I.O.S.

DISPERSION OF TRACERS
BY THE OCEANIC EDDY FIELD MODELLING PROGRAMME
FINAL REPORT, OCTOBER 1987

BY
K.J. RICHARDS & S.P. O'FARRELL

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OCEAN DISPOSAL OF HIGH LEVEL RADIOACTIVE WASTE
A RESEARCH REPORT PREPARED FOR
THE DEPARTMENT OF THE ENVIRONMENT

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DEACON LABORATORY

INSTITUTE OF OCEANOGRAPHIC SCIENCES

DEACON LABORATORY

**Wormley, Godalming,
Surrey, GU8 5UB, U.K.**

**Telephone: 0428 79 4141
Telex: 858833 OCEANS G
Telefax: 0428 79 3066**

Director: Sir Anthony Laughton, Ph.D., F.R.S.

Natural Environment Research Council

INSTITUTE OF OCEANOGRAPHIC SCIENCES

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Dispersion of tracers
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K.J. Richards* & S.P. O'Farrell*

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** Present address:*

Department of Oceanography
The University
Highfield
SOUTHAMPTON, SO9 5NH

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Author/Affiliations etc. K.J. Richards and S.P. O'Farrell
Department of Oceanography, Southampton University

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Abstract **(100-200 words as desired)**

A numerical model has been developed to study the dispersion of tracers by the oceanic eddy field.

The present study is designed to study the dispersion of particles in a mesoscale eddy field produced by the numerical model. Dispersion rates are calculated for flows above three types of topography, a flat bottom, a random collection of hills and a ridge. The presence of topography is found to significantly affect the flow. The effective diffusion coefficient of the flow near the bottom is reduced by 20% for the random topography and 60% for the ridge from that for the flat bottom case. Estimates are given of the number of float years required to obtain a given accuracy for the diffusion coefficient. At the surface a modest number of floats (order 5) are required to obtain a 50% accuracy. However at the bottom, to be within a factor of 2 of the true value for the flows considered requires respectively 26, 42 and 103 float years for the flat, random and ridge cases.

The results will be of use to the planning of future dispersion experiments in the deep ocean using floats.

Keywords **(suggested maximum of five to be taken from DOE standard keyword list provided**
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This work has been commissioned by the Department of the Environment as part of its radioactive waste management research programme. The results will be used in the formulation of Government policy, but at this stage they do not necessarily represent Government policy.

CONTENTS

Preface

The research described in this report is concerned with a small part of the scientific assesment of the feasibility of the disposal of heat generating radioactive waste (HGW) into the deep sea environment. A presentation is given of research aimed at understanding the mechanisms of dispersal of radionuclides in the water column by large scale (100 km) oceanic eddies. In this particular study the dispersion of particles is simulated using a numerical model and the statistics of the dispersion calculated. The work has implications for the planning of dispersion experiments to be carried out at sea.

The Natural Environment Research Council, through the Institute of Oceanographic Sciences, Deacon Laboratory, has a contract with the Department of the Environment (DOE, PECD7/9/216) to examine the diffusion of tracers by the oceanic eddy field both by direct measurement within the deep ocean and also by numerical modelling. The emphasis of the investigations has been placed on studying the processes relevant to radionuclide dispersal in the water column in order that realistic predictive models can be developed. This report covers work carried out in the Department of Oceanography, Southampton University under contract to the Natural Environment Research Council (contract F60/B1/12).

Introduction

The dispersion of a tracer in the deep ocean on scales of order 100km and larger is dominated by mesoscale eddies. Mesoscale eddies form a major contribution to currents in the deep ocean. In the deep N.E.Atlantic[1] eddies have a diameter of approximately 50 km and an average speed of 1 to 2 cm/s. Direct mearurements of the flow in the deep ocean are difficult to make and expensive. Most are restricted to a few longterm current meter moorings[2]. The exception has been the series of experiments conducted by the IOSDL designed to study the the flow close to the ocean floor in three topographically different locations[3]. The IOSDL experiments show the charateristics of the eddy field to vary greatly with the type of topography of the region.

Current meter measurements are taken at a fixed location and known as Eulerian measurements. Even for the mooring arrays of IOSDL the spatial extent is sub-mesoscale. An alternative approach is to release a float at a prescribed depth and to track the path of the float as it is advected by the current, a Lagrangian measurement. Floats which have been developed can be tracked for a relatively long time (up to 2 years) and can sample many eddy 'events'[4]. Released in clusters, a direct measurement of dispersion by the mesoscale eddy field can be made. Such an experiment has been conducted by IOSDL. A cluster of 9 floats were released with an initial separation between the floats of 25km and at a nominal depth of 3000m. Only 6 of the 9 floats functioned properly. The first six months of the float tracks is shown in Figure 1. During this time the floats separate from each other due to the action of mesoscale motions whilst being advected as a group by a larger scale flow feature. The topography of the mid-Atlantic ridge to the west has a noticable effect on the motion of the floats. A measure of the dispersion of the floats is given by the way the separation between pairs of floats varies with time averaged over all particle pairs in the group (two particle statistics). The average squared separation between the IOSDL floats is shown in Figure 2. During two periods the floats separated, on average, at an approximately constant rate. However, between these two periods the average separation actually decreases. This example highlights the difficulty in estimating dispersion rates with a relatively small number of floats. With a larger number of floats spread over a larger area and tracked for longer the averaged separation should increase monotonically.

