Strain Measurement on Composites: Errors due to Rosette Misalignment

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ABSTRACT: Electrical resistance strain gauges are increasingly used for the determination of the strain field in composite components. The effect of the angular misalignment of a strain gauge rosette on the determination of the strains in a composite material is investigated in this paper. The theoretical analysis shows that the strain error along the principal material directions depends on the difference of principal strains, on the angular misalignment of the rosette and on the angle between the maximum principal strain and the fibre direction. The paper also shows experimental evidence for the theoretical analysis.

KEY WORDS: *composites, strain gauges, misalignment error*

NOTATION

E_1, E_2	Young's moduli along the principal material axes
G_{12}	Shear modulus
r	Ratio between principal strains ($= \varepsilon_q/\varepsilon_p$)
β	Misalignment angle of the rosette
ε_1, ε_2, γ_{12}	Strains along the principal directions 1, 2 of the composite lamina
ε'_1, ε'_2, γ'_{12}	Apparent values of strains ε_1, ε_2, γ_{12} when a misaligned rosette is used
ε_a, ε_b, ε_c	Strains measured by grids a, b and c of an aligned rosette
ε'_a, ε'_b, ε'_c	Strains measured by grids a, b and c of a misaligned rosette
ε_p, ε_q	Principal strains ($\varepsilon_p \geq \varepsilon_q$)
ε_x, ε_y, γ_{xy}	Cartesian components of strains
φ_ε	Angle between the fibre direction 1 and the maximum principal strain ε_p
φ'_ε	Apparent value of angle φ_ε when a misaligned rosette is used
φ_σ	Angle between the fibre direction 1 and the maximum principal stress σ_p
ν_{12}, ν_{21}	Major and minor Poisson's ratios along the principal material axes
θ_1	Angle between the fibre direction and the rosette reference axis (gauge a axis)
θ_p	Angle between the maximum principal strain ε_p and the gauge a axis
θ'_p	Apparent value of angle θ_p when a misaligned rosette is used
σ_p, σ_q	Principal stresses ($\sigma_p \geq \sigma_q$)

Introduction

Composite materials are increasingly used in structural components. As a consequence, experimental stress analysis methods are more often used in order to determine the mechanical properties of these materials and to measure the strain field in composite components. In this context, the use of electrical resistance strain gauges for testing composite materials is also increasing. A review of strain gauge technology as applied to composite materials is reported in Ref. [1] as far as the gauge bonding procedure, transverse sensitivity effect, errors due to gauge misalignment and temperature sensitivity are concerned. Other aspects related to the use of strain gauges on composites are reported in Refs [2–6].

This paper is concerned with the misalignment of gauges. In particular, the influence of the angular misalignment on strain measurement is well known [7–11] as far as single strain gauges and rosettes, both plane and three-dimensional are concerned. Although the strain is a purely geometric quantity, that is independent of the material properties, the anisotropic behaviour of composite materials requires special consideration, as shown by Refs [1, 12, 13] for single strain gauges and for two element rectangular rosettes. The purpose of this research is to extend the conclusions of the previous cited papers

[1, 12, 13] to the case of a misaligned three gauge rosette mounted on a composite material.

It is well known that, independent of the nature (isotropic or anisotropic) of the material, the misalignment of a three gauge rosette as a whole influences:

- the measured strains ε_a, ε_b, ε_c;
- the Cartesian components of strains ε_x, ε_y, γ_{xy} inferred from the gauges measurement;
- the angle θ_p between the gauge *a* axis and the maximum principal strain ε_p;

whereas it does not influences the values of the principal strains ε_p and ε_q [9].

In composite materials however the strains of interest are usually those along the principal axes of the material. Since the principal material directions, 1, 2, do not coincide in general with the principal strain directions, *p*, *q*, an error occurs on the determination of the strains ε_1, ε_2, γ_{12} along the material directions 1, 2.

The paper focuses on the errors not considered in the previous literature. To this end both theoretical analysis and experimental evidence of the effect of misaligned rosettes are shown. In particular, formulas of strain error for three gauge rosettes are given; furthermore, some experimental results obtained with fibreglass specimens instrumented with both aligned and misaligned rosettes are reported.

