BRITISH GEOLOGICAL SURVEY

TECHNICAL REPORT WC/00/17
Overseas Geology Series

A manual for BGSPT: programs to simulate and analyse pumping tests in large-diameter wells

Author
J A Barker
University College London, Department of Geological Sciences
D M J Macdonald
British Geological Survey, Wallingford

DFID classification: W5
Subsector: Water and Sanitation
Theme: Water for Food Production
Project title: Productive Water Point Handbook
Project reference: R7131

This document is an output from a project funded by the Department of International Development (DFID) for the benefit of developing countries. The views expressed are not necessarily those of the DFID.

Bibliographical reference
Barker, J A, and Macdonald, D M J. 2000. A manual for BGSPT: programs to simulate and analyse pumping tests in large-diameter wells. *BGS Technical Report* WC/00/17. DFID Project No. R7131. 19pp.

Keyworth, Nottingham British Geological Survey 2000

BRITISH GEOLOGICAL SURVEY

The full range of survey publications is available from the BGS Sales Desk at the Survey headquarters, Keyworth, Nottingham. The more popular maps and books may be purchased from BGS-approved stockists and agents and over the counter at the Bookshop, Gallery 37, Natural History Museum (Earth Galleries), Cromwell Road, London. Sales desks are also located at the BGS London Information Office, and at Murchison House, Edinburgh. The London Information Office maintains a reference collection of BGS publications including maps for consultation. Some BGS books and reports may also be obtained from the Stationery Office Publications Centre or from the Stationery Office bookshops and agents.

The Survey publishes an annual catalogue of maps, which lists published material and contains index maps for several of the BGS series.

The British Geological Survey carries out the geological survey of Great Britain and Northern Ireland (the latter as an agency service for the government of Northern Ireland), and of the surrounding continental shelf, as well as its basic research projects. It also undertakes programmes of British technical aid in geology in developing countries as arranged by the Department for International Development and other agencies.

The British Geological Survey is a component body of the Natural Environment Research Council.

Keyworth, Nottingham NG12 5GG
Tel: 0115-936 3100 Fax: 0115-936 3200
e-mail: sales@bgs.ac.uk
www.bgs.ac.uk

Murchison House, West Mains Road, Edinburgh EH9 3LA
Tel: 0131-667 1000 Fax: 0131-668 2683

London Information Office at the Natural History Museum (Earth Galleries), Exhibition Road, South Kensington, London SW7 2DE
Tel: 020-7589 4090 Fax: 020-7584 8270
Tel: 020-7938 9056/57

Forde House, Park Five Business Centre, Harrier Way, Sowton, Exeter, Devon EX2 7HU
Tel: 01392-445271 Fax: 01392-445371

Geological Survey of Northern Ireland, 20 College Gardens, Belfast BT9 6BS
Tel: 028-9066 6595 Fax: 028-9066 2835

Maclean Building, Crowmarsh Gifford, Wallingford, Oxfordshire OX10 8BB
Tel: 01491-838800 Fax: 01491-692345

Parent Body
Natural Environment Research Council
Polaris House, North Star Avenue, Swindon, Wiltshire SN2 1EU
Tel: 01793-411500 Fax: 01793-411501

Contents

Preface

BGSPT was developed over a long period by John Barker whilst working on a number of projects at the British Geological Survey. (He now holds the Chair of Hydrogeology in the Department of Geological Sciences, University College London.)

The original version of the programme was developed for the analysis of pumping tests performed in Bangladesh as part of the IDA 4000 Deep Tubewell II Project (Barker et al., 1989). At that stage the program was being used on a mainframe computer. But as personal computers were becoming widely available, the author decided to produce a somewhat simpler, easy-to-use version of the code specifically for PC use (Barker, 1989). Further development was undertaken within the Basement Aquifer Research Project (1984-1989), funded by the UK's Department for International Development (DFID). In the years since 1989, the program has been further developed to allow more flexibility (details of the improved functionality are given within this manual).

The PC version of the program was developed primarily due to the absence of any available computer code to analyse pumping tests from large-diameter wells. Also, few similar tools offered the facility of automatic parameter estimation and where they did, error estimates were not produced. That latter capability is still very rare amongst similar software tools.

The BGSPT code is DOS-based, it does not presently have a Windows interface - input and output is via ASCII text files. It is hoped that a Windows interface will be developed in the near future. Updates will be posted on the web site http://www.bgs.ac.uk/bgspt.

The development of this program was carried out at the suggestion of, and with encouragement from, Robin Herbert, previously Head of Overseas Hydrogeology, British Geological Survey. Funds to make the software available over the Internet were provided by DFID through project R7131, Productive Water Point Handbook (Lovell, 2000).

1. INTRODUCTION

Pumping tests are traditionally analysed using simple graphical or type-curve methods. Such techniques become impractical when more than two or three parameters have to be determined and then computer based techniques need to be employed. This manual describes two computer programs (the two elements of BGSPT): one for analysing pumping tests (PTFIT), and the other to simulate time-drawdown behaviour for a specified set of parameters (PTSIM). The programs were developed for large-diameter wells but can be used for any size of well.

A significant limitation on the software is that no facilities for graphical display of the results are provided. Graphical displays of pumping-test data often provide valuable insights to the behaviour of the groundwater system. Also, it cannot be over-emphasized that the results from the test analysis program should be checked graphically, by plotting the well hydrograph predicted using the parameter estimates alongside the observed drawdowns.

