GENERALISED HYBRID FUZZY MULTI CRITERIA DECISION MAKING BASED ON INTUITIVE MULTIPLE CENTROID DEFUZZIFICATION

by

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Abstract

The concept of fuzzy multi criteria decision making process has received significant attention from research community due to its successful applications for human based decision making problems under fuzzy environment. It complements the decision makers to evaluate their subjective judgements under situations that are vague, imprecise, random and uncertain in nature. Inspired by such real applications, in this research study, the theoretical foundation of a hybrid fuzzy multi criteria decision making model based on new centroid defuzzification method is proposed. The proposed model tackles some issues that may be associated with the selection problems of the multi criteria decision making such as deriving decision criteria important weights, ranking various alternatives, suitable combination of fuzzy multi criteria decision making techniques and proper defuzzification method used. In developing the hybrid model, two multi criteria decision making techniques are integrated which are; 1) consistent fuzzy preference relations and; 2) fuzzy technique for order of preference by similarity to ideal solution. It is also incorporated together with new defuzzification method namely intuitive multiple centroid.

In the view of evidence outlined in this study, the proposed model serves as a generic multi criteria decision making procedure, particularly when fuzzy sets are involved in the decision process. The two major contributions from this study are that:

1) The intuitive multiple centroid defuzzification capable to cater all possible representations of fuzzy sets reasonably and consistent with human intuition or judgment.

2) The generalised hybrid fuzzy multiple decision making model using intuitive multiple centroid gives better computation to evaluate criteria and alternatives in decision making problems under different uncertain environment.

Furthermore, an empirical validation of the proposed model is investigated through conducting a case study of staff recruitment in MESSRS SAPRUDIN, IDRIS & CO, Malaysia. In this case study, a group of three decision makers, and four finalist of candidates are selected to take part of this case study. Their involvement achieved the first objective of the case study. At the end of the case study, a sensitivity analysis is conducted to indicate the robustness and the consistency of the results obtained. It is concluded that the proposed model is indeed beneficial under different environment.

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Author’s Declaration

Whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not submitted for any other academic award.

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List of Symbols

$=$	Equal to
$\neq$	Not equal to
$>$	Greater than
$<$	Less than
$\geq$	Greater than or equal to
$\leq$	Less than or equal to
$+$	Addition
$-$	Subtraction/ negative
$\times$	Scala multiplication
$/$	Division
max	Maximum
min	Minimum
$\in$	Element of
Δ_p	Changes
w_p	Weight of criteria
w'_p	Weight changes of criteria
$-\infty$	Negative infinity
∞	Infinity
φ	Varphi
$\sum$	Summation
$\int$	Integral
$\sqrt{\ }$	Square root
log	Logarithm
$\Re$	Real number
Fuz	Fuzzification
Def	Defuzzification
τ	Domain of fuzzy sets
IE	Inference engine
T	T-norm
T^*	T-conorm
$\tilde{x}$	*x* tilde
A_i	Fuzzy number
$\mu_{A_i}(x)$	Membership function of fuzzy number, A_i
h	Degree of membership function
$\mu_{A_i \cup A_j}(x)$	Fuzzy union of A_i and A_j

$\mu_{A_i \cap A_j}(x)$	Fuzzy intersection of A_i and A_j
$\mu_{\bar{A}_i}(x)$	Fuzzy complement of A_i
$Z_{\tilde{A},\tilde{R}}$	Z-number of A_i
$E_{\tilde{Z}'}$	Fuzzy expectation
$(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$	Centroid of coordinate point for fuzzy number $\tilde{A}$
$\tilde{x}_{\tilde{A}}$	Centroid of horizontal x – axis fuzzy number $\tilde{A}$
$\tilde{y}_{\tilde{A}}$	Centroid of vertical y – axis fuzzy number $\tilde{A}$
$\alpha_{\tilde{A}}$	Sub centroid of left triangle shape
$\beta_{\tilde{A}}$	Sub centroid of rectangle shape
$\gamma_{\tilde{A}}$	Sub centroid of right triangle shape
$IMC(\tilde{A})$	Intuitive multiple centroid for fuzzy number $\tilde{A}$
$R(\tilde{A})$	Centroid index of intuitive multiple centroid for fuzzy number $\tilde{A}$
r_i	Aggregated process
w_i	Weighted process
v_i	Normalised process
r_{ij}	Preference ratio
p_{ij}	Proposition
$f:[-c, 1+c]$	Transformation function
A^+	Fuzzy positive ideal solution
A^-	Fuzzy negative ideal solution
$\bar{d}_i^+$	Distance from fuzzy positive ideal solution
$\tilde{d}_i^-$	Distance from fuzzy negative ideal solution
CC_i	Closeness coefficient
f_{ij}	Fuzzy performance decision matrix
f_j^*	Fuzzy best value
f_j^-	Fuzzy worst value

List of Acronyms

Analytic Heirarchy Model (AHP), 25
Analytic Network Process (ANP), 25
Best Non-Fuzzy Performance (BNP), 53
Centre of area (COA), 18
Centre of gravity (COG), 18
Decision Making Trial and Evaluation Laboratory (DEMATEL), 25
ELimination and Choice Expressing Reality (ELECTRE), 25
Emotional steadiness (ES), 122
Footprint of uncertainty (FOU), 29
Geographic information systems (GISs), 17
Mean of maxima (MOM), 18
Mean of maximum (MoM), 16
Midpoint of area (LOM), 18
Multi criteria decision making (MCDM), 3
Multidisciplinary optimization compromise solution (VIKOR), 25
Oration (O), 122,
Past experience (PE), 122
Personality (P), 122
Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETEE), 25
Self-confidence (S-C), 122, 124
Simple Additive Weighting Method (SAW), 25
Technique for order of preference by similarity to ideal solution (TOPSIS), 4

CHAPTER 1

INTRODUCTION

1.1 Overview

This chapter provides a prologue to the research work presented in this thesis. It describes the research background and explains the problems for pursuing this work. Likewise, it briefly summarises chapters review and illustrates the structure of the thesis. Details on those points above in chapters review are broadly discussed in following chapters.

1.2 Research Background

In real world phenomena, much of the decision making take place in an environment which the goals, the constraints and the consequences of possible actions are not known precisely. Most of the decisions are taken from an intuitionistic perspective or only with some very basic information. The reality is, the information is often not so easy to handle and it is necessary to analyse in more detail. Currently, contemporary science is presented in decision making environment as handling and solving current decision making problems that are significant and essential. It proposes the development and application of computerised simulations or mathematical models to solve numerous decision making problems appropriately. Due to growth in computational capability and technology development, data are being generated for understanding in detail of real world problems, especially in human based decision making problems. The availability of subjective data has become the essential challenge for any new mathematical approach to modelling.

While much of the literature (Mardani, Jusoh, & Zavadskas, 2015) in decision making has focused on the applications of established mathematical models to solve decision making problems. The application based approach of decision making is undoubtedly pointed out as much easier than developing a mathematical model. This is because the previous studies involve only the use of an appropriate established mathematical model while the latter studies require new mathematical model development to handle the problem. Despite the fact that the development of a novel mathematical model is challenging, it suggests improved quality in terms of describing and observing the situation than applying established models. In developing the novel mathematical model, it is best if the model follows some basic principles, then the experiment is only used to validate or verify that either the model or formulation is compatible with the structure of the phenomenon studied. Precisely, the mathematical formulations or models should be simple in presenting and easy to compute.

Due to concerns expressed about decision environment nowadays, the involvement of human perception in mathematical based decision models is pointed out as one of the most important factors in many research areas such as computer science, engineering, artificial intelligent, economy, psychology, philosophy, even linguistic and so forth. Literally, human perception refers to the process of perceiving something with human senses. Human perception is defined as a generic way of human expressions towards a situation perceived using subjective judgements and preferences. For that reason, the development of an effective mathematical model for decision making problems is expected to have the capability in representing linguistic terms appropriately because the human perception is commonly associated with natural languages. Also, the model is expected to produce correct decision results such that the results obtained are consistent with the human intuition or human judgement. Nevertheless, both expectations are hard to achieve using a mathematical forms because it is impractical and unrealistic for the human nature.

For this reason, linguistic terms are used to solve the problem using mathematical model is not idealistic, but it still represents rapid improvement regarding human knowledge. This indicates when the fuzzy set theory was introduced as the medium of representation of human perception. Fuzzy set theory is a mathematical field that is capable of dealing effectively with situations that are vague, imprecise and ambiguous in nature like human decision making. It provides proper representation for the mathematical model in signifying human perception appropriately. Since the application of the fuzzy set theory in human decision making is relevant and suitable, this study aims at developing a fuzzy based mathematical decision methodology that is capable of representing linguistic terms and producing decision results that are consistent with the human intuition. The model is also expected to serve as a generic decision making model for any human based decision making problem.

Multi criteria decision making (MCDM) was introduced as a promising field of study in early 1970’s. It represents a prominent class of such decision making problems in operations research. The typical MCDM problem deals with the evaluation of a set of alternatives regarding a set of decision criteria. The challenge facing practitioners in some of the methodological problems in MCDM models is how to deal with human based decision making problems under fuzzy environment. In this sense, much of the literature studies the implementation of fuzzy set theory in

MCDM models in handling the fuzzy event. When fuzzy set theory was introduced into MCDM research, the associated methods were basically developed along the same line where decision making in practice presented that fuzzy set theory allowed decision making with estimated values in spite of incomplete information. Under such circumstances, the fuzzy systems often outperformed the classical MCDM methods. Hence, fuzzy set theory has powerful features to be incorporated into optimization techniques as MCDM.

In this research study, the problems considered are: 1) the development of generalised hybrid fuzzy multi criteria decision making model based on intuitive multiple centroid defuzzification and 2) an evaluation process for the staff selection in a law firm in Malaysia case study. The proposed fuzzy decision making model is developed generally in order to solve under different fuzzy environment. It capable to deal with imprecision, vagueness and uncertain problems in human based decision making assessments. Since the research problem is considered as an evaluation process of decision making problem, this process involves a group of decision makers who have expertise and knowledge in law field of study to select the best candidate. The group of decision makers is comprised of different decision makers with different levels of expertise and different perceptions. Several criteria are considered that may affect the selection of potential candidate from a group of candidate. In dealing with human perception, fuzzy linguistic scales are used as the medium of representation of human perception or judgement. The proposed fuzzy decision making is applied in this case study in order to solve the evaluation process.

Sensitivity analysis is utilised as validation method in order to evaluate how robust the optimal solution produced when different circumstances are considered with making changes in parameters. Thus, in this research study, sensitivity analysis acts as an instrument for the assessment of the input parameters to apply the model efficiently and to enable a focused planning of future research and field evaluation.

1.3 Chapters Review

This section illustrates some reviews in terms of organisation of the thesis. Fig. 1.1 illustrates the thesis structure. Seven chapters are presented in the thesis. The remaining six are described as follows.

Chapter 2 presents the concept or idea of research interests whereby it describes in detail the history and chronology of research study. It identifies the research problems, research questions and objectives, also research contributions. It illustrates the gaps and limitations of established works done by previous researchers in human decision making based problems. At the end, the chapter previews a summary whether the literature is reviewed and identified a reasonable direction for the thesis.

Chapter 3 outlines the theoretical preliminaries of the thesis such that the definitions and formulations used in this study are given. It describes details of the relevant theories, definitions and methods to be used for data analysis including figures, tables, diagrams and procedures.

Chapter 4 describes in detail the process of the development of intuitive multiple centroid defuzzification method for fuzzy sets that includes official models, elementary operations, basic properties and advanced applications. It considers three types of fuzzy sets which are type-1, type-2 and the z-number. The extensions of the proposed centroid method are discussed in the next two sections for type-2 fuzzy sets and z-numbers. The validation process for the proposed intuitive multiple centroid are discussed theoretically and empirically.

For Chapter 5, it illustrates in detail the process of the development of a hybrid MCDM model that consist of consistent fuzzy preference relations and fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) that incorporated with intuitive multiple centroid method as discussed in Chapter 4. It discusses how generically the proposed hybrid fuzzy MCDM model can be implemented in type-2 fuzzy set and the z-number. Those hybrid fuzzy MCDM model are validated using sensitivity analysis that is discussed in Section 6.6.2.

Chapter 6 describes an application of the proposed methodology for hybrid fuzzy MCDM model that incorporates with intuitive multiple centroid. It demonstrates how the proposed methodology can be used in the evaluation process for the selection of a right employee for one of law firms in Malaysia. This chapter presents a comparative study between the proposed methodology for with hybrid fuzzy MCDM model with intuitive multiple centroid against established models by previous studies.

The final chapter summarises the whole thesis whereby it illustrates the contributions of the research work, the concluding remarks and recommendations for future work. It presents a summary of all the works contributing to acknowledge in every chapter of the thesis.

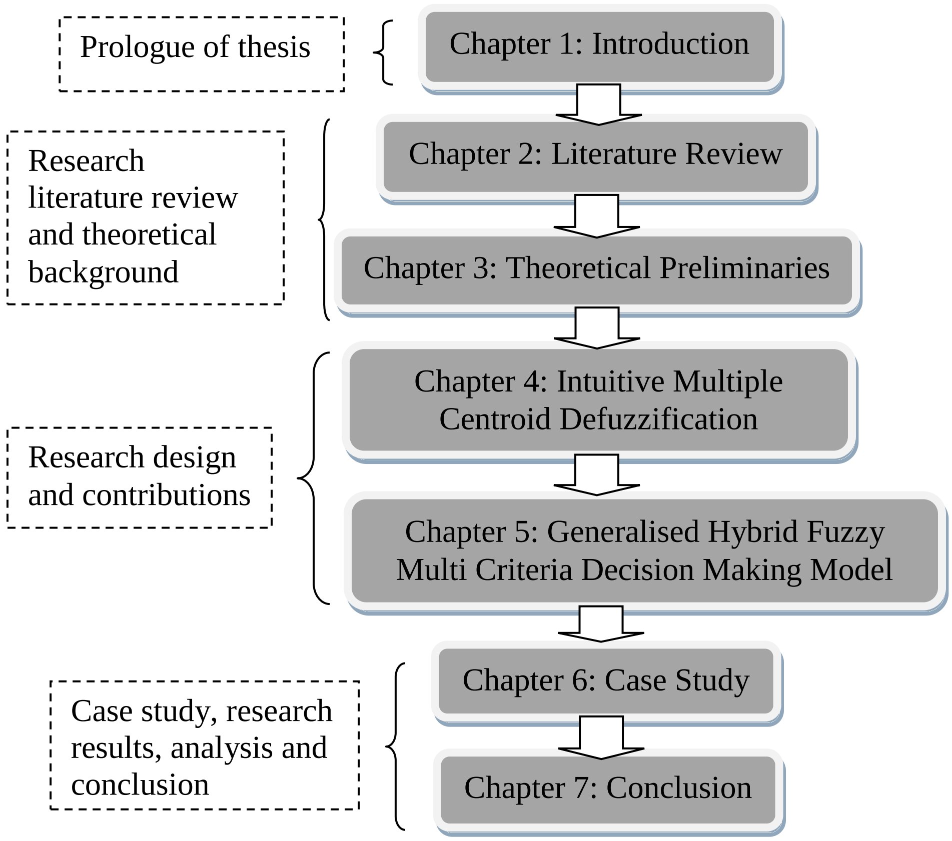


Fig. 1. 1: Thesis structure

1.4 Summary of the Chapter

The introductory chapter briefly discusses the review of the whole thesis. This is later followed by Chapter 2 for literature review discussion. It also illustrates the research problems, research questions, research objectives and research contributions of the thesis.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter discusses the background of research study by reviewing the literature and prior researches that are related to the research questions and objectives. The first part of this chapter corresponds to review on basic concept of fuzzy sets which justify the applicability of fuzzy sets in human decision making. Then, the chronological development of fuzzy set tools are highlighted where provide a comprehensive overview on type-1 fuzzy sets and its extensions known as type-2 fuzzy sets and z-numbers are enclosed. This chapter mainly focuses to address established works found in previous defuzzification methods for type-1 fuzzy sets, type-2 fuzzy sets and z-numbers within human based decision making problems in real world case studies. A brief review of the history and chronology of defuzzification methods to understand the development of defuzzification process for fuzzy sets. The following main focus here is to discuss some MCDM techniques which are applied by previous researchers in many field of disciplines under fuzzy environment. The theoretical foundations and applicability in particular decision making problems of established models are thoroughly discussed in this chapter.

2.2 Basic Concepts on Fuzzy Sets

In the modern age, the American philosopher Charles Pierce pointed out that "Logicians have too much neglect the study of vagueness, not suspecting the important part it plays in mathematical thought" (Peirce, 1931). Some discussions have been made to study the links between logics and vagueness are not unusual in the philosophical literature in the first half of the century (Copilowish, 1939). Vagueness is restricted to fuzziness sides that can be accounted for by attaching to any situation a grade of applicability of a given concept to it. A proposition or statement is said to be vague if it contains a gradual predicates. In modelling the gradual features enable some paradoxes of classical logic to be solved. In particular, a certain number of words or sentences refer to supposedly continuous numerical scales instead of discrete. A more realistic point of view, the motivation of fuzzy sets is to provide a formal setting for incomplete and gradual information, as expressed by human in natural language.

This section deliberates the suitability and reliability of fuzzy sets when dealing with human decision making. Natural language is regularly used in human decision making processes as the medium of indications towards a situation perceived. While it is true because subjective perceptions that expressed by human are only appropriate when they are described using linguistic hedges as part of natural language (Yeh, Deng, & Chang, 2000). Some research works have done in applying linguistic scale as natural language which are (Kangari & Riggs, 1989), (Lee-Kwang & Lee, 1999), (R.-C. Wang & Chuu, 2004), (L. Lee & Chen, 2008), (S.-M. Chen & Chen, 2009), (Zamri & Abdullah, 2013), (Azadeh, Saberi, Atashbar, Chang, & Pazhoheshfar, 2013), (Abdullah & Najib, 2014) and (J. Wang, Wang, Zhang, & Chen, 2015). They utilised fuzzy set theory that are brought out as a suitable tool to deal with natural language. Despite the fact that fuzzy set theory underpins three basic concepts namely graduality, epistemic uncertainty and bipolarity factors which are capable to present natural language appropriately (Dubois & Prade, 2012). These three notions of fuzzy sets are interacted closely each other. In order to clarify the significant of fuzzy sets in practice, the description on these three basic concepts that underlying the fuzzy sets are as follows.

2.2.1 Graduality

In Zadeh's perception, the idea that many categories in natural language expresses by human is a matter of degree, including truth (L.A. Zadeh, 1965). Considering that, natural language uses by human on portraying a subject is distinguished by different degree of beliefs. Fuzzy set is an extension of gradual predicate, where the transition between membership and non-membership are, in the words of its inventor, "gradual rather than abrupt" (Dubois & Prade, 2012). For instance in the case of temperature, if the temperature is considered as 'cold' with 47 Fahrenheit, then 45 Fahrenheit is not regarded as 'cold' but classified as 'very cold', where 'very cold' is another natural language is used to describe the coldness. The employment of both 'cold' and 'very cold' in this instance, suggest that there is a transition process occurs in terms of degree of belief used when information about the subject perceived is changed. This is expressed when degree of belief 'cold' increases and degree of belief 'very cold' decreases as values of temperature approaches 45 Fahrenheit. The continuous but alternate pattern transition between the degrees of belief implies that natural languages convey by human are gradual rather than abrupt as mentioned before.

2.2.2 Epistemic Uncertainty

The treatment of uncertainty in the analysis of any computerised or mathematical model is essential for understanding possible ranges of scenario implications. The capability in quantifying the impact of uncertainty in the decision making context is critical. Epistemic uncertainty represents a lack of knowledge about the appropriate value to use for a quantity. Sometimes it is referred to as state of knowledge uncertainty, subjective uncertainty, reducible uncertainty, where it means that the uncertainty can be reduce through increased understanding of research, increased the information and more relevant data are needed (Swiler, Paez, & Mayes, 2009). Fuzzy set may justify for epistemic uncertainty since it extends the notion of a classical set. In addition, it is gradual since belief is often a matter of degree. Epistemic uncertainty in fuzzy sets is viewed as representation of incomplete information about a situation (Dubois & Prade, 2012). Among examples of the human decision making situations involve in this case are forecasting and group decision making (S.-M. Chen & Chen, 2009). The misunderstanding between graduality and uncertainty pervading fuzzy set theory is actually a variant of a misunderstanding pervading some parts of the literature in logic between truth values and belief degrees or information states.

2.2.3 Bipolarity

In human decision making analysis, especially multi-agent decision making analysis, is based on bipolar or double sided judgmental thinking on a positive side or negative side such as effect or side effect and feedforward or feedback (Zhang, 1994). The argument that positive and negative causal relationship should not be buried or eliminated in a summation if they are not counteractive at the same time, or not from same source, or not through same paths (Zhang, Chen, Chen, Zhang, & Bezdek, 1988). This expresses the fact that regardless of the possibility if enough information about a decision is collected, human sometimes relies on their corresponding positive, negative or neutral effects on a circumstance. For instance, there are options under consideration that are separated based on good or bad alternatives and a decision is made accordance to the strongest criterion or attribute produces by one of the alternatives. Comparable to unipolar crisp value, where the real valued bipolar representation suffers from the deficiency that can’t be used to represent high order fuzziness (Zhang, 1994). Besides, bipolarity perspective complements the capability of membership functions in fuzzy sets in representing both causal relations of positive and negative effects appropriately. For simplicity, bipolar fuzzy set theory formalises a unified approach to polarity fuzziness and captures the bipolar or double-sided effect of human perception and cognition.

2.3 Fuzzy Set Theory

As discussed in a previous section, the description on basic concepts that underlying the fuzzy sets are briefly explained in order to clarify the nature of human being. This section illustrates the chronological development of fuzzy set theory. It was specifically designed to mathematically represent uncertainty and vagueness to provide formalised tools for dealing with imprecision intrinsic to many problems in human based decision making. Aforementioned in Section 2.2, fuzzy sets are pointed out as a suitable knowledge for human decision making where this is justified when basic concepts of fuzzy sets capable in representing the natural language very well, but it is not easy to distinguish two or more natural languages are used in a decision making problems as they are all defined qualitatively. Due to this concern, Zadeh (1965) suggested a quantitative definition for fuzzy sets which are well – suited for natural language known as fuzzy numbers. While much of the literature of fuzzy sets discuss there are three kinds of fuzzy sets found namely type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. In this study, these three fuzzy sets are considered in different situations in human based decision making where it is not easy to distinguish natural languages that are very subjectively.

Among those three, type-1 fuzzy sets are the most applied fuzzy sets in research studies followed by type-2 fuzzy sets and z-numbers. Rationally, type-1 fuzzy sets are most usable fuzzy numbers as compared to two others because the chronology of fuzzy sets was started with type-1 fuzzy set in 1965, while type-2 fuzzy sets in 1975 and z-numbers were introduced in 2011. The different types of fuzzy sets are not utilised simultaneously in representing the natural language. This is because all of them have different representations in theoretical nature, that indicate only one fuzzy set is applied at one time. Applying fuzzy sets in human decision making problems is a straightforward process due to the flexibility of using linguistic terms as variables to access the human’s judgements. Therefore, fuzzy sets have attracted the attention of many researchers and practitioners in modelling imprecision, vagueness and uncertainty in their decision making systems. Details on type-1 fuzzy sets, type-2 fuzzy sets and z-numbers are described as follows.

2.3.1 Type-1 Fuzzy Sets

Since its inception in 1965, fuzzy set theory has been applied and advanced in variety ways and in many disciplines. Type-1 fuzzy sets or classical fuzzy sets are the first fuzzy numbers introduced in literature of fuzzy set theory. It has been widely used in many research fields such as artificial intelligent, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operational research, pattern recognition, robotic and so forth. The terms of type-1 fuzzy sets are used in discussion of many established research studies done by (Yager, 1980), (L. Chen & Lu, 2002), (X.-W. Liu & Han, 2005), (Li, 2013) and so on. Originally, the term of type-1 fuzzy sets was changed from fuzzy set or fuzzy number only when type-2 fuzzy sets was introduced in the literature of fuzzy set theory. Even both types of fuzzy sets are fuzzy numbers, but they have difference representation in nature. Type-1 fuzzy set is a uniquely defined by a membership function. It consist of both membership and spread features as a range for information that are corresponding to confidence level and opinion of decision makers respectively (S.-M. Chen & Chen, 2009). Considering that, type-1 fuzzy sets are widely applied in decision making problems such as selection of beneficial project investment (Jiao, Lian, & Qunxian, 2009), proposing fuzzy risk analysis method using generalised fuzzy numbers (S.-M. Chen & Chen, 2009), improving out patient service for elderly patient for Healthcare Failure Mode and Effect Analysis (HFMEA) in Taiwan (Kuo, Wu, & Hsu, 2012), solving image processing and image understanding problems in dealing with imprecise information and knowledge (Bloch, 2015).

2.3.2 Type-2 Fuzzy Sets

Type-2 fuzzy sets notion was introduced by Zadeh as an extension of the type-1 fuzzy sets (L.A. Zadeh, 1975). According to (Karnik & Mendel, 2001a), type-2 fuzzy sets can be considered as fuzzy membership function where the membership value for each element in type-2 fuzzy set is a fuzzy set in interval range of [0,1], different with type-1 fuzzy set where the membership value is in crisp condition between [0,1]. Due to this concerned, the uncertainty representation of type-1 fuzzy set on natural language is insufficient enough to model perception (Dereli, Durmusoglu, & Daim, 2011). Plus, the imprecision level about a situation increases when number is translated into word which means natural language and finally to perceptions. There would be some uncertainty in associating the perception in the information. This indicates that the representation capability of type-1 fuzzy set on uncertainty is arguable. The participation of higher level of uncertainty in type-2 fuzzy sets compared to type-1 fuzzy sets, they are provided additional degree of freedom to represent the uncertainty in human based decision making problems. Uncertainty can be divided into two types which are inter and intra personal uncertainties, in provisioning the representation of type-1 fuzzy sets in the literature of fuzzy sets (Wallsten & Budescu, 1995). Due to this, type-2 fuzzy sets are utilised in many decision making research studies such as modelling data uncertainty on electric load forecasting using type-2 fuzzy set theory (Lou & Dong, 2012), proposing a new type-2 fuzzy set of linguistic variable for fuzzy analytic hierarchy process for work safety evaluation (Abdullah & Najib, 2014), evaluation of criteria and dimensions of human resource management problems (Abdullah & Zulkifli, 2015) and developing MCDM method for robot selection with interval type-2 fuzzy sets (Keshavarz Ghorabaee, 2016).

2.3.3 Z-Numbers

The concept of z-number was suggested by Zadeh in (Lotfi A. Zadeh, 2011a). Compared to type-1 and type-2 fuzzy sets, z-numbers present the latest version of fuzzy sets in the literature. The idea of z-numbers is intended to provide a basis for computation with numbers which are not totally reliable. Z-numbers can be represented as an extension of type-1 fuzzy sets in term of membership function, but completely differ from type-2 fuzzy sets. It concerns the reliability component in fuzzy numbers to make sure the information are in reliable state. Z-numbers enhances the capability of both type-1 and type-2 fuzzy sets by taking into account the reliability component of fuzzy numbers are used (Lotfi A. Zadeh, 2011a). More concretely, a z-number is an ordered pair of two type-1 fuzzy numbers. The first component is a restriction of on the values which a real-valued variable can take, while the second component is a restriction on a degree of certainty of that real-valued (Lotfi A. Zadeh, 2011b). According to (Lotfi A. Zadeh, 2011a), z-number is a new notion in fuzzy set theory that has more capability in describing the uncertain and complex knowledge. The idea of z-numbers is to provide a basis for computation with numbers which are not complete reliable and more intelligent to describe the knowledge of human being, also capable to cater uncertain information. In literature, some applications of z-numbers in human based decision making problems have been found which are, the evaluation of vehicle selection under uncertain environment (Kang, Wei, Li, & Deng, 2012b), the evaluation of best universities (Azadeh et al., 2013) and selection of facility location using PROMETHEE under a fuzzy environment (Kamiński, Kersten, & Szapiro, 2015).

2.4 Defuzzification

This section illustrates a thorough review on defuzzification of fuzzy sets approach which is one of the important process in fuzzy logic system. A related point to consider is that descriptions made in this section is only focus on discussion of defuzzification process of fuzzy sets only. This indicates in detail associated defuzzification of type-1 fuzzy sets are applicable for type-2 fuzzy sets and z-numbers. Thus, all discussions made on defuzzification of type-1 fuzzy sets are also relevant for defuzzification of type-2 fuzzy sets and z-numbers.

2.4.1 General Overview of Defuzzification

In dealing with complex systems in approximate reasoning, it is difficult to precisely describe the behavior of a complex system as there are many factors which influence it. One way to deal with these uncertain behaviors of the system is to use fuzzy logic. Fuzzy logic is an approach in computing fuzzy set theory based on degree of truth rather than the regular true or false Boolean logic using classical set theory. Defuzzification is an ultimate process in fuzzy logic system. Basically, in fuzzy logic system, it consists of the following five steps (Naaz S., Alam A., 2011):

1. Fuzzification: The process of converting crisp or regular inputs to membership functions which comply with intuitive perception of system status.

2. Rules processing: The process of computing the response from system status inputs according to the pre-defined rule matrix algorithm.

3. Inference: The process of evaluating each case for all fuzzy rules to the fuzzy output.

4. Composition: The process of combining the information from fuzzy rules.

5. Defuzzification: The process of converting results of fuzzy output of the inference engine using membership function to crisp or regular values.

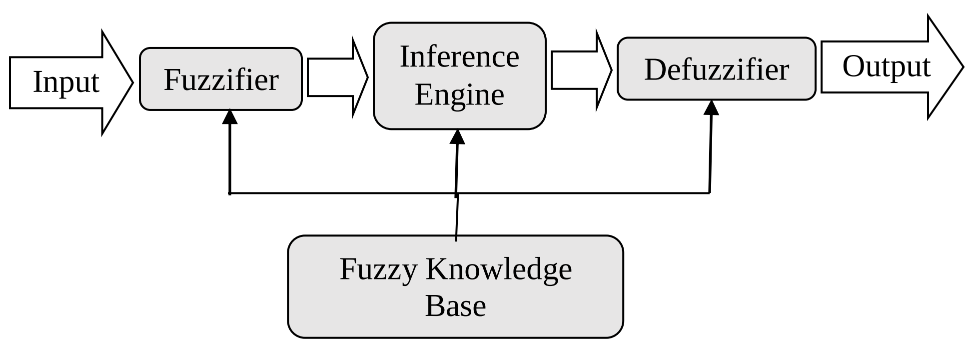


Fig. 2. 1: Fuzzy inference system

Fig. 2.1 depicts the fuzzy inference system where it illustrates the process of formulating the mapping from a given input to an output using fuzzy logic operators. The process involves the membership functions to convert the crisp inputs using linguistic variable that stored in the fuzzy knowledge base, then fuzzy logic operator acts as an inference engine to operate the system in what practitioners need with rules in evaluating all fuzzy rules before combine the information from rules in composition process and finally, defuzzification process in converting the fuzzy outputs of the inference engine to crisp using membership functions analogous to the ones used by fuzzifier. As previously mentioned, the descriptions made in this section only focusing on the discussion of defuzzification process.

Defuzzification plays as a key role in the performance of fuzzy systems modelling techniques. Generally, defuzzification process is guided by the output fuzzy subset in possibility distribution that one value would be selected as a single crisp value as the system output. It is the last step in generating an output from a fuzzy inference system. There are variation defuzzification methods have largely developed. However, they have different performances in different applications according to membership function for output variable, but there is no general method that can gain satisfactory performance in all condition (Mogharreban & Dilalla, 2006). Most of the practitioners of fuzzy system in human decision making problems utilised the defuzzification process in order to access the final results at the same time fulfill the human perception. According to (Saletic, Velasevic, & Mastorakis, 2002), in discussion on fuzzy systems, defuzzification process often is not treated as much in details as the other processes in the system. It seem that in the domain of defuzzification a practitioner has too wide possibilities of choices, so that some indicators in connection of defuzzification approach are welcome. The defuzzification method selection essentially influence the output value determined by selected method, so it is important to use an appropriate method in order to consider the need of human perception.

In the representation of fuzzy sets, the most typical fuzzy set membership function used in the graph are triangular and trapezoidal. But there are others representation of fuzzy set membership function such as singleton, Gaussian, generalised bell-shape, s-shape, sigmoidal, z-shape and pi-shape (π). In this research work, the linear fuzzy membership functions are considered because of the non-linear fuzzy sets are too complex to handle and they are normally transformed into linear type for convenience (M. Y. Chen & Linkens, 2004).

2.4.2 Characteristics of Defuzzification

The task of developing a general theory of defuzzification methods consider several characteristics or features that are important in solving human based decision making problems. From an application point of view the following characteristics of defuzzification methods are considered (Saletic et al., 2002): defuzzification result continuity, computational efficiency, design suitability and compatibility of fuzzy system.

Under first character of defuzzification regarding the continuity result of defuzzification, it considers the small changes in membership values of the output fuzzy set should not give large changes in the results of defuzzification. In fuzzy system, this character is seem to be very important because it requires input and output continuity where if there are small changes in input parameters, it should give or should effect small changes as well of output values. In this respect, the defuzzification methods must be continuous because assuming overlapping output membership functions, the best compromise does not jump to a different value with a small change to the inputs. Some defuzzification methods like centre of maximum (COM) and mean of maximum (MOM) are discontinuous, because an arbitrary small change in the input values of the fuzzy system can cause the output value to switch to another to get more plausible output. The resulting behavior of fuzzy system using any of defuzzification methods have been studied to establish the automatic and computational determination of fuzzy membership functions. This is in order to come out the optimal solutions in fuzzy systems (Mitsuishi, Sawada, & Shidama, 2009).

Computational efficiency refers to the dependency of defuzzification method in computing mostly on the types and a number of operations that required for obtaining the results of defuzzification. Consistency refers to the process in computing and the results' achieve of defuzzification process possible for all cases fuzzy numbers that considers the need of human intuition as well. The several defuzzification methods exploit the pre-calculation to achieve an excellent approximation with substantially fewer computations. The efficiency of defuzzification computation is important because it is the main parameter for the choice, since, for instant in real time systems the number of operations required to evaluate a defuzzified value should be strongly reduced to concretely achieve the maximum efficiency. Many researchers have been working on the choice of defuzzification methods for execution of fuzzy systems and their impacts for the performance's systems. This is describes that the computational efficiency in defuzzification method is utmost important in for fuzzy system design.

Design suitability expresses the impact of a defuzzification method on a software or hardware implementation and tuning of fuzzy system. The fuzzy systems will achieve better prediction accuracy than the classical counterpart, by incorporating fuzzy suitability membership of environment factors in the modelling process. Moreover, these fuzzy systems also produce more informative fuzzy suitability system through the choice of defuzzification process. The suitable design of defuzzification methods selection in fuzzy systems can be converted into conventional systems with clearly defined the problems. Qiu et al., (2013) applied fuzzy suitability in modelling the suitability of land to support specific land uses using Geographic information systems (GISs) (Qiu, Chastain, Zhou, Zhang, &

Sridharan, 2013). The authors examined the classical models within a more general framework defined by fuzzy logic concept. Through a defuzzification procedure based on the model calibration procedure proposed in the study, the fuzzy suitability maps is converted into conventional suitability maps with clearly defined boundaries were also derived.

Compatibility of fuzzy system refers to operation that can be used in fuzzy systems like inference and composition. According to (Oussalah, 2002), previous studies were accomplished on the topological level as well as on the parametrization level that include Left – Right type (Dubois & Parade, 1980) in order to improve the foundation of the theory and to simplify the performance of different combination operations in fuzzy sets. Later, in many practical applications, the need for a permanent switch from a fuzzy representation to numerical representation that is carried out by the defuzzification process which is easily recognizable. The benefit of such analysis in compatibility of fuzzy system is when the matter is the determination of the defuzzified value affecting to the results of some manipulation of fuzzy quantities, the explicit determination of the resulting fuzzy sets or fuzzy distribution can be removed, while the process may be restricted to a standard computation over single values corresponding to defuzzify initial inputs.

It seem that in the domain of defuzzification, these characteristics abovementioned are described and discussed the defuzzification features. These characteristics are presented for defuzzification of type-1 fuzzy sets, type-2 fuzzy sets and z-numbers as well. The general ideas underlying defuzzification representation of type-1 fuzzy sets are applicable for type-2 fuzzy sets and z-numbers. However, the defuzzification computation process is different for each type of fuzzy set. Some of the defuzzification methods for type-1 fuzzy sets are not compatible for type-2 fuzzy sets and z-numbers and instead.

2.4.3 Defuzzification Methods

This section briefly identifies the most commonly used defuzzification mechanisms in the literature of fuzzy system. Fundamentally, it is worth noting that the defuzzification methods focus on geometric area-based computation applications. There are four most often used defuzzification methods in the literature of fuzzy set theory as follows.

Centre of gravity (COG) is most prevalent and is often used as a standard defuzzification method in experimental as well as decision making models. This method also referred to as a centre of area (COA) method in fuzzy literature. It returns the centre of area under the curve that make sure the shape of fuzzy set would balance along *x*-axis and *y*-axis. Recently, most of researchers use word ‘centroid’ in representing COG or COA. The term centroid is widely applied in many human decision making based problems especially ranking problems. Ranking problem plays an important role in practical use of MCDM techniques. Mean of maxima (MOM) is a defuzzification method that computes the centre of gravity of the area under the maxima of fuzzy set. According to (Filev & Yager, 1991), MOM method generates poor steady-state performance and yields a less smooth response curve compare with the COG method. In general, the MOA is not similar to the centroid method. This is

Zbecause, for symmetric regions, the concept of MOA is more or less overlap, apart from some technical issues. These methods are quite popular as they are computationally inexpensive and easy to implement within fuzzy systems.

2.4.4 Centroid Defuzzification

According to (Y.-M. Wang, 2009), centroid point can be defined as a point which is situated at a middle of fuzzy number which is reflects as a representation of fuzzy number using crisp number. Centroid defuzzification process is significant in computing ranking fuzzy numbers since most of the centroid methods were developed for ranking purposes. Ranking fuzzy numbers is one of the important task that use defuzzification process has become an essential role in real-world use such as in approximate reasoning, decision making, optimizing, forecasting, control and other usage. In fuzzy decision analysis, fuzzy numbers are frequently applied to describe the performance of criteria and alternatives in modelling a real world human decision making based problem (Ramli & Mohamad, 2009). The centroid concept has been applied in various disciplines since hundred years ago, the involvement of centroid concept in ranking fuzzy numbers only started in 1980 by Yager. Other than (Yager, 1980), a number of researchers like (Murakami & Meada, 1984), (Cheng, 1998), (Shi-Jay Chen & Chen, 2002) (Chu & Tsao, 2002), (Shi-Jay Chen & Chen, 2003), (Y. M. Wang, Yang, Xu, & Chin, 2006), (Liang, Wu, & Zhang, 2006), (Shieh, 2007), (S. J. Chen & Chen, 2007) and (Y. J. Wang & Lee, 2008) have also implemented centroid concept in developing ranking method for fuzzy numbers. For each of centroid defuzzification method, researchers present their own definition and representation where some of the centroid methods are based on the value of x alone while some are based on contribution of both x and y values (Ramli & Mohamad, 2009).

In type-1 fuzzy logic system, the output is type-1 fuzzy set. This set is normally defuzzified using the useful defuzzification methods involve a centroid calculation (Mendel, 1995). Type-2 fuzzy logic system has been developed, where the output is a type-2 fuzzy set. A major calculation in a type-2 fuzzy logic system is type-reduction (Karnik & Mendel, 2001a), where it can be represented as an extension of type-1 defuzzification process. Meaning that, before the conversion process of type-2 fuzzy numbers into crisp numbers, we need to reduce type-2 fuzzy numbers into type-1 fuzzy numbers. This concept of type-reduction has implemented in practical applications for generalised type-2 fuzzy sets, interval type-2 fuzzy sets, also Gaussian type-2 fuzzy sets. Several researchers applied type-reduction procedure in their study such as, (F. Liu, 2008) proposed an efficient centroid type-reduction strategy for generalised type-2 fuzzy logic system, (Hsiao, Li, Lee, Chao, & Tsai, 2008) designed of interval type-2 fuzzy sliding-mode controller form linear and non-linear system, (Figueroa, 2012) proposed an approximate method for type-reduction of an interval type-2 fuzzy set based on $\alpha - cut$. Later, some researchers have developed direct method to defuzzify fuzzy numbers without type-reduction procedure such as (Nie & Tan, 2008), (Greenfield, Chiclana, Coupland, & John, 2009), (Gong, 2013), (Gong, Hu, Zhang, Liu, & Deng, 2015), (Abu Bakar & Gegov, 2015a).

The notion of z-numbers has more capability to describe the uncertain information in human decision making based problems. The theory of z-number is still pre-mature where to convert z-numbers into crisp numbers are significant in real world case studies. Defuzzification of z-numbers are quite tricky because the consideration of two components (fuzzy restriction and reliability of fuzzy restriction) for one z-number representation. Under this situation, (Kang et al., 2012b) proposed a conversion method for z-numbers to classical fuzzy numbers which are type-1 fuzzy sets according to the multiplication operation of triangular fuzzy numbers. Later, (Kang, Wei, Li, & Deng, 2012a) proposed a method of converting z-numbers to classical fuzzy numbers that is according Fuzzy Expectation. This conversion method has more influence to describe the knowledge of human being and widely used in uncertain information. In solving human decision making based problems, much of the information on which decision are based in uncertain condition. Z-numbers are extension of type-1 fuzzy sets that has capability to describe the knowledge of human being in uncertain environment. There is number of researchers employed z-numbers in their research works such as, (Kang et al., 2012a), (Azadeh et al., 2013), (Xiao, 2014) and (Yaakob & Gegov, 2016).

A fuzzy MCDM model is used to assess the alternatives versus selected criteria through a group of decision makers, where suitability of alternatives versus criteria, and the importance weights of criteria, can be evaluated in linguistic values represented by fuzzy numbers (Hadi-vencheh & Mirjaberi, 2011). Most of MCDM methods apply ranking operation for criteria or alternatives selection. In dealing with ranking fuzzy numbers in fuzzy MCDM models, centroid methods are commonly used in and often require membership functions to be known. There are numerous fuzzy MCDM techniques based on centroid of fuzzy numbers have been proposed thus far such as (Sun, 2010), (Rostamzadeh & Sofian, 2011), (Hadi-Vencheh &

Mokhtarian, 2011), (Azadeh et al., 2013), (Kahraman, ßar, ßi, Sarı, & Turanog ˘lu, 2014), (Abdullah & Zulkifli, 2015), (Salehi, 2015), (Yaakob & Gegov, 2016) and so forth. There are many researchers and practitioners have attempted to understand the logic and the workings of the defuzzification operation. It has been found that in many cases that the choice of defuzzification method in human decision making based problems can be critical in designing the fuzzy system in MCDM models. Due to this sense, the next section briefly overviews decision making process followed by decision making process under fuzzy environment.

2.5 Decision Making

This section explains a comprehensive review on decision making process which regular part of human being life with thousands of decisions having to be made in many fields every single day. All of us make decision of varying important every day, thus the idea of decision making study is to give best solution in evaluating the available alternatives in order to choose the most desirable one.

2.5.1 General Overview of Decision Making

Decision making can be described as the study of identifying and choosing alternatives that based on the values and preferences of the decision makers. It can be considers as a cognitive process of ranking and evaluating available alternatives from a list of opinions in order to choose the most desirable solution (Zimmermann, 1987). Decision making is crucial factor to succeed in any discipline, particularly in a field which requires handling large amounts of information and knowledge (Jato-Espino & Canteras-Jordana, 2014). Another definition from (Ribeiro, 1996), he defined that decision making can be justified as a process of choosing or electing sufficiently good alternative or course of action, from a set of alternatives to attain a goal or goals. From these definition, for every decision making process, the consideration of a decision goal, a set of criteria and a set of alternatives. It can be emerged as a sub-discipline of operation research intended to facilitate the resolution of these issues.

Every single decision is made within a decision environment, which is defined as the gathering of information, criteria, values, preferences and alternatives available at the time of decision. In decision making environment, the state of nature and alternatives are confronted decisions involve a choice of one or more alternatives from among an arrangement of possible outcomes, the choice is being based on how well each alternative is measured up to a set of predefined criteria. Much decision making environments involve uncertainty. Hence, in dealing with imprecision, vague, random and uncertain information, one of the most important aspects for a useful decision making process incorporate with fuzzy set theory is implemented in many decision making problems. According to (Bellman & Zadeh, 1970), much of the decision making problems in real world phenomena take place in an environment where the goals, the constraints and the consequences of possible actions are not known precisely. In this research work, the integration of decision making process with fuzzy set theory is considered in order to deal quantitatively with decision making problems under fuzzy environment.

2.5.2 Multi Criteria Decision Making

Multi criteria decision making (MCDM) is a one of the most popular branches of decision making studies. The field of MCDM can be traced to Benjamin Franklin (1706-1790), who was the American statesman (Steuer & Zionts, 2016). He specifically designed as a simple paper system for deciding important issues. MCDM was introduced in the early 1970'es as one of the important study in dealing with decision making problems. It is standout amongst the most perceived branches of decision making over the last four decades for solving decision problems in the presence of multiple criteria and alternatives. It has become one of the most important and fastest growing subfields operation research and management science. There are a lot of modern researchers have considered MCDM problems that represented as an evaluation problem, where the decision maker selects among a finite set of discrete alternatives as a design problem (Köksalan, Wallenius, & Zionts, 2011). The set of decision alternatives in MCDM is described with a mathematical model.

The motivations for developing the MCDM techniques to decision making problems emerged from the limitation of the classical decision making techniques to the study of single criterion decisions (Banville, Landry, Martel, & Boulaire, 1998). It also capable in handling massive real decision making problems that involve many criteria and decision makers. It is worth noting that descriptions of MCDM is to guide decision makers in determining the course of action that best achieves the long term goals. Saaty was motivated to develop a simple way to help lay people make complex decisions in MCDM problems (Saaty, 2008). According to (Dooley, Sheath, & Smeaton, 2005) several advantages of MCDM can be concluded: provide people with a quantitative means to assist with decision making where there are multiple and conflicting goals measured in different units, making a decision more transparent to others, providing a means of problems structuring and working through the information, providing a focus for discussion, helping people better understand a problem from their own and other's viewpoints and so forth. The field of MCDM is used in discussion of many established research studies done by (Zimmermann, 1987), (E Triantaphyllou, Shu, Sanchez, & Ray, 1998), (Satapathy & Bijwe, 2004), (Shyur & Shih, 2006), (W. S. Lee, Tzeng, Guan, Chien, & Huang, 2009), (Mamaghani, 2012), (Zolfani, Esfahani, Bitarafan, Zavadskas, & Arefi, 2013), (Jato-Espino & Canteras-Jordana, 2014) and so forth.

On the classical decision making approaches, there is no uniform classification of MCDM techniques. As a consequence, there are many ways to classify them, such as the form of the model linearity (e.g. linear, non-linear or stochastic), the characteristics of the decision space (e.g. finite or infinite, and the solution process (prior specification of preferences or interactive). Later, (Hwang & Yoon, 1981) and (Zimmermann, 1987) provided a general classification of the MCDM fields into two categories that based on different purposes and different data types which are multi objective decision making (MODM) and multi criteria decision making (MCDM). Multi objective decision making studies the decision problems is which the decision space is in continuous condition. It is therefore not associated with problems in which alternatives have been predetermined. The decision makers are primarily concern to design the most promising alternative with respect to limited resources. Multi objective decision making (MODM) is used for design, dealing with the problem or resolving a set of conflicting goals that cannot be achieved simultaneously. A typical example is mathematical programming problems with multiple objective functions (Evangelos Triantaphyllou, 2000).

Multi criteria decision making (MCDM) is associated with problems together with a discrete decision space where it involves evaluation of a definite set of alternatives according to a predefined set of evaluation attributes. An attribute or criterion is a property, quality or characteristic of an alternative. For evaluating an alternative, ‘a criterion is set up for each of its attribute and the attribute is examined against the criterion'. Because of the one to one correspondence between an attribute and a criterion, sometimes attributes are also referred to as criteria. In the context of MCDM, the word attributes and criteria are used interchangeably (Xu & Yang, 2001). The terms MADM and MCDM are normally used the same class of models. Some researchers or practitioners used MADM and MCDM as the same class of models in their studies. In MCDM problems, there are two typical categories of problems and the distinction between the two categories is based on the number of alternatives under evaluation: the first involves a finite number of alternative solutions and another one having an infinite number of solutions (Xu & Yang, 2001). Commonly, in problems related to evaluation and selection, the number of alternative solutions is limited whereas, in problems associated with design, the potential alternative solutions could be infinite. If this is the case, the problem is referred to as MODM instead of MCDM problem. Looking back to the research problem, one can be noticed that it is a decision selection problem with a finite number of alternatives available. Therefore, the problem tackled in this study should be considered as a MCDM problem.

Generally, MCDM problems are complex and ill-structured. When the MCDM technique is used, the decision making processes are composed of three main steps, as suggested by (Belton & Stewart, 2002) which are; 1) Problem identification and structuring; 2) Model building and utilisation and; 3) Model testing and taking action. According to (Evangelos Triantaphyllou, 2000), in handling MCDM process, the main quantified and formal procedures that should be utilised for any decision making involving multiple criteria and finite alternatives are identified as follows: 1) Determine the relevant criteria and alternatives associated with the problems considered; 2) Attach numerical measures to the relative importance of the criteria and the impacts of the alternatives on these criteria and; 3) Process the numerical values to determine a ranking for each alternative. Most of MCDM techniques proposed in the literature are applied to judge group decision making.

A group decision making can be defined as 'two or more people who are jointly responsible for detecting a problem, elaborating on the nature of the problem, generating possible solution, evaluating potential solutions, or formulating strategies for implementing solutions' (DeSanctis & Gallupe, 1987). In group decision making process, in finding mathematical model for aggregating the information preferences expressed by the group members and to determine the weights or priorities or ranking for the decision alternatives are core problem (Indrani & Saaty, 1993). This group decision making is used as basis for establishing the MCDM techniques for judging group decision supports approaches. While much of the literature of MCDM techniques discuss the capability of each technique to handle and solve the variety decision making problems with a group of decision makers. Among of them are Analytic Hierarchy Model (AHP), Analytic Network Process (ANP), preference relation, ELimination and Choice Expressing Reality (ELECTRE), Preference Ranking Organization METhod for Enrichment of Evaluations (PROMETEE), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Simple Additive Weighting Method (SAW), multidisciplinary optimization compromise solution (VIKOR), Decision Making Trial and Evaluation Laboratory (DEMATEL), and so forth. These MCDM techniques have been applied in many situations in real world decision making problems. In this research work, only several MCDM techniques are considered which are AHP, TOPSIS, preference relation and VIKOR. It has been found that in many cases that the choice of MCDM techniques in human based decision making problems under uncertain or imprecise environment. Considering this sense, next section discusses modelling uncertainty in MCDM techniques processes.

2.5.3 *Modelling Uncertainty in Multi Criteria Decision Making*

This section provides a review the tools that can model the uncertainty issue in MCDM problems. Uncertainty can be defined as the situation which is incomplete or conflicting information. According to (Zimmermann, 2000), he defined uncertainty in the context of practical application in MCDM as; Uncertainty implies that in a certain situation a person does not dispose about information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristic. There are two different types of uncertainty which are external and internal uncertainties (Durbach & Stewart, 2012). External uncertainty denotes concern regarding issues outside the control of the decision makers. It refers to results from lack of understanding or knowledge about the consequences of a particular choices (Stewart, 2005). While internal uncertainty relates to the process of problem structuring and analysis, as well as to ignorance, complexity of information, subjective judgements and imprecision in human judgements (Durbach & Stewart, 2012). In human based decision making problems, it is common that people may not be 100 percent sure when making subjective judgements.

Since the problem of selecting the best alternatives in decision making problems is subject to uncertainty due to imprecision and subjectivity in the decision makers' judgements, method for representing uncertain information in decision making are summarised and analysed in the following: probability theories, the Dempster-Shafer (D-S) theory, rough set theory and fuzzy set theory. The previously reported review indicates very clearly that probability theories, the Dempster-Shafer theory, rough set theory and fuzzy set theory are the most frequently used frameworks for handling information about uncertainty in decision making. Despite the aforementioned limitations, fuzzy set theory can provides a vital alternative to probability theories, the Demspter-Shafer or rough set for modelling uncertainty. The subject nature of human's opinions, incomplete judgements and ranking evaluations in many researches are major barriers against using the tools of probability theories, the Dempster-Shafer and rough set theory. Since there are several limitations from Demspter-Shafer or rough set, fuzzy set theory offers great potential in modelling uncertainty in this study.

Generally, the probability theory has several shortcomings of providing a comprehensive methodology for dealing with uncertainty and imprecision (Lotfi A Zadeh, 1994), there are: 1) Probability does not support the concept of fuzzy event; 2) Probability offers no techniques for dealing with fuzzy quantifiers like many, most, several, few: 3) Probability theory does not provide a system for computing with fuzzy probabilities expressed as likely, unlikely, not very likely, and so forth; 4)

Probability theory does not provide methods for estimating fuzzy probabilities; 5) Probability theory is not sufficiently expressive as a meaning-representation language; 6) The limited expressive power of probability theory makes it difficult to analyse problems in which the data are described in fuzzy terms. Fuzzy logic is pretty much the same tools as probability theory. In any case, it is utilising them trying to capture a very different idea. According to (Lotfi A. Zadeh, 2003), to enable probability theory to deal with perceptions, it is necessary to add fuzzy component drawn from semantics of natural languages. Without this fuzzy component, there are many situations in which probability theory cannot answer questions that arise when everyday decisions have to be made on the basis of perception-based information. Fuzzy logic is all about the degrees of truth, where it is about fuzziness and partial or relative truths. Probability theory is keen on trying to make predictions about events from the state of partial knowledge. But, probability theory says nothing about how to reason about things that are not entirely true or false.

Its advantage over these other theories is its ability to represent imprecise and incomplete judgement, which is a typical problems in the evaluation process for selection of alternatives. Obviously, fuzzy set theory is the most applicable of these tools for the modelling of uncertainty due to the huge numbers of papers published in the literature. Moreover, fuzzy set theory requires less time regarding the computation process because there are many software programmes that can be applied in analysing and designing fuzzy set theory concepts. Therefore, for this research work, the fuzzy set theory approach is considered as the most appropriate and practical tools to handle uncertainty.

2.5.4 Fuzzy Multi Criteria Decision Making

This section discusses the consideration fuzzy set theory in MCDM techniques in solving real world problems under fuzzy environment. It is not surprising to see that uncertainty always exist in the human based decision making problems (Shu-Jen Chen & Hwang, 1992). In human based decision making problems, uncertain and imprecise judgements by decision makers are taken into account through the application of fuzzy numbers instead of crisp numbers by using linguistic scales. Moreover, there are some difficulties to make decisions where experts or decision makers are unable to give exact numerical values to their preferences. In such cases, in evaluating the alternatives, linguistic assessments are used instead of numerical values to express preferences (Adamopoulos & Pappis, 1996). Due to flexibility of using linguistic variables, incorporating fuzzy set theory in MCDM problems is a straightforward process to assess decision makers' judgements.

Fuzzy set theory has attracted the attention of many researchers and practitioners for modelling the uncertainty in MCDM problems. Bellman and Zadeh were the first who proposed the decision making under a fuzzy environment, then they initiated fuzzy multi criteria decision making (FMCDM) (Bellman & Zadeh, 1970). In decision making situation, making choices which depends on numerous factors limited to human ability that is very difficult to deal with (T. C. Wang & Chen, 2008). The consideration of fuzzy aspect in MCDM techniques is significant in order to solve these issues. In the literature, there are huge amount of research studies integrating fuzzy set theory with MCDM techniques to deal with uncertainty aspect of any decision making problem (Van Laarhoven & Pedrycz, 1983), (Buckley, 1985) (Boender, de Graan, & Lootsma, 1989), (Chang, 1996), (E Triantaphyllou et al., 1998), (Yeh et al., 2000), (Kahraman, Cebeci, & Ruan, 2004), (T.-C. Wang & Chang, 2007), (T. C. Wang & Chen, 2008), (Kang et al., 2012b), (S. Chen & Wang, 2013), (Abdullah & Najib, 2014), (Mardani et al., 2015), (J. Wang et al., 2015) and (Keshavarz Ghorabaee, 2016).

Most of the research utilising fuzzy aspect in MCDM techniques consider fuzzy numbers in linguistic variables to express decision makers' preferences in order to evaluate the criteria and alternatives. Numerous studies have attempted to discuss about fuzzy numbers as linguistic variables. Most of the researchers implemented triangular fuzzy numbers instead of trapezoidal fuzzy numbers as linguistic variables to express natural languages (Kahraman et al., 2004), (T.-C. Wang & Chang, 2007), (Lin & Wu, 2008), (Hadi-Vencheh & Mokhtarian, 2011) and so forth. This is because the representation of triangular fuzzy numbers is not as complicated as trapezoidal fuzzy numbers. However, studies on trapezoidal fuzzy numbers for MCDM techniques have aroused lately. Zheng et al., (2012) adopted trapezoidal fuzzy numbers in fuzzy AHP for work safety in hot and humid environment in Taiwan (Zheng, Zhu, Tian, Chen, & Sun, 2012). Salehi developed a hybrid fuzzy MCDM approach for project selection problem (Salehi, 2015). Fu et al., (2011) presented fuzzy AHP and VIKOR methodology to perform a benchmarking analysis in the hotel industry (Fu, Chu, Chao, Lee, & Liao, 2011). Thus, this research study aims to investigate the implementation of trapezoidal fuzzy numbers that are preferred instead of triangular fuzzy numbers.

Aforementioned in Section 2.3, there are three different types of fuzzy sets found in literature namely type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Each of these fuzzy sets has their own capability in dealing with imprecision, vagueness, randomness and uncertainty. Type-1 fuzzy sets have limited capability in modelling uncertainty because the representation of type-1 fuzzy sets is on natural language which is insufficient enough to model human perception. Recently, type-2 fuzzy sets are used predominantly in MCDM problems. Several studies have been discussed in this thesis to implement type-2 fuzzy sets in MCDM problems. Gong investigated the fuzzy multi-attribute group decision making problems to solve global supplier selection which all the information given is expressed in interval type-2 fuzzy sets (Gong, 2013). Kahraman lead his research team to develop an interval type-2 fuzzy AHP method based on new defuzzification method proposed to a supplier selection problem in Turkey (Kahraman et al., 2014). Abdullah and Zulkifli integrated fuzzy AHP and fuzzy DEMATEL together with interval type-2 fuzzy sets and was tested this proposed methodology to a case of human resource management (Abdullah & Zulkifli, 2015).

Decision are based on information given. Most of the information on which decisions are based is uncertain. In describing the uncertain information, information must be reliable. Basically, the concept of z-numbers relates to the issues of reliability of information as mentioned in Section 2.3.3. In MCDM problems, the consideration of z-numbers as a linguistic variable are lesser than type-1 and type-2 fuzzy sets. This is because the z-numbers present the latest version of fuzzy sets in the literature. Several studies have done to implement z-numbers with MCDM techniques. Kang et al., (2012) studied vehicle selection for journey under uncertain environment (Kang et al., 2012b). Moreover, Azadeh et al., (2013) proposed new AHP method based on z-numbers to search the criteria's for the evaluation of best universities (Azadeh et al., 2013). Xiao (2014) proposed new multi criteria fuzzy decision making method using z-numbers by converted z-numbers to the interval-valued fuzzy set with footprint of uncertainty (FOU) (Xiao, 2014). Kaminski et al., (2015) proposed fuzzy MCDM model to select the facility location using PROMETHEE under a fuzzy environment (Kamiński et al., 2015). Later, Yaakob & Gegov (2016) proposed interactive TOPSIS method using z-number to rank stock selection (Yaakob & Gegov, 2016).

2.6 Sensitivity Analysis

This section discusses the sensitivity analysis application in decision making models for validation purposes. In this research work, sensitivity analysis is utilised as validation method in order to evaluate how robust the optimal solution produced when different circumstances are considered with making changes in parameters. Thus, sensitivity analysis acts as an instrument for the assessment of the input parameters to apply the model efficiently and to enable a focused planning of future research and field evaluation.

