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ORIGINAL PAPER

# Exploring the uncertainty in capacity estimation at roundabouts

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**Abstract**
*Purpose* In gap-acceptance theory the critical and the follow-up headways have a significant role in determining roundabout entry capacities which in turn depend on circulating flow rates under a specified arrival headway distribution. Calculation considers single mean values of the gap-acceptance parameters, neglecting the inherent variations in these random variables and providing a single value of entry capacity. The purpose of this paper is to derive the entry capacity distribution which accounts for the variations of the contributing (random) variables and suggest how to consider this issue in the operational analysis of the roundabouts.
*Methods* We performed a Monte Carlo simulation to get the distribution of entry capacity and found Crystal Ball software effective for performing the random sampling from the probability density functions of each contributing parameter. A steady-state model of capacity was used for performing many runs; in each run, the values of each contributing parameter were randomly drawn from the corresponding distributions.
*Results* The paper presents the first simulations and the entry capacity distributions at roundabouts, once the probability distributions of the headways were assumed. The results of the analysis were expressed probabilistically, meaning that the probability distributions of capacity rather than the simple point estimates were obtained.
*Conclusions* Comparing the capacity values based on a meta-analytic estimation of critical and follow-up headways and the capacity functions based on the probability distributions of the model parameters, more insights in developing an appropriate approach to capacity estimation at roundabouts can be gained.

**Keywords** Roundabout · Entry capacity · Gap-acceptance parameter · Operations · Uncertainty

## 1 Introduction

### 1.1 The background

The transportation decision-making process about a road facility or a transport system, as a consequence of planning and design activities or operational analysis, often exposes planners and designers to many sources of variability and uncertainty [1]. In transportation engineering, despite considerable information can be derived from new technologies and can be incorporated into the traditional performance measurements (see e.g. [2–8]), the effect of variability and uncertainty in input parameters on outputs is not often taken into account in the capacity analysis of roads and intersections. The assessment of the effects of a design choice on one or more parameters that are used when an operational analysis is being carried out, requests information on the sources of uncertainty that have affected them and the relation among them [9]. Since the variability is a chance-caused variation and depends on the facility or the system that is being considered, whereas uncertainty is the lack in the analyst's knowledge of the parameters which define the physical system to be modeled, the combination of variability and uncertainty can erode the ability for making

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predictions about the future [10]; moreover, high levels of uncertainty can characterize long-term predictions [11]. The analysts typically produce a single number that explains the performance of the road facility, but usually do not give a statement of a likely range of variation in the result nor try to quantify the impact of this uncertainty on capacity estimation [12–14]. When the deterministic models are developed, and used to characterize a whatever process governing the road traffic phenomena, they should be applied for many iterations; for each iteration, rather than selecting the mean or the median value for each parameter, the values of model parameters should be randomly drawn from the corresponding probability distributions. By this way, the results can be expressed in probabilistic terms [15] [16]. Despite the required tasks may be complex, at least they include: i) to identify the possible sources of uncertainty for the problem under consideration; ii) to determine the main variables involved in the probabilistic analysis; iii) to assign the probability distributions to these variables.

The uncertainty analysis, indeed, aims to assess various aspects of a model as the statistical properties of the outputs when stochastic input parameters are considered [17]. Many methods exist for incorporating uncertainty into the quantitative estimates of the performance parameters. Monte Carlo simulation is commonly used by researchers and practicing engineers as a method for propagating uncertainties in model inputs into uncertainties in results; lots of “add-ins” can be now inserted in spreadsheets, or computer programming can help to develop custom solutions [1].

In the case of the capacity analysis at intersections and roundabouts, the impact of uncertainty depends on the kind of problem to be faced and/or solved. The analysts may need to identify how many lanes are required for a given approach of a roundabout, or know which control type (stop or traffic signal) is most appropriate for a given intersection, etc. According to Kyte et al. [13], the analysts should account for uncertainty when the capacity and level-of-service of a given intersection and/or roundabout should be estimated, and should explain how this component can affect the problem or decision under consideration. Moreover, the analyst should be aware of the large observed variation in driver behavior at intersections and roundabouts [18, 19].

### 1.2 Review of related literature on roundabouts

Over the last decades, considerable advances in highway operational analysis have been made to develop and apply statistical methods adequate for accommodating different traffic conditions that can be hard to handle through conventional statistical models and methods. Operational analyses often need a large volume of data that have to be generated via experimental measurements, each being appropriate for performing different investigations. However, the observations that we make never exactly match the surveyed processes, since several factors can interfere with the measurement; as a consequence, many sources of uncertainty in the measured variables can arise and increase the risk of misapplying the statistical techniques that are used in the subsequent analysis. In the context of typical experimental measurements at intersections and roundabouts, the analysts often need data which should be qualitative and/or quantitative: the qualitative data are usually considered descriptive and can result subjective in comparison to the quantitative data that are gathered in an experimentally repeatable manner. However, qualitative information is usually closer to phenomenon under examination, but can be subjected to interpretation by individual analyst. Sources of disagreement among the data can arise for different kinds of intersection (unsignalized intersection, signalized intersection and roundabout), since they are so different for geometric design, operations and driver behavior, as well as for the same kind of intersection; in the last case, indeed, the influence of the geometric design features on operations and driver behavior must be clearly taken into consideration especially when regression analysis is used to develop the relationship between the variables under examination.

When calculation of entry capacity - or whatever efficiency measures - at roundabouts, for a chosen observation period, is performed, steadiness and variability in traffic demand, as well as saturated or oversaturated conditions of one or more entries, have to be specified. Thus, the analysis with and without statistical equilibrium should be required and, based on traffic conditions at entries, probabilistic, deterministic, or time-dependent models can be used. Although many studies have been done to analyze the operations of (existing or planned) roundabouts, even now the topical discussion between gap acceptance theory and empirical regression models characterizes the general situation about the estimation of entry capacity at roundabouts [20]. Based on their specific advantages and drawbacks, empirical and analytical models for capacity estimation at steady-state roundabouts are used in different countries. Empirical regression models are generated from saturated roundabout entries and, based on a wide data collection, establish relationships between capacity and geometric design features [21]. Based on the concept of gap acceptance, analytical models, in turn, can be developed starting from uncongested conditions; they require that the behavioral parameters are specified [8].