The reader who would like to begin using the programs immediately should:

(i) Study Figure 1 in Section 3

(ii) Follow the installation and testing procedures described in Section 2

(iii) Set up data files as described in Section 4.

(iv) Run the programs as instructed in Section 2.

A few applications of the fitting program are described in Herbert et al. (1992).

2. INSTALLING, RUNNING AND TESTING THE PROGRAMS

2.1 Installation

The installation procedure is simple:

(i) Create a directory for the BGSPT code on your PC.

(ii) The executable files and sample input and output files are compressed into one zip file. Copy this to the BGSPT directory on your PC.

(iii) Unzip the file by double-clicking on it.

Check that the directory contains the following files:

PTFIT.EXE
FIT_DEMO.DAT
FIT_DEMO.OUT

PTSIM.EXE
SIM_DEMO.DAT
SIM_DEMO.OUT

2.2 Running the Programs

BGSPT includes two programs. PTSIM simulates pumping tests. It requires the input file, PTSIM.DAT and produces the output file, PTSIM.OUT. PTFIT analyses pumping tests. It requires the input file, PTFIT.DAT and produces the output file, PTFIT.OUT. The programs can be run within the Windows environment by double-clicking on the executable file. To avoid setting paths, and as the executable files are relatively small, it is suggested that they are copied into the working directory.

2.3 Testing

A demonstration input data set is provided for each program (FIT_DEMO.DAT and SIM_DEMO.DAT). It is suggested that the two programs are run using these input files (by first giving them the required input file name) and the results compared with the contents of the corresponding output files (FIT_DEMO.OUT and SIM_DEMO.OUT). The user's results should only differ in the least-significant digits from the numbers in these files. The demonstration input files provide useful templates for creating data files.

3. THE MODEL

The well and aquifer system being simulated in PTFIT and PTSIM is shown in Figure 1. It is assumed that the well fully penetrates a semi-confined aquifer of infinite extent. The aquitard is unconfined but, for simplicity, it is assumed that the water table remains horizontal during the pumping test. It is also assumed that water flows horizontally in the aquifer and vertically in the aquitard. Darcy's law is assumed to be applicable throughout the system. The model allows for well losses. The model parameters can be in any units as long as they are used consistently.

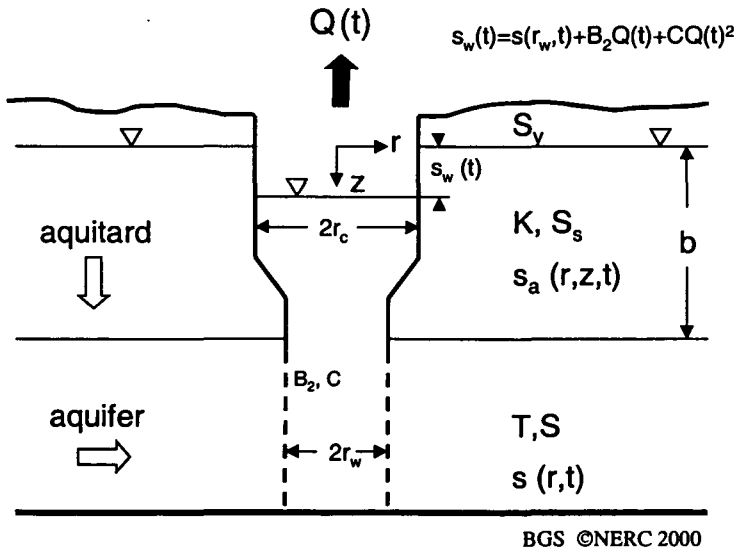

Figure 1 Schematic diagram of a large-diameter well in a leaky aquifer with a horizontal water table.

where:

		dimensions
b	thickness of the aquitard	[L]
B_2	well-loss parameter	$[L^{-2}T]$
C	well-loss parameter	$[L^{-5}T^2]$
K	hydraulic conductivity of the aquitard	$[L\ T^{-1}]$

$Q(t)$	pumping rate	$[L^3T^{-1}]$
r	radial distance from the centre of the well	[L]
r_c	radius of the well casing (see Figure 1)	[L]
r_w	radius of the well screen	[L]
$s(r,t)$	drawdown in the aquifer	[L]
$s_a(r,z,t)$	drawdown in the aquitard	[L]
$s_w(t)$	drawdown in the well	[L]
S	storage coefficient of the aquifer	[-]
S_s	specific storage of the aquitard	$[L^{-1}]$
S_y	specific yield of the aquitard	[-]
t	time since the start of pumping	[T]
T	transmissivity of the aquifer	$[L^2T^{-1}]$
z	vertical depth from top of aquitard	[L]

Mathematical details providing a strict definition of the model are given in Appendix A, along with inputs for special cases.

BGSPT may also be used to analyse pumping tests and simulate time-drawdown behaviour for double-porosity systems. Details are provided in Appendix B.

4. THE PROGRAMS

4.1 PTSIM

4.1.1 Purpose

PTSIM produces a table of drawdowns at specified times and radii, for any given set of input parameters. The time ranges and intervals are given in either linear or logarithmic form of either the time since the start of pumping or since the start of recovery. Any number of time ranges and radii can be given for a single set of aquifer parameters, and any number of sets of parameters can be processed in a single program run. The method involves the inversion of the Laplace transform given by equation (A15) in Appendix A using a numerical method of integration in the complex plane (Talbot, 1979).