The aim of the present study is to establish the amount of information contained in a small sample of float tracks and give estimates on the reliability of the statistics obtained. It is hoped that a full two years of data will be collected from the IOSDL floats giving approximately 12 float years of data. Floats are expensive and any experiment will be restricted to a small finite number. An alternative approach, the one taken here, is to simulate the flow using a numerical model. Particles or 'numerical' floats can then be tracked using the calculated flow field. The big advantage is that numerical floats are relatively inexpensive in terms of computer time. A large number can be seeded in the flow and tracked for long periods giving more robust statistics. Also, the large number allows a large areal coverage. Thus in a region where topography causes inhomogeneities in the eddy field a representative figure of dispersion rates for the region can be obtained.

The first problem is to simulate the eddy field. As work on this has been reported on in an early report[5] this will be only briefly referred to here. The presented results will concentrate on the statistics of simulated float tracks, the errors involved in employing a limited number of floats and how this varies with the topography of the region. Estimates will be given of the number of float years required to obtain a given accuracy in the determination of dispersion coefficients.

The Eddy Field

In order to simulate a field of mesoscale eddies it is convenient to idealise the problem and consider a small region of the ocean, or box, which is assumed to be embedded in a larger region where the character of the eddies is horizontally uniform. The flow is then assumed to be periodic over the size of the box, here taken to be a 1000km square. An eddy travelling out of one side of the box comes in on the opposite side (see for example figure 5). Eddies are generated within the box by some prescribed process and the system allowed to attain some statistically steady state. Such models are referred to as periodic or process models[6].

A number of authors have studied the effect of topography on the structure of the eddy field[5,7,8]. The results of these studies have shown that the resultant eddy field is very dependent on the horizontal length scale of both the forcing and bottom topography. Broadly speaking however the topography has the effect of decoupling the upper and bottom flows, the bottom flow has a tendency to be steered along height contours and the ratio of the kinetic energy of the eddies at the surface to that at the bottom is increased as the height of the topography is increased.

The model used to simulate the eddy fields reported here is that used by Richards[5]. The vertical density structure of the model is taken to be typical of the eastern N. Atlantic, figure 3, with the vertical structure of the horizontal flow being decomposed into its first three normal modes. Three different topographies have been studied (Figure 4), a flat bottom, a random topography having smaller scale hills of lengthscale 50km and average height 125m and a north/south ridge of width 333km and height 1000m. Typical flow patterns for the three cases are shown in Figure 5. Notable features are the reduction in the horizontal length scale of the eddies with the introduction of topography and the greater correlation between the flow and the topography for the random topography compared to the ridge case. A summary of the variation of the kinetic energy and horizontal length scales of the three cases with depth is given in Table 1. The two topography cases show an increase in the ratio of the surface to bottom kinetic energy whilst only the random case shows a reduction in horizontal length scale with depth, due to the action of the small scale topographic features. A measure of the steering effect of the topography is given by the ratio of the energy of the flow across slope to that along. For the random topography case this is 0.21 and for the ridge 0.42. These are average figures for the whole box and they will be greatly reduced if only areas with high slopes are considered.

Some note is required on the longterm stability of the flow. The introduction of topography into the model produced some long time scale (order several months) fluctuations in the overall barotropic and baroclinic energies of the flow. This was particularly marked in the ridge case. This slow variation in energy levels with time means that very long averages (several years) are required to obtain stable statistics. This has implications for the required record length of field observations.

Particle Statistics

In each of the 3 flows 1089 particles where seeded with a uniform horizontal distribution at 3 levels, the surface, 2000m and the bottom. The initial separation between the particles was 30km. The particles were tracked for 3.5 years giving a total number of particle/float years at each level of 3812.The effect of topography on the motion of the particles is graphically illustrated in Figure 6 which shows the tracks of a group of 9 particles at the 3 levels, initially above a small hill, for a period of 1.8 years. During this period the surface particles are widely dispersed over the area of the box. The bottom particles, on the other hand, are confined to the region of the hill and slowly rotate about it in a clockwise (anticyclonic) sense.

The particle tracks can be analysed in two ways, either individually or by looking at pairs of particles and calculating the separation between them. The two methods are known as one and two particle statistics, respectively.