Roundabouts present interesting challenges in gap acceptance modelling, since roundabouts typically use gap acceptance rules. Indeed, capacity and service times at roundabout entries rely upon the possibilities of the minor street drivers to meet enough gaps between the circulating vehicles and safely merge (or cross) the conflict spaces. These possibilities are depending on the flow rate of circulating streams and the arrival headway distributions, as well as individual drivers, vehicle and environment characteristics that affect each individual gap acceptance behaviour. The reader interested in modern roundabouts design and technologies, as well as calculation of roundabout performances, is referred to [8, 22], respectively. Since vehicles enter a roundabout using the gaps between circulating vehicles, the entry capacity is correlated to the driver gap acceptance and its estimation is based on the critical headway and the follow-up headway when the analytical (gap-acceptance) models are used to analyze roundabout operations and performances. Thus, the accurate estimation of roundabout capacity is widely dependent on the equally accurate estimation of the critical headway and the follow-up headway; however, the analyst should manage the confidence intervals around the estimates of critical headway and the follow-up headway, but there are no procedures for assessing and measuring the uncertainty in capacity estimation at roundabouts.

The most commonly used methods (although not the only ones) for estimating the critical headway - or the minimum acceptable gap during which a minor-street vehicle can enter an unsignalized intersection or a roundabout - are the methods of Raff [23] and Troutbeck [24, 25]. The method of Raff [23] is based on macroscopic model and it is used in many countries because of its simplicity. In Troutbeck's microscopic model the probability of the critical headway is calculated through the maximum likelihood method which requires an iteration process. Based on the outcome of a comprehensive analysis of technical literature (see e.g. [13, 19, 20, 26]), the maximum likelihood method has been suggested and used to estimate the critical headway; see e.g. the best-known manuals for traffic and transportation engineering [18, 27]. Differently from the critical headway, the follow-up headway - or the time between two successive departures of minor-street vehicles which use the same major-street gap when a continuous queue on the minor street is observed - can be directly measured on the field [28]. According to the gap acceptance theory, drivers are considered consistent and uniform. Gap acceptance models provide estimates of entry capacity based on constant values of the critical headway and follow-up headway which, in turn, represent average values for all observed drivers. However, since variability and heterogeneity characterize drivers' population, the assumptions above introduced can produce erroneous or unreasonably high estimates of roundabout capacity. Tian et al. [26] highlighted that the accurate estimation of the critical headway and the follow-up headway can be reached when one considers the specific conditions of the site, as the geometric design of the intersection and the approach grade, the types of vehicle and traffic movements. Respecting this, Kyte et al. [29] found that data relating to a specific site usually produce higher forecasts than the forecasts based on general values; thus, a high-level uncertainty may be associated with the gap acceptance parameters and great variability in estimation of site-specific critical headway and follow-up headway can be observed.

Empirical evidence shows that the measurements of the critical headway and the follow-up headway at roundabouts vary depending on the geometric elements of the intersection layout and the circulating stream bunching characteristics; under changes in traffic demand, the measurements of the gap acceptance parameters at a multi-lane roundabout can be also influenced by dominant and/or subdominant arrival flows at entry lanes [24, 25]. Considering constant values of the critical headway and the follow-up headway, the capacity calculation always represents average conditions. However, the critical headway and the follow-up headway being stochastically distributed cannot be considered as constant values but each of them should be represented by a distribution of a set of values.

### 1.3 Research objectives and organization of the paper

Based on the considerations above, the purpose of this paper is to consider which variables significantly affect the entry capacity estimation and suggest how to investigate this problem in the operational analysis at roundabouts. The paper presents the results of a research study aimed at finding the probability distributions of entry capacity for single-lane, double-lane and turbo roundabouts, once the probability distributions of the critical headway and the follow-up headway were assumed. On this regards, we proposed a methodological path based on an application of Crystal Ball software in order to perform a Monte Carlo simulation (see [30] for further application of Crystal-Ball for performing a Monte-Carlo simulation also in other fields of interest). The use of Crystal-Ball is illustrated with three working examples of roundabout (i.e. the single-lane roundabout, the double-lane roundabout and the turbo roundabout), dealing with a capacity model of non-linear features and the correlated variables. Thus, a capacity model at steady-state conditions was then used for performing many iterations; in each iteration, the values of the model parameters were randomly drawn from the corresponding probability distributions. The results of the analysis were expressed probabilistically, meaning that the probability distributions of the capacity at each entry lane rather than the simple point estimates of the performance measure were obtained. At last, the comparison was also done between the capacity estimations based on the mean values of behavioral parameters - as derived from a systematic review performed in a previous study by the Authors [31] - and capacity functions based on the probability distributions of the model parameters.

Starting from the results of the meta-analysis of the systematic review briefly summarized in section 2, only for what we deem congruent with the goal of the research presented in this paper, section 3 will describe the procedure and the analysis developed to determine the probability distributions of the entry capacity at single-lane, double-lane and turbo roundabouts, will present the computational approach that incorporates uncertainty in roundabout capacity analysis, and will discuss the main results. At last, conclusions will be summarized in section 4.

**Table 1** The meta-analytic estimates for critical headway at roundabouts [31]

| Roundabout | Entry lane | Circulating lane | Random estimate (se) | 95% LL and UL | *Z*-value | *p*-value | *Q* | $I^2$ |
|---|---|---|---|---|---|---|---|---|
| Single-lane | | | 4.27 (0.11) | (4.05; 4.49) | 37.46 | 0.000 | 23.28 | 1.19 |
| Double lane | Right | | 3.82 (0.13) | (3.56; 4.08) | 28.79 | 0.00 | 15.12 | 0.00 |
| | Left | | 4.17 (0.13) | (3.90; 4.42) | 31.24 | 0.00 | 16.78 | 0.00 |
| Turbo (major road) | Left | | 3.60 (0.06) | (3.49; 3.72) | 61.22 | 0.00 | 4.84 | 37.98 |
| Turbo (major road) | Right | | 3.91 (0.25) | (3.42; 4.40) | 15.66 | 0.00 | 1.00 | 0.00 |
| Turbo (minor road) | Left | Outer | 3.07 (0.15) | (2.78; 3.36) | 20.85 | 0.00 | 2.91 | 31.22 |
| Turbo (minor road) | Left | Inner | 3.20 (0.03) | (3.15; 3.26) | 106.36 | 0.00 | 1.87 | 0.00 |
| Turbo (minor road) | Right | | 3.83 (0.20) | (3.43; 4.23) | 18.70 | 0.00 | 10.66 | 81.23 |

95% LL and UL stand for the 95% lower and upper limits for the summary effect; Z-value corresponding to the confidence limits of 95% tests the null hypothesis that the common true effect is zero; p – value tells us only that the effect may be or is probably not zero; Q stands for the Cochran's Q test [34], that represents a measure of heterogeneity and is the sum of the squared deviation of each effect size from the mean, weighted by the inverse-variance for each study; $I^2$ stand for the Higgin's index, or the ratio of true heterogeneity to total observed variation [35].