4.1.2 Data input

Data are input from a file which has the structure shown below. The parameters must be in consistent units. It is recommended that the demonstration datasets provided are studied to help understand the formats.

Contents	Format	See notes
Title for (first) parameter set	Up to 75 characters	
r_c,r_w, T,S,K,S_s,S_y,b, B_2, C	Real numbers	1
Q1,t1	Real numbers	1,2
Q2,t2	Real numbers	1,2
Qn,tn	Real numbers	1,2
END or **REPEAT=** j k		3,7
ICASE, TLO,THI,DT, r	One integer and four reals	1,4,5
NEW or **END**		7
(if NEW) Title for second parameter set		
etc.		6,7

NOTES:

1 Separated by blanks or commas.
2. Up to 2000 pumping periods allowed (in total, including other parameter sets).
   Q1 from time 0 to t1
   Q2 from time t1 to t2
   Qn from time tn-1 to tn
   0 from time tn to ∞
3. Repeat the last j pumping periods, k times.
4. The time range (TLO to THI) and interval (DT) can be specified in four different ways, depending on the setting of ICASE:
   ICASE=1 Linear time since the start of pumping.
   ICASE=2 Log time (log10) since the start of pumping.
   ICASE=3 Linear time since the start of recovery.
   ICASE=4 Log time (log10) since the start of recovery.
   (e.g. '2 -2 1 1 r' gives times 0.01, 0.1, 1.0 & 10.0)
5. Any number of sets of time intervals and radii can be used.
6. Any number of parameter sets (and time intervals) can follow.
7. Text should be left justified.

Any realistic combination of parameters can be achieved but certain restrictions must be observed. In particular, all of the parameters:

$$T, S, S_s, b, r_w \text{ and } r_c$$

must be greater than zero.

The results from model runs are presented in the form of a table of times and drawdowns.

4.1.3 Special cases

(a) In order to generate the large-diameter well function of Papadopulos and Cooper (1967):

$$F(u_w, \alpha)$$

use

$$Q = 4\pi, t = 0.25/u_w, S = \alpha, r_w = r_c = T = S_s = b = 1, K = S_y = 0$$

(b) For a simple confined aquifer analysis (i.e. the Papadopulos and Cooper model), use:

$$K = 0, S_s = 1, b = 1, S_y = 1$$

(c) For a confined aquitard, use:

$$S_y = 0$$

(d) For a fixed head (zero drawdown) at the top of the aquitard an approximate result can be obtained by setting a very large value for the specific yield (a value of 10 000 is suggested).

4.2 PTFIT

4.2.1 Purpose

This program attempts to fit the well function to a set of drawdown data by automatic variation of the aquifer parameters. Any (physically realistic) subset of the aquifer parameters can be varied, and the

data can be from either the pumping well or observation wells penetrating the aquifer, or both. The program allows for the analysis of step tests. Automatic fitting of the well radius allows effective radius to be estimated.

4.2.2 Minimisation methods

The program attempts to minimise R^2: the sum of squares of residuals (differences between observed, s_{obs} and calculated drawdowns, s_{calc}).

$$R^2 = \sum_i \left[s_i^{obs} - s(r_i, t_i) \right]^2$$

Two methods of function minimisation are employed, with user control of their interaction.

Random-search method

(a) The user specifies a search range (minimum and maximum values) for each parameter to be fitted to the drawdown data.

(b) A random set of parameter values, all within the search range, is generated and the R^2 value found.

(c) This is repeated NTRIAL times and the parameter set with the minimum value of R^2 identified. The search range is then halved (with respect to each parameter) and centred on the minimum point (but kept within the original limits).

(d) Steps (b) and (c) are repeated NRNDIT times.

Because of the large uncertainty in many aquifer parameters, the search is actually carried out in terms of the logarithms of the parameter values (consequently, zero values cannot be used internally so a small value, 10^{-18}, is added).

If it is desired to keep a parameter constant the minimum and maximum values specified for that parameter should be equal.

Marquardt iteration

This extremely efficient method is based on a search, over the residual space for small values using the gradient with respect to the aquifer parameters to guide the search. The required derivatives (or rather their Laplace transforms) are given in Appendix B. The user only has to specify the number of iterations, NMRQIT. The method has been implemented in such a way that it will report on confidence intervals for the fitted parameters at the end of the search. These intervals are only meaningful if a minimum has been reached and the parameter values are not restricted by the specified search range (Section 4.2.5).

Combined use of the random-search and Marquardt iterations

As a general rule, it is much more efficient to use the Marquardt method of search rather than the random-search method. However, if the user has little idea of the correct parameter values for a particular problem, and therefore makes the search range very large, parameter values sometimes combine to produce unresolvable numerical problems in the Marquardt algorithm. It is therefore best to perform a short random search, to get rough estimates of parameter values, followed by a Marquardt search to refine those values.

Because the random-search method is very robust it can be used entirely on its own for refining the parameter values if the Marquardt method persistently fails. However:

(i) such persistent failure indicates a fundamental problem (such inadequate data or inadequacy of the model);

(ii) the random search method is very slow; and

(iii) the final result may be quite different from the best possible.