One particle statistics

A measure of the spread of the particles in the north (x) and east (y) directions is given by the quantities R_x and R_y defined by

$$R_x^2 = <(x-x_0)^2> \quad , \quad R_y^2 = <(y-y_0)^2>$$

where x_0 and y_0 are the initial position of the particle and the carat brackets indicates an average over all particles. After a sufficiently long time when the displacements of the particles from their initial positions is large compared to the eddy lengthscale we can defined effective eddy diffusion coefficients K_x and K_y by

$$K_x = 1/2\ dR_x^2/dt \quad , \quad K_y = 1/2\ dR_y^2/dt$$

Two particle statistics

A measure of the spread of a cloud of particles is given by the quantity defined by

$$D^2 = <<(x_1-x_2)^2 + (y_1-y_2)^2>>$$

where x_1,y_2 and x_2,y_2 are the positions of a pair of particles initially a given distance apart and the double carat brackets indicates an average over all such particle pairs. Again an effective diffusion coefficient can be defined by

$$K_m = 1/4 \, dD^2/dt$$

Note that the definition of diffusion coefficients is meaningful for both the single and two particle statistics only if the quantities R_x^2, R_y^2 and D^2 vary approximately linearly with time. In the case of D^2 it was found that the rate of increase for the flat, random and ridge topographies was, respectively, slightly greater than linear, close to linear and slightly less than linear.

A summary of the various calculated diffusion coefficients for the three flow cases at the three levels is given in Table 2. Points to note are the general rapid decrease in the diffusion coefficients with depth and the reduction in the bottom diffusion coefficients for the random and ridge topographies compared to the flat bottom. The reduction of D^2 is 20% and 60% respectively. The magnitude of the bottom values compares well with values estimated from observations in the N E Atlantic. The larger surface values of K_x compared to K_y is due to the effect of the curvature of the Earth's surface which causes the currents to be more zonal. The reverse is seen at the bottom for the ridge case where the flow has a tendency to be along the ridge rather than across it.

It is interesting to note that the values of K_m derived from the IOSDL floats during the two periods in which D^2 is increasing (see figure 2) are 416 and 319 $m^2 \, s^{-1}$. These values are close to that at the bottom for the random topography case.

Estimates of the diffusion coefficients can be derived from the properties of the flow itself, namely

$$K_x \, , \, K_y \sim I_L(u'^2 \, ,v'^2)$$

where I_L is the integral Lagrangian timescale and u'^2, v'^2 the varience of the velocity components[9]. For the flows here I_L was found to be around 18 days. The estimates are given in Table 2. Comparison with the particle data shows the estimates are reasonable and show the correct variation with topography. The largest differences are at the surface where the estimates in the worst case underpredict by a factor of 4.

Implications for field observations

As well as obtaining the mean values of the rate of change of quantities such as D^2 the variance about this value of individual particle pairs can be calculated. Because of the large number of floats involved in the calculation the variance is a relatively robust statistic. The ratio of the standard deviation to the mean value of the diffusion coefficients is found to increase with depth by a number between 3 and 8, i.e. the uncertainty in the estimated mean from a small number of particles increses with depth. As an example, the ratio for D^2 for all three cases is around 3 at the surface and increases to 9, 11 and 20 respectively for the flat bottom, random and ridge topographies, respectively. The larger value of the ratio at the bottom for the ridge is due to the existence of low period oscillations in the flow, thought to be topographically trapped Rossy waves. The ratio of the standard deviation to mean also allows us to estimate the number of independent observations, N, needed for the calculated mean to be within certain bounds of the true value. Thus for example, for a 95% chance for the estimated mean to be within a fraction F of the true mean T with a standard deviation S, then

$$N = 4\ S^2/(F^2T^2)$$

The values of N required to estimate the diffusion coefficient K_m to a required accuracy are given in Table 3. The number of independent observations increases sharply towards the bottom but these values are reduced markedly as larger errors are tolerated.

As the length of time the particles or floats are tracked increases the number of independent observations increases. Assuming that observations are independent after an integral time scale I_L then the equivalent number of float years is given in brackets in Table 3. If the lifetime of a float is 2 years then the number of floats required is half this number. Thus to get within in factor of 2 of the true value for the diffusion coefficient for the bottom flows requires approximately 13 floats for the flat bed case. This number is increased to 20 for random topography and to 50 for the ridge. The larger of these numbers would be inpractical with present technology. Surface values, however, are more encouraging and suggest that a modest number of floats (order 5) are needed to obtain a 50% accuracy.

Conclusions and recommendation for further work

The particle dispersion experiments using a numerical model reported here provide estimates of dispersion rates due to mesoscale eddies in the ocean for a variety of locations with differing topographic features. The presence of topography and the form it takes is found to significantly affect the dispersion rates in the deep ocean with the rates being decreased from those of a flat bottomed ocean. The dispersion coefficients derived from the numerical model compare favourably with those estimated from deep sea measurements. Estimates are given for the number of floats required to obtain a given accuracy for the dispersion coefficients. It is found that with a realistic number of floats (order 10) for an experiment in a given region the accuracy will be satisfactory for the surface flow but near the bottom the errors involved will be at least 100% and probably larger.

The figures presented here are only valid for the numerical flows considered and care is required in applying them to the ocean environment. However they should act as guidelines and suggest that under certain conditions the amount of information gained from float experiments in the deep ocean may be severely limited.

