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# Teaching Ultrasonics Using Spreadsheets

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## Abstract

*Whenever an ultrasonic wave encounters a boundary between two media it is partially reflected and refracted, as any acoustic wave would be. Unlike light, the wave also undergoes mode conversion so that in the general case a single incident wave could produce two reflected waves and two refracted waves. The angles which define the path of the wave are determined by Snell's law and are easily calculated. The relative amplitudes, on the other hand, require quite complicated formula when the angle of incidence is anything other than 0 degrees. This problem gets compounded when the angle of the incident wave goes beyond the first critical angle. At this point the angle of the refracted wave becomes imaginary and the equations to calculate the relative amplitudes become complex. This paper describes a tool that has been developed, using a spreadsheet, which performs the calculations for all incident angles. The user selects the media and the type of incident wave and the resulting waves are shown graphically as well as numerically. The tool was developed primarily as part of an undergraduate course on ultrasonic testing, but could be used more widely.*

**Keywords:** *Ultrasonics, spreadsheets, complex angles.*

## 1. Introduction

Any course on ultrasonics or ultrasonic testing has to include the behaviour of waves at boundaries. Most courses or textbooks on ultrasonics (or acoustics in general) [1,2] will show waves at boundaries being reflected, refracted and mode converted. In the general case, shown in Fig. 1, the incident wave could be compressive (longitudinal) or shear (transverse). At the boundary, mode conversion takes place which means that there is a reflected compression and shear wave, and a refracted compressive and shear wave.

The angles of the reflected and refracted waves are calculated using Snell's law. So:

$$\frac{c_{1c}}{\sin \theta_{ci}} = \frac{c_{1s}}{\sin \theta_{si}} = \frac{c_{1c}}{\sin \theta_{cr}} = \frac{c_{1s}}{\sin \theta_{sr}} = \frac{c_{2c}}{\sin \theta_{ct}} = \frac{c_{2s}}{\sin \theta_{st}} \quad (1)$$

where  $c_{pq}$  is the velocity of the waveform in media  $p$  and type of wave  $q$  ('s' for shear or 'c' for compression) and  $\theta_{qr}$  is the angle from the vertical for wave of type  $q$  and  $r$  (where it can be 'i' for incident, 'r' for reflected and 't' for transmitted).

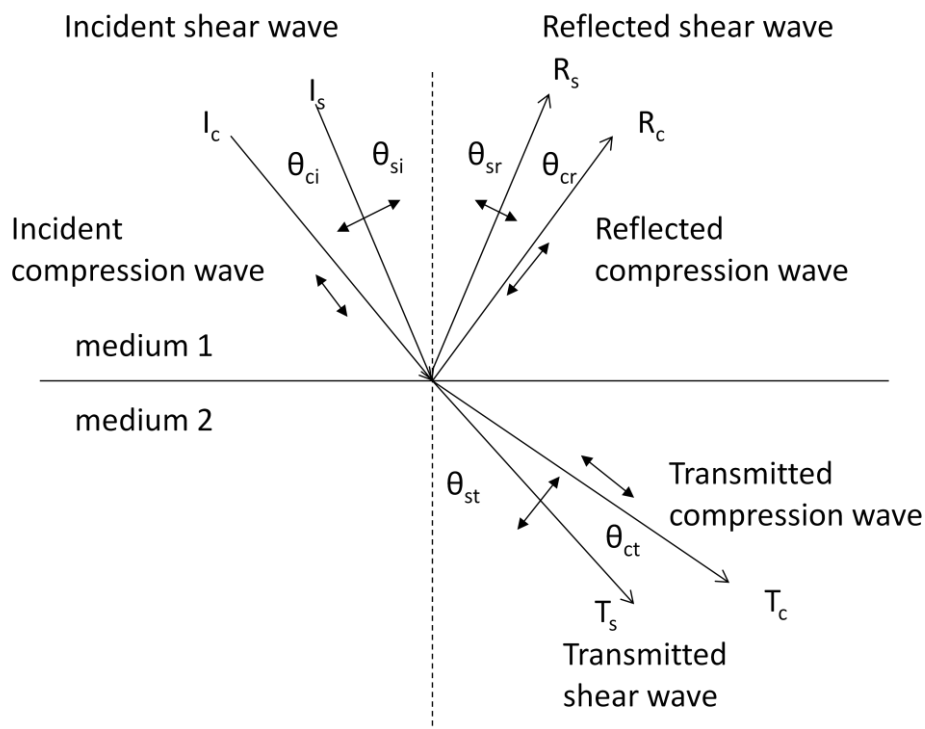


Figure 1. The general case of an ultrasonic wave at a boundary.