Theory

Review of strain analysis on composites

In a homogeneous, elastic and orthotropic lamina subjected to a plane stress field, the principal strain (ε_p, ε_q) directions no longer coincide in general with either the principal stress (σ_p, σ_q) directions, or the principal material directions 1, 2. In particular, the angle φ_ε between the fibre direction 1 and the maximum principal strain, ε_p, is related to the angle φ_σ, between the fibre direction and the maximum principal stress, σ_p (Figure 1) by the following relation [14, 15]:

$$\tan 2\varphi_\varepsilon = \frac{(1-\sigma_q/\sigma_p)G_{12}^{-1}\tan\varphi_\sigma}{E_1^{-1}(1+\nu_{12})\left[1+(\sigma_q/\sigma_p)\tan^2\varphi_\sigma\right]-E_2^{-1}(1+\nu_{21})\left[(\sigma_q/\sigma_p)+\tan^2\varphi_\sigma\right]} \tag{1}$$

where E_1, E_2 are the Young's moduli along the principal material axes, ν_{12} is the major Poisson's ratio, ν_{21} is the minor Poisson's ratio ($\nu_{21} = \nu_{12}E_2/E_1$) and G_{12} is the shear modulus.

The analysis, by means of rosettes, of the strain field in a composite material is more complex than in isotropic materials. Various methods are available; a possible procedure is based on the following steps [4, 16]:

1. measurement of the strains ε_a, ε_b, ε_c given by the rosette grids;
2. calculation of the principal strains ε_p, ε_q and of the angle θ_p (Figure 1) between the rosette reference axis (gauge *a* axis in this case) and the maximum principal strain, ε_p, using the standard rosette relationships [17];
3. determination of the angle φ_ε between the maximum principal strain ε_p and the fibre direction 1:

$$\varphi_\varepsilon = \theta_p - \theta_1; \tag{2}$$

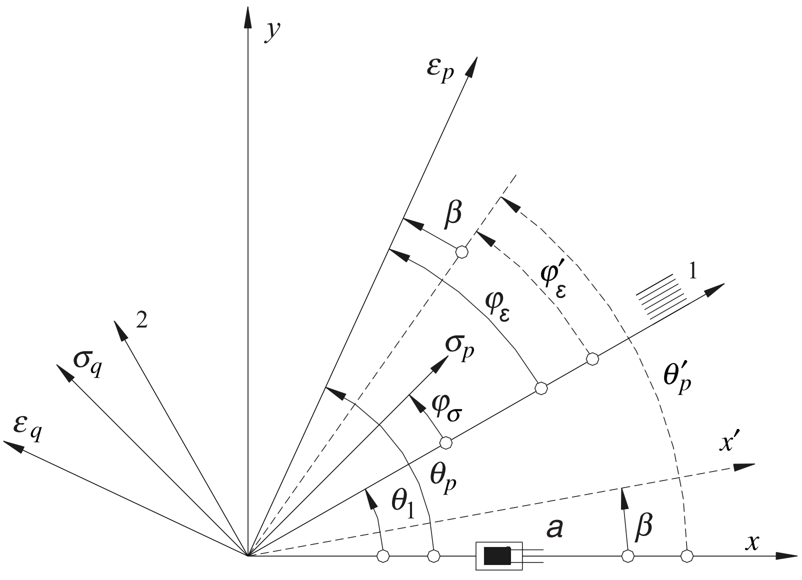

Figure 1: Angle notations: β is the misalignment angle of the rosette having gauge *a* along the *x* axis, other definitions are given in the *Notation* section (angles positive in the counterclockwise direction)

where θ_1 is the angle between the fibre direction and the rosette reference axis (gauge *a*);

4 evaluation of the strains along the principal material directions by the strain transformation relationship [18]:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} = [T(-\varphi_\varepsilon)] \begin{bmatrix} \varepsilon_p \\ \varepsilon_q \\ 0 \end{bmatrix} \tag{3}$$

where the transformation matrix is given by

$$[T(\theta)] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \tag{4}$$

with $\theta = -\varphi_\varepsilon$, $m = \cos\theta$, $n = \sin\theta$.