## 2 Preliminary research activities

### 2.1 Searching for studies for a systematic review

Without wanting make a judgment on the validity of the methods of capacity estimation (see section 1), and recognizing that the efforts attributable to field observations are always praiseworthy, the Authors have set themselves the goal of exploring the advantages of using statistical methods to synthesize data rather than taking the results collected for one or more case studies of a same kind of roundabout (single-lane, double-lane or turbo roundabout) where field observations had been made.

Bearing in mind that the synthesis of empirical estimates of the behavioral parameters at roundabouts should be based on published researches, in a previous work the Authors undertook an extensive review of the benchmark studies currently available in the worldwide literature to collect the mean values of the critical headway and the follow-up headway at roundabouts and address the variation of each parameter across studies [31]. Thus, we focused on a meta-analysis of effect sizes, that is the analysis where each (primary) study yields an estimate of statistical mean values of the critical headway and the follow-up headway (hereinafter the effect sizes), assessed the dispersion in these effects and then computed a summary effect [32]. The choice was inspired by the results of applications of meta-analysis carried out in other research areas and for other aspects of transportation data analysis; see e.g. [33].

The reader is referred to Giuffrè et al. [31] both for further details on the set of rules used for doing the literature search and determining which studies should be included in or excluded from the analysis, the total data set extracted from the available relevant literature on the topic, the methods applied to obtain the values of the critical and follow up headways considered in the calculations, and for a more effective presentation of the results of the meta-analysis of effect sizes that was performed as part of the literature review through the random-effects model. It is noteworthy that some important studies were excluded from our investigation, since experiences in capacity estimation at roundabouts carried out in countries as Great Britain and France were based on a capacity formula which is not a gap acceptance-based model [31]. It should be also noted that the research did not attempt to answer the question of how drivers change their acceptance characteristics and adapt themselves to accept shorter or large headways based on traffic levels of the circulating volume. Indeed, the choice of the behavioural parameters that we selected for the meta-analysis was also based on how arrivals in the circulating stream were estimated in primary studies.

**Table 2** The meta-analytic estimates for follow-up headway at roundabouts [31]

| Roundabout | Entry lane | Random estimate (se) | 95% LL and UL | *Z*-value | *p*-value | *Q* | $I^2$ |
|---|---|---|---|---|---|---|---|
| Single-lane | | 3.10 (0.07) | (2.96; 3.25) | 41.82 | 0.00 | 33.90 | 20.37 |
| Double lane | Left | 2.85 (0.10) | (2.66; 3.04) | 29.58 | 0.00 | 22.32 | 19.36 |
| | Right | 2.72 (0.08) | (2.57; 2.87) | 35.74 | 0.00 | 17.77 | 9.98 |

95% LL and UL stand for the 95% lower and upper limits for the summary effect; Z-value corresponding to the confidence limits of 95% tests the null hypothesis that the common true effect is zero; p – value tells us only that the effect may be or is probably not zero; Q stands for the Cochran's Q test [34], that represents a measure of heterogeneity and is the sum of the squared deviation of each effect size from the mean, weighted by the inverse-variance for each study; $I^2$ stand for the Higgin's index, or the ratio of true heterogeneity to total observed variation [35].

### 2.2 The random-effect meta-analysis

Based on the effect size collected from each selected study, the meta-analysis performed the statistical synthesis of the data and produced a single summary effect of which the statistical significance was also assessed [34]. According to [33] the mean values of the two parameters (or the effect sizes), the standard deviations and sample size were the data input of meta-analysis. Since each effect size varied from a study to another study, the meta-analysis of effect sizes was carried out to combine the data used in all selected studies through the random-effects model. In order to produce a more precise estimation of the mean value of the distribution of effect sizes (namely the summary effect) both the original variance within-study and the variance between-studies were considered [31, 32]. Thus, we computed a weighted mean value, assuming that the weight for each study was the inverse of the study's variance; the last one is the sum of the two components of the variance as above introduced. Based on the dispersion in the effects across studies, we computed the summary effect which represented the weighted mean of the single effects [32, 33].

Tables 1 and 2 show the (quantitative) meta-analytic estimate for each behavioral parameter at roundabouts. The tables report the summary effect (namely the random estimate), the 95% lower and upper limits for the summary effect, the values of the Cochran's $Q$ test and the Higgin's index $I^2$ [34, 35], $p$-values and $Z$-values. It should be noted that, the $p$-value close to zero and $I^2$ less than 25% for both headways, confirmed the absence of heterogeneity for the single-lane and double-lane roundabouts, whereas moderate-to-high values of $I^2$ at turbo roundabouts highlighted that more studies should be carried out. Figure 1 shows, by way of example, the forest plot for the critical headway at single-lane roundabouts which provides context for the analysis of the set of studies [31]. In the figure, each point represents a single study and it is bounded by the 95% confidence interval for the effect size as reported by each study.

Comparing the summary effect (random) and the effect size in each single study, one can observe that the summary effect may also differ significantly from the estimation of each study; indeed, it is independent and not based on similar real world data. Thus, the meta-analytic estimate for the critical headway was found nearly consistent across all studies and, compared to single studies, provided a more reliable result for the parameters of interest.

## 3 Incorporating uncertainty in capacity analysis for roundabouts: The case study

### 3.1 The starting point hypothesis

In the following we refer to the Hagring's model [39] that has been particularized for the three roundabouts here studied. Within the values of the inscribed circle diameter and other single-lane roundabout's dimensions, we can mainly identify the mini roundabouts and the compact roundabouts, whereas within the values of the inscribed circle diameter and other double-lane roundabout's dimensions we can also recognize the compact roundabouts and the large roundabouts [28, 31]. For the selected case study of turbo roundabout, the basic turbo roundabout geometry was considered with an inner radius of 12 m, an outer radius of 22.45 m, an inside roadway width of 5.30 m and an outsider roadway width of 5.00 m [40].