2.6.1 General Overview of Sensitivity Analysis

It is recognized that sensitivity analysis is one of the validation of the results of mathematical models or systems. Sensitivity analysis can be defined as the study how uncertainty in the output of a model can be attributed to different sources of uncertainty in the model input (Saltelli, 2004). Sensitivity analysis is broadly defined, is the investigation of these potential changes and errors, and their impacts for the conclusion to be drawn from the model (Pannell, 1997). Sensitivity analysis is valuable tool for identifying important models parameters, testing the model conceptualization, and improving the model structure (Bahremand & Smedt, 2008). Sensitivity can be beneficial for the wide range of purposes including (Pannell, 1997); test the robustness of the results of a model or system in the presence of uncertainty; increased understanding of the relationship between input and output variable in a model or system; uncertainty reduction; ease the calibration stage. The sensitivity analysis after the problem solving can effectively contribute to making accurate decisions. The conduction of sensitivity analysis is to indicate how important the model make the changes to management suggested by the changing optimal solution. The robustness and reliability of the results obtained which mean that the model insensitive to changes in parameters.

2.6.2 Sensitivity Analysis in Multi Criteria Decision Making

Sensitivity analysis for MCDM techniques is one of the discussed issues in MCDM fields. Many researchers studied regarding this technique about a couple of decades ago. The MCDM techniques always deal with unbalanced and changeable data inputs. Therefore, the sensitivity analysis after problem solving can effectively contribute to making accurate decisions by assuming that a set of weights for attributes or alternatives then obtained a new round of weights for them, so that the efficiency of alternatives has become equal or their order has changed. It focused on determining the most sensitive criteria and the least value of the modification. It clearly indicates that the sensitivity analysis is calculated the changing in the final score of alternatives when a changes occur in the weight of one alternative. The results of MCDM techniques is crucially needed to validate and calibrate in analysing the quality and how robustness of MCDM techniques to reach a right decision under different conditions.

Sensitivity analysis is performed by changing the specific input parameter on model to determine the impact of such changes on the evaluation of the outcomes and to test the strength of the results of the proposed model. It, therefore, provides the information on the stability of the final ranking in MCDM techniques. Of the ranking is highly sensitive to small changes in the parameter values, a careful review of those parameters is recommended. Thus, sensitivity analysis after the problem solving can effectively contribute to making accurate decisions. In this research study, the scenario was investigated to examine the stability of the final ranking under varying weights of criteria which are the criteria weights. When a change occurs in the weight of one criterion, the change in the score and final ranking of alternatives are calculated. In doing so, a sensitivity analysis method proposed by (Amini & Alinezhad, 2011) is applied here. In varying the weight of one criterion is accompanied by decreasing the weights of others criteria by certain amounts such that the total of all criteria weights is equal to one.

Many studies in the literature where the application of sensitivity analysis in MCDM techniques have been found. Bevilacqua & Braglia, (2000) improved the effectiveness of the application of AHP model for selecting the maintenance strategy for important Italian oil refinery using sensitivity analysis (Bevilacqua & Braglia, 2000). Besides, Memariani et al., (2009) proposed a new sensitivity analysis of MADM problems for SAW technique (Memariani, Amini, & Alinezhad, 2009). Amini & Alinezhad, (2011) developed a new sensitivity analysis MADM problems for TOPSIS technique (Amini & Alinezhad, 2011). Likewise, Rezaie et al., (2014) developed integrated fuzzy AHP and VIKOR to evaluate the performance of 27 Iranian cement firms in the Tehran stock exchange market for two years which from 2008 until 2009, separately (Rezaie, Ramiyani, Nazari-Shirkouhi, & Badizadeh, 2014).

2.7 Research Problems

In this research work, the main problems are considered in developing of new hybrid MCDM model that is incorporated with new defuzzification method in evaluating decision making problems in fuzzy environment. This research addresses the evaluation and selection processes in order to understand the generic phenomena in decision making problems, whereby at the same time it considers the need for human intuition or human judgement in computational process. This section discusses research problems that cover gaps and limitations faced by established centroid defuzzification methods and established hybrid fuzzy MCDM models in evaluating the weights and alternatives. The following details signify gaps and limitations of the established methods in literature of centroid defuzzification methods and hybrid fuzzy MCDM models.

The first and foremost gap in the literature of centroid defuzzification methods is the inability of established centroid methods in converting fuzzy numbers into regular numbers with considering the degree of membership component. The centroid proposed by (Yager, 1980) only considers horizontal axis in Cartesian plane whereby he did not consider the vertical axis which represents a membership degree component. He made no assumption on the normality and convexity of fuzzy number. Hence, the centroid method that proposed by (Yager, 1980) is biassed and irrelevant in finding the correct centre point of fuzzy number. Another main gap in the literature for centroid defuzzification methods is there are some established centroid methods such as (Y. M. Wang et al., 2006), that are not appropriately applicable in solving decision making problems. With regards to the discussion made on the first gap in term of considering y – axis or vertical axis in fuzzy numbers representation and, it is worth considering that all aforementioned centroid methods are unable to deal with real decision making problems appropriately. This is because all of them are limited to give correct centre point in certain cases of fuzzy numbers theoretically which implies that all of them are unable to solve decision making problems in real world situations under fuzzy environment.

In literature, most of centroid defuzzification methods unable to present human intuition or human judgement properly in their computational formulation such as (Yager, 1980), (Murakami & Meada, 1984), (Cheng, 1998), (Shi-Jay Chen & Chen, 2002) (Chu & Tsao, 2002), (Shi-Jay Chen & Chen, 2003) and (Liang et al., 2006). This is refer to the capability of producing the correct centroid formulae for all possible cases of fuzzy numbers. In real world phenomena, the representation of fuzzy events are too varies. Even, some of the established centroid defuzzification methods are developed to focus on particular type of fuzzy numbers. The applicability of centroid defuzzification for all types of fuzzy sets are limited since most of the established centroid methods are developed for type-1 fuzzy sets only. As it should be pointed out that only several established centroid defuzzification methods such as (Karnik & Mendel, 2001a), (Wu & Mendel, 2007), (Nie & Tan, 2008), (Gong, 2013), (Kang et al., 2012a, 2012b), (Azadeh et al., 2013) and (Abu Bakar & Gegov, 2015b) are developed or extended for type-2 fuzzy sets or z-numbers. In developing mathematical formulation, validation process is needed to prove the theoretical and empirical foundation in formulating ideas and identify underlying assumptions.

While much of the literature for fuzzy MCDM techniques has their special ability in solving decision making problems. Some of fuzzy MCDM techniques are integrated or combined several techniques together to give better results in evaluating criteria and alternatives under fuzzy environment. This is because in some cases in decision making problems, not only just making the final decision. There are some phases are steps in solving decision making problems especially massive cases that need more decision making techniques to work together such as; evaluation of the performance of global top four notebook computer Original Design Manufacturers (ODM) companies (Sun, 2010); selection the best plastic recycling methods (Vinodh, Prasanna, & Hari Prakash, 2014); determination of certain number of projects for investment from among twenty research and development (R&D) projects (Collan, Fedrizzi, & Luukka, 2015); evaluation of the dimensions and several criteria of human resource management (HRM) problems (Abdullah & Zulkifli, 2015) and so forth. The second primary gap here is most of researchers that integrate fuzzy MCDM techniques are unable to cope fuzzy entities appropriately in terms of imprecision, vagueness and uncertainty.

There are also several limitations in previous studies performed in combining fuzzy MCDM techniques together which are; most of researchers and practitioners abandoned to consider *y*-axis part of Cartesian in fuzzy representation; focused on particular case of representation of fuzzy number only such as triangular fuzzy numbers; focused on particular type of fuzzy set only such as type-1 fuzzy set; inappropriate defuzzification method used in converting fuzzy numbers into regular numbers; no validation process after final results appear to evaluate the robustness of the model. In this research study, the development of new centroid defuzzification is developed in order to make sure the final results are consistent with human intuition or human judgment in evaluating human based decision making problems. All of these gaps and limitations are identified in order to improve decision making evaluation with considering human intuition and capable to solve uncertain information.

However, in the literature centroid defuzzification methods and hybrid fuzzy MCDM models indicate that both of them are extensive, gaps and limitations faced by established research works are still unsolved. Consequently, the study is carried out to solve these gaps and limitations appropriately.

2.8 Research Questions

The research work identifies seven main research questions on the development of novel centroid defuzzification method and hybrid fuzzy MCDM model. All research questions are identified based on underlying the research problems. This indicates that research problems underpin research questions of the thesis. The questions are listed below:

1. Is there any established centroid defuzzification method that considers human intuition or human judgement in their computational formulation which is capable of producing the correct centroid formulae for all possible cases of fuzzy numbers that considered in literature?
2. Does the proposed centroid defuzzification method has only limited to type-1 fuzzy sets?
3. Is there any validation process for the proposed centroid defuzzification method to verify how consistent there are with human intuition?

4. Is there any hybrid fuzzy MCDM model that integrates consistent fuzzy preference relations and fuzzy TOPSIS together whereby incorporate with proposed centroid method for defuzzification process purposes?
5. Does the proposed hybrid fuzzy MCDM model have only limited to type-1 fuzzy sets?
6. Is there any validation process for the proposed hybrid fuzzy MCDM model to verify how robust and consistent they are?
7. Does the implementation of centroid defuzzification method in any fuzzy MCDM technique give impact the final results?

2.9 Research Objectives

The specific objectives of this research that will realise in answering the research questions where the research aims are:

1. To develop a new defuzzification method for type-1 fuzzy sets which is named as intuitive multiple centroid that considers all possible cases of fuzzy numbers which represents the correct centroid formulae from the viewpoint of median analytical geometry such that the results are consistent with human intuition.
2. To extend the proposed intuitive multiple centroid defuzzification method of type-1 fuzzy sets on type-2 fuzzy sets and z-numbers.
3. To validate the reliability and consistency of intuitive multiple centroid defuzzification method of type-1 fuzzy sets and its extension of type-2 fuzzy sets and z-numbers theoretically and empirically respectively.
4. To develop the new generic hybrid fuzzy MCDM model that consist of consistent fuzzy preference relations and fuzzy TOPSIS, together with the implementation of the proposed intuitive multiple centroid.
5. To extend the methodology for hybrid fuzzy MCDM model that consists of consistent fuzzy preference relations and fuzzy TOPSIS on type-2 fuzzy sets and z-numbers.
6. To validate reliability and consistency of the proposed hybrid fuzzy MCDM model for type-1 fuzzy sets and its extension on type-2 fuzzy sets and z-numbers theoretically and empirically.

7. To apply the proposed hybrid fuzzy MCDM model that is incorporated with intuitive multiple centroid for type-1 fuzzy sets and its extension on type-2 fuzzy sets and z-numbers in decision making case study, then validate the proposed models with the established hybrid fuzzy MCDM models using sensitivity analysis.

2.10 Research Contributions

This research study has contributed to knowledge in the form of theoretical and practical contributions. Research questions and research objectives are underpinned the research contributions. This study makes some significant theoretical contributions in the following ways.

The first main theoretical contribution of this study is proposed a new centroid defuzzification method for type-1 fuzzy sets and its extension for type-2 fuzzy sets and z-number to comprehend the centroid defuzzification formulation that consistent with human intuition in decision making problems. In the literature, there are many centroid defuzzification methods for fuzzy numbers proposed, but there is no research that extends the centroid method into type-2 fuzzy sets and z-numbers. Moreover, the proposed centroid defuzzification methods might be applied to any decision making model with uncertainty in different areas. The theoretical validations for the proposed centroids are broadly illustrated in Chapter 4.

In ongoing effort, the second theoretical contribution is the development of a novel hybrid fuzzy MCDM model based on consistent fuzzy preference relations and fuzzy TOPSIS as well which takes into account the application of new centroid defuzzification method that is mentioned in point one. The consistent fuzzy preference relations has been extended by including the defuzzification process to get the weight of criteria before the final ranking evaluation process is implemented. The improvising of fuzzy TOPSIS has been extended by considering several additional steps compare to classical one including the application of different normalisation method used. The proposed centroid defuzzification method is applied for alternatives evaluation phase to get the final rank. The novel hybrid fuzzy MCDM model is validated using sensitivity analysis to evaluate the robustness of the results of a model or system in the presence of uncertainty.

The third of theoretical contribution of this study is that there are several prototypes of decision making tools and sensitivity analysis calculator are developed in Microsoft Excel. The decision making tools present the proposed hybrid fuzzy MCDM model that consist of consistent fuzzy preference relations and fuzzy TOPSIS together with new centroid defuzzification method. The prototypes are developed in order to assist decision makers to implement the proposed methodology for ranking alternatives with considering the criteria's weight. Moreover, the implementation of the prototypes are necessary as in fuzzy decision making environment where fuzzy numbers are utilised as data representation. Thus, this indicates that these prototypes are developed not only to rank the alternatives but capable to solve decision making problems.

The practical contributions consider the case study application and some recommendation for the future works. There are presented as follows.

Referring to the first theoretical contribution, the proposed centroid defuzzification method is extended to type-2 fuzzy sets and z-numbers. All of them are compared numerically in order to evaluate the consistency of proposed method and established methods in literature. Both proposed and established centroid defuzzification methods for all fuzzy sets are compared with all possible cases in representing fuzzy numbers. In order to do so, the reliable centroid defuzzification has capability to calculate all possible cases of fuzzy numbers that consistent with human intuition. Also capable to deal with imprecision, vagueness and uncertainty. The numerical validation for the proposed centroids are discussed in Chapter 4.

For the second practical contribution, this study provide a case study and briefly explains the decision making process for staff recruitment in a company in Malaysia. The evaluation process of selecting a right employee includes; identifying the selection criteria, deriving the criteria weights and ranking the available alternatives. The results or findings of the case study provide some recommendations in enabling the decision makers in the company to develop decision making model in searching the right employees which might increase the productivity of the company. The results and findings also provide practical study in order to be applied for other companies or industries cases. The proposed hybrid fuzzy MCDM model is extended to type-2 fuzzy sets and z-numbers, where are compared with other several established hybrid fuzzy MCDM models in the literature in evaluating the results and findings.

The third of practical contribution refers to the validation process of hybrid fuzzy MCDM model using sensitivity analysis. It evaluates the proposed and established hybrid fuzzy MCDM models in order to study the robustness of the model. Likewise, sensitivity analysis assists the decision makers to implement the proposed methodology for ranking alternatives with considering the criteria's weight. The sensitivity analysis evaluation method is used to validate the robustness of the proposed and established hybrid fuzzy MCDM models in a case study of the thesis.

As a final point, this research contributes to knowledge of the subject of human based decision making problems under fuzzy environment, since there is a lack of literature in this area.

2.11 Summary of the Chapter

This chapter broadly discusses a review of the literature. A brief review of the basic concept of fuzzy sets is first discussed in this chapter and followed by the development of fuzzy sets. The literature on defuzzification process is then reviewed whereby thorough reviews on characteristics of defuzzification process and some of the defuzzification methods are illustrated. Afterwards, Section 2.5 broadly reviews regarding decision making study which covers an overview of decision making in modern perspective, MCDM techniques, modelling uncertainty in MCDM and fuzzy MCDM. In addition, the validation of proposed methodology which using sensitivity analysis method is explicitly discussed. Later on, the results of literature review are presented the research problems, research questions, research objectives and research contributions such that all of them illustrate the gaps, targets and contributions by this study respectively. In Chapter 3, the thesis discusses theoretical preliminaries regarding some theories, methods or tools applied in this research work.

CHAPTER 3
THEORETICAL PRELIMINARIES

3.1 Overview

The purpose of this chapter is to review some definitions and theoretical background that are used in the thesis. It illustrates some notions and basic concepts of fuzzy set theory and MCDM techniques throughout the thesis. The necessary condition of terminology used where some of the concepts are defined using experts' definitions while some are provided with theoretical proves. In addition, details on those aforementioned points are broadly explained in sections and subsections of this chapter.

3.2 Basic Concepts of Fuzzy Set Theory

In the literature of decision making, researchers show that the classical set theory serves as a useful tool in solving decision making problems. Classical sets are sets with crisp boundaries where usually is called a collection of elements which have some properties distinguishing them from other elements which do not possess these properties. The membership degree of elements in a classical set is in binary or bivalent conditions where representing either 0 or 1 to indicate whether an element is not a member or a member of a set respectively. If weather condition for today is considered as an example, then today weather is either 'hot' or 'not hot' when the classical sets are used. However, consideration only to two binary terms by classical sets is inadequate as human perceptions are vary among people, as different people employ different types of perceptions which are vague and fuzzy (Cheng, 1998).

Due to limitation of the classical sets, fuzzy set theory was introduced in decision making environment as dealing with situations that are fuzzy in nature is important. In contrast with classical sets, fuzzy sets allow gradual assessments of an element's degree of belongingness in the interval of 0 and 1 where these values indicate variation in terms of human perceptions about a situation perceived. The definition of fuzzy set theory is given as follows.

Definition 3.1 (Cheng, 1998) A fuzzy set A_i in a universe of discourse U is characterised by a membership function $\mu_{A_i}(x)$ which maps each element x in U such that x is real number in the interval [0,1].

Membership function for A_i , $\mu_{A_i}(x)$ is given as

$$\mu_{A_i}(x): X \rightarrow [0,1] \tag{3.1}$$

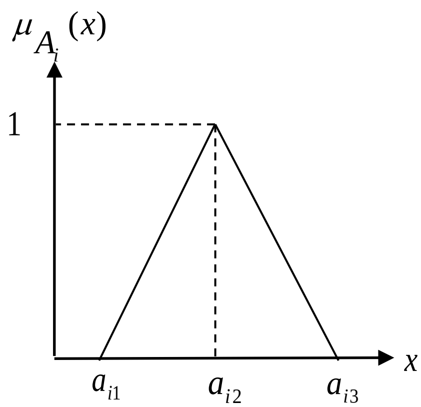


Fig. 3. 1: Membership function of a fuzzy set

Equation (3.1) and Fig. 3.1 indicate that value of membership of fuzzy set is defined within interval [0,1]. For instance, if $\mu_{cold}(x)$ is defined as membership function of 'cold' as weather condition for today and the membership value is approaching 0, then x is closer to 'not cold' or 'very hot'. In contrary, x is closer to 'cold' when the membership value is approaching 1. The following Table 3.1 illustrates differences between classical set theory and fuzzy set theory.

Table 3. 1: The representation of classical set theory and fuzzy set theory

Set theory	Representation	Membership degree
Classical	Binary	0 and 1
Fuzzy	Gradual	[0,1]

3.3 Fuzzy Sets Operations

There are three basic operations of fuzzy sets defined in the literature of fuzzy sets namely fuzzy union, fuzzy intersection and fuzzy complement. All of these operations are defined in by (Klir, Clair, & Yuan, 1997) the following definitions.

Let A_i and A_j be two fuzzy subsets of the universal interval U with membership functions for A_i and A_j are denoted by $\mu_{A_i}(x)$ and $\mu_{A_j}(x)$ respectively. Definition of fuzzy union, fuzzy intersection and fuzzy complement based on (Klir et al., 1997) are given as

a) Fuzzy union of A_i and A_j is denoted by $A_i \cup A_j$ such that the membership function is defined as

$$\mu_{A_i \cup A_j}(x) = \max\lfloor \mu_{A_i}(x), \mu_{A_{ji}}(x) \rfloor, \text{ for all } x \in U$$

b) Fuzzy intersection of A_i and A_j is denoted by $A_i \cap A_j$ such that the membership function is defined as

$$\mu_{A_i \cap A_j}(x) = \min\lfloor \mu_{A_i}(x), \mu_{A_{ji}}(x) \rfloor, \text{ for all } x \in U$$

c) Fuzzy complement of A_i is denoted by $\mu_{\bar{A}_i}(x)$ such that the membership function is defined as

$$\mu_{\bar{A}_i}(x) = 1 - \mu_{A_i}(x), \text{ for all } x \in U$$

3.4 Fuzzy sets

As discussed in Section 2.3, three types of fuzzy sets are pointed out in the literature of fuzzy sets namely type-1 fuzzy sets, type-2 fuzzy sets and z-numbers where all of them are defined chronologically as follows.

3.4.1 Type-1 Fuzzy Sets

In Section 2.3.1, type-1 fuzzy sets are chronologically developed as the first fuzzy numbers are established in literature of fuzzy sets (Lotfi A. Zadeh, 1965). As fuzzy numbers are actually type-1 fuzzy sets, definition of fuzzy numbers given by the (Dubois & Parade, 1983) which reflects as the definition of type-1 fuzzy sets, is as follows.

Definition 3.2: (Dubois & Parade, 1983) A type-1 fuzzy set A_i is a fuzzy subset of the real line $\Re$ that is both convex and normal that must satisfies the following properties:

i. μ_{A_i} is a continuous mapping from $\Re$ to the closed interval $[0, h]$, $0 \le h \le 1$,
ii. $\mu_{A_i}(x) = 0$, for all $x \in [-\infty, a_1]$,
iii. μ_{A_i} is strictly increasing on $[a_1, a_2]$,
iv. $\mu_{A_i}(x) = h$, for all $x \in [a_2, a_3]$ where *h* is a constant and $0 \le h \le 1$,
v. μ_{A_i} is strictly decreasing on $[a_3, a_4]$,
vi. $\mu_{A_i}(x) = 0$ for all $x \in [a_4, \infty]$,

where $a_1 \leq a_2 \leq a_3 \leq a_4$; a_1, a_2, a_3 and a_4 are component of a type-1 fuzzy set and real number $\Re$, while *h* represents the height or membership degree of a type-1 fuzzy set.

3.4.2 Type-2 Fuzzy Sets

Type-2 fuzzy sets are developed in the literature of fuzzy sets as extension of type-1 fuzzy sets, as the capability of type-1 fuzzy sets to represent human perception is inadequate (Wallsten & Budescu, 1995). As the type-2 fuzzy sets are used in this research work, thus the definition of type-2 fuzzy sets by (Mendel, John, & Liu, 2006) is as follows.

Definition 3.3: (Mendel et al., 2006) A type-2 fuzzy set A_i in a universe of discourse U is characterised by a type-2 membership function $\mu_{A_i}(x)$ which maps each element *x* in *U* a real number in the interval $[0,1]$.

The membership function for A_i, $\mu_{A_i}(x)$ is given as

$$A_i = \left\{ \left((x,u), \mu_{A_i}(x,u) \right) \middle| \forall x \in U, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{A_i}(x,u) \leq 1 \right\} \tag{3.2}$$

where J_x represents an interval in $[0,1]$.

According to (Mendel et al., 2006), another representation of type-2 fuzzy set is given in the following equation depicted as

$$A_i = \int_{x \in U} \int_{u \in J_x} \mu_A(x,u) / (x,u) \tag{3.3}$$

where $J_x \subseteq [0,1]$ and $\int\int$ represents the union over all allowable *x* and *u*.

It has to be noted that from equation (3.3), if $\mu_A(x,u) = 1$, then A_i is known as an interval type-2 fuzzy set. It is worth mentioning here that interval type-2 fuzzy set is a special case of type-2 fuzzy set (Mendel et al., 2006) where it can be represented by the following equation

$$A_i = \int_{x \in U} \int_{u \in J_x} 1/(x,u) \tag{3.4}$$

where $J_x \subseteq [0,1]$.

Interval type-2 fuzzy sets are utilised in the research works as the frequent used type-2 fuzzy set in the literature. According to (L.A. Zadeh, 1975), representation of interval type-2 fuzzy sets using number is called as interval type-2 fuzzy numbers. The following Fig. 3.2 illustrates interval type-2 fuzzy set.

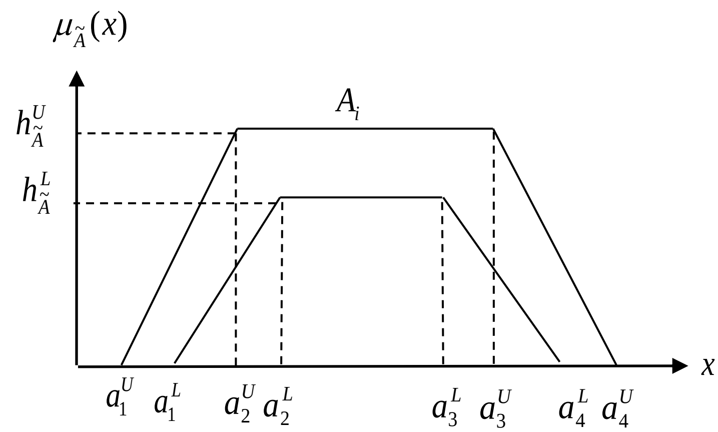


Fig. 3. 2: Interval type-2 fuzzy set

It is noticeable that type-2 fuzzy set in Fig. 3.2 is more complex in terms of representation where this indicates that type-2 fuzzy set needs more complicated computational technique than type-1 fuzzy set. According to (Greenfield & Chiclana, 2011), there are numerous defuzzification strategies developed in the literature of fuzzy sets which plan on converting type-2 fuzzy numbers into type-1 fuzzy numbers. This strategy is intentionally introduced to reduce the complexity of type-2 fuzzy numbers without losing information on the computational results. Among them that consider this strategy are (Karnik & Mendel, 2001a), (Nie & Tan, 2008), (Wu & Mendel, 2009) and (Greenfield & Chiclana, 2011). Nevertheless, based on a thorough comparative analysis made by (Greenfield & Chiclana, 2011) on all the aforementioned methods, (Nie & Tan, 2008) reduction method outperforms other approaches on reducing type-2 fuzzy set into type-1 fuzzy set. Therefore, without loss of generality of the reduction method is as follows (Nie & Tan, 2008).

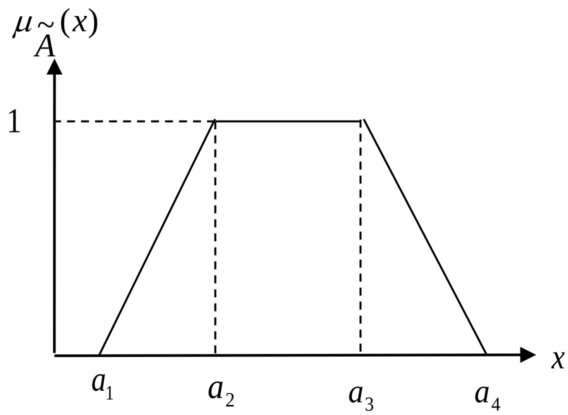
$$\mu_T(x_A) = \frac{1}{2}\left(\mu_L(x_A) + \mu_U(x_A)\right) \tag{3.5}$$

where T is the resultant type-1 fuzzy set.

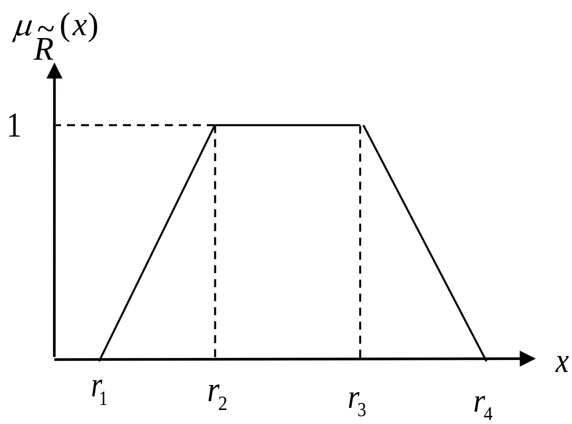
3.4.3 Z-Numbers

According to (Lotfi A. Zadeh, 2011a), z-numbers are the latest type of fuzzy numbers introduced in the literature of fuzzy sets. Definition of z-numbers given by (Kang et al., 2012a) is as follows.

Definition 3.4: (Kang et al., 2012a) A z-number is an ordered pair of fuzzy set denoted as $Z = (\tilde{A}, \tilde{R})$. The first component, $\tilde{A}$ is known as the restriction component where it is real-valued uncertain on *X* whereas the second component $\tilde{R}$, is a measure of reliability for $\tilde{A}$. The following Fig. 3.3 illustrates z-number based on (Kang et al., 2012a) definition.



$$\tilde{A} = (a_1, a_2, a_3, a_4; 1)$$



$$\tilde{R} = (r_1, r_2, r_3, r_4; 1)$$

Fig. 3. 3: A z-number, $Z = (\tilde{A}, \tilde{R})$

As mentioned in Chapter 2, z-numbers describe better representation as compared to type-1 fuzzy sets and type-2 fuzzy sets. This is due to the fact that z-numbers (level 3) are classified as the highest level in terms of generalised types than type-1 and type-2 fuzzy sets which level 2 (Lotfi A. Zadeh, 2011a). Therefore, (Lotfi A. Zadeh, 2011a) suggests any computational work involving z-numbers that need to be reduced into certain level without losing the informativesness of the computational results. This suggestion is taken into account by (Kang et al., 2012b) where a method of converting z-numbers into type-1 fuzzy sets or regular fuzzy sets based on Fuzzy Expectation of a fuzzy set is proposed. With no loss of generality of (Kang et al., 2012b) work, the conversion process of z-numbers into regular fuzzy numbers is as follows.

Step 1: Convert the reliability component, $\tilde{R}$ into crisp value, α using the following equation.

$$\alpha = \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{R}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\tilde{R}}(x)dx} \tag{3.6}$$

Note that, α represents the weight of the reliability component of a z-number.

Step 2: Add the weight of the reliability component, $\tilde{R}$ to the restriction component, $\tilde{A}$. The z-number is now defined as weighed restriction of z-number and can be denoted as

$$\tilde{Z}^{\alpha} = \left\{\left\langle x, \mu_{\tilde{A}'}(x)\right\rangle \middle| \mu_{\tilde{A}^{\alpha'}}(x) = \alpha\mu_{\tilde{A}}(x), x \in [0,1]\right\} \tag{3.7}$$

Step 3: Convert the weighted restriction of z-number into a fuzzy number which can be represented as

$$\tilde{Z}' = \left\{\left\langle x, \mu_{Z'}(x)\right\rangle \middle| \mu_{Z'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), x \in [0,1]\right\} \tag{3.8}$$

In (Kang et al., 2012b), it is shown that the process of converting z-numbers into regular fuzzy numbers was sensible and logical because the results obtained by the study indicates that a z-number is reduced into a lower level of generality which is a regular fuzzy number, but the computational informative is unaffected. Moreover, the conversation of a z-number into regular fuzzy number is reasonable due to the fact that both $\tilde{Z}^{\alpha}$ and $\tilde{Z}'$ are basically the same when the Fuzzy Expectation Theorem is applied.

3.5 Forms of Fuzzy Sets

This section covers discussion in terms of several forms of fuzzy numbers which are found in the literature of fuzzy sets. It has to be noted that all descriptions provided in this section focus only in type-1 fuzzy sets. As for type-2 fuzzy sets and z-numbers, their discussion are similar to in type-1 fuzzy sets as both type-2 fuzzy sets and z-numbers are extension of type-1 fuzzy sets. Therefore, any description of type-1 fuzzy sets is provided in the following subsections that are applicable to type-2 fuzzy sets and z-numbers as well. Therefore, a generic term of fuzzy numbers is used in this case to indicate that it covers type-1 fuzzy sets, type-2, fuzzy sets and z-numbers.

3.5.1 Linear Fuzzy Sets

According to (S.-M. Chen & Chen, 2009), fuzzy numbers are divided into two types namely linear and non-linear. Nevertheless, linear fuzzy numbers are often used in many decision making situations since non-linear fuzzy numbers are too complex to handle and they are normally transformed into linear type for convenience (M. Y. Chen & Linkens, 2004). In literature of fuzzy sets, there are two linear types of fuzzy numbers which are often utilised namely triangular and trapezoidal fuzzy numbers. However, there is another fuzzy number that is rather extensively used in the literature of decision making which is a singleton fuzzy number. It is worth mentioning here that all of these mentioned fuzzy numbers are used throughout the thesis. Thus, the following Definition 3.5 and Fig. 3.4 are definition and illustrations of triangular fuzzy number respectively while Definition 3.6 and Fig. 3.5 are definition and illustration for trapezoidal fuzzy number respectively.

Definition 3.5: (Van Laarhoven & Pedrycz, 1983) A triangular fuzzy number A_i is represented by the following membership function. Fig. 3.4 illustrates the representation of triangular fuzzy number.

$$\mu_{\tilde{A}}(x) = (a_1, a_2, a_3; 1) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & if \quad a_1 \le x \le a_2 \\ \dfrac{(x-a_3)}{(a_2-a_3)} & if \quad a_2 \le x \le a_3 \\ 0 & otherwise \end{cases} \tag{3.9}$$

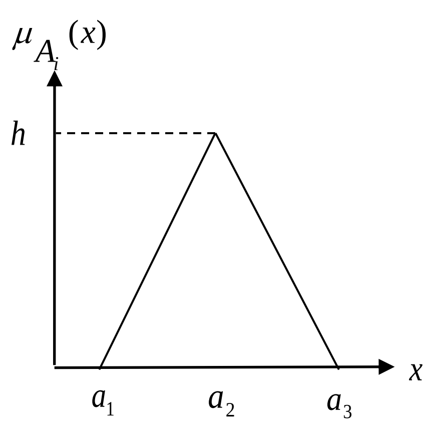


Fig. 3. 4: A triangular fuzzy number

Definition 3.6: A trapezoidal fuzzy number A_i is represented by the following membership function given by

$$\mu_{\tilde{A}}(x) = (a_1, a_2, a_3, a_4; 1) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} & if \quad a_1 \le x \le a_2 \\ h & if \quad a_2 \le x \le a_3 \\ \dfrac{(x - a_4)}{(a_3 - a_4)} & if \quad a_3 \le x \le a_4 \\ 0 & otherwise \end{cases} \tag{3.10}$$

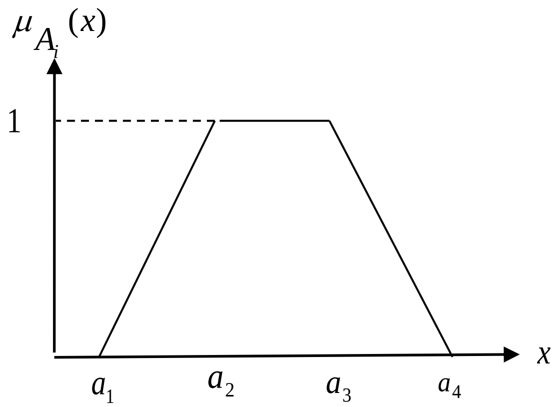


Fig. 3. 5: A trapezoidal fuzzy number

It has to be noted here that for trapezoidal fuzzy number, if $a_2 = a_3$, then a fuzzy number is in the form of a triangular fuzzy number (Cheng, 1998). While, if $a_1 = a_2 = a_3 = a_4$ or $a_1 = a_2 = a_3$ for both trapezoidal and triangular fuzzy numbers, respectively, then both are in the form of singleton fuzzy number (S.-M. Chen & Chen, 2009). The following Fig. 3.6 illustrates singleton fuzzy numbers.

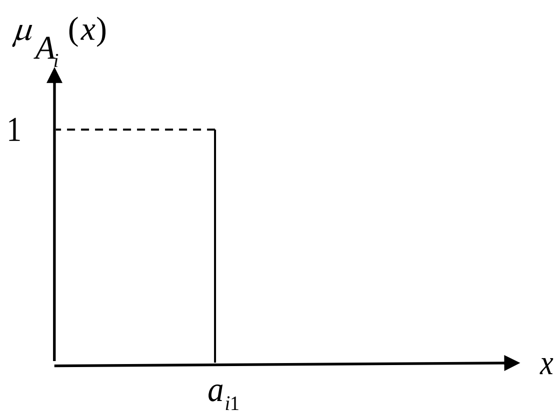


Fig. 3. 6: A singleton fuzzy number

3.5.2 Generalised Fuzzy Sets

This subsection provides discussion on another form of fuzzy numbers which is generalised fuzzy numbers. According to (S. J. Chen & Chen, 2003), a fuzzy number is better represented by generalised fuzzy numbers. This is because generalised fuzzy numbers provide a consistent representation for any fuzzy number even if any shape of fuzzy number is utilised. Starting from this point until the last part of this chapter, only trapezoidal fuzzy numbers are utilised as medium of representation. This is due to the fact that both triangular and singleton fuzzy numbers are special cases of trapezoidal fuzzy numbers ((Cheng, 1998) and (S. J. Chen & Chen, 2003)). Therefore, without loss of generality, definition of generalised trapezoidal fuzzy numbers is as follows.

Definition 3.7: (S. J. Chen & Chen, 2003) Generalised trapezoidal fuzzy number A_i is a fuzzy number $A_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; h_{A_i})$ where $0 \leq a_{i1} \leq a_{i2} \leq a_{i3} \leq a_{i4} \leq 1$ with height $h_{A_i} \in [0,1]$.

It is worth pointing out here that in this research work, the consideration of generalised trapezoidal is utilised in term of linguistic scales of type-1 fuzzy sets can be implemented for type-2 fuzzy sets and z-numbers as well.

3.6 Defuzzification

This section defines the defuzzification of fuzzy sets that is represented the process of defuzzification of type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Thus, all definition made on defuzzification of type-1 fuzzy sets are also significant for defuzzification of type-2 fuzzy sets and z-numbers.

3.6.1 Defuzzification Operation

The generic fuzzy system can be represented by the following transformation of real number to another real number (Roychowdhury & Pedrycz, 2001):

$$\Re \rightarrow Fuz(\Re) \rightarrow \tau \rightarrow IE(\tau) \rightarrow \tau' \rightarrow Def(\tau') \rightarrow \Re \qquad (3.11)$$

where:

$\Re$: real domain of real numbers

$Fuz(\Re) \rightarrow \tau$: Fuzzification process

τ : domain of fuzzy sets

$IE(\tau) \rightarrow \tau^{'}$: Inference engine

$Def(\tau) \rightarrow \Re$: Defuzzification process

The used of defuzzification operation is geared toward studies of a linguistic reconstruction mechanism from fuzzification operation. A defuzzification operation can be denoted by the following transformation fuzzy numbers into real number as follows:

$$\Re^{'} = Def\ /(Fuz(\Re)) \tag{3.12}$$

3.6.2 Properties of Defuzzification

The properties of defuzzification summarised by (Roychowdhury & Pedrycz, 2001) are identified as follows:

Property 1: *A defuzzification operator always computes to one numeric value.*

Any defuzzified value must has single or unique value, not ambiguity. The defuzzification operator is always injective. Clearly, two fuzzy sets can have same defuzzified value. It is assumed that, the defuzzified value is always within the support set of the original fuzzy set.

Property 2: *The membership function determines the defuzzified value.*

The membership function is important in determining the defuzzified value, not only core area. Some of the researchers ignore the membership function while running the defuzzification process. In this sense, concentration of fuzzy set monotonically leads to the normal or non-normal fuzzy sets.

Property 3: *The defuzzified value of two triangular-operated fuzzy sets is always continued within the bounds of individual defuzzified values.*

If fuzzy set $C_f = T(A_f, B_f)$ A_f where A_f and B_f are fuzzy sets and *T* is the *T-norm,* $Def(A_f) \leq Def(C_f) \leq Def(B_f)$, and so it is true for *T-conorm (T*)* $C_f = T^*(A_f, B_f)$.

Property 4: *In the case of prohibitive information, the defuzzified value should fall in the permitted zone.*

In many application of defuzzification operation in specific situations, the centre of the largest area strategies was found to be effective. The defuzzified value must be fall in the permitted zone in area of *x*-axis.

3.7 Fuzzy Multi Criteria Decision Making

3.7.1 Fuzzy Analytic Hierarchy Process

The steps in fuzzy AHP are presented as follows (Vinodh et al., 2014):

Step 1: Building the evaluation hierarchy systems for evaluating the best alternative among given alternatives considering the various criteria involved.

Step 2: Determining the assessment dimensions weights using fuzzy numbers.

The reason for using fuzzy numbers is that is intuitively easy for decision makers to use and calculate the evaluation for questionnaire.

Step 3: Determining the weights for the criteria involved.

The pairwise comparison matrix is constructed in order to present the preference of one criterion over the other by entering the judgement values by the decision makers. The formulation of aggregated process is calculated using Geometric mean method r_i:

$$\tilde{r}_{ij} = (\tilde{a}_{ij}^1 \times \tilde{a}_{ij}^2 \times \ldots \times \tilde{a}_{ij}^n)^{1/k} \qquad (3.13)$$

where k is the number of decision makers and $i=1,2,\ldots m; j=1,2,\ldots n$.

Step 4: The weight of each alternative is determined using normalising the matrix

This is done by using equation (3.14):

$$w_i = r_i \times (r_1 + r_2 + r_3 + \ldots + r_n)^{-1} \qquad (3.14)$$

Step 5: Defuzzify each weight from Step 4 using defuzzification method proposed by Best Non-Fuzzy Performance (BNP).

The defuzzification process is utilised in order to handle fuzzy sets. Then, normalization process is followed after defuzzification process. This is done by normalizing the matrix.

$$BNP \quad value = \left[(u-l) + (m-l)\right]/3 + l \qquad (3.15)$$

Step 6: Criteria are ranked based on the BNP values. The alternative that having larger BNP value is considered to have a greater impact when compared with other alternatives.

3.7.2 Consistent Fuzzy Preference Relations

Consistent fuzzy preference relations was proposed by (Herrera-Viedma, Herrera, Chiclana, & Luque, 2004) for constructing the decision matrices of pairwise comparisons based on additive transitivity property. Referring to (Kamis, Abdullah, Mohamed, Sudin, & Ishak, 2011), a fuzzy preference relation R on the set of the criteria or alternatives A is a fuzzy set stated on the Cartesian product set $A \times A$ with the membership function $\mu_R : A \times A \to [0,1]$. The preference relation is denoted by $n \times n$ matrix $R = (r_{ij})$ where $r_{ij} = \mu_y(a_i, a_j)$ $\forall i,\ j \in \{1,...,n\}$. The preference ratio, r_{ij} of the alternative a_i to a_j is determined by

$$r_{ij} = \begin{cases} 0.5 & a_i \text{ is different to } a_j \\ (0.5,1) & a_i \text{ is preferred than } a_j \\ 1 & a_i \text{ is absolutely preferred than } a_j \end{cases} \tag{3.16}$$

The preference matrix R is presumed to be additive reciprocal, $p_{ij} + p_{ji} = 1,\ \forall i,\ j \in \{1,...,n\}$. Several propositions are associated to the consistent additive preference relations as follows:

Proposition 4.1 (T. C. Wang & Chen, 2007): *Consider a set of criteria or alternatives,* $X = \{x_1,...,x_n\}$, *and associated with a reciprocal multiplicative preference relation* $A = (a_{ij})$ *for* $a_{ij} \in \left[\frac{1}{9}, 9\right]$. *Then, the corresponding reciprocal fuzzy preference relation,* $P = (p_{ij})$ *with* $p_{ij} \in [0,1]$ *associated with A is given by the following formulation*

$$p_{ij} = g(a_{ij}) = \frac{1}{2}(1 + \log_9 a_{ij}) \tag{3.17}$$

Generally, if $a_{ij} \in \left[\frac{1}{n}, n\right]$, *then* $\log_n a_{ij}$ *is used, in particular, when* $a_{ij} \in \left[\frac{1}{9}, 9\right]$; $\log_9 a_{ij}$ *is considered as in the above proposition because* a_{ij} is between $\frac{1}{9}$ *and 9. If* a_{ij} is between $\frac{1}{7}$ *and 7, then* $\log_7 a_{ij}$ *is used.*

Proposition 4.2 (T. C. Wang & Chen, 2007): *For a reciprocal fuzzy preference relation* $P = (p_{ij})$ *, the following statements are equivalent*

$$(i) \qquad p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \ \forall i, j, k \tag{3.18}$$

$$(ii) \qquad p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \ \forall i < j < k \tag{3.19}$$

Proposition 4.3 (T. C. Wang & Chen, 2007): *For a reciprocal fuzzy preference relation* $P = (p_{ij})$*, the following statements are equivalent*

$$(i) \qquad p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \ \forall i < j < k \tag{3.20}$$

$$(ii) \qquad p_{i(i+1)} + p_{(i+1)(i+2)} + \ldots + p_{(j-1)j} + p_{ji} = \frac{j-i+1}{2}, \ \forall i < j \tag{3.21}$$

Proposition 4.3 is crucial because it can be used to construct a consistent fuzzy preference relations form the set of $n-1$ values $\{p_{12}, p_{23}, \ldots, p_{n-1}\}$. A decision matrix with entries that are not in the interval $[0,1]$, but in an interval $[-c, 1+c]$, $c > 0$, can be obtained by transforming the obtained values using a transformation function that preserves reciprocity and additive consistency with the function

$$f : [-c, 1+c] \rightarrow [0,1], \ f(x) = \frac{(x+c)}{(1+2c)} \tag{3.22}$$

3.7.3 Fuzzy Technique for Order of Preference by Similarity to Ideal Solution

The formal conventional fuzzy TOPSIS procedure is illustrates as follows (Sun, 2010):

Step 1: Create fuzzy performance or decision matrix.

The judgmental values from decision makers for each decision alternatives corresponding to each alternative are tabulated with fuzzy numbers as entries.

Step 2: Normalised the fuzzy performance matrix.

The normalised fuzzy decision matrix denoted by $\tilde{R}$ is shown as following formula:

$$\tilde{R} = \left[\tilde{r}_{ij}\right]_{m \times n} \tag{3.23}$$

where *i=1,2,…m; j=1,2,…n*.

Then, the normalization process can be performed by following formula:

$$\tilde{r}_{ij} = \left(\frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+}\right), \quad u_j^+ = \max{}_i \left\{u_{ij} \middle| i = 1,2,\ldots,n\right\} \tag{3.24}$$

The best aspired level is 1, otherwise the worst is 0.

The normalised , $\tilde{r}_{ij}$ is still triangular fuzzy numbers. For trapezoidal fuzzy numbers, the normalization process can be conducted in the same way.

Step 3: Construct the weighted normalised decision matrix.

Multiply each column of normalised decision matrix by its associated weight. An element of the new matrix is:

$$v_{ij} = w_j \times r_{ij} \tag{3.25}$$

where, $i = 1,\ldots,m$ and $j = 1,\ldots,n$.

Step 4: Determine the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS).

The FPIS (A^+) and FNIS (A^-) are defined as aspiration levels and worst levels respectively in terms of the weighted normalised values. The range belong to the closed interval [0,1].

Fuzzy Positive Idea Solution (FPIS):

$$A^+ = \left\{v^*{}_1,\ldots,v^*{}_j,\ldots,v^*{}_n\right\} \tag{3.26}$$

Fuzzy Negative Idea Solution (FNIS):

$$A^- = \left\{v^-{}_1,\ldots,v^-{}_j,\ldots,v^-{}_n\right\} \tag{3.27}$$

where

$$\tilde{v}^{+}{}_{j} = (1,1,1) \times \tilde{w}_{j} = (lw_{j}, mw_{j}, uw_{j}) \text{ and}$$

$$\tilde{v}^{-}{}_{j} = (0,0,0) \times \tilde{w}_{j} = (lw_{j}, mw_{j}, uw_{j}), \ j = 1,2,...,n.$$

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^{+}$ and $\tilde{d}_i^{-}$ of each alternative from formulation A^{+} and A^{-} can be calculated by the area of compensation method:

$$\bar{d}_i^{+}(\tilde{v}_{ij}, \tilde{v}_j^{+}) = \sum_{j=1}^{n} d(\tilde{v}_{ij}, v_{ij}^{+}) \tag{3.28}$$

$$\bar{d}_i^{-}(\tilde{v}_{ij}, \tilde{v}_j^{-}) = \sum_{j=1}^{n} d(\tilde{v}_{ij}, v_{ij}^{-}) \tag{3.29}$$

Step 6: Find the closeness coefficient, CC_i (relative gaps degree) and improve alternatives for achieving aspiration levels in each criteria.

$$\overline{CC}_i = \frac{\bar{d}_i^{-}}{\bar{d}_i^{+} + \bar{d}_i^{-}} = 1 - \frac{\bar{d}_i^{+}}{\bar{d}_i^{+} + \bar{d}_i^{-}} \tag{3.30}$$

where, $\frac{\bar{d}_i^{-}}{\bar{d}_i^{+} + \bar{d}_i^{-}}$ is satisfaction degree in *ith* alternative and $\frac{\bar{d}_i^{+}}{\bar{d}_i^{+} + \bar{d}_i^{-}}$ is fuzzy gaps degree in *ith* alternative.

3.7.4 Fuzzy Multidisciplinary Optimization Compromise Solution

The generalised fuzzy VIKOR can be computed as steps follows (Salehi, 2015):

Step 1: Construct the fuzzy performance decision matrix for alternatives' evaluation.

Step 2: Compute normalised fuzzy performance deicision matrix by using equation as follows

Assume *m* alternatives and *n* criteria.

$$f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^{2}}} \tag{3.31}$$

where, $i = 1,2,..., m; \quad j = 1,2,..., n$.

Step 3: Determine the fuzzy best value (FBV= f_j^*) and the fuzzy worst value (FWV= f_j^-).

If we assume the *j*th function represents a benefit, then $f_j^* = \max f_{ij}$ (or setting an inspired level) and $f_j^- = \min f_{ij}$ (or setting a tolerate level). Alternatively, if we assume the *j*th function represents a cost/ risk, the then $f_j^* = \min f_{ij}$ (or setting an inspired level) and $f_j^- = \max f_{ij}$ (or setting a tolerate level).

Step 4: Compute the values of S_i and $R_i; i = 1,2,...m$ by the equations below

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - \tilde{x}_{ij}) / (f_j^* - f_{ij}) \tag{3.32}$$

$$R_i = Max\left[w_j (f_j^* - \tilde{x}_{ij}) / (f_j^* - f_{ij})\right] \tag{3.33}$$

where w_j are the weights of criteria, that expressing their relative importance.

Step 5: Compute the index values of Q_i;1,2,..., by the equation below.

$$Q_i = \upsilon\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-\upsilon)\left[\frac{R_i - R^*}{R^- - R^*}\right] \tag{3.34}$$

where, $S^- = \max value_i S_i$, $S^* = \min_i S_i$, $R^- = \max value_i R_i$, $R^* = \min_i R_i$ and υ is introduced as the weight of the strategy "the majority of criteria" (or "the maximum group utility") and usually $\upsilon = 0.5$.

Step 6: Defuzzify fuzzy number Q_i and rank the alternatives, sorting by the values S_i, R_i and Q_i in decreasing order.

Defuzzification process is computed using (S. H. Chen & Hsieh, 1999) based on graded mean integration method.

$$mean(a) = \frac{(a_1 + 4a_2 + a_3)}{6} \tag{3.35}$$

Consequently, the smaller the value of Q_i, the better the alternative.

3.8 Sensitivity Analysis

3.8.1 Introduction

The MCDM techniques always deal with unstable and changeable data inputs. Therefore, the sensitivity analysis after problem solving that can effectively contribute to making accurate decision by assume that a set of weights for criteria or alternatives then obtained a new set of weights for them, so that the efficiency of alternatives has become equal or their order has changed. It is focused on determining the most sensitive criteria and the least value of the change. It is clearly indicate that the sensitivity analysis is calculated the changing in the final score of alternatives when change occurs in the weight of one alternative. The results of MCDM techniques is crucially needed to validate and calibrate in analysing the quality and how robustness of MCDM techniques to reach a good decision under different conditions. In doing so, a sensitivity analysis method proposed by (Amini & Alinezhad, 2011) is applied in this research work.

3.8.2 Computational Process

Assume that the vector for the weights of criteria is $W^t = (w_1, w_2, ..., w_k)$ where in weights are normalised and sum of them is 1, that is

$$\sum_{j=1}^{k} w_t = 1 \tag{3.36}$$

From this assumption, if the weight of one criterion changes, then the weight of other criteria change accordingly, and the new vector of weights transformed into

$$W'^t = (w'_1, w'_2, ..., w'_k) \tag{3.37}$$

The following theorem shows changes in the weight of criteria.

Theorem 4.4.

In MCDM technique, if the weight of criteria P^{th}, changes as Δ_p, the the weight of other criteria change as Δ_j; $j = 1,2,...,k$.

$$\Delta_j = \frac{\Delta_p \times w_j}{w_p - 1} \tag{3.38}$$

where $j = 1,2,...,k$, $j \neq p$.

Proof:

If the new weight of criteria are w'_j and the new weight of criterion P^{th} changes are

$$w'_p = w_p + \Delta_p \tag{3.39}$$

Then the new weight of other criteria would change as

$$w'_j = w_j + \Delta_j \tag{3.40}$$

where $j = 1,2,...,k$, $j \neq p$.

The sum of weight must be equal to 1 then

$$\sum_{j=1}^{k} w'_j = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_j \Rightarrow \sum_{j=1}^{k} \Delta_j = 0 \tag{3.41}$$

Therefore

$$\Delta_p = -\sum_{\substack{j=1 \\ j \neq p}}^{k} \Delta_p \tag{3.42}$$

Where from equation (3.38)

$$\Delta_j = \frac{\Delta_p \times w_j}{w_p - 1}, \; j = 1,2,...,k \; , \; j \neq p$$

Since

$$\begin{aligned} -\Delta_p &= \sum_{\substack{j=1 \\ j \neq p}}^{k} \Delta_j = \sum_{\substack{j=1 \\ j \neq p}}^{k} \frac{\Delta_p \times w_j}{w_p - 1} \\ &= \frac{\Delta_p}{w_p - 1} \sum_{\substack{j=1 \\ j \neq p}}^{k} w_j \\ &= \frac{\Delta_p}{w_p - 1}(1 - w_p) = -\Delta_p \end{aligned} \tag{3.43}$$

In a MCDM problem, of the weight of the P^{th} criteria changes from w_p to w'_p as

$$w'_p = w_p + \Delta_p \tag{3.44}$$

Then, the weight of other criteria would change as

$$w'_j = \frac{1 - w_p - \Delta_p}{1 - w_p} \times w_j = \frac{1 - w'_p}{1 - w_p} \times w_j \tag{3.45}$$

where $j = 1,2,...,k$, $j \neq p$.

Since, for $j = 1,2,...,k$, $j \neq p$ we have

$$w'_j = w_j + \Delta_j = w_j + \frac{\Delta_p \times w_j}{w_p - 1} = \frac{w_j(w_p - 1) + (\Delta_p \times w_j)}{w_p - 1}$$

$$\Rightarrow w'_j = \frac{(1 - w_p - \Delta_p) \times w_j}{1 - w_p} = \frac{1 - w'_p}{1 - w_p} \times w_j \tag{3.46}$$

where $j = 1,2,...,k$, $j \neq p$.

Then, new vector for weights of criteria would be $W'^t = (w'_1, w'_2, ..., w'_k)$, that is

$$w'_j = \begin{cases} w_j + \Delta_p & j = p \\ \dfrac{1 - w'_p}{1 - w_p} \times w_j & j \neq p, \quad j = 1,2,...,k \end{cases} \tag{3.47}$$

$$w'_p = w_p + \Delta_p \Rightarrow \begin{cases} if & w'_p > w_p \Rightarrow w'_j < w_j \\ if & w'_p < w_p \Rightarrow w'_j > w_j \end{cases} \tag{3.48}$$

where $j = 1,2,...,k$, $j \neq p$.

The sum of new weights of criteria that are obtained in (3.48) is 1, because

$$\sum_{j=1}^{k} w'_j = \sum_{\substack{j=1 \\ j \neq p}}^{k} w'_j + w'_p = \sum_{\substack{j=1 \\ j \neq p}}^{k} \frac{w_j(1 - w_p - \Delta_p)}{1 - w_p} + w_p + \Delta_p$$

$$= \frac{(1 - w_p - \Delta_p)}{1 - w_p} \sum_{\substack{j=1 \\ j \neq p}}^{k} w_j + w_p + \Delta_p$$

$$= \frac{(1 - w_p - \Delta_p)}{1 - w_p}(1 - w_p) + w_p + \Delta_p$$

$$= 1 - w_p - \Delta_p + w_p + \Delta_p = 1 \tag{3.49}$$

Corollary: In the new vector of weights that is obtained from (3.49), the weight's ratio is constant which except of criterion P^{th}, because new weights for criteria except for P^{th} is obtained by multiplying the constant value $\frac{(1 - w_p - \Delta_p)}{1 - w_p}$ to old weight of them, then the ratio of new weight of attributes C_i to new weight of criterion C_i for $i, j = 1,2,...,k$, $i, j \neq p$ is equal to the ration of old ones. That is shown below

$$\frac{w'_i}{w'_j} = \frac{w_i}{w_j};\ i, j = 1,2,...,k \ \ i, j \neq p \tag{3.50}$$

3.9 Summary of the Chapter

This chapter briefly discusses the technical part of tools and methods applied in this thesis work. It covers the definitions, basic notions and terminologies of fuzzy sets that consist of type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. In addition, the conventional fuzzy MCDM techniques and sensitivity analysis validation method are discussed in order to be applied in developing the proposed methodology and case studies for comparative analysis in Chapter 4, 5 and 6. Next chapter discusses the development of the first novelty in this research study which is intuitive multiple centroid defuzzification method in order to implement in fuzzy MCDM problems.

CHAPTER 4

INTUITIVE MULTIPLE CENTROID DEFUZZIFICATION

4.1 Overview

This chapter describes in detail the process of development of intuitive multiple centroid defuzzification method for fuzzy sets. In developing defuzzification method, a novel manner of computing intuitive multiple centroid method is presented in formulae based on the perspective of analytic geometric principle which consider the coordinate on the vertical axis is as important as the coordinate on the horizontal axis. In dealing with intuitionistic part, the defuzzification results obtained must be reasonable and consistent with human intuition or judgment. Most of the defuzzification methods capable to defuzzify the fuzzy numbers, but ignore the imprecision and fuzziness of the quantity of numbers themselves. Unlike other defuzzification centroid methods, the proposed defuzzification method defuzzify the fuzzy numbers and at the same time obtained the imprecision and fuzziness of original quantity. Likewise, it's presented very efficient computational procedures for fuzzy sets. The exact computational procedures are provided for type-1 fuzzy sets and its' extension for type-2 fuzzy sets and z-numbers.

The proposed intuitive multiple centroid defuzzification method is validated theoretically and empirically which determine reliability and consistency respectively. Reliability is a theoretical based – validation method in evaluating the novel intuitive multiple centroid using several properties that are considered for justifying the applicability of centroid for fuzzy numbers. The consistency represents an empirical based – validation evaluates the capability of the novel centroid method to correctly formulae that are consistent or better with established methods in considering human perception aspect. Both theoretical and empirical validations stated are thoroughly defined in this chapter but the implementations of intuitive multiple centroid are demonstrated in the following chapter. Thus, this chapter supports the next two chapters of the thesis. Details on those aforementioned points are extensively discussed in sections of this chapter. In the following chapter, the proposed intuitive multiple centroid defuzzification method would be applied in developing the new hybrid fuzzy MCDM model.

4.2 Intuitive Multiple Centroid for Type-1 Fuzzy Sets

In this section, a novel formulae on computing the intuitive multiple centroid defuzzification of fuzzy sets are developed. The formulation includes official models, elementary operations, basic properties and advanced applications. The presented formulae justify the proposed defuzzification method from the perspective of analytic geometry. In analytic geometry principle, also known as coordinate geometry or Cartesian geometry, the geometry study uses a coordinate system in Cartesian plane by a pair of numerical coordinates. Likewise, the intuitive multiple centroid should be determined naturally like the way of determining the coordinate on the horizontal axis. Following this method, some fundamental centroid formulae for fuzzy sets are proposed and derived based on simplified expressions for trapezoidal, triangular and singleton fuzzy numbers. In developing the intuitive multiple centroid method, the coordinates of the centroid on Cartesian plane are simply the average of the coordinates of the vertices. The vertices in intuitive multiple centroid are considering the median points among several parts of trapezoid shape of fuzzy numbers.

The word 'intuitive' refers to on feelings rather than facts or proof. Most people have an intuitive sense in making their judgements either right or wrong. While the word 'multiple' is formally defined as very many of the same type, or of different types. The 'centroid' refers to the centre of mass of a geometric object of uniform density. The select of fuzzy set functions affects how well fuzzy systems approximate function (Mitaim & Kosko, 1996). Since this has been the primary motivation for the proposed defuzzification method to be developed. In this sense, the intuitive multiple centroid defuzzification is relevant in context of human intuition or judgement that considers all possible fuzzy sets. The concept is similar to other centroid methods application, where the aim is to find the best centre point of a fuzzy set that is represented in crisp or single value. This proposed method is compared with other established centroid methods, (Shi-Jay Chen & Chen, 2002), (Y. M. Wang et al., 2006), (Liang et al., 2006) and (Shieh, 2007) regarding consistency.