The next step is to find the amplitude of the reflected and refracted waves. Most books give the equations for the relative sizes of the waves only when the incident wave is at zero degrees i.e. orthogonal to the boundary. In some of the more advanced books, equations are given for the relative energy or amplitude up to the first critical angle, but not beyond. The reason for this is that the equations become complex, i.e. have real and imaginary parts [3], after the first critical angle, and for the beginner in ultrasonics this is too much to handle.

Previous work [4,5] has shown how these equations can be calculated using Excel spreadsheets. These spreadsheets were chosen because most students have access to Microsoft Office, and therefore there is no need to purchase or learn to use bespoke software. The previous papers showed how a spreadsheet can be used to produce plots of the relative amplitude/energy with respect to the incident angle, or in a different spreadsheet how matrices can be used to calculate the relative amplitude/energy for a specific angle. In this paper the same spreadsheet will be used to solve the underlying equations. Feedback from students was received about the original spreadsheets, where the output showed a graph of the amplitudes/energies plotted against incident angle. The students were expected to read from these graphs to get solutions to specific incident angles. They also said that they found it a little difficult to picture the problem. A new graphical output was developed to provide a simple visual interpretation of the problem, and give the solution to a specific incident angle.

## 1.1. Matrix Solution

The relative amplitude can be calculated using matrix notation for which only one equation is used.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} R_c \\ T_c \\ R_s \\ T_s \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \text{ or } \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \quad (2)$$

The  $[B]$  matrix is used if the incident wave is compression, whereas the  $[C]$  matrix is used if the incident wave is shear. The elements of the matrices are shown below.

$$[A] = \begin{bmatrix} -\cos \theta_{c1} & -\cos \theta_{c2} & -\sin \theta_{s1} & \sin \theta_{s2} \\ -\sin \theta_{c1} & \sin \theta_{c2} & \cos \theta_{s1} & \cos \theta_{s2} \\ -Z_{c1} \cos 2\theta_{s1} & Z_{d2} \cos 2\theta_{s2} & -Z_{s1} \sin 2\theta_{s1} & -Z_{s2} \sin 2\theta_{s2} \\ -Z_{s1} \frac{c_{s1}}{c_{c1}} \sin 2\theta_{c1} & -Z_{s2} \frac{c_{s2}}{c_{c2}} \sin 2\theta_{c2} & Z_{s1} \cos 2\theta_{s1} & -Z_{s2} \cos 2\theta_{s2} \end{bmatrix} \quad (3)$$

$$[B] = \begin{bmatrix} -\cos \theta_{ci} \\ \sin \theta_{ci} \\ Z_{c1} \cos 2\theta_{si} \\ -Z_{s1} \frac{c_{s1}}{c_{c1}} \sin 2\theta_{ci} \end{bmatrix} \quad (4)$$

$$[C] = \begin{bmatrix} \sin \theta_{si} \\ \cos \theta_{si} \\ -Z_{s1} \sin 2\theta_{si} \\ -Z_{s1} \cos 2\theta_{si} \end{bmatrix} \quad (5)$$

To find the values of the reflected and transmitted waves, the  $[A]$  matrix needs to be inverted.

$$[A] \begin{bmatrix} R_c \\ T_c \\ R_s \\ T_s \end{bmatrix} = [B] \text{ or } [C] \quad (6)$$

$$\begin{bmatrix} R_c \\ T_c \\ R_s \\ T_s \end{bmatrix} = [A]^{-1}[B] \text{ or } [A]^{-1}[C] \quad (7)$$

The inversion is carried out on Excel as described in a previous publication [5].