Based on the general Hagring's model for multi–lane intersections [39], entry capacity estimation depends on behavioral parameters and conflicting flows as follows:

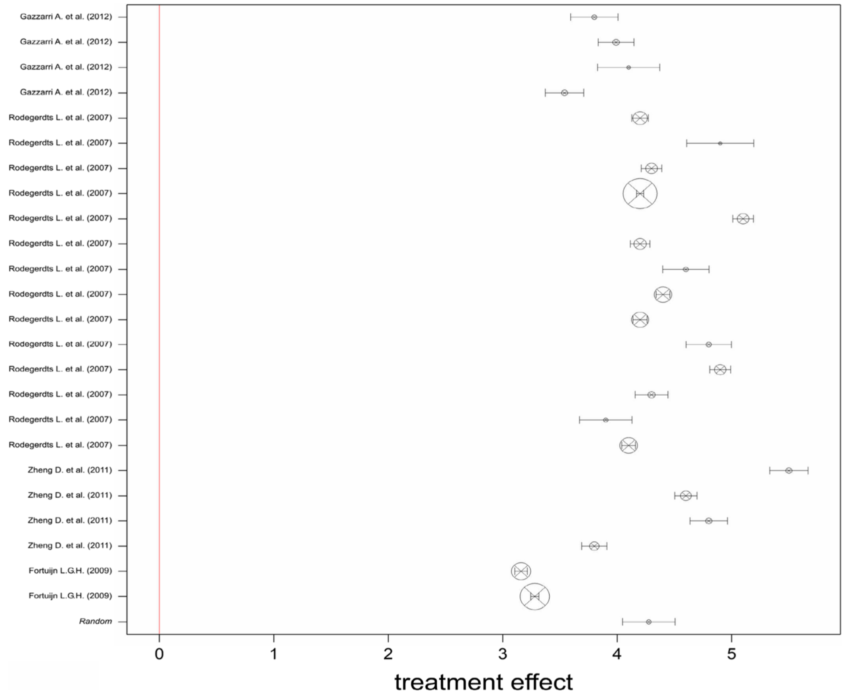
$$C_e = 3600\cdot\sum_{\mathrm{j}}\frac{\phi_j\cdot Q_{c,j}}{3600-\Delta\cdot Q_{c,j}}\cdot\prod_k\left(\frac{3600-\Delta_k\cdot Q_{c,k}}{3600}\right)\cdot\frac{\exp\left[-\sum_i\frac{\phi_i\cdot Q_{c,l}}{3600-\Delta l\cdot Q_{c,l}}\cdot\left(T_{c,l}-\Delta_l\right)\right]}{1-\exp\left[-\sum_m\frac{\phi_m\cdot Q_{c,m}}{3600-\Delta_m\cdot Q_{c,m}}\cdot T_{f,m}\right]} \quad (1)$$

where:

$C_e$ = entry-lane capacity [pcu/h];
$\varphi_j$ = Cowan's M3 parameter, i.e. the proportion of the free traffic on the circulating stream;
$Q_c$ = circulating traffic flow [pcu/h];
$T_c$ = critical headway [s];
$T_f$ = follow –up headway [s];
$\Delta$ = minimum headway of major flow [s];
$j, k, l, m$ = indices for the lanes on the circulatory roadway.

Note that a Cowan's M3 headway distribution - that explicitly takes into account the number of bunched vehicles through the $\varphi$ parameter (equal to $1-\Delta\cdot q_c,$ where $q_c$ is the circulating

**Fig. 1** Random-effect model: the forest plot showing the relative weights for critical headways at single-lane roundabouts based on Rodegerdts et al. [28]; Gazzarri et al. [36]; Zheng et al. [37]; Fortuijn [38]



traffic flow in pcu/s) representing the proportion of free vehicles - was assumed for each circulating stream; according to literature, the parameter $\Delta$ was assumed equal to 2.10 s.

The Hagring's model [40] in eq. (1) was then specified for single-lane roundabout as follows:

$$C_e = Q_c\cdot\left(1-\frac{\Delta\cdot Q_c}{3600}\right)\cdot\frac{\exp\left[\frac{-Q_c}{3600}\cdot(T-\Delta)\right]}{1-\exp\left[\frac{-Q_c}{3600}\cdot T_f\right]} \qquad (2)$$

where notations mean the exact same thing as above. For the double-lane roundabouts, eq. (1) was adapted to right- and left-entry lane separately, considering that the conflict schemes for these entry lanes are different. The conflict scheme of the right-entry lane at double-lane roundabouts is the same for the single-roundabouts, since vehicles must yield only to a single antagonist stream; in turn, the vehicles entering from the left-entry lane at double-lane roundabouts must yield to the two antagonist streams: one of them uses the outer circulating lane close to the entry, and the other uses the inner circulating lane close to the central island of the roundabout.