4.2.1 Intuitive Multiple Centroid for Type-1 Fuzzy Sets Methodology

Let consider $\tilde{A} = (a_1, a_2, a_3, a_4; h_A)$ as the generalised trapezoidal fuzzy number and $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ be the centroid point for $\tilde{A}$ such that $\tilde{x}_{\tilde{A}}$ and $\tilde{y}_{\tilde{A}}$ are the horizontal x – axis and vertical y – axis of generalised fuzzy number $\tilde{A}$ respectively. The complete process for intuitive multiple centroid point, $IMC(\tilde{A})$ computation is signified as follows.

Step 1: Find the centroids of the three parts of α, β and γ in trapezoid plane representation as shown in Fig. 4.1. The trapezoid shape is divided into three parts, which are: 1) left triangle shape; 2) rectangle shape and; 3) right triangle shape. The sub centroids of right triangle shape, rectangle shape and left triangle shape represent as $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ respectively.

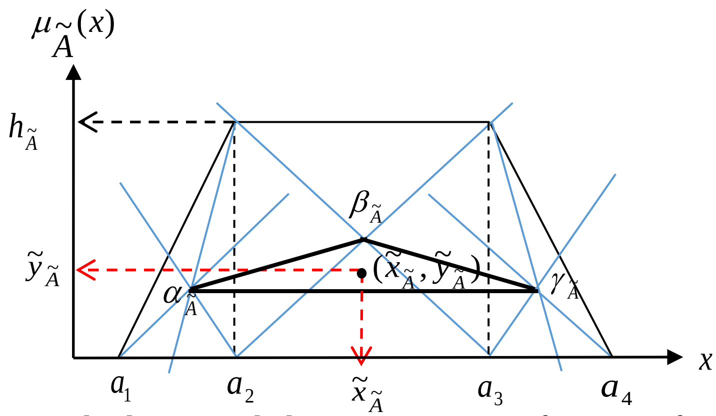


Fig. 4. 1: Intuitive multiple centroid plane representation for type-1 fuzzy set

The blue lines represent the median lines for sub centroid points. Theoretically, the intuitive multiple centroid defuzzification is based on the median points that cover centralised of the each shape properly. The partition of shapes are presented in Fig. 4.2, Fig. 4.3 and Fig. 4.4, where the sub centroid points of $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ based - median computation before there are connected each other to create another triangle plane.

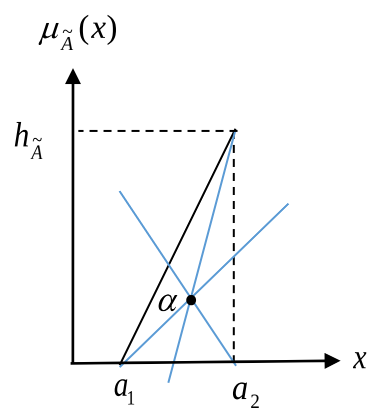


Fig. 4. 2: Sub centroid of left triangle $\alpha_{\tilde{A}}$

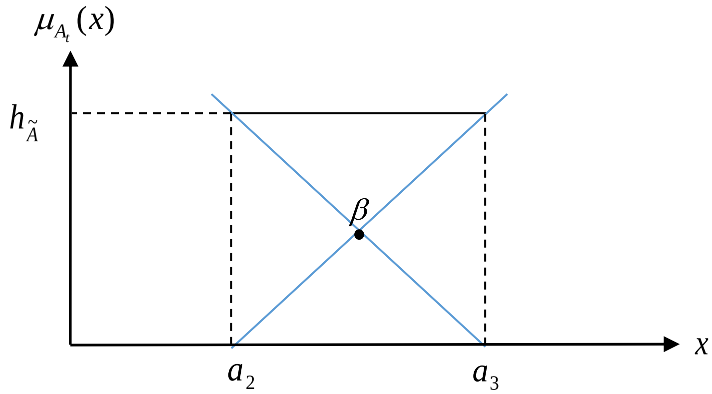


Fig. 4. 3: Sub centroid of rectangle, $\beta_{\tilde{A}}$

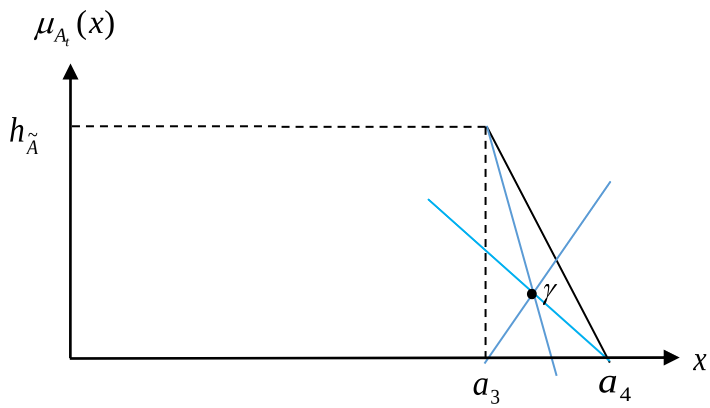


Fig. 4. 4: Sub centroid of right triangle, $\gamma_{\tilde{A}}$

The formulation for median point is based on the representation of the shape itself. A median of a triangle is a line segment from a vertex of the line triangle to the midpoint of the opposite side of vertex. Line segment represents the straight line which associates two points without extending beyond them (Fig. 4.5). Vertex is the common endpoint of two or more line segments (Fig. 4.6). The median point or centre point of rectangle shape is a half of the length of its shape. The key to prove for all rectangles, the midpoints of the diagonals are coincidental at the centre of the rectangle as shown in Fig. 4.3.

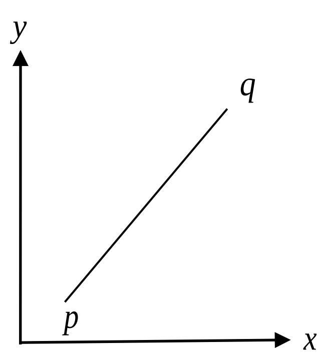


Fig. 4. 5: Line segment *pq*

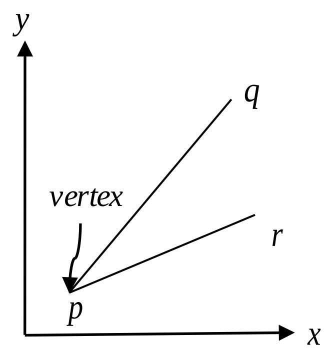


Fig. 4. 6: Vertex of line segment *pq* and *pr*

Step 2: Connect all vertices sub centroid points of $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ each other, where it will create another triangular plane inside of trapezoid plane as represented in Fig. 4.1. The formulation of sub centroid points of $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ are computed as follows.

1) *Sub centroid points of* $\alpha_{\tilde{A}}$ *formula.*

$$\alpha(x_{\tilde{A}}, y_{\tilde{A}}) = \left(a_1 + \left[\frac{2}{3}(a_2 - a_1) \right], \frac{h_{\tilde{A}}}{3} \right) \tag{4.1}$$

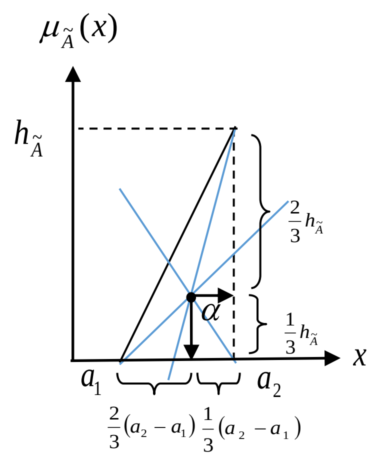


Fig. 4. 7: The intercept of median lines for sub centroid of left triangle, $\alpha_{\tilde{A}}$

Fig. 4.7 presents how the formulation for $\alpha(x_{\tilde{A}}, y_{\tilde{A}})$, sub centroid point of left triangle, $\alpha_{\tilde{A}}$ is produced. The centroid is a point of concurrency of the triangle. The centroid point is formed by all three medians are intersected each other. This point is often described as the triangle's centre of gravity. One of the properties of the centroid, it must be always located inside the shape. The centroid divides each median in a ratio of 2:1. In other words, the centroid always be 2/3 of the way long any given median.

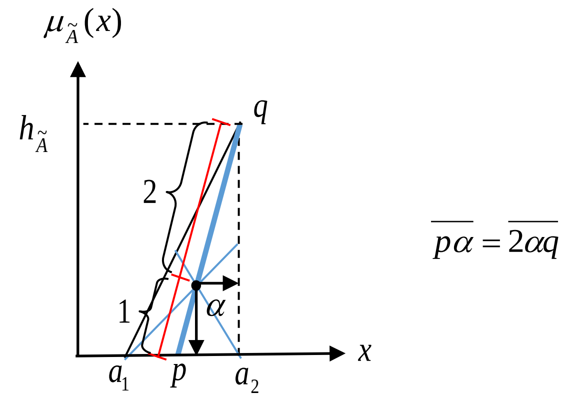


Fig. 4. 8: The divided segment line

From the Fig. 4.8, the centroid, α divides each median into two segments, the segment joining the centroid to the vertex multiplied by two is equal to the length of the line segment joining the midpoint to the opposite side where $\overline{p\alpha} = \overline{2\alpha q}$.

2) Sub centroid points of $\beta_{\tilde{A}}$ formula.

$$\beta(x_{\tilde{A}}, y_{\tilde{A}}) = \left(\frac{a_2 + a_3}{2}, \frac{h_{\tilde{A}}}{2} \right) \tag{4.2}$$

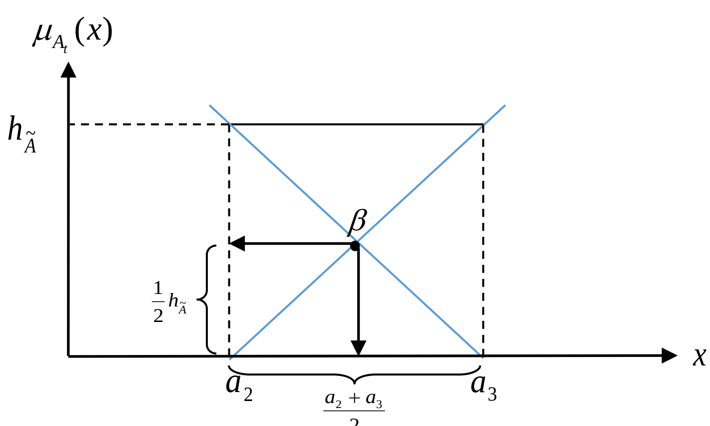


Fig. 4. 9: The intercept of median line for sub centroid of rectangle, $\beta_{\tilde{A}}$

Fig. 4.9 depicts the sub centroid point for of left rectangle, $\beta(x_{\tilde{A}}, y_{\tilde{A}})$. The centroid of rectangle is formed when two axes of symmetry intercept each other and the intersection locates the centroid by half.

3) Sub centroid points of $\gamma_{\tilde{A}}$ formula.

$$\gamma(x_{\tilde{A}}, y_{\tilde{A}}) = \left(a_4 + \left[\frac{2}{3}(a_3 - a_4) \right], \frac{h_{\tilde{A}}}{3} \right) \tag{4.3}$$

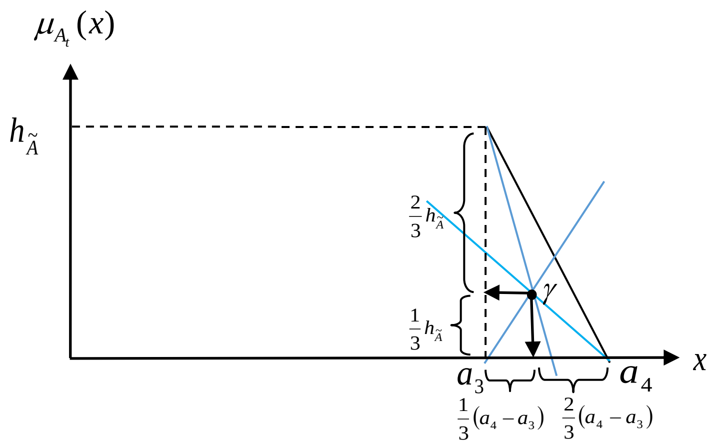


Fig. 4. 10: The intercept of median line of right triangle, $\gamma_{\tilde{A}}$

Fig. 4.10 depicts the representation of formulation developed for $\gamma(x_{\tilde{A}}, y_{\tilde{A}})$, sub centroid point of left triangle, $\gamma_{\tilde{A}}$. The explanation for centroid, $\gamma_{\tilde{A}}$ is same as sub centroid points of $\alpha_{\tilde{A}}$ formula.

The sub centroid points of $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ are calculated in coordinate point $(\tilde{x}, \tilde{y})$ because the consideration of the degree of membership values in dealing with subjective events. About this, (Cheng, 1998) claims that x value on the horizontal axis is the most important index. He also stated that in certain cases, the value of x can be act as minor index and y becomes the major index in fuzzy numbers. Thus, the consideration of y – axis plays an important role as x – axis.

Step 3: The centroid coordinate points of intuitive multiple centroid, $(\tilde{x}, \tilde{y})$ of fuzzy number $\tilde{A}$ with vertices $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ can be calculated as

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\beta(\tilde{x}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{x}_{\tilde{A}}) + \gamma(\tilde{x}_{\tilde{A}})}{2} - \beta(\tilde{x}_{\tilde{A}}) \right) \right], \beta(\tilde{y}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{y}_{\tilde{A}}) + \gamma(\tilde{y}_{\tilde{A}})}{2} - \beta(\tilde{y}_{\tilde{A}}) \right) \right] \right) \tag{4.4}$$

Intuitive multiple centroid can be summarised as

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}, \frac{7h_{\tilde{A}}}{18} \right) \tag{4.5}$$

where

$\tilde{x}_{\tilde{A}}$: the centroid on the horizontal x-axis

$\tilde{y}_{\tilde{A}}$: the centroid on the vertical y-axis

$(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$: the centroid point of fuzzy number $\tilde{A}$

The processes of getting the final centroid coordinate $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ are illustrated as follows.

Proving:

$$IMC(\tilde{x}_{\tilde{A}}) = \frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}$$

$$= \beta(\tilde{x}_{\tilde{A}}) + \left[\frac{2}{3} \left(\frac{\alpha(\tilde{x}_{\tilde{A}}) + \gamma(\tilde{x}_{\tilde{A}})}{2} - \beta(\tilde{x}_{\tilde{A}}) \right) \right]$$

$$= \left(\frac{a_2 + a_3}{2} \right) + \left[\frac{2}{3} \left(\frac{\left[a_1 + \left(\frac{2}{3}(a_2 - a_1) \right) \right] + \left[a_4 + \left(\frac{2}{3}(a_3 - a_4) \right) \right]}{2} - \left(\frac{a_2 + a_3}{2} \right) \right) \right]$$

$$= \left(\frac{a_2 + a_3}{2} \right) + \left[\frac{2}{3} \left(\frac{\left[a_1 + \left(\frac{2a_2}{3} - \frac{2a_1}{3} \right) \right] + \left[a_4 + \left(\frac{2a_3}{3} - \frac{2a_4}{3} \right) \right]}{2} - \left(\frac{a_2 + a_3}{2} \right) \right) \right]$$

$$= \left(\frac{a_2 + a_3}{2} \right) + \left[\frac{2}{3} \left(\frac{\left(\frac{a_1}{3} + \frac{2a_2}{3} \right) + \left(\frac{a_4}{3} + \frac{2a_3}{3} \right)}{2} - \left(\frac{a_2 + a_3}{2} \right) \right) \right]$$

$$= \left(\frac{a_2 + a_3}{2} \right) + \left[\frac{2}{3} \left(\left(\frac{a_1}{6} + \frac{2a_2}{6} \right) + \left(\frac{a_4}{6} + \frac{2a_3}{6} \right) - \left(\frac{a_2 + a_3}{2} \right) \right) \right]$$

$$= \frac{a_2}{2} + \frac{a_3}{2} + \left[\frac{2}{3}\left(\frac{a_1}{6} + \frac{2a_2}{6} + \frac{a_4}{6} + \frac{2a_3}{6} - \frac{a_2}{2} - \frac{a_3}{2}\right)\right]$$

$$= \frac{a_2}{2} + \frac{a_3}{2} + \left(\frac{2a_1}{18} + \frac{4a_2}{18} + \frac{2a_4}{18} + \frac{4a_3}{18} - \frac{2a_2}{6} - \frac{2a_3}{6}\right)$$

$$= \frac{2a_1}{18} + \frac{7a_2}{18} + \frac{7a_3}{18} + \frac{2a_4}{18}$$

$$= \frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}$$

Proving:

$$IMC(\tilde{y}_{\tilde{A}}) = \frac{7h_{\tilde{A}}}{18}$$

$$= \beta(\tilde{y}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{y}_{\tilde{A}}) + \gamma(\tilde{y}_{\tilde{A}})}{2} - \beta(\tilde{y}_{\tilde{A}})\right)\right]$$

$$= \frac{h_{\tilde{A}}}{2} + \left[\frac{2}{3}\left(\frac{\left(h_{\tilde{A}} - \frac{2}{3}h_{\tilde{A}}\right) + \left(h_{\tilde{A}} - \frac{2}{3}h_{\tilde{A}}\right)}{2} - \frac{h_{\tilde{A}}}{2}\right)\right]$$

$$= \frac{h_{\tilde{A}}}{2} + \left[\frac{2}{3}\left(\frac{1}{3}h_{\tilde{A}} - \frac{h_{\tilde{A}}}{2}\right)\right]$$

$$= \frac{h_{\tilde{A}}}{2} + \left[\frac{2}{3}\left(\frac{-h_{\tilde{A}}}{6}\right)\right]$$

$$= \frac{h_{\tilde{A}}}{2} + \left[\frac{-h_{\tilde{A}}}{9}\right]$$

$$= \frac{h_{\tilde{A}}}{2} - \frac{h_{\tilde{A}}}{9}$$

$$= \frac{7h_{\tilde{A}}}{18}$$

Centroid index of intuitive multiple centroid can be generated using Euclidean Distance by (Cheng, 1998) as below.

$$R(\tilde{A}) = \sqrt{\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}} \tag{4.7}$$

Hence

$$IMC(\tilde{A}) = \sqrt{\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}} \tag{4.8}$$

4.2.2 Illustrative Example

This section illustrates a numerical – based example which is used to demonstrate the utilisation of the intuitive multiple centroid method developed in Section 4.2. A complete illustration of utilising the intuitive multiple centroid method in this example is as follows.

Let $\tilde{A} = (12,13,15,16;0.9)$ be a generalised trapezoidal fuzzy number to calculate the centroid point of $\tilde{A}$, then the centroid point is computed using equation (4.5) and (4.8) as follows.

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}, \frac{7h_{\tilde{A}}}{18} \right)$$

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(12+16) + 7(13+15)}{18}, \frac{7(0.9)}{18} \right)$$

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = (14, 0.35)$$

Hence, the centroid index of intuitive multiple centroid for $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ type-1 fuzzy set can be calculated as

$$IMC(\tilde{A}) = \sqrt{\left(\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}\right)}$$

$$IMC(\tilde{A}) = \sqrt{\left(14^2 + 0.35^2\right)}$$

$$IMC(\tilde{A}) = 14.0044$$

4.2.3 Theoretical Validation

This section validates theoretically in term of properties of defuzzification and properties of centroid. The properties of defuzzification summarised by (Roychowdhury & Pedrycz, 2001) as mentioned in Chapter 3 are applied while the properties of centroid are developed in order to fulfill the reliability requirement. The relevant properties of defuzzification and centroid are illustrated on the next pages.

Let $\tilde{A}$ and $\tilde{B}$ are be trapezoidal and triangular fuzzy numbers respectively.

The properties of defuzzification summarised by (Roychowdhury & Pedrycz, 2001) are identified as follows.

Property 1: *A defuzzification operator always computes to one numeric value.*

Proof: *Since $\tilde{A}$ and $\tilde{B}$ are different types of fuzzy numbers, both of them must have single or unique defuzzified values, not ambiguity. The defuzzification operator is always injective. Clearly, two fuzzy sets can have same defuzzified value. It is assumed that, the defuzzified value is always within the support set of the original fuzzy set.*

Property 2: *The membership function determines the defuzzified value.*

Proof: *All fuzzy numbers represent together with membership function (y-axis). The membership function is important in determining the defuzzified value, not only core area (x-area). In this sense, defuzzification process must considers normal or non-normal fuzzy sets even the weight of core area (x-axis) is greater than membership function (y-axis).*

Property 3: *The defuzzified value of two triangular-operated fuzzy sets is always continued within the bounds of individual defuzzified values.*

Proof: *If fuzzy set $C_f = T(B_{f1}, B_{f2})$ B_{f1} where B_{f1} and B_{f2} are fuzzy sets and T and T-norm, $Def(B_{f1}) \leq Def(C_f) \leq Def(B_{f2})$, and so it is true for T-conorm (T*) $C_f = T*(B_{f1}, B_{f2})$.*

Property 4: *In the case of prohibitive information, the defuzzified value should fall in the permitted zone.*

Proof: *The defuzzified values of any fuzzy numbers must be fall in the permitted zone in core area of x-axis.*

The relevant properties of centroid are considered for justifying the applicability of centroid for fuzzy numbers, where they depend on the practically within the area of research. However, they shall not be regarded as complete. Therefore, with no loss of generality, the relevant properties of the centroid are as follows.

Let $\tilde{A}$ and $\tilde{B}$ are be trapezoidal and triangular fuzzy numbers respectively, while the coordinate intuitive multiple centroid, $IMC_{\tilde{A}}(\tilde{x}, \tilde{y})$ and $IMC_{\tilde{B}}(\tilde{x}, \tilde{y})$ be centroid for $\tilde{A}$ and $\tilde{B}$ respectively. Centroid index of intuitive multiple centroid represents the crisp value of centroid point that is denoted as $IMC(\tilde{A}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$ and $IMC(\tilde{B}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$.

Property 1: *If $\tilde{A}$ and $\tilde{B}$ are embedded and symmetry, then $IMC(\tilde{A}) > IMC(\tilde{B})$.*

Proof: *Since $\tilde{A}$ and $\tilde{B}$ are embedded and symmetry, hence we know that $\tilde{x}_{\tilde{A}} = \tilde{x}_{\tilde{B}}$ and $\tilde{y}_{\tilde{A}} > \tilde{y}_{\tilde{B}}$. Then, from equation (4.8) we have $\sqrt{\tilde{x}_{\tilde{A}}^{\ 2} + \tilde{y}_{\tilde{A}}^{\ 2}} > \sqrt{\tilde{x}_{\tilde{B}}^{\ 2} + \tilde{y}_{\tilde{B}}^{\ 2}}$. Therefore, $IMC(\tilde{A}) > IMC(\tilde{B})$.*

Property 2: *If $\tilde{A}$ and $\tilde{B}$ are embedded with $h_{\tilde{A}} > h_{\tilde{B}}$, then $IMC(\tilde{A}) > IMC(\tilde{B})$.*

Proof: *Since $\tilde{A}$ and $\tilde{B}$ are embedded and with $h_{\tilde{A}} > h_{\tilde{B}}$, hence we know that $\tilde{x}_{\tilde{A}} = \tilde{x}_{\tilde{B}}$ and $\tilde{y}_{\tilde{A}} > \tilde{y}_{\tilde{B}}$. Then, from equation (4.8) we have $\sqrt{\tilde{x}_{\tilde{A}}^{\ 2} + \tilde{y}_{\tilde{A}}^{\ 2}} > \sqrt{\tilde{x}_{\tilde{B}}^{\ 2} + \tilde{y}_{\tilde{B}}^{\ 2}}$. Therefore, $IMC(\tilde{A}) > IMC(\tilde{B})$.*

Property 3: *If $\tilde{A}$ is fuzzy singleton number, then $IMC(\tilde{A}) = \sqrt{\tilde{x}_{\tilde{A}}^{\ 2} + \tilde{y}_{\tilde{A}}^{\ 2}}$.*

Proof: *For any crisp (real) numbers, we know that $a_1 = a_2 = a_3 = a_4 = \tilde{x}_{\tilde{A}}$ and $\tilde{y}_{\tilde{A}} < 1$ which are equivalent to equation (4.5). Therefore, $IMC(\tilde{A}) = \sqrt{\tilde{x}_{\tilde{A}}^{\ 2} + \tilde{y}_{\tilde{A}}^{\ 2}}$.*

Property 4: *If $\tilde{A}$ and $\tilde{B}$ are any fuzzy symmetrical or asymmetrical number, then $a_1 < IMC(\tilde{A}) < a_4$ and $b_1 < IMC(\tilde{B}) < b_4$.*

Proof: *Since $\tilde{A}$ and $\tilde{B}$ are any fuzzy symmetrical or asymmetrical numbers, hence $a_1 < IVC_{\tilde{A}}(\tilde{x}, \tilde{y}) < a_4$ and $b_1 < IVC_{\tilde{B}}(\tilde{x}, \tilde{y}) < b_4$. Therefore, $a_1 < IMC(\tilde{A}) < a_4$ and $b_1 < IMC(\tilde{B}) < b_4$ respectively.*

All properties are related with computation for single crisp value $IMC(\tilde{A})$, where $\tilde{A}$ is any possible generalised type-1 fuzzy sets.

4.2.4 Empirical Validation

The empirical validation of centroid method is extensively discussed. Discussions of this validation are made in accordance with case studies found in the literature of fuzzy sets.

There are several possible cases in representing type-1 fuzzy numbers which are:

1) Trapezoidal normal symmetry
2) Trapezoidal normal asymmetry
3) Trapezoidal non – normal symmetry
4) Trapezoidal non – normal asymmetry
5) Triangular normal symmetry
6) Triangular normal asymmetry
7) Triangular non – normal symmetry
8) Triangular non – normal asymmetry
9) Singleton normal
10) Singleton non – normal

Representation of all possible cases for type-1 fuzzy sets:

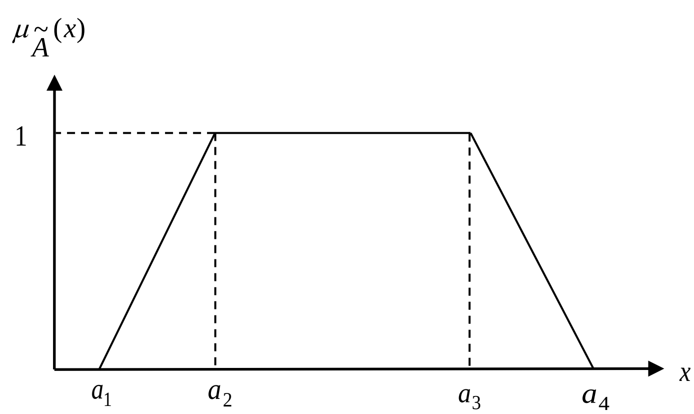


Fig. 4. 11: Trapezoidal normal symmetry of type-1 fuzzy number, $\tilde{A} = (a_1, a_2, a_3, a_4; 1)$

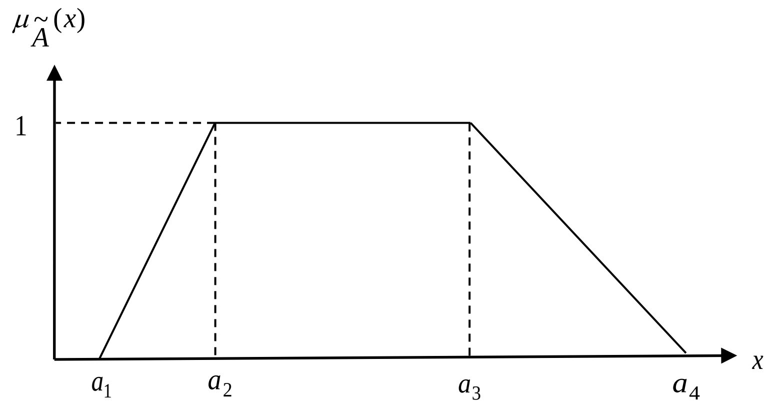


Fig. 4. 12: Trapezoidal normal asymmetry of type-1 fuzzy number,
$\tilde{A} = (a_1, a_2, a_3, a_4; 1)$

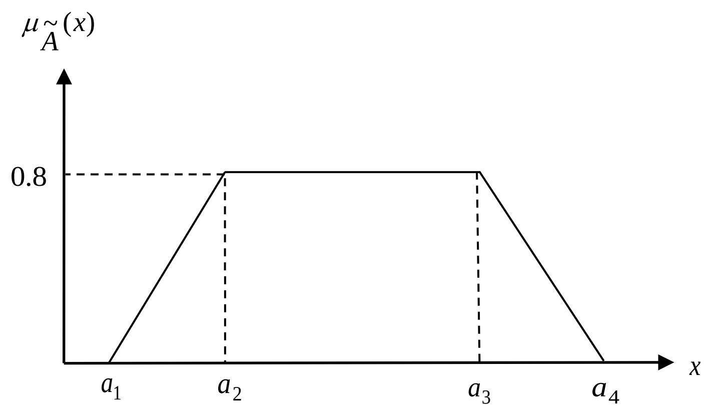


Fig. 4. 13: Trapezoidal non – normal symmetry of type-1 fuzzy number,
$\tilde{A} = (a_1, a_2, a_3, a_4; 0.8)$

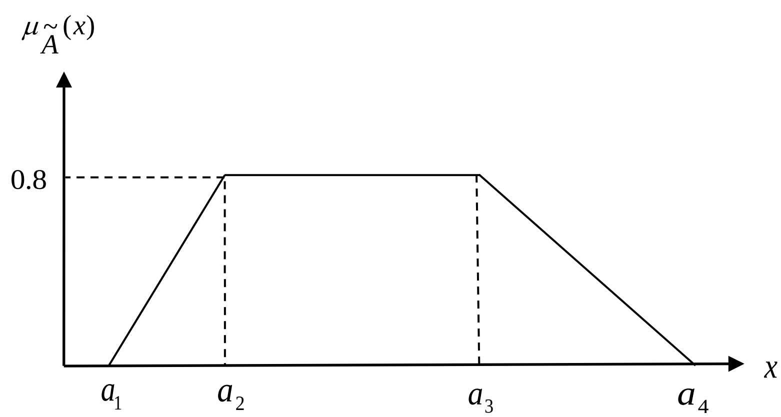


Fig. 4. 14: Trapezoidal non – normal asymmetry of type-1 fuzzy number,
$\tilde{A} = (a_1, a_2, a_3, a_4; 0.8)$

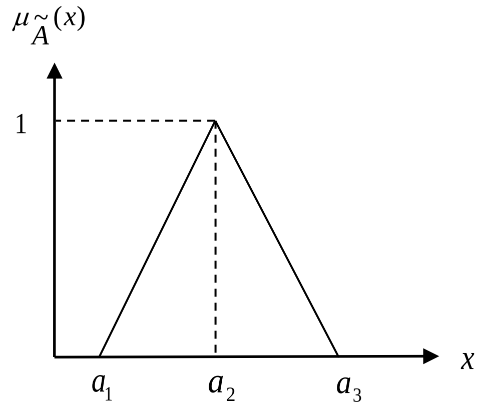


Fig. 4. 15: Triangular normal symmetry of type-1 fuzzy number, $\tilde{A} = (a_1, a_2, a_3; 1)$

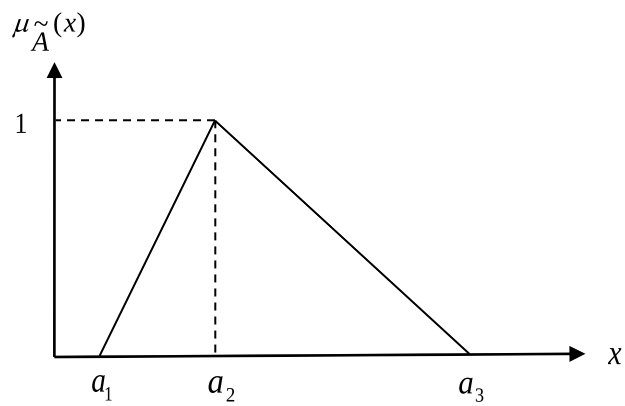


Fig. 4. 16: Triangular normal asymmetry of type-1 fuzzy number, $\tilde{A} = (a_1, a_2, a_3; 1)$

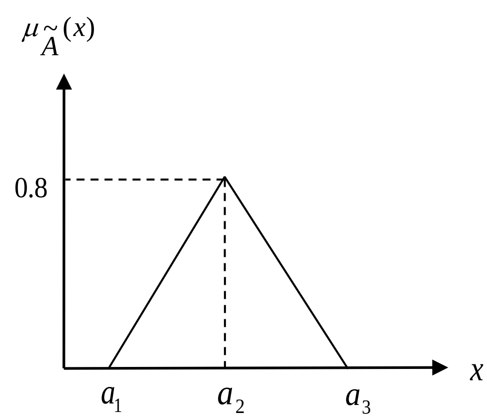


Fig. 4. 17: Triangular non – normal symmetry of type-1 fuzzy number,
$\tilde{A} = (a_1, a_2, a_3; 0.8)$

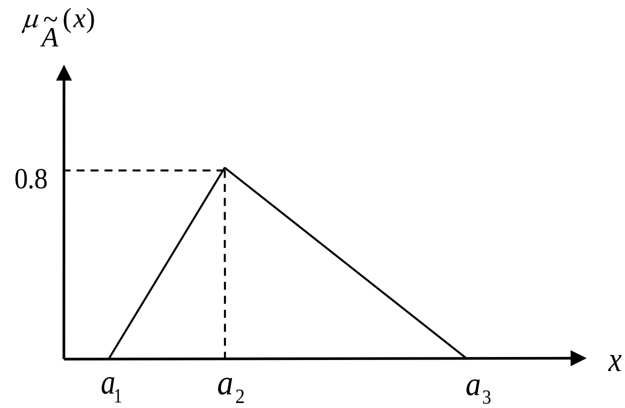


Fig. 4. 18: Triangular non – normal asymmetry of type-1 fuzzy number, $\tilde{A} = (a_1, a_2, a_3; 0.8)$

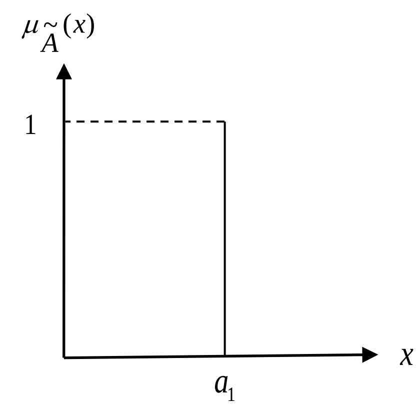


Fig. 4. 19: Singleton normal of type-1 fuzzy number, $\tilde{A} = (a_1; 1)$

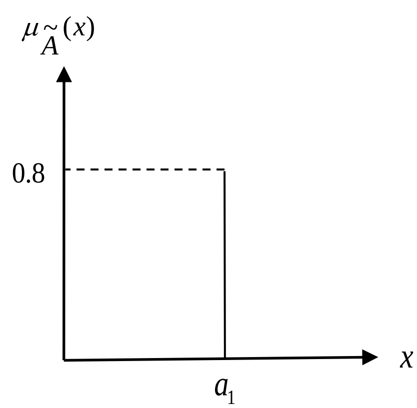


Fig. 4. 20: Singleton non – normal of type-1 fuzzy number, $\tilde{A} = (a_1; 0.8)$

Application:

The elementary problem of temperature mensuration based on arithmetic operation of centroid defuzzification methods of the proposed intuitive multiple centroid and established methods , (Shi-Jay Chen & Chen, 2002), (Y. M. Wang et al., 2006), (Liang et al., 2006) and (Shieh, 2007) are compared.

Let the temperature (C°) of a room is measured by each possible cases of type-1 fuzzy numbers as presented in Table 4.1. All of possible cases of fuzzy numbers are defuzzified using five different defuzzification methods and the results are presented in table below.

Table 4. 1. Comparative empirical – based validation study for centroid defuzzification of type-1 fuzzy sets

Case	Generalised Fuzzy Numbers					Chen & Chen (2002)			Wang et al. (2006)			Liang et al. (2006)			Shieh (2007)			Ku Khalif & Gegov (proposed)		
	(a1,	*a2,*	*a3,*	*a4;*	*h)*	*x*	*y*	Score Index	*x*	*y*	Score Index	*x*	*y*	Score Index	*x*	*y*	Score Index	*x*	*y*	Score Index
1	10	12	14	16	1	13.0000	0.6111	13.0144	13.0000	0.4167	13.0067	13.0000	0.4167	13.0067	13.0000	0.4167	13.0067	13.0000	0.3889	13.0058
2	10	12	14	17	1	13.3095	0.6190	13.3239	13.2963	0.4074	13.3025	13.2963	0.4074	13.3025	13.2963	0.4074	13.3025	13.1111	0.3889	13.1169
3	10	12	14	16	0.9	13.0000	0.5500	13.0116	13.0000	0.3750	13.0054	13.0000	0.3750	13.0054	13.0000	0.3750	13.0054	13.0000	0.3500	13.0047
4	10	12	14	17	0.9	13.3095	0.5571	13.3212	13.2963	0.3667	13.3014	13.2963	0.3667	13.3014	13.2963	0.3667	13.3014	13.1111	0.3500	13.1158
5	10	12	12	14	1	12.0000	0.6667	12.0185	12.0000	0.3333	12.0046	12.0000	0.3333	12.0046	12.0000	0.3333	12.0046	12.0000	0.3889	12.0063
6	10	12	12	15	1	12.3333	0.6667	12.3513	12.3333	0.3333	12.3378	12.3333	0.3333	12.3378	12.3333	0.3333	12.3378	12.1111	0.3889	12.1174
7	10	12	12	14	0.9	12.0000	0.6000	12.0150	12.0000	0.3000	12.0037	12.0000	0.3000	12.0037	12.0000	0.3000	12.0037	12.0000	0.3500	12.0051
8	10	12	12	15	0.9	12.3333	0.6000	12.3479	12.3333	0.3000	12.3370	12.3333	0.3000	12.3370	12.3333	0.3000	12.3370	12.1111	0.3500	12.1162
9	10	10	10	10	1	10.0000	0.5000	10.0125	10.0000	0.3333	10.0056	N/A	N/A	N/A	13.3333	0.3333	13.3375	10.0000	0.3889	10.0076
10	10	10	10	10	0.9	10.0000	0.4500	10.0101	10.0000	0.3000	10.0045	N/A	N/A	N/A	13.3333	0.3000	13.3367	10.0000	0.3500	10.0061

Footnotes: N/A - Not available

Based on the concept that the coordinate on the vertical axis is as important as the coordinate on the horizontal axis. According to (Y. J. Wang & Lee, 2008), they assumed that multiplying the value of *x* and *y* will degrade the importance of the value *x* whereby the importance of the degree of *x* should be higher than *y*. The core area (*x*-axis) contributes greater weight than membership function (*y*-axis). Moreover, this point is supported by several researchers in literature such as (Yager, 1980) and (Murakami & Meada, 1984). This is because the *x*-axis represents the certain values of fuzzy numbers while *y*-axis only represent the membership function or the confident level (uncertain) of certain values of fuzzy numbers. Logically, the range of membership function is between 0 until 1, but the values of core area or *x*-axis are varies. It can be positive value or negative value, with different units.

As can be seen in Table 4.1, several centroid defuzzification methods of type-1 fuzzy sets are compared with different possible cases of fuzzy numbers representation. The proposed intuitive multiple centroid for type-1 fuzzy sets, $\tilde{x}_{\tilde{A}} = \frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}$, $\tilde{y}_{\tilde{A}} = \frac{7h_{\tilde{A}}}{18}$ is compared with established centroid methods which are from:

1) (Shi-Jay Chen & Chen, 2002)

$$\tilde{y}_{\tilde{A}} = \begin{cases} \dfrac{h_{\tilde{A}} \times \left(\dfrac{a_3 - a_2}{a_4 - a_1}\right) + 2}{6}, & if\ a_1 \neq a_4 \quad and \quad 0 < h_{\tilde{A}} \leq 1, \\ \dfrac{h_{\tilde{A}}}{2} & if\ a_1 = a_4 \quad and \quad 0 < h_{\tilde{A}} \leq 1, \end{cases} \tag{4.9}$$

$$\tilde{x}_{\tilde{A}} = \frac{\tilde{y}_{\tilde{A}}(a_3 + a_2) + (a_4 + a_1)(h_{\tilde{A}} - \tilde{y}_{\tilde{A}})}{2h_{\tilde{A}}} \tag{4.10}$$

2) (Y. M. Wang et al., 2006)

Trapezoidal:

$$\tilde{x}_{\tilde{A}} = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{(a_4 \times a_3) - (a_1 \times a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right] \tag{4.11}$$

$$\tilde{y}_{\tilde{A}} = h_{\tilde{A}} \frac{1}{3}\left[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)}\right] \tag{4.12}$$

Triangular:

$$\tilde{x}_{\tilde{A}} = \frac{1}{3}(a_1 + a_2 + a_3) \tag{4.13}$$

$$\tilde{y}_{\tilde{A}} = \frac{1}{3} \tag{4.14}$$

3) (Liang et al., 2006)

$$\tilde{x}_{\tilde{A}} = \frac{d^2 + c^2 + cd - a^2 - b^2 - ab}{3(d + c - a - b)} \tag{4.15}$$

$$\tilde{y}_{\tilde{A}} = \frac{h_{\tilde{A}}(d + 2c - a - 2b)}{3(d + c - a - b)} \tag{4.16}$$

4) (Shieh, 2007)

$$\tilde{x}_{\tilde{A}} = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{(a_4 \times a_3) - (a_1 \times a_2)}{(a_4 + a_3) - (a_1 + a_2)} \right] \tag{4.17}$$

$$\tilde{y}_{\tilde{A}} = h_{\tilde{A}} \frac{1}{3}\left[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)} \right] \tag{4.18}$$

There are ten all possible cases representing fuzzy numbers as mentioned earlier. Case 1, 3, 5 and 7 are symmetry cases where all type-1 fuzzy numbers are in symmetry condition in which the length of $[a_1, a_2]$ and $[a_3, a_4]$ are same. All established methods and the proposed method give same values of horizontal *x*-axis for symmetry cases. It is depicts that the proposed method produces consistent results with established methods for symmetry cases of horizontal *x*-axis values. For vertical *y*-axis, the formulation given by (Y. M. Wang et al., 2006), is same as (Liang et al., 2006) and (Shieh, 2007). This is because (Liang et al., 2006) and (Shieh, 2007) use same formula produced by (Y. M. Wang et al., 2006) in computing *y*-axis. Nevertheless, when dealing with triangular or singleton fuzzy numbers, (Y. M. Wang et al., 2006) use different formulation for *x*-axis and *y*-axis as well. Since triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers, the centroid formulation proposed by them are inappropriately able to deal with triangular cases.

There are two singleton cases are considered here, which are singleton normal and singleton non-normal. For (Liang et al., 2006) centroid method, the formulation given cannot deal with singleton cases. This is because the numerator values for *x* and *y* formulation give zero value of results. As mentioned in Property 4 in defuzzification properties and Property 3 in centroid properties, logically, the defuzzification values should be fall in permitted zone and the horizontal *x*-axis values must be $a_1 = a_2 = a_3 = a_4 = \tilde{x}_{\tilde{A}}$. (Shieh, 2007) centroid method give *x*-values quite dispersed away from permitted zone. In this sense, (Shieh, 2007) centroid method deviates from Property 4 in defuzzification and Property 3 in centroid properties.

This follows the assumption as mentioned in early point where, the weight of *x*-axis is greater than *y*-axis. Thus, the utilising of intuitive multiple centroid is more reasonable than other established centroid methods.

4.3 Intuitive Multiple Centroid for Type-2 Fuzzy Sets

This section illustrates the proposed formulae on computing the extension of intuitive multiple centroid defuzzification of type-2 fuzzy sets. The theoretical and empirical foundations of the extension of intuitive multiple centroid method for type-2 fuzzy sets are introduced. The intuitive multiple centroid method for type-2 fuzzy sets takes a broad view by examples labelled by a classical intuitive multiple centroid defuzzification method for type-1 fuzzy sets. In real world applications, the implementation of fuzzy events is widely broad. Not just limited to classical fuzzy sets, but various types of fuzzy sets are applied. Aforementioned in Chapter 2, type-2 fuzzy sets let us incorporate the uncertainty of membership functions into the fuzzy set theory.

The development of intuitive multiple centroid for type-2 fuzzy sets is limited to interval type-2 fuzzy sets. Since generalised type-2 fuzzy set requires complex and huge computational difficult operations, the vast spread of generalised type-2 fuzzy systems has not occurred. The intuitive multiple centroid for type-1 fuzzy sets cannot be processed parallel for type-2 fuzzy sets. A similar analysis applies to the centroid for type-2 fuzzy sets is irrational compared to type-1 fuzzy sets. Presently, parallel processing is not available for most researchers, so the computational process is somewhat complex of the centroid for type-2 fuzzy sets. This issue motivates us to develop and simplify the new centroid for type-2 fuzzy sets. Currently, interval type-2 fuzzy sets are extensively used and have been successfully applied in numerous fields of study. The extension of proposed centroid defuzzification method for all possible interval type-2 fuzzy sets that consist of trapezoidal, triangular and singleton fuzzy numbers that are incorporated into the development as well. This extension of proposed intuitive multiple centroid for type-2 fuzzy sets is compared with other established centroid methods in literature which are (Karnik & Mendel, 2001a), (Wu & Mendel, 2009), (Gong, 2013) and (Abu Bakar & Gegov, 2015a).

4.3.1 *Extension of Intuitive Multiple Centroid for Type-2 Fuzzy Sets*

Let consider by $\tilde{\tilde{A}} = (\tilde{A}^U, \tilde{A}^L) = ((a_1^U, a_2^U, a_3^U, a_4^U; h_{\tilde{A}}^U), (a_1^L, a_2^L, a_3^L, a_4^L; h_{\tilde{A}}^L))$ as an interval type-2 fuzzy number. The complete method process of intuitive multiple centroid for interval type-2 fuzzy set is signified as follows.

Step 1: Find the centroids of the three parts of $\alpha_{\bar{\alpha},\underline{\alpha}}$, $\beta_{\bar{\beta},\underline{\beta}}$ and $\gamma_{\bar{\gamma},\underline{\gamma}}$ in interval type-2 fuzzy set representation as shown in Fig. 4.21. We divide the trapezoidal interval type-2 fuzzy set into three parts which are: 1) two right triangle shapes of $\bar{\alpha}$ and $\underline{\alpha}$; 2) two rectangle shapes of $\bar{\beta}$ and $\underline{\beta}$ and; 3) two left triangle shapes of $\bar{\gamma}$ and $\underline{\gamma}$. The sub centroids of right triangle shape, rectangle shape and left triangle shape represent as $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ respectively.

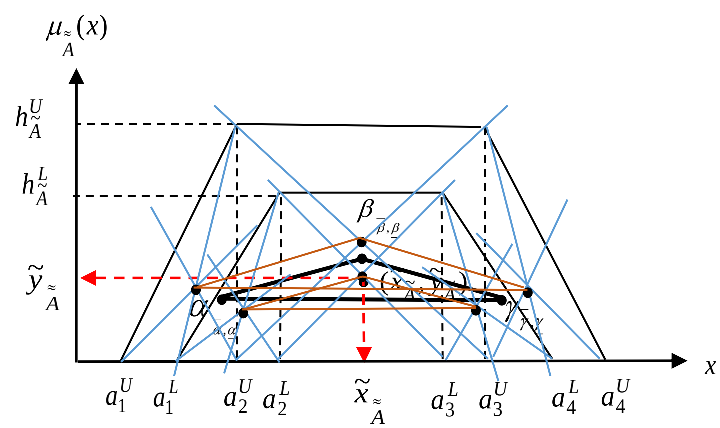


Fig. 4. 21: Intuitive multiple centroid plane representation of type-2 fuzzy set

Theoretically, intuitive multiple centroid defuzzification is based on median point that covers centralised of the shape properly. Fig. 4.22, Fig. 4.23 and Fig. 4.24 depict the sub centroid points separately to represent the median points of $\alpha_{\bar{\alpha},\underline{\alpha}}$, $\beta_{\bar{\beta},\underline{\beta}}$ and $\gamma_{\bar{\gamma},\underline{\gamma}}$ respectively.

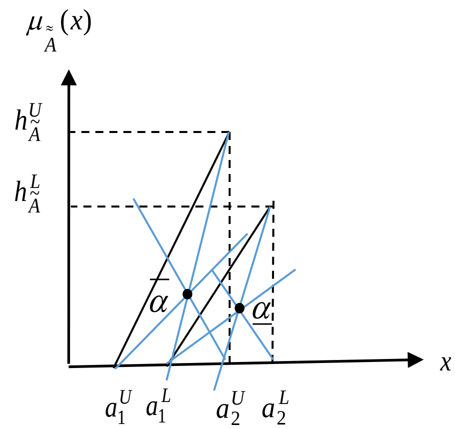


Fig. 4. 22: Centroid for upper, $\bar{\alpha}$ and lower forms, $\underline{\alpha}$ of left triangles

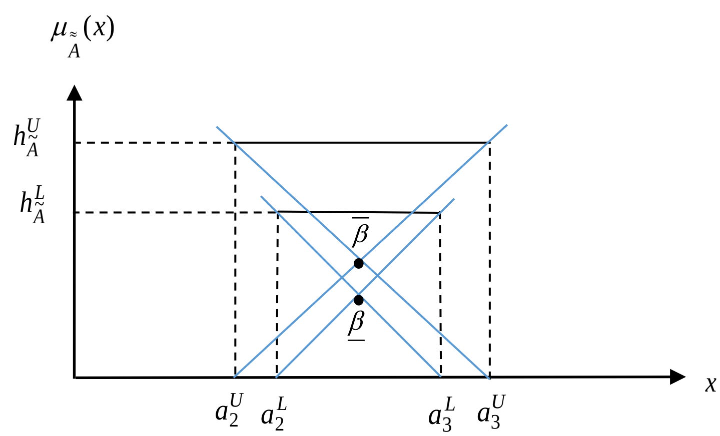


Fig. 4. 23: Centroid for upper, $\bar{\beta}$ and lower forms, $\underline{\beta}$ of rectangles

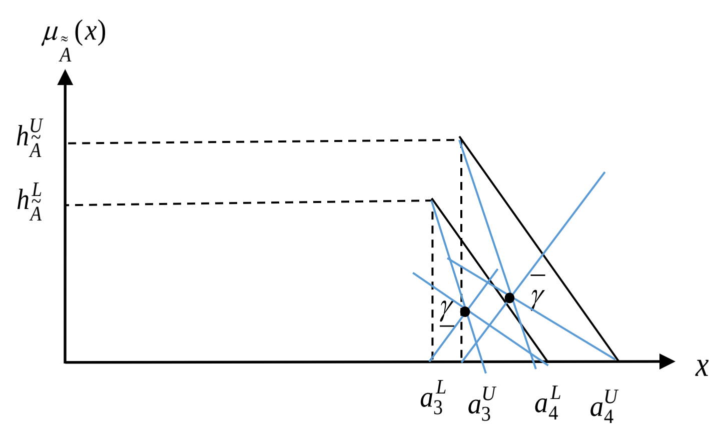


Fig. 4. 24: Centroid for upper, $\bar{\gamma}$ and lower forms, $\underline{\gamma}$ of right triangles

1) Sub centroid points of $\alpha_{\bar{\alpha},\underline{\alpha}}$ formula.

$$\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\frac{1}{6}(a_1^U+a_1^L)+\frac{1}{3}\left(a_2^U+a_2^L\right),\frac{1}{6}(h_{\tilde{A}}^U+h_{\tilde{A}}^L)\right) \tag{4.19}$$

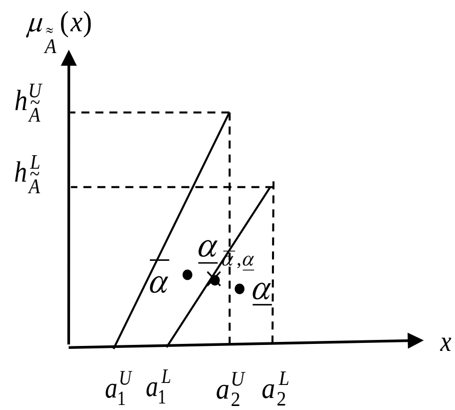


Fig. 4. 25: Sub centroid of left triangle, $\alpha_{\bar{\alpha},\underline{\alpha}}$

Fig. 4.25 presents the $\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})$, sub centroid point of $\alpha_{\bar{\alpha},\underline{\alpha}}$ left shape of triangle are developed. Basically, the sub centroid for $\alpha_{\bar{\alpha},\underline{\alpha}}$ is same concept from Fig. 4.7, $\alpha_{\tilde{A}}$ for type-1 fuzzy sets. Aforementioned in Section 3.4.2, an interval type-2 fuzzy number has upper, $\bar{\alpha}$ and lower, $\underline{\alpha}$ form, where it has two type-1 fuzzy numbers in one representation on a plane. In getting the centroid for interval type-2 fuzzy sets, most of the researchers in literature (Karnik & Mendel, 2001a), (Wu & Mendel, 2009), (Gong, 2013), (Abu Bakar & Gegov, 2015a), there are straightforwardly find the midpoint between upper and lower form. The extension of proposed intuitive multiple centroid apply the same process in getting the middle points between $\bar{\alpha}$ and $\underline{\alpha}$. Equation (4.19) is formulated for the middle points between $\bar{\alpha}$ and $\underline{\alpha}$:

$$\bar{\alpha}(x_{\tilde{A}},y_{\tilde{A}})=\left(a_1^U+\left[\frac{2}{3}\left(a_2^U-a_1^U\right)\right],\frac{h_{\tilde{A}}^U}{3}\right)$$

$$\underline{\alpha}(x_{\tilde{A}},y_{\tilde{A}})=\left(a_1^L+\left[\frac{2}{3}\left(a_2^L-a_1^L\right)\right],\frac{h_{\tilde{A}}^L}{3}\right)$$

Then,

$$\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\frac{\left(a_1^U+\left[\frac{2}{3}\left(a_2^U-a_1^U\right)\right]\right)+\left(a_1^L+\left[\frac{2}{3}\left(a_2^L-a_1^L\right)\right]\right)}{2},\frac{\frac{h_{\tilde{A}}^U}{3}+\frac{h_{\tilde{A}}^L}{3}}{2}\right)$$

$$\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\frac{\left(a_1^U+\frac{2a_2^U}{3}-\frac{2a_1^U}{3}\right)+\left(a_1^L+\frac{2a_2^L}{3}-\frac{2a_1^L}{3}\right)}{2},\frac{h_{\tilde{A}}^U}{6}+\frac{h_{\tilde{A}}^L}{6}\right)$$

$$\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\frac{\left(\frac{a_1^U}{3}+\frac{2a_2^U}{3}\right)+\left(\frac{a_1^L}{3}+\frac{2a_2^L}{3}\right)}{2},\frac{1}{6}\left(h_{\tilde{A}}^U+h_{\tilde{A}}^L\right)\right)$$

$$\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\left(\frac{a_1^U}{6}+\frac{a_1^L}{6}\right)+\left(\frac{2a_2^U}{6}+\frac{2a_2^L}{6}\right),\frac{1}{6}\left(h_{\tilde{A}}^U+h_{\tilde{A}}^L\right)\right)$$

Hence,

$$\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\frac{1}{6}(a_1^U+a_1^L)+\frac{1}{3}\left(a_2^U+a_2^L\right),\frac{1}{6}(h_{\tilde{A}}^U+h_{\tilde{A}}^L)\right)$$

2) Sub centroid points of $\beta_{\bar{\beta},\underline{\beta}}$ formula.

$$\beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})=\left(\frac{1}{4}(a_2^U+a_3^U+a_2^L+a_3^L),\frac{1}{4}(h_{\tilde{A}}^U+h_{\tilde{A}}^L)\right) \qquad (4.20)$$

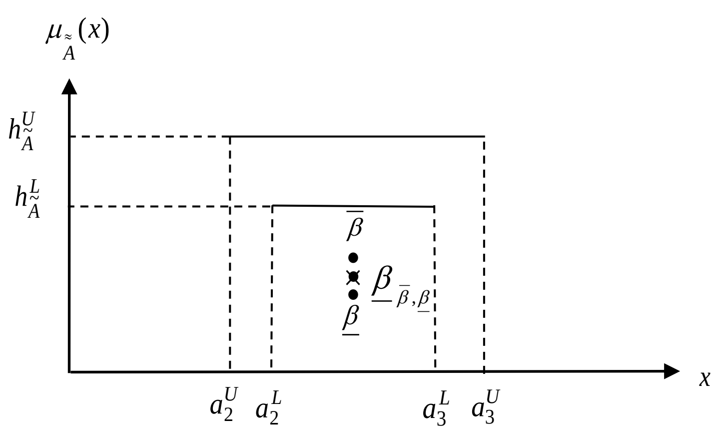


Fig. 4. 26: Sub centroid of rectangle, $\beta_{\bar{\beta},\underline{\beta}}$

Same goes to $\beta_{\overline{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})$, the middle point between $\overline{\beta}$ and $\underline{\beta}$ is computed. Fundamentally, the sub centroid for $\beta_{\overline{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}})$ is same concept from Fig. 4.9, $\beta_{\tilde{A}}$ for type-1 fuzzy sets. Equation (4.19) is formulated for the middle points between $\overline{\alpha}$ and $\underline{\alpha}$:

$$\beta_{\overline{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}}) = \left(\frac{\left(\frac{a_2^U + a_3^U}{2}\right) + \left(\frac{a_2^L + a_3^L}{2}\right)}{2}, \frac{\frac{h_{\tilde{A}}^U}{2} + \frac{h_{\tilde{A}}^L}{2}}{2} \right)$$

$$\beta_{\overline{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}}) = \left(\left(\frac{a_2^U + a_3^U}{4}\right) + \left(\frac{a_2^L + a_3^L}{4}\right), \frac{h_{\tilde{A}}^U}{4} + \frac{h_{\tilde{A}}^L}{4} \right)$$

Hence,

$$\beta_{\overline{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}}) = \left(\frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L), \frac{1}{4}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right)$$

3) Sub centroid points of $\gamma_{\overline{\gamma},\underline{\gamma}}$ formula.

$$\gamma_{\overline{\gamma},\underline{\gamma}}(\tilde{x}_{\tilde{A}},\tilde{y}_{\tilde{A}}) = \left(\frac{1}{6}(a_4^U + a_4^L) + \frac{1}{3}(a_3^U + a_3^L), \frac{1}{6}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right) \qquad (4.21)$$

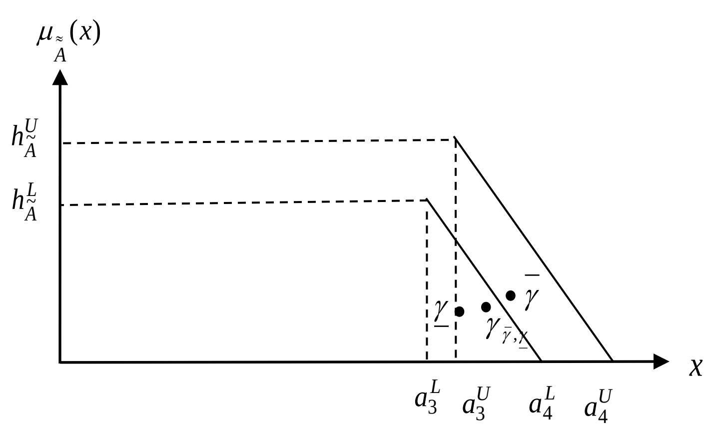


Fig. 4. 27: Sub centroid of right triangle, $\gamma_{\overline{\gamma},\underline{\gamma}}$

Fig. 4.27 depicts the representation of formulation developed for $\gamma(x_{\tilde{A}}, y_{\tilde{A}})$, sub centroid point of $\gamma_{\bar{\gamma},\underline{\gamma}}$ left shape of triangle. The explanation for equation (4.21) are generated is similar way as sub centroid points of $\alpha_{\bar{\alpha},\underline{\alpha}}$ formula from equation (4.19).

Step 2: Connect all vertices centroids points of $\alpha_{\bar{\alpha},\underline{\alpha}}$, $\beta_{\bar{\beta},\underline{\beta}}$ and $\gamma_{\bar{\gamma},\underline{\gamma}}$ each other, where it will create another triangular plane inside of trapezoid plane.