In most models with more than a couple of parameters there are several local minima in the residual surface, giving different sets of parameter estimates. Which of these minima is found by the Marquardt algorithm depends on the starting position. One approach to overcoming this problem is to use various starting points and then to choose the overall minimum of the solutions (if more than one) found by the Marquardt algorithm. This method has been facilitated by providing an option for looping over the random-search/Marquardt iteration sequence a number of times (Figure 2), which is specified by the parameter NLOOPS. A different set of random points is generated in each loop, and it is probably best to use NTRIAL=1 and NRNDIT=1 to prevent the random search always homing-in on one particular minimum.

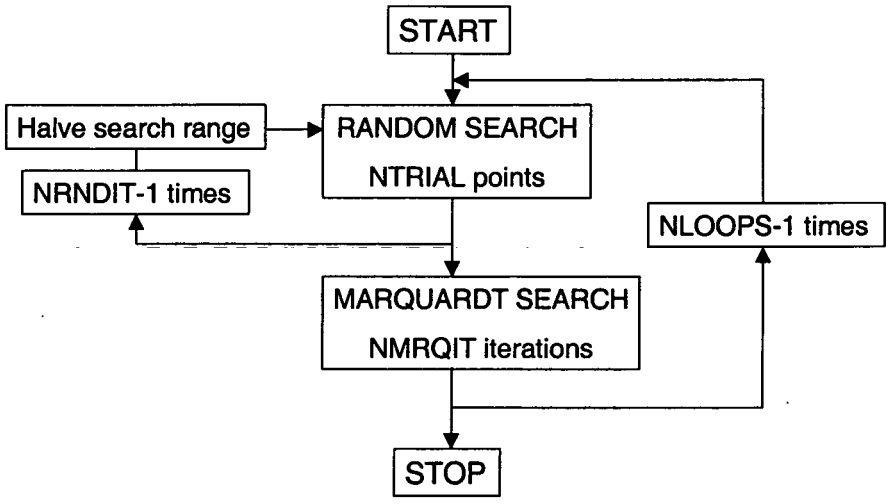

Figure 2 Flow diagram of search procedure with control parameters.

The user should be constantly aware of this problem of non-uniqueness of the parameter set obtained, even when the fit to the data is excellent. The user should also be aware that there are both advantages and disadvantages to increasing the values of NTRIAL, NRNDIT, NMRQIT and NLOOPS and therefore this should be undertaken with due care. As no weighting options are built into the program, the user must also make a judicious choice of range and density of data points.

4.2.3 Hints

- The simultaneous estimation of a large number of parameters presents a very formidable task, which cannot be left entirely to the software to resolve. Sensible restrictions on the parameter ranges will greatly reduce the computing time (especially during random searching), and help to prevent fatal execution errors arising.
- It is always wise to attempt to fit a simple model to any set of data, initially. So it is recommended that a confined aquifer fit is sought first (Section 4.1.3 (b)). A fairly wide range of transmissivities and storage coefficients can be used. A small resultant storage coefficient will indicate that the aquifer is essentially confined, while a large value will indicate that it is leaky, or unconfined. The unconfined case can then be investigated with sensible bounds on the specific storage and yield.

- No facility is provided for weighting the data points, but some control can be exerted by the choice of points. It is always wise to distribute the points over all stages of the test.
- If only large-time data are available or most of the data are long-time, the 'Jacob approximation' tends to apply. Where well losses are introduced in this situation, varying B_2 is equivalent to varying r_w or S (see Appendix A). So simultaneous fitting of more than one of those three parameters is not advisable. Similarly, if Q does not vary much, the parameter C is difficult to determine accurately. These difficulties may be overcome if both pumping well and observation well data are used simultaneously in fitting.

4.2.4 Data input

Data are input from a file which has the structure shown below. The parameters must be in consistent units. It is recommended that the demonstration datasets provided are studied to help understand the above formats.

Contents	Format	See notes
Title of run 1	Up to 75 characters	1
END		2
Q1, t1	Real numbers	3,4
Q2, t2	Real numbers	3,4
Qn, tn	Real numbers	3,4
END		2
Tmin, Tmax	Real numbers	3,5
Smin, Smax	Real numbers	3,5
Kmin, Kmax	Real numbers	3,5,6,7
$S_s$min, $S_s$max	Real numbers	3,5,6
$S_y$min, $S_y$max	Real numbers	3,5,6
bmin, bmax	Real numbers	3,5,6
$r_c$min, $r_c$max	Real numbers	3,5,8,9
$r_w$min, $r_w$max or **=Rc**	Real numbers	3,5,8,9,10 (2)
B_2min, B_2max	Real numbers	3,5
Cmin, Cmax	Real numbers	3,5
NTRIAL, NRNDIT, NMRQIT, NLOOPS	Two integers and a real	3,11,12,13
R= r1	Real numbers	2,14
t1, s1(r1, t1)	Real numbers	3,15,16,17,18
t2, s2(r1, t2)	Real numbers	3,15,16,17,18
etc.		
R= r2	Real numbers	2,14
t1, s1(r2, t1)	Real numbers	3,15,16,17,18
t2, s2(r2, t2)	Real numbers	3,15,16,17,18
etc.		
END		2
Title of run 2		1
etc.		

NOTES:

1. As many lines of text (up to 75 characters per line) as required - end of title identified by **END** on following line.
2. Text should be left justified.
3. Separated by blanks or commas.
4. Up to 100 pumping periods allowed.