## 2. Graphical Representation

### 2.1. Relative Energy

Although the spreadsheet calculates the relative amplitude and energy of all the waves at a boundary, it is much easier to interpret the situation using graphics. The choice of

graphics in Excel is quite wide, such as line graphs, bar charts, pie charts, and scatter diagrams. To illustrate the relative energy it was worth noting that as no energy can be created or destroyed, the energy of the incident wave must equal the sum of the energy of the reflected and refracted waves. As we have calculated relative energies, that is the energy of the reflected and refracted waves divided by the energy of the incident wave, the sum of the relative energies should be 1 or 100%. Therefore, a pie chart which shows how the 100% is divided among the reflected and refracted waves is a good choice. Figure 2 shows a typical result.

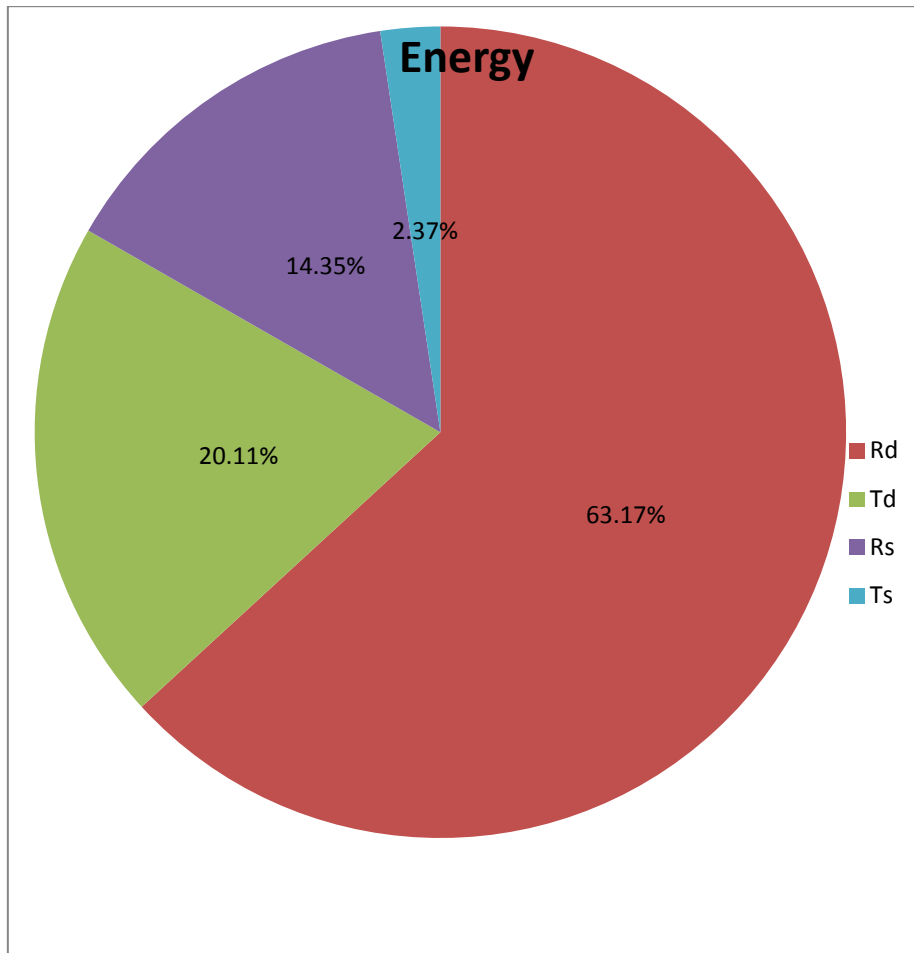


Figure 2. Relative energy shown as a pie chart

The pie chart shown in Figure 2 shows the relative energies of the reflected and refracted waves as percentages and as coloured segments. In this example the incident wave is a compression wave at an angle of 20 degrees at a boundary between Perspex and Inconel 625, a steel alloy. It shows the relative energy of the reflected compression wave, Rd in red, the reflected shear wave, Rs in purple, the transmitted compression wave, Td in green, and the transmitted shear wave, Ts in light blue.

## 2.2. Relative Amplitude

Since the angles of the various waves can be easily calculated, and in fact have to be calculated as part of the bigger calculation of the relative amplitude, it makes sense to use them in the graphical display of the relative amplitude. The decision was made to show the waves as a vector, with the angle of the vector corresponding wave, and the length of the vector corresponding to the relative amplitude. First, a table had to be constructed with the coordinate pairs for the beginning and end of each vector. Then the plot could be created, as in Figure 3.