Errors along the principal material axes

If the rosette is misaligned as a whole the new measured strains, ε'_a, ε'_b, ε'_c, still give exact values of the principal strains ε_p, ε_q, while the apparent angle between the *x* axis (presumed position of gauge *a*) and the maximum principal strain becomes (Figure 1):

$$\theta'_p = \theta_p - \beta \tag{5}$$

where β is the misalignment angle of the rosette.

The apparent angle between the principal strain ε_p and the fibre direction 1 is now:

$$\varphi'_\varepsilon = \theta'_p - \theta_1 \tag{6}$$

Therefore from Equations (2) and (5) Equation (6) becomes:

$$\varphi'_\varepsilon = \varphi_\varepsilon - \beta \tag{7}$$

The strains along the principal material axes are affected by errors because they are now given by the following relationship:

$$\begin{bmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \gamma'_{12}/2 \end{bmatrix} = \left[T(-\varphi'_\varepsilon)\right] \begin{bmatrix} \varepsilon_p \\ \varepsilon_q \\ 0 \end{bmatrix} \tag{8}$$

where the transformation matrix is still given by Equation (4), whereas the angle θ is now:

$$\theta = -\varphi'_\varepsilon$$

The strain errors along the principal material directions from Equations (3) and (8) are:

$$\varepsilon'_1 - \varepsilon_1 = \frac{\varepsilon_p - \varepsilon_q}{2}[\cos 2(\varphi_\varepsilon - \beta) - \cos 2\varphi_\varepsilon] \tag{9}$$

$$\varepsilon'_2 - \varepsilon_2 = -\frac{\varepsilon_p - \varepsilon_q}{2}[\cos 2(\varphi_\varepsilon - \beta) - \cos 2\varphi_\varepsilon] \tag{10}$$

$$\gamma'_{12} - \gamma_{12} = (\varepsilon_p - \varepsilon_q)[\sin 2(\varphi_\varepsilon - \beta) - \sin 2\varphi_\varepsilon] \tag{11}$$

The previous relations show that the magnitude of strain errors along the principal material axes depend upon three factors:

- the difference between principal strains $\varepsilon_p - \varepsilon_q$;
- the misalignment installation error β of the rosette;
- the angle φ_ε between the maximum principal strain ε_p and the fibre direction 1.

The error is independent of rosette type and of rosette orientation with respect to the principal material axes. Figure 2 shows the strain errors versus the angle φ_ε for the following misalignment angles: β = ±5°, ±7.5°, ±10°. The maximum errors in normal strains are for $\varphi_\varepsilon = \pm 45° + \beta/2$, while the maximum errors in shear

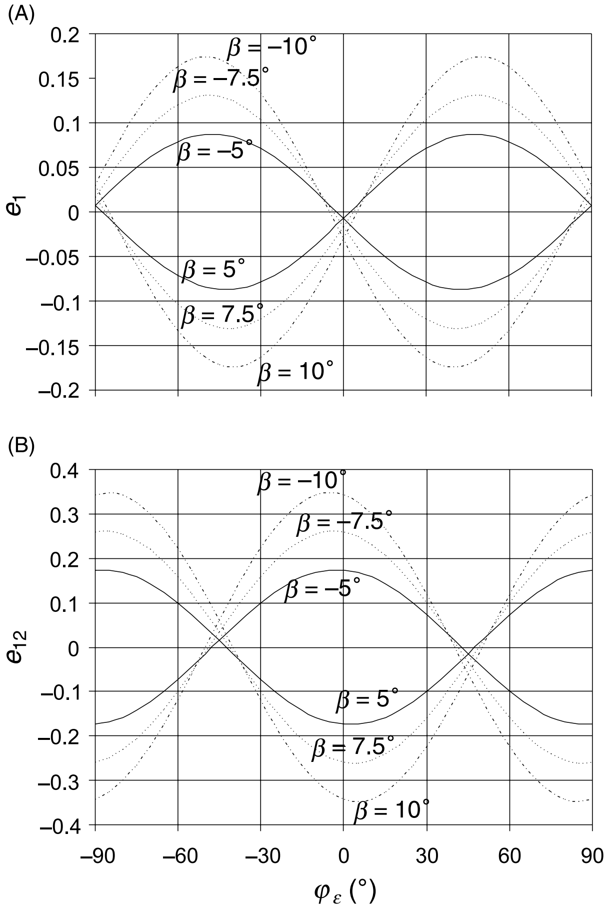

Figure 2: Errors, referred to the difference of principal strains, in (A) normal $\left(e_1 = \frac{\varepsilon'_1 - \varepsilon_1}{\varepsilon_p - \varepsilon_q} = -\frac{\varepsilon'_2 - \varepsilon_2}{\varepsilon_p - \varepsilon_q}\right)$ and (B) shear $\left(e_{12} = \frac{\gamma'_{12} - \gamma_{12}}{\varepsilon_p - \varepsilon_q}\right)$ strains versus the angle φ_ε between the maximum principal strain and the fibre axis for various values of the misalignment angle β