Q1	from time	0	to	t1
Q2	from time	t1	to	t2
Qn	from time	tn-1	to	tn
0	from time	tn	to	∞

5. The values can be identical (fixed parameter).
6. If K = 0, no use is made of Ss, Sy or b.
7. Choose units such that a non-zero K is significantly greater than 10^{-18}.
8. Radii r_c and r_w can be dealt with as best fit parameters.
9. The maximum of r_c and r_w should be set less than the radius for any observation well data used.
10. To keep $r_c = r_w$ while varying both put '=Rc' on the data record for r_w instead of the maximum and minimum.
11. See Sections 4.2.2 and Figure 2. If in doubt try NTRIAL=10, NRNDIT=1, NMRQIT=10, NLOOPS=1.
12. If NTRIAL<0 on entry, the random search will use a random number sequence initiated by the clock time (this may be useful if a run is to be repeated using a different random search pattern). The absolute value of NTRIAL is used for the number of random points.
13. NLOOPS will be set (internally) to at least one.
14. If r is less than 1.001 r_w, it is set equal to r_w.
15. Times should be measured from the start of pumping, even if only recovery data are being used.
16. A maximum of 1000 drawdown records can be input.
17. At least as many data points as variables.
18. If t is either zero or negative the data are ignored.

4.2.5 Results

As the search for the best fit to the drawdown data proceeds, each improved parameter set is reported along with the sum-of-squares value. These results appear in the output file and also on the computer screen. The process could be terminated when a suitable fit has been achieved, but such termination will prevent the confidence intervals on the final parameters being reported. No automatic restart procedure is provided.

The program provides warnings indicating whether a parameter is held fixed, or whether it is at either the upper or lower value of the search range: **FIXED**, **HIGH** or **LOW** is printed below each parameter value affected.

The confidence limits on the parameter estimates should not be over-interpreted. They relate to the local minimum which will often not be the global minimum (best possible fit of the model to the data), which could even be outside the confidence limits reported. The interested reader is directed to texts on nonlinear least-squares techniques for a deeper discussion of these formidable problems.

It is strongly recommended that the fitted parameters are used in the PTSIM program to produce a theoretical well hydrograph which can be compared with the original drawdown data. (No automatic transfer facility has been provided.)

5. ERROR MESSAGES

As the programs run it is likely that various warning messages will appear. An attempt has been made to anticipate fatal errors and issue meaningful messages to the user. However, fatal errors are not uncommon. These normally take the form:

Error code at address ***

These messages are run-time error messages from the Prospero compiler, which was used to compile the original Fortran 77 code. The code is a letter or number, the codes most likely to be encountered are:

Z indicating divide by zero

and

7 indicating (probably) that an underflow has occurred.

It has been found that nearly all such errors are associated with aquifer parameters becoming unrealistically large or small, and can therefore be avoided by restricting the ranges of the parameter values.

REFERENCES

BARKER, J A. 1985. Generalized well-function evaluation for homogeneous and fissured aquifers. *Journal of Hydrology*, Vol. 76, 143-154.

BARKER, J A, DAVIES, J, and HERBERT, R. 1989. A Pilot Study into Optimum Well Design: IDA 4000 Deep Tubewell II Project. Volume 4 Part I: A multi-layered model for long-term pumping tests. *British Geological Survey Technical Report,* WD/89/11.

BARKER, J A. 1989. Programs to simulate and analyse pumping tests in large-diameter wells. *British Geological Survey Technical Report,* WD/89/24.

HERBERT, R, BARKER, J A, and KITCHING, R. 1992. New approaches to pumping test interpretation for dug wells constructed in shallow crystalline basement terrains. 221-242 *in* The Hydrogeology of Crystalline Basement Aquifers in Africa, WRIGHT, E P and BURGESS, W G (editors). *Special Publication No. 66, The Geological Society, London.*

LOVELL C J. 2000. *Productive Water Points in Dryland Areas.* Intermediate Technology Publications, London.

PAPADOPULOS, I S, and COOPER H H. 1967. Drawdown in a well of large diameter. *Water Resources Research*, Vol. 3, 241-244.

TALBOT, A 1979. The accurate numerical inversion of Laplace transforms. *Journal. Institute of Mathematics and Its Applications.*, Vol. 23, 97-120.

NOTATION

		dimensions
b	thickness of the aquitard	$[L]$
B_1	aquifer-loss parameter (see A(19))	$[L^{-2}T]$
B_2	well-loss parameter (see A(19))	$[L^{-2}T]$
C	well-loss parameter (see A(19))	$[L^{-5}T^2]$
D	fractures per unit depth of aquifer in double-porosity model	$[L^{-1}]$
$F(u_w,\alpha)$	well function for a large-diameter well	$[-]$
K	hydraulic conductivity of the aquitard	$[LT^{-1}]$
$K_0(z)$	modified Bessel function	$[-]$
$K_1(z)$	modified Bessel function	$[-]$
K_m	matrix hydraulic conductivity in double-porosity model	$[LT^{-1}]$
L	aquifer depth in double-porosity model	$[L]$
N	number of fractures in double-porosity model	$[-]$
p	Laplace transform parameter (see (A15))	$[-]$
Q	pumping rate	$[L^3T^{-1}]$
r	radial distance from the centre of the well	$[L]$
r_c	radius of the well casing (see Figure 1)	$[L]$
r_w	radius of the well screen	$[L]$
R^2	sum of squares of residuals	$[L^2]$
$s(r,t)$	drawdown in the aquifer	$[L]$
s_o	initial drawdown in the well	$[L]$
$s_a(r,z,t)$	drawdown in the aquitard	$[L]$
s_f	drawdown in fractures in double-porosity model	$[L]$
s_m	drawdown in matrix in double-porosity model	$[L]$
s^{obs}	drawdown measured during pumping test	$[L]$
$s_w(t)$	drawdown in the well	$[L]$
S	storage coefficient of the aquifer	$[-]$
S_f	fracture storativity in double-porosity model	$[-]$
S_s	specific storage of the aquitard	$[L^{-1}]$
S_{sm}	matric specific storage double-porosity model	$[L^{-1}]$
S_y	specific yield of the aquitard	$[-]$
t	time since the start of pumping	$[T]$
t_p	period of pumping	$[T]$
T	transmissivity of the aquifer	$[L^2T^{-1}]$
T_f	fracture transmissivity in double-porosity model	$[L^2T^{-1}]$

