer program. Comp Mech Engng 1985;4:20–8.
[11] Cappello F. Progettazione di strutture mediante ottimizzazione topologica. Proceedings of the XI International Conference ADM, Palermo; 1999.
[12] Goldberg DE. Genetic algoritms in search, optimisation and machine learning. Reading, MA: Addison-Wesley; 1989.
[13] Holland JH. Adaptation in natural and artificial system. Ann Arbor: University of Michigan Press; 1975.
[14] Vanderplats Garret N. Numerical optimisation techniques for engineering design, Monterey, California; 1984.
[15] Patnaik LM, Mandavilli S. Adaptation in genetic algorithms. In: Sankar KP, Wang PP, editors. Genetic algorithms for pattern recognition. New York: CRC Press; 1996. p. 45–64.
[16] Michalewicz Z. Genetic algorithms–data structures–evolution programs, Department of Computer Science, University of North Carolina, Charlotte, USA; 1994.
[17] Wright AH. Genetic Algorithms for real parameter optimisation. Foundation of genetic algorithms, first on the foundation of genetic algorithms and classifier systems, Los Altos, CA: Morgan Kaufmann; 1991. pp. 205–18.

[18] De S, Ghosh A, Pal SP. Fitness evaluation in genetic algorithms with ancestors influence. In: Sankar KP, Wang PP, editors. Genetic algorithms for pattern recognition. New York: CRC Press; 1996. p. 1–23.
[19] Sankar KP, Wang PP, editors. Genetic algorithms for pattern recognition. Boca Raton, FL: CRC Press; 1996.
[20] Arabas J, Michalewicz Z, Mulawka J. GAVaPS—a genetic algorithm with varying population size. Orlando: IEEE Press; 1994.
[21] Ansys Users' Manual, Swanson Analysis System, Inc; 1992.
[22] Chung J, Lee K. Optimal design of rib structures using the topology optimisation technique. Proc Inst Mech Engrs 1997;211.
[23] Yang RJ, Chuang CH. Optimal topology design using linear programming. Comput Struct 1994;52(2):265–75.
[24] Mlejnek HP, Schirrmacher R. An engineering approach to optimal material distribution and shape finding. Comp Meth Appl Mech Engng 1993;106:1–26.
[25] Krog LA, Olhoff N. Topology and reinforcement layout optimisation of disk, plate, and shell structures. Topology optimisation in structural mechanics, New York: Springer; 1997.
[26] Allaire G. The homogenisation method for topology and shape optimisation. Topology optimisation in structural mechanics, New York: Springer; 1997.
[27] Mlejnek HP. Some approach for shape finding in optimal structural design. Proc Int Conf Engrg Optimiz in Design Proc, vol. 63. Berlin: Springer; 1991. Lecture notes in English.
[28] Tenek LH, Hagiwara I. Optimal rectangular plate and shallow shell topologies using thickness distribution or homogenisation. Comp Meth Appl Mech Engng 1994;115:111–24.

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