In this work the numerical float dataset has been considered as a whole and global statistics have been calculated. By sub-dividing the dataset the figures for the required number of floats can be tested and different sampling stratigies can be explored. For instance is it better to spread the limited number of floats available uniformly over a region or to deploy them in small clusters? The ocean floor is not uniformly rough and floats will migrate between areas with differing topographic features. The effect on the float statistics needs to be investigated and again the best sampling stratigies determined under such circumstances.

The present work has concentrated on dispersion by eddying flows. The mean flows in the cases considered are either indistinct or on the small scale of the topography. Mean flows, even if weak, can significantly affect the transport of a tracer in the ocean[3]. The ability of determining mean flows using floats in a highly turbulent region needs to be studied.

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Table 1

Eddy statistics for the 3 different topographies

Topography	Depth (m)	Kinetic Energy (cm^2s^{-2})	Horizontal Length Scale* (km)
Flat	0	17.2	23
	1000	5.3	23
	2000	2.6	24
	3000	2.5	25
	4000	2.5	26
	5000	2.5	26
Random	0	29.8	25
	1000	9.8	24
	2000	2.1	23
	3000	1.6	18
	4000	1.7	16
	5000	1.7	16
Ridge	0	26.6	23
	1000	8.0	24
	2000	2.2	24
	3000	2.0	24
	4000	2.1	25
	5000	2.1	25

* The horizontal length scale is taken as the zero crossing point of the transverse velocity correlation.

Table 2

Estimated diffusion coefficients from one and two particle statistics (see text for definitions)

Topography	Depth m	K_m	K_x	K_y	$I_L u'^2$	$I_L v'^2$
			(m^2 s^{-1})			
Flat	0	3580	3975	955	972	850
	2000	994	1352	72	329	162
	5000	477	796	64	258	178
Random	0	6764	1194	1592	1679	1148
	2000	875	954	199	242	160
	5000	398	254	175	158	138
Ridge	0	4774	4774	1194	1577	905
	2000	557	358	239	162	227
	5000	159	95	223	95	236

Table 3

Number of independent observations required to obtain given accuracy

(numbers in brackets are the equivalent number of float years)

	Error (%)	10	50	100
Topography	Depth			
Flat	0	4740 (205)	190 (8)	47 (2)
	2000	11520 (994)	460 (40)	115 (10)
	5000	33333 (2596)	1333 (104)	333 (26)
Random	0	2007 (89)	80 (4)	20 (1)
	2000	11570 (1096)	462 (44)	116 (10)
	5000	52000 (4169)	2080 (167)	520 (42)
Ridge	0	3889 (163)	156 (7)	39 (2)
	2000	24490 (1956)	979 (79)	244 (20)
	5000	162500 (10300)	6500 (412)	1625 (103)