strain are for $\varphi_\varepsilon = \beta/2$ or $\varphi_\varepsilon \pm 90° + \beta/2$. However, considering only small misalignment errors, it is possible to evaluate the maximum errors that correspond to the following values of the angle φ_ε: 0°, ±45°, 90°. Therefore, for $\varphi_\varepsilon = 0°$ or 90°, Equations (9)–(11) give

$$\varepsilon'_1 - \varepsilon_1 = \mp(\varepsilon_p - \varepsilon_q)\sin^2\beta \tag{12}$$

$$\varepsilon'_2 - \varepsilon_2 = \pm(\varepsilon_p - \varepsilon_q)\sin^2\beta \tag{13}$$

$$\gamma'_{12} - \gamma_{12} = \mp(\varepsilon_p - \varepsilon_q)\sin 2\beta \tag{14}$$

where the upper and lower signs refer to 0° and 90° respectively; for $\varphi_\varepsilon = \pm 45°$ Equations (9)–(11) give

$$\varepsilon'_1 - \varepsilon_1 = \pm\frac{\varepsilon_p - \varepsilon_q}{2}\sin 2\beta \tag{15}$$

$$\varepsilon'_2 - \varepsilon_2 = \mp\frac{\varepsilon_p - \varepsilon_q}{2}\sin 2\beta \tag{16}$$

$$\gamma'_{12} - \gamma_{12} = \mp 2(\varepsilon_p - \varepsilon_q)\sin^2\beta \tag{17}$$

The previous relations and Figure 2 show that for small misalignments:

- the strain error in normal strains is maximum when the principal strain directions cross the principal material directions ($\varphi_\varepsilon = \pm 45°$);
- the strain error in shear strains is maximum when the principal strain directions and the principal material directions are coincident ($\varphi_\varepsilon = 0°$ or 90°).

It is interesting to note that Equations (9) and (12) are equal to those obtained for a single strain gauge [7] provided that the angle φ_ε is substituted by the angle between the chosen measurement direction and the maximum principal strain.

Taking into account Equations (3) and (9)–(11) and setting $r = \varepsilon_q/\varepsilon_p$, the relative errors are:

$$\frac{\varepsilon'_1 - \varepsilon_1}{\varepsilon_1} = \frac{\cos 2(\varphi_\varepsilon - \beta) - \cos 2\varphi_\varepsilon}{(1+r)/(1-r) + \cos 2\varphi_\varepsilon} \tag{18}$$

$$\frac{\varepsilon'_2 - \varepsilon_2}{\varepsilon_2} = \frac{\cos 2(\varphi_\varepsilon - \beta) - \cos 2\varphi_\varepsilon}{-(1+r)/(1-r) + \cos 2\varphi_\varepsilon} \tag{19}$$

$$\frac{\gamma'_{12} - \gamma_{12}}{\gamma_{12}} = \frac{\sin 2(\varphi_\varepsilon - \beta)}{\sin 2\varphi_\varepsilon} - 1 \tag{20}$$

These errors diverge when the reference strain tends to zero. For example, if $r = -1$ the relative error in normal strains diverges when $\varphi_\varepsilon = \pm 45°$, while the relative error associate with the shear strain diverges when $\varphi_\varepsilon = 0°$ or 90° as it is shown in Figure 3 where the relative errors associated with the normal (for $r = -1$) and shear strains versus the angle φ_ε are reported for the following misalignment angles: $\beta = 5°$, 7.5°, 10°. The previous analysis confirms that, although the strain measurement error is independent of the material, errors occur because the strains of interest are those along the principal axes of the material and not the principal strains, as for isotropic materials.