Step 3: The centroid index of intuitive multiple centroid of $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ with vertices $\alpha_{\bar{\alpha},\underline{\alpha}}$, $\beta_{\bar{\beta},\underline{\beta}}$ and $\gamma_{\bar{\gamma},\underline{\gamma}}$ can be calculated as

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}}) + \gamma_{\bar{\gamma},\underline{\gamma}}(\tilde{x}_{\tilde{A}})}{2} - \beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}}) \right) \right], \beta_{\bar{\beta},\underline{\beta}}(\tilde{y}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{y}_{\tilde{A}}) + \gamma_{\bar{\gamma},\underline{\gamma}}(\tilde{y}_{\tilde{A}})}{2} - \beta_{\bar{\beta},\underline{\beta}}(\tilde{y}_{\tilde{A}}) \right) \right] \right) \tag{4.22}$$

Intuition multiple centroid can be summarised as

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}, \frac{7}{36}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right) \tag{4.23}$$

where

$\alpha_{\bar{\alpha},\underline{\alpha}}$, : the centroid coordinate of first triangle plane

$\beta_{\bar{\beta},\underline{\beta}}$: the centroid coordinate of rectangle plane

$\gamma_{\bar{\gamma},\underline{\gamma}}$: the centroid coordinate of second triangle plane

$(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$: the centroid coordinate of fuzzy number $\tilde{\tilde{A}}$

The process of getting the final centroid coordinate $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ are illustrated as follows.

Proving:

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}) = \frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}$$

$$= \beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}}) + \gamma_{\bar{\gamma},\underline{\gamma}}(\tilde{x}_{\tilde{A}})}{2} - \beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}})\right)\right]$$

$$= \frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L) + \left[\frac{2}{3}\left(\frac{\left(\frac{1}{6}(a_1^U + a_1^L) + \frac{1}{3}\left(a_2^U + a_2^L\right)\right) + \left(\frac{1}{6}(a_4^U + a_4^L) + \frac{1}{3}\left(a_3^U + a_3^L\right)\right)}{2} - \left(\frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L)\right)\right)\right]$$

$$= \frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L) + \left[\frac{2}{3}\left(\left(\frac{1}{12}(a_1^U + a_1^L) + \frac{1}{6}\left(a_2^U + a_2^L\right)\right) + \left(\frac{1}{12}(a_4^U + a_4^L) + \frac{1}{6}\left(a_3^U + a_3^L\right)\right) - \left(\frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L)\right)\right)\right]$$

$$= \frac{1}{4}(a_2^U + a_2^L) + \frac{1}{4}(a_3^U + a_3^L) + \left[\frac{2}{3}\left(\left(\frac{1}{12}(a_1^U + a_1^L) + \frac{1}{6}\left(a_2^U + a_2^L\right)\right) + \left(\frac{1}{12}(a_4^U + a_4^L) + \frac{1}{6}\left(a_3^U + a_3^L\right)\right) - \frac{1}{4}(a_2^U + a_2^L) - \frac{1}{4}(a_3^U + a_3^L)\right)\right]$$

$$= \frac{1}{4}(a_2^U + a_2^L) + \frac{1}{4}(a_3^U + a_3^L) + \left[\frac{2}{3}\left(\frac{1}{12}(a_1^U + a_1^L) - \frac{1}{12}\left(a_2^U + a_2^L\right) - \frac{1}{12}(a_3^U + a_3^L) + \frac{1}{12}(a_4^U + a_4^L)\right)\right]$$

$$= \frac{1}{4}(a_2^U + a_2^L) + \frac{1}{4}(a_3^U + a_3^L) + \frac{1}{18}(a_1^U + a_1^L) - \frac{1}{18}\left(a_2^U + a_2^L\right) - \frac{1}{18}(a_3^U + a_3^L) + \frac{1}{18}(a_4^U + a_4^L)$$

$$= \frac{1}{18}(a_1^U + a_1^L) + \frac{7}{36}(a_2^U + a_2^L) + \frac{7}{36}(a_3^U + a_3^L) + \frac{1}{18}(a_4^U + a_4^L)$$

$$= \frac{1}{18}(a_1^U + a_1^L + a_4^U + a_4^L) + \frac{7}{36}(a_2^U + a_2^L + a_3^U + a_3^L)$$

$$= \frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}$$

Proving:

$$IMC_{\tilde{\tilde{A}}}(\tilde{y}_{\tilde{A}}) = \frac{7}{36}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})$$

$$= \beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) + \left[\frac{2}{3}\left(\frac{\alpha_{\bar{\alpha},\underline{\alpha}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) + \gamma_{\bar{\gamma},\underline{\gamma}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})}{2} - \beta_{\bar{\beta},\underline{\beta}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})\right)\right]$$

$$= \frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) + \left[\frac{2}{3}\left(\frac{\frac{1}{6}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) + \frac{1}{6}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})}{2} - \left(\frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})\right)\right)\right]$$

$$= \frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) + \left[\frac{2}{3}\left(\frac{\frac{2}{6}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})}{2} - \left(\frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})\right)\right)\right]$$

$$= \frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) + \left[\frac{2}{3}\left(\frac{1}{6}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) - \frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})\right)\right]$$

$$= \frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) + \left[\frac{2}{3}\left(-\frac{1}{12}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})\right)\right]$$

$$= \frac{1}{4}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L}) - \frac{1}{18}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})$$

$$= \frac{7}{36}(h_{\tilde{A}}^{U} + h_{\tilde{A}}^{L})$$

Centroid index of intuitive multiple centroid can be generated using Euclidean distance by (Cheng, 1998):

$$R(\tilde{\tilde{A}}) = \sqrt{\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}} \tag{4.24}$$

Hence

$$IMC(\tilde{\tilde{A}}) = \sqrt{\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}} \tag{4.25}$$

4.3.2 Illustrative Example

This subsection illustrates a numerical – based example which is used to demonstrate the utilisation of the extension of intuitive multiple centroid method for fuzzy set of interval type-2 is developed in Section 4.3. A complete illustration of utilising the proposed method in this example is as follows.

Let $\tilde{\tilde{A}} = (\tilde{A}^U, \tilde{A}^L) = ((10,12,14,16;1),(11,12.5,13.5,15;0.9))$ be an interval type-2 fuzzy number to be calculated the centroid point of $\tilde{\tilde{A}}$, then the centroid point is computed using equation (4.23) and (4.25) as follows.

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}, \frac{7}{36}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right)$$

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(10+11+16+15)+7(12+12.5+14+13.5)}{36}, \frac{7}{36}(1+0.9) \right)$$

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = (13, 0.3694)$$

Hence, the centroid index of intuitive multiple centroid for $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ fuzzy set of interval type-2 can be computed as

$$IMC(\tilde{\tilde{A}}) = \sqrt{\left(\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}\right)}$$

$$IMC(\tilde{\tilde{A}}) = \sqrt{\left(13^2 + 0.3694^2\right)}$$

$$IMC(\tilde{\tilde{A}}) = 13.0053$$

4.3.3 Theoretical Validation

The properties of defuzzification summarised by (Roychowdhury & Pedrycz, 2001) as mentioned in Chapter 3 for type-1 fuzzy sets are extended while the properties of centroid are improvised in order to fulfill the reliability of type-2 fuzzy sets requirement. The relevant properties of defuzzification and centroid for type-2 fuzzy sets are illustrated as follows.

Let $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are be trapezoidal and triangular type-2 fuzzy number respectively.

The properties of defuzzification summarised by (Roychowdhury & Pedrycz, 2001) are identified as follows.

Property 1: *A defuzzification operator always computes to one numeric value.*

Proof: *Since $\tilde{A}$ and $\tilde{B}$ are different types of type-2 fuzzy numbers, both of them must have single or unique defuzzified values, not ambiguity. The footprint of uncertainty (FOU) is considered in order to reduce the uncertainty. The defuzzification operator is always injective. Clearly, two fuzzy sets can have same defuzzified value. It is assumed that, the defuzzified value is always within the support set of the original fuzzy set.*

Property 2: *The membership function determines the defuzzified value.*

Proof: *All type-2 fuzzy numbers represent together with membership function (y-axis) with two bounds, which are lower bound and upper bound. The area between both bounds is named as footprint of uncertainty (FOU). Here is where the uncertainty is located. The membership function is important in determining the defuzzified value, not only core area (x-area). In this sense, defuzzification process must considers normal or non-normal fuzzy sets even the weight of core area (x-axis) is greater than membership function (y-axis).*

Property 3: *The defuzzified value of two triangular-operated fuzzy sets is always continued within the bounds of individual defuzzified values.*

Proof: *If type-2 fuzzy set $C_f = T(B_{f1}, B_{f2})$ B_{f1} where B_{f1} and B_{f2} are fuzzy sets and T and T-norm, $Def(B_{f1}) \leq Def(C_f) \leq Def(B_{f2})$, and so it is true for T-conorm (T*) $C_f = T^*(B_{f1}, B_{f2})$.*

Property 4: *In the case of prohibitive information, the defuzzified value should fall in the permitted zone.*

Proof: *The defuzzified values of any type-2 fuzzy numbers must be fall in the permitted zone in core area of x-axis.*

The relevant properties are considered for qualifying the applicability of centroid for interval type-2 fuzzy sets, where they depend on the practicality within the area of research, however, they are not regarded as complete. Therefore, without loss of generality, the relevant properties of the centroid are as follows:

Let $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are be trapezoidal and triangular interval type-2 fuzzy number respectively, while $IMC_{\overset{\approx}{A}}(\tilde{x},\tilde{y})$ and $IMC_{\overset{\approx}{B}}(\tilde{x},\tilde{y})$ be centroid points for $\overset{\approx}{A}$ and $\overset{\approx}{B}$ respectively. Centroid index of intuitive multiple centroid shows the crisp value of centroid point that is denoted as $IMC(\overset{\approx}{A}) = \sqrt{\tilde{x}_{\tilde{A}}^{\,2} + \tilde{y}_{\tilde{A}}^{\,2}}$ and $IMC(\overset{\approx}{B}) = \sqrt{\tilde{x}_{\tilde{B}}^{\,2} + \tilde{y}_{\tilde{B}}^{\,2}}$.

Property 1: *If $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded and symmetry, then* $IMC(\overset{\approx}{A}) > IMC(\overset{\approx}{B})$**.**

Proof: *Since $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded and symmetry, hence from equation (4.15) we have* $\sqrt{\tilde{x}_{\overset{\approx}{A}}^{\,2} + \tilde{y}_{\overset{\approx}{A}}^{\,2}} > \sqrt{\tilde{x}_{\overset{\approx}{B}}^{\,2} + \tilde{y}_{\overset{\approx}{B}}^{\,2}}$. *Therefore***,** $IMC(\overset{\approx}{A}) > IMC(\overset{\approx}{B})$.

Property 2: *If $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded with* $(h^U, h^L)_{\overset{\approx}{A}} > (h^U, h^L)_{\overset{\approx}{B}}$, *then* $IMC(\overset{\approx}{A}) > IMC(\overset{\approx}{B})$**.**

Proof: *Since $\overset{\approx}{A}$ and $\overset{\approx}{B}$ are embedded and with* $(h^U, h^L)_{\overset{\approx}{A}} >$ $(h^U, h^L)_{\overset{\approx}{B}}$, *hence we know that* $\tilde{y}_{\tilde{A}} > \tilde{y}_{\tilde{B}}$**.** *Then, from equation (4.13) we have* $\sqrt{\tilde{x}_{\overset{\approx}{A}}^{\,2} + \tilde{y}_{\overset{\approx}{A}}^{\,2}} > \sqrt{\tilde{x}_{\overset{\approx}{B}}^{\,2} + \tilde{y}_{\overset{\approx}{B}}^{\,2}}$. *Therefore***,** $IMC(\overset{\approx}{A}) > IMC(\overset{\approx}{B})$**.**

Property 3: *If $\overset{\approx}{A}$ is fuzzy singleton number, then* $IMC(\overset{\approx}{A}) = \sqrt{\tilde{x}_{\overset{\approx}{A}}^{\,2} + \tilde{y}_{\overset{\approx}{A}}^{\,2}}$.

Proof: *For any crisp (real) interval type-2 fuzzy set, we know that* $a_1^U = a_2^U = a_3^U = a_4^U = a_1^L = a_2^L = a_3^L = a_4^L = \tilde{x}_{\overset{\approx}{A}}$ *which are equivalent to equation (4.15). Therefore,* $IMC(\overset{\approx}{A}) = \sqrt{\tilde{x}_{\overset{\approx}{A}}^{\,2} + \tilde{y}_{\overset{\approx}{A}}^{\,2}}$.

Property 4: *If $\overset{\approx}{A}$ is any symmetrical or asymmetrical interval type-2 fuzzy number, then* $a_1^U < IMC(\overset{\approx}{A}) < a_4^U$.

Proof: *Since any symmetrical or asymmetrical interval type-2 fuzzy set* has $a_1^U \le a_2^U \le a_3^U \le a_4^U$, hence $a_1^U < IMC_{\overset{\approx}{A}}(\tilde{x},\tilde{y}) < a_4^U$**.** *Therefore,* $a_1^U < IMC(\overset{\approx}{A}) < a_4^U$.

All properties are related with computation for single crisp value $IMC(\tilde{\tilde{A}})$, where $\tilde{\tilde{A}}$ is any possible interval type-2 fuzzy set.

4.3.4 Empirical Validation

The empirical validation of centroid method is extensively discussed. Discussions of this validation are made in accordance with case studies found in the literature of fuzzy sets.

There are several possible cases in representing interval fuzzy sets of type-2 which are:

1) Trapezoidal normal symmetry
2) Trapezoidal normal asymmetry
3) Trapezoidal non – normal symmetry
4) Trapezoidal non – normal asymmetry
5) Triangular normal symmetry
6) Triangular normal asymmetry
7) Triangular non – normal symmetry
8) Triangular non – normal asymmetry
9) Singleton normal
10) Singleton non – normal

Representation of all possible cases for interval type-2 fuzzy sets:

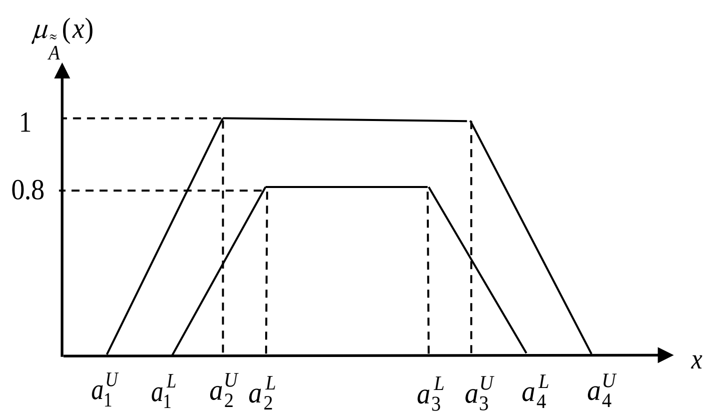


Fig. 4. 28: Trapezoidal normal symmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U, a_4^U; 1), (a_1^L, a_2^L, a_3^L, a_4^L; 0.8))$$

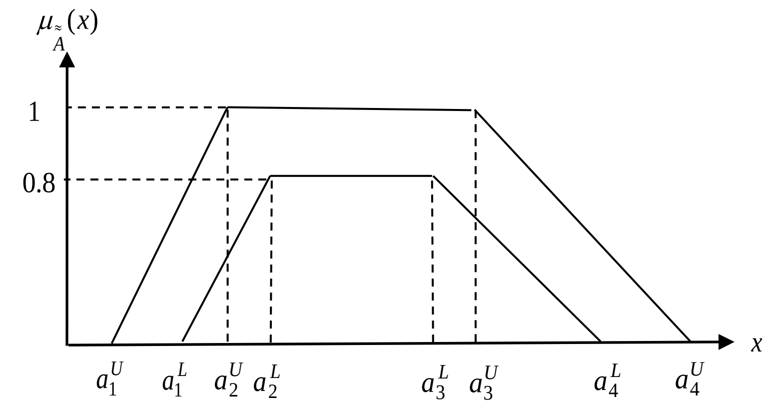


Fig. 4. 29: Trapezoidal normal asymmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U, a_4^U; 1), (a_1^L, a_2^L, a_3^L, a_4^L; 0.8))$$

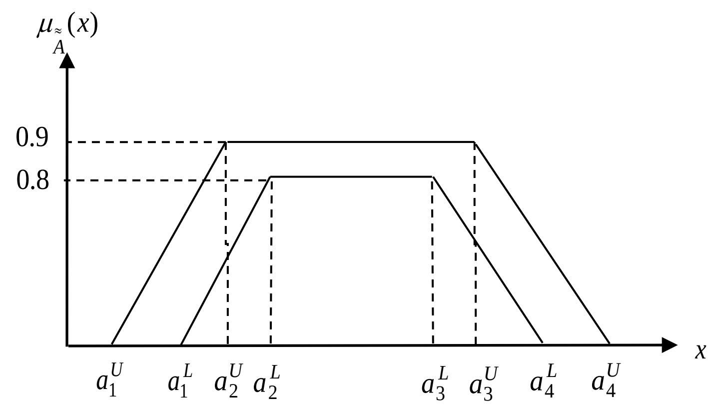


Fig. 4. 30: Trapezoidal non – normal symmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U, a_4^U; 0.9), (a_1^L, a_2^L, a_3^L, a_4^L; 0.8))$$

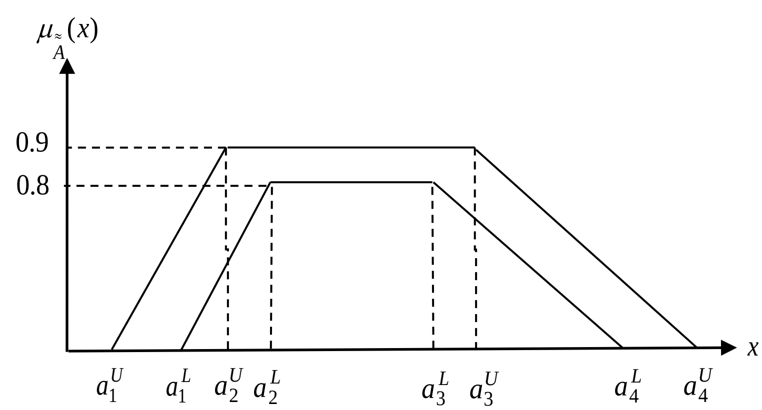


Fig. 4. 31: Trapezoidal non – normal asymmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U, a_4^U; 0.9), (a_1^L, a_2^L, a_3^L, a_4^L; 0.8))$$

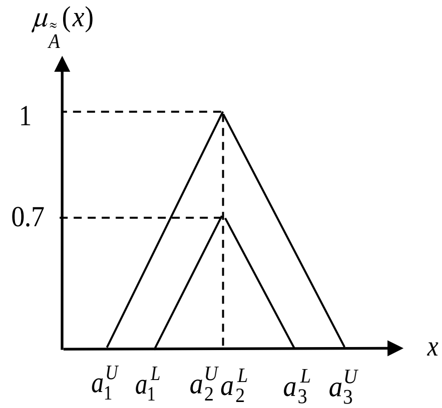


Fig. 4. 32: Triangular normal symmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U; 1), (a_1^L, a_2^L, a_3^L; 0.7))$$

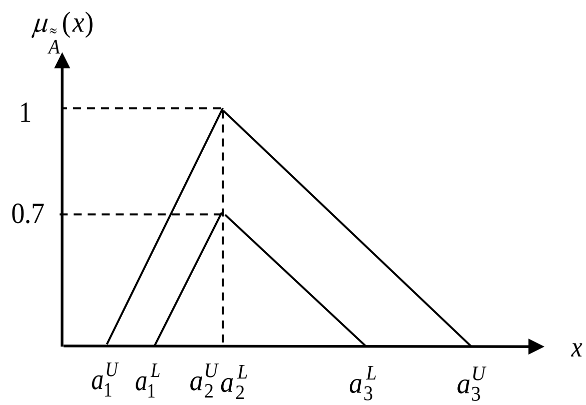


Fig. 4. 33: Triangular normal asymmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U; 1), (a_1^L, a_2^L, a_3^L; 0.7))$$

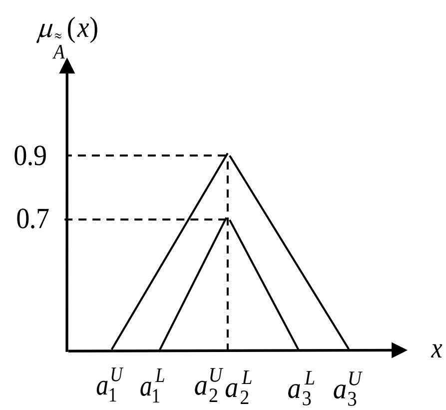


Fig. 4. 34: Triangular non – normal symmetry of interval type-2 fuzzy number,

$$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U; 0.9), (a_1^L, a_2^L, a_3^L; 0.7))$$

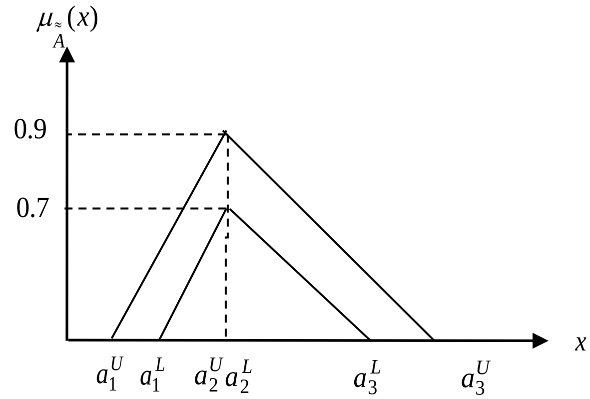


Fig. 4. 35: Triangular non – normal asymmetry of interval type-2 fuzzy number,

$\tilde{\tilde{A}} = ((a_1^U, a_2^U, a_3^U; 0.9), (a_1^L, a_2^L, a_3^L; 0.7))$

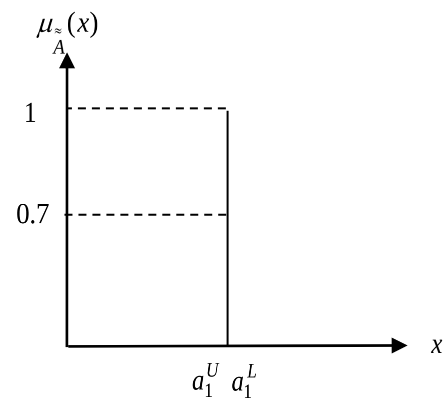


Fig. 4. 36: Singleton normal of interval type-2 fuzzy number, $\tilde{\tilde{A}} = ((a_1^U; 1), (a_1^L; 0.7))$

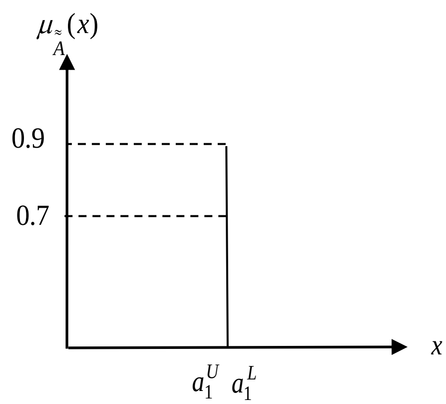


Fig. 4. 37: Singleton non – normal of interval type-2 fuzzy number,

$\tilde{\tilde{A}} = ((a_1^U; 0.9), (a_1^L; 0.7))$

Application:

The elementary problem of temperature mensuration based on arithmetic operation of centroid defuzzification methods of the proposed of extension intuitive multiple centroid for type-2 fuzzy sets and established methods, (Karnik & Mendel, 2001a), (Wu & Mendel, 2009), (Gong, 2013) and (Abu Bakar & Gegov, 2015a) are compared.

Let the temperature (C°) of a room is measured by each possible cases of interval type-2 fuzzy numbers as presented in Table 4.2. All of possible cases of fuzzy numbers are defuzzified using four different defuzzification methods and the results are presented in table below. (Karnik & Mendel, 2001b)

Table 4. 2. Comparative empirical – based validation study for centroid defuzzification of interval type-2 fuzzy sets

Case	Interval Type-2 Fuzzy Numbers										Karnik & Mendel (2001)			Wu & Mendel (2009)			Gong et al(2013)	Abu Bakar & Gegov (2015)			Ku Khalif & Gegov (proposed)		
	(aU1,	*aU2,*	*aU3,*	*aU4;*	*hU)*	*(aL1,*	*aL2,*	*aL3,*	*aL4;*	*hL)*	$g(L_A)$	$g(R_A)$	Score Index	*C(left)*	*C(right)*	Score Index	PMV	*x*	*y*	Score Index	*x*	*y*	Score Index
1	10	12	14	16	1	11	12	14	15	1	12.0000	14.0000	13.0000	12.0000	14.0000	13.0000	13.0000	13.0000	0.4306	13.0071	13.0000	0.3889	13.0058
2	10	12	14	17	1	11	12	14	16	1	12.0000	14.0000	13.0000	12.0000	14.0000	13.0000	13.1667	13.2910	0.4180	13.2976	13.2222	0.3889	13.2279
3	10	12	14	16	1	11	12	14	15	0.9	12.0556	13.9444	13.0000	12.0556	13.9444	13.0000	12.3500	13.0000	0.4083	13.0064	13.0000	0.3694	13.0052
4	10	12	14	17	1	11	12	14	16	0.9	12.0556	13.8889	12.9722	12.0556	13.8889	12.9722	12.5083	13.2910	0.3966	13.2969	13.2222	0.3694	13.2274
5	10	13	13	15	1	12	13	13	14	1	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	12.9167	12.8333	0.3333	12.8377	12.8889	0.3889	12.8948
6	10	13	13	16	1	12	13	13	15	1	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0833	13.1667	0.3333	13.1709	13.1111	0.3889	13.1169
7	10	13	13	15	1	12	13	13	14	0.9	13.0556	12.9444	13.0000	13.0556	12.9444	13.0000	12.2667	12.8333	0.3167	12.8372	12.8889	0.3694	12.8942
8	10	13	13	16	1	12	13	13	15	0.9	13.0556	12.8889	12.9722	13.0556	12.8889	12.9722	12.4250	13.1667	0.3167	13.1705	13.1111	0.3694	13.1163
9	10	10	10	10	1	10	10	10	10	1	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	13.3333	0.3333	13.3375	10.0000	0.3889	10.0076
10	10	10	10	10	1	10	10	10	10	0.9	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	9.5000	13.3333	0.3167	13.3371	10.0000	0.3694	10.0068

Referring to Table 4.2, four centroid defuzzification methods for interval type-2 fuzzy sets are compared with different possible cases of fuzzy numbers representation. The extension of proposed intuitive multiple centroid, $\tilde{x}_{\tilde{A}} = \dfrac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}$, $\tilde{y}_{\tilde{A}} = \dfrac{7}{36}(h_{\tilde{A}}^U + h_{\tilde{A}}^L)$ is compared with established centroid methods for interval type-2 fuzzy sets which are from:

1. (Karnik & Mendel, 2001a)

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x_1}} \cdots \int_{\theta_N \in J_{x_N}} 1 \Bigg/ \frac{\sum_{i}^{N} x_i \theta_i}{\sum_{=1i}^{N} \theta_i} = [c_l, c_r] \tag{4.26}$$

For crisp value,

$$c(\tilde{A}) = \frac{c_l(\tilde{A}) + c_r(\tilde{A})}{2} \tag{4.27}$$

2. (Wu & Mendel, 2009)

$$c_l(\tilde{A}) = \frac{\sum_{i=1}^{L} x_i \overline{\mu}_{\tilde{A}}(x_i) + \sum_{i=L+1}^{N} x_i \underline{\mu}_{\tilde{A}}(x_i)}{\sum_{i=1}^{L} \overline{\mu}_{\tilde{A}}(x_i) + \sum_{i=L+1}^{N} \underline{\mu}_{\tilde{A}}(x_i)} \tag{4.28}$$

$$c_r(\tilde{A}) = \frac{\sum_{i=1}^{R} x_i \underline{\mu}_{\tilde{A}}(x_i) + \sum_{i=R+1}^{N} x_i \overline{\mu}_{\tilde{A}}(x_i)}{\sum_{i=1}^{R} \underline{\mu}_{\tilde{A}}(x_i) + \sum_{i=R+1}^{N} \overline{\mu}_{\tilde{A}}(x_i)} \tag{4.29}$$

For crisp value,

$$c(\tilde{A}) = \frac{c_l(\tilde{A}) + c_r(\tilde{A})}{2}$$

3. (Gong, 2013)

$$\underline{M}(\approx{A}) = \int_0^{h_U} \alpha \left(a_1^U + \frac{a_2^U - a_1^U}{h_u} \alpha \right) d\alpha + \int_0^{h_L} \beta \left(a_1^L + \frac{a_2^L - a_1^L}{h_L} \beta \right) d\beta \tag{4.30}$$

$$\overline{M}(\approx{A}) = \int_0^{h_U} \alpha \left(a_4^U + \frac{a_3^U - a_4^U}{h_u} \alpha \right) d\alpha + \int_0^{h_L} \beta \left(a_4^L + \frac{a_3^L - a_4^L}{h_L} \beta \right) d\beta \tag{4.31}$$

For crisp value,

$$M(\approx{A}) = \frac{1}{12} h_U^2 \left(a_1^U + 2a_2^U + 2a_3^U + a_4^U \right) + \frac{1}{12} h_L^2 \left(a_1^L + 2a_2^L + 2a_3^L + a_4^L \right) \tag{4.32}$$

4. (Abu Bakar & Gegov, 2015a)
   The authors apply (Shieh, 2007) centroid method from type-1 fuzzy sets for interval type-2 fuzzy sets,

$$\tilde{x}_{\bar{A}} = \frac{1}{3}\left[a_1^U + a_2^U + a_3^U + a_4^U - \frac{\left(a_4^U \times a_3^U\right) - \left(a_1^U \times a_2^U\right)}{\left(a_4^U + a_3^U\right) - \left(a_1^U + a_2^U\right)} \right] \tag{4.33}$$

$$\tilde{y}_{\bar{A}} = h_{\bar{A}} \frac{1}{3}\left[1 + \frac{a_3^U - a_2^U}{\left(a_4^U + a_3^U\right) - \left(a_1^U + a_2^U\right)} \right] \tag{4.34}$$

$$\tilde{x}_{\underline{A}} = \frac{1}{3}\left[a_1^L + a_2^L + a_3^L + a_4^L - \frac{\left(a_4^L \times a_3^L\right) - \left(a_1^L \times a_2^L\right)}{\left(a_4^L + a_3^L\right) - \left(a_1^L + a_2^L\right)} \right] \tag{4.35}$$

$$\tilde{y}_{\underline{A}} = h_{\underline{A}} \frac{1}{3}\left[1 + \frac{a_3^L - a_2^L}{\left(a_4^L + a_3^L\right) - \left(a_1^L + a_2^L\right)} \right] \tag{4.36}$$

   For crisp value,

$$Centroid\,(\tilde{x}_{\bar{A},\underline{A}}, \tilde{y}_{\bar{A},\underline{A}}) = \sqrt{\left(\frac{\tilde{x}_{\bar{A}} + \tilde{x}_{\underline{A}}}{2}\right)^2 + \left(\frac{\tilde{y}_{\bar{A}} + \tilde{y}_{\underline{A}}}{2}\right)^2} \tag{4.37}$$

There are ten all possible cases same as type-1 fuzzy sets representing fuzzy numbers as mentioned earlier. Centroid defuzzification methods that proposed by (Karnik & Mendel, 2001a) and (Wu & Mendel, 2009) are actually same, but different in representation. (Wu & Mendel, 2009) improvised the computational process of (Karnik & Mendel, 2001a)'s centroid method to make easy to understand and compute. Representing both (Karnik & Mendel, 2001a) and (Wu & Mendel, 2009) centroid methods produce good results for normal symmetry cases (case 1, 5 and 9) only but not for the others cases. As can be seen at case 1, 2, 3, 5, 6 and 7, all of them are represented different representation of fuzzy numbers, but both (Karnik & Mendel, 2001a) and (Wu & Mendel, 2009) centroid methods give same value of defuzzification. Case 9 and 10 as well. This is irrational, illogic and not consider human judgment in their computations.

(Gong, 2013; Gong et al., 2015) centroid method does not compute *x* and *y* values to get the defuzzification values. In order to get the defuzzification value, the authors defined possibility degree which are the upper and lower possibility mean values. The fuzzy number is divided into two parts which are lower and upper part. Hence, to get the defuzzification value, average between lower and upper possibility mean part is computed, $M(\tilde{\tilde{A}}) = \frac{\underline{M} + \overline{M}}{2}$. Overall, this centroid method produces better results compare to (Karnik & Mendel, 2001a) and (Wu & Mendel, 2009). From Table 4.2, (Gong, 2013; Gong et al., 2015) give rational results for case 1, 2, 5 and 6. The remaining cases are irrational because the defuzzification values are deviated away too much. For instance, case 10 (singleton non-normal), the defuzzification value produced by (Gong, 2013; Gong et al., 2015) is about 0.5 from permitted zone 10. As can be seen here, (Gong, 2013; Gong et al., 2015) method is inappropriately able to deal with non-normal cases since produce such that results.

Centroid method for interval type-2 fuzzy sets that proposed by (Abu Bakar & Gegov, 2015a) is an extension of centroid method for type-1 fuzzy sets from (Shieh, 2007). As we know, the representation of interval type-2 fuzzy set is a pair of type-1 fuzzy sets (refer Section 3.4.2). (Abu Bakar & Gegov, 2015a) applied (Shieh, 2007) centroid method for both shapes presented in type-2 fuzzy sets, where at the end of calculation, the average between two centroid points is computed. The results that are produced by (Abu Bakar & Gegov, 2015a) are good in terms of centre point of *x*-axis. Aforementioned, *x*-axis represents greater weight in defuzzification process compared to *y*-axis. This due to the fact that *x*-axis represents actual value regarding the information but *y*-axis represents membership degree of the data values. Both play important role in defuzzification process, but the *x*-axis plays greater role. This method inappropriately deal with singleton cases either for normal or non – normal. The results depict that the centroid method proposed by (Abu Bakar & Gegov, 2015a) are dispersed away too much from the permitted zone for singleton cases which is 10.

The extension of the proposed intuitive multiple centroid gives consistent results for all cases compared to established centroid methods for interval type-2 fuzzy sets in literature. There is no one result from all these cases gives worse defuzzification value. In addition, it gives better defuzzification results that consistent with the original values of fuzzy numbers (core value; *x*-axis). This proposed centroid method follows the assumption as mentioned in early point where, the weight of *x*-axis is greater than *y*-axis. Thus, the utilising of intuitive multiple centroid is more reasonable, rational and logic than other established centroid methods for interval type-2 fuzzy sets for all possible cases.

4.4 Intuitive Multiple Centroid for Z-Numbers

In this section, the proposed formulae on computing the extension of intuitive multiple centroid defuzzification of z-numbers is developed. The proposed of the extension of intuitive multiple centroid method is justified the presented formulae from the classical one, which is intuitive multiple centroid for the type-1 fuzzy sets. The consideration of reliability component in fuzzy sets has more ability to describe knowledge of human being and uncertain information process.

In real applications, the decision makers may give their opinions by fuzzy numbers. However, typically, a basic question arises which is how reliable are the numbers that we deal with. It plays a particularly significant role in decision analysis. The concept of intuitive multiple centroid defuzzification for z-number has remarkable capability than type-1 fuzzy set to make rational decisions regarding considering the reliability of the numbers.

4.4.1 Extension of Intuitive Multiple Centroid for Fuzzy Set Z-Numbers

Let consider a z-number, $Z=(\tilde{A},\tilde{R})$ is an ordered pair of fuzzy numbers with $Z_{\tilde{A},\tilde{R}}=((a_1,a_2,a_3,a_4;h_A),(R_1,R_2,R_3,R_4;h_R))$. The first component, $\tilde{A}=\{\langle x,u_{\tilde{A}}(x)\rangle | x\in[0,1]\}$ is known as restriction component whereby it is a real-valued uncertain on X while the second component, $\tilde{R}=\{\langle x,u_{\tilde{R}}(x)\rangle | x\in[0,1]\}$ is a measure of reliability for $\tilde{A}$ as mentioned in Chapter 2. Let assume $\tilde{A}=(a_1,a_2,a_3,a_4;h_A)$ as the generalised trapezoidal type-1 fuzzy number and $(\tilde{x}_{\tilde{A},\tilde{R}},\tilde{y}_{\tilde{A},\tilde{R}})$ be the centroid point for $(\tilde{A},\tilde{R})$ such that $\tilde{x}_{\tilde{A},\tilde{R}}$ and $\tilde{y}_{\tilde{A},\tilde{R}}$ are the horizontal x – axis and vertical y – axis of z-number of $(\tilde{A},\tilde{R})$ respectively. The complete process for intuitive multiple centroid point, $IMC(Z_{\tilde{A},\tilde{R}})$ computation is signified as follows.

Step 1: Converting the reliability component on x – coordinate into crisp number as a weight for restriction component, $\tilde{A}$. Find the sub centroids of the three parts of $\alpha_{\tilde{R}}$, $\beta_{\tilde{R}}$ and $\gamma_{\tilde{R}}$ in trapezoid plane of reliability $\tilde{R}$, representation as shown in Fig. 4.38. Trapezoid shape is divided into three parts which are: 1) the left triangle; 2) the rectangle and; 3) the right triangle. The sub centroids of left triangle shape, rectangle shape and right triangle shape represent as $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ respectively. Converting the reliability component, $\tilde{R}$ on x – coordinate into crisp number or weight using equation (4.5).

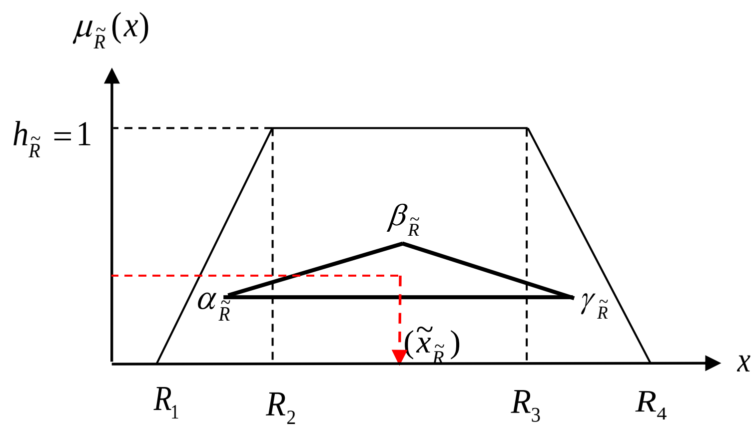


Fig. 4. 38: Intuitive multiple centroid plane representation for Reliability, $\tilde{R}$ component.

Step 2: Add the weight of reliability component into the restriction component, $\tilde{A}$ with multiplicative operation. The weighted z-number can be denoted as $\tilde{Z}^{\varphi} = \{\langle x, \mu_{\tilde{A}^{\varphi}}(x)\rangle | \mu_{\tilde{A}^{\varphi}}(x) = \varphi\mu_{\tilde{A}^{\varphi}}(x), x \in [0,1]\}$. Theorem 1 illustrates the process in getting weighted z-number.

Theorem 4.1:

$$E_{\tilde{A}^{\varphi}}(x) = \alpha E_{\tilde{A}}(x), \quad x \in X \tag{4.38}$$

Subject to:

$$\mu_{\tilde{A}^{\varphi}}(x) = \alpha\mu_{\tilde{A}^{\varphi}}(x), \quad x \in X \tag{4.39}$$

Proof:

$$E_{\tilde{A}^{\varphi}}(x) = \left[a_1, a_2, a_3, a_4; \frac{2(R_1 + R_4) + 7(R_2 + R_3)}{18}\right] = [a_1, a_2, a_3, a_4; \varphi] = \varphi E_{\tilde{A}}(x) \tag{4.40}$$

which can be denoted by the Fig. 4. 39 as next page.

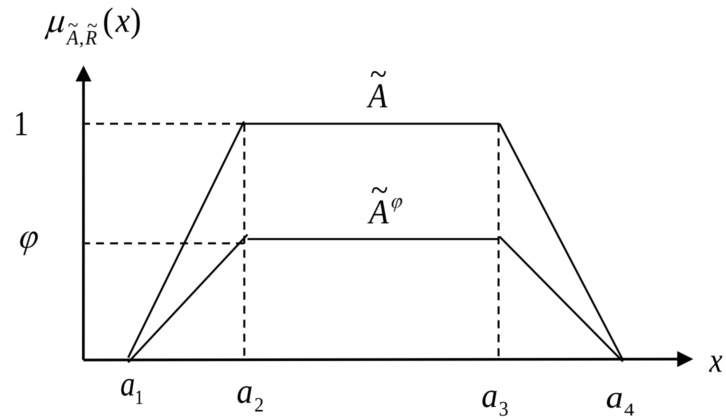


Fig. 4. 39: Z-number after multiplying the reliability

Step 3: Convert the irregular fuzzy number (weighted restriction) to regular fuzzy number that denoted as $\tilde{Z}' = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\sqrt{\varphi}x\right), x \in [0,1]\right\}$. In accordance with the Theorem 4.1, the conclusion can be made that $\tilde{Z}'$ has the same fuzzy expectation with $\tilde{Z}^{\varphi}$ where both are equal with fuzzy expectation.

Theorem 4.2:

$$E_{\tilde{Z}'}(x) = \alpha E_{\tilde{A}}(x), \quad x \in X \tag{4.41}$$

Subject to:

$$\mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\sqrt{\alpha}x\right), \quad x \in \sqrt{\alpha}X \tag{4.42}$$

Proof:

$$E_{\tilde{Z}}(x) = \left[a_1, a_2, a_3, a_4; \sqrt{\frac{2(R_1 + R_4) + 7(R_2 + R_3)}{18}}\right] = \left[a_1, a_2, a_3, a_4; \sqrt{\varphi}\right] = \sqrt{\varphi}E_{\tilde{A}}(x) \tag{4.43}$$

which can be denoted by the Fig. 4.40 as next page.

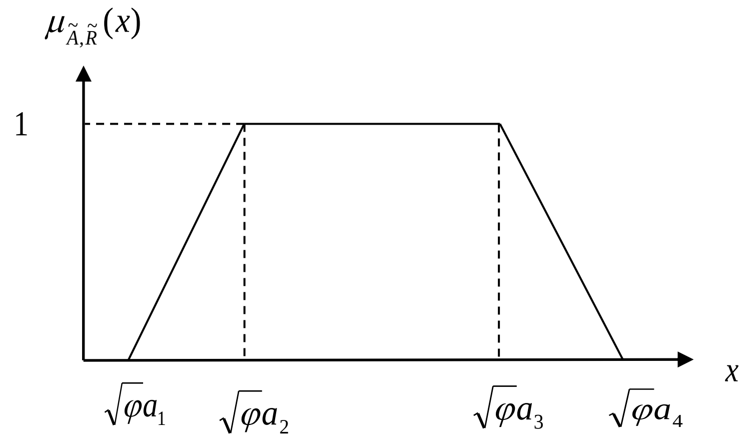


Fig. 4. 40: The regular fuzzy number transforms from z-number

Theorem 4.3:

$$E_{\tilde{Z}'}(x) = E_{\tilde{A}^{\varphi}}(x) \tag{4.44}$$

Proof:

$$E_{\tilde{A}^{\varphi}}(x) = \varphi E_{\tilde{A}}(x) \tag{4.45}$$

$$E_{\tilde{Z}'}(x) = \varphi E_{\tilde{A}}(x) \tag{4.46}$$

$$E_{\tilde{Z}'}(x) = E_{\tilde{A}^{\varphi}}(x) \tag{4.47}$$

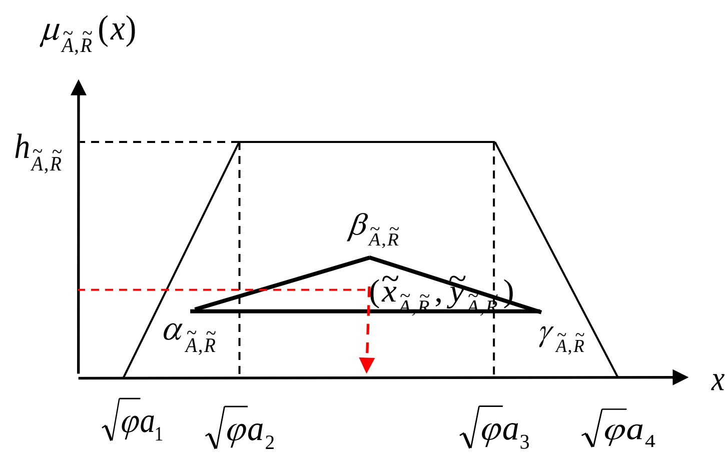


Fig. 4. 41: Intuitive multiple centroid plane representation for z-number

Step 4: Connect all vertices sub centroid points of $\alpha_{\tilde{A},\tilde{R}}$, $\beta_{\tilde{A},\tilde{R}}$ and $\gamma_{\tilde{A},\tilde{R}}$ each other, where it will create another triangular plane inside of trapezoid plane as represented in Fig. 4.41. The formulation of sub centroid points of $\alpha_{\tilde{A},\tilde{R}}$, $\beta_{\tilde{A},\tilde{R}}$ and $\gamma_{\tilde{A},\tilde{R}}$ are computed same as intuitive multiple centroid for type-1 fuzzy sets as below.

$$\alpha(x_{\tilde{A},\tilde{R}}, y_{\tilde{A},\tilde{R}}) = \left(\sqrt{\varphi}a_1 + \left[\frac{2}{3}\left(\sqrt{\varphi}a_2 - \sqrt{\varphi}a_1\right)\right], \frac{h_{\tilde{A},\tilde{R}}}{3} \right) \tag{4.48}$$

$$\beta(x_{\tilde{A},\tilde{R}}, y_{\tilde{A},\tilde{R}}) = \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}, \frac{h_{\tilde{A},\tilde{R}}}{2} \right) \tag{4.49}$$

$$\gamma(x_{\tilde{A},\tilde{R}}, y_{\tilde{A},\tilde{R}}) = \left(\sqrt{\varphi}a_4 + \left[\frac{2}{3}\left(\sqrt{\varphi}a_3 - \sqrt{\varphi}a_4\right)\right], \frac{h_{\tilde{A},\tilde{R}}}{3} \right) \tag{4.50}$$

The sub centroid points of $\alpha_{\tilde{A},\tilde{R}}$, $\beta_{\tilde{A},\tilde{R}}$ and $\gamma_{\tilde{A},\tilde{R}}$ are calculated in coordinate point, $(\tilde{x},\tilde{y})$ because the consideration of degree of membership values in dealing with subjective events for reliability component.

Step 3: The centroid coordinate points of intuitive multiple centroid, $(\tilde{x},\tilde{y})$ of fuzzy number $\tilde{A}$ with vertices $\alpha_{\tilde{A}}$, $\beta_{\tilde{A}}$ and $\gamma_{\tilde{A}}$ can be calculated as

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \left(\beta(\tilde{x}_{\tilde{A},\tilde{R}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{x}_{\tilde{A},\tilde{R}}) + \gamma(\tilde{x}_{\tilde{A},\tilde{R}})}{2} - \beta(\tilde{x}_{\tilde{A},\tilde{R}}) \right)\right], \beta(\tilde{y}_{\tilde{A},\tilde{R}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{y}_{\tilde{A},\tilde{R}}) + \gamma(\tilde{y}_{\tilde{A},\tilde{R}})}{2} - \beta(\tilde{y}_{\tilde{A},\tilde{R}}) \right)\right] \right) \tag{4.51}$$

Intuitive multiple centroid can be summarised as

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \left(\frac{2(\sqrt{\varphi}a_1 + \sqrt{\varphi}a_4) + 7(\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3)}{18}, \frac{7h_{\tilde{A},\tilde{R}}}{18} \right) \tag{4.52}$$

where

$\tilde{x}_{\tilde{A},\tilde{R}}$: the centroid on the horizontal *x*-axis

$\tilde{y}_{\tilde{A},\tilde{R}}$: the centroid on the vertical *y*-axis

$(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}})$: the centroid point of fuzzy number $\tilde{A}$

The process of getting the final centroid coordinate $(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}})$ are illustrated as next page.

Proving:

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}) = \frac{2(\sqrt{\varphi}a_1 + \sqrt{\varphi}a_4) + 7(\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3)}{18}$$

$$= \beta(\tilde{x}_{\tilde{A},\tilde{R}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{x}_{\tilde{A},\tilde{R}}) + \gamma(\tilde{x}_{\tilde{A},\tilde{R}})}{2} - \beta(\tilde{x}_{\tilde{A},\tilde{R}})\right)\right]$$

$$= \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right) + \left[\frac{2}{3}\left(\frac{\left[\sqrt{\varphi}a_1 + \left(\frac{2}{3}\left(\sqrt{\varphi}a_2 - \sqrt{\varphi}a_1\right)\right)\right] + \left[\sqrt{\varphi}a_4 + \left(\frac{2}{3}\left(\sqrt{\varphi}a_3 - \sqrt{\varphi}a_4\right)\right)\right]}{2} - \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right)\right)\right]$$

$$= \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right) + \left[\frac{2}{3}\left(\frac{\left[\sqrt{\varphi}a_1 + \left(\frac{2\sqrt{\varphi}a_2}{3} - \frac{2\sqrt{\varphi}a_1}{3}\right)\right] + \left[\sqrt{\varphi}a_4 + \left(\frac{2\sqrt{\varphi}a_3}{3} - \frac{2\sqrt{\varphi}a_4}{3}\right)\right]}{2} - \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right)\right)\right]$$

$$= \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right) + \left[\frac{2}{3}\left(\frac{\left(\frac{\sqrt{\varphi}a_1}{3} + \frac{2\sqrt{\varphi}a_2}{3}\right) + \left(\frac{\sqrt{\varphi}a_4}{3} + \frac{2\sqrt{\varphi}a_3}{3}\right)}{2} - \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right)\right)\right]$$

$$= \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right) + \left[\frac{2}{3}\left(\left(\frac{\sqrt{\varphi}a_1}{6} + \frac{2\sqrt{\varphi}a_2}{6}\right) + \left(\frac{\sqrt{\varphi}a_4}{6} + \frac{2\sqrt{\varphi}a_3}{6}\right) - \left(\frac{\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3}{2}\right)\right)\right]$$

$$= \frac{\sqrt{\varphi}a_2}{2} + \frac{\sqrt{\varphi}a_3}{2} + \left[\frac{2}{3}\left(\frac{\sqrt{\varphi}a_1}{6} + \frac{2\sqrt{\varphi}a_2}{6} + \frac{\sqrt{\varphi}a_4}{6} + \frac{2\sqrt{\varphi}a_3}{6} - \frac{\sqrt{\varphi}a_2}{2} - \frac{\sqrt{\varphi}a_3}{2}\right)\right]$$

$$= \frac{\sqrt{\varphi}a_2}{2} + \frac{\sqrt{\varphi}a_3}{2} + \left(\frac{2\sqrt{\varphi}a_1}{18} + \frac{4\sqrt{\varphi}a_2}{18} + \frac{2\sqrt{\varphi}a_4}{18} + \frac{4\sqrt{\varphi}a_3}{18} - \frac{2\sqrt{\varphi}a_2}{6} - \frac{2\sqrt{\varphi}a_3}{6}\right)$$

$$= \frac{2\sqrt{\varphi}a_1}{18} + \frac{7\sqrt{\varphi}a_2}{18} + \frac{7\sqrt{\varphi}a_3}{18} + \frac{2\sqrt{\varphi}a_4}{18}$$

$$= \frac{2(\sqrt{\varphi}a_1 + \sqrt{\varphi}a_4) + 7(\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3)}{18}$$

Proving:

$$IMC(\tilde{y}_{\tilde{A},\tilde{R}}) = \frac{7h_{\tilde{A},\tilde{R}}}{18}$$

$$= \beta(\tilde{y}_{\tilde{A},\tilde{R}}) + \left[\frac{2}{3}\left(\frac{\alpha(\tilde{y}_{\tilde{A},\tilde{R}}) + \gamma(\tilde{y}_{\tilde{A},\tilde{R}})}{2} - \beta(\tilde{y}_{\tilde{A},\tilde{R}})\right)\right]$$

$$= \frac{h_{\tilde{A},\tilde{R}}}{2} + \left[\frac{2}{3}\left(\frac{\left(h_{\tilde{A},\tilde{R}} - \frac{2}{3}h_{\tilde{A},\tilde{R}}\right) + \left(h_{\tilde{A},\tilde{R}} - \frac{2}{3}h_{\tilde{A},\tilde{R}}\right)}{2} - \frac{h_{\tilde{A},\tilde{R}}}{2}\right)\right]$$

$$= \frac{h_{\tilde{A},\tilde{R}}}{2} + \left[\frac{2}{3}\left(\frac{1}{3}h_{\tilde{A},\tilde{R}} - \frac{h_{\tilde{A},\tilde{R}}}{2}\right)\right]$$

$$= \frac{h_{\tilde{A},\tilde{R}}}{2} + \left[\frac{2}{3}\left(\frac{2h_{\tilde{A},\tilde{R}} - 3h_{\tilde{A},\tilde{R}}}{6}\right)\right]$$

$$= \frac{h_{\tilde{A},\tilde{R}}}{2} + \left[\frac{2}{3}\left(\frac{-h_{\tilde{A},\tilde{R}}}{6}\right)\right]$$

$$= \frac{h_{\tilde{A},\tilde{R}}}{2} + \left[\frac{-h_{\tilde{A},\tilde{R}}}{9}\right]$$

$$= \frac{h_{\tilde{A},\tilde{R}}}{2} - \frac{h_{\tilde{A},\tilde{R}}}{9}$$

$$= \frac{7h_{\tilde{A},\tilde{R}}}{18}$$

Centroid index of intuitive multiple centroid can be generated using Euclidean Distance by (Cheng, 1998) as

$$R(\tilde{A},\tilde{R}) = \sqrt{\tilde{x}_{\tilde{A},\tilde{R}}{}^{2} + \tilde{y}_{\tilde{A},\tilde{R}}{}^{2}} \tag{4.53}$$

Hence

$$IMC(\tilde{A},\tilde{R}) = \sqrt{\tilde{x}_{\tilde{A},\tilde{R}}{}^{2} + \tilde{y}_{\tilde{A},\tilde{R}}{}^{2}} \tag{4.54}$$

4.4.2 Illustrative Example

This section illustrates a numerical – based example which is used to demonstrate the utilisation of the extension intuitive multiple centroid method for z-number is developed in Section 4.4. A complete illustration of utilising the extension of intuitive multiple centroid method for z-numbers on this example is as follows.

Let $Z=(\tilde{A},\tilde{R})=((10,12,14,16;1),(0.75,1,1,1;1))$ be z-number to be calculated the centroid point of $Z=(\tilde{A},\tilde{R})$, then the centroid point is computed using equation (4.40), (4.43), (4.52) and (4.54) as below. At first, the reliability component, $\tilde{R}$ should be converted into crisp value for x – axis as a weightage for restriction component, $\tilde{A}$.

$$IMC_{\tilde{R}}(\tilde{x}_{\tilde{R}})=\left(\frac{2(\tilde{R}_1+\tilde{R}_4)+7(\tilde{R}_2+\tilde{R}_3)}{18}\right)$$

$$IMC_{\tilde{R}}(\tilde{x}_{\tilde{R}})=\left(\frac{2(0.75+1)+7(1+1)}{18}\right)$$

$$IMC_{\tilde{R}}(\tilde{x}_{\tilde{R}(\varphi)})=(0.9722)$$

Add the weight of the reliability to the constraint. Convert the weighted z-number to regular fuzzy number.

$$\tilde{Z}^{\varphi}=(10,12,14,16;0.9722)$$

$$\tilde{Z}^{'}=\left(\sqrt{\varphi}\times 10,\sqrt{\varphi}\times 12,\sqrt{\varphi}\times 14,\sqrt{\varphi}\times 16;1\right)$$

$$\tilde{Z}^{'}=\left(\sqrt{0.9722}\times 10,\sqrt{0.9722}\times 12,\sqrt{0.9722}\times 14,\sqrt{0.9722}\times 16;1\right)$$

$$\tilde{Z}^{'}=(9.86,10.846,11.832,13.804,15,776;1)$$

Hence, use equation (4.30) and (4.32) for final defuzzification calculation

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}},\tilde{y}_{\tilde{A},\tilde{R}})=\left(\frac{2(\sqrt{\varphi}a_1+\sqrt{\varphi}a_4)+7(\sqrt{\varphi}a_2+\sqrt{\varphi}a_3)}{18},\frac{7h_{\tilde{A},\tilde{R}}}{18}\right)$$

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}},\tilde{y}_{\tilde{A},\tilde{R}})=\left(\frac{2(9.86+15.776)+7(10.846+11.832)}{18},\frac{7(1)}{18}\right)$$

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = (11.6677, 0.3889)$$

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = (11.6677, 0.3889)$$

Hence, the centroid index of intuitive multiple centroid for $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ z-number can be computed as

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \sqrt{\tilde{x}_{\tilde{A},\tilde{R}}{}^2 + \tilde{y}_{\tilde{A},\tilde{R}}{}^2}$$

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \sqrt{11.6677^2 + 0.3889^2}$$

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = 11.6742$$

4.4.3 Theoretical Validation

The properties of defuzzification summarised by (Roychowdhury & Pedrycz, 2001) as mentioned in Section 4.2.3 are applied while the properties of centroid are developed in order to fulfill the reliability requirement. The relevant properties of defuzzification are same as type-1 fuzzy sets while properties of centroid for z-number are presented as follows.

The relevant properties considered for justifying the applicability of centroid for fuzzy numbers, where they depend on the practically within the area of research. However, they are not regarded as complete. Therefore, with no loss of generality, the relevant properties of the centroid are as follows.

Let $Z = (\tilde{A}, \tilde{R})$ and $Z = (\tilde{B}, \tilde{R})$ are be trapezoidal and triangular z-number respectively, while the coordinate intuitive multiple centroid**,** $IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}})$ and $IMC(\tilde{x}_{\tilde{B},\tilde{R}}, \tilde{y}_{\tilde{B},\tilde{R}})$ be centroid for $\tilde{A}$ and $\tilde{B}$ respectively. Centroid index of intuitive multiple centroid represents the crisp value of centroid point that is denoted as $IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}})$ and $IMC(\tilde{x}_{\tilde{B},\tilde{R}}, \tilde{y}_{\tilde{B},\tilde{R}})$.

Property 1: *If* $\tilde{A}$ *and* $\tilde{B}$ *are embedded and symmetry, then* $IMC(\tilde{A},\tilde{R}) > IMC(\tilde{B},\tilde{R})$.

Proof: *Since* $\tilde{A}$ *and* $\tilde{B}$ *are embedded and symmetry, hence we know that* $\tilde{x}_{\tilde{A},\tilde{R}} = \tilde{x}_{\tilde{B},\tilde{R}}$ *and* $\tilde{y}_{\tilde{A},\tilde{R}} > \tilde{y}_{\tilde{B},\tilde{R}}$. *Then, from equation (4.32) we have* $\sqrt{\tilde{x}_{\tilde{A},\tilde{R}}^{\ 2} + \tilde{y}_{\tilde{A},\tilde{R}}^{\ 2}} > \sqrt{\tilde{x}_{\tilde{B},\tilde{R}}^{\ 2} + \tilde{y}_{\tilde{B},\tilde{R}}^{\ 2}}$. *Therefore,* $IMC(\tilde{A},\tilde{R}) > IMC(\tilde{B},\tilde{R})$.

Property 2: *If* $\tilde{A}$ *and* $\tilde{B}$ *are embedded with* $h_{\tilde{A}} > h_{\tilde{B}}$, *then* $IMC(\tilde{A},\tilde{R}) > IMC(\tilde{B},\tilde{R})$.

Proof: *Since* $\tilde{A}$ *and* $\tilde{B}$ *are embedded and with* $h_{\tilde{A}} > h_{\tilde{B}}$,*, hence we know that* $\tilde{x}_{\tilde{A},\tilde{R}} = \tilde{x}_{\tilde{B},\tilde{R}}$ *and* $\tilde{y}_{\tilde{A},\tilde{R}} > \tilde{y}_{\tilde{B},\tilde{R}}$. *Then, from equation (4.32) we have* $\sqrt{\tilde{x}_{\tilde{A},\tilde{R}}^{\ 2} + \tilde{y}_{\tilde{A},\tilde{R}}^{\ 2}} > \sqrt{\tilde{x}_{\tilde{B},\tilde{R}}^{\ 2} + \tilde{y}_{\tilde{B},\tilde{R}}^{\ 2}}$. *Therefore,* $IMC(\tilde{A},\tilde{R}) > IMC(\tilde{B},\tilde{R})$.