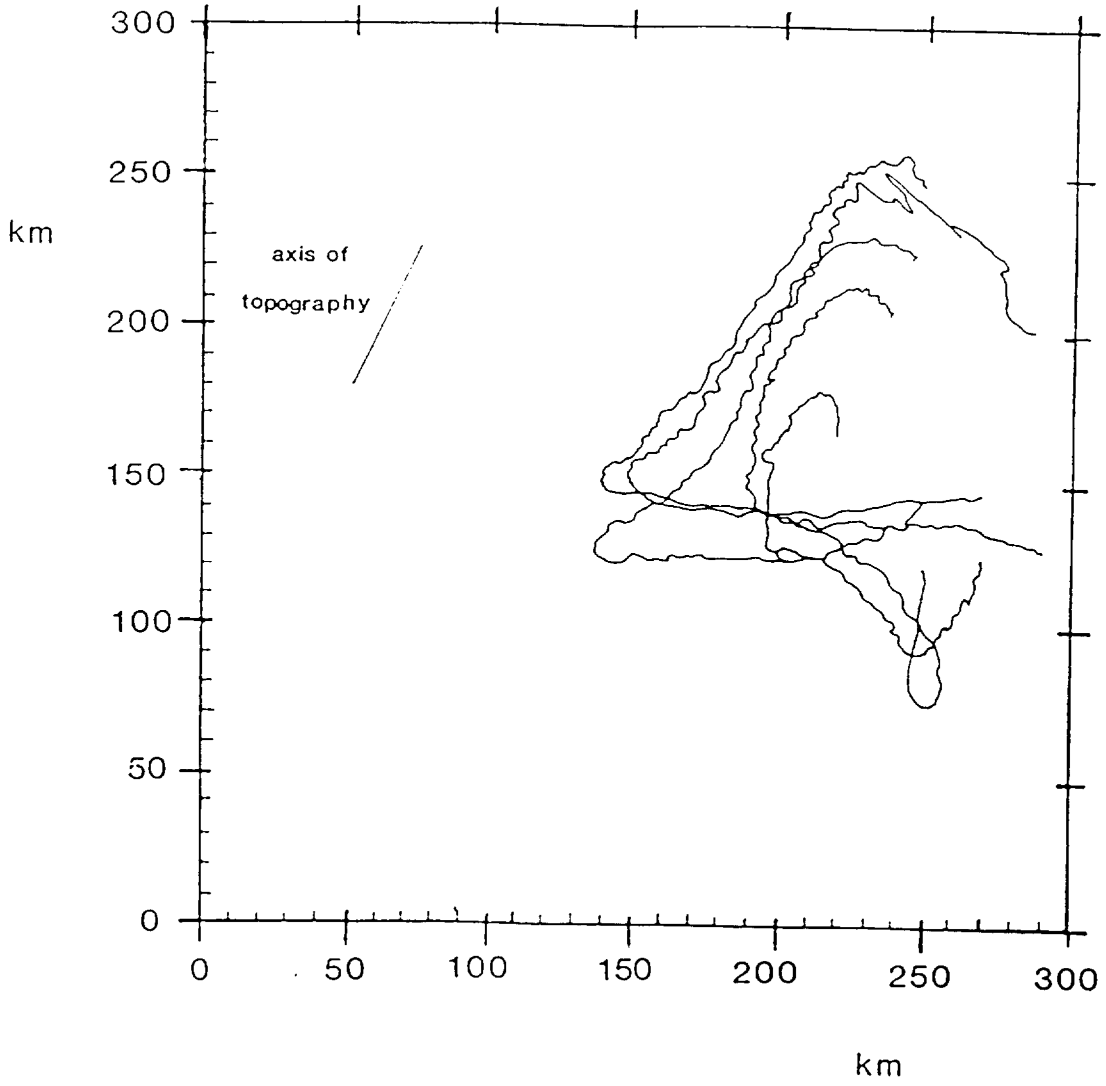

Figure 1 Tracks of 6 neutrally buoyant floats over 6 months deployed by IOSDL at a depth of 3000m in the GME area.

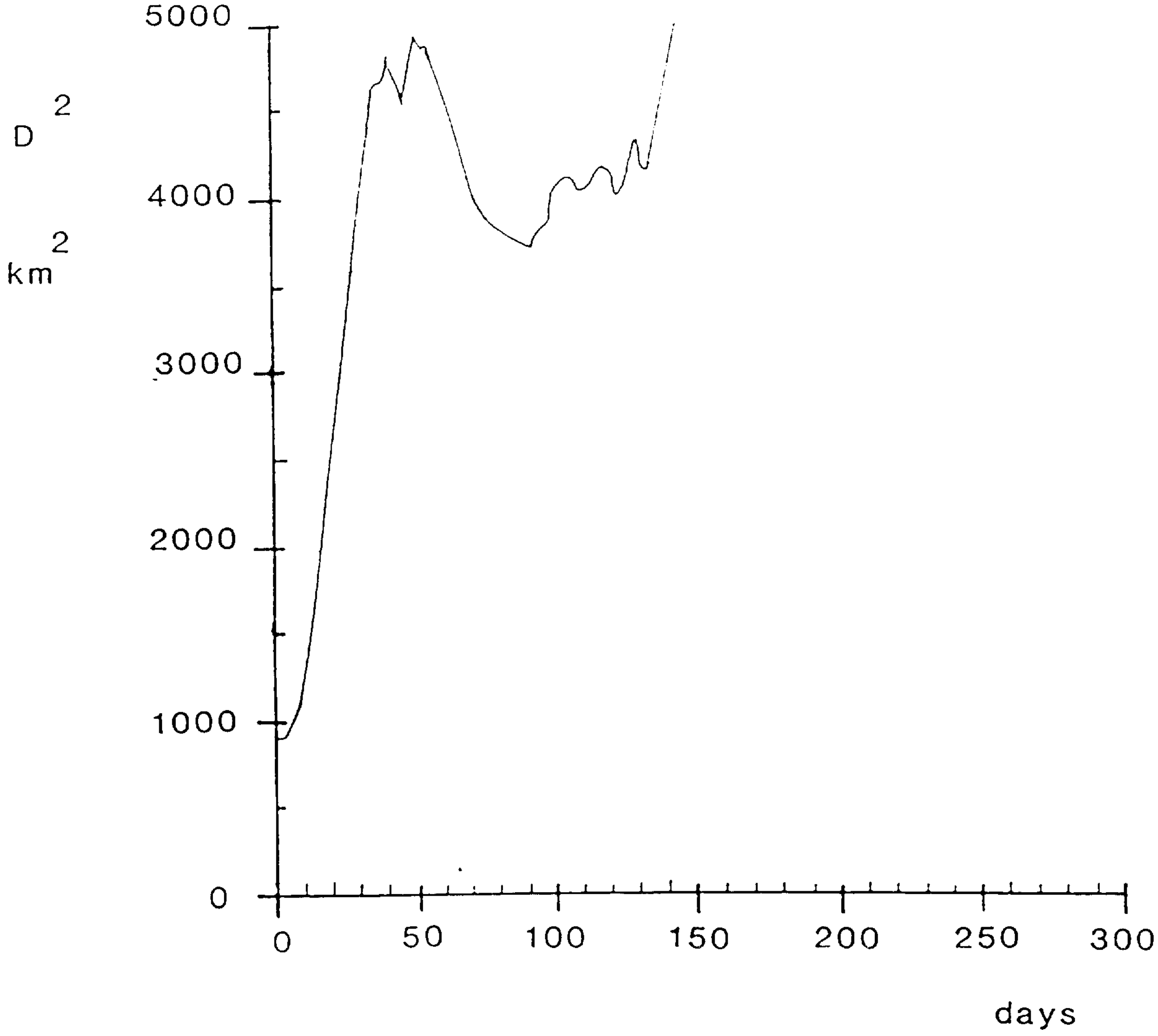

Figure 2 Average squared separation between pairs of the IOSDL floats as a function of time.