x	one of the aquifer parameters, $x \in \{T, S, K, S_s, b, S_y\}$.	
u_w	$= Sr_w^2 / 4Tt$	[-]
α	$= Sr_w^2 / r_c^2$	[-]
β	$= bS_s / S$	[-]
γ	$= Kr_w^2 / Tb$	[-]
δ	$= S_y / bS_s$	[-]
λ	see (A17)	[-]
μ	$= \sqrt{p\beta / \gamma}$	[-]
ρ	$= r / r_w$	[-]
τ	$= Tt / Sr_w^2$	[-]
f_x	$= \partial f / \partial x$	[-]

APPENDIX A DEFINITION OF THE MODEL

A.1 Flow Equations

Combining Darcy's law with the conservation equation for the aquifer:

$$S\frac{\partial s}{\partial t} = \frac{T}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) - K\left.\frac{\partial s_a}{\partial z}\right|_{z=b} \qquad \text{(A1)}$$

(refer to Figure 1).

Similarly, for the aquitard

$$S_s\frac{\partial s_a}{\partial t} = K\frac{\partial^2 s_a}{\partial z^2} \qquad \text{(A2)}$$

At the boundary between the aquifer and the aquitard the drawdowns will be identical:

$$s(r,t) = s_a(r,b,t) \qquad \text{(A3)}$$

At the water table the downward flow of water must balance the rate of release of water from storage:

$$S_y\frac{\partial s_a}{\partial t} = K\frac{\partial s_a}{\partial z} \qquad z = 0 \qquad \text{(A4)}$$

The change in the amount of water in the well is equal to the difference between the abstraction and inflow from the aquifer:

$$\pi r_c^2\frac{\partial s_w}{\partial t} = Q + 2\pi r_w T\frac{\partial s}{\partial r}(r_w,t) \qquad \text{(A5)}$$

The initial condition is that of zero drawdown throughout the system:

$$s_w(0) = s(r,0) = s_a(r,z,0) = 0 \qquad \text{(A6)}$$

Also the aquifer is assumed to be of infinite horizontal extent so the drawdown tends to zero away from the well:

$$\lim_{r\to\infty} s(r,t) = \lim_{r\to\infty} s_a(r,t) = 0 \qquad \text{(A7)}$$

A.2 Solution of the Equations

The solution of the flow equations can be obtained by taking the Laplace transform with respect to time. Normally, only the drawdown in the aquifer is of concern, and this is best expressed in terms of non-dimensional parameter groups:

$$\frac{4\pi Ts(r,t)}{Q} = F(\tau,\rho,\alpha,\beta,\gamma,\delta) \tag{A8}$$

where

$$\tau = \frac{1}{4u_w} = \frac{Tt}{Sr_w^2} \tag{A9}$$

$$\rho = r/r_w \tag{A10}$$

$$\alpha = \frac{Sr_w^2}{r_c^2} \tag{A11}$$

$$\beta = \frac{bS_s}{S} \tag{A12}$$

$$\gamma = \frac{Kr_w^2}{Tb} \tag{A13}$$

$$\delta = \frac{S_y}{bS_s} \tag{A14}$$

and the Laplace transform of the well function is given by

$$\overline{F}(p,\rho,\alpha,\beta,\gamma,\delta) = \int_0^\infty e^{-p\tau} F(\tau,\rho,\alpha,\beta,\gamma,\delta)\ d\tau \tag{A15}$$

$$= \frac{4K_0(\lambda\rho)}{p[pK_0(\lambda)/\alpha + 2\lambda K_1(\lambda)]} \tag{A16}$$

where

$$\lambda^2 = p + \gamma\mu \left(\frac{\tanh\mu + S\mu}{1 + \delta\mu\tanh\mu}\right) \tag{A17}$$

and

$$\mu^2 = p\beta/\gamma \tag{A18}$$

The drawdown can be evaluated by numerical inversion of the transform given by (A14), see Barker (1985) for details.

A.3 Well Losses

The drawdown in the above equations applies to the aquifer. Due to 'well losses' the drawdown in the pumping well will normally differ from the drawdown predicted by the solution given by (A8). Following the normal convention, it is assumed that the drawdown in the pumping well is of the form:

$$s_w(t) = [B_1(t) + B_2]Q + CQ^2 \tag{A19}$$

The function $B_1(t)$ is related to the original model drawdown through:

$$B_1(t)Q = s(r_w, t) = \frac{Q}{4\pi T} F(\tau, 1, \alpha, \beta, \gamma, \delta) \tag{A20}$$

Thus two further parameters are introduced: B2 and C.