Property 3: *If* $\tilde{A}$ *is fuzzy singleton number, then* $IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \sqrt{\tilde{x}_{\tilde{A},\tilde{R}}^{\ 2} + \tilde{y}_{\tilde{A},\tilde{R}}^{\ 2}}$.

Proof: *For any crisp (real) numbers, we know that* $a_1 = a_2 = a_3 = a_4 = \tilde{x}_{\tilde{A}}$ *and* $\tilde{y}_{\tilde{A}} < 1$ *which are equivalent to equation (4.30). Therefore,* $IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \sqrt{\tilde{x}_{\tilde{A},\tilde{R}}^{\ 2} + \tilde{y}_{\tilde{A},\tilde{R}}^{\ 2}}$.

Property 4: *If* $\tilde{A}$ *and* $\tilde{B}$ *are any symmetrical or fuzzy asymmetrical number, then* $a_1 < IMC(\tilde{A},\tilde{R}) < a_4$ *and* $b_1 < IMC(\tilde{B},\tilde{R}) < b_4$.

Proof: *Since* $\tilde{A}$ *and* $\tilde{B}$ *are any symmetrical or asymmetrical fuzzy numbers, hence* $a_1 < IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) < a_4$ *and* $b_1 < IMC(\tilde{x}_{\tilde{B},\tilde{R}}, \tilde{y}_{\tilde{B},\tilde{R}}) < b_4$. *Therefore,* $a_1 < IMC(\tilde{A},\tilde{R}) < a_4$ *and* $b_1 < IMC(\tilde{B},\tilde{R}) < b_4$ *respectively.*

All properties are related with computation for single crisp value $IMC(\tilde{A},\tilde{R})$, where $\tilde{A}$ and $\tilde{R}$ is any possible generalised type-1 fuzzy set.

4.4.4 Empirical Validation

The empirical validation of centroid method is extensively discussed. Discussions of this validation are made in accordance with case studies found in the literature of fuzzy sets.

There are several possible cases in representing z-number which are:

1) Trapezoidal normal symmetry
2) Trapezoidal normal asymmetry
3) Trapezoidal non – normal symmetry
4) Trapezoidal non – normal asymmetry
5) Triangular normal symmetry
6) Triangular normal asymmetry
7) Triangular non – normal symmetry
8) Triangular non – normal asymmetry
9) Singleton normal
10) Singleton non – normal

Representation of all possible cases in z-numbers:

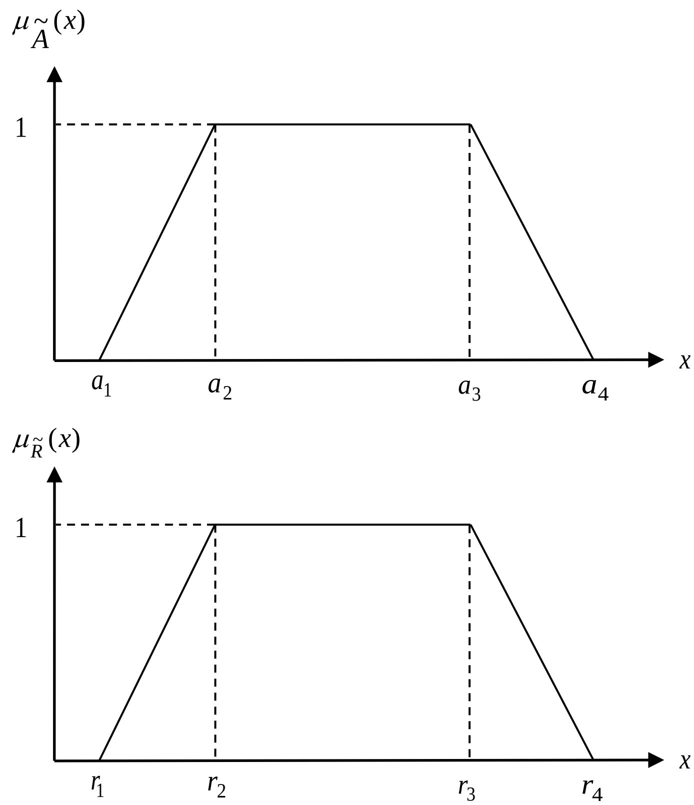


Fig. 4. 42: Trapezoidal normal symmetry of z-number, $Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3, a_4; 1), (R_1, R_2, R_3, R_4; 1))$

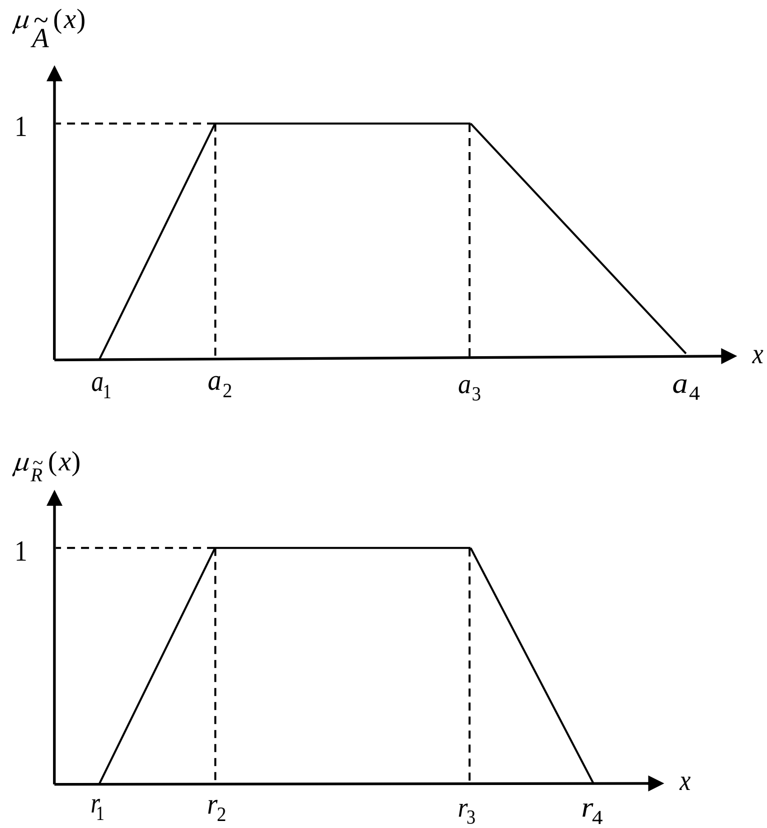


Fig. 4. 43: Trapezoidal normal asymmetry of z-number,

$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3, a_4; 1), (R_1, R_2, R_3, R_4; 1))$

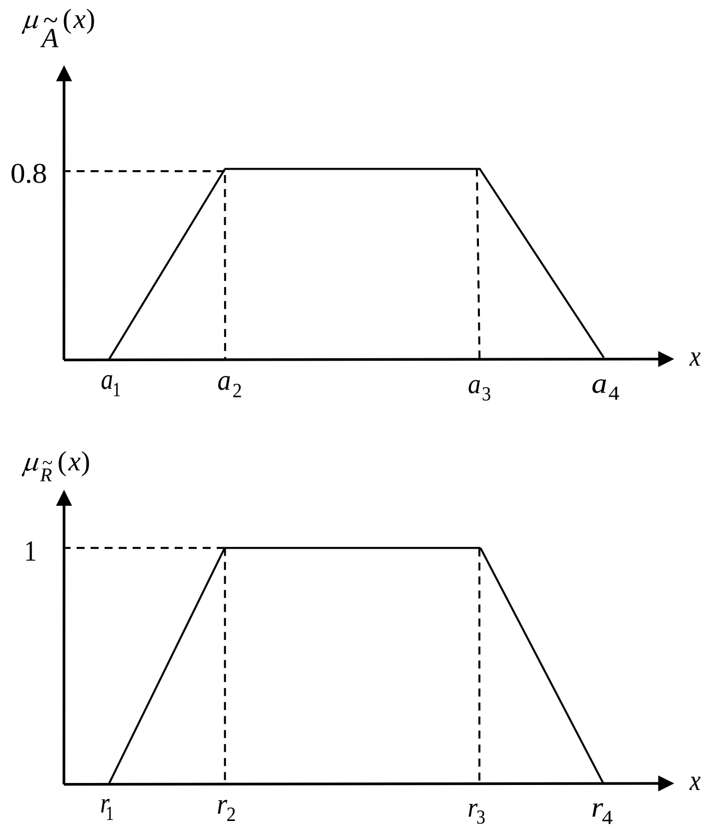


Fig. 4. 44: Trapezoidal non – normal symmetry of z-number,

$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3, a_4; 0.8), (R_1, R_2, R_3, R_4; 1))$

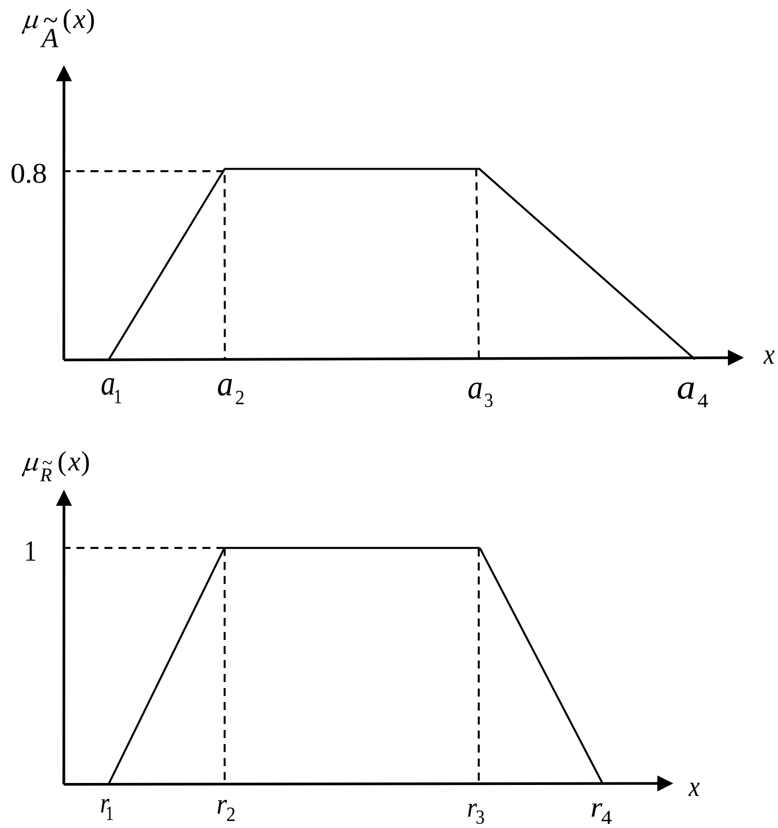


Fig. 4. 45: Trapezoidal non – normal asymmetry of z-number,
$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3, a_4; 0.8), (R_1, R_2, R_3, R_4; 1))$

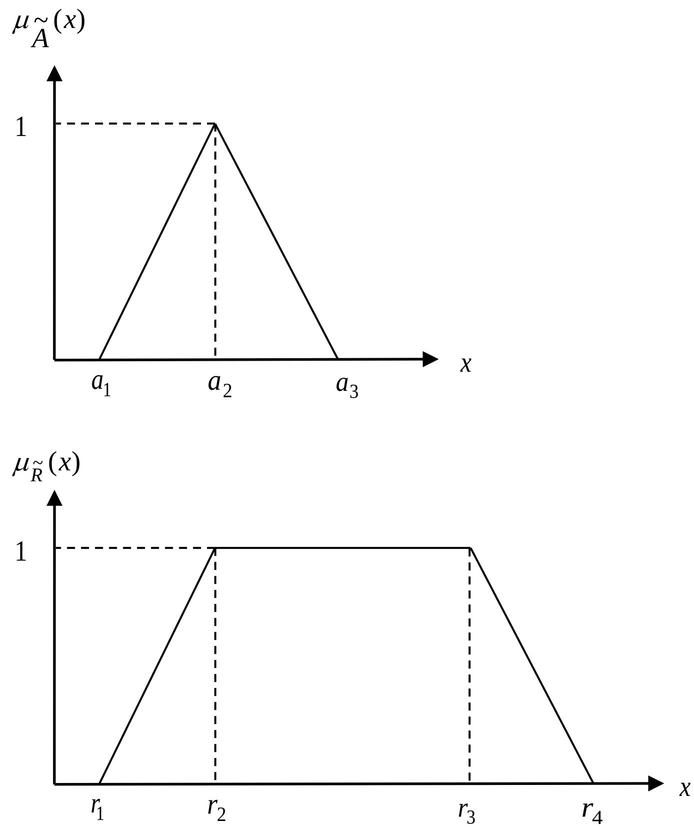


Fig. 4. 46: Triangular normal symmetry of z-number,

$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3;1), (R_1, R_2, R_3, R_4;1))$

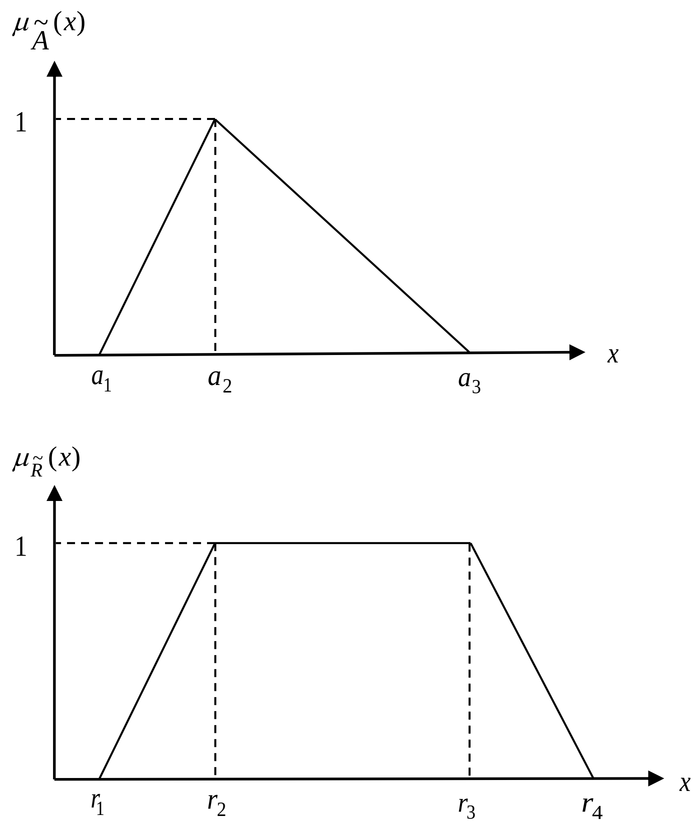


Fig. 4. 47: Triangular normal asymmetry of z-number,
$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3;1),(R_1, R_2, R_3, R_4;1))$

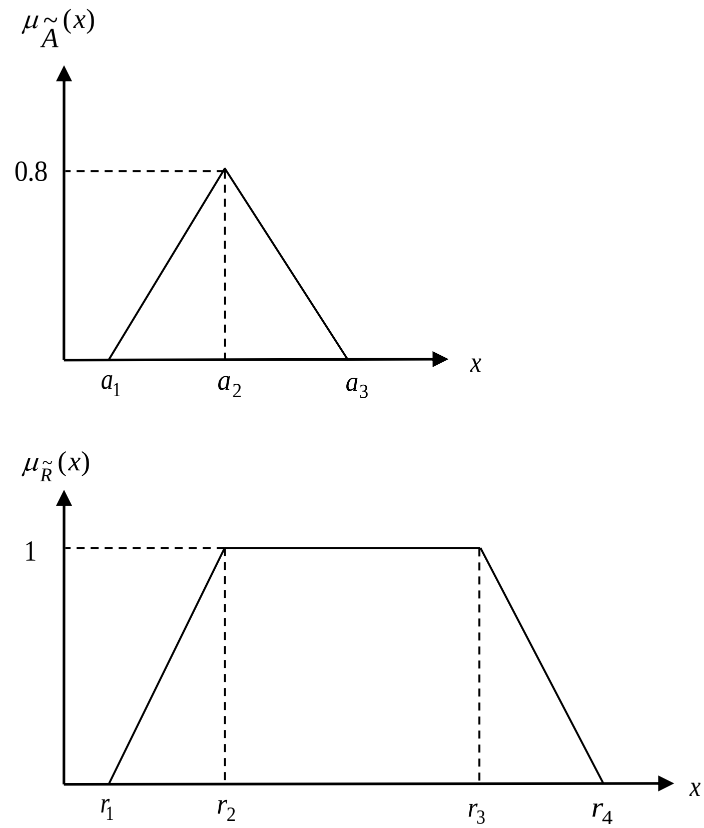


Fig. 4. 48: Triangular non – normal symmetry of z-number,
$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3; 0.8), (R_1, R_2, R_3, R_4; 1))$

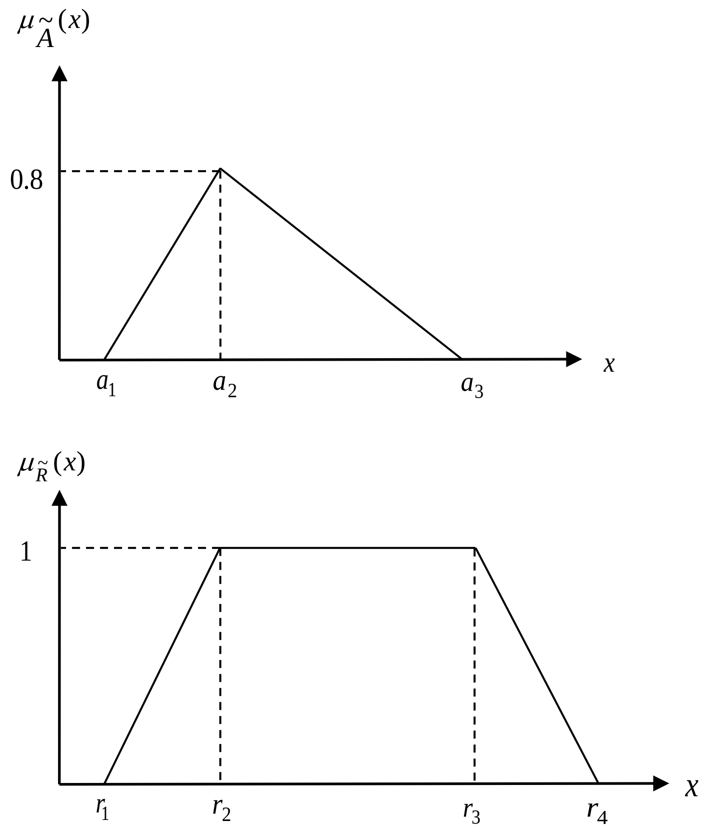


Fig. 4. 49: Triangular non - normal asymmetry of z-number,
$Z_{\tilde{A},\tilde{R}} = ((a_1, a_2, a_3; 0.8), (R_1, R_2, R_3, R_4; 1))$

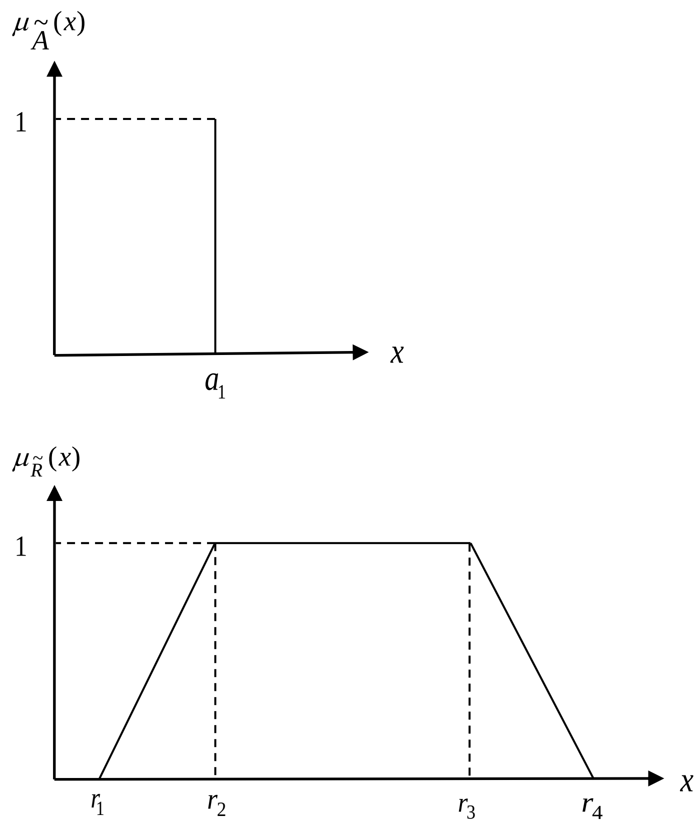


Fig. 4. 50: Singleton normal of z-number, $Z_{\tilde{A},\tilde{R}} = ((a_1;1),(R_1, R_2, R_3, R_4;1))$

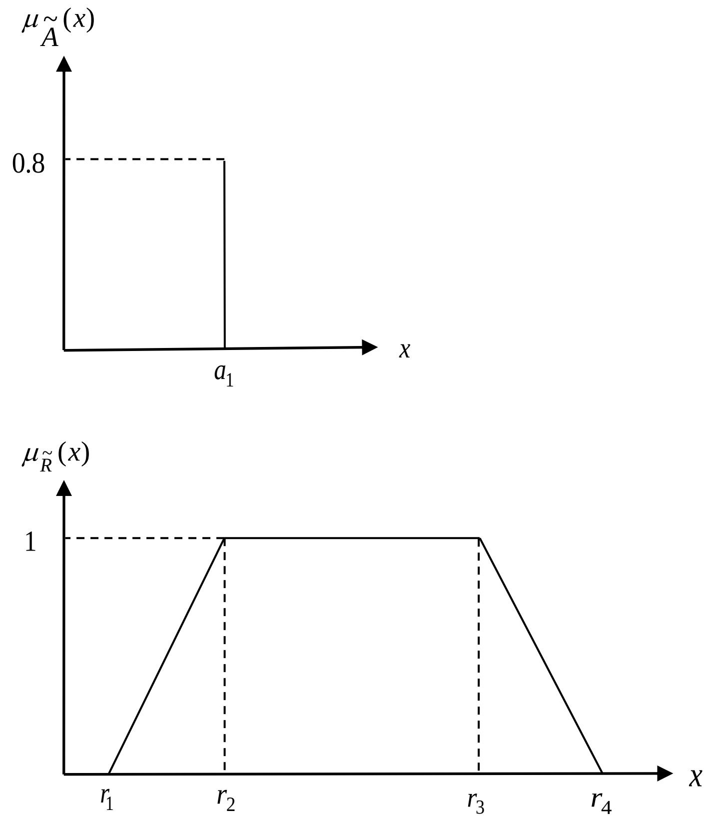


Fig. 4. 51: Singleton non – normal of z-number, $Z_{\tilde{A},\tilde{R}} = ((a_1;0.8),(R_1,R_2,R_3,R_4;1))$

Application:

The elementary problem of temperature mensuration based on arithmetic operation of centroid defuzzification methods of the proposed of extension intuitive multiple centroid for z-numbers and established methods, (Kang et al., 2012b) and (Abu Bakar & Gegov, 2015b) are compared.

Let the temperature (C°) of a room is measured by each possible cases of z-numbers as presented in Table 4.3. All of possible cases of fuzzy numbers are defuzzified using two different defuzzification methods and the results are presented in table below.

Table 4. 3. Conversion process from z-numbers to classical type-1 fuzzy sets

Case	Z-numbers									
	(a1,	a2,	a3,	a4;	h)	(R1,	R2,	R3,	R4;	hR)
1	(10,	12,	14,	16;	1)	(0.75,	1,	1,	1;	1)
2	(10,	12,	14,	17;	1)	(0.75,	1,	1,	1;	1)
3	(10,	12,	14,	16;	0.9)	(0.75,	1,	1,	1;	1)
4	(10,	12,	14,	17;	0.9)	(0.75,	1,	1,	1;	1)
5	(10,	13,	13,	15;	1)	(0.75,	1,	1,	1;	1)
6	(10,	13,	13,	16;	1)	(0.75,	1,	1,	1;	1)
7	(10,	13,	13,	15;	0.9)	(0.75,	1,	1,	1;	1)
8	(10,	13,	13,	16;	0.9)	(0.75,	1,	1,	1;	1)
9	(10,	10,	10,	10;	1)	(0.75,	1,	1,	1;	1)
10	(10,	10,	10,	10;	0.9)	(0.75,	1,	1,	1;	1)

Table 4. 4. Comparative empirical – based validation study for centroid defuzzification of z-numbers

Case	Generalised Fuzzy Numbers					Kang et al. (2012b)			Abu Bakar & Gegov (2015b)			Ku Khalif & Gegov (proposed)		
	(a1,	a2,	a3,	a4;	h)	*x*	*y*	Score Index	*x*	*y*	Score Index	*x*	*y*	Score Index
1	9.71825	11.6619	13.6056	15.5492	1	12.6337	0.4167	12.6406	12.6337	0.4167	12.6406	12.6337	0.3889	12.6397
2	9.71825	11.6619	13.6056	16.521	1	12.9217	0.4074	12.9281	12.9217	0.4074	12.9281	12.8497	0.3889	12.8556
3	9.71825	11.6619	13.6056	15.5492	0.9	12.6337	0.3750	12.6393	12.6337	0.3750	12.6393	12.6337	0.3500	12.6386
4	9.71825	11.6619	13.6056	16.521	0.9	12.9217	0.3667	12.9269	12.9217	0.3667	12.9269	12.8497	0.3500	12.8545
5	9.71825	12.6337	12.6337	14.5774	1	12.3098	0.3333	12.3143	12.3098	0.3333	12.3143	12.4178	0.3889	12.4239
6	9.71825	12.6337	12.6337	15.5492	1	12.6337	0.3333	12.6381	12.6337	0.3333	12.6381	12.6337	0.3889	12.6397
7	9.71825	12.6337	12.6337	14.5774	0.9	12.3098	0.3000	12.3134	12.3098	0.3000	12.3134	12.4178	0.3500	12.4227
8	9.71825	12.6337	12.6337	15.5492	0.9	12.6337	0.3000	12.6373	12.6337	0.3000	12.6373	12.6337	0.3500	12.6386
9	9.71825	9.71825	9.71825	9.71825	1	9.7183	0.3333	9.7240	12.9577	0.3333	12.9620	9.7183	0.3889	9.7260
10	9.71825	9.71825	9.71825	9.71825	0.9	9.7183	0.3000	9.7229	12.9577	0.3000	12.9611	9.7183	0.3500	9.7246

Defuzification for z-numbers is still new in the literature of z-numbers. Z-number can be represented as an extension of type-1 fuzzy set in term of membership function, but completely differ from type-2 fuzzy sets. There is a pair of type-1 fuzzy set in representing z-number as mentioned early. The defuzzification methods proposed by researchers are lesser than type-1 and type-2 fuzzy sets. Defuzzification of z-numbers is quite complicated because the consideration of two components (fuzzy restriction and reliability of fuzzy restriction) for one z-number. Under this situation, (Kang et al., 2012b) proposed a conversion method for z-numbers to classical fuzzy numbers which are type-1 fuzzy sets according to the multiplication operation of triangular fuzzy numbers. Later, (Kang et al., 2012a) proposed a method of converting z-numbers to classical fuzzy numbers that is according Fuzzy Expectation. Most of the researchers or practitioners used this conversion method in dealing with z-numbers. This conversion method has more influence to describe the knowledge of human being and widely used in uncertain information.

As can be seen in Table 4.4, three centroid defuzzification methods for z-numbers are compared with different possible cases of fuzzy numbers representation. The extension of proposed intuitive multiple centroid for z-numbers, $\tilde{x}_{\tilde{A},\tilde{R}} = \frac{2(\sqrt{\varphi}a_1 + \sqrt{\varphi}a_4) + 7(\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3)}{18}$, $\tilde{y}_{\tilde{A},\tilde{R}} = \frac{7h_{\tilde{A},\tilde{R}}}{18}$ is compared with established centroid methods for z-numbers which are from:

1. (Kang et al., 2012b)

   The authors proposed conversion process for z-numbers into regular fuzzy numbers by reduce the reliability component into crisp number as a weight for restriction component using (Y. M. Wang et al., 2006), equation (3.6).

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)dx}{\int \mu_{\tilde{R}}(x)dx}$$

2. (Abu Bakar & Gegov, 2015b)

   The authors apply conversion process proposed by (Kang et al., 2012b) and reduce the z-numbers into regular fuzzy numbers by converting reliability using centroid defuzzification (Shieh, 2007) from equation (4.17) and (4.18) into crisp number as a weight for restriction component.

There are ten all possible cases same as type-1 and type-2 fuzzy sets representing fuzzy numbers as mentioned earlier. Representing all centroid defuzzification methods for z-numbers follow (Kang et al., 2012b) conversion process according Fuzzy Expectation. The different is centroid method used. Referring to Table 4.4, (Kang et al., 2012b) and (Abu Bakar & Gegov, 2015b) produce eight same results for case 1 until case 8, except case 9 and 10. Seeing that results, (Kang et al., 2012b) applied (Y. M. Wang et al., 2006) centroid method for defuzzification process to get crisp value. Instead (Abu Bakar & Gegov, 2015b) apply (Shieh, 2007) centroid method for defuzzification process to get the crisp value. In the literature, (Shieh, 2007) centroid method is an improvised from (Y. M. Wang et al., 2006) in the representation and properties. No wonder both (Kang et al., 2012b) and (Abu Bakar & Gegov, 2015b) give same results for case 1 until 8. But, (Abu Bakar & Gegov, 2015b) deal inappropriately with singleton cases which produce defuzzification results too far from the permitted zone which is should be close to 10.

The extension of the proposed intuitive multiple centroid for z-numbers gives consistent results for all cases compared to (Kang et al., 2012b) centroid method. Moreover, both of them give almost similar results for all possible cases of representation of z-numbers. Thus, the implementation of intuitive multiple centroid is more reasonable, rational and logic and consistent with established one.

4.5 Summary of the Chapter

This chapter presents in detail the process of development of intuitive multiple centroid defuzzification method for fuzzy sets. Reviewing the advantages and limitations of the established centroid defuzzification methods for fuzzy sets are very useful task to investigate the proposed intuitive multiple centroid method that should be adopted in this research work and used in proposed hybrid fuzzy MCDM model in the next chapter. A novel intuitive multiple centroid for fuzzy sets is developed in this chapter that covers type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Furthermore, the theoretical and empirical validations are broadly discussed in this chapter. All relevant properties are considered on differentiating fuzzy numbers for justifying the applicability of centroid appropriately.

The novel proposed intuitive multiple centroid defuzzification method for fuzzy sets are technically discussed in detail in this chapter. Moreover, several numerical examples are presented to show the applicability and performance of the proposed methods. As an application of the concepts of proposed method are introduced, it will be applied in the development of hybrid fuzzy MCDM model in Chapter 5. Descriptions on proposed intuitive multiple centroid defuzzification in this chapter underpin applications on the proposed hybrid fuzzy MCDM model for next two chapters. This indicates that Chapter 4 underpins Chapter 5 and Chapter 6 of the thesis. In Chapter 5, the development of hybrid fuzzy MCDM model is presented that is incorporated with intuitive multiple centroid methods in different fuzzy environment.

CHAPTER 5

GENERALISED HYBRID FUZZY MULTI CRITERIA DECISION MAKING MODEL

5.1 Overview

This chapter illustrates the detail process on the development of generalised hybrid MCDM model that consist of consistent fuzzy preference relations and fuzzy TOPSIS. The novel hybrid MCDM model is developed by improvising several steps in computing the consistent fuzzy preference relations and fuzzy TOPSIS to make sure both techniques are perfectly integrated. This model capable to interact or cooperate with unlimited criteria in dealing with real world decision making problems. In developing the proposed hybrid MCDM model, the intuitive multiple centroid method is applied as defuzzification process in converting fuzzy values into crisp or single values. As mentioned in Chapter 2, it is important to use proper defuzzification method in order to consider the need of human perception even representing in regular numbers.

The proposed intuitive multiple centroid defuzzification and hybrid MCDM model are validated theoretically and empirically which determine reliability, consistency and sensitivity analysis. Reliability, a theoretical based – validation, validates the: 1) novel intuitive multiple centroid using several properties that are considered for justifying the applicability of centroid for fuzzy numbers and: 2) novel hybrid MCDM model with several improvement steps from the classical one. The other two criteria namely consistency and sensitivity analysis, which are two distinct empirical based – validation, compute: 1) the capability of the novel centroid method to correctly formulae that are consistent with other established models and: 2) the proficiency of novel hybrid MCDM model using approval status table (Luukka, 2011) and sensitivity analysis (Amini & Alinezhad, 2011). Both theoretical and empirical validations stated are thoroughly defined in this chapter but the implementations are demonstrated in the Chapter 6. Details on those aforementioned points are extensively discussed in sections and subsections of this chapter.

5.2 Development of Hybrid Consistent Fuzzy Preference Relations and Fuzzy Techniques for Order of Preference by Similarity to Ideal Solution

5.2.1 Introduction

The latest trend with respect to MCDM is to combine two or more techniques to make up or handle shortcomings appropriately in any single particular method (Velasquez & Hester, 2013). In this research study, consistent fuzzy preference relations and fuzzy TOPSIS are integrated in dealing with uncertain judgements. In MCDM concept, to make best decision under different circumstances for the alternatives that based on criteria provided is focusing on the main objective to achieve is. Evaluation process for criteria and alternatives play important role in MCDM techniques requirement. In identifying the best decision making to be made among the various alternatives with several criteria, the methodology has study the relationship or preference among the criteria to make sure the weights of criteria are reliable enough to be implemented in the selection of alternatives. Both extended MCDM techniques include synthesis of uncertainty into group decision making by applying fuzzy set theory concept. Considering that, it takes the fact that each decision maker in the decision making group could have individual importance power within the group. This represents a new step and a new field of study for the existing MCDM techniques.

Fuzzy set theory was introduced to rationalise uncertainty associated with imprecision or vagueness and plus thus applicable to human thought. To express the experts' opinions, classical MCDM techniques can be used but unable to cater human thinking. Because of that reason, fuzzy MCDM techniques are developed to hierarchy imprecise problems. Consistency is crucial for achieving correct solutions in decision process. Due to each positive reciprocal matrix is described by fuzzy numbers in fuzzy linguistic terms, so to satisfy the consistency is very difficult (T.-C. Wang & Chen, 2006). Besides, establishing a fuzzy positive reciprocal matrix requires $\frac{n \times (n-1)}{2}$ judgements to be made for a level with n criteria. Hence, the number of comparisons increase with the numbers of criteria, so inconsistent conditions are likely to occur. To solve the consistency problem, the consistent fuzzy preference relations technique is adopted in order to construct fuzzy decision matrix instead of fuzzy positive reciprocal matrix. The utilisation of consistent fuzzy preference relations in this phase yields decision matrices for making pairwise comparison matrices using additive transitivity. There are only $n-1$ comparison judgements are required to ensure consistency on a level that contains n criteria.

According to (Zanakis, Solomon, Wishart, & Dublish, 1998), TOPSIS provides unique way to approach problems, intuitively appealing and easy to understand. In additional, it also represents the rationale of individual choice a scalar value that records both the best and worst alternatives concurrently a straightforward computation algorithm. Fuzzy TOPSIS is an extended classical TOPSIS with considers fuzzy component as an added value in order to deal human perceptions. In ongoing effort, fuzzy TOPSIS is particularly useful for those problems in which the valuations of the alternatives on the basis of the criteria are not represented in the same units (Lima Junior, Osiro, & Carpinetti, 2014). In addition, through another viewpoint, the TOPSIS technique is a good decision making approach due to its simplicity and ability to consider a non-limited number of alternatives and criteria in the decision making process. Regarding to the level of interaction with decision makers to imprecise data collection, fuzzy TOPSIS technique provides good agility in the decision process. Concerning the agility in the decision process, fuzzy TOPSIS performs better in most cases. The increase or decrease number of criteria or alternatives does not affect the agility of fuzzy TOPSIS.

The combination of consistent fuzzy preference relations and fuzzy TOPSIS in this study gives better computation to evaluate criteria and alternatives in MCDM problems. In the development of hybrid fuzzy MCDM model, particularly, consistent fuzzy preference relations plays role in evaluating criteria while fuzzy TOPSIS is utilised in evaluating alternatives. This hybrid fuzzy MCDM model incorporates together with intuitive multiple centroid that discussed in Chapter 4. Concerning in dealing with fuzzy linguistic scales, defuzzification process is needed in order to access the final results as regular numbers at the same time fulfill the need of human perception in decision making problems. Still, the development of hybrid fuzzy MCMD model is not only limited for type-1 fuzzy sets, but it does covers for type-2 fuzzy sets and z-numbers. While much of the literature regarding fuzzy MCDM nowadays not only focusing on type-1 fuzzy sets, researchers and experts have initiate in using different fuzzy numbers in order to deal with different uncertain events appropriately.

In conclusion, this combination of consistent fuzzy preference relations and fuzzy TOPSIS that incorporating with intuitive multiple centroid provide better selection in human based decision making problems where at the same time capable to deal with imprecision, vagueness and uncertainty under fuzzy environment. Due to access information and availability of the huge amount of data, it is hard to make right decision. In this sense, it is important for the decision making problems to extend the classical decision making techniques, adding intuitive reasoning, human subjectivity and imprecision. In traditional decision making processes, the researchers or practitioners only consider single criteria problems, the decision making is extremely intuitive. The decision makers only need to choose the alternative with highest preference rating without considering the multiple criteria involves, the weights of criteria, preference dependence, conflict among criteria, how complicated the problems and decision making methods to be used. While it is true that human intuition in MCDM problems provide huge weightage in order to understand the imprecision, vagueness and uncertainty. As a consequence, the development of hybrid fuzzy MCDM model is developed to design the robust and reliable methodology in order to give the most promising alternative with respect to resources.

5.2.2 Methodology

This hybrid fuzzy MCDM methodology considers general steps for any fuzzy set's evaluations. The new hybrid consistent fuzzy preference relations – fuzzy TOPSIS methodology consist of four phases is illustrated as below.

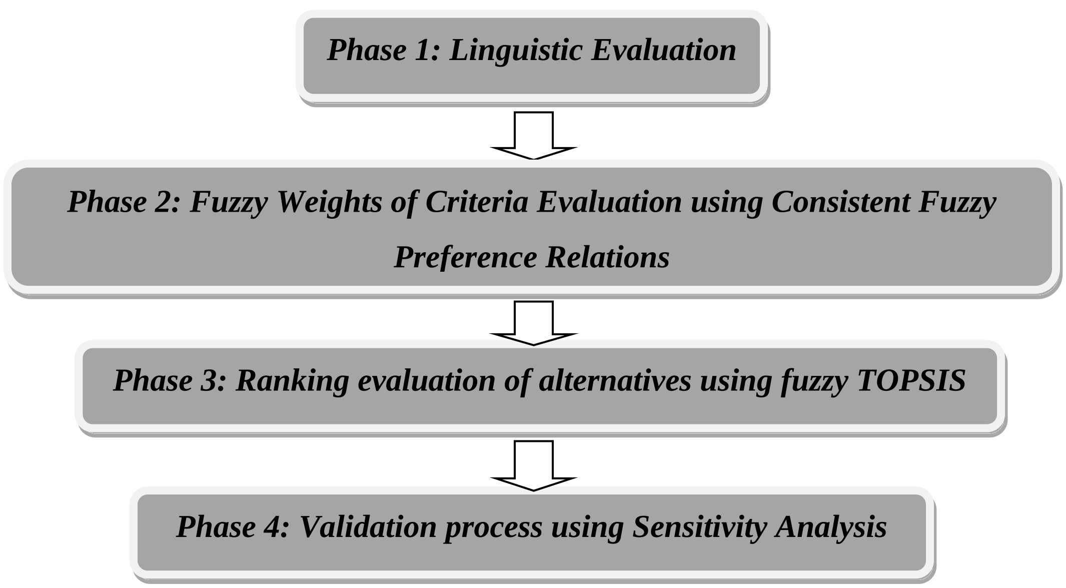


Fig. 5. 1: Hybrid consistent fuzzy preference relations – fuzzy TOPSIS framework

Phase 1: Linguistic Evaluation

The decision makers will use the linguistic terms to present the weights of criteria using consistent fuzzy preference relations evaluation based on type of fuzzy sets. The linguistic terms present the important of criteria preferences. For fuzzy TOPSIS evaluation, the another linguistic terms are used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively.

Phase 2: Fuzzy Weights of Criteria Evaluation using Consistent Fuzzy Preference Relations

Step 1: Construct a hierarchy structure.

The construction of hierarchy model needs judgement matrix that filled by decision makers about the evaluation of all criteria.

Step 2: Construct a pairwise comparison matrices

Consistent fuzzy preference relations is adopted to evaluate the weights of difference criteria for the performance of alternatives. The pairwise comparison matrices are constructed among all criteria in the dimension of the hierarchy systems based on the decision makers' preferences in phase 1 as following matrix *A*:

$$A = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & \cdots & 1 \end{bmatrix} \qquad (5.1)$$

Step 3: Aggregate the decision makers' preferences.

The pairwise comparison matrices of decision makers' preferences are aggregated using equation:

$$\tilde{a}_{ij} = (\tilde{a}_{ij}^{1} \times \tilde{a}_{ij}^{2} \times \ldots \times \tilde{a}_{ij}^{n})^{1/k} \qquad (5.2)$$

where k is the number of decision makers and $i=1,2,\ldots m; j=1,2,\ldots n$.

Step 4: Defuzzify the fuzzy numbers of aggregation's results of decision makers' preferences using intuitive multiple centroid.

Defuzzify trapezoidal fuzzy weights using intuitive multiple centroid using equation (4.5), (4.23) and (4.52) for x - axis and y – axis, then get the crisp value using Euclidean Distance by (Cheng, 1998). For evaluation of criteria for this stage, it depend on the case study that handled, the degree of confidence of the decision makers' opinions are agreed either as normal which is highest degree with $h=1$, or non-normal case with $0<h<1$. Some of the linguistic scales are representing the membership degree with $h=1$. Since all evaluations are $h=1$, most of the researchers in decision making analysis (Sun, 2010), (Rostamzadeh & Sofian, 2011), (Vinodh et al., 2014), only consider the defuzzification of x – axis and ignore the y – axis. Below are the intuitive multiple centroid defuzzification methods for type-1 fuzzy sets, interval type-2 fuzzy sets and z-numbers respectively.

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}, \frac{7h_{\tilde{A}}}{18} \right)$$

$$IMC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}, \frac{7}{36}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right)$$

$$IMC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \left(\frac{2(\sqrt{\varphi}a_1 + \sqrt{\varphi}a_4) + 7(\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3)}{18}, \frac{7h_{\tilde{A},\tilde{R}}}{18} \right)$$

Step 5: Compute the criteria values as weightage for alternatives' evaluation using consistent fuzzy preference relations.

In order to avoid misleading solutions in expressing the decision makers' opinions, the study of consistency by means of preference relations becomes a very important aspect. In decision making problems based on fuzzy preference relations, the study of consistency is associated with the study of transitivity properties. In this chapter, the new characterisation of consistency property is defined by the additive transitivity property of fuzzy preference relation is developed.

Referring to (Kamis et al., 2011), a fuzzy preference relation R on the set of the criteria or alternatives A is a fuzzy set stated on the Cartesian product set $A \times A$ with the membership function $\mu_R : A \times A \to [0,1]$. The preference relation is denoted by $n \times n$ matrix $R = (r_{ij})$ where $r_{ij} = \mu_y(a_i, a_j)$ $\forall i,\ j \in \{1,...,n\}$. The preference ratio, r_{ij} of the alternative a_i to a_j is determined by equation (3.16):

$$r_{ij} = \begin{cases} 0.5 & a_i \ \textit{is different to} \ a_j \\ (0.5,1) & a_i \ \textit{is preferred than} \ a_j \\ 1 & a_i \ \textit{is absolutely preferred than} \ a_j \end{cases}$$

The preference matrix R is presumed to be additive reciprocal, $p_{ij} + p_{ji} = 1,\ \forall i,\ j \in \{1,...,n\}$. Several propositions are associated to the consistent additive preference relations as follows:

Proposition (4.1) (T. C. Wang & Chen, 2007): *Consider a set of criteria or alternatives,* $X = \{x_1,...,x_n\}$*, and associated with a reciprocal multiplicative preference relation* $A = (a_{ij})$ *for* $a_{ij} \in \left[\frac{1}{9}, 9\right]$*. Then, the corresponding reciprocal fuzzy preference relation,* $P = (p_{ij})$ *with* $p_{ij} \in [0,1]$ *associated with A is given by the equation (3.17):*

$$p_{ij} = g(a_{ij}) = \frac{1}{2}(1 + \log_9 a_{ij})$$

Generally, if $a_{ij} \in \left[\frac{1}{n}, n\right]$*, then* $\log_n a_{ij}$ *is used, in particular, when* $a_{ij} \in \left[\frac{1}{9}, 9\right]$*;* $\log_9 a_{ij}$ *is considered as in the above proposition because* a_{ij} is between $\frac{1}{9}$ *and 9. If* a_{ij} is between $\frac{1}{7}$ *and 7, then* $\log_7 a_{ij}$ *is used.*

Proposition (4.2) (T. C. Wang & Chen, 2007): *For a reciprocal fuzzy preference relation* $P = (p_{ij})$*, the following statements are equivalent (equation (3.18), (3.19) and (3.21)):*

(i) $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \ \forall i, j, k$

(ii) $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \ \forall i < j < k$

(iii) $p_{i(i+1)} + p_{(i+1)(i+2)} + ... + p_{(j-1)j} + p_{ji} = \frac{j-i+1}{2}, \ \forall i < j$

Proposition (4.2) (ii) and (iii) are crucial because it can be used to construct a consistent fuzzy preference relations form the set of $n-1$ values $\{p_{12}, p_{23},..., p_{n-1}\}$. A decision matrix with entries that are not in the interval $[0,1]$, but in an interval $[-c, 1+c]$, $c > 0$, can be obtained by transforming the obtained values using a transformation function that preserves reciprocity and additive consistency with the function (equation 3.22):

$$f : [-c, 1+c] \to [0,1], \ f(x) = \frac{(x+c)}{(1+2c)}$$

Phase 3: Ranking evaluation of alternatives using fuzzy TOPSIS

Step 1: Determine the weights of evaluation criteria.

The weighting of evaluation criteria are employed from consistent fuzzy preference relations process before.

Step 2: Construct the fuzzy decision matrix for alternatives' evaluation using fuzzy TOPSIS.

Concept of TOPSIS technique originally proposed by (Hwang & Yoon, 1981). They claimed that the alternative should not be chosen based on having the shortest distance from the positive ideal reference point (PIRT) only, but also have the longest distance from the negative ideal reference point (NIRP) in solving the MCDM problems. Here, the proposed methodology for fuzzy TOPSIS is illustrated differ from others in terms of the usage of defuzzification method, normalization process and ranking.

The fuzzy decision matrix is constructed and the linguistic terms from fuzzy numbers are used to evaluate the alternatives with respect to criteria. Then, aggregate DMs' preferences:

$$\overline{DM} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{5.3}$$

$$i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n \quad ,\tilde{x}_{ij} = \frac{1}{K}\left(\tilde{x}_{ij}^{1} \oplus,,, \oplus \tilde{x}_{ij}^{k} \oplus,,, \tilde{x}_{ij}^{K}\right)$$

where x_{ij} is the performance rating of alternatives, A_i with respect to criterion C_i evaluated by *kth* experts and $\tilde{x}_{ij} = (a_1^k, a_2^k, a_3^k, a_4^k; h^k)$.

Step 3: Fuzzy decision matrix is weighted and normalised. Then, defuzzify the standardised generalised fuzzy numbers into coordinate form, $(\tilde{x}, \tilde{y})$. The weighted fuzzy normalised decision matrix is denoted by $\tilde{V}$ as depicted below with numerical example:

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{m \otimes n}; \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n \tag{5.4}$$

where

$$\tilde{v}_{ij} = \tilde{x}_{ij} \times \tilde{w}_j \tag{5.5}$$

Normalised each generalised trapezoidal fuzzy numbers into standardised generalised fuzzy numbers using (Zuo, Wang, & Yue, 2013):

$$a'_1 = \frac{a_1 - \min(a_1, b_1)}{\max(a_4, b_4) - \min(a_1, b_1)},$$

$$a'_2 = \frac{a_2 - \min(a_1, b_1)}{\max(a_4, b_4) - \min(a_1, b_1)},$$

$$a'_3 = \frac{a_3 - \min(a_1, b_1)}{\max(a_4, b_4) - \min(a_1, b_1)},$$

$$a'_4 = \frac{a_4 - \min(a_1, b_1)}{\max(a_4, b_4) - \min(a_1, b_1)}, \tag{5.6}$$

The weights from consistent fuzzy preference relations are adopted here. Defuzzify the standardised generalised fuzzy numbers using intuitive multiple centroid $IMC(x_{\tilde{A}_i^*}, y_{\tilde{A}_i^*})$, then translate them into the index point proposed by (Yong & Qi, 2005), shown as follows:

$$x_{\tilde{A}_i^*} = x_{\tilde{A}_i^*} \tag{5.7}$$

$$y_{\tilde{A}_i^s} = 0.5 \times h_{\tilde{A}_i} - y_{\tilde{A}_i^*} \times STD_{\tilde{A}_i} \tag{5.8}$$

where spread, $STD_{\tilde{A}_i} = \sqrt{\dfrac{\sum_{j=1}^{4}(a_{ij}^* - x_{\tilde{A}_i^*})}{n-1}}$ $, n = number \quad of \quad observation$ (5.9)

Use the new point of $y_{\tilde{A}_i^*}$ to compute the index centroid point of standardised generalised trapezoidal fuzzy numbers using Euclidean distance equation (4.7):

$$R(\tilde{A}_i^*) = \sqrt{\tilde{x}_i^{*2} + \tilde{y}_i^{S\,2}}$$

Step 4: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

Referring to normalise trapezoidal fuzzy weights, the FPIS, A^+ represents the compromise solution while FNIS, A^- represents the worst possible solution. The range belong to the closed interval [0,1]. The FPIS A^+ (aspiration levels) and FNIS A^- (worst levels) as following below:

$$A^+ = [1,1,1,1;1] \qquad A^- = [-1,-1,-1,-1;1]$$

The FPIS, A^+ and FNIS, A^- can be obtained by centroid method for (x_{A^+}, y_{A^+}) and (x_{A^-}, y_{A^-}).

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative from formulation A^+ and A^- can be calculated by the area of compensation method:

$$\bar{d}_i^+(\tilde{v}_{ij}, \tilde{v}_j^+) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^+})^2 + (y_{\tilde{A}_i^*} - y_{A^+})^2} \tag{5.10}$$

$$\bar{d}_i^-(\tilde{v}_{ij}, \tilde{v}_j^-) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^-})^2 + (y_{\tilde{A}_i^*} - y_{A^-})^2} \tag{5.11}$$

Step 6: Find the closeness coefficient, CC_i and improve alternatives for achieving aspiration levels in each criteria. Notice that the highest CC_i value is used to determine the rank.

$$\overline{CC}_i = \frac{\bar{d}_i^-}{\bar{d}_i^+ + \bar{d}_i^-} = 1 - \frac{\bar{d}_i^+}{\bar{d}_i^+ + \bar{d}_i^-} \tag{5.12}$$

where, $\frac{\bar{d}_i^-}{\bar{d}_i^+ + \bar{d}_i^-}$ is satisfaction degree in *ith* alternative and $\frac{\bar{d}_i^+}{\bar{d}_i^+ + \bar{d}_i^-}$ is fuzzy gaps degree in *ith* alternative.

Fuzzy gap should be improvised for reaching aspiration levels and get the best mutually beneficial strategy from among a fuzzy set of feasible alternatives.

Phase 4: Validation process using Sensitivity Analysis

Aforementioned in Chapter 2, sensitivity analysis can effectively contributes to making accurate decisions by assuming that a set of weights for criteria or alternatives then obtained a new round of weights for them, so that the efficiency of alternatives has become equal or their order has changed. The results of MCDM techniques are importantly needed to validate using sensitivity analysis method to analyse the quality and how robustness of MCDM technique to reach a right decision under different conditions. The computational process for sensitivity analysis is calculated in Section 3.8.2.

5.3 Summary of the Chapter

This chapter comprehensively discusses the development of hybrid fuzzy MCDM model based on the extended method of consistent fuzzy preference relations (used to derive the weight of criteria) and the extended of fuzzy TOPSIS (used to rank the alternatives). The proposed methodology capable to apply all possible fuzzy sets as the linguistic terms. Computation and description details of results and sensitivity analysis in this chapter would be underlined in empirical validation for case study in the next chapter. In Chapter 6, the thesis discusses the capability of hybrid consistent fuzzy preference relations and fuzzy TOPSIS that incorporating with intuitive multiple centroid for the staff recruitment in a company in Malaysia with different fuzzy environments.

CHAPTER 6

CASE STUDY

6.1 Introduction

Idris Zain & Co was established on the 2nd of January 2005. It is located in Damansara, Selangor Darul Ehsan, Malaysia and was operating with only two staff, a lawyer and a legal clerk. After two months of no progress, Idris Zain & Co formed a partnership with Saprudin & Co. Saprudin & Co has 14 years of experience in legal matters. They have been practicing since 1991 in Seremban, Negeri Sembilan Darul Khusus, Malaysia. Saprudin & Co concentrates on conveyancing and litigation while Idris Zain & Co concentrates on conveyancing only. The partnership is only in name as both branches handle its own account. Since then, Saprudin, Idris & Co has been an established firm and it is insured for 15 million Ringgit Malaysia.

From year 2010, Saprudin, Idris & Co was planning to open another branch is the state of Selangor. After several years, they changed the name from Saprudin, Idris & Co into MESSRS SAPRUDIN, IDRIS & CO. At least six more staffs are needed for this new branch. During 12 years of the company operation, at least seven more staffs have resigned. Work stress, inexperience worker or unable to adapt might be the factor. Once a staff resigned, recruitment new staff is not only time consuming but also involves financial implication especially for a new company. Operating a legal firm is not an easy thing to do. A legal firm usually needs three to six years to stabilise or to reach a breakeven. Hence, selecting and hiring a capable and dedicated staff with the lowest risk of him/ her resigning is very important task. To tackle this problem, the development of new hybrid fuzzy MCDM model is used in selecting the right employee for MESSRS SAPRUDIN, IDRIS & CO.

This section briefly summarises the background of the company and review the staff selection problems faced by MESSRS SAPRUDIN, IDRIS & CO Company. Details on staff selection problem above are broadly discussed in following section on the next page.

6.2 Staff Selection in MESSRS SAPRUDIN, IDRIS & CO

6.2.1 Aim

The purpose of the case study is to demonstrate how the proposed hybrid consistent fuzzy preference relations – fuzzy TOPSIS model that incorporated with intuitive multiple centroid defuzzification method may be used in the evaluation process for the selection of a right employee for MESSRS Saprudin, Idris & Co with the lowest of him/ her to resign. As stated by (Yin, 2014), a case study approach is generally used for the validation of a new proposed model. It is expected to enable more effective knowledge and information regarding the phenomenon under study based on the experts' viewpoints (Bryman, 2008). The computation is fully figured out using Microsoft Excel. The main objectives of the case study are:

i. Investigate of the evaluation process of selecting right employee, including identifying the selection criteria, deriving the criteria weights and ranking the available alternatives.

ii. Application of the new hybrid consistent fuzzy preference relations – fuzzy TOPSIS that is incorporated together with new intuitive multiple centroid defuzzification method under different fuzzy environment. Sensitivity analysis is used to validate the proposed methodology in this study.

6.2.2 Background

The challenges faced by most employers, with regards to the quality of employees they hire, how loyal the employees to their company and are the employees' performance achieve the employers' satisfaction level in performance index. The quality of employees depend on an effective recruitment and selection strategy. Nevertheless, the process isn't always smooth sailing. Most of employers face tangible problems such as the cost of advertising job openings and intangible obstacles such as improving communication between recruiters and hiring top managements. Finding the right candidates is a big challenge recruiting companies today. The clients need skilled, focused workers, and these people are not easy to find. Considering that, MESSRS Saprudin, Idris & Co realise regarding this challenging issues in recruiting new employee. Several strategies, frameworks and plans have been adopted in order to recruit the best employee.

Broadly construed, in human based decision making problems, decision makers play important role to give the right or best selection regarding their knowledge and experiences. Several criteria have been studied as performance evaluation to support selection process. In order to do so, questionnaire has been constructed for decision makers to evaluate their preferences towards criteria and candidates. In this case study, three decision makers are considered based on their position in MESSRS Saprudin, Idris & Co. There are four finalist candidates are evaluated after filtering stages. Questionnaires are constructed in regular numbers, then are translated into fuzzy linguistic terms in order to handle imprecision, vagueness and uncertainty in human based decision making process. The sample of questionnaire is presented in Appendix A.

There are five criteria are considered which consist of emotional steadiness (ES), oration (O), past experience (PE), personality (P) and self-confidence (S-C). Four candidates were screened for the final interview. Since the research problem is considered as an evaluation process, the process should involve a group of people who have expertise and knowledge in the legal company. This group is comprised of different decision makers with different level of expertise and different perceptions. Each of decision maker has unique characteristics with regard to the evaluation process. Alongside, the decision makers usually make diverging decisions due to their different perceptions and judgements. Due to imprecise and vagueness information and the subjective nature of decision makers' judgements, which are common problems in the selection problem, uncertainty exists in the process of selecting a good staff. In other words, the decision makers are unable to make reliable judgements regarding the evaluation procedure. Consequently, the evaluation and selection problem could be expressed as a group decision making problem under uncertain environments.

6.3 Hybrid Fuzzy Multi Criteria Decision Making for Type-1 Fuzzy Sets

This section illustrates computational process of proposed and established hybrid fuzzy MCDM models regarding case study of staff selection in MESSRS Saprudin, Idris & Co. for type-1 fuzzy sets. Two established fuzzy MCDM models from literature are considered which are fuzzy AHP - TOPSIS (Vinodh et al., 2014) and fuzzy AHP – VIKOR (Rezaie et al., 2014) in order to do comparative study.

6.3.1 Consistent Fuzzy Preference Relations – Fuzzy Technique for Order of Preference by Similarity to Ideal Solution for Type-1 Fuzzy Sets

The proposed hybrid fuzzy MCDM model is discussed in detail in Chapter 5. In the context of methodology, it considers all possible fuzzy numbers in order to solve most of imprecision based human intuition problems from information given.

The computational process of hybrid fuzzy MCDM model based consistent fuzzy preference relations – fuzzy TOPSIS are as follows.

Phase 1: Linguistic Evaluation

The decision makers used the linguistic terms that proposed by (Zheng et al., 2012) as shown in Table 6.1 in presenting the weights of criteria using consistent fuzzy preference evaluation for type-1 fuzzy sets. The linguistic terms with the crisp scale of relative important present the important of criteria preferences namely equally important (1), intermediate value (2), moderately more important (3), intermediate value (4), strongly more important (5), intermediate value (6), very strong more important (7), intermediate important (8) and extremely more important (9). For fuzzy TOPSIS evaluation, the linguistic terms and the corresponding of fuzzy numbers that proposed by (Zheng et al., 2012) is used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively. The scales consist of absolutely-low (1), very-low (2), low (3), fairly-low (4), medium (5), fairly-high (6), high (7), very-high (8) and absolutely-high (9). The linguistic scales for alternatives evaluation are depicted in Table 6.2 that are measure from 0 until 1.