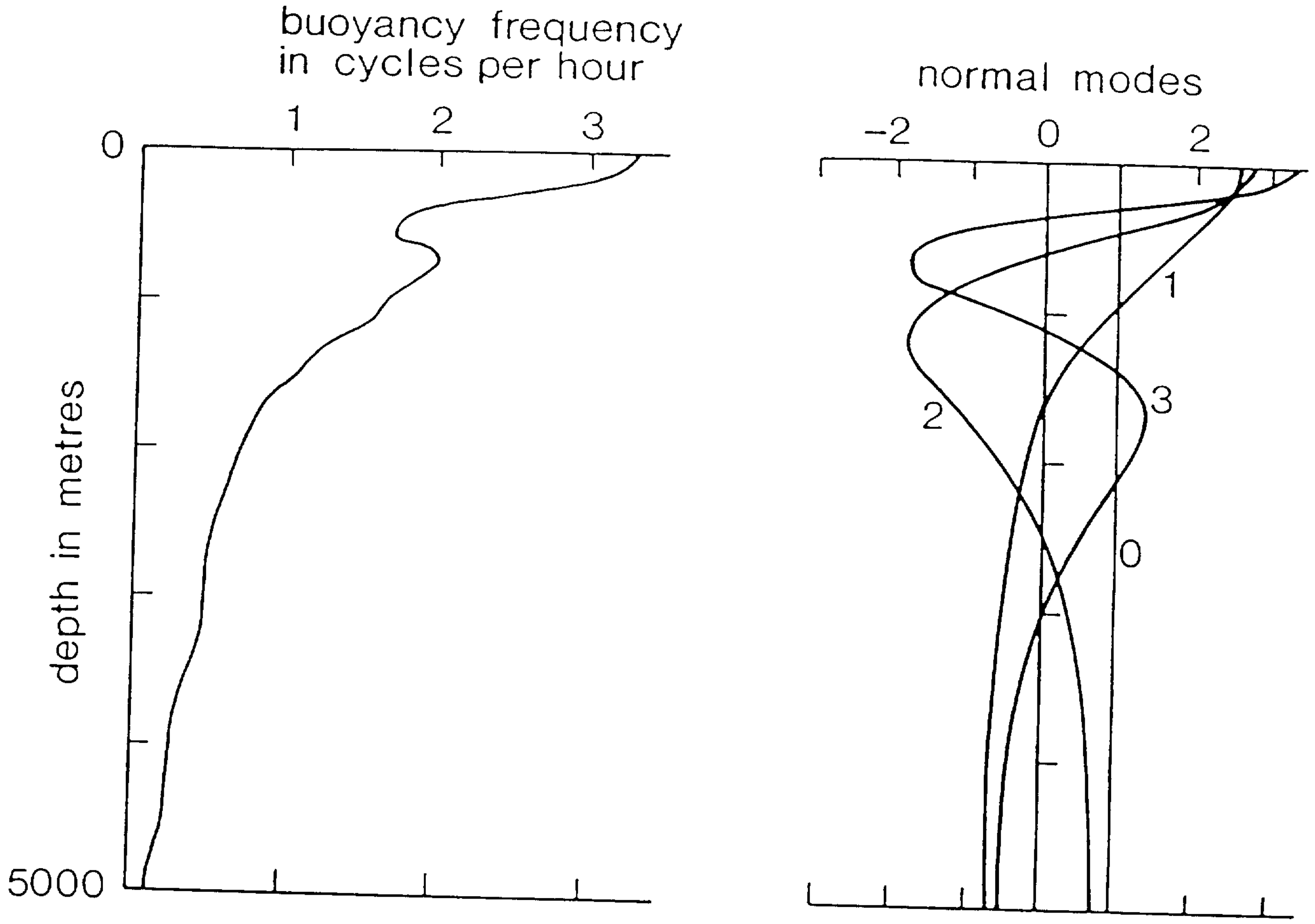

Figure 3 Typical profile of the buoyancy frequency $N = \left(-g\frac{d\rho}{dz}/\rho\right)^{0.5}$ for the Eastern N. Atlantic and the associated barotropic and baroclinic normal modes for the horizontal flow.