**Fig. 2** Log-normal distribution of $T_c$ vs normal distribution for the mean of $T_c$

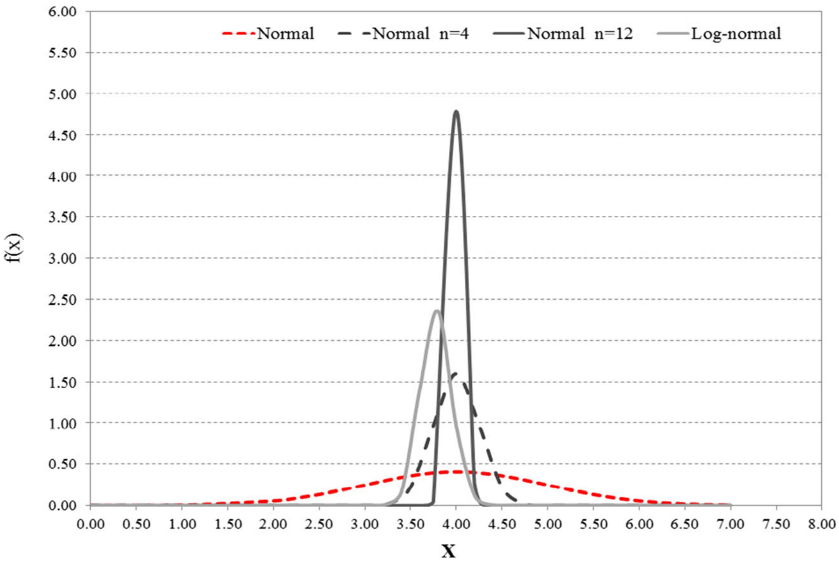


**Table 3** The parameters of the sample distribution for the critical and the follow-up headways ($Q_e = 0.5{\cdot}C$)

| Roundabout | Entry | Entry lane | Circulating lane | Mean [s] | $\frac{\sigma}{\sqrt{n}}$ [s] |
|---|---|---|---|---|---|
| *Critical headway* | | | | | |
| Single-lane | | | | 4.27 | 0.43 |
| Double-lane | | Right | | 3.82 | 0.49 |
| | | Left | Inner | 4.17 | 0.49 |
| | | | Outer | 3.81 | 0.49 |
| Turbo | Major | Left | | 3.60 | 0.31 |
| Turbo | Major | Right | | 3.91 | 0.47 |
| Turbo | Minor | Left | Inner | 3.20 | 0.18 |
| | | | Outer | 3.07 | 0.27 |
| Turbo | Minor | Right | | 3.83 | 0.41 |
| *Follow-up headway* | | | | | |
| Single-lane | | | | 3.10 | 0.53 |
| Double-lane | | Left | | 2.85 | 0.45 |
| Double-lane | | Right | | 2.72 | 0.44 |

Thus, eq. (2) was also applied to estimate the right-entry capacity at double-lane roundabouts, in which the circulating flow $Q_c$ is the outer circulating flow $Q_{c,e}$. The left-entry lane capacity was determined by the equation below as follows:

$$C_e = \left(Q_{c,e}+Q_{c,i}\right)\cdot\left(1-\frac{\Delta\cdot Q_{c,e}}{3600}\right)\cdot\left(1-\frac{\Delta\cdot Q_{c,i}}{3600}\right)\frac{\exp\left[-\frac{Q_{c,e}}{3600}\cdot\left(T_{c,e}-\Delta\right)-\frac{Q_{c,i}}{3600}\cdot\left(T_{c,i}-\Delta\right)\right]}{1-\exp\left[\frac{-\left(Q_{c,e}+Q_{c,i}\right)}{3600}\cdot T_f\right]} \tag{3}$$

where the circulating traffic flow ($Q_c$) is split in two streams $Q_{c,i}$ and $Q_{c,e}$ representing the inner circulating flow and the outer circulating flow, respectively.

It is noteworthy that drivers entering a roundabout are not required to pre-select the entry lane; moreover, vehicles using the outer circulating lane usually leave the roundabout at the next exit. The Hagring's model [39] was also applied to the turbo roundabouts: for major entries, eq. (2) is applied to estimate the capacity of each entry lane, considering $Q_{c,e}$ instead of $Q_c$; for minor entries, eq. (2) is applied to estimate the capacity of the right-entry lane, whereas eq. (3) is applied to estimate the capacity of the left-entry lane.

In order to reach a broad-based assessment of the variability of the behavioral parameters and incorporate uncertainty into the entry capacity estimation, we assumed that the critical headway and the follow-up headway could be captured over an observation period short enough to ensure a persistent steady-state condition and long enough to overstep the transient state. Under this hypothesis, the headways experienced by users during the observation period can be considered as sampled from the entire population; in this sense, they assume mean values that are within the distribution of the mean.

Based on the probability theory, if the initial (normal distributed) population (X) has mean $\mu$ and variance $\sigma^2$, the sampling

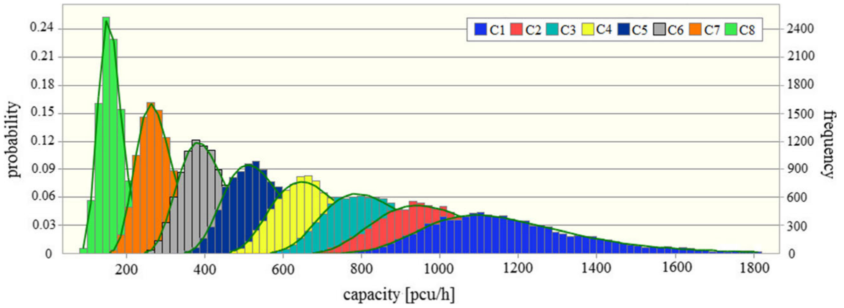


**Fig. 3** Probability distributions of entry capacity at single-lane roundabouts. Note that C1–8 are the probability distributions of entry capacity where each of them is corresponding to a value of the circulating flow around the ring ranging from 0 to 1400 pcu/h with step 200 pcu/h

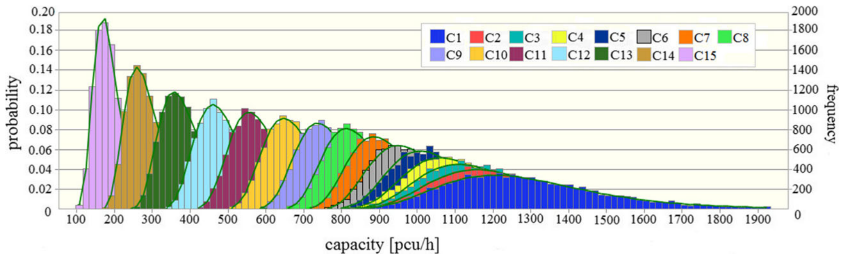


**Fig. 4** Probability distributions of entry capacity for the left-lane at double-lane roundabouts. Note that C1–15 are the probability distributions of entry capacity where each of them is corresponding to a value of the circulating flow around the ring (where $Q_{c,i} = Q_{c,e}$) ranging from 0 to 2800 pcu/h with step 200 pcu/h

distribution of the sample mean $\overline{X}$ from samples of size $n$ is assumed normally distributed $\overline{X} \sim N(\mu, \sigma^2/n)$.

This is also true for a population that is not normally distributed - namely the sampling distributions may also be assumed approximately normally distributed, regardless of the population distribution that one samples from - if the sample size is not too small ($n \geq 30$), and the population size, $N$, is at least twice the sample size. When the distribution of $\overline{X}$ is unknown or differs from the normal distribution, according to the central limit theorem, $\overline{X}$ assumes a normal asymptotic distribution. In fact, as $n$ increases, the density function of $\overline{X}$ approaches a normal distribution very rapidly, although the population distribution is strongly asymmetric; see e.g. [41].