If the majority of the data are long-time, the 'Jacob approximation' tends to apply for drawdown in the pumping well:

$$s(r_w, t) \approx \frac{Q}{4\pi T} \ln\left(\frac{2.25Tt}{Sr_w^2}\right) + B_2 Q + CQ^2 = \frac{Q}{4\pi T} \ln\left(\frac{2.25Tt\ e^{4\pi TB_2}}{Sr_w^2}\right) + CQ^2 \tag{A21}$$

As B_2, r_w and S appear only once in the Q coefficient term, varying B_2 is equivalent to varying r_w or S. So simultaneous fitting of more than one of those three parameters is not advisable.Similarly, if Q does not vary much, the parameter C is difficult to determine accurately.

These difficulties may be overcome if both pumping well and observation well data are used simultaneously in fitting.

A.4 Special Cases

The model reverts to the Papadopulos and Cooper (1967) model (confined aquifer) when:

(i) K = 0, so b, S_s and S_y have no effect; and

(ii) K = ∞, so b and S_s have no effect.

In the latter case the total effective storage is the sum of S and S_y, so these two parameters are not then independent.

The model approximates an ideal slug test when the time of pumping t_p, tends to zero, but the effective pumping rate tends to infinity in such a way that the volume pumped, $Q.t_p$, remains finite. The correct choice of pumping rate is

$$Q = \frac{\pi r_c^2}{t_p} s_0 \tag{A22}$$

where s_0 is the initial drawdown in the well. (A choice of 1 second for t_p should be effective in most cases.)

APPENDIX B APPLICATION OF BGSPT TO A DOUBLE-POROSITY SYSTEM

BGSPT may be used to analyse pumping tests and simulate time-drawdown behaviour for double-porosity systems. The double-porosity aquifer system has equally spaced horizontal fractures separated by matrix blocks (Figure B1) both being homogeneous, isotropic and of infinite extent. The flow equations for the system described by Figure B1 are defined below and then the parameters are transformed to obtain the set of equations detailed in Appendix A.

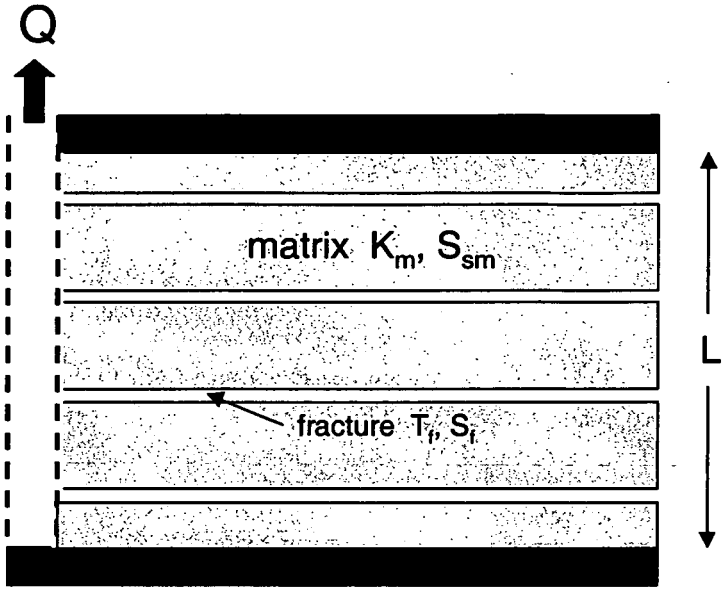

Figure B1 Schematic diagram of double-porosity system approximated by BGSPT.

B.1 Flow Equations

Consider a well tapping N fractures each with transmissivity T_f and storativity S_f.

Flow in the well:

$$\pi r_c^2 \frac{\partial s_w}{\partial t} = Q + 2\pi r_w N T_f \left.\frac{\partial s_f}{\partial r}\right|_{r_w} \qquad \text{(B1)}$$

To convert this to Equation A5, replace NT_f by T (the aquifer transmissivity).

Flow in a single fracture with leakage both from above and below:

$$S_f \frac{\partial s_f}{\partial t} = \frac{T_f}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s_f}{\partial r}\right) - 2K_m \left.\frac{\partial s_m}{\partial z}\right|_{z=0} \qquad \text{(B2)}$$

To convert this to the Equation A1, multiply through by N and then replace NT_f by T (as above); NS_f by S; and $2NK_m$ by K.

Flow within any matrix block (taken to occur in the vertical z direction only):

$$S_m \frac{\partial s_m}{\partial t} = K_m \frac{\partial^2 s_m}{\partial z^2} \tag{B3}$$

To convert this to the Equation A2, multiply through by $2N$ and then replace $2NS_{sm}$ by S_s and $2NK_m$ by K (as above).

If the fracture frequency (number per unit depth) is D. Clearly:

$$D = N/L. \tag{B4}$$

The other parameters needed are the thickness of the aquitard, b, and the specific yield, S_y. If we set $S_y=0$ then this represents the condition (no flow) at the centre of a matrix block in the double-porosity model. Now, the aquitard thickness is half the thickness of a matrix block and if the fractures are taken to have negligible apertures then:

$$2b=L/N. \tag{B5}$$

The relationship between the double-porosity model and the main model is summarised in Table B1.

Table B1 Summary of the relationship between the double-porosity model and the BGSPT model.