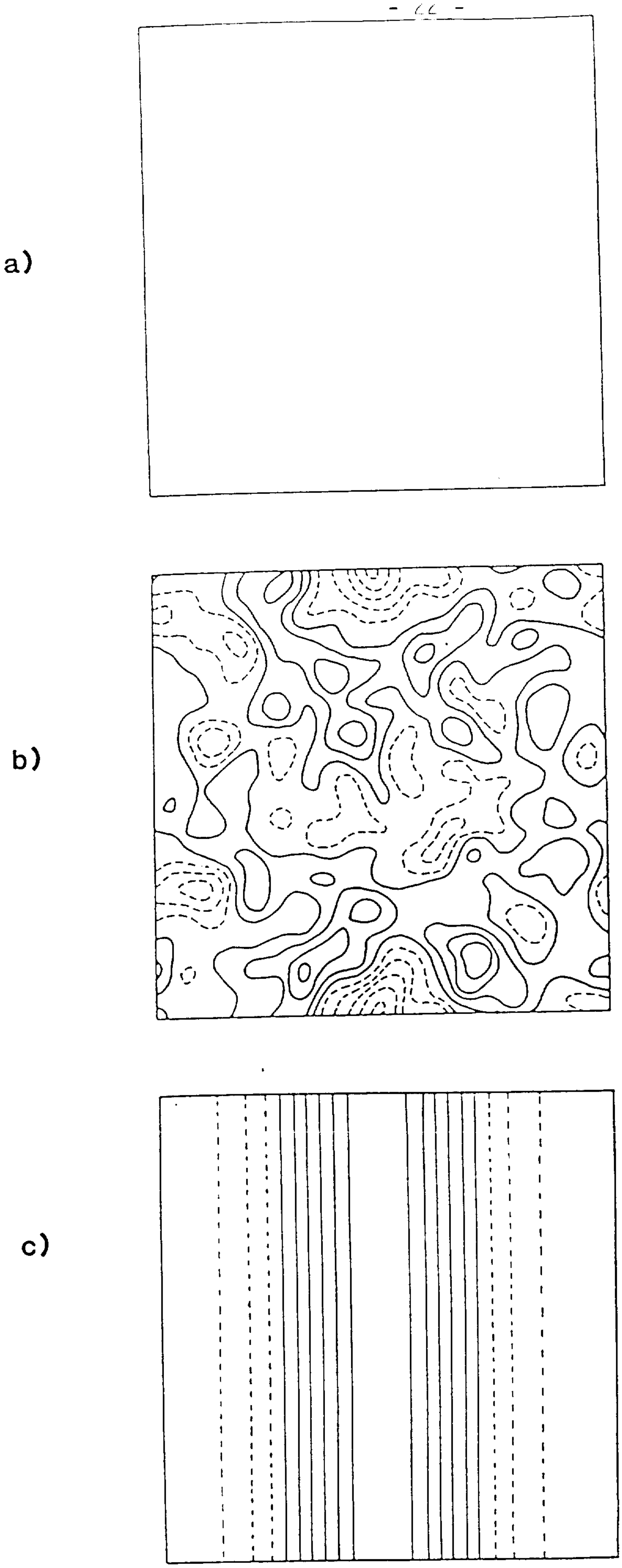

Figure 4 The three topographies used in the experiments; (a) a flat bottom, (b) a random topography, (c) a north/south ridge.