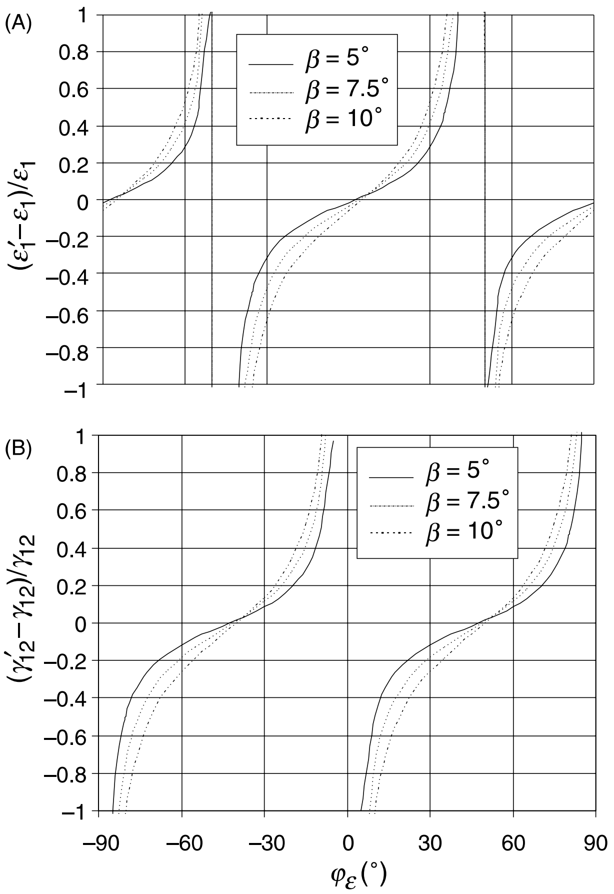

Figure 3: Relative errors associated with (A) normal (for $r = \varepsilon_q/\varepsilon_p = -1$) and (B) shear strains versus the angle φ_ε for various values of the misalignment angle β

Experimental Analysis

The experiments were performed using three specimens obtained from the same GFRP unidirectional lamina, 2.5 mm thick. The plate used for specimens 1 and 2 was subjected to twisting loading (Figure 4). The first plate, referred to as specimen 1, has the principal material directions parallel to the edges (Figure 5A). In this condition $\varphi_\varepsilon = \varphi_\sigma = 45°$, $r = \varepsilon_q/\varepsilon_p = \sigma_q/\sigma_p = -1$. This specimen was instrumented with three rectangular rosettes M-M, type

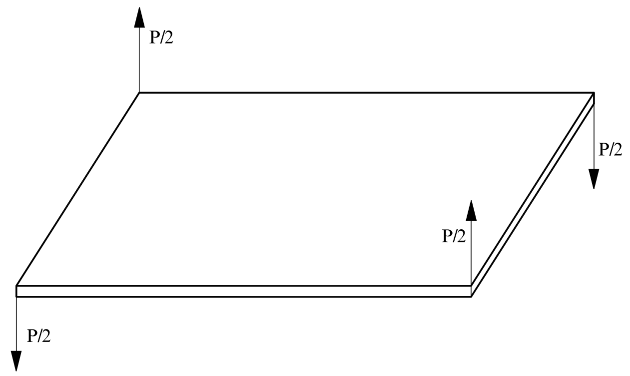

Figure 4: Loading for plate twisting specimen, i.e. for plates shown in Figure 5

CEA-05-250UR-350, having an active grid length of $Lo = 6.35$ mm and an electrical resistance $Re = 350\ \Omega$. Figure 5A shows that rosette 1 is aligned with the chosen measurement direction ($\beta = 0°$), while rosettes 2 and 3 are bonded with angular mounting errors $\beta = +10°$ and $\beta = -10°$ respectively.

After the test the plate was machined in order to obtain specimen 2 (Figure 5B); small reinforcements were added in order to load the plate in the same manner as specimen 1. In this case, the principal material directions were oriented along the diagonals of the plate. In this condition, $\varphi_\varepsilon = \varphi_\sigma = 0°$, $\sigma_q/\sigma_p = -1$. An additional rosette (0) was bonded to the plate with grid a aligned without error ($\beta = 0°$) along the fibre axis. Finally specimen 3 is a 10° off-axis bar subjected to tensile loading. The specimen (Figure 6) was instrumented with two pair of rosettes installed on both sides of the test surface without misalignment error (rosettes 4, 4′) and with misalignment error (rosettes 5, 5′).