Double-porosity model	**BGSPT**	**Relation (DP to BGSPT)**	**Relation (BGSPT to DP)**
T_f	T	$T=NT_f$	$T_f=T/N$
S_f	S	$S=NS_f$	$S_f=S/N$
K_m	K	$K=2NK_m$	$K_m=K/(2N)$
S_{sm}	S_s	$S_s=2NS_{sm}$	$S_{sm}=S_s/(2N)$
No flow across block centre	S_y	$Sy=0$	*'Sy=0 in DP model'*
(L and N) or D (=N/L)	b	$b=L/(2N)=1/(2D)$	$N=L/(2b)$ *or* $D=1/(2b)$

Given a set of parameters for the double-porosity model the relations in column 3 of Table B1 give the parameters to be used in PTFIT and, especially, PTSIM. Given a set of parameters from PTSIM and, especially, PTFIT, the double-porosity model parameters can be calculated as in column 4 of Table B1.

APPENDIX C DERIVATIVES WITH RESPECT TO AQUIFER PARAMETERS

In order to apply the Marquardt algorithm (Section 4.2.3) for fitting the model to the observed drawdowns, the derivatives of the drawdown with respect to the aquifer parameters are required. In this appendix the Laplace transforms of these derivatives are given so that they can be evaluated, along with the drawdown, by numerical inversion of the transform.

The problem is to find the Laplace transform:

$$\bar{\sigma}_x \equiv \frac{\partial \sigma}{\partial x} \qquad x \in \{T, S, K, S_s S_y, b\} \tag{C1}$$

such that the derivatives of the drawdown, s_x, is given by the inverse Laplace transform of $\bar{\sigma}_x$:

$$\frac{4\pi T}{Q_w} s_x = \frac{1}{2\pi i} \int_c e^{p\tau} \bar{\sigma}_x \, dp \tag{C2}$$

when the drawdown is given by:

$$\frac{4\pi T}{Q_w} s = \frac{1}{2\pi i} \int_c e^{p\tau} \bar{F} \quad dp \tag{C3}$$

and the Laplace transform $\bar{F}$ is given by (A16).

The result $(\bar{\sigma}_x)$ is obtained by successive use of the chain rule for differentiation, which produces the sequence of results given below:

$$\bar{\sigma}_x = (p\tau_x - T_x / T)\bar{F} + \bar{F}_x \tag{C4}$$

$$\bar{F}_x = \bar{F}_\lambda \lambda_x + \bar{F}_\alpha \alpha_x \tag{C5}$$

$$\bar{F}_\lambda = \frac{4}{p\phi}\left[-\rho K_1(\lambda\rho) + \frac{K_0(\lambda\rho)}{\phi}\left(pK_1(\lambda)/\alpha + 2\lambda K_0(\lambda)\right)\right] \tag{C6}$$

$$\phi = pK_0(\lambda)/\alpha + 2\lambda K_1(\lambda) \tag{C7}$$

$$\bar{F}_\alpha = -\frac{4K_0(\lambda\rho)K_0(\lambda)}{(\alpha\phi)^2} \tag{C8}$$

$$\lambda_x = \frac{\lambda_x^2}{2\lambda} \tag{C9}$$

$$\lambda_x^2 = \lambda_\gamma^2\gamma_x + \lambda_\mu^2\mu_x + \lambda_\delta^2\delta_x \tag{C10}$$

$$\lambda_\gamma^2 = \mu\psi/\chi \tag{C11}$$

$$\psi = \tanh\mu + \delta\mu \tag{C12}$$

$$\chi = \delta\mu \quad \tanh\mu + 1 \tag{C13}$$

$$\lambda_\mu^2 = \frac{\gamma\psi\mu}{\chi}\left(\frac{1}{\mu} + \frac{\psi_\mu}{\psi} - \frac{\chi_\mu}{\chi}\right) \tag{C14}$$

$$\psi_\mu = \operatorname{sech}^2\mu + \delta \tag{C15}$$

$$\chi_\mu = \delta(\tanh\mu + \mu\operatorname{sech}^2\mu) \tag{C16}$$

$$\lambda_\delta^2 = \gamma\mu^2(\chi - \psi\tanh\mu)/\chi^2 \tag{C17}$$

$$\mu_x = \frac{\mu_x^2}{2\mu} \tag{C18}$$

$$\mu_x^2 = \frac{p\beta}{\gamma}\left(\frac{\beta_x}{\beta} - \frac{\gamma_x}{\gamma}\right) \tag{C19}$$

The computational procedure can be outlined as follows:

1. For a given set of aquifer parameters and transform variable p, evaluate the parameters and functions given in (A9)-(A18).

2. Evaluate the derivatives given in Table C1.

3. Evaluate the quantities given in (C4)-(C19), but in such an order that required values are always available.

The major computational effort is in the evaluation of the Bessel functions. Since three out of four of these have to be evaluated to obtain the drawdown function, the additional effort required to obtain the six derivatives only (approximately) doubles the computation time.

Table C1 Partial derivatives of fundamental parameters with respect to aquifer parameters.

	χ					
	T	**S**	**K**	**S_s**	**S_y**	**b**
$\tau_x =$	τ/T	τ/S	0	0	0	0
$T_x =$	1	0	0	0	0	0
$\alpha_x =$	0	α/S	0	0	0	0
$\beta_x =$	0	$-\beta$	0	β/S_s	0	β/b
$\gamma_x =$	$-\gamma/T$	0	γ	0	0	$-\gamma/b$
$\delta_x =$	0	0	0	$-\delta/S_s$	$-\delta/S_y$	$-\delta/b$