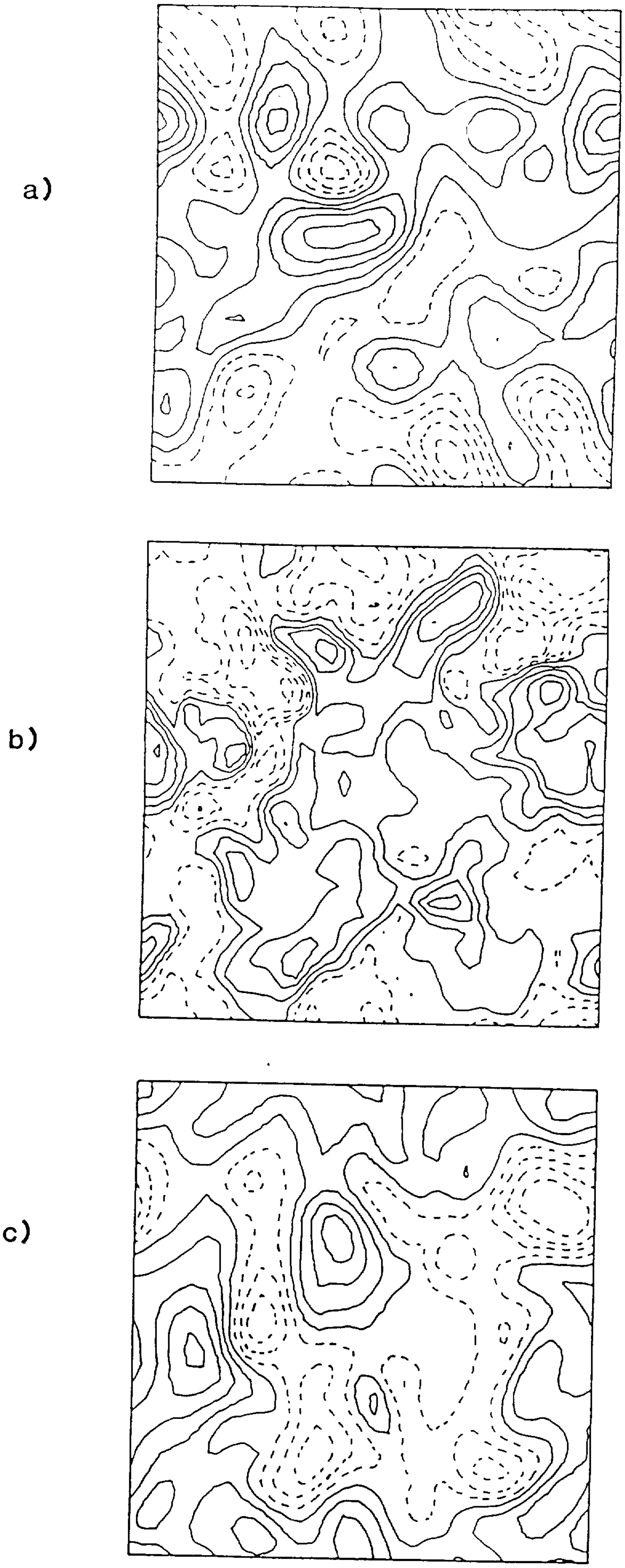

Figure 5 Typical stream function maps of the bottom flow for the three cases (a) flat bottom, (b) random topography and (c) a north/south ridge.

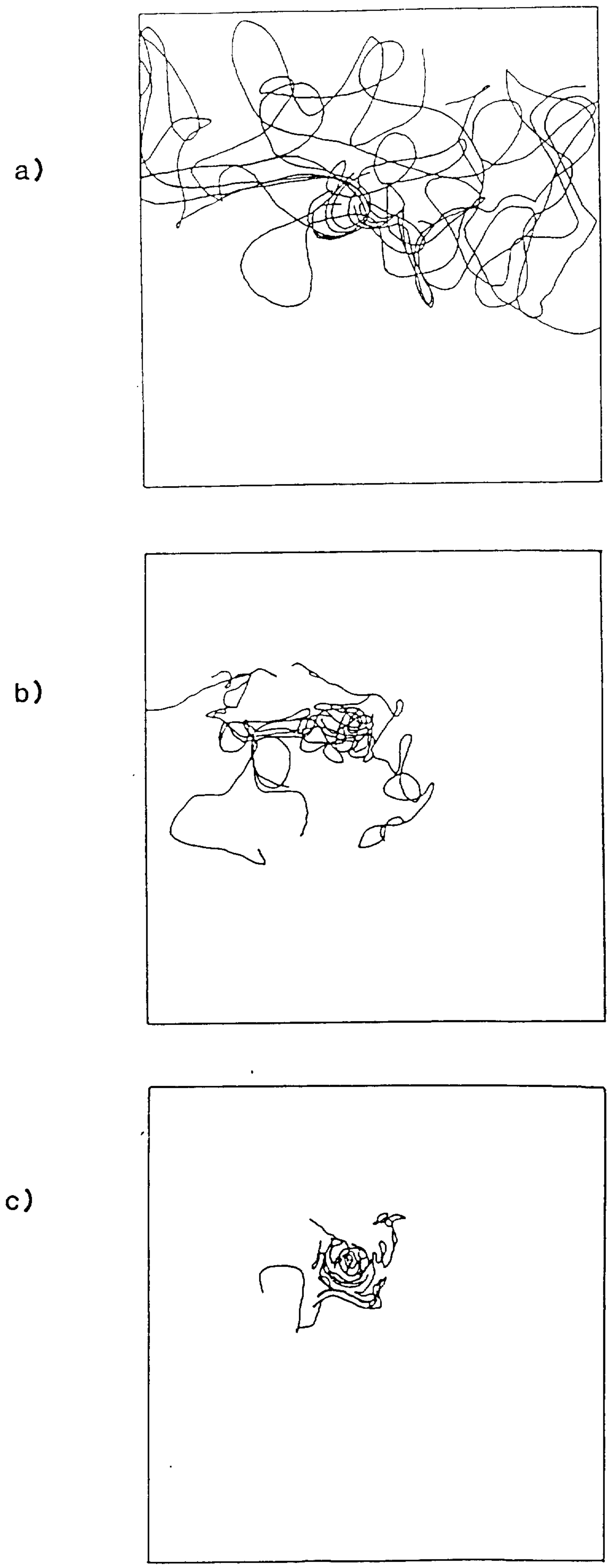

Figure 6 Tracks of a group of 9 particles initially 30km apart over a period of 1.75 years at depths of (a) surface, (b) 2000m and (c) bottom. The floats were placed initially over a small hill of the random topography (Figure 4b).

DOCUMENT DATA SHEET

AUTHOR RICHARDS, K.J. & O'FARRELL, S.P.

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ISSUING ORGANISATION

Institute of Oceanographic Sciences
Deacon Laboratory
Wormley, Godalming
Surrey GU8 5UB. UK.

Director: Dr A S Laughton FRS

TELEPHONE
0428 79 4141

TELEX
858833 OCEANS G

TELEFAX
0428 79 3066

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