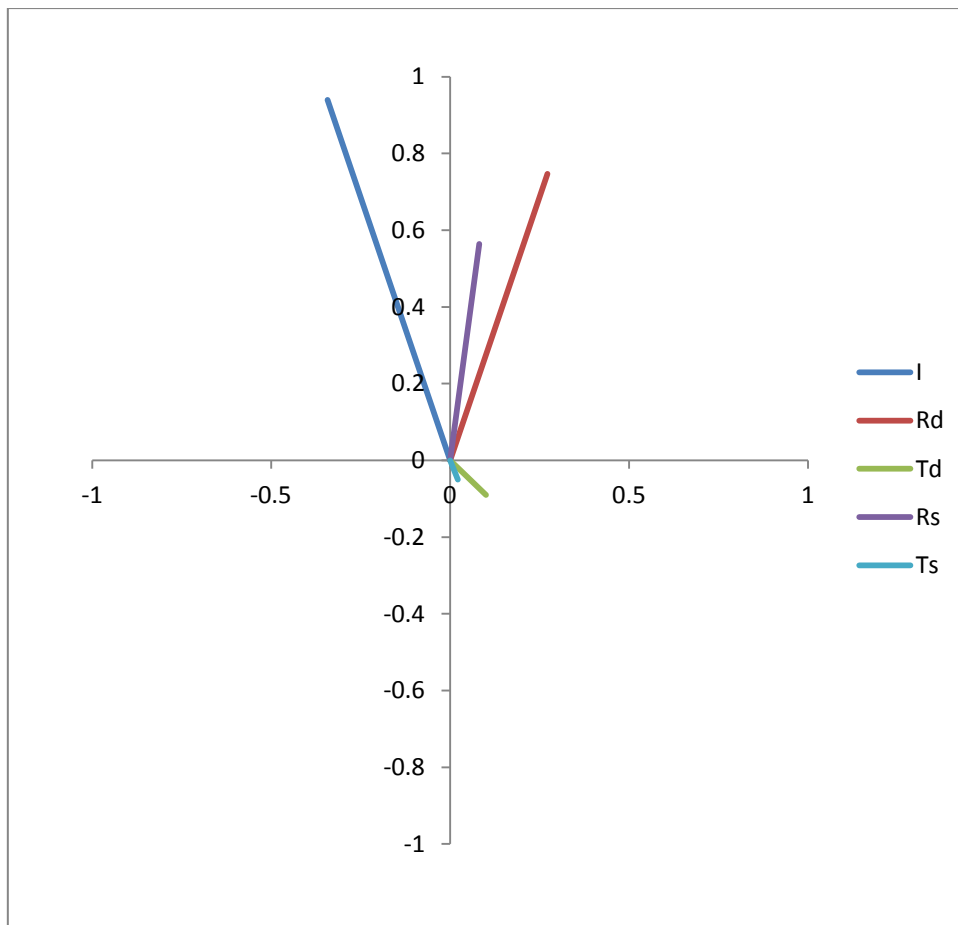


Figure 3. Relative amplitudes shown as a linear vector plot

The plot in Figure 3 corresponds to the same set up as in Figure 2. The incident wave is shown in the top-left quadrant, while the reflected waves are in the top-right quadrant, and the transmitted or refracted waves are in the bottom right quadrant.

### 3. Additional Features

As well as the graphical display, some additional features were added to the original spreadsheets to make it easier to use. The first was a drop-down menu to select the type of incident wave, compression or shear. The second was a slider input for the angle of the incident wave. This means that the incident angle can be selected using the slider in steps of 1 degree, or a specific angle can be entered directly into the box. These features are shown in Figure 4.