After the preliminary checks and loading cycles, at least three measurement cycles were performed for each specimen. The maximum load was 21.4 N for specimens 1 and 2 and 1430 N for specimen 3. The strain readings were reproducible within 1–2 μm m^{-1}.

Table 1 shows the experimental results which are based on the average readings, at the maximum load, of three measurement cycles. The data for specimen 3 represent membrane strains obtained by the average values from rosettes 4–4′ and 5–5′ respectively. Rows (4)–(6) show the measured strains, already corrected for transverse sensitivity effect. Rows (7)–(9) show the strains ε_1, ε_2, γ_{12} along the principal directions 1, 2 of the composite lamina, obtained for the aligned rosettes by means of Equation (3), while rows (10)–(12) give the values for the misaligned rosette obtained by means of Equation (8). Rows (13)–(15) show the experimental errors obtained by difference between misaligned and aligned strain values, whereas the rows (16)–(18) show the theoretical errors obtained using Equation (9)–(11), where the data from the aligned rosettes 1, 0 and 4/4′, were used for the calculation concerning specimen 1, 2 and 3 respectively. The agreements between theory and experimental results is satisfactory; the differences between experimental and theoretical errors are due to spurious influences.

As expected, for specimen 1 ($\varphi_\varepsilon = 45°$) the larger error is for the normal strains, whereas for specimen 2 ($\varphi_\varepsilon = 0°$) the larger error is for the shear strain. For the 10° off-axis bar (specimen 3, $\varphi_\varepsilon = -22°$) both normal and shear strains experience large errors.

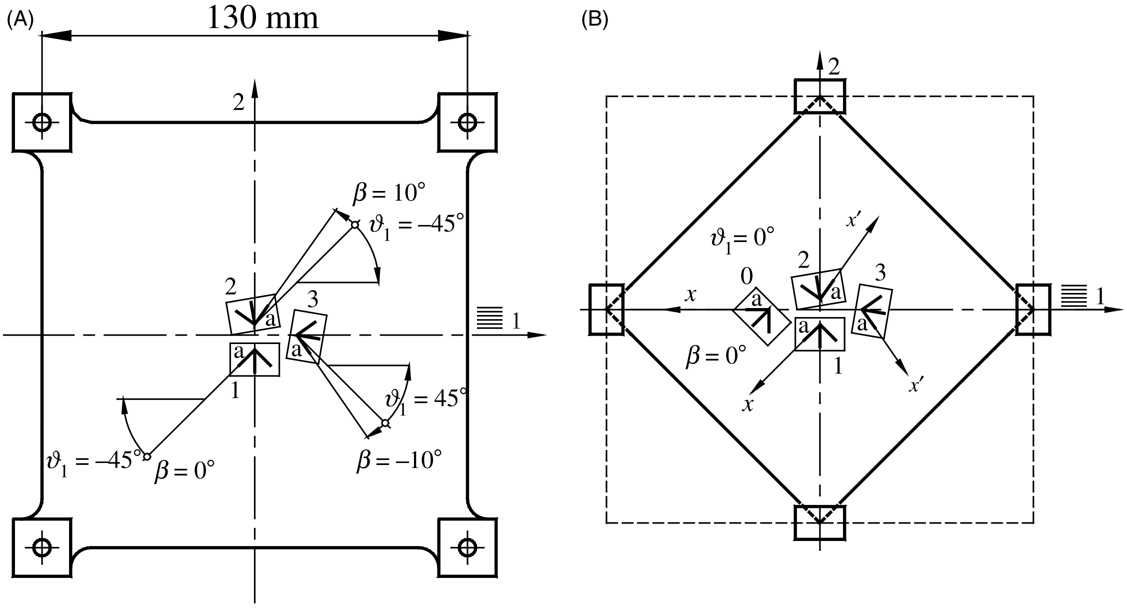

Figure 5: Geometry of the plates instrumented with aligned and misaligned rosettes: (A) specimen 1 with principal material directions along the edges, (B) specimen 2 with principal material directions along the diagonals (ϑ_1 is the fibre angle, β is the rosette misalignment, as it is shown in Figure 1)