In our applications, independently from the sample size, we assumed that the sampling distribution of the sample mean $\overline{X}$ is approximately normally distributed. Based on literature data, Fig. 2 shows the log-normal distribution for the critical headway vs the normal distribution for the mean of the critical headway. Sample size $n$, as it will be better explained below, was obtained under specific hypothesis on the degree of saturation (namely the ratio of the entry flow to the entry capacity) and the time duration of the observation period.

Based on the above said, in order to characterize the sampling distribution of the sample mean $\overline{X}$ from samples of size $n$, the sample size $n$ has to be defined. On this regard, we remember that the number of entering vehicles during the period of observation will depend on the length of steady-state condition, that is not immediately known. On the contrary, it is possible to get an appropriate measurement of the time $T$ that the system needs in order to move from a steady-state condition to another subsequent steady-state condition. The transient time $T$ can be calculated through the Morse's inequality [42]:

$$T > \max\left\{ 1 \Big/ \left( \sqrt{\frac{C_i}{3600}} - \sqrt{\frac{Q_{ei}}{3600}} \right)^2 \right\} \tag{4}$$

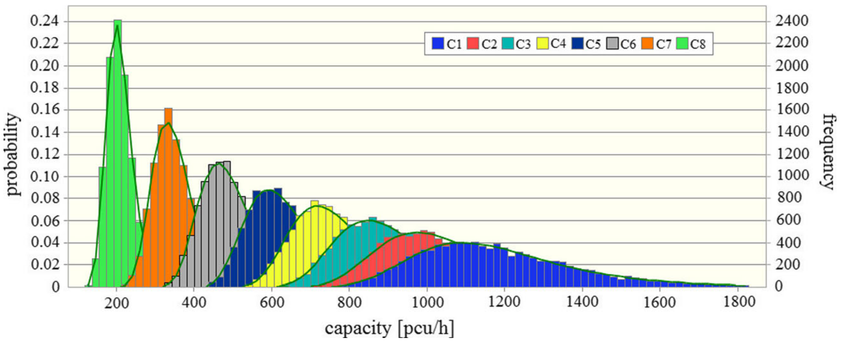
where:

$C_i$ = capacity at entry lane $i$, pcu/s;
$Q_{ei}$ = entry flow at the lane $i$, pcu/s.

It is noteworthy that this formula can be applied only when the ratio $(Q_{ei}/C_i) < 1$. Besides, it must be said that the steady-state models of entry capacity are only a useful approximation if the duration of the analysis period is considerably greater than the duration calculated using the Morse's expression [42]. Thus, in our application we assumed a period of observation equal to twice the time of the transient phenomenon and we calculated the number of entering vehicles over this period that we considered as sample size.

In order to apply the Morse's formula (thereby determining the sample size $n$), an entering flow rate should be set, or the ratio of the entry flow to the entry capacity ($Q_e/C$) should be specified upon the condition $(Q_{ei}/C_i) < 1$; this means that only undersaturated conditions shall be considered. Thus, we considered three different values of the ratio of the entry flow to the entry capacity, i.e. 0.25, 0.50 and 0.70. Under these

**Fig. 5** Probability distributions of entry capacity for the left-lane on major entries at turbo roundabouts. Note that C1–8 are the probability distributions of entry capacity where each of them is corresponding to a value of the circulating flow around the ring ranging from 0 to 1400 pcu/h with step 200 pcu/h



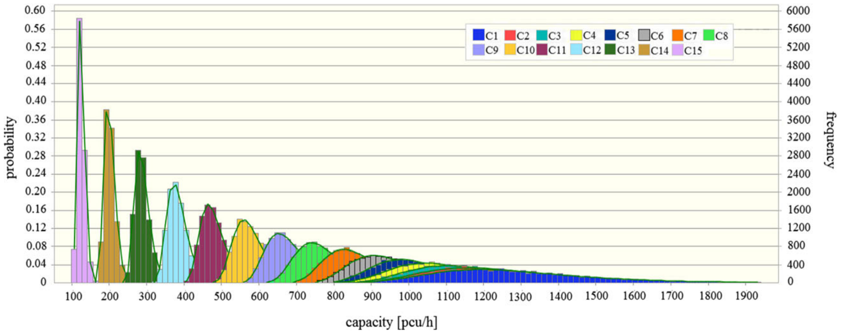


**Fig. 6** Probability distributions of entry capacity for the left-lane on minor entries at turbo roundabouts. Note that C1–15 are the probability distributions of entry capacity where each of them is corresponding to a value of the circulating flow around the ring (where $Q_{c,i} = Q_{c,e}$) ranging from 0 to 2800 pcu/h with step 200 pcu/h

hypotheses, the corresponding values of *n* were found equal to 4, 12, 37, respectively, whereas half of them can be considered in steady-state conditions.

### 3.2 Uncertainty analysis

To understand uncertainty in roundabout capacity estimation, the probability distributions of the random variables of the capacity model had to be identified. Crystal Ball software was used to find the probability distributions of each parameter contributing to entry capacity of the roundabouts under examination. For this purpose, literature data sources were used to hypothesize the probability distributions of each contributing parameter, as described in the previous section 3.1. Since sample sizes generally affect precision, the weighted average value of the standard deviation σ of each single (primary) study used in the meta-analysis was calculated, multiplying each standard deviation by the corresponding sample size (adding those values and then dividing the total value by the total number of sample sizes).

For each roundabout and each contributing parameter, the normal distribution best seemed to fit the data; the (random) summary effect, or the meta-analytic estimation for each headway, is the mean of the distribution, whereas the standard deviation $\boldsymbol{\sigma}$ is weighted with regard to the sample size, as reported in the different primary studies [31].

With reference to the case $Q_e/C = 0.5$ ($n = 12/2 = 6$), Table 3 shows the parameters of the sampling distribution for the critical headway and the follow-up headway for the single-lane roundabout, the double-lane roundabout and the turbo roundabout.

A major task in our applications was to perform preliminary simulations in order to know how many iterations were needed. Thus, a quite high number of iterations was tried until very slight differences in the outputs led to the searched distributions; lastly, we opted for 10,000 trials. Once the probability distributions for the contributing parameters were set, simulation started. Indeed, the simulation methods generate sequences of random numbers to conduct simulation runs; thus, the probability density functions can be used to describe the physical system. As introduced in section 1, we used the Monte Carlo method and ran simulations with Crystal Ball software. The Monte Carlo method, indeed, selects a random set of input data values drawn from their individual probability distributions; these values are then used in the simulation model to obtain some output values.