Table 6. 1. Trapezoidal fuzzy numbers preference scale (Zheng et al., 2012)

Linguistic variables	Scale of relative important of crisp numbers	Trapezoidal fuzzy numbers	Reciprocal trapezoidal fuzzy number
Equally important (EI)	1	(1, 1, 1, 1)	(1, 1, 1, 1)
Intermediate value (IV)	2	(1, 3/2, 5/2, 3)	(1/3, 2/5, 2/3, 1)
Moderately more important (MMI)	3	(2, 5/2, 7/2, 4)	(1/4, 2/9, 2/5, 1/2)
Intermediate value (IV)	4	(3, 7/2, 9/2, 5)	(1/5, 2/9, 2/7, 1/3)
Strongly more important (SMI)	5	(4, 9/2, 11/2, 6)	(1/6, 2/11, 2/9, 1/4)
Intermediate value (IV)	6	(5, 11/2, 13/2, 7)	(1/7, 2/13, 2/11, 1/5)
Very strong more important (VSMI)	7	(6, 13/2, 15/2, 8)	(1/8, 2/15, 2/13, 1/6)
Intermediate value (IV)	8	(7, 15/2, 17/2, 9)	(1/9, 2/17, 2/15, 1/7)
Extremely more important (EMI)	9	(8, 17/2, 9, 9)	(1/9, 1/9, 2/17, 1/8)

Table 6. 2. Linguistic terms and their corresponding generalised fuzzy numbers (Zheng et al., 2012)

Linguistic terms	Scale of preference crisp numbers	Generalised fuzzy numbers
Absolutely-low (AL)	1	(0.0, 0.0, 0.0, 0.0; 1.0)
Very-low (VL)	2	(0.0,0.0, 0.02, 0.07;1.0)
Low (L)	3	(0.04, 0.10, 0.18, 0.23; 1.0)
Fairly-low (FL)	4	(0.17, 0.22, 0.36, 0.42; 1.0)
Medium (M)	5	(0.32, 0.41, 0.58, 0.6; 1.0)
Fairly-high (FH)	6	(0.58, 0.63, 0.80, 0.86; 1.0)
High (H)	7	(0.72, 0.78, 0.92, 0.97; 1.0)
Very-high (VH)	8	(0.93, 0.98, 1.0, 1.0; 1.0)
Absolutely-high (AH)	9	(1.0, 1.0, 1.0, 1.0; 1.0)

Phase 2: Fuzzy Weights of Criteria Evaluation using Consistent Fuzzy Preference Relations

Step 1: Construct a hierarchy structure.

The hierarchy model below as shown in Fig. 6.1 is illustrated the connection of criteria and alternatives, which are the candidates to be interviewed. There are five criteria are considered which consist of emotional steadiness (ES), oration (O), past experience (PE), personality (P) and self-confidence (S-C). Four candidates were screened for the final interview. These criteria below were evaluated by decision makers and represented in judgement matrix to measure the weight of each criterion in the next step.

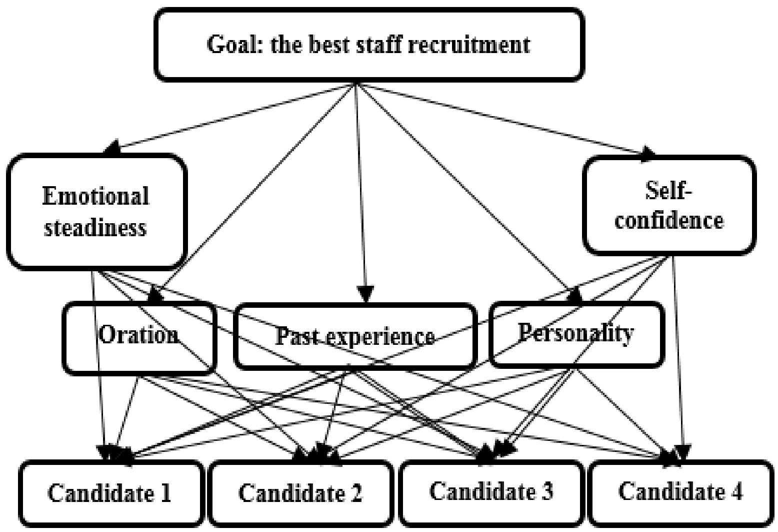


Fig. 6. 1. The hierarchy of staff recruitment problem

Step 2: Construct a pairwise comparison matrices.

The pairwise comparison matrices are constructed among all criteria in the dimension of the hierarchy systems based on the decision makers' preferences in phase 1 using equation (5.1). The linguistic evaluations of pairwise comparison matrices are based on regular numbers are depicted in equation (6.1), (6.2) and (6.3), then are translated into trapezoidal fuzzy numbers using Table 6.1. The linguistic ratings of criteria fuzzy numbers – based given by decision maker 1 (DM1), decision maker 2 (DM2) and decision maker 3 (DM3) are shown in equation (6.4), (6.5) and (6.6) respectively.

$$
DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\begin{bmatrix} 1 & 1/6 & 5 & 1/6 & 1/2 \\ 6 & 1 & 5 & 1/2 & 3 \\ 1/5 & 1/5 & 1 & 1/6 & 1/5 \\ 6 & 2 & 6 & 1 & 4 \\ 2 & 1/3 & 5 & 1/4 & 1 \end{bmatrix} \end{array} \tag{6.1}
$$

Pairwise comparison matrix of criteria evaluation from DM1

$$
DM2 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\begin{bmatrix} 1 & 1/5 & 4 & 1/6 & 1/2 \\ 5 & 1 & 6 & 3 & 3 \\ 1/4 & 1/6 & 1 & 1/6 & 1/3 \\ 6 & 1/3 & 6 & 1 & 4 \\ 2 & 1/3 & 3 & 1/4 & 1 \end{bmatrix} \end{array} \tag{6.2}
$$

Pairwise comparison matrix of criteria evaluation from DM2

$$
DM3 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\begin{bmatrix} 1 & 1/5 & 4 & 1/5 & 1/3 \\ 5 & 1 & 4 & 1/2 & 4 \\ 1/4 & 1/4 & 1 & 1/5 & 1/4 \\ 5 & 2 & 5 & 1 & 1/3 \\ 3 & 1/4 & 4 & 3 & 1 \end{bmatrix} \end{array} \tag{6.3}
$$

Pairwise comparison matrix of criteria evaluation from DM3

$$DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{ccccc} ES & O & P & PE & S-C \\ \left[\begin{array}{c} (1,1,1,1;1) \\ (5,5.5,6.5,7;1) \\ (0.1667,0.1818,0.2222,0.25;1) \\ (5,5.5,6.5,7;1) \\ (1,1.5,2.5,3;1) \end{array} \right. & \begin{array}{c} (0.1429,0.1538,0.1818,0.2;1) \\ (1,1,1,1;1) \\ (0.1667,0.1818'0.2222,0.25;1) \\ (1,1.5,2.5,3;1) \\ (0.25,0.2222,0.4,0.5;1) \end{array} & \begin{array}{c} (4,4.5,5.5,6;1) \\ (4,4.5,5.5,6;1) \\ (1,1,1,1;1) \\ (5,5.5,6.5,7;1) \\ (4,4.5,5.5,6;1) \end{array} & \begin{array}{c} (0.1429,0.1538,0.1818,0.2;1) \\ (0.3333,0.4,0.6667;1) \\ (0.1429,0.1538,0.1818,0.2;1) \\ (1,1,1,1;1) \\ (0.2,0.2222,0.2857,0.3333;1) \end{array} & \left. \begin{array}{c} (0.3333,0.4,0.6667,1;1) \\ (2,2.5,3.5,4;1) \\ (0.1667,0.1818,0.2222,0.25;1) \\ (3,3.5,4.5,5;1) \\ (1,1,1,1;1) \end{array} \right] \end{array} \tag{6.4}$$

Type-1 fuzzy pairwise comparison matrix of criteria evaluation from DM1

$$DM2 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{ccccc} ES & O & P & PE & S-C \\ \left[\begin{array}{c} (1,1,1,1;1) \\ (4,4.5,5.5,6;1) \\ (0.2,0.2222,0.2857,0.3333;1) \\ (5,5.5,6.5,7;1) \\ (1,1.5,2.5,3;1) \end{array} \right. & \begin{array}{c} (0.1667,0.1818'0.2222,0.25;1) \\ (1,1,1,1;1) \\ (0.1429,0.1538,0.1818,0.2;1) \\ (0.25,0.2222,0.4,0.5;1) \\ (0.25,0.2222,0.4,0.5;1) \end{array} & \begin{array}{c} (3,3.5,4.5,5;1) \\ (5,5.5,6.5,7;1) \\ (1,1,1,1;1) \\ (5,5.5,6.5,7;1) \\ (2,2.5,3.5,4;1) \end{array} & \begin{array}{c} (0.1429,0.1538,0.1818,0.2;1) \\ (2,2.5,3.5,4;1) \\ (0.1429,0.1538,0.1818,0.2;1) \\ (1,1,1,1;1) \\ (0.2,0.2222,0.2857,0.3333;1) \end{array} & \left. \begin{array}{c} (0.3333,0.4,0.6667,1;1) \\ (2,2.5,3.5,4;1) \\ (0.25,0.2222,0.4,0.5;1) \\ (3,3.5,4.5,5;1) \\ (1,1,1,1;1) \end{array} \right] \end{array} \tag{6.5}$$

Type-1 fuzzy pairwise comparison matrix of criteria evaluation from DM2

$$DM3 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{ccccc} ES & O & P & PE & S-C \\ \left[\begin{array}{c} (1,1,1,1;1) \\ (4,4.5,5.5,6;1) \\ (0.2,0.2222,0.2857,0.3333;1) \\ (4,4.5,5.5,6;1) \\ (2,2.5,3.5,4;1) \end{array} \right. & \begin{array}{c} (0.1667,0.1818'0.2222,0.25;1) \\ (1,1,1,1;1) \\ (0.2,0.2222,0.2857,0.3333;1) \\ (1,1.5,2.5,3;1) \\ (0.2,0.2222,0.2857,0.3333;1) \end{array} & \begin{array}{c} (3,3.5,4.5,5;1) \\ (3,3.5,4.5,5;1) \\ (1,1,1,1;1) \\ (4,4.5,5.5,6;1) \\ (3,3.5,4.5,5;1) \end{array} & \begin{array}{c} (0.1667,0.1818,0.2222,0.25;1) \\ (0.3333,0.4,0.6667,1;1) \\ (0.1667,0.1818,0.2222,0.25;1) \\ (1,1,1,1;1) \\ (2,2.5,3.5,4;1) \end{array} & \left. \begin{array}{c} (0.25,0.2222,0.4,0.5,1) \\ (3,3.5,4.5,5;1) \\ (0.2,0.2222,0.2857,0.3333;1) \\ (0.25,0.2222,0.4,0.5;1) \\ (1,1,1,1;1) \end{array} \right] \end{array} \tag{6.6}$$

Type-1 fuzzy pairwise comparison matrix of criteria evaluation from DM3

Step 3: Aggregate the decision makers' preferences.

The fuzzy pairwise comparison matrices for criteria's judgement of decision makers (DM1, DM2 and DM3) preferences as listed in equation (6.4), (6.5) and (6.6) are aggregated using equation (5.2). The results of aggregated pairwise comparison matrix is shown in equation (6.8) on next page.

$$\tilde{a}_{ij} = (\tilde{a}_{ij}^{1} \times \tilde{a}_{ij}^{2} \times \ldots \times \tilde{a}_{ij}^{n})^{1/k}$$

where k is the number of decision makers and $i=1,2,\ldots m; j=1,2,\ldots n$.

Step 4: Defuzzify the fuzzy numbers of aggregation's result of decision makers' preferences.

The aggregation's result of decision maker's preferences are defuzzify using intuitive multiple centroid for type-1 fuzzy sets using equation (4.5). The defuzzification results are presented in equation (6.7) below.

$$IMC(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1 + a_4) + 7(a_2 + a_3)}{18}, \frac{7h_{\tilde{A}}}{18} \right)$$

$$Defuzzify = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 1 & 0.1911 & 4.3084 & 0.1795 & 0.4684 \\ 5.313 & 1 & 4.9312 & 0.9808 & 3.3008 \\ 0.2376 & 0.20647 & 1 & 0.1795 & 0.2562 \\ 5.6459 & 1.0899 & 5.6459 & 1 & 1.7275 \\ 2.2861 & 0.2998 & 3.9123 & 0.5825 & 1 \end{bmatrix} \end{array} \qquad (6.7)$$

Defuzzification results of aggregated matrix comparison

$$
AGGREGATED = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} (1,1,1,1;1) & (0.1583,0.1720,0.2078,0.2321;1) & (3.3019,3.8058,4.8113,5.3133;1) & (0.1504,0.1627,0.1944,0.2154;1) & (0.3029,0.3288,0.5623,0.7937,1) \\ (4.3089,4.8113,5.8150,6.3164;1) & (1,1,1,1;1) & (3.9149,4.4247,5.4387,5.9439;1) & (0.6057,0.7368,1.1587,1.5874;1) & (2.2894,2.7967,3.8058,4.3089;1) \\ (0.1882,0.2078,0.2628,0.3029;1) & (0.1682,0.1839,0.2260,0.2554;1) & (1,1,1,1;1) & (0.1504,0.1627,0.1944,0.2154;1) & (0.2027,0.2078,0.2939,0.3467;1) \\ (4.6416,5.1441,6.1479,6.6494;1) & (0.63,0.7937,1.3572,1.6510;1) & (4.6416,5.1441,6.1479,6.6494;1) & (1,1,1,1;1) & (1.3104,1.3963,2.0083,2.3208;1) \\ (1.2599,1.7784,2.7967,3.3019;1) & (0.2321,0.2222,0.3576,0.4368;1) & (2.8845,3.4020,4.4247,4.9324;1) & (0.4309,0.4979,0.6586,0.7631;1) & (1,1,1,1;1) \end{bmatrix} \end{array} \quad (6.8)
$$

The aggregated type-1 fuzzy pairwise comparison matrix of decision makers for criteria evaluation

Step 5: Compute the weights of criteria values for alternatives' evaluation using consistent fuzzy preference relations.

The weights of aggregated matrix comparison of criteria are calculated using consistent fuzzy preference relations which based on additive transitivity property using equation (3.16-3.22) in equation (6.9) below.

$$Fuzzy\quad weights = \begin{array}{c|ccccc} & ES & O & P & PE & S-C \\ \hline ES & 0.5 & 0.1234 & 0.4865 & 0.09562 & 0.22 \\ O & 0.8766 & 0.5 & 0.8631 & 0.4722 & 0.5966 \\ P & 0.5135 & 0.1369 & 0.5 & 0.1092 & 0.2335 \\ PE & 0.9044 & 0.5278 & 0.8909 & 0.5 & 0.6244 \\ S-C & 0.78 & 0.4034 & 0.7665 & 0.3756 & 0.5 \end{array} \tag{6.9}$$

The consistent type-1 fuzzy preference relations matrix for criteria

By having five criteria, $n = 5$ so only $(n-1) = 5 - 1 = 4$ entry values $(p_{12}, p_{23}, p_{34} \quad and \quad p_{45})$ are required in constructing the consistent fuzzy preference relations matrix from equation (6.7) where:

$$p_{12} = \frac{1}{2}(1 + \log_9 0.911) = 0.1234$$

$$p_{23} = \frac{1}{2}(1 + \log_9 4.9312) = 0.8631$$

$$p_{34} = \frac{1}{2}(1 + \log_9 0.1795) = 0.1092$$

$$p_{45} = \frac{1}{2}(1 + \log_9 1.7275) = 0.6244$$

The remains of the entries can be determine by utilising Proposition 2 and 3 presented as follows.

$$p_{21} = 1 - p_{12} = 1 - 0.1234 = 0.8766$$

$$p_{32} = 1 - p_{23} = 1 - 0.8631 = 0.1369$$

$$p_{43} = 1 - p_{34} = 1 - 0.1092 = 0.8908$$

$$p_{54} = 1 - p_{45} = 1 - 0.6244 = 0.3756$$

$$p_{31} = \frac{3}{2} - p_{12} - p_{23} = \frac{3}{2} - 0.1234 - 0.8631 = 0.5135$$

$$p_{42} = \frac{3}{2} - p_{23} - p_{34} = \frac{3}{2} - 0.8631 - 0.1092 = 0.5278$$

$$p_{53} = \frac{3}{2} - p_{34} - p_{45} = \frac{3}{2} - 0.1092 - 0.6244 = 0.7665$$

$$p_{41} = \frac{j-i+1}{2} - p_{12} - p_{23} - p_{34} = \frac{4-1+1}{2} - 0.1234 - 0.8631 - 0.1092 = 0.9044$$

$$p_{51} = \frac{j-i+1}{2} - p_{12} - p_{23} - p_{34} - p_{45} = \frac{5-1+1}{2} - 0.1234 - 0.8631 - 0.1092 - 0.6244 = 0.78$$

$$p_{52} = \frac{j-i+1}{2} - p_{23} - p_{34} - p_{45} = \frac{5-2+1}{2} - 0.8631 - 0.1092 - 0.6244 = 0.4034$$

$$p_{13} = 1 - p_{31} = 1 - 0.5135 = 0.4865$$

$$p_{14} = 1 - p_{41} = 1 - 0.9044 = 0.0956$$

$$p_{15} = 1 - p_{51} = 1 - 0.78 = 0.22$$

$$p_{24} = 1 - p_{42} = 1 - 0.5278 = 0.4722$$

$$p_{25} = 1 - p_{52} = 1 - 0.4034 = 0.5966$$

$$p_{35} = 1 - p_{53} = 1 - 0.7665 = 0.2335$$

Then, the average and weight of each criterion from equation (6.9) are illustrated in Table 6.3 below. These results of criteria's weight are implemented in following phase to evaluate alternatives selection.

Table 6. 3. The type-1 fuzzy average and weights of criteria

Criteria	***ES***	***O***	***P***	***PE***	***S-C***	***Average***	***Weight***	***Rank***
ES	0.5	0.1234	0.4865	0.0956	0.2200	0.2851	0.1140	5
O	0.8766	0.5	0.8631	0.4722	0.5966	0.6617	0.2647	2
P	0.5135	0.1369	0.5	0.1091	0.2335	0.2986	0.1195	4
PE	0.9044	0.5278	0.8909	0.5	0.6244	0.6895	0.2758	1
S-C	0.7710	0.4034	0.7664	0.3756	0.5	0.5651	0.2260	3
Total						2.5	1	

Phase 3: Ranking evaluation of alternatives using fuzzy TOPSIS

Step 1: Determine the weights of evaluation criteria.

The weights of evaluation criteria are employed from consistent fuzzy preference relations process before. Refer Table 6.3.

Step 2: Construct the fuzzy decision matrix for alternatives evaluation using fuzzy TOPSIS.

The construction of fuzzy decision matrix for alternatives evaluation are utilised linguistic terms by (Zheng et al., 2012) presented on Table 6.4. This table presents the evaluations of linguistic terms of the alternatives given by the decision makers with respect to different criteria.

Table 6. 4. Evaluating linguistic terms of the alternatives given by the decision makers with respect to different criteria

Criteria	Alternatives/ Candidates	Decision Maker		
		DM1	DM2	DM3
Emotional Steadiness	*x1*	FH	H	VH
	x2	H	H	FH
	x3	VH	H	VH
	x4	M	FH	M
Oration	*x1*	VH	H	VH
	x2	H	H	VH
	x3	VH	VH	H
	x4	FH	M	FH
Personality	*x1*	VH	VH	VH
	x2	H	H	VH
	x3	VH	VH	VH
	x4	H	H	H
Past Experience	*x1*	FL	L	FL
	x2	M	M	M
	x3	H	M	H
	x4	FL	FL	FL
Self-Confidence	*x1*	H	FH	FH
	x2	VH	H	H
	x3	VH	VH	VH
	x4	M	FH	FH

Table 6. 5. Evaluating type-1 fuzzy values of the alternatives given by the decision makers with respect to different criteria

Criteria	Alternatives (Candidates)	Decision Maker 1 DM1	Decision Maker 2 DM2	Decision Maker 3 DM3
Emotional Steadiness	*x1*	(0.58 0.63 0.80 0.86; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x2	(0.72 0.78 0.92 0.97; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.58 0.63 0.80 0.86; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x4	(0.32 0.41 0.58 0.65; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.32 0.41 0.58 0.65; 1.00)
Oration	*x1*	(0.93 0.98 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x2	(0.72 0.78 0.92 0.97; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x4	(0.58 0.63 0.80 0.86; 1.00)	(0.32 0.41 0.58 0.65; 1.00)	(0.58 0.63 0.80 0.86; 1.00)
Personality	*x1*	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x2	(0.72 0.78 0.92 0.97; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x4	(0.72 0.78 0.92 0.97; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
Past Experience	*x1*	(0.17 0.22 0.36 0.42; 1.00)	(0.04 0.10 0.18 0.23; 1.00)	(0.17 0.22 0.36 0.42; 1.00)
	x2	(0.32 0.41 0.58 0.65; 1.00)	(0.32 0.41 0.58 0.65; 1.00)	(0.32 0.41 0.58 0.65; 1.00)
	x3	(0.72 0.78 0.92 0.97; 1.00)	(0.32 0.41 0.58 0.65; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x4	(0.17 0.22 0.36 0.42; 1.00)	(0.17 0.22 0.36 0.42; 1.00)	(0.17 0.22 0.36 0.42; 1.00)
Self-Confidence	*x1*	(0.72 0.78 0.92 0.97; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.58 0.63 0.80 0.86; 1.00)
	x2	(0.93 0.98 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x4	(0.32 0.41 0.58 0.65; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.58 0.63 0.80 0.86; 1.00)

Step 3: Fuzzy decision matrix is weighted using equation (5.5) and normalised each generalised fuzzy numbers into standardised generalised fuzzy numbers using (Zuo et al., 2013).

Equation (6.10), (6.11) and (6.12) represent the fuzzy pairwise comparison matrices of decision makers for alternatives evaluation. Then, aggregated result is depicted in equation (6.13). The weighted fuzzy normalised decision matrix is computed using equation (5.6). The results of weighted and normalisation process are presented in equation (6.14) and equation (6.15) respectively. Defuzzify the standardised generalised fuzzy numbers using intuitive multiple centroid (equation (6.16)), then translate them into the index point proposed by (Yong & Qi, 2005) as presented in equation (6.17), then do the average computational process depicted in equation (6.18).

Step 4: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

Referring to normalise trapezoidal fuzzy weights, the FPIS, A^+ represents the compromise solution while FNIS, A^- represents the worst possible solution. The range belong to the closed interval [0,1]. The FPIS A^+ (aspiration levels) and FNIS A^- (worst levels) as following below:

$$A^+ = (1,1,1,1;1) \qquad A^- = (-1,-1,-1,-1;1)$$

The FPIS, A^+ and FNIS, A^- can be obtained by centroid method for (x_{A^+}, y_{A^+}) and (x_{A^-}, y_{A^-}).

$$DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{bmatrix} (0.58,0.63,0.8,0.86;1) & (0.72,0.798,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.32,0.41,0.58,0.65;1) \\ (0.93,0.98,1,1;1) & (0.72,0.798,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.58,0.63,0.8,0.86;1) \\ (0.93,0.98,1,1;1) & (0.72,0.798,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.72,0.78,0.92,0.97;1) \\ (0.17,0.22,0.36,0.42;1) & (0.32,0.41,0.58,0.65;1) & (0.72,0.78,0.92,0.97;1) & (0.17,0.22,0.36,0.42;1) \\ (0.72,0.78,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.93,0.98,1,1;1) & (0.32,0.41,0.58,0.65;1) \end{bmatrix} \end{array} \tag{6.10}$$

Type-1 fuzzy pairwise comparison matrix of alternatives evaluation from DM1

$$DM2 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{bmatrix} (0.72,0.798,0.92,0.97;1) & (0.72,0.798,0.92,0.97;1) & (0.72,0.798,0.92,0.97;1) & (0.58,0.63,0.8,0.86;1) \\ (0.72,0.798,0.92,0.97;1) & (0.72,0.798,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.32,0.41,0.58,0.65;1) \\ (0.93,0.98,1,1;1) & (0.72,0.798,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.72,0.78,0.92,0.97;1) \\ (0.04,0.1,0.18,0.23;1) & (0.32,0.41,0.58,0.65;1) & (0.32,0.41,0.58,0.65;1) & (0.17,0.22,0.36,0.42;1) \\ (0.58,0.63,0.80,0.86;1) & (0.72,0.798,0.92,0.97;1) & (0.93,0.98,1,1;1) & (0.58,0.63,0.8,0.86;1) \end{bmatrix} \end{array} \tag{6.11}$$

Type-1 fuzzy pairwise comparison matrix of alternatives evaluation from DM2

$$DM3 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{bmatrix} (0.93,0.98,1,1;1) & (0.58,0.63,0.80,0.86;1) & (0.93,0.98,1,1;1) & (0.32,0.41,0.58,0.65;1) \\ (0.93,0.98,1,1;1) & (0.93,0.98,1,1;1) & (0.72,0.78,0.92,0.97;1) & (0.58,0.63,0.8,0.86;1) \\ (0.93,0.98,1,1;1) & (0.93,0.98,1,1;1) & (0.93,0.98,1,1;1) & (0.72,0.78,0.92,0.97;1) \\ (0.17,0.22,0.36,0.42;1) & (0.32,0.41,0.58,0.65;1) & (0.72,0.78,0.92,0.97;1) & (0.17,0.22,0.36,0.42;1) \\ (0.58,0.63,0.80,0.86;1) & (0.93,0.98,1,1;1) & (0.93,0.98,1,1;1) & (0.58,0.63,0.8,0.86;1) \end{bmatrix} \end{array} \tag{6.12}$$

Type-1 fuzzy pairwise comparison matrix of alternatives evaluation from DM3

$$
Aggregated = \begin{array}{c c}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
\begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.7433,0.7967,0.9067,0.9433;1) & (0.6733,0.73,0.88,0.9333;1) & (0.86,0.9133,0.9733,0.99;1) & (0.4067,0.4833,0.6533,0.72;1) \\
(0.86,0.9133,0.9733,0.99;1) & (0.79,0.8467,0.9467,0.98;1) & (0.86,0.9133,0.9733,0.99;1) & (0.4933,0.5567,0.7262;0.79;1) \\
(0.93,0.98,1,1;1) & (0.79,0.8467,0.9467,0.98;1) & (0.93,0.98,1,1;1) & (0.72,0.78,0.92,0.97;1) \\
(0.1267,0.18,0.3,0.3567;1) & (0.32,0.41,0.58,0.65;1) & (0.5867,0.6567,0.8067,0.8633;1) & (0.17,0.22,0.36,0.42;1) \\
(0.6267,0.68,0.84,0.8967;1) & (0.79,0.8467,0.9467,0.98;1) & (0.93,0.98,1,1;1) & (0.4933,0.5567,0.7267,0.79;1)
\end{bmatrix}
\end{array}
\tag{6.13}
$$

The aggregated type-1 fuzzy pairwise comparison matrix for alternatives evaluation

$$
Weighted = \begin{array}{c c}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
\begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.0848,0.09085,0.1034,0.1076;1) & (0.0768,0.08325,0.1004,0.1064;1) & (0.0981,0.1042,0.1111,0.1129;1) & (0.04638,0.05512,0.0745,0.0821;1) \\
(0.2276,0.2418,0.2576,0.2620;1) & (0.2091,0.2241,0.2506,0.2594;1) & (0.2276,0.2417,0.2576,0.262;1) & (0.1306,0.1473,0.1923,0.2091;1) \\
(0.1111,0.1171,0.1195,0.1185;1) & (0.0944,0.1011,0.1131,0.1171;1) & (0.1111,0.1171,0.1195,0.1195;1) & (0.086,0.09317,0.1099,0.1159;1) \\
(0.0349,0.04964,0.0827,0.09837;1) & (0.0883,0.11301,0.16,0.1793;1) & (0.1618,0.1811,0.2225,0.2381;1) & (0.04688,0.06067,0.0993,0.1158;1) \\
(0.1416,0.1537,0.1899,0.2027;1) & (0.1786,0.1914,0.214,0.2215;1) & (0.2102,0.2215,0.226,0.226;1) & (0.1151,0.1258,0.1643,0.1786;1)
\end{bmatrix}
\end{array}
\tag{6.14}
$$

The weighted aggregated type-1 fuzzy pairwise comparison matrix for alternatives evaluation

$$
Normalised = \begin{array}{c c}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
\begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.2194,0.2462,0.3014,0.3199;1) & (0.1843,0.2127,0.2881,0.3148;1) & (0.278,0.3048,0.3349,0.3433;1) & (0.05038,0.08888,0.1742,0.2077;1) \\
(0.8485,0.9107;0.9806,1;1) & (0.7669,0.8330,0.9495,0.9883;1) & (0.8485,0.9106,0.9806,1;1) & (0.4212,0.4950,0.6931,0.7669;1) \\
(0.3353,0.3616,0.3721,0.3722;1) & (0.2617,0.2915,0.3441,0.3616;1) & (0.3353,0.3616,0.3721,0.3721;1) & (0.2249,0.2564,0.3301,0.3564;1) \\
(0,0.06477,0.2105,0.2793;1) & (0.2348,0.3441,0.5505,0.6355;1) & (0.5586,0.6436,0.8258,0.8946;1) & (0.05262,0.1133,0.2834,0.3562;1) \\
(0.4699,0.523,0.6822,0.7386;1) & (0.6324,0.6888,0.7884,0.8215;1) & (0.7718,0.8215,0.8414,0.8414;1) & (0.3372,0.4002,0.5694,0.6324;1)
\end{bmatrix}
\end{array}
\tag{6.15}
$$

The normalised weighted aggregated type-1 fuzzy pairwise comparison matrix for alternatives evaluation

$$Defuzzifiied = \begin{array}{cc} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} x=0.2729,\ y=0.3889 & x=0.2502,\ y=0.3889 & x=0.3178,\ y=0.3889 & x=0.131,\ y=0.3889 \\ x=0.9409,\ y=0.3889 & x=0.8882,\ y=0.3889 & x=0.9409,\ y=0.3889 & x=0.5940,\ y=0.3889 \\ x=0.3640,\ y=0.3889 & x=0.3164,\ y=0.3889 & x=0.364,\ y=0.3889 & x=0.2927,\ y=0.3889 \\ x=0.1381,\ y=0.3889 & x=0.4446,\ y=0.3889 & x=0.7329,\ y=0.3889 & x=0.1997,\ y=0.3889 \\ x=0.6030,\ y=0.3889 & x=0.7360,\ y=0.3889 & x=0.826,\ y=0.3889 & x=0.4848,\ y=0.3889 \end{bmatrix} \end{array} \tag{6.16}$$

The defuzzified type-1 fuzzy pairwise comparison matrix for alternatives evaluation

$$Translate\ \ defuzzifiied = \begin{array}{cc} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} x=0.2729,\ y=0.4818 & x=0.2502,\ y=0.4761 & x=0.3178,\ y=0.4883 & x=0.131,\ y=0.4716 \\ x=0.9409,\ y=0.4729 & x=0.8882,\ y=0.4601 & x=0.9409,\ y=0.4759 & x=0.5940,\ y=0.4367 \\ x=0.3640,\ y=0.4931 & x=0.3164,\ y=0.482 & x=0.364,\ y=0.493 & x=0.2927,\ y=0.4761 \\ x=0.1381,\ y=0.45 & x=0.4446,\ y=0.4284 & x=0.7329,\ y=0.4393 & x=0.1997,\ y=0.4447 \\ x=0.6030,\ y=0.4504 & x=0.7360,\ y=0.4659 & x=0.826,\ y=0.4868 & x=0.4848,\ y=0.446 \end{bmatrix} \end{array} \tag{6.17}$$

The translate defuzzified type-1 fuzzy pairwise comparison matrix for alternatives evaluation

$$Average\ \ Translate\ \ defuzzifiied = \begin{array}{c} \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{bmatrix} x=0.4638,\ y=0.4696 & x=0.5271,\ y=0.4625 & x=0.6363\ \ y=0.4761 & x=0.3404,\ y=0.455 \end{bmatrix} \end{array} \tag{6.18}$$

The average translate defuzzified type-1 fuzzy pairwise comparison matrix for alternatives evaluation

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative from formulation A^+ and A^- can be calculated by the area of compensation method.

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^+})^2 + (y_{\tilde{A}_i^*} - y_{A^+})^2}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{(0.4638-1)^2 + (0.4696-0.5)^2}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = 0.5371$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^-})^2 + (y_{\tilde{A}_i^*} - y_{A^-})^2}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{(0.4638+1)^2 + (0.4696-0.5)^2}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = 1.4641$$

Step 6: Find the closeness coefficient, CC_i and improve alternatives for achieving aspiration levels in each criteria.

The decision rules for five classes are depicted in Table 6.7. Notice that the highest CC_i value is used to determine the rank.

$$\overline{CC}_i = \frac{\bar{d}_i^-}{\bar{d}_i^+ + \bar{d}_i^-} = 1 - \frac{\bar{d}_i^+}{\bar{d}_i^+ + \bar{d}_i^-}$$

$$\overline{CC}_i = \frac{1.4641}{0.5371+1.4641}$$

$$\overline{CC}_i = 0.7316$$

After several processes, referring to Table 6.6, the CC_i values shows candidate 3 represents the highest rank with 0.8178 followed by candidate 2 with 0.7630, candidate 1 with 0.7316 and candidate 4 with 0.6698 for the last ranked. The results reveal that the candidate 3 is most suitable for this recruitment post because based on approval status from (Luukka, 2011) table in Table 6.7, the score is in approved and preferred range.

Table 6. 6. Closeness coefficients computation for type-1 fuzzy sets

Alternative	**Closeness Coefficient, *CCi***
Candidate 1	0.7316 (Rank 3)
Candidate 2	0.7630 (Rank 2)
Candidate 3	0.8178 (Rank 1)
Candidate 4	0.6698 (Rank 4)

Table 6. 7. Approval status table (Luukka, 2011)

***CCi* value**	**Assessment status**
$CC_i \in [0,0.2)$	Do not recommend
$CC_i \in [0.2,0.4)$	Recommend with high risk
$CC_i \in [0.4,0.6)$	Recommend with low risk
$CC_i \in [0.6,0.8)$	Approved
$CC_i \in [0.8,1]$	Approved and preferred

6.3.2 *Fuzzy Analytic Hierarchy Process – Fuzzy Technique for Order of Preference by Similarity to Ideal Solution*

This section presents established hybrid fuzzy MCDM based fuzzy AHP – fuzzy TOPSIS proposed by (Vinodh et al., 2014). In this methodology, the authors only consider triangular fuzzy numbers. But, in order to make compatibility in information given, trapezoidal fuzzy sets are used. Several steps are replaced in order to fulfil the requirement of trapezoidal fuzzy sets such as, linguistic scale used, defuzzification step and area of compensation process. There are several phases in computing hybrid fuzzy MCDM model based fuzzy AHP – fuzzy TOPSIS are as follows.

Phase 1: Linguistic Evaluation

The decision makers used the linguistic terms that proposed by (Zheng et al., 2012) as shown in Table 6.1 in presenting the weights of criteria using fuzzy AHP evaluation for type-1 fuzzy sets. For fuzzy TOPSIS evaluation, the linguistic terms and the corresponding of fuzzy numbers that proposed by (Zheng et al., 2012) as depicted in Table 6.2 which is used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively.

Phase 2: Fuzzy Weights Evaluation using Fuzzy AHP

Step 1: Building the evaluation hierarchy systems.

The hierarchy model is presented in Fig. 6.1. It illustrates the connection of criteria and alternatives where the candidates to be interviewed. Five criteria are considered which consist of emotional steadiness, oration, past experience, personality and self-confident.

Step 2: Determining the evaluation dimensions weights of pairwise comparison matrix to find the fuzzy weights.

The pairwise comparison matrix showing the preference of one criterion over the others which is built by entering the judgement values by the decision makers. Since the values of linguistic variables are quadruplet trapezoidal fuzzy numbers are entered.

Step 3: Determining the weights for the criteria involved.

The synthetic pairwise comparison matrices for criteria’s judgement of decision makers (DM1, DM2 and DM3) preferences as listed in equation (6.4), (6.5) and (6.6) are aggregated using geometric mean method, refer equation (3.13). The result of aggregated pairwise comparison matrix is shown in equation (6.19).

$$\tilde{r}_{ij} = (\tilde{a}_{ij}^{1} \times \tilde{a}_{ij}^{2} \times \ldots \times \tilde{a}_{ij}^{n})^{1/k}$$

where k is the number of decision makers and $i=1,2,\ldots m;\ j=1,2,\ldots n$.

Step 4: The weight of each criterion is determined using normalising the matrix

This is done by using equation (3.14) and the results are presented in equation (6.20). The results of normalising process is presented in equation (6.21).

$$w_i = r_i \times (r_1 + r_2 + r_3 + \ldots + r_n)^{-1}$$

Step 5: Defuzzify each weight from Step 4 using defuzzification method proposed by (Y. M. Wang et al., 2006).

The defuzzification method proposed by (Y. M. Wang et al., 2006) is utilised in order to compute trapezoidal fuzzy sets.

$$\bar{x}_0(\tilde{A}) = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)}\right] \tag{6.22}$$

$$\bar{y}_0(\tilde{A}) = h_A \frac{1}{3}\left[1 + \frac{a_4 a_3 - a_1 a_2}{(a_4 + a_3) - (a_1 + a_2)}\right] \tag{6.23}$$

Then, normalization process is followed after defuzzification process. This is done by normalizing the matrix using (Sun, 2010) normalise equation.

$$
Aggregated = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\begin{bmatrix}
(1,1,1,1;1) & (0.1583,0.172,0.2078,0.2321;1) & (3.3019,3.8058,4.8113,5.3133;1) & (0.1504,0.1627,0.1944,0.2154;1) & (0.3029,0.3288,0.5623,0.7937;1) \\
(4.3089,4.8113,5.815,6.3164;1) & (1,1,1,1;1) & (3.9149,4.4247,5.4387,5.9439;1) & (0.6057,0.7368,1.1587,1.5874;1) & (2.2894,2.7967,3.8058,4.3089;1) \\
(0.1882,0.2078,0.2628,0.3029;1) & (0.1682,0.1839,0.226,0.2554;1) & (1,1,1,1;1) & (0.1504,0.1627,0.1944,0.2154;1) & (0.2027,0.2078,0.2939,0.2467;1) \\
(4.6415.1446.1476.6494916;1) & (0.63,0.79,1.3572,1.651;1) & (4.6416,5.1441,6.1479,6.6494;1) & (1,1,1,1;1) & (1.3104,1.3963,2.0083,2.3208;1) \\
(1.2599,1.7784,2.7967,3.3019;1) & (0.2321,0.222,0.3576,0.4368;1) & (2.8845,3.402,4.4247,4.9324;1) & (0.4309,0.4979,0.6586,0.7631;1) & (1,1,1,1;1)
\end{bmatrix}
\end{array}
\tag{6.19}
$$

The aggregated pairwise comparison matrix of decision makers for criteria evaluation

$$
Fuzzy \quad geometric \quad mean = \begin{array}{c} r(ES) \\ r(O) \\ r(P) \\ r(PE) \\ r(S-C) \end{array}
\begin{bmatrix}
0.4735 & 0.5115 & 0.6423 & 0.7325 \\
1.8785 & 2.1304 & 2.6847 & 3.033 \\
0.2494 & 0.2644 & 0.3207 & 0.3567 \\
1.7783 & 1.9654 & 2.5269 & 2.7912 \\
0.8167 & 0.9229 & 1.2385 & 1.4026
\end{bmatrix}
\tag{6.20}
$$

The geometric mean of decision makers for criteria evaluation

$$
Fuzzy \quad normalised \quad weights = \begin{array}{c} w(ES) \\ w(O) \\ w(P) \\ wPE) \\ w(S-C) \end{array}
\begin{bmatrix}
0.0569 & 0.069 & 0.1109 & 0.141 \\
0.2259 & 0.2874 & 0.4633 & 0.5837 \\
0.0299 & 0.0357 & 0.05535 & 0.0687 \\
0.2138 & 0.2651 & 0.4361 & 0.5371 \\
0.0982 & 0.1245 & 0.2138 & 0.2699
\end{bmatrix}
\tag{6.21}
$$

The normalised weighted for each criteria

Then, the average and weightage of each criterion from equation (6.21) are illustrated in Table 6.8 below as follows:

Table 6. 8. The weights of criteria

Criteria	*Weight*	*New Weight*	*Rank*
ES	0.0949	0.08823	4
O	0.3917	0.3641	1
P	0.0476	0.04426	5
PE	0.3643	0.3386	2
S-C	0.1774	0.1649	3
Total	2.5	1	

These results of criteria's weightages are implemented in following phase to evaluate for alternatives selection.

Phase 3: Fuzzy TOPSIS Evaluation for Alternatives Selection

Step 1: Obtain the weighting of evaluation criteria from fuzzy AHP evaluation.

The weighting of evaluation criteria are employed from fuzzy AHP evaluation process before. Refer Table 6.8.

Step 2: Create fuzzy evaluation matrix for alternatives' evaluation.

The construction of fuzzy decision matrix for alternatives' evaluation are utilised linguistic terms by (Zheng et al., 2012) presented on Table 6.5.

Step 3: Fuzzy decision matrix is weighted and normalised each generalised fuzzy numbers into standardised generalised fuzzy numbers.

The pairwise comparison matrices for decision makers for alternatives evaluation are presented before in equation (6.10), (6.11) and (6.12). Then, aggregated results is depicted in equation (6.13). The weighted fuzzy decision matrix is computed using equation (5.11). The normalization process is computed as follows.

$$\tilde{r}_i = \frac{a_1}{u_j^+}, \frac{a_2}{u_j^+}, \frac{a_3}{u_j^+}, \frac{a_4}{u_j^+} \qquad (6.24)$$

where, u_j^+ is the maximum value in entire fuzzy decision matrix.

The results of weighted and normalisation process are presented in equation (6.25) and (6.26) respectively. The average of weighted normalised process is presented in equation (6.27).

Step 4: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

Referring to normalise trapezoidal fuzzy weights, the FPIS, A^+ represents the compromise solution while FNIS, A^- represents the worst possible solution. The range belong to the closed interval [0,1]. The FPIS A^+ (aspiration levels) and FNIS A^- (worst levels) as following below:

$$A^+ = [1,1,1,1;1] \qquad A^- = [-1,-1,-1,-1;1]$$

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative from formulation A^+ and A^- can be calculated by the area of compensation method.

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{\frac{1}{4}\left[(x_{\tilde{A}_1^*} - x_{A_1^+})^2 + (x_{\tilde{A}_2^*} - x_{A_2^+})^2 + (x_{\tilde{A}_3^*} - x_{A_3^+})^2 + (x_{\tilde{A}_4^*} - x_{A_4^+})^2\right]}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{\frac{1}{4}\left[(0.314-1)^2 + (0.344-1)^2 + (0.399-1)^2 + (0.42-1)^2\right]}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = 0.6324$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{\frac{1}{4}\left[(x_{\tilde{A}_1^*} - x_{A_1^-})^2 + (x_{\tilde{A}_2^*} - x_{A_2^-})^2 + (x_{\tilde{A}_3^*} - x_{A_3^-})^2 + (x_{\tilde{A}_4^*} - x_{A_4^-})^2\right]}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{\frac{1}{4}\left[(0.314+1)^2 + (0.344+1)^2 + (0.399+1)^2 + (0.42+1)^2\right]}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = 1.3697$$

$$
\begin{array}{lc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Weighted = \begin{array}{r} ES \\ O \\ P \\ PE \\ S-C \end{array} & \left[\begin{array}{cccc}
(0.0656,0.0703,0.08,0.0832;1) & (0.0594,0.0644,0.0776,0.0824;1) & (0.0759,0.0806,0.0859,0.0874;1) & (0.03589,0.04265,0.0576,0.0635;1) \\
(0.3131,0.3325,0.3544,0.3604;1) & (0.2876,0.3082,,0.3447,0.3568;1) & (0.3131,0.3325,0.3544,0.3604;1) & (0.1796,0.2027,0.2646,0.2876;1) \\
(0.0412,0.0434,0.0443,0.0443;1) & (0.035,0.0375,0.0419,0.0434;1) & (0.0412,0.0434,0.0443,0.0443;1) & (0.03187,0.0345,0.0407,0.0429;1) \\
(0.0429,0.0610,0.1016,0.1208;1) & (0.1083,0.1388,0.1964,0.2201;1) & (0.1986,0.2223,0.2731,0.2923;1) & (0.05756,0.0745,0.1219,0.1422;1) \\
(0.1033,0.1121,0.1385,0.1478;1) & (0.1302,0.1396,0.1561,0.1616;1) & (0.1533,0.1616,0.1648,0.1648;1) & (0.0813,0.0918,0.1198,0.1302;1)
\end{array}\right]
\end{array}
\qquad (6.25)
$$

The weighted pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{lc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Normalised = \begin{array}{r} ES \\ O \\ P \\ PE \\ S-C \end{array} & \left[\begin{array}{cccc}
(0.182,0.1950,0.2219,0.2309;1) & (0.1648,0.1787,0.2154,0.2285;1) & (0.2105,0.2236,0.2383,0.2423;1) & (0.0996,0.1183,0.1599,0.1763;1) \\
(0.8687,0.9226,0.9832,1;1) & (0.798,0.8552,0.9562,0.9899;1) & (0.8687,0.9226,0.9832,1;1) & (0.4983,0.5623,0.734,0.798;1) \\
(0.1142,0.1203,0.1228,0.1228;1) & (0.097,0.104,0.1162,0.1203;1) & (0.1142,0.1203,0.1228,0.1228;1) & (0.08841,0.0958,0.113,0.1191;1) \\
(0.119,0.1691,0.2818,0.3351;1) & (0.3006,0.3852,0.5448,0.6106;1) & (0.5511,0.6169,0.7578,0.811;1) & (0.1597,0.2067,0.3382,0.3945;1) \\
(0.2866,0.31100.3842,0.4101;1) & (0.3613,0.3872,0.433,0.4482;1) & (0.4253,0.4482,0.4574,0.4574;1) & (0.2256,0.2546,0.3324,0.3613;1)
\end{array}\right]
\end{array}
\qquad (6.26)
$$

The weighted normalised pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{lc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Average\ \ normalised = & \left[\begin{array}{cccc}
(0.314,0.344,0.399,0.42;1) & (0.344,0.382,0.453,0.48;1) & (0.434,0.466,0.512,0.527;1) & (0.2143,0.248,0.335,0.37;1)
\end{array}\right]
\end{array}
\qquad (6.27)
$$

The average weighted normalised pairwise comparison matrix for alternatives evaluation

Step 6: Find the closeness coefficient, CC_i and improve alternatives for achieving aspiration levels in each criteria.

The decision rules for five classes are depicted in Table 6.9. Notice that the highest CC_i value is used to determine the rank.

$$\overline{CC}_i = \frac{\overline{d}_i^-}{\overline{d}_i^+ + \overline{d}_i^-} = 1 - \frac{\overline{d}_i^+}{\overline{d}_i^+ + \overline{d}_i^-}$$

$$\overline{CC}_i = \frac{1.3697}{0.6324 + 1.3697}$$

$$\overline{CC}_i = 0.6842$$

After several processes, referring to Table 6.9, the CC_i values shows candidate 3 represents the highest rank with 0.7419 followed by candidate 2 with 0.7067, candidate 1 with 0.6842 and candidate 4 with 0.6453 for the last ranked. The results reveal that the candidate 3 is most suitable for this recruitment post because based on approval status table in Table 6.7, the score is in approved range.

Table 6. 9. Closeness coefficients computation for type-1 fuzzy sets.

Alternative	**Closeness Coefficient, CC_i**
Candidate 1	0.6842 (Rank 3)
Candidate 2	0.7067 (Rank 2)
Candidate 3	0.7419 (Rank 1)
Candidate 4	0.6453 (Rank 4)

6.3.3 Fuzzy Analytic Hierarchy Process – Fuzzy Multidisciplinary Optimization Compromise Solution

This established hybrid fuzzy MCDM based fuzzy AHP – fuzzy VIKOR proposed by (Rezaie et al., 2014). In this study, the authors only consider triangular fuzzy numbers. But, in order to make compatibility in information given, trapezoidal fuzzy sets are used. Several steps are replace in order to fulfil the requirement of trapezoidal fuzzy sets such as, linguistic terms used, defuzzification step and area of compensation process. There are several phases in computing hybrid fuzzy MCDM model based fuzzy AHP – fuzzy VIKOR are as follows.

Phase 1: Linguistic Evaluation

The decision makers used the linguistic terms that proposed by (Zheng et al., 2012) as shown in Table 6.1 in presenting the weights of criteria using fuzzy AHP evaluation for type-1 fuzzy sets. For fuzzy TOPSIS evaluation, the linguistic terms and the corresponding of fuzzy numbers that proposed by (Zheng et al., 2012) as depicted in Table 6.2 is used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively.

Phase 2: Fuzzy Weights Evaluation using Fuzzy AHP

Same calculation as Section 6.3.2 in phase 2.

Phase 3: Fuzzy VIKOR Evaluation for Alternatives Selection

Step1: Create fuzzy evaluation matrix for alternatives' evaluation.

The construction of fuzzy decision matrix for alternatives' evaluation are utilised linguistic terms by (Zheng et al., 2012) presented on Table 6.5.

Step 2: Compute normalised quantities by using equation as follows.

Assume *m* alternatives and *n* criteria.

$$f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$

where, $i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n$.

Step 3: Determine the best (f_j^*) and the worst (f_j^-) quantities in each criterion.

If we assume the *j*th function represents a benefit, then $f_j^* = \max f_{ij}$ (or setting an inspired level) and $f_j^- = \min f_{ij}$ (or setting a tolerate level). Alternatively, if we assume the *j*th function represents a cost/ risk, the then $f_j^* = \min f_{ij}$ (or setting an inspired level) and $f_j^- = \max f_{ij}$ (or setting a tolerate level). Refer equation (6.28).

Step 4: Determine the weights of the criteria.

The weighting of evaluation criteria are employed from fuzzy AHP evaluation process before. Refer Table 6.8.

Step 5: Compute the values of S_i and $R_i; i = 1,2,...m$ by the equations (3.32) and (3.33).

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - \tilde{x}_{ij}) / (f_j^* - f_{ij})$$

$$R_i = Max\left[w_j (f_j^* - \tilde{x}_{ij}) / (f_j^* - f_{ij})\right]$$

Where w_j are the criteria's weights, that expressing their relative important from fuzzy AHP evaluation.

Step 6: Compute the values of Q_i;1,2,..., by the equation below:

$$Q_i = \upsilon\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-\upsilon)\left[\frac{R_i - R^*}{R^- - R^*}\right]$$

Where, $S^- = \max value_i S_i$, $S^* = \min_i S_i$, $R^- = \max value_i R_i$, $R^* = \min_i R_i$ and υ is introduced as the weight of the strategy "the majority of criteria" (or "the maximum group utility") and usually $\upsilon = 0.5$ (Bevilacqua & Braglia, 2000). The whole computation are represented in equation (6.29), (6.30), (6.31) and (6.32). Then find the S^-, $S^* = \min_i S_i$, $R^- = \max value_i R_i$, $R^* = \min_i R_i$ and υ.

$$S^- = \max value_i S_i = 0.9801$$

$$S^* = \min_i S_i = (-0.276, -0.1317, 0.1317, 0.276; 1)$$

$$R^- = \max value_i R_i = 0.3641$$

$$R^* = \min_i R_i = (-0.0111, -0.0032, 0.0689, 0.1272; 1)$$

$$\upsilon = 0.5$$

Hence, the calculation of Q for candidate 1 is illustrated as follows.

$$Q_i = \upsilon\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-\upsilon)\left[\frac{R_i - R^*}{R^- - R^*}\right]$$

$$Q_{x1(a1,a2,a3,a4)} = \upsilon\left[\frac{S_{x1(a1,a2,a3,a4)} - S^*}{S^- - S^*}\right] + (1-\upsilon)\left[\frac{R_{x1(a1,a2,a3,a4)} - R^*}{R^- - R^*}\right]$$

$$Q_{x1(a1)} = 0.5\left[\frac{0.0163-0.276}{(0.9801-(-0.276))}\right]+(1-0.5)\left[\frac{0.1057-0.1272}{0.3641-(-0.0111)}\right]=-0.1319$$

$$Q_{x1(a2)} = 0.5\left[\frac{0.1797-0.1317}{(0.9801-(-0.276))}\right]+(1-0.5)\left[\frac{0.1639-0.0689}{0.3641-(-0.0111)}\right]=0.1457$$

$$Q_{x1(a3)} = 0.5\left[\frac{0.4760-(-0.1317)}{(0.9801-(-0.276))}\right]+(1-0.5)\left[\frac{0.2880-(-0.0032)}{0.3641-(-0.0111)}\right]=0.63$$

$$Q_{x1(a54)} = 0.5\left[\frac{0.6113-(-0.276)}{(0.9801-(-0.276))}\right]+(1-0.5)\left[\frac{0.3386-(-0.0111)}{0.3641-(-0.0111)}\right]=-0.8192$$

$$Q_{x1(a1,a2,a3,a4)} = (-0.1319, 0.1457, 0.63, 0.8192; 1)$$

The defuzzification process is utilised using (Y. M. Wang et al., 2006) converts S_i, R_i and Q_i into regular number are depicted in Table 6.10. The defuzzification computational process for $Q_{x1(a1,a2,a3,a4)}$ is presented below

$$Defuzzified \quad Q_{x1(a1,a2,a3,a4)} = \frac{1}{3}\left(a_1+a_2+a_3+a_4-\left(\frac{(a_4\times a_3)-(a_1\times a_2)}{a_4+a_3-a_1-a_2}\right)\right)$$

$$Defuzzified \quad Q_{x1(a1,a2,a3,a4)} = \frac{1}{3}\left(-0.1319+0.1457+0.63+0.8192-\left(\frac{(0.8192\times 0.63)-(-0.1319\times 0.1457)}{0.8192+0.63-(-0.1319)-0.1457}\right)\right)$$

$$Defuzzified \quad Q_{x1(a1,a2,a3,a4)} = 0.3634$$

Table 6. 10. Defuzzification computations for type-1 fuzzy sets.

	Candidate 1	Candidate 2	Candidate 3	Candidate 4
Q	0.3634 (Rank 3)	0.1721 (Rank 2)	0.0000 (Rank 1)	0.4702 (Rank 4)
S	0.3201	0.2022	0.0000	0.5986
R	0.2239	0.1154	0.0468	0.2207

Step 7: Rank the alternatives, sorting by the values S_i, R_i and Q_i in decreasing order.

After several processes, referring to Table 6.10, the rank values shows candidate 3 represents the highest rank with the lowest *Q* and *S* with 0.00 followed by candidate 2 with 0.1721, candidate 1 with 0.3634 and candidate 4 with 0.4702 for the last ranked. The results show that the candidate 3 is most suitable for this recruitment post. For VIKOR analysis, the lowest the *Q* value, the better rank.

Step 8: Investigate as a compromise solution the alternative A', which is ranked the best alternative according to the measure Q(Minimum) if the following two conditions are satisfied:

Condition 1. Acceptable advantage:

$$Q(A'') - Q(A') \geq \frac{1}{m-1}$$

where A'' the alternative with second position in the ranking list by $Q; m$ is the number of alternative.

Condition 2. Acceptable stability:

Alternative A', must be also the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be "voting by the majority rule" (when $\upsilon > 0.5$ is needed), or "by consensus" $\upsilon \cong 0.5$, "with veto" $(\upsilon = 0.5)$. Here, υ is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility") (Fu et al., 2011).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consist of:

Alternative A', and A'' if only condition 2 is not satisfied, or;

Alternative $A', A'', ..., A^{(M)}$ if condition 1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A') < \frac{1}{m-1}$ for maximum m (the positions of these alternatives are "in closeness").

$$
Best \;\; (f_j^*) \;\; and \;\; Worst \;\; (f_j^-) = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{cc} f^* & f^- \end{array} \\ \begin{bmatrix} (0.86,0.9133,0.9733,0.99;1) & (0.4067,0.4833,0.6533,0.72;1) \\ (0.86,0.9133,0.9733,0.99;1) & (0.4933,0.5567,0.7267,0.79;1) \\ (0.93,0.98,1,1;1) & (0.72,0.78,0.92,0.97;1) \\ (0.5867,0.6567,0.8067,0.8633;1) & (0.1267,0.18,0.3,0.3567;1) \\ (0.93,0.98,1,1;1) & (0.4933,0.5567,0.7267,0.79;1) \end{bmatrix} \end{array} \tag{6.28}
$$

The best (f_j^*) and worst (f_j^-) quantities in each criteria.

$$
\frac{f_j^* - \tilde{x}_{ij}}{f_j^* - f_j^-} = \begin{array}{c|cccc} & x1 & x2 & x3 & x4 \\ \hline ES & (0.0696,0.1965,0.4164,0.5088;1) & (-0.1257,0.0571,0.4171,0.5429;1) & (-0.2229,-0.1029,0.1029,0.2229;1) & (0.24,0.4457,0.84,1;1) \\ O & (-0.2617,-0.1208,0.1208,0.2617;1) & (-0.2416,-0.0671,0.255,0.4027;1) & (-0.2617,-0.1208,0.1208,0.2617;1) & (0.1409,0.3758,0.8389,1;1) \\ P & (-0.25,-0.0714,0.07143,0.25;1) & (-0.1786,0.119,0.5476,0.75;1) & (-0.25,-0.0714,0.0714,0.25;1) & (-0.1429,0.2143,0.7857,1;1) \\ PE & (0.3122,0.4842,0.8507,1;1) & (-0.086,0.1041,0.5385,0.7376;1) & (-0.3756,-0.2036,0.2036,0.3756;1) & (0.2262,0.4027,0.7934,0.9412;1) \\ S-C & (0.0658,0.2763,0.6316,0.7368;1) & (-0.0987,0.0658,0.3026,0.4145;1) & (-0.1382,-0.0395,0.0395,0.1382;1) & (0.2763,0.5,0.875,1;1) \end{array} \tag{6.29}
$$

The fuzzy difference for alternatives evaluation

$$
w_j \times \frac{f_j^* - \tilde{x}_{ij}}{f_j^* - f_j^-} = \begin{array}{c|cccc} & x1 & x2 & x3 & x4 \\ \hline ES & (0.0061,0.0173,0.0367,0.0449;1) & (-0.0111,0.005,0.03681,0.0479;1) & (-0.0197,-0.0091,0.00908,0.0197;1) & (0.0212,0.0393,0.0714,,0.0882;1) \\ O & (-0.09953,-0.044,0.044,0.0953;1) & (-0.088,-0.0244,0.0929,0.1466;1) & (-0.0953,0.044,0.0440,0.0953;1) & (0.0513,0.1368,0.3054,0.3641;1) \\ P & (-0.0111,-0.0032,0.00316,0.0111;1) & (-0.0079,0.0053,0.02424,0.0332;1) & (-0.0111,-0.00032,0.0032,0.0111;1) & (-0.0063,0.0095,0.0348,0.0443;1) \\ PE & (0.1057,0.1639,0.2880,0.3386;1) & (-0.0291,0.0352,0.1823,0.2497;1) & (-0.1272,-0.0689,0.0689,0.1272;1) & (0.0766,0.1364,0.2696,0.3187;1) \\ S-C & (0.0108,0.0456,0.1041,0.1215;1) & (-0.0163,0.0108,0.0499,.0.0683;1) & (-0.0228,,-0.0065,0.0065,0.0028;1) & (0.0456,0.0824,0.1442,0.1649;1) \end{array} \tag{6.30}
$$

The weighted of fuzzy difference for alternatives evaluation.

$$
\begin{array}{ccccc}
 & x1 & x2 & x3 & x4 \\
S_i = & [(0.0163,0.1797,0.4760,0.6113;1) & (-0.1523,0.032,0.3861,0.5458;1) & (-0.276,-0.1317,0.1317,0.2760;1) & (0.1883,0.4044,0.8282,0.9801;1)]
\end{array}
\tag{6.31}
$$

The S_i weighted pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{ccccc}
 & x1 & x2 & x3 & x4 \\
R_i = & [(0.1057,0.1639,0.2880,0.3386;1) & (-0.0079,0.0352,0.1823,0.2497;1) & (-0.0111,-0.0032,0.0689,0.1272;1) & (0.0766,0.1368,0.3054,0.3641;1)]
\end{array}
\tag{6.32}
$$

The R_i weighted pairwise comparison matrix for alternatives evaluation

6.4 Hybrid Fuzzy Multi Criteria Decision Making for Type-2 Fuzzy Sets

This section illustrates computational process of proposed and established hybrid fuzzy MCDM models regarding staff selection in MESSRS Saprudin, Idris & Co for interval type-2 fuzzy sets. The proposed hybrid fuzzy MCDM model is compared with fuzzy AHP – TOPSIS (Kiliç & Kaya, 2015) model from literature for comparative study.