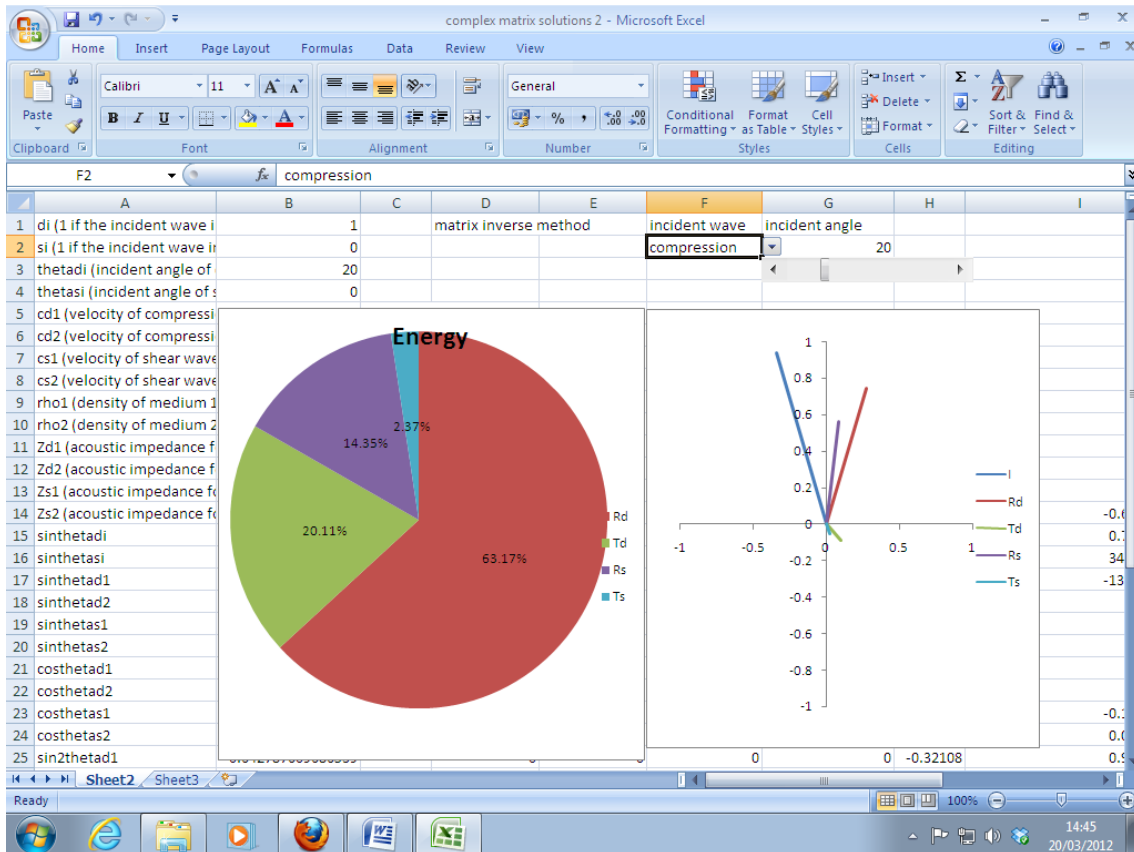


Figure 4 Whole spreadsheet showing additional features

### 5. Conclusion

This paper has developed a spreadsheet application in which all the ultrasonic waves at a boundary can be calculated, both the relative energy and amplitude. However, for teaching purposes, and indeed applications to real-world situations, a visual display of the results is more effective. The relative energy can be displayed as a pie chart which then easily shows the relative amount of energy in each of the reflected and transmitted waves. In addition, the relative amplitude and angle is displayed as a vector diagram, which incorporates the angle of reflection/refraction with the relative amplitude. So far, students have given favourable comments about the new spreadsheet and have found it useful both in learning about ultrasonics and in real-world applications.

## References

- [1] D. Ensinger, *Ultrasonics: Fundamentals , Technology, Applications*, 2<sup>nd</sup> edition, CRC,1988.
- [2] J. Krautkramer and H. Krautkramer , *Ultrasonic Testing of Materials*, Springer Verlag, 1969.
- [3] W. G. Meyer, “Energy Partition of Ultrasonic Waves at Flat Boundaries”, *Ultrasonics*, April-June, 1965, pp. 62-68.
- [4] P. D. Picton and S. Kaczmarczyk, “A software tool for calculating the relative amplitude of ultrasonic waves at boundaries”, Proc. 18<sup>th</sup> International Congress on Sound and Vibration, 10-14 July 2011, Rio De Janeiro, Brazil, CD-ROM, ISBN 978-85-63243-01-0, Ricardo E Musafir (Ed).
- [5] P. D. Picton and S. Kaczmarczyk, “Calculating the Relative Amplitude of Ultrasonic Waves at Boundaries Using a Spreadsheet”, submitted to the *International Journal of Sound and Vibration*, 2012