Thus, we performed the random sampling from the probability density functions for each random variable based on the adopted capacity formulation. The Hagring model was then used for performing many runs; in each run, the values of the contributing parameters, namely the critical headway and the follow-up headway, were randomly drawn from the corresponding probability distributions. The Crystal Ball software provided, for each type of roundabout, the "overlay" graph which depicts, in a single graph, the probability distributions of entry capacity when varying the circulating flow. Based on

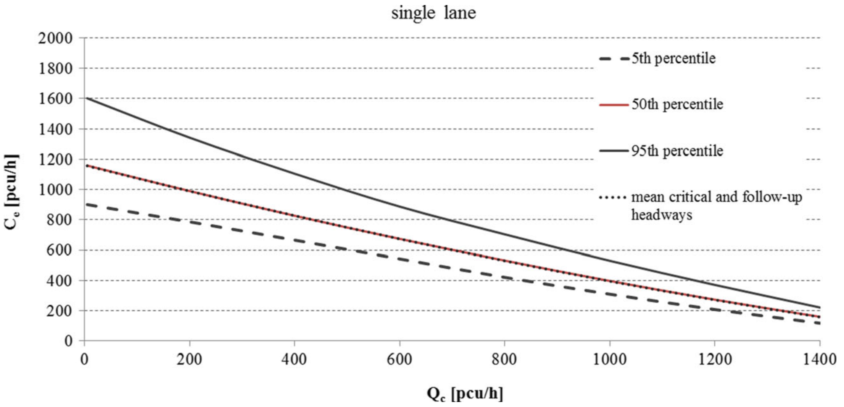
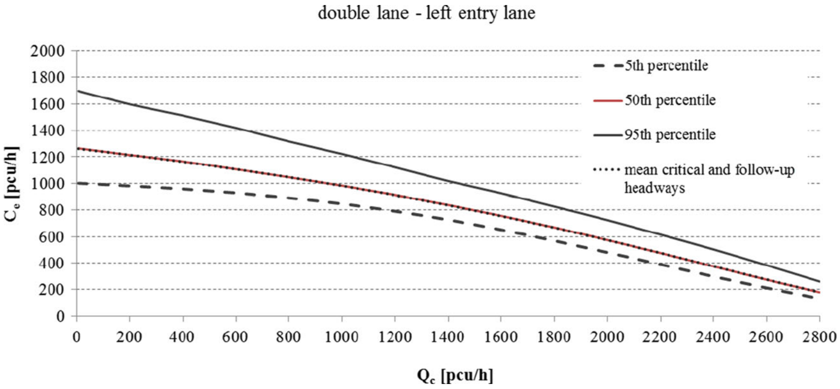


**Fig. 7** Entry capacity functions for single-lane roundabouts

**Fig. 8** Entry capacity functions for the left-lane at double-lane roundabouts



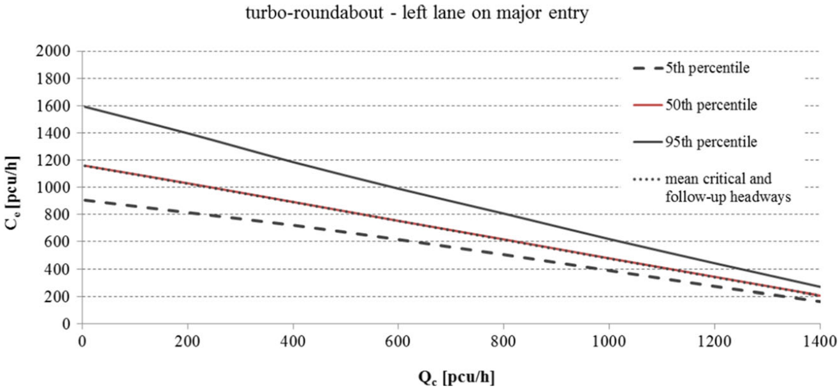
such output, one can analyze variations in capacity and then make a comparison with the results given by the deterministic model. In detail, Fig. 3 depicts the probability distributions of capacity at single-lane roundabouts, where eight values of the circulating flow from 0 to 1400 veh/h with step 200 veh/h were considered in the single circulating lane. In the same way, Fig. 4 contains the probability distributions of the left-lane capacity for double-lane roundabouts; the probability distributions of right-lane capacity for the double-lane roundabouts were about the same of the single-lane roundabouts and for reasons of synthesis have not been reported. Figure 5 shows the probability distributions of left-lane capacity for major entries at turbo roundabouts, whereas Fig. 6 shows, by way of example, the probability distributions of entry capacity only for the left-lane on minor entries at turbo roundabouts; in this case entering vehicles face two antagonist traffic streams for which we made the assumption that $Q_{ce} = Q_{ci}$.

In any overlay graph, that is for whatever roundabout examined, one can see bell-shaped and symmetrical histograms, in which the central column represents the (mean) capacity corresponding to a specified value of the circulating flow; more numerous measures around the mean value can be observed. It should be noted that, when the circulating flow is low, the capacity distribution turns out to be "squashed" with respect to the abscissa axis. Such distribution is characterized by a high variance, or values highly dispersed, whereby the degree of uncertainty of the output in this case is of some importance. It should be noted again that, if one considers gradually more and more high values in the circulating flow, the distribution of capacity takes a higher and narrow shape, with values quite concentrated around the mean; so the result is found to be more stable.

Figures from 7, 8, 9 and 10 show the capacity functions which incorporate the mean values of the critical headway and the follow-up headway, as derived from the meta-analysis, for each type of roundabout under study. In the same figures one can also see the 5th and 95th percentiles that, for a specified set of values, represent a measure that expresses what percent of the total frequency is falling below that measure. The capacity functions which were based on the adopted capacity model matched the median function or the 50th percentile below which the 50% of the resulting measures of capacity falls below. As one can expect, for all the cases the capacity functions - built running the steady state model and assuming for each behavioral parameter a single (mean) value representative of the entire population - tend to overlap with the 50th percentile curve.

The results of the simulations indicated that the uncertainty in capacity estimation could be high, especially when the opposing flow is low; in these cases, estimation through the mean values of the individual parameters can be far from the real value, providing a rough underestimation/overestimation of the latter value. The results indicated, indeed, that the actual capacity of the roundabout may be, with a probability of about

**Fig. 9** Entry capacity functions for the left-lane on major entries at turbo roundabouts



50%, higher than the capacity which can be estimated deterministically; based on this result, the traffic conditions could be better than the expected conditions.