6.4.1 Consistent Fuzzy Preference Relations – Fuzzy Technique for Order of Preference by Similarity to Ideal Solution for Type-2 Fuzzy Sets

The extended hybrid fuzzy MCDM model for type-2 fuzzy sets is discussed in detail in Chapter 5. The proposed model is limited to interval type-2 fuzzy sets. In order to solve the vagueness and uncertainty based human decision making problems from information given, interval type-2 fuzzy sets are used to enhance judgement in the fuzzy decision making environment. The computational process of hybrid fuzzy type-2 MCDM model based consistent fuzzy preference relations – fuzzy TOPSIS are as follows.

Phase 1: Linguistic Evaluation

The decision makers used the linguistic terms that proposed by (Zheng et al., 2012) as shown in Table 6.11 in presenting the weights of criteria using consistent fuzzy preference evaluation for interval type-2 fuzzy sets. The linguistic terms with the crisp scale of relative important present the important of criteria preferences namely equally important (1), intermediate value (2), moderately more important (3), intermediate value (4), strongly more important (5), intermediate value (6), very strong more important (7), intermediate important (8) and extremely more important (9). For fuzzy TOPSIS evaluation, the linguistic terms and the corresponding of fuzzy numbers that proposed by (S. M. Chen & Lee, 2010) is used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively. The scales consist of very-low (1), low (2), medium-low (3), medium (4), medium-high (5), high (6) and very-high (7). The linguistic scales for alternatives evaluation are depicted in Table 6.12 that are measure from 0 until 1.

Table 6. 11. Trapezoidal interval type-2 fuzzy numbers preference scale based type-1 fuzzy numbers (Zheng et al., 2012)

Linguistic variables	Scale of relative important of crisp numbers	Trapezoidal interval type-2 fuzzy numbers	Reciprocal trapezoidal fuzzy number
Equally important (EI)	1	(1,1,1,1;1) (1,1,1,1;0.9)	(1,1,1,1;1)(1,1,1,1;0.9)
Intermediate value (IV)	2	(1,1.5,2.5,3;1)(1.25,2,2.75,3;0.9)	(0.33,0.4,0.67,1;1)(0.37,0.53,0.6,0.83;0.9)
Moderately more important (MMI)	3	(2,2.5,3.5,4;1)(2.25,3,3.75,4;0.9)	(0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9)
Intermediate value (IV)	4	(3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9)	(0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;0.9)
Strongly more important (SMI)	5	(4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9)	(0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9)
Intermediate value (IV)	6	(5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9)	(0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9)
Very strong more important (VSMI)	7	(6,6.5,7.5,8;1)(6.25,7,7.75,8;0.9)	(0.13,0.13,0.15,0.17;1)(0.13,0.14,0.15,0.16;0.9)
Intermediate value (IV)	8	(7,7.5,8.5,9;1)(7.25,8,8.75,9;0.9)	(0.11,0.12,0.13,0.14;1)(0.11,0.13,0.13,0.14;0.9)
Extremely more important (EMI)	9	(8,8.5,9,9;1)(8.25,9,9,9;0.9)	(0.11,0.11,0.12,0.13;1)(0.11,0.11,0.12,0.12;0.9)

Table 6. 12. Linguistic terms and their corresponding interval type-2 fuzzy numbers (S. M. Chen & Lee, 2010)

Linguistic Terms	Scale of preferences of crisp numbers	Interval type-2 fuzzy numbers
Very-low (VL)	1	(0,0,0,0.1;1)(0,0,0,0.5;0.9)
Low (L)	2	(0,0.1,0.1,0.3;1)(0.05,0.1,0.1,0.2;0.9)
Medium-low (ML)	3	(0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9)
Medium (M)	4	(0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9)
Medium-high (MH)	5	(0.5,0.7,0.7,0.9;1)(0.8,0.9,0.9,0.95;0.9)
High (H)	6	(0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9)
Very-high (VH)	7	(0.9,1,1,1;1)(0.95,1,1,1;0.9)

Phase 2: Fuzzy Weights of Criteria Evaluation using Consistent Fuzzy Preference Relations

Step 1: Construct a hierarchy structure.

The hierarchy model as shown in Fig. 6.1 at Section 6.3.1 is illustrated the connection of criteria and alternatives, which all candidates have to be interviewed.

Step 2: Construct pairwise comparison matrices.

The pairwise comparison matrices are constructed among all criteria in the dimension of the hierarchy systems based on the decision makers' preferences in phase 1 using equation (5.1). The linguistic evaluations of pairwise comparison matrices are based on regular numbers are depicted in equation (6.1), (6.2) and (6.3) from Section 6.3.1, then are translated into interval type-2 fuzzy numbers using Table 6.11. The linguistic ratings of criteria fuzzy numbers – based given by DM1, DM2 and DM3 are shown in equation (6.33), (6.34) and (6.35) respectively.

$$
DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{ccc}
ES & O & P \\
\left[\begin{array}{c} (1,1,1,1;1)(1,1,1,1;0.9) \\ (5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9) \\ (0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) \\ (5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9) \\ (1,1.5,2.5,3;1)(1.25,2,2.75,3;0.9) \end{array}\right.
&
\begin{array}{c} (0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9) \\ (1,1,1,1;1)(1,1,1,1;0.9) \\ (0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) \\ (1,1.5,2.5,3;1)(1.25,2,2.75,3;0.9) \\ (0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9) \end{array}
&
\begin{array}{c} (4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9) \\ (4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9) \\ (1,1,1,1;1)(1,1,1,1;0.9) \\ (5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9) \\ (4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9)\ldots \end{array}
\end{array}
$$

$$
\begin{array}{cc}
PE & S-C \\
\begin{array}{c} (0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9) \\ (0.33,0.4,0.67,1;1)(0.37,0.53,0.6,0.83;0.9) \\ (0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9) \\ (1,1,1,1;1)(1,1,1,1;0.9) \\ \ldots(0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;9) \end{array}
&
\left.\begin{array}{c} (0.33,0.4,0.67,1;1)(0.37,0.53,0.6,0.83,0.9) \\ (2,2.5,3.5,4;1)(2.25,3,3.75,4;0.9) \\ (0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) \\ (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9) \\ (1,1,1,1;1)(1,1,1,1;0.9) \end{array}\right]
\end{array}
\quad (6.33)
$$

Type-2 fuzzy pairwise comparison matrix of criteria evaluation from DM1

$$
DM2 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccc} ES & O & P \end{array} \\
\left[\begin{array}{ccc}
(1,1,1,1;1)(1,1,1,1;0.9) & (0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) & (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9) \\
(4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9) & (1,1,1,1;1)(1,1,1,1;0.9) & (5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9) \\
(0.20,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;0.9) & (0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9) & (1,1,1,1;1)(1,1,1,1;0.9) \\
(5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9) & (0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9) & (5,5.5,6.5,7;1)(5.25,6,6.75,7;0.9) \\
(1,1.5,2.5,3;1)(1.25,2,2.75,3;0.9) & (0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9) & (2,2.5,3.5,4;1)(2.25,3,3.75,4;0.9)\ldots
\end{array}\right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} PE & S-C \end{array} \\
\left.\begin{array}{cc}
(0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9) & (0.33,0.4,0.67,1;1)(0.37,0.53,0.6,0.83,0.9) \\
(2,2.5,3.5,4;1)(2.25,3,3.75,4;0.9) & (2,2.5,3.5,4;1)(2.25,3,3.75,4;0.9) \\
(0.14,0.15,0.18,0.2;1)(0.15,0.17,0.17,0.19;0.9) & (0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9) & (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9) \\
\ldots(0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;9) & (1,1,1,1;1)(1,1,1,1;0.9)
\end{array}\right]
\end{array} \tag{6.34}
$$

Type-2 fuzzy pairwise comparison matrix of criteria evaluation from DM2

$$
\begin{array}{cc}
 & \begin{array}{ccc} ES & O & P \end{array} \\
DM3 = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\left[\begin{array}{ccc}
(1,1,1,1;1)(1,1,1,1;0.9) & (0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) & (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9) \\
(4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9) & (1,1,1,1;1)(1,1,1,1;0.9) & (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9) \\
(0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;0.9) & (0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;0.9) & (1,1,1,1;1)(1,1,1,1;0.9) \\
(4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9) & (1,1.5,2.5,3;1)(1.25,2,2.75,3;0.9) & (4,4.5,5.5,6;1)(4.25,5,5.75,6;0.9) \\
(2,2.5,3.5,4;1)(2.25,3,3.75,4;0.9) & (0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;0.9) & (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9)\ldots
\end{array} \right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} PE & S-C \end{array} \\
\left. \begin{array}{cc}
(0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) & (0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9) \\
(0.33,0.4,0.67,1;1)(0.17,0.2,0.21,0.24;0.9) & (3,3.5,4.5,5;1)(3.25,4,4.75,5;0.9) \\
(0.17,0.18,0.22,0.25;1)(0.17,0.2,0.21,0.24;0.9) & (0.2,0.22,0.29,0.33;1)(0.21,0.25,0.27,0.31;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9) & (0.25,0.22,0.4,0.5;1)(0.24,0.31,0.36,0.45;0.9) \\
\ldots(2,2.5,3.5,4;1)92.25,3,3.75,4;9) & (1,1,1,1;1)(1,1,1,1;0.9)
\end{array} \right]
\end{array}
\qquad (6.35)
$$

Type-2 fuzzy pairwise comparison matrix of criteria evaluation from DM3

Step 3: Aggregate the decision makers' preferences.

The pairwise comparison matrices for criteria's judgement of decision makers' (DM1, DM2, DM3) preferences as listed in equation (6.33), (6.34) and (6.35) are aggregated using equation (5.2). The result of aggregated pairwise comparison matrix is shown in equation (6.37) on the next page.

$$\tilde{a}_{ij} = (\tilde{a}_{ij}^{1} \times \tilde{a}_{ij}^{2} \times \ldots \times \tilde{a}_{ij}^{n})^{1/k}$$

where k is the number of decision makers and $i=1,2,\ldots m; j=1,2,\ldots n$.

Step 4: Defuzzify the trapezoidal fuzzy numbers of aggregation's result of decision makers' preferences.

The aggregation's result of decision maker's preferences are defuzzify using intuitive multiple centroid for type-2 fuzzy sets using equation (4.13) and presented in equation (6.36).

$$IVC_{\tilde{\tilde{A}}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}, \frac{7}{36}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right)$$

$$Defuzzify = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 1 & 0.1925 & 4.4690 & 0.1808 & 0.4741 \\ 5.4733 & 1 & 5.0933 & 1.00513 & 3.4621 \\ 0.2397 & 0.2081 & 1 & 0.1808 & 0.2594 \\ 5.8063 & 1.1529 & 5.8063 & 1 & 1.7848 \\ 2.4491 & 0.3042 & 4.0758 & 0.5960 & 1 \end{bmatrix} \end{array} \qquad (6.36)$$

Defuzzification type-2 fuzzy results of aggregated matrix comparison

$$
\begin{array}{cc}
 & \begin{array}{cc} ES & O \end{array} \\
\begin{array}{c} Aggregated = \end{array}
\begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\left[\begin{array}{cc}
(1,1,1,1;1)(1,1,1,1;0.9) & (0.1583,0.1720,0.2078,0.2321;1)(0.1651,0.1899,0.1989,0.22;0.9) \\
(4.3089,4.8113,5.8150,6.3164;1)(4.5601,5.3133,6.066,6.3164;0.9) & (1,1,1,1;1)(1,1,1,1;0.9) \\
(0.1882,0.2078,0.2628,0.3029;1)(0.1980,0.2353,0.2490,0.2828;0.9) & (0.1682,0.1839,0.2260,0.2554;1)(0.1761,0.2050,0.2155,0.2407;0.9) \\
(4.6416,5.1441,6.1479,6.6494;1)(4.8629,5.6462,6.3987,6.6494;0.9) & (0.63,0.7937,1.3572,1.651;1)(0.7172,1.0756,1.3906,1.5940;0.9) \\
(1.2599,1.7784,2.7967,3.3019;1)(1.5206,2.2894,3.0495,3.3019;0.9) & (0.2321,0.2222,0.3576,0.4368;1)(0.2275,0.2908,0.3243,0.3972;0.9)\ldots
\end{array}\right.
\end{array}
$$

$$
\begin{array}{cc}
P & PE \\
(3.3019,3.8058,4.8113,5.3133;1)(3.5540,4.3089,5.0623,5.3133;0.9) & (0.1504,0.1627,0.1944,0.2154;1)(0.1565,0.1785,0.1865,0.2049;0.9) \\
(3.9149,4.4247,5.4387,5.9439;1)(4.1701,4.9324,5.6914,5.9439;0.9) & (0.6057,0.7368,1.1587,1.5874;1)(0.6713,0.9485,1.1052,1.4057;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9) & (0.1504,0.1627,0.1944,0.2154;1)(0.1565,0.1785,0.1865,0.2049;0.9) \\
(4.6416,5.1441,6.1479,6.6494;1)(4.8929,5.6462,6.3987,6.6494;0.9) & (1,1,1,1;1)(1,1,1,1;0.9) \\
\ldots(2.8845,3.4020,4.4247,4.9324;1)(3.1440,3.9149,4.6788,4.9324;0.9) & (0.4309,0.4979,0.6586,0.7631;1)(0.4646,0.5784,0.6488,0.7264;0.9)\ldots
\end{array}
$$

$$
\begin{array}{c}
S-C \\
\left.\begin{array}{c}
(0.3029,0.3288,0.5623,0.7937;1)(0.3166,0.4456,0.5040,0.6786;0.9) \\
(2.2894,2.7967,3.8058,4.3089;1)(2.5434,3.3019,4.0574,4.3089;0.9) \\
(0.2027,0.2078,0.2939,0.3467;1)(0.2056,0.2518,0.2730,0.3204;0.9) \\
(1.3104,1.3963,2.0083,2.3208;1)(1.3561,1.7074,2.0019,2.2407;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9)
\end{array}\right]
\end{array}
\tag{6.37}
$$

The aggregated type-2 fuzzy pairwise comparison matrix of decision makers for criteria evaluation

Step 5: Compute the weights of criteria values for alternatives' evaluation using consistent fuzzy preference relations.

The weights of aggregated matrix comparison of criteria are calculated using consistent preference relations which based on additive transitivity property using equation (3.16-3.22) in equation (6.38) below:

$$Fuzzy\ \ weights = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 0.5 & 0.1251 & 0.4956 & 0.1063 & 0.2382 \\ 0.8749 & 0.5 & 0.8705 & 0.4812 & 0.6131 \\ 0.5045 & 0.1296 & 0.5 & 0.1108 & 0.2426 \\ 0.8937 & 0.5188 & 0.8892 & 0.5 & 0.6318 \\ 0.7618 & 0.3869 & 0.7574 & 0.3682 & 0.5 \end{bmatrix} \end{array} \tag{6.38}$$

The consistent type-2 fuzzy preference relations matrix for criteria

By having five criteria, $n = 5$ so only $(n-1) = 5-1 = 4$ entry values $(p_{12}, p_{23}, p_{34}\ \ and\ \ p_{45})$ are required in constructing the consistent fuzzy preference relations matrix from equation (6.36) where:

$$p_{12} = \frac{1}{2}(1 + \log_9 0.1925) = 0.1251$$

$$p_{23} = \frac{1}{2}(1 + \log_9 5.0933) = 0.8705$$

$$p_{34} = \frac{1}{2}(1 + \log_9 0.1808) = 0.1108$$

$$p_{45} = \frac{1}{2}(1 + \log_9 1.785) = 0.6318$$

The remains of the entries can be determine by utilizing Proposition 2 and 3 are presented as follows.

$$p_{21} = 1 - p_{12} = 1 - 0.1251 = 0.8749$$

$$p_{32} = 1 - p_{23} = 1 - 0.8705 = 0.1296$$

$$p_{43} = 1 - p_{34} = 1 - 0.1108 = 0.8892$$

$$p_{54} = 1 - p_{45} = 1 - 0.6318 = 0.3682$$

$$p_{31} = \frac{3}{2} - p_{12} - p_{23} = \frac{3}{2} - 0.1251 - 0.8705 = 0.5045$$

$$p_{42} = \frac{3}{2} - p_{23} - p_{34} = \frac{3}{2} - 0.8705 - 0.1108 = 0.5188$$

$$p_{53} = \frac{3}{2} - p_{34} - p_{45} = \frac{3}{2} - 0.1108 - 0.6318 = 0.7574$$

$$p_{41} = \frac{j-i+1}{2} - p_{12} - p_{23} - p_{34} = \frac{4-1+1}{2} - 0.1251 - 0.8705 - 0.1108 = 0.8937$$

$$p_{51} = \frac{j-i+1}{2} - p_{12} - p_{23} - p_{34} - p_{45} = \frac{5-1+1}{2} - 0.1251 - 0.8705 - 0.1108 - 0.6318 = 0.7618$$

$$p_{52} = \frac{j-i+1}{2} - p_{23} - p_{34} - p_{45} = \frac{5-2+1}{2} - 0.8705 - 0.1108 - 0.6318 = 0.3869$$

$$p_{13} = 1 - p_{31} = 1 - 0.5045 = 0.4955$$

$$p_{14} = 1 - p_{41} = 1 - 0.8937 = 0.1063$$

$$p_{15} = 1 - p_{51} = 1 - 0.7618 = 0.2382$$

$$p_{24} = 1 - p_{42} = 1 - 0.5188 = 0.4812$$

$$p_{25} = 1 - p_{52} = 1 - 0.3869 = 0.6131$$

$$p_{35} = 1 - p_{53} = 1 - 0.7574 = 0.2426$$

Then, the average and weight of each criterion from equation (6.38) are illustrated in Table 6.13 as follows.

Table 6. 13. The type-2 fuzzy average and weightage of criteria

Criteria	*ES*	*O*	*P*	*PE*	*S-C*	*Average*	*Weights*	*Rank*
ES	0.5	0.1251	0.4956	0.1063	0.2382	0.2930	0.1172	5
O	0.8749	0.5	0.8705	0.4812	0.6131	0.6679	0.2672	2
P	0.5045	0.1296	0.5	0.1108	0.2426	0.2975	0.1190	4
PE	0.8937	0.5188	0.8892	0.5	0.6318	0.6867	0.2747	1
S-C	0.7318	0.3869	0.7574	0.3688	0.5	0.5549	0.2219	3
Total						2.5	1	

These results of criteria's weights are implemented in following phase to evaluate for alternatives selection.

Phase 3: Ranking evaluation of alternatives using fuzzy TOPSIS

Step 1: Determine the weights of evaluation criteria.

The weights of evaluation criteria are employed from consistent fuzzy preference relations process before. Refer Table 6.13.

Step 2: Construct the fuzzy decision matrix for alternatives' evaluation using fuzzy TOPSIS.

The construction of fuzzy decision matrix for alternatives' evaluation are utilised linguistic terms by (S. M. Chen & Lee, 2010) presented on Table 6.14.

Table 6. 14. Evaluating linguistic terms of the alternatives given by the decision makers with respect to different criteria

Criteria	Alternatives	Decision Maker		
		DM1	**DM2**	**DM3**
Emotional Steadiness	*x1*	H	MH	H
	x2	H	H	MH
	x3	VH	H	VH
	x4	M	MH	M
Oration	*x1*	H	H	VH
	x2	H	H	VH
	x3	VH	VH	H
	x4	MH	MH	MH
Personality	*x1*	VH	VH	VH
	x2	H	H	VH
	x3	VH	VH	VH
	x4	H	H	H
Past Experience	*x1*	ML	L	ML
	x2	M	M	M
	x3	H	M	H
	x4	ML	ML	ML
Self-Confidence	*x1*	H	MH	MH
	x2	VH	H	H
	x3	VH	VH	VH
	x4	M	MH	MH

Table 6. 15. Evaluating type-2 fuzzy values of the alternatives given by the decision makers with respect to different criteria

Criteria	**Alternatives (Candidates)**	**Decision Maker 1**		**Decision Maker 2**
		DM1		**DM2**
Emotional Steadiness	*x1*	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.50 0.70 0.70 0.90; 1.00)
	x2	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
	x3	(0.90 1.00 1.00 1.00; 1.00)	(0.95 1.00 1.00 1.00; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
	x4	(0.30 0.50 0.50 0.70; 1.00)	(0.40 0.50 0.50 0.60; 0.90)	(0.50 0.70 0.70 0.90; 1.00)
Oration	*x1*	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
	x2	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
	x3	(0.90 1.00 1.00 1.00; 1.00)	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)
	x4	(0.50 0.70 0.70 0.90; 1.00)	(0.60 0.70 0.70 0.80; 0.90)	(0.50 0.70 0.70 0.90; 1.00)
Personality	*x1*	(0.90 1.00 1.00 1.00; 1.00)	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)
	x2	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
	x3	(0.90 1.00 1.00 1.00; 1.00)	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)
	x4	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
Past Experience	*x1*	(0.10 0.30 0.30 0.50; 1.00)	(0.20 0.30 0.30 0.40; 0.90)	(0.00 0.10 0.10 0.30; 1.00)
	x2	(0.30 0.50 0.50 0.70; 1.00)	(0.40 0.50 0.50 0.60; 0.90)	(0.30 0.50 0.50 0.70; 1.00)
	x3	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.30 0.50 0.50 0.70; 1.00)
	x4	(0.10 0.30 0.30 0.50; 1.00)	(0.20 0.30 0.30 0.40; 0.90)	(0.10 0.30 0.30 0.50; 1.00)
Self-Confidence	*x1*	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 0.95; 0.90)	(0.50 0.70 0.70 0.90; 1.00)
	x2	(0.90 1.00 1.00 1.00; 1.00)	(0.95 1.00 1.00 1.00; 0.90)	(0.70 0.90 0.90 1.00; 1.00)
	x3	(0.90 1.00 1.00 1.00; 1.00)	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)
	x4	(0.30 0.50 0.50 0.70; 1.00)	(0.40 0.50 0.50 0.60; 0.90)	(0.50 0.70 0.70 0.90; 1.00)

Table 6. 12. Evaluating type-2 fuzzy values of the alternatives given by the decision makers with respect to different criteria (cont.)

Criteria	Alternatives (Candidates)	Decision Maker 2	Decision Maker 3	
		DM2	DM3	
Emotional Steadiness	*x1*	(0.60 0.70 0.70 0.80; 0.90)	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 1.00; 0.90)
	x2	(0.80 0.90 0.90 0.95; 0.90)	(0.50 0.70 0.70 0.90; 1.00)	(0.60 0.70 0.70 0.80; 0.90)
	x3	(0.80 0.90 0.90 0.95; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x4	(0.60 0.70 0.70 0.80; 0.90)	(0.30 0.50 0.50 0.70; 1.00)	(0.40 0.50 0.50 0.60; 0.90)
Oration	*x1*	(0.80 0.90 0.90 0.95; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x2	(0.80 0.90 0.90 0.95; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x3	(0.95 1.00 1.00 1.00; 0.90)	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 1.00; 0.90)
	x4	(0.60 0.70 0.70 0.80; 0.90)	(0.50 0.70 0.70 0.90; 1.00)	(0.60 0.70 0.70 0.80; 0.90)
Personality	*x1*	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x2	(0.80 0.90 0.90 0.95; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x3	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x4	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 1.00; 0.90)
Past Experience	*x1*	(0.05 0.10 0.10 0.20 0.90)	(0.10 0.30 0.30 0.50; 1.00)	(0.20 0.30 0.30 0.40; 0.90)
	x2	(0.40 0.50 0.50 0.60; 0.90)	(0.30 0.50 0.50 0.70; 1.00)	(0.40 0.50 0.50 0.60; 0.90)
	x3	(0.40 0.50 0.50 0.60; 0.90)	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 1.00; 0.90)
	x4	(0.20 0.30 0.30 0.40; 0.90)	(0.10 0.30 0.30 0.50; 1.00)	(0.20 0.30 0.30 0.40; 0.90)
Self-Confidence	*x1*	(0.60 0.70 0.70 0.80; 0.90)	(0.50 0.70 0.70 0.90; 1.00)	(0.60 0.70 0.70 0.80; 0.90)
	x2	(0.80 0.90 0.90 0.95; 0.90)	(0.70 0.90 0.90 1.00; 1.00)	(0.80 0.90 0.90 1.00; 0.90)
	x3	(0.95 1.00 1.00 1.00; 0.90)	(0.90 1.00 1.00 1.00; 1.00)	(0.95 100 1.00 1.00; 0.90)
	x4	(0.60 0.70 0.70 0.80; 0.90)	(0.50 0.70 0.70 0.90; 1.00)	(0.60 0.70 0.70 0.80; 0.90)

Step 3: Fuzzy decision matrix is weighted using equation (5.5) and normalised each generalised fuzzy numbers into standardised generalised fuzzy numbers using (Zuo et al., 2013).

Equation (6.39), (6.40) and (6.41) represent the pairwise comparison matrices for decision makers for alternatives evaluation. Then, aggregated results is depicted in equation (6.42). The weighted fuzzy normalised decision matrix is computed using equation (5.6) (Zuo et al., 2013). The results of weighted and normalisation process are presented in equation (6.43) and (6.44) respectively. Defuzzify the standardised generalised fuzzy numbers using proposed intuitive multiple centroid method for type-2 fuzzy sets, then translate them into the index point proposed by (Yong & Qi, 2005) as presented in equation (6.45) and equation (6.46) respectively, then find average computational process as depicted in equation (6.47).

Step 4: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

Referring to normalise trapezoidal fuzzy weights, the FPIS, A^+ represents the compromise solution while FNIS, A^- represents the worst possible solution. The range belong to the closed interval [0,1]. The FPIS A^+ (aspiration levels) and FNIS A^- (worst levels) as following below:

$$A^+ = (1,1,1,1;1)(1,1,1,1;0.9) \qquad A^- = (-1,-1,-1,-1;1)(-1,-1,-1,-1;0.9)$$

The FPIS, A^+ and FNIS, A^- can be obtained by centroid method for (x_{A^+}, y_{A^+}) and (x_{A^-}, y_{A^-}).

$$
DM1 = \begin{array}{c c} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) \\ (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) \\ (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) \\ (0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9) \\ (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.9,1,1,1;1)(0.95,0.9,0.9,0.95;0.9) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) \end{bmatrix} \end{array} \tag{6.39}
$$

The pairwise comparison matrix of *DM1* for alternatives evaluation

$$
DM2 = \begin{array}{c c} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) \\ (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) \\ (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) \\ (0,0.1,0.1,0.3;1)(0.05,0.1,0.1,0.2;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) & (0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9) \\ (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) \end{bmatrix} \end{array} \tag{6.40}
$$

The pairwise comparison matrix of *DM2* for alternatives evaluation

$$
DM3 = \begin{array}{c c} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) \\ (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.9,1,1,1;1)(0.95,0.9,0.9,0.95;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) \\ (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.9,1,1,1;1)(0.95,0.9,0.9,0.95;0.9) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) \\ (0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95) & (0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9) \\ (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) & (0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9)) \end{bmatrix} \end{array} \tag{6.41}
$$

The pairwise comparison matrix of *DM3* for alternatives evaluation

$$
\begin{array}{ccc}
 & & \begin{array}{cc} x1 & x2 \end{array} \\
Aggregated = & \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \left[\begin{array}{ll}
(0.63,0.83,0.83,0.97;1)(0.73,0.83,0.83,0.9;0.9) & (0.63,0.83,0.83,0.97;1)(0.73,0.83,0.83,0.9;0.9) \\
(0.77,0.93,0.93,1;1)(0.85,0.93,0.93,0.97;0.9) & (0.77,0.93,0.93,1;1)(0.85,0.93,0.93,0.97;0.9) \\
(0.9,1,1,1;1)(0.95,1,1,1;0.9) & (0.77,0.93,0.93,1;1)(0.85,0.93,0.93,0.97;0.9) \\
(0.07,0.23,0.23,0.43;1)(0.15,0.23,0.23,0.33;0.9) & (0.3,0.5,0.5,0.7;1)(0.4,0.5,0.5,0.6;0.9) \\
(0.57,0.77,0.77,0.93;1)(0.67,0.77,0.77,0.85;0.9) & (0.77,0.93,0.93,1;1)(0.85,0.93,0.93,0.97;0.9) \ldots
\end{array}\right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} x3 & x4 \end{array} \\
\left.\begin{array}{ll}
(0.83,0.97,0.97,1;1)(0.9,0.97,0.97,0.98;0.9) & (0.37,0.57,0.57,0.77;1)(0.47,0.57,0.57,0.67;0.9) \\
(0.83,0.97,0.97,1;1)(0.9,0.97,0.97,0.98;0.9) & (0.5,0.7,0.7,0.9;1)(0.6,0.7,0.7,0.8;0.9) \\
(0.9,1,1,1;0.9)(0.95,1,1,1;0.9) & (0.7,0.9,0.9,1;1)(0.8,0.9,0.9,0.95;0.9) \\
(0.57,0.77,0.77,0.9;1)(0.67,0.77,0.77,0.83;0.9) & (0.1,0.3,0.3,0.5;1)(0.2,0.3,0.3,0.4;0.9) \\
\ldots((0.9,1,1,1;0.9)(0.95,1,1,1;0.9) & (0.43,0.63,0.63,0.83;1)(0.53,0.63,0.63,0.73;0.9)
\end{array}\right]
\end{array}
\quad (6.42)
$$

The aggregated pairwise comparison matrix for alternatives evaluation

$$
Weighted \quad aggregated = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} x1 \\ \left[\begin{array}{l} (0.0742,0.0977,0.0977,0.1133;1)(0.08596,0.0977,0.0977,0.1055;0.9) \\ (0.2048,0.2494,0.2494,0.2672;1)(0.2271,0.2494,0.2494,0.2583;0.9) \\ (0.1071,0.119,0.119,0.119;1)(0.113,0.119,0.119,0.119;0.9) \\ (0.0183,0.0641,0.0641,0.119;1)(0.0412,0.0641,0.0641,0.0916;0.9) \\ (0.1258,0.1702,0.1702,0.2072;1)(0.148,0.1702,0.1702,0.1887;0.9) \end{array}\right. \end{array} \begin{array}{c} x2 \\ \begin{array}{l} (0.0742,0.0977,0.0977,0.1133;1)(0.086,0.0977,0.0977,0.1055;0.9) \\ (0.2048,0.2494,0.2494,0.2672;1)(0.2271,0.2494,0.2494,0.2583;0.9) \\ (0.0912,0.1111,0.1111,0.119;1)(0.1011,0.1111,0.1111,0.115;0.9) \\ (0.0824,0.1373,0.1373,0.1923;1)(0.1099,0.1373,0.1373,0.1648;0.9) \\ (0.1702,0.2071,0.2071,0.2219;1)(0.1887,0.2072,0.2072,0.2145;0.9)\ldots \end{array} \end{array}
$$

$$
\begin{array}{c} x3 \\ \left.\begin{array}{c} (0.0977,0.1133,0.1133,0.1172;1)(0.1055,0.1133,0.1133,0.1153;0.9) \\ (0.2226,0.2583,0.2583,0.2672;1)(0.2405,0.2583,0.2583,0.2627;0.9) \\ (0.1071,0.119,0.119,0.119;1)(0.113,0.119,0.119,0.119;0.9) \\ (0.1556,0.2106,0.2016,0.2472;1)(0.1831,0.2106,0.2106,0.2289;0.9) \\ \ldots(0.1998,0.2219,0.2219,0.2219;1)(0.2108,0.2219,0.2219,0.2219;0.9) \end{array}\right. \end{array} \begin{array}{c} x4 \\ \left.\begin{array}{l} (0.0043,0.0664,0.0664,0.0899;1)(0.0547,0.0664,0.0664,0.0781;0.9)) \\ (0.1336,0.187,0.187,0.2405;1)(0.1603,0.187,0.187,0.2137;0.9) \\ (0.0833,0.1071,0.1071,0.119;1)(0.0952,0.1071,0.1071,0.113;0.9) \\ (0.0275,0.0824,0.0824,0.1373;1)(0.1184,0.1406,0.1406,0.1628;0.9) \\ (0.0962,0.1406,0.1406,0.185;1)(0.1184,0.1406,0.1406,0.1628;0.9) \end{array}\right] \end{array} \tag{6.43}
$$

The weighted aggregated pairwise comparison matrix for alternatives evaluation

$$
Normalised\quad weighted\quad aggregated = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{cc} x1 & x2 \\ \left[\begin{array}{c} (0.2247,0.3189,0.3189,0.3817;1)(0.2718,0.3189,0.318,3503;0.9) \\ (0.7495,0.9284,0.9284,1;1)(0.839,0.9284,0.9284,0.9642;0.9) \\ (0.3568,0.4046,0.4046,0.4046;1)(0.3807,0.4046,0.4046,0.4046;0.9) \\ (0,0.184,0.184,0.4047;1)(0.092,0.184,0.184,0.2943;0.9) \\ (0.4318,0.6102,0.6102,0.7588;1)(0.521,0.6102,0.6102,0.6845;0.9) \end{array}\right. & \begin{array}{c} (0.2247,0.3189,0.3189,0.3817;1)(0.2718,0.3189,0.3189,3503;0.9) \\ (07495,0.9284,0.9284,1;1)(0.839,0.9284,0.9284,0.9642;0.9) \\ (0.293,0.3727,0.3727,0.4046;1)(0.3329,0.9284,0.9284,0.9642.;0.9) \\ (0.2575,0.4783,0.4783,0.699;1)(0.3679,0.4783,0.4783,0.5887;0.9) \\ (0.6102,0.7588,0.7588,0.3189,0.3817;1)(0.6845,0.7588,0.7588;0.9)\ldots \end{array} \end{array}
$$

$$
\begin{array}{cc} x3 & x4 \\ \begin{array}{c} (0.3189,0.3817,0.3817,0.3974;1)(0.3503,0.3817,0.3817,0.3896;0.9) \\ (0.8211,0.9642,0.9642,1;1)(0.8926,0.9642,0.9642,0.9821;0.9) \\ (0.3568,0.4046,0.4046,0.4046;1)(0.38007,0.4046,0.4046,0.4046;0.9) \\ (0.5519,0.7726,0.7726,0.9198;1)(0.6622,0.7726,0.7726,0.8462;0.9) \\ \ldots(0.7291,0.8183,0.8183,0.8183;1)(0.7737,0.8183,0.8183,0.8183;0.9) \end{array} & \left.\begin{array}{c} (0.0991,0.1933,0.1933,0.2875;1)(0.1462,0.1933,0.2404;0.9) \\ (0.4632,0.6779,0.6779,0.8926;1)(0.5706,0.6779,0.6779,0.7853;0.9) \\ (0.2611,0.3568,0.3568,0.4046;0.9)(0.3089,0.3568,0.3568,0.3807;0.9) \\ (0.0368,0.2575,0.2575,0.4783;1)(0.1472,0.2575,0.2575,0.3679;0.9) \\ (0.3129,0.4913,0.4913,0.6696;1)(0.4021,0.4913,0.4913,0.5804;0.9) \end{array}\right] \end{array} \tag{6.44}
$$

The normalised weighted aggregated pairwise comparison matrix for alternatives evaluation

$$
Defuzzifiied = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
\begin{bmatrix}
x=0.3163, \ y=0.3694 & x=0.3163, \ y=0.3694 & x=0.3778, \ y=0.3694 & x=0.1933, \ y=0.3694 \\
x=0.9195, \ y=0.3694 & x=0.9195, \ y=0.3694 & x=0.9553, \ y=0.3694 & x=0.6779, \ y=0.3694 \\
x=0.4006, \ y=0.3694 & x=0.3687, \ y=0.3694 & x=0.4006, \ y=0.3694 & x=0.3528, \ y=0.3694 \\
x=0.187, \ y=0.3694 & x=0.4783, \ y=0.3694 & x=0.7665, \ y=0.3694 & x=0.2575, \ y=0.3694 \\
x=0.6077, \ y=0.3694 & x=0.7514, \ y=0.3694 & x=0.8208, \ y=0.3694 & x=0.4913, \ y=0.3694
\end{bmatrix}
\end{array}
\tag{6.45}
$$

The defuzzified type-2 fuzzy pairwise comparison matrix for alternatives evaluation

$$
Translate \ \ defuzzifiied = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
\begin{bmatrix}
x=0.3163, \ y=0.4570 & x=0.3163, \ y=0.4570 & x=0.3778, \ y=0.4651 & x=0.1933, \ y=0.4537 \\
x=0.9195, \ y=0.4450 & x=0.9195, \ y=0.4450 & x=0.9553, \ y=0.4525 & x=0.6779, \ y=0.4264 \\
x=0.4006, \ y=0.4680 & x=0.3687, \ y=0.4616 & x=0.4006, \ y=0.4680 & x=0.3528, \ y=0.4582 \\
x=0.187, \ y=0.4291 & x=0.4783, \ y=0.4251 & x=0.7665, \ y=0.4328 & x=0.2575, \ y=0.4251 \\
x=0.6077, \ y=0.4379 & x=0.7514, \ y=0.4478 & x=0.8208, \ y=0.4620 & x=0.4913, \ y=0.4346
\end{bmatrix}
\end{array}
\tag{6.46}
$$

The translate defuzzified type-2 fuzzy pairwise comparison matrix for alternatives evaluation

$$
Average \ \ Translate \ \ defuzzifiied =
\begin{array}{c}
\begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
\begin{bmatrix}
x=0.4862, \ y=0.4474 & x=0.5668, \ y=0.4478 & x=0.6622 \ \ y=0.4561 & x=0.3946, \ y=0.4396
\end{bmatrix}
\end{array}
\tag{6.47}
$$

The average translate defuzzified fuzzy pairwise comparison matrix for alternatives evaluation

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative from formulation A^+ and A^- can be calculated by the area of compensation method:

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^+})^2 + (y_{\tilde{A}_i^*} - y_{A^+})^2}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{(0.4862-1)^2 + (0.4474-0.5)^2}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = 0.5165$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^-})^2 + (y_{\tilde{A}_i^*} - y_{A^-})^2}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{(0.4862+1)^2 + (0.4474-0.5)^2}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = 1.4871$$

Step 6: Find the closeness coefficient, CC_i and improve alternatives for achieving aspiration levels in each criteria.

The decision rules for five classes are depicted in Table 6.7. Notice that the highest CC_i value is used to determine the rank.

$$\overline{CC}_i = \frac{\bar{d}_i^-}{\bar{d}_i^+ + \bar{d}_i^-} = 1 - \frac{\bar{d}_i^+}{\bar{d}_i^+ + \bar{d}_i^-}$$

$$\overline{CC}_i = \frac{1.4871}{0.5165 + 1.4871}$$

$$\overline{CC}_i = 0.7422$$

After several processes, referring to Table 6.16, the CC_i values shows candidate 3 represents the highest rank with 0.83 followed by candidate 2 with 0.7823, candidate 1 with 0.7422 and candidate 4 with 0.6964 for the last ranked. The results reveal that the candidate 3 is most suitable for this recruitment post because based on approval status table, the score is in approved and preferred range.

Table 6. 16. Closeness coefficients computation for type-2 fuzzy sets.

Alternative	**Closeness Coefficient, *CCi***
Candidate 1	0.7422 (Rank 3)
Candidate 2	0.7823 (Rank 2)
Candidate 3	0.83 (Rank 1)
Candidate 4	0.6964 (Rank 4)

Table 6. 7. Approval status table (Luukka, 2011)

***CCi* value**	**Assessment status**
$CC_i \in [0,0.2)$	Do not recommend
$CC_i \in [0.2,0.4)$	Recommend with high risk
$CC_i \in [0.4,0.6)$	Recommend with low risk
$CC_i \in [0.6,0.8)$	Approved
$CC_i \in [0.8,1]$	Approved and preferred

6.4.2 *Fuzzy Analytic Hierarchy Process – Fuzzy Technique for Order of Preference by Similarity to Ideal Solution*

This section presents established hybrid fuzzy MCDM for type- 2 fuzzy sets based fuzzy AHP – fuzzy TOPSIS proposed by (Kiliç & Kaya, 2015). In this research study, the authors combined AHP and TOPSIS based on interval type-2 fuzzy sets. There are several phases in computing hybrid fuzzy MCDM model based fuzzy AHP – fuzzy TOPSIS are as follows.

Phase 1: Linguistic Evaluation

The decision makers used the linguistic terms that proposed by (Zheng et al., 2012) as shown in Table 6.11 in presenting the weights of criteria using fuzzy AHP evaluation for interval type-2 fuzzy set. For fuzzy TOPSIS evaluation, the linguistic terms and the corresponding of fuzzy numbers that proposed by (S. M. Chen & Lee, 2010) as depicted in Table 6.12 is used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively.

Phase 2: Fuzzy Weights Evaluation using Fuzzy AHP

Step 1: Building the evaluation hierarchy systems.

The hierarchy model is presented in Fig. 6.1 is illustrated the connection of criteria and alternatives, which are the candidates to be interviewed. Five criteria are considered which consist of emotional steadiness, oration, past experience, personality and self-confidence.

Step 2: Determining the evaluation dimensions weights of pairwise comparison matrix to find the fuzzy weights.

The pairwise comparison matrix showing the preference of one criterion over the other is built by entering the judgement values by the decision makers. Since the values of linguistic variables are quadruplet trapezoidal fuzzy numbers are entered.

Step 3: Determining the weights for the criteria involved.

The synthetic pairwise comparison matrices for criteria's judgement of decision makers (DM1, DM2 and DM3) preferences as listed in equation (6.33), (6.34) and (6.35) are aggregated using geometric mean method, refer equation (5.2). The result of aggregated pairwise comparison matrix and fuzzy geometric mean are shown in equation (6.50) and (6.51) respectively.

$$\tilde{r}_{ij} = (\tilde{a}_{ij}^{1} \times \tilde{a}_{ij}^{2} \times \ldots \times \tilde{a}_{ij}^{n})^{1/k}$$

where k is the number of decision makers and $i=1,2,\ldots m; j=1,2,\ldots n$.

Step 4: The fuzzy weight of each criterion is determined using normalising equation.

This is done by using equation (3.14). The results of fuzzy normalised weighted of each criterion are presented in equation (6.52).

$$w_i = r_i \times (r_1 + r_2 + r_3 + \ldots + r_n)^{-1}$$

Step 5: Defuzzify each weight from Step 4 using best nonfuzzy performance (BNP).

The best nonfuzzy performance (BNP) based centre of area (COA) defuzzification method is utilised in order to handle interval type-2 fuzzy sets to find arithmetic mean of upper bound and lower bound. This is represented in equation (6.53).

$$\tilde{w}_j = \frac{\int xu(x)dx}{\int u(x)dx} \tag{6.48}$$

$$\tilde{w}_j = \frac{-w_{j1} \times w_{j2} + w_{j3} \times w_{j4} + 1/3(w_{j4} - w_{j3})^2 - 1/3(w_{j2} - w_{j1})^2}{-w_{j1} - w_{j2} + w_{j3} + w_{j4}} \tag{6.49}$$

Compute the criteria values as weights for alternatives' evaluation using fuzzy AHP.

$$
Aggregated = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{cc} ES & O \\
\left[\begin{array}{c}
(1,1,1,1;1)(1,1,1,1;0.9) \\
(4.3089,4.8113,5.8150,6.3164;1)(4.5601,5.3133,6.066,6.3164;0.9) \\
(0.1882,0.2078,0.2628,0.3029;1)(0.1980,0.2353,0.2490,0.2828;0.9) \\
(4.6416,5.1441,6.1479,6.6494;1)(4.8629,5.6462,6.3987,6.6494;0.9) \\
(1.2599,1.7784,2.7967,3.3019;1)(1.5206,2.2894,3.0495,3.3019;0.9)
\end{array}\right. &
\begin{array}{c}
(0.1583,0.1720,0.2078,0.2321;1)(0.1651,0.1899,0.1989,0.22;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9) \\
(0.1682,0.1839,0.2260,0.2554;1)(0.1761,0.2050,0.2155,0.2407;0.9) \\
(0.63,0.7937,1.3572,1.651;1)(0.7172,1.0756,1.3906,1.5940;0.9) \\
(0.2321,0.2222,0.3576,0.4368;1)(0.2275,0.2908,0.3243,0.3972;0.9)\ldots
\end{array}
\end{array}
$$

$$
\begin{array}{cc} P & PE \\
\begin{array}{c}
(3.3019,3.8058,4.8113,5.3133;1)(3.5540,4.3089,5.0623,5.3133;0.9) \\
(3.9149,4.4247,5.4387,5.9439;1)(4.1701,4.9324,5.6914,5.9439;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9) \\
(4.6416,5.1441,6.1479,6.6494;1)(4.8929,5.6462,6.3987,6.6494;0.9) \\
\ldots(2.8845,3.4020,4.4247,4.9324;1)(3.1440,3.9149,4.6788,4.9324;0.9)
\end{array} &
\begin{array}{c}
(0.1504,0.1627,0.1944,0.2154;1)(0.1565,0.1785,0.1865,0.2049;0.9) \\
(0.6057,0.7368,1.1587,1.5874;1)(0.6713,0.9485,1.1052,1.4057;0.9) \\
(0.1504,0.1627,0.1944,0.2154;1)(0.1565,0.1785,0.1865,0.2049;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9) \\
(0.4309,0.4979,0.6586,0.7631;1)(0.4646,0.5784,0.6488,0.7264;0.9)\ldots
\end{array}
\end{array}
$$

$$
\begin{array}{c} S-C \\
\left.\begin{array}{c}
(0.3029,0.3288,0.5623,0.7937;1)(0.3166,0.4456,0.5040,0.6786;0.9) \\
(2.2894,2.7967,3.8058,4.3089;1)(2.5434,3.3019,4.0574,4.3089;0.9) \\
(0.2027,0.2078,0.2939,0.3467;1)(0.2056,0.2518,0.2730,0.3204;0.9) \\
(1.3104,1.3963,2.0083,2.3208;1)(1.3561,1.7074,2.0019,2.2407;0.9) \\
(1,1,1,1;1)(1,1,1,1;0.9)
\end{array}\right]
\end{array}
\tag{6.50}
$$

The aggregated type-2 fuzzy pairwise comparison matrix of decision makers for criteria evaluation

$$Fuzzy\quad geometric\quad mean = \begin{array}{r} r(ES) \\ r(O) \\ r(P) \\ r(PE) \\ r(S-C) \end{array}\begin{bmatrix} (0.4735,0.5115,0.6423,0.7325;1)(0.4929,0.5791,0.6240,0.6953;0.9) \\ (1.8785,2.1302,2.6847,3.0333;1)(2.0058,2.4146,2.7413,2.9605;0.9) \\ (0.2494,0.2644,0.3207,0.3567;1)(0.2570,0.2932,0.3071,0.3389;0.9) \\ (1.7783,1.9654,2.5269,2.7912;1)(1.8768,2.2569,2.5785,2.7522;0.9) \\ (0.8167,0.9229,1.2385,1.4026;1)(0.8724,1.0855,1.2459,1.3627;0.9) \end{bmatrix} \tag{6.51}$$

The type-2 fuzzy geometric mean of decision makers for criteria evaluation

$$Normalised\quad fuzzy\quad weight = \begin{array}{r} w(ES) \\ w(O) \\ w(P) \\ w(PE) \\ w(S-C) \end{array}\begin{bmatrix} (0.1561,0.1686,0.2117,0.2415;1)(0.1625,0.1909,0.2057,0.2292;0.9) \\ (0.6193,0.7023,0.8851,1.0000;1)(0.6613,0.7960,0.9037,0.9760;0.9) \\ (0.0822,0.0872,0.1057,0.1176;1)(0.0847,0.0967,0.1012,0.1117;0.9) \\ (0.5863,0.6479,0.8330,0.9202;0.9)(0.6187,0.7440,0.85,0.9073;0.9) \\ (0.2693,0.3042,0.4083,0.4624;1)(0.2876,0.3579,0.4107,0.4492;0.9) \end{bmatrix} \tag{6.52}$$

The type-2 fuzzy geometric mean of decision makers for criteria evaluation

$$Defuzzified\quad fuzzy\quad weights = \begin{array}{r} w(ES) \\ w(O) \\ w(P) \\ wPE) \\ w(S-C) \end{array}\begin{bmatrix} 0.1959 \\ 0.8171 \\ 0.0984 \\ 0.7624 \\ 0.3683 \end{bmatrix} \tag{6.53}$$

The weight for each criteria

Then, the average and weights of each criterion from equation (6.53) are illustrated in Table 6.17 as follows:

Table 6. 17. The weights of criteria

Criteria	*Weight*	*New Weight*	*Rank*
ES	0.1959	0.087	4
O	0.8171	0.364	1
P	0.0984	0.044	5
PE	0.7624	0.34	2
S-C	0.3683	0.164	3
Total	2.2422	1	

These results of criteria's weights are implemented in following phase to evaluate for alternatives selection.

Phase 3: Fuzzy TOPSIS Evaluation for Alternatives Selection

Step 1: Obtain the weighting of evaluation criteria from fuzzy AHP evaluation.

The weights of evaluation criteria are employed from fuzzy AHP evaluation process before. Refer Table 6.17.

Step 2: Create fuzzy evaluation matrix for alternatives' evaluation.

The construction of fuzzy decision matrix for alternatives' evaluation are utilised linguistic terms by (Zheng et al., 2012) presented on Table 6.4. Fig. 6.33, 6.34 and 6.35 represent the pairwise comparison matrices for decision makers for alternatives evaluation. Then, aggregated results.

Step 3: Weighted normalised decision matrix can be obtained by multiplying normalised matrix with the weights of criteria.

The results of weighted normalised process are presented in equation (6.57).

$$r_{ij} = \begin{cases} \dfrac{r_{ij}}{r^{+}}, & a_i \in A_1 \\ \dfrac{r^{-}}{r_{ij}}, & a_i \in A_2 \end{cases} \tag{6.54}$$

Where A_1 denotes the set of benefit criteria, A_2 denotes the set of cost criteria, and $1 \le i \le m$.

Step 4: The fuzzy evaluation matrices are defuzzified.

The best nonfuzzy performance (BNP) based centre of area (COA) defuzzification method is utilised in order to handle interval type-2 fuzzy sets to find arithmetic mean of upper bound and lower bound.

$$\tilde{w}_j = \frac{\int xu(x)dx}{\int u(x)dx}$$

$$\tilde{w}_j = \frac{-w_{j1} \times w_{j2} + w_{j3} \times w_{j4} + 1/3(w_{j4} - w_{j3})^2 - 1/3(w_{j2} - w_{j1})^2}{-w_{j1} - w_{j2} + w_{j3} + w_{j4}} \qquad (6.55)$$

The results of defuzzified process are presented in equation (6.56) as follows.

$$Defuzzified = \begin{array}{c c} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} 0.0713 & 0.07135 & 0.0823 & 0.0495 \\ 0.331 & 0.3310 & 0.3432 & 0.2551 \\ 0.0428 & 0.0399 & 0.0428 & 0.0384 \\ 0.0822 & 0.17 & 0.255 & 0.0102 \\ 0.1246 & 0.1492 & 0.1601 & 0.104 \end{bmatrix} \end{array} \qquad (6.56)$$

The defuzzified values of type-2 fuzzy numbers

$$
Weighted \quad normalised \quad aggregated = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{cc} x1 & x2 \\ \left[\begin{array}{l} (0.0553,0.0728,0.0728,0.0845;1)(0.0641,0.0728,0.0728,0.0786;0.9) \\ (0.2794,0.3401,0.3401,0.3644;1)(0.3098,0.3401,0.3401,0.3523;0.9) \\ (0.0395,0.0439,0.0439,0.0439;1)(0.0417,0.0439,0.0439,0.0439;0.9) \\ (0.0227,0.0793,0.0793,0.1474;1)(0.0510,0.0793,0.0793,0.1133;0.9) \\ (0.0931,0.1259,0.1259,0.1533;1)(0.1095,0.1259,0.1259,0.1396;.9) \end{array}\right. & \begin{array}{l} (0.0553,0.0728,0.0728,0.0845;1)(0.0641,0.0728,0.0728,0.0786;0.9) \\ (0.2794,0.3401,0.3401,0.3644;1)(0.3098,0.3401,0.3401,0.3523;0.9) \\ (0.0337,0.0410,0.0410,0.0439;1)(0.0373,0.0410,0.0410,0.0424;0.9) \\ (0.1020,0.1700,0.1700,0.2380;1)(0.1360,0.1700,0.1700,0.2040;0.9) \\ (0.1259,0.1533,0.1533,0.1642;1)(0.1396,0.1533,0.1533,0.1588;0.9).. \end{array} \end{array}
$$

$$
\begin{array}{cc} x3 & x4 \\ \begin{array}{l} (0.0728,0.0845,0.0845,0.0874;1)(0.0786,0.0845,0.0845,0.0859;0.9) \\ (0.3037,0.3523,0.3523,0.3644;1)(0.3280,0.3523,0.3523,0.3584;0.9) \\ (0.0395,0.0439,0.0439,0.0439;1)(0.0417,0.0439,0.0439,0.0439;0.9) \\ (0.1927,0.2607,0.2607,0.3060;1)(0.2267,0.2607,0.2607,0.2834;0.9) \\ ...(01478,0.1642,0.1642,0.1642;1)(0.1560,0.1642,0.1642,0.1642;0.9) \end{array} & \left.\begin{array}{l} (0.0320,0.0495,0.0495,0.0670;1)(0.0408,0.0495,0.0495,0.0582;0.9) \\ (0.1822,0.2551,0.2551,0.3280;1)(0.2187,0.2551,0.2551,0.916;0.9) \\ (0.0307,0.0395,0.0395,0.0439;1)(0.0351,0.0395,0.0395,0.0417;0.9) \\ (0.03 \\ (0.0340,0.1020,0.1020,0.1700;1)(0.0680,0.1020,0.1020,0.1360;0.9) \\ (0.0712,0.1040,0.1040,0.1369;1)(0.0876,0.1040,0.1040,0.1204;0.9) \end{array}\right] \end{array} \tag{6.57}
$$

The weighted normalised aggregated pairwise comparison matrix for alternatives evaluation

Step 5: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

Referring to normalise trapezoidal fuzzy weights, the FPIS, A^{+} represents the compromise solution while FNIS, A^{-} represents the worst possible solution. The range belong to the closed interval [0,1]. The FPIS A^{+} (aspiration levels) and FNIS A^{-} (worst levels) as following below:

$$A^{+} = (1,1,1,1;1)(1,1,1,1;0.9) \qquad A^{-} = (-1,-1,-1,-1;1)(-1,-1,-1,-1;0.9)$$

Step 6: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^{+}$ and $\tilde{d}_i^{-}$ of each alternative from formulation A^{+} and A^{-} can be calculated by the area of compensation method:

$$d^{+}(p_j) = \sqrt{\sum_{i=1}^{n} (f_{i1} - v_i^{+})^2}$$

$$d^{+}(p_j) = \sqrt{(0.0713-1)^2 + (0.331-1)^2 + (0.0428-1)^2 + (0.0822-1)^2 + (0.1246-1)^2}$$

$$d^{+}(p_j) = 1.9583$$

$$d^{+}(p_j) = \sqrt{\sum_{i=1}^{n} (f_{i1} - v_i^{+})^2}$$

$$d^{-}(p_j) = \sqrt{(0.0713+1)^2 + (0.331+1)^2 + (0.0428+1)^2 + (0.0822+1)^2 + (0.1246+1)^2}$$

$$d^{+}(p_j) = 2.5382$$

Step 7: Find the closeness coefficient, CC_i and improve alternatives for achieving aspiration levels in each criteria.

The decision rules for five classes are depicted in Table 6.7. Notice that the highest CC_i value is used to determine the rank.

$$\overline{CC}_i = \frac{\overline{d}_i^{-}}{\overline{d}_i^{+} + \overline{d}_i^{-}} = 1 - \frac{\overline{d}_i^{+}}{\overline{d}_i^{+} + \overline{d}_i^{-}}$$

$$\overline{CC}_i = \frac{2.5544}{1.9472 + 2.5544}$$

$$\overline{CC}_i = 0.5674$$

Table 6. 7. Approval status table (Luukka, 2011)

***CCi* value**	**Assessment status**
$CC_i \in [0,0.2)$	Do not recommend
$CC_i \in [0.2,0.4)$	Recommend with high risk
$CC_i \in [0.4,0.6)$	Recommend with low risk
$CC_i \in [0.6,0.8)$	Approved
$CC_i \in [0.8,1]$	Approved and preferred

After several processes, referring to Table 6.18, the CC_i values shows candidate 3 represents the highest rank with 0.5872 followed by candidate 2 with 0.5754, candidate 1 with 0.5645 and candidate 4 with 0.5546 for the last ranked. The results reveal that the candidate 3 is most suitable for this recruitment post because based on approval status table, the score is in recommended with low risk range.

Table 6. 18. Closeness coefficients computation for type-2 fuzzy sets.

Alternative	**Closeness Coefficient, *CCi***
Candidate 1	0.5645 (Rank 3)
Candidate 2	0.5754 (Rank 2)
Candidate 3	0.5872 (Rank 1)
Candidate 4	0.5546 (Rank 4)

6.5 Hybrid Fuzzy Multi Criteria Decision Making for Z-Numbers

This section demonstrates computational process of proposed hybrid fuzzy MCDM model regarding case study of staff selection in MESSRS Saprudin, Idris & Co. for z-numbers. There is no comparative study for z-numbers. This is because the established hybrid fuzzy MCDM for z-numbers is not found in literature so far.

6.5.1 Consistent Fuzzy Preference Relations – Fuzzy Technique for Order of Preference by Similarity to Ideal Solution for Z-Numbers

Phase 1: Linguistic Evaluation

The decision makers used the linguistic terms that proposed by (Zheng et al., 2012) as shown in Table 5.1 in presenting the weights of criteria using consistent fuzzy preference evaluation using trapezoidal type-1 fuzzy sets. The linguistic terms with the crisp scale of relative important present the important of criteria preferences namely equally important (1), intermediate value (2), moderately more important (3), intermediate value (4), strongly more important (5), intermediate value (6), very strong more important (7), intermediate important (8) and extremely more important (9). For fuzzy TOPSIS evaluation, the linguistic terms and the corresponding of fuzzy numbers that proposed by (Zheng et al., 2012) is used to represent the evaluating values of the alternatives with respect to difference criteria with degree of confidence respectively. The scales consist of absolutely-low (1), very-low (2), low (3), fairly-low (4), medium (5), fairly-high (6), high (7), very-high (8) and absolutely-high (9). The linguistic scales for alternatives evaluation are depicted in Table 6.2 that are measure from 0 until 1. Both linguistic scales as mentioned are supported with reliability linguistic terms that is proposed by (Kang et al., 2012b) to represent z-numbers in measuring reliability for the first components. Reliability linguistic scales is presented in Table 6.19.

Table 6. 19. Reliability linguistic terms and their corresponding z-numbers (Kang et al., 2012b)

Linguistic Terms	Scale of reliability of crisp numbers	Generalised fuzzy numbers
Very-low (VL)	1	(0,0,0,0.25;1)
Low (L)	2	(0,25,0.25,0.5;1)
Medium (M)	3	(0.25,0.5,0.5,0.75;1)
High (H)	4	(0.5,0.75,0.75,1;1)
Very-high (VH)	5	(0.75,1,1,1;1)

Phase 2: Fuzzy Weights Evaluation using Consistent Fuzzy Preference Relations

Step 1: Construct a hierarchy structure.

The hierarchy model as shown in Fig. 5.1 in Section 6.3.1 which is illustrated the connection of criteria and alternatives, which are the candidates to be interviewed.

Step 2: Construct a pairwise comparison matrices.