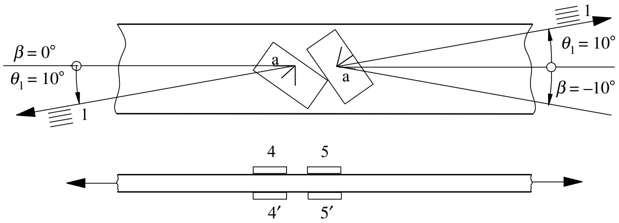

Figure 6: Off-axis tensile specimen, 3, instrumented with aligned (4/4′) and misaligned (5/5′) rosettes (θ_1 is the fibre angle, β is the rosette misalignment, as it is shown in Figure 1)

Conclusions

In this paper the error due to the misalignment of a three gauge rosette on the determination of strains from composites was considered. Although the strain is a purely geometric entity, which therefore does not depend on the material properties, the determination of strains on composites requires attention as usually the information of interest relies on the strains along the material axes more than on the principal strains.

The theoretical analysis developed in this paper shows that the errors in the strains along the principal material directions depend on:

- the misalignment mounting angle of the rosette β;
- the difference between the principal strains, and
- the angle φ_ε between the maximum principal strain and the fibre direction.

The analysis also shows that:

- the strain errors associated with the normal strains (ε_1, ε_2) are maxima and opposite when the principal strains directions are at 45° to the principal material directions;
- the strain error associated with the shear strain (γ_{12}) is maximum when the principal strains and the material directions are parallel.

The previous analysis confirms that errors occur because the strains of interest are those along the principal axes of the material and not the principal strains as for isotropic materials.

The experimental results obtained with GFRP unidirectional specimens instrumented with rectangular

Table 1: Experimental results for aligned (0, 1, 4/4′) and misaligned (2, 3, 5/5′) rosettes

		Specimen 1 (Figure 5A) Rosettes			Specimen 2 (Figure 5B) Rosettes				Specimen 3 (Figure 6) Rosettes	
Row	Experimental results	1	2	3	0	1	2	3	4/4′	5/5′
1	Fibre angle, θ_1	−45°	−45°	45°	0°	−45°	−45°	45°	10°	10°
2	Rosette misalignment, β	0°	10°	−10°	0°	0°	10°	−10°	0°	−0°
3	Principal angle from fibre axis, φ_ε	45°			0°				−22°	
	Measured strains (μm m^{-1})									
4	ε_a	329	288	−311	118	−101	−156	−156	522	561
5	ε_b	−5	−120	−112	−82	−328	−292	117	22	143
6	ε_c	−329	−304	307	−293	−100	−21	−6	−166	−213
	Strains along material axes 1, 2 (μm m^{-1}) (aligned rosettes)									
7	ε_1	4	–	–	118	127	–	–	448	–
8	ε_2	−5	–	–	−293	−328	–	–	−93	–
9	γ_{12}	659	–	–	11	0	–	–	−527	–
	Strains along material axes 1, 2 (μm m^{-1}) (misaligned rosettes)									
10	ε'_1	–	104	−112	–	–	115	117	–	527
11	ε'_2	–	−120	108	–	–	−292	−279	–	−179
12	γ'_{12}	–	591	618	–	–	−135	150	–	−323
	Experimental error (μm m^{-1})									
13	$\varepsilon'_1 - \varepsilon_1$	–	100	−116	–	–	−3	−1	–	79
14	$\varepsilon'_2 - \varepsilon_2$	–	−115	113	–	–	1	14	–	−86
15	$\gamma'_{12} - \gamma_{12}$	–	−68	−41	–	–	−146	139	–	204
	Theoretical error (μm m^{-1})									
16	$\varepsilon'_1 - \varepsilon_1$	–	113	−113	–	–	−12	−12	–	73
17	$\varepsilon'_2 - \varepsilon_2$	–	−113	113	–	–	12	12	–	−73
18	$\gamma'_{12} - \gamma_{12}$	–	−40	−40	–	–	−141	141	–	217

rosettes mounted with and without misalignment error, confirm the theoretical predictions.

Both theory and experiments confirm the need for precise rosette alignment for reliable strain measurements from composite materials.

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