At the same time, however, with the same probability, the capacity estimation based on the deterministic model may be an overestimation of the actual capacity, with the result that, for a given traffic demand, the oversaturated conditions at entries are not highlighted. Based on the results obtained, the deterministic estimation of capacity is not cautionary, but rather the risk of poor performances at roundabouts, especially when the circulating flow is low, is quite significant. However, the conclusions that are drawn from this research could be affected by the choice of one or another capacity model. The reader is advised that the use of other models that incorporate different processes could further improve understanding of uncertainty in capacity estimation at roundabouts.

## 4 Conclusions

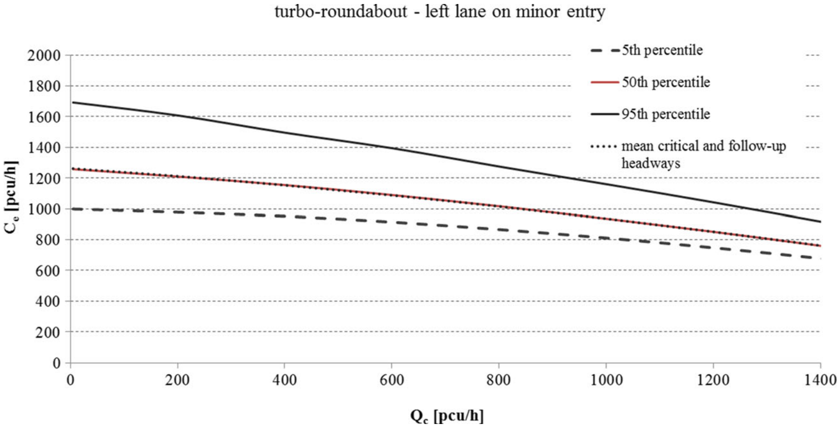
The concept of gap acceptance is inherent in the traffic interaction which takes place when the minor-street vehicles enter the intersection merging into or crossing a traffic major stream. Gap acceptance models are aimed at representing to what extent the minor-street vehicles entering a roundabout will be able to use an acceptable gap between two consecutive vehicles in the major traffic stream. When a gap acceptance model is going to be developed, assumptions need to be made both for the psycho-technical headways (or the critical headway and the follow-up headway), and for the arrival headway distribution (or the distribution of the gaps between the vehicles in the different circulating streams), as well as for the distribution of traffic flows among the circulating lanes. The accuracy of the capacity estimation is primarily determined by the accuracy of the estimation of the critical headway and the follow-up headway. In calculation process, single mean values usually replace these random variables, disregarding their inherent variations, and thus providing a single-value of entry capacity. However, the model performance in predicting capacities can be limited, although when one is assessing the operational performance of an existing roundabout or deciding whether (or not) a roundabout should be planned, designed or built, an important role is played by the capacity estimations and level-of-service determinations. Hence, analysts would like to perform estimations as complete and correct as possible.

The paper analyzes how variations in behavioral factors affect the capacity estimates for different types of roundabouts. Uncertainty analysis in estimation of roundabout capacity was performed by using the Monte Carlo sampling and simulation procedure for finding the distributions of output variable values based on the distributions of input data values. A Monte-Carlo simulation was then developed as statistical simulation method to obtain the probability distributions of the entry capacity at roundabouts. For this purpose, Crystal Ball software was found very suitable for performing Monte-Carlo simulation and estimating uncertainty; thus it was used for the random sampling from the probability density functions that were chosen for the contributing parameters according to the adopted entry capacity model of non-linear features.

The paper presents the results of first simulations which provided estimates of entry capacity distributions for different roundabouts, once the probability distributions of the critical headways and the follow-up headways were assumed. A deterministic capacity model was then used for performing many runs; in each run, the values of the contributing parameters were randomly drawn from the corresponding distributions. For the examined roundabouts, simulations with the Crystal Ball software provided the probability distributions of entry capacity when varying the circulating flow. Thus, the results of the analysis were expressed probabilistically, meaning that the probability distributions of capacity at each entry lane rather than the simple point estimates of the performance measure were obtained.

Based on these outputs, one can analyze variations in capacity and then make a comparison with the results derived from the application of the deterministic model. The end results

**Fig. 10** Entry capacity functions for the left-lane on minor entries at turbo roundabouts



were, indeed, the probability distributions of entry capacity for each roundabout, the capacity functions including the 5th percentile, the median (50th percentile) and the 95th percentile, the capacity functions which were built based on the adopted capacity formulation; the last overlapped the capacity functions corresponding to the median value below which the 50% of the outputs may be found. Although variation in capacity values are expected due to variations in parameters, and the deterministic estimate represent the average value, the results provide some insights about the situations in which the uncertainty of the capacity estimates might be particularly relevant. Indeed, the results of the simulations indicated that uncertainty in capacity estimates could be high, especially when the opposing flow was low; in these cases, estimation through the mean values of the individual parameters can be far from the real value, providing a rough underestimation/overestimation of the latter. The results, indeed, indicated that the actual capacity of the roundabouts may be, with a probability of about 50%, higher than the capacity which can be estimated deterministically. Based on this result, the traffic conditions could be better than the expected ones. However, at the same time, with the same probability, the capacity estimated by using the deterministic model may be an overestimation of the actual capacity, with the result that, for a given traffic demand, the oversaturated conditions at entries are not highlighted. Based on the results obtained, the deterministic capacity appears to be not cautionary, but rather the risk of poor performance at roundabouts, especially when the circulating flow is low, is quite significant. The reader should be advised that the conclusions drawn from this research were based on calculations performed by a particular capacity model and the use of other models that incorporate different processes could further improve the understanding of uncertainty in roundabout capacity estimation.

Further work in this analysis would extend the uncertainty analysis to better understand variations in drivers' psychotechnical attitudes based on geometric design of the roundabouts and the bunched vehicles in the circulating traffic flows. The analysis can be extended by including:

- more traffic demand scenarios to reflect the different constraints (namely environmental and of context); especially at multi-lane roundabouts, dominant (or subdominant) arrival flows could influence the estimation of the gap acceptance parameters;
- different assumptions about the arrival headway distributions and the distribution of vehicles among the circulating lanes (where possible and consistently with the chosen layout of the roundabout);
- uncertainty analysis on the central island size (and/or the entry width) of the roundabout, which we have assumed as being not influent; this analysis can be included as well to reflect the different needs of the built environment.

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