The pairwise comparison matrices are constructed among all criteria in the dimension of the hierarchy systems based on the decision makers' preferences in phase 1 using equation (5.1). The linguistic evaluations of pairwise comparison matrices are based on regular numbers are depicted in equation (6.58), (6.59) and (6.60), then are translated into z-numbers using Table 6.1 (Section 6.3) and Table 6.19 for reliability component. The linguistic ratings of criteria fuzzy numbers – based given by DM1, DM2 and DM3 are shown in Fig. 6.54, 6.55 and Fig 6.56 respectively.

$$DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 1(VH) & 1/6(VH) & 5(VH) & 1/6(VH) & 1/2(H) \\ 6(VH) & 1(VH) & 5(VH) & 1/2(VH) & 3(VH) \\ 1/5(VH) & 1/5(VH) & 1(VH) & 1/6(VH) & 1/5(VH) \\ 6(VH) & 2(VH) & 6(VH) & 1(VH) & 4(VH) \\ 2(H) & 1/3(H) & 5(H) & 1/4(VH) & 1(VH) \end{bmatrix} \end{array} \tag{6.58}$$

Pairwise comparison matrix of criteria with reliability component from DM1

$$DM2 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 1(VH) & 1/5(VH) & 4(VH) & 1/6(VH) & 1/2(VH) \\ 5(VH) & 1(VH) & 6(VH) & 3(VH) & 3(VH) \\ 1/4(VH) & 1/6(VH) & 1(VH) & 1/6(VH) & 1/3(VH) \\ 6(H) & 1/3(VH) & 6(VH) & 1(VH) & 4(VH) \\ 2(VH) & 1/3(H) & 3(VH) & 1/4(H) & 1(VH) \end{bmatrix} \end{array} \tag{6.59}$$

Pairwise comparison matrix of criteria with reliability component from DM2

$$DM3 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 1(VH) & 1/5(VH) & 4(VH) & 1/5(VH) & 1/3(VH) \\ 5(VH) & 1(VH) & 4(VH) & 1/2(H) & 4(VH) \\ 1/4(H) & 1/4(VH) & 1(VH) & 1/5(VH) & 1/4(VH) \\ 5(VH) & 2(VH) & 5(VH) & 1(VH) & 1/3(VH) \\ 3(VH) & 1/4(VH) & 4(H) & 3(VH) & 1(VH) \end{bmatrix} \end{array} \tag{6.60}$$

Pairwise comparison matrix of criteria with reliability component from DM3

Step 3: Convert decision makers' preferences from z-numbers into type-1 fuzzy sets.

The pairwise comparison matrices for criteria's judgement of decision makers' (DM1, DM2, DM3) preferences as listed in equation (6.61), (6.62) and (6.63) are converted into regular fuzzy numbers which is type-1 fuzzy sets using equation (4.21). The results of conversion process of pairwise comparison matrix are shown in equation (6.64), (6.65) and (6.66).

$$
DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccc} ES & O & P \end{array} \\
\left[\begin{array}{ccc}
(1,1,1,1;1)(0.75,1,1,1;1) & (0.1429,0.1538,0.1818,0.2;1)(0.75,1,1,1;1) & (4,4.5,5.5,6;1)(0.75,1,1,1;1) \\
(5,5.5,6.5,7;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1) & (4,4.5,5.5,6;1)(0.75,1,1,1;1) \\
(0.1667,0.1818,0.2222,0.25;1)(0.75,1,1,1;1) & (0.1667,0.1818,0.2222,0.25;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1) \\
(5,5.5,6.5,7;1)(0.75,1,1,1;1) & (1,1.5,2.5,3;1)(0.75,1,1,1;1) & (5,5.5,6.5,7;1)(0.75,1,1,1;1) \\
(1,1.5,2.5,3;1)(0.5,0.75,0.75,1;1) & (0.25,0.22,0.4,0.5;1)(0.75,1,1,1;1) & (4,4.5,5.5,6;1)(0.5,0.75,0.75,1;1)\ldots
\end{array}\right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} PE & \end{array} \\
\left.\begin{array}{cc}
(0.1429,0.1538,0.1818,0.2;1)(0.75,1,1,1;1) & (0.3333,0.4,0.6667,1;1)(0.75,1,1,1;1) \\
(0.3333,0.4,0.6667,1;1)(0.75,1,1,1;1) & (2,2.5,3.5,4;1)(0.75,1,1,1;1) \\
(0.1429,0.1538,0.1818,0.2;1)(0.75,1,1,1;1) & (0.1667,0.1818,0.2222,0.25;1)(0.75,1,1,1;1) \\
(1,1,1,1;1)(1,1,1,1;0.9) & (3,3.5,4.5,5;1)(30.75,1,1,1;1) \\
\ldots(0.2,0.2222,0.2857,0.33;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1)
\end{array}\right]
\end{array}
\qquad (6.61)
$$

The z-numbers of pairwise comparison matrix of DM1 for criteria evaluation

$$
\begin{array}{cc}
 & \begin{array}{ccc} ES & O & P \end{array} \\
DM2 = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \left[\begin{array}{ccc}
(1,1,1,1;1)(0.75,1,1,1;1) & (0.1667,0.1818,,0.2222,0.25;1)(0.75,1,1,1;1) & (3,3.5,4.5,5;1)(0.75,1,1,1;1) \\
(4,4.5,5.5,6;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1) & (5,5.5,6.5,7;1)(0.75,1,1,1;1) \\
(0.2,0.2222,0.2857,0.3333;1)(0.75,1,1,1;1) & (0.1429,0.1538,0.1818,0.2;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1) \\
(5,5.5,6.5,7;1)(0.5,0.75,0.75,1;1) & (0.25,0.22,0.4,0.5;1)(0.75,1,1,1;1) & (5,5.5,6.5,7;1)(0.75,1,1,1;1) \\
(1,1.5,2.5,3;1)(0.75,1,1,1;1) & (0.25,0.22,0.4,0.5;1)(0.5,0.75,0.75,1;1) & (2,2.5,3.5,4;1)(0.75,1,1,1;1)\ldots
\end{array} \right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} PE & S-C \end{array} \\
\left. \begin{array}{cc}
(0.1429,0.1538,0.1818,0.2;1)(0.75,1,1,1;1) & (0.3333,0.4,0.6667,1;1)(0.75,1,1,1;1) \\
(2,2.5,3.5,4;1)(0.75,1,1,1;1) & (2,2.5,3.5,4;1)(0.75,1,1,1;1) \\
(0.1429,0.1538,0.1818,0.2;1)(0.75,1,1,1;1) & (0.25,0.22,0.4,0.5;1)(0.75,1,1,1;1) \\
(1,1,1,1;1)(1,1,1,1;0.9) & (3,3.5,4.5,5;1)(30.75,1,1,1;1) \\
\ldots(0.2,0.2222,0.2857,0.33;1)(0.5,0.75,0.75,1;1) & (1,1,1,1;1)(0.75,1,1,1;1)
\end{array} \right]
\end{array}
\tag{6.62}
$$

The z-numbers pairwise comparison matrix of DM2 for criteria evaluation

$$
DM3 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccc} ES & O & P \end{array} \\
\left[\begin{array}{ccc}
(1,1,1,1;1)(0.75,1,1,1;1) & (0.1667,0.1818,,0.2222,0.25;1)(0.75,1,1,1;1) & (3,3.5,4.5,5;1)(0.75,1,1,1;1) \\
(4,4.5,5.5,6;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1) & (3,3.5,4.5,5;1)(0.75,1,1,1;1) \\
(0.2,0.2222,0.2857,0.3333;1)(0.5,0.75,0.75,1;1) & (0.2,0.2222,0.2857,0.333;1)(0.75,1,1,1;1) & (1,1,1,1;1)(0.75,1,1,1;1) \\
(4,4.5,5.5,6;1)(0.75,1,1,1;1) & (1,1.5,2.5,3;1)(0.75,1,1,1;1) & (4,4.5,5.5,6;1)(0.75,1,1,1;1) \\
(2,2.5,3.5,4;1)(0.75,1,1,1;1) & (0.2,0.2222,0.2857,0.333;1)(0.75,1,1,1;1) & (3,3.5,4.5,5;1)(0.5,0.75,0.75,1;1)\ldots
\end{array}\right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} PE & S-C \end{array} \\
\left.\begin{array}{cc}
(0.1667,0.1818,0.2222,0.25;1)(0.75,1,1,1;1) & (0.25,0.22,0.4,0.5;1)(0.75,1,1,1;1) \\
(0.3333,0.4,0.6667,1;1)(0.5,0.75,0.75,1;1) & (3,3.5,4.5,5;1)(30.75,1,1,1;) \\
(0.1667,0.1818,0.2222,0.25;1)(0.75,1,1,1;1) & (0.2,0.2222,0.2857,0.3333;1)(0.75,1,1,1;1) \\
(1,1,1,1;1)(1,1,1,1;0.9) & (0.25,0.22,0.4,0.5;1)(0.75,1,1,1;11) \\
.\ \ ..(2,2.5,3.5,41)(0.5,0.75,0.75,1;1) & (1,1,1,1;1)(0.75,1,1,1;1)
\end{array}\right]
\end{array}
\quad (6.63)
$$

The z-numbers of pairwise comparison matrix of DM3 for criteria evaluation

$$
DM1 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\left[\begin{array}{ccccc}
(0.986,0.986,0.986,0.986;1) & (0.1409,0.1517,0.1793,0.1972;1) & (3.944,4.437,5.423,5.9161;1) & (0.1409,0.1517,0.1793,0.1972;1) & (0.2887,0.3464,0.5774,0.866;,1) \\
(4.9301,5.4231,6.4091,6.9021;1) & (0.986,0.986,0.986,0.986;1) & (3.944,4.4371,5.423,5.916;1) & (0.3287,0.3944,0.6573,0.9860;1) & (1.972,2.465,3.451,3.944;1) \\
(0.1643,0.1793,0.2191,0.2465;1) & (0.16,0.1793,0.2191,0.2465;1) & (0.986,0.986,0.986,0.986;1) & (0.1409,0.1517,0.1793,0.1972;1) & (0.1643,0.1793,0.2191,0.2465;1) \\
(4.9301,5.4231,6.4091,6.9021;1) & (0.986,1.479,2.465,2.958;1) & (4.930,5.423,6.409,6.902;1) & (0.986,0.986,0.986,0.986;1) & (2.958,3.451,4.437,4.930;1) \\
(0.866,1.299,2.1651,2.5981;1) & (0.2157,0.1917,0.3451,0.4314;1) & (3.464,3.8971,4.763,5.196;1) & (0.1972,0.2191,0.2817,0.2817,0.3287;1) & (0.986,0.986,0.986,0.98;1)
\end{array}\right]
\end{array}
\tag{6.64}
$$

The fuzzy pairwise comparison matrix of DM1 for criteria evaluation after conversion process

$$
DM2 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\left[\begin{array}{ccccc}
(0.986,0.986,0.986,0.986;1) & (0.1643,0.1793,0.2191,0.2465;1) & (2.958,3.451,4.4371,4.9301;1) & (0.1409,0.1517,0.1793,0.1972;1) & (0.3287,0.3944,0.6573,0.986;,1) \\
(3.944,4.437,5.423,5.9161;1) & (0.986,0.986,0.986,0.986;1) & (4.9301,5.4231,6.4091,6.9021;1) & (1.972,2.465,3.451,3.9144;1) & (1.972,2.465,3.451,3.944;1) \\
(0.1972,0.2191,0.22817,0.3287;1) & (0.1409,0.1517,0.1793,0.1972;1) & (0.986,0.986,0.986,0.986;1) & (0.1409,0.1517,0.1793,0.1972;1) & (0.2465,0.2191,0.3944,0.493;1) \\
(4.3301,4.7631,5.6292,6.0622;1) & (0.2465,0.2191,0.3944,0.493;1) & (4.9301,5.4231,6.4091,6.9021;1) & (0.986,0.986,0.986,0.986;1) & (2.958,3.451,4.4371,4.9301;1) \\
(0.9860,1.4790,2.4650,2.958;1) & (0.2165,0.1925,0.3464,0.4330;1) & (1.972,2.465,3.451,3.9441;1) & (0.1732,0.125,0.2474,0.2887;1) & (0.986,0.986,0.986,0.986;1)
\end{array}\right]
\end{array}
\tag{6.65}
$$

The fuzzy pairwise comparison matrix of DM2 for criteria evaluation after conversion process

$$
DM3 = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\
\left[\begin{array}{ccccc}
(0.986,0.986,0.986,0.986;1) & (0.1643,0.1793,0.2191,0.2465;1) & (2.958,3.451,4.437,4.930;1) & (0.1643,0.1793,0.2191,0.2465;1) & (0.2465.0.2191.0.3944.0.4930;1) \\
(3.9441,4.4371,5.4231,5.9161;1) & (0.986,0.986,0.986,0.986;1) & (2.958,3.451,4.437,4.930;1) & (0.2887,0.3464,0.5774,0.8660;1) & (2.958,3.451,4.4371,4.9301;1) \\
(0.1732,0.1925,0.2474,0.2887;1) & (0.1972,0.2191,0.2817,0.3287;1) & (0.986,0.986,0.986,0.986;1) & (0.1643,0.1793,0.2191,0.2465;1) & (0.1972,0.2191,0.2817,0.3287;1) \\
(3.9441,4.4371,5.4231,5.9161;1) & (0.986,1.479,2.465,2.958;1) & (3.9441,4.4371,5.4231,5.9161;1) & (0.986,0.986,0.986,0.986;1) & (0.2465,0.2191,0.3944,0.4930;1) \\
(1.9720,2.465,3.451,3.9441;1) & (0.1972,0.2191,0.2817,0.3287;1) & (2.5981,3.0311,3.8971,4.3301;1) & (1.972,2.465,3.451,3.9441;1) & (0.986,0.986,0.986,0.98;1)
\end{array}\right]
\end{array}
\tag{6.66}
$$

The fuzzy pairwise comparison matrix of DM3 for criteria evaluation after conversion process

Step 4: Aggregate the decision makers' preferences.

The converted pairwise comparison matrices for criteria's judgement of decision makers' (DM1, DM2, DM3) preferences as listed in equation (6.64), (6.65) and (6.66) are aggregated using equation (5.2). The result of aggregated pairwise comparison matrix is shown in equation (6.68) on next page.

$$\tilde{a}_{ij} = (\tilde{a}_{ij}^{1} \times \tilde{a}_{ij}^{2} \times \ldots \times \tilde{a}_{ij}^{n})^{1/k}$$

where k is the number of decision makers and $i=1,2,\ldots m;\ j=1,2,\ldots n$.

Step 5: Defuzzify the trapezoidal fuzzy numbers of aggregation's result of decision makers' preferences

The aggregation's result of decision maker's preferences are defuzzify using intuitive multiple centroid for z-numbers using equation (4.30) is presented in equation (6.67).

$$IVC(\tilde{x}_{\tilde{A},\tilde{R}}, \tilde{y}_{\tilde{A},\tilde{R}}) = \left(\frac{2(\sqrt{\varphi}a_1 + \sqrt{\varphi}a_4) + 7(\sqrt{\varphi}a_2 + \sqrt{\varphi}a_3)}{18}, \frac{7h_{\tilde{A},\tilde{R}}}{18} \right)$$

$$Defuzzify = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array} \begin{array}{c} \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{bmatrix} 0.986 & 0.1884 & 4.2481 & 0.1770 & 0.4423 \\ 5.2387 & 0.986 & 4.8622 & 0.9262 & 3.2546 \\ 0.2243 & 0.2036 & 0.986 & 0.1770 & 0.2526 \\ 5.3313 & 1.0747 & 5.567 & 0.986 & 1.7033 \\ 2.1587 & 0.2708 & 3.5379 & 0.55 & 0.986 \end{bmatrix} \end{array} \qquad (6.67)$$

Defuzzification results of aggregated matrix comparison

$$
Aggregated = \begin{array}{c} \\ ES \\ O \\ P \\ PE \\ S-C \end{array}
\begin{array}{c}
\begin{array}{ccc} ES & O & P \end{array} \\
\left[\begin{array}{ccc}
(0.986,0.986,0.986,0.986;1) & (0.1561,0.1696,0.2049,0.2288;1) & (3.2557,3.7526,4.744,5.239;1) \\
(4.2486,4.744,5.7336,6.228;1) & (0.986,0.986,0.986,0.986;1) & (3.8601,4.3628,5.3626,5.8608;1) \\
(4.3829,4.8575,5.8054,6.2789;1) & (0.1659,0.1813,0.2228,0.2519;1) & (0.986,0.986,0.986,0.986;1) \\
(4.3829,4.8575,5.8054,6.2789;1) & (0.6211,0.7826,1.3382,1.6279;1) & (4.5767,5.0722,6.062,6.5564;1) \\
(1.1897,1.6793,2.6409,3.1179;1) & (0.2096,0.2007,0.3229,0.3945;1) & (2.6085,3.0765,4.0013,4.4604;1)\ldots
\end{array}\right.
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cc} PE & S-C \end{array} \\
\left.\begin{array}{cc}
(0.1483,0.1604,0.1917,0.2124;1) & (0.286,0.3105,0.531,0.7495;1) \\
(0.5720,0.6957,1.0941,1.4989;1) & (2.2574,2.7576,3.7526,4.2486;1) \\
(0.1483,0.1604,0.1917,0.2124;1) & (0.1999,0.2049,0.2898,0.3418;1) \\
(0.986,0.986,0.986,0.986;1) & (1.292,1.3768,1.9802,2.2883;1) \\
\ldots(0.4069,0.4702,0.6219,0.7206;1) & (0.986,0.986,0.986,0.986;1)
\end{array}\right]
\end{array}
\quad (6.68)
$$

The aggregated fuzzy pairwise comparison matrix for criteria evaluation after conversion process

Step 5: Compute the criteria values as weights for alternatives' evaluation using consistent fuzzy preference relations

The weights of aggregated matrix comparison of criteria are calculated using consistent preference relations which based on additive transitivity property using equation (4.35-4.40) in equation (6.69) below.

$$Fuzzy\ \ weights = \begin{array}{c|c} & \begin{array}{ccccc} ES & O & P & PE & S-C \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} 0.5 & 0.1202 & 0.4801 & 0.086 & 0.2072 \\ 0.8798 & 0.5 & 0.8599 & 0.4658 & 0.587 \\ 0.5199 & 0.1401 & 0.5 & 0.1059 & 0.2271 \\ 0.914 & 0.5342 & 0.9412 & 0.5 & 0.6212 \\ 0.7928 & 0.4130 & 0.7729 & 0.3788 & 0.5 \end{bmatrix} \end{array} \tag{6.69}$$

The consistent fuzzy preference relations matrix for criteria

By having five criteria, $n = 5$ so only $(n-1) = 5-1 = 4$ entry values $(p_{12}, p_{23}, p_{34}\ \ and\ \ p_{45})$ are required in constructing the consistent fuzzy preference relations matrix from equation (6.67) where:

$$p_{12} = \frac{1}{2}(1 + \log_9 0.1884) = 0.1202$$

$$p_{23} = \frac{1}{2}(1 + \log_9 4.8622) = 0.8599$$

$$p_{34} = \frac{1}{2}(1 + \log_9 0.1770) = 0.1059$$

$$p_{45} = \frac{1}{2}(1 + \log_9 1.7033) = 0.6212$$

The remains of the entries can be determine by utilizing Proposition 2 and 3 presented as follows.

$$p_{21} = 1 - p_{12} = 1 - 0.1202 = 0.8798$$

$$p_{32} = 1 - p_{23} = 1 - 0.8599 = 0.1401$$

$$p_{43} = 1 - p_{34} = 1 - 0.1059 = 0.8941$$

$$p_{54} = 1 - p_{45} = 1 - 0.6212 = 0.0.3788$$

$$p_{31} = \frac{3}{2} - p_{12} - p_{23} = \frac{3}{2} - 0.1202 - 0.8599 = 0.5199$$

$$p_{42} = \frac{3}{2} - p_{23} - p_{34} = \frac{3}{2} - 0.8599 - 0.1059 = 0.5342$$

$$p_{53} = \frac{3}{2} - p_{34} - p_{45} = \frac{3}{2} - 0.1059 - 0.6212 = 0.7729$$

$$p_{41} = \frac{j-i+1}{2} - p_{12} - p_{23} - p_{34} = \frac{4-1+1}{2} - 0.1202 - 0.8599 - 0.1059 = 0.914$$

$$p_{51} = \frac{j-i+1}{2} - p_{12} - p_{23} - p_{34} - p_{45} = \frac{5-1+1}{2} - 0.1202 - 0.8599 - 0.1059 - 0.6212 = 0.7928$$

$$p_{52} = \frac{j-i+1}{2} - p_{23} - p_{34} - p_{45} = \frac{5-2+1}{2} - 0.8599 - 0.1059 - 0.6210 = 0.413$$

$$p_{13} = 1 - p_{31} = 1 - 0.5199 = 0.4801$$

$$p_{14} = 1 - p_{41} = 1 - 0.914 = 0.086$$

$$p_{15} = 1 - p_{51} = 1 - 0.7928 = 0.2072$$

$$p_{24} = 1 - p_{42} = 1 - 0.5342 = 0.4658$$

$$p_{25} = 1 - p_{52} = 1 - 0.413 = 0.587$$

$$p_{35} = 1 - p_{53} = 1 - 0.7729 = 0.2271$$

Then, the average and weights of each criterion from equation (6.69) are illustrated in Table 6.10 as follows.

Table 6. 20. The average and weightage of criteria

Criteria	*ES*	*O*	*P*	*PE*	*S-C*	*Average*	*Weights*	*Rank*
ES	0.5	0.1202	0.4801	0.086	0.2072	0.2787	0.1115	5
O	0.8798	0.5	0.8599	0.4658	0.587	0.6585	0.2634	2
P	0.5199	0.1401	0.5	0.1059	0.2271	0.2986	0.1195	4
PE	0.914	0.5342	0.8941	0.5	0.6212	0.6927	0.2771	1
S-C	0.7928	0.413	0.7729	0.3788	0.5	0.5715	0.2286	3
Total						2.5	1	

These results of criteria's weightage are implemented in following phase to evaluate for alternatives selection.

Phase 3: Fuzzy TOPSIS Evaluation for Alternatives Selection

Step 1: Determine the weights of evaluation criteria.

The weights of evaluation criteria are employed from consistent fuzzy preference relations process before. Refer Table 6.20.

Step 2: Construct the fuzzy decision matrix for alternatives' evaluation using fuzzy TOPSIS.

The construction of fuzzy decision matrix for alternatives' evaluation are utilised linguistic terms by (Zheng et al., 2012) presented on Table 6.21 and Table 6.22.

Step 3: Convert decision makers' preferences from z-numbers into type-1 fuzzy sets.

The pairwise comparison matrices for criteria's judgement of decision makers' (DM1, DM2, DM3) preferences as listed in equation (6.70), (6.71) and (6.72) are converted into regular fuzzy numbers which is type-1 fuzzy sets using equation (4.21). The results of conversion process of pairwise comparison matrix are shown in equation (6.73), (6.74) and (6.75).

Table 6. 21. Evaluating linguistic terms of the alternatives with reliability components given by the decision makers with respect to different criteria

Criteria	Alternatives/ Candidates	Decision Maker		
		DM1	***DM2***	***DM3***
Emotional Steadiness	*x1*	FH (VH)	H (VH)	VH (H)
	x2	H (H)	H (VH)	FH (VH)
	x3	VH (VH)	H (VH)	VH (VH)
	x4	M (VH)	FH (VH)	M (VH)
Oral	*x1*	VH (VH)	H (VH)	VH (VH)
	x2	H (VH)	H (H)	VH (VH)
	x3	VH (VH)	VH (VH)	H (VH)
	x4	FH (H)	M (VH)	FH (VH)
Personality	*x1*	VH (VH)	VH (VH)	VH (VH)
	x2	H (VH)	H (VH)	VH (VH)
	x3	VH (VH)	VH (VH)	VH (VH)
	x4	H (VH)	H (VH)	H (VH)
Past Experience	*x1*	FL (VH)	L (VH)	FL (VH)
	x2	M (VH)	M (H)	M (VH)
	x3	H (H)	M (VH)	H (VH)
	x4	FL (VH)	FL (VH)	FL (VH)
Self-Confidence	*x1*	H (VH)	FH (VH)	FH (VH)
	x2	VH (H)	H (VH)	H (H)
	x3	VH (VH)	VH (VH)	VH (VH)
	x4	M (VH)	FH (VH)	FH (VH)

Table 6. 22. Evaluating values of the alternatives with reliability components given by the decision makers with respect to different criteria

Criteria	Alternatives/ Candidates	Decision Maker 1		Decision Maker 2
		DM1		DM2
Emotional Steadiness	*x1*	(0.58 0.63 0.80 0.86; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x2	(0.72 0.78 0.92 0.97; 1.00)	(0.50 0.75 0.75 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x4	(0.32 0.41 0.58 0.65; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)
Oration	*x1*	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x2	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x4	(0.58 0.63 0.80 0.86; 1.00)	(0.50 0.75 0.75 1.00; 1.00)	(0.32 0.41 0.58 0.65; 1.00)
Personality	*x1*	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x2	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.50 0.75 0.75 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x4	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
Past Experience	*x1*	(0.17 0.22 0.36 0.42; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.04 0.10 0.18 0.23; 1.00)
	x2	(0.32 0.41 0.58 0.65; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.32 0.41 0.58 0.65; 1.00)
	x3	(0.72 0.78 0.92 0.97; 1.00)	(0.50 0.75 0.75 1.00; 1.00)	(0.32 0.41 0.58 0.65; 1.00)
	x4	(0.17 0.22 0.36 0.42; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.17 0.22 0.36 0.42; 1.00)
Self-Confidence	*x1*	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)
	x2	(0.93 0.98 1.00 1.00; 1.00)	(0.50 0.75 0.75 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)
	x3	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)
	x4	(0.32 0.41 0.58 0.65; 1.00)	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)

Table 6. 22. Evaluating values of the alternatives given by the decision makers with respect to different criteria (cont.)

Criteria	Alternatives/ Candidates	Decision Maker 2 DM2		Decision Maker 3 DM3
Emotional Steadiness	*x1*	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.50 0.75 0.75 1.00; 0.90)
	x2	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x3	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x4	(0.75 1.00 1.00 1.00; 1.00)	(0.32 0.41 0.58 0.65; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
Oration	*x1*	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x2	(0.50 0.75 0.75 1.00; 0.90)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x3	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x4	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
Personality	*x1*	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x2	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x3	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x4	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
Past Experience	*x1*	(0.75 1.00 1.00 1.00; 1.00)	(0.10 0.30 0.30 0.50; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x2	(0.50 0.75 0.75 1.00; 0.90)	(0.32 0.41 0.58 0.65; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x3	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x4	(0.75 1.00 1.00 1.00; 1.00)	(0.10 0.30 0.30 0.50; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
Self-Confidence	*x1*	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x2	(0.75 1.00 1.00 1.00; 1.00)	(0.72 0.78 0.92 0.97; 1.00)	(0.50 0.75 0.75 1.00; 0.90)
	x3	(0.75 1.00 1.00 1.00; 1.00)	(0.93 0.98 1.00 1.00; 1.00)	(0.75 1.00 1.00 1.00; 1.00)
	x4	(0.75 1.00 1.00 1.00; 1.00)	(0.58 0.63 0.80 0.86; 1.00)	(0.75 1.00 1.00 1.00; 1.00)

Step 3: Fuzzy decision matrix is weighted using equation (4.43) and normalised each generalised fuzzy numbers into standardised generalised fuzzy numbers using (Zuo et al., 2013).

Equation (6.73), (6.74) and (6.75) represent the conversion of z-numbers into regular fuzzy numbers of pairwise comparison matrices of decision makers for alternatives evaluation. Then, aggregated result is depicted in equation (6.76). The weighted fuzzy normalised decision matrix is computed using equation (5.12) (Zuo et al., 2013). The results of weighted and normalisation process are presented in equation (6.77) and (6.78) respectively. Defuzzify the standardised generalised fuzzy numbers using proposed intuitive multiple centroid method using equation (4.30) that is depicted in equation (6.79), then translate them into the index point proposed by (Yong & Qi, 2005) as presented in equation (6.80), then do the average computational process depicted in equation (6.81).

Step 4: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

Referring to normalise trapezoidal fuzzy weights, the FPIS, A^{+} represents the compromise solution while FNIS, A^{-} represents the worst possible solution. The range belong to the closed interval [0,1]. The FPIS A^{+} (aspiration levels) and FNIS A^{-} (worst levels) as following below:

$$A^{+} = (1,1,1,1;1) \qquad A^{-} = (-1,-1,-1,-1;1)$$

The FPIS, A^{+} and FNIS, A^{-} can be obtained by centroid method for $(x_{A^{+}}, y_{A^{+}})$ and $(x_{A^{-}}, y_{A^{-}})$.

$$DM1 = \begin{array}{cc} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} (0.58,0.63,0.8,0.86;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.5,0.75,0.75,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) \\ (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.58,0.63,0.8,0.86;1)(0.5,0.75,0.751;1) \\ (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.78,0.92,0.97;1)(0.75,1,1,1;1) \\ (0.17,0.22,0.36,0.42;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) & (0.72,0.78,0.92,0.97;1)(0.5,0.75,0.75,1;1) & (0.17,0.22,0.36,0.42;1)(0.75,1,1,1;1) \\ (0.72,0.78,0.92,0.97;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.5,0.75,0.75,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) \end{bmatrix} \end{array} \tag{6.70}$$

The z-numbers of pairwise comparison matrix of DM1 for alternatives evaluation

$$DM2 = \begin{array}{cc} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.58,0.63,0.8,0.86;1)(0.75,1,1,1;1) \\ (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.5,0.75,0.75,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) \\ (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.78,0.92,0.97;1)(0.75,1,1,1;1) \\ (0.04,0.1,0.18,0.23;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.5,0.75,0.75,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) & (0.17,0.22,0.36,0.42;1)(0.75,1,1,1;1) \\ (0.58,0.63,0.80,0.86;1)(0.75,1,1,1;1) & (0.72,0.798,0.92,0.97;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.58,0.63,0.8,0.86;1)(0.75,1,1,1;1) \end{bmatrix} \end{array} \tag{6.71}$$

The z-numbers of pairwise comparison matrix of DM2 for alternatives evaluation

$$DM3 = \begin{array}{cc} & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\ \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix} (0.93,0.98,1,1;1)(0.5,0.75,0.75,1;1) & (0.58,0.63,0.80,0.86;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) \\ (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.78,0.92,0.97;1)(0.75,1,1,1;1) & (0.58,0.63,0.8,0.86;1)(0.75,1,1,1;1) \\ (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.72,0.78,0.92,0.97;1)(0.75,1,1,1;1) \\ (0.17,0.22,0.36,0.42;1)(0.75,1,1,1;1) & (0.32,0.41,0.58,0.65;1)(0.75,1,1,1;1) & (0.72,0.78,0.92,0.97;1)(0.75,1,1,1;1) & (0.17,0.22,0.36,0.42;1)(0.75,1,1,1;1) \\ (0.58,0.63,0.80,0.86;1)(0.75,1,1,1;1) & (0.93,0.98,1,1;1)(0.5,0.75,0.75,1;1) & (0.93,0.98,1,1;1)(0.75,1,1,1;1) & (0.58,0.63,0.8,0.86;1)(0.75,1,1,1;1) \end{bmatrix} \end{array} \tag{6.72}$$

The z-numbers of pairwise comparison matrix of DM3 for alternatives evaluation

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
DM1 = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.5719,0.6212,0.7888,0.8480;1) & (0.6235,0.6755,0.7967,0.84;1) & (0.917,0.9663,0.986,0.986;1) & (0.3155,0.4043,0.5719,0.6409;1) \\
(0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.917,0.9663,0.986,0.986;1) & (0.5023,0.5456,0.6928,0.7448;1) \\
(0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) \\
(0.1676,0.2169,0.355,0.4141;1) & (0.3155,0.4043,0.5719,0.6409;1) & (0.6235,0.6755,0.7967,0.84;1) & (0.1676,0.2169,0.355,0.4141;1) \\
(0.7099,0.7691,0.9071,0.9564;1) & (0.8054,0.8487,0.866,0.866;1) & (0.917,0.9663,0.986,0.986;1) & (0.3155,0.4043,0.5719,0.6409;1)
\end{bmatrix}
\end{array}
\tag{6.73}
$$

The fuzzy pairwise comparison matrix of DM1 for alternatives evaluation after conversion process

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
DM2 = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.7099,0.7691,0.9071,0.9564;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.5719,0.6212,0.7888,0.848;1) \\
(0.7099,0.7691,0.9071,0.9564;1) & (0.6235,0.6755,0.7976,0.84;1) & (0.917,0.9663,0.986,0.986;1) & (0.3155,0.4043,0.5719,0.6409;1) \\
(0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) \\
(0.0394,0.0986,0.1775,0.2268;1) & (0.2771,0.3551,0.5023,0.5629;1) & (0.3155,0.4043,0.5719,0.6409;1) & (0.1676,0.2169,0.355,0.4141;1) \\
(0.5719,0.6212,0.7888,0.848;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.917,0.9663,0.986,0.986;1) & (0.5719,0.6212,0.7888,0.848;1)
\end{bmatrix}
\end{array}
\tag{6.74}
$$

The fuzzy pairwise comparison matrix of DM2 for alternatives evaluation after conversion process

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
DM3 = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.8054,0.8487,0.866,0.866;1) & (0.5719,0.6212,0.7888,0.848;1) & (0.917,0.9663,0.986,0.986;1) & (0.3155,0.4043,0.5719,0.6409;1) \\
(0.917,0.9663,0.986,0.986;1) & (0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.5719,0.6212,0.7888,0.848;1) \\
(0.917,0.9663,0.986,0.986;1) & (0.917,0.9663,0.986,0.986;1) & (0.917,0.9663,0.986,0.986;1) & (0.7099,0.7691,0.9071,0.9564;1) \\
(0.1676,0.2169,0.355,0.4141;1) & (0.3155,0.4043,0.5719,0.6409;1) & (0.7099,0.7691,0.9071,0.9564;1) & (0.1676,0.2169,0.355,0.4141;1) \\
(0.5719,0.6212,0.7888,0.848;1) & (0.6235,0.6755,0.7967,0.840;1) & (0.9170,0.9663,0.986,0.986;1) & (0.5719,0.6212,0.7888,0.848;1)
\end{bmatrix}
\end{array}
\tag{6.75}
$$

The fuzzy pairwise comparison matrix of DM3 for alternatives evaluation after conversion process

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Aggregated = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.6889,0.7401,0.8526,0.8889;1) & (0.6326,0.6859,0.8292,0.8799;1) & (0.8420,0.8955,0.9590,0.9761;1) & (0.3847,0.4665,0.6366,0.7036;1) \\
(0.8420,0.8955,0.9590,0.9761;1) & (0.7404,0.7948,0.8932,0.9253;1) & (0.8420,0.8955,0.9590,0.9761;1) & (0.5188,0.5944,0.7449,0.8015;1) \\
(0.9170,0.9663,0.9860,0.9860;1) & (0.7732,0.8299,0.5477,0.6138;1) & (0.9170,0.9663,0.9860,0.9860;1) & (0.7099,0.7691,0.9071,0.9564;1) \\
(0.1035,0.1668,0.2817,0.3388;1) & (0.3022,0.3872,0.5477,0.6138;1) & (0.5188,0.5944,0.7449,0.8015;1) & (0.1676,0.2169,0.3550,0.4141;1) \\
(0.6146,0.6670,0.8264,0.8827;1) & (0.7091,0.7611,0.8554,0.8861;1) & (0.9170,0.9663,0.986,0.986;1) & (0.469,0.5383,0.7086,0.7724;1)
\end{bmatrix}
\end{array}
\tag{6.76}
$$

The aggregated fuzzy pairwise comparison matrix for alternatives evaluation after conversion process

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Weighted = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.0768,0.0825,0.095,0.0991;1) & (0.0705,0.0765,0.0924,0.0981;1) & (0.0939,0.0998,0.1069,0.1088;1) & (0.0429,0.052,0.071,0.0784;1) \\
(0.2218,0.2359,0.2526,0.2571;1) & (0.195,0.2093,0.2353,0.2437;1) & (0.2218,0.2359,0.2526,0.2571;1) & (0.1367,0.1484,0.1885,0.2026;1) \\
(0.1059,0.1154,0.1178,0.1178;1) & (0.0924,0.0991,0.1114,0.1154;1) & (0.1095,0.1154,0.1178,0.1178;1) & (0.0848,0.0919,0.1084,0.1143;1) \\
(0.0287,0.0462,0.0781,0.0939;1) & (0.0837,0.1073,0.1517,0.1701;1) & (0.1438,0.1647,0.2064,0.2221;1) & (0.0464,0.0601,0.0984,0.1147;1) \\
(0.1405,0.1525,0.1889,0.2018;1) & (0.1621,0.174,0.1955,0.2026;1) & (0.2096,0.2209,0.2254,0.2254;1) & (0.1072,0.1231,0.162,0.1766;1)
\end{bmatrix}
\end{array}
\tag{6.77}
$$

The weighted fuzzy pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Normalised = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} &
\begin{bmatrix}
(0.2107,0.2357,0.2905,0.3083;1) & (0.1832,0.2092,0.2791,0.3039;1) & (0.2854,0.3115,0.3425,0.3508;1) & (0.0622,0.1021,0.1851,0.2178;1) \\
(8454,0.9071,0.9803,1;1) & (0.7283,0.7910,0.9045,0.9415;1) & (0.8454,0.9071,0.9803,1;1) & (0.4727,0.5243,0.6996,0.7615;1) \\
(0.354,0.3798,0.3901,0.3901;1) & (0.2788,0.3085,0.3622,0.3797;1) & (0.354,0.3798,0.3901,0.3901;1) & (0.2457,0.2767,0.3488,0.3746;1) \\
(0,0.0768,0.2162,0.2855;1) & (0.241,0.3441,0.5388,0.619;1) & (0.5038,0.5955,0.778,0.8467;1) & (0.0778,0.1376,0.305,3768;1) \\
(0.4896,0.542,0.7015,0.7578;1) & (0.5841,0.6362,0.7305,0.76131) & (0.7922,0.8415,0.8612,0.8612;1) & (0.3439,0.4132,0.5836,0.6475;1)
\end{bmatrix}
\end{array}
\tag{6.78}
$$

The normalised fuzzy pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Defuzzifiied = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix}
x=0.2623,\ y=0.3889 & x=0.2440,\ y=0.3889 & x=0.3250,\ y=0.3889 & x=0.1428,\ y=0.3889 \\
x=0.939,\ y=0.3889 & x=0.8449,\ y=0.3889 & x=0.939,\ y=0.3889 & x=0.6131,\ y=0.3889 \\
x=0.3821,\ y=0.3889 & x=0.3340,\ y=0.3889 & x=0.3821,\ y=0.3889 & x=0.3122,\ y=0.3889 \\
x=0.1457\ \ y=0.3889 & x=0.4389,\ y=0.3889 & x=0.6842,\ y=0.3889 & x=0.2227,\ y=0.3889 \\
x=0.6222,\ y=0.3889 & x=0.6810,\ y=0.3889 & x=0.8459,\ y=0.3889 & x=0.4978,\ y=0.3889
\end{bmatrix}
\end{array}
\tag{6.79}
$$

The defuzzified pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Translate\ \ defuzzifiied = \begin{array}{c} ES \\ O \\ P \\ PE \\ S-C \end{array} & \begin{bmatrix}
x=0.2623,\ y=0.4822 & x=0.2440,\ y=0.4779 & x=0.3250,\ y=0.4883 & x=0.1428,\ y=0.4720 \\
x=0.939,\ y=0.4723 & x=0.8449,\ y=0.4615 & x=0.939,\ y=0.4723 & x=0.6131,\ y=0.4463 \\
x=0.3821,\ y=0.4932 & x=0.3340,\ y=0.4818 & x=0.3821,\ y=0.4932 & x=0.3122,\ y=0.4765 \\
x=0.1457\ \ y=0.4496 & x=0.4389,\ y=0.4324 & x=0.6842,\ y=0.4383 & x=0.2227,\ y=0.4456 \\
x=0.6222,\ y=0.4505 & x=0.6810,\ y=0.468 & x=0.8459,\ y=0.4867 & x=0.4978,\ y=0.4447
\end{bmatrix}
\end{array}
\tag{6.80}
$$

The translate defuzzified pairwise comparison matrix for alternatives evaluation

$$
\begin{array}{cc}
 & \begin{array}{cccc} x1 & x2 & x3 & x4 \end{array} \\
Average\ \ Translate\ \ defuzzifiied = & \begin{bmatrix} x=0.4703,\ y=0.4695 & x=0.5086,\ y=0.4643 & x=0.6352\ \ y=0.4758 & x=0.3577,\ y=0.457 \end{bmatrix}
\end{array}
\tag{6.81}
$$

The average translate defuzzified pairwise comparison matrix for alternatives evaluation

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distance $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative from formulation A^+ and A^- can be calculated by the area of compensation method:

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^+})^2 + (y_{\tilde{A}_i^*} - y_{A^+})^2}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = \sqrt{(0.4703-1)^2 + (0.4695-0.5)^2}$$

$$\bar{d}_i^+(\tilde{v}_{ij},\tilde{v}_j^+) = 0.5306$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{(x_{\tilde{A}_i^*} - x_{A^-})^2 + (y_{\tilde{A}_i^*} - y_{A^-})^2}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = \sqrt{(0.4603+1)^2 + (0.4695-0.5)^2}$$

$$\bar{d}_i^-(\tilde{v}_{ij},\tilde{v}_j^-) = 1.4706$$

Step 6: Find the closeness coefficient, CC_i and improve alternatives for achieving aspiration levels in each criteria.

The decision rules for five classes are depicted in Table 6.6. Notice that the highest CC_i value is used to determine the rank.

$$\overline{CC}_i = \frac{\bar{d}_i^-}{\bar{d}_i^+ + \bar{d}_i^-} = 1 - \frac{\bar{d}_i^+}{\bar{d}_i^+ + \bar{d}_i^-}$$

$$\overline{CC}_i = \frac{1.4706}{0.5306 + 1.4706}$$

$$\overline{CC}_i = 0.7348$$

After several processes, referring to Table 6.23, the CC_i values shows candidate 3 represents the highest rank with 0.81735 followed by candidate 2 with 0.7538, candidate 1 with 0.7348 and candidate 4 with 0.6785 for the last ranked. The results reveal that the candidate 3 is most suitable for this recruitment post because based on approval status table from Table 6.7, the score is in approved and preferred range.

Table 6. 23. Closeness coefficients computation for z-numbers fuzzy sets

Alternative	Closeness Coefficient, *CC*
Candidate 1	0.7348 (Rank 3)
Candidate 2	0.7538 (Rank 2)
Candidate 3	0.8173 (Rank 1)
Candidate 4	0.6785 (Rank 4)

Table 6. 7. Approval status table (Luukka, 2011)

CC_i value	Assessment status
$CC_i \in [0,0.2)$	Do not recommend
$CC_i \in [0.2,0.4)$	Recommend with high risk
$CC_i \in [0.4,0.6)$	Recommend with low risk
$CC_i \in [0.6,0.8)$	Approved
$CC_i \in [0.8,1]$	Approved and preferred

6.6 Comparative Study

6.6.1 Ranking Analysis

This section discusses the consistency and robustness of the proposed hybrid fuzzy MCDM model and established models considered in the study in solving staff recruitment as mentioned before. The following Table 6.24 and Table 6.25 signify the ranking results for criteria and alternatives of the proposed hybrid fuzzy MCDM models and established models which are considered in this study.

Table 6. 24. Ranking results of criteria for hybrid fuzzy MCDM models

Hybrid Fuzzy MCDM Model	*Criteria weight values*					*Ranking Results*
	(ES)	*(O)*	*(P)*	*(PE)*	*(S-C)*	
Type-1 Fuzzy Sets						
Fuzzy AHP – TOPSIS (Vinodh et al., 2014)	0.0882	0.3641	0.0443	0.3386	0.1649	O>PE>S-C>ES>P
Fuzzy AHP – VIKOR (Rezaie et al., 2014)	0.0882	0.3641	0.0443	0.3386	0.1649	O>PE>S-C>ES>P
Proposed Hybrid Fuzzy MCDM Model	0.1140	0.2647	0.1195	0.2758	0.2260	PE>O>S-C>P>ES
Type-2 Fuzzy Sets						
Fuzzy AHP – TOPSIS (Kilic & Kaya, 2015)	0.087	0.364	0.044	0.34	0.164	O>PE>S-C>ES>P
Extension of Hybrid Fuzzy MCDM Model	0.1172	0.2672	0.1190	0.2747	0.2219	PE>O>S-C>P>ES
Z-Numbers						
Extension of Hybrid Fuzzy MCDM Model	0.1115	0.2634	0.1195	0.2771	0.2286	PE>O>S-C>P>ES

Table 6. 25. Ranking results of alternatives for hybrid fuzzy MCDM models

Hybrid Fuzzy MCDM Model	***Alternative ranking values***				***Ranking Results***
	(Alt1)	***(Alt2)***	***(Alt3)***	***(Alt4)***	
Type-1 Fuzzy Sets					
Fuzzy AHP – TOPSIS (Vinodh et al., 2014)	0.6842	0.7067	0.7419	0.6453	Alt3>Alt2>Alt1>Alt4
Fuzzy AHP – VIKOR (Rezaie et al., 2014)	0.3634	0.1721	0.0000	0.4702	Alt3>Alt2>Alt1>Alt4
Proposed Hybrid Fuzzy MCDM Model	0.7314	0.7630	0.8178	0.6698	Alt3>Alt2>Alt1>Alt4
Type-2 Fuzzy Sets					
Fuzzy AHP – TOPSIS (Kilic & Kaya, 2015)	0.5497	0.5543	0.5616	0.5413	Alt3>Alt2>Alt1>Alt4
Extension of Hybrid Fuzzy MCDM Model	0.7422	0.7823	0.83	0.6964	Alt3>Alt2>Alt1>Alt4
Z-Numbers					
Extension of Hybrid Fuzzy MCDM Model	0.7348	0.7538	0.8173	0.6785	Alt3>Alt2>Alt1>Alt4

Table 6.24 depicts the ranking results of criteria for hybrid fuzzy MCDM model that are considered in this study. The proposed hybrid fuzzy MCDM model and its extension for type-2 fuzzy sets and z-numbers give same ranking results for criteria weight values with PE>O>S-C>P>ES. All of them give past experience, (PE), as highest ranking, followed by, oration, (O), self-confidence, (S-C), personality, (P) and emotional steadiness, (ES). While all established hybrid fuzzy MCDM models give O>PE>S-C>ES>P. All three of them give oration, (O) as highest weightage, followed by past experience, (PE) as second one, self-confidence, (S-C), emotional steadiness, (ES), and personality, (P). The last three criteria give same rank but not the early two. Comparing both, the evaluation of criteria for proposed models is computed using consistent fuzzy preference relations while the other established models is fuzzy AHP. Most of hybrid fuzzy MCDM models in the literature used fuzzy AHP in evaluating the criteria before proceed to ranking alternatives. We have been concerned with the invalidity of fuzzy theory apply to AHP technique.

According to (Zhü, 2014), there are several flaws in fuzzy AHP which are: the application of fuzzy AHP violates the main logic of fuzzy set theory that the membership grade function used in the arithmetic operation and definition of fuzzy numbers are improper; the arithmetic operation of fuzzy AHP violates basic principles of the AHP including the reciprocal and continuity axioms, the operational rule of consistency; fuzzy judgement of fuzzy AHP are less effective than the 1-9 scale of the classical AHP and; when dealing with outcomes, fuzzy AHP cannot give a generally accepted method to rank fuzzy numbers and a valid method to check the validity of the results. Based on these evidences, it can be conclude that the use of fuzzy AHP for solving decision making problems is an inappropriate tool to be used under fuzzy environment. Considering these flaws, the proposed hybrid fuzzy MCDM model does not use fuzzy AHP to evaluate criteria weights even there are a lot studies in literature literally thousand papers have been published about it. Therefore, in this research study, we prefer to use consistent fuzzy preference relations technique in order to avoid misleading solution in expressing the decision makers' opinions by means of preference relations. Applying this method, it is possible to assure better consistency of the fuzzy preference relations provided by the decision makers and in a such away, to avoid the inconsistent solutions in the decision making process of criteria evaluation.

As can be seen in Table 6.25, ranking results of alternatives or candidates for hybrid fuzzy MCDM models are depicted. Overall, all hybrid fuzzy MCDM models give same rank for alternatives with different ranking values. All of hybrid fuzzy MCDM models here applying fuzzy TOPSIS in evaluating the final ranking for alternatives except (Rezaie et al., 2014) where they used fuzzy VIKOR. Fuzzy VIKOR doesn't use closeness coefficient, CC_i, in evaluating the final results of ranking. The range value of fuzzy VIKOR ranking is same with closeness coefficient, *CC* which from 0 to 1, but the highest ranking is close to zero. Instead of closeness coefficient, CC_i, the closer to one is better ranking. Since all the hybrid fuzzy MCDM models give same final rank, it can be say that the proposed hybrid fuzzy MCDM model and its extension are consistent with other established hybrid fuzzy MCDM models. The alternatives ranking values using fuzzy TOPSIS as final evaluation is based on closeness coefficient, *CC* from approval status Table 6.7 proposed by (Luukka, 2011).

Referring to the Table 6.7, the proposed and its extension models give 'approved' and 'approved and preferred' assessment status for the final results. However, (Vinodh et al., 2014) and (Kiliç & Kaya, 2015) hybrid fuzzy MCDM models give 'approved' and 'recommended with low risk' assessment status respectively. This represents that the proposed and its extension models provide better assessment status of closeness coefficient, *CC* values than other established hybrid fuzzy MCDM models in this study. Concerning the computational complexity, agility and easy decision making using both positive and negative criteria in the decision process, fuzzy TOPSIS performs better than the other MCDM techniques. Moreover, it has ability to consider unlimited number of criteria and alternatives in decision making process. The concept of fuzzy TOPSIS technique is the most preferred alternative should have the shortest distance from the fuzzy positive ideal solution (FPIS) and the longest distance from the fuzzy negative ideal solution (FNIS). Consequently, fuzzy TOPSIS is recommended in solving human based decision making problems under fuzzy environment. Fuzzy TOPSIS at present offers a solution for decision makers when dealing with real world data that are usually multi criteria and involves a complex decision making process.

As mentioned in previous section, no comparative study for z-numbers because the established hybrid fuzzy MCDM model for z-numbers is not found in literature so far. Nevertheless, the ranking results of proposed model for z-numbers produces same ranking results as type-1 and type-2 fuzzy sets. All of these model are evaluated the robustness using sensitivity analysis in Section 6.6.2.

6.6.2 Sensitivity Analysis Computation

In sensitivity analysis evaluation, the focus is to test the effect of the criteria weight on the ranking of the results. The tests are proceed by increasing each original criteria weight by 50%, 100% and 150%. While one criterion is increased, the values of the remaining criteria are decreased by certain amount, such that the total amount of criteria are equal to one. A series of evaluation runs is conducted where each criterion's weight is altered by 50%, 100% and 150%. The scenario consist of 15 evaluation runs for each fuzzy sets. The computational process is illustrated on the next page.

The vector for the original weights of criteria is $W^t = (w_1, w_2, ..., w_k)$ where in weights are normalised and sum of them is 1, $\sum_{j=1}^{k} w_t = 1$. Both tables below represent the original weight of criteria and original alternatives results of proposed hybrid fuzzy MCDM model for type-1 fuzzy sets.

Table 6. 26. The original weight of criteria of type-1 fuzzy sets.

Criteria	**Weight, *w***
Emotional steadiness	0.1140
Oration	0.2647
Personality	0.1195
Past experience	0.2758
Self-confidence	0.2260
Total	**1**

Table 6. 6. The original results of closeness coefficients computation for type-1 fuzzy sets.

Alternative	**Closeness Coefficient, *CCi***
Candidate 1	0.7316 (Rank 3)
Candidate 2	0.7630 (Rank 2)
Candidate 3	0.8178 (Rank 1)
Candidate 4	0.6698 (Rank 4)

The weight of one criterion changes, then the weight of other criteria change accordingly, and the new vector of weights transformed into equation (4.51).

$$W'^t = (w'_1, w'_2, ..., w'_k)$$

The theorem 4.4 shows changes in the weight of criteria P^{th}, changes as Δ_p, then the weight of other criteria change as Δ_j; $j = 1,2,...,k$. By using equation (4.52), the changes are computed in order to increase the percentage of criteria weight. Here, assume that the weight of the 'emotional steadiness' is altered by 50%, 100% and 150%.

Changes for 'emotional steadiness' criterion.

Increase 50% for weight of 'emotional steadiness' criterion, then the new vector of weights if criteria would be produced.

By applying equation (4.53), the new weight of 'emotional steadiness' would change as below.

$$w'_p = w_p + \Delta_p$$

$$w'_{ES} = w_{ES} + \Delta_{ES}$$

$$w'_{ES} = 0.1140 + (50\% \times 0.1140)$$

$$w'_{ES} = 0.1711$$

Then, the weight of other criteria would change using equation (4.59) as shown below.

$$w'_j = \frac{1 - w_p - \Delta_p}{1 - w_p} \times w_j = \frac{1 - w'_p}{1 - w_p} \times w_j$$

$$w'_O = \frac{1 - w_{ES} - \Delta_{ES}}{1 - w_{ES}} \times w_O = \frac{1 - w'_{ES}}{1 - w_{ES}} \times w_O$$

$$w'_O = \frac{1 - 0.1711}{1 - 0.1140} \times 0.2647$$

$$w'_O = 0.2477$$

Then, new vector for other weights of criteria are presented in Table 6.27

Table 6. 27. New weights of criteria of type-1 fuzzy sets.

Criteria	**Weight, *w***
Emotional steadiness	0.1711
Oration	0.2477
Personality	0.1118
Past experience	0.2580
Self-confidence	0.2115
Total	**1**

Hence, the results of alternatives' ranking would change as well. Below depicted the new results of all criteria weights when 'emotional steadiness' criterion's weight is increased by 50% changes.

Table 6. 28. New closeness coefficients computation for type-1 fuzzy sets.

Alternative	**Closeness Coefficient, *CC***
Candidate 1	0.7572 (Rank 3)
Candidate 2	0.7872 (Rank 2)
Candidate 3	0.8462 (Rank 1)
Candidate 4	0.6869 (Rank 4)

This sensitivity analysis computational process are evaluated for every hybrid fuzzy MCDM model. In this analysis, the evaluation process are required for type-1 fuzzy sets, type-2 fuzzy sets and z-numbers for comparative study.

6.6.3 Discussion

This section discusses the sensitivity analysis evaluation results of proposed and established hybrid fuzzy MCDM models for all fuzzy sets. Sensitivity analysis investigates the effect of criteria weights on the ranking of alternatives.

Consistent fuzzy preference relations – fuzzy TOPSIS (Proposed)

Fig. 6.2 shows the analysis results of changing the criteria weights for proposed hybrid fuzzy MCDM model which combined consistent fuzzy preference relations and fuzzy TOPSIS for type-1 fuzzy sets. The horizontal axis represents the percentage increases in the criteria weights and the vertical axis represents the new values for the closeness coefficient, CC_i of the alternatives. Fig. 6.75 illustrates that when the weights of the criteria change, the values of the CC_i vary slightly. According to sensitivity analysis results here, alternative 3 or candidate 3, Alt_3 is determined to be the most appropriate to be selected as a potential staff, because he or she always has a maximum CC_3 value after the weight changes are applied. Alternative or candidate 3 has the highest CC_3 value of 0.8786 when criterion emotional steadiness, (ES), is increased by 100%, whereas it has lowest value of 0.6220 when criterion past experience, (PE) is raised by 150%. In addition, it can be observed that the CC_3 values of candidate 3 show upward tendency when the weight of emotional steadiness, (ES) and personality, (P) are increased by 50% to 100%. However, when criterion oration, (O), past experience, (PE), and self-confidence, (S-C), are increased by 50%, 100% and 150%, values of the CC_3 shows downward tendency.

The second best candidate is candidate 2, Alt_2, with the maximum CC_2 value of 0.8149, obtained when criterion *ES* is increased by 100%. It has the smallest CC_2 value of 0.5892 when criterion *PE* is raised by 150%. Moreover, the CC_2 values of candidate 2, Alt_2 depict slight upward trend when the weights of criterion *ES* and *P* are increased by 50% and 100%. Though, when criterion O, PE and S-C, are increased by 50%, 100% and 150%, values of the CC_2 shows downward tendency. Candidate 1, Alt_1 has the third ranking with the maximum CC_1 value, 0.7866, when criterion ES is increased by 100%. While, the lowest value, 0.5588, when criterion PE is increased by 150%. Additionally, it can be noted that when criterion ES and P are increased by 50% and 100%, values of the CC_1 show upward tendency. Yet, when criterion O, PE and S-C, are increased by 50%, 100% and 150%, values of the CC_1 shows downward tendency. The last ranking of the case study is candidate 4, Alt_4 with the maximum CC_4 value, 0.7147, when criterion P is increased by 100%. While the lowest CC_4 value is 0.5534, when criterion PE is increased by 150%. It can be concluded that the CC_4 values are upward tendency when criterion ES and P are increased by 50% and 100%. Moreover, when criterion O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_4 shows downward tendency.

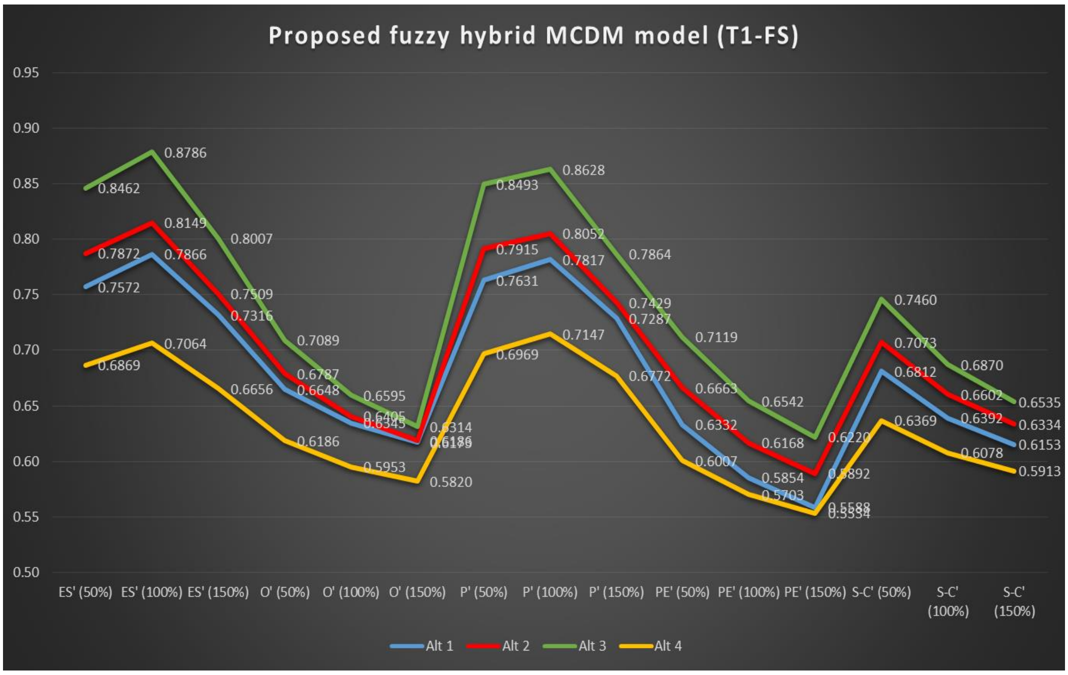


Fig. 6. 2. Sensitivity analysis results caused by varying the weights of the criteria by proposed fuzzy hybrid MCDM model for type-1 fuzzy sets

As a result, the proposed hybrid fuzzy MCDM model for type-1 fuzzy sets is robust and stable, since changes in the criteria weights do not significantly affect the final ranking order of the alternatives candidates. As related before, referring to the Table 6.29, the consistency of correct ranking order based on original rank presents 100% level of consistency. Even the ranking values are changed, but the ranking order are significantly consistent with the original ranking. In the context of sensitivity analysis evaluation, it presents that the proposed hybrid fuzzy MCDM model for type-1 fuzzy sets is definitely consistent even the weights of criteria are changed. From the consistency results of Table 6.29, the proposed hybrid fuzzy MCDM model for type-1 fuzzy sets is recommended to deal with bigger case study in real world phenomena in order to solve human based decision making problems under fuzzy environment.

Table 6. 29. Sensitivity analysis results of proposed hybrid fuzzy MCDM model for type-1 fuzzy sets

Changes of criteria (%)	Alt1	Alt2	Alt3	Alt4	Ranking results	Consistency based on original result
ES' (50%)	0.7572	0.7872	0.8462	0.6869	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (100%)	0.7866	0.8149	0.8786	0.7064	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (150%)	0.7316	0.7509	0.8007	0.6656	Alt3>Alt2>Alt1>Alt4	Consistent
O' (50%)	0.6648	0.6787	0.7089	0.6186	Alt3>Alt2>Alt1>Alt4	Consistent
O' (100%)	0.6345	0.6405	0.6595	0.5953	Alt3>Alt2>Alt1>Alt4	Consistent
O' (150%)	0.6173	0.6186	0.6314	0.5820	Alt3>Alt2>Alt1>Alt4	Consistent
P' (50%)	0.7631	0.7915	0.8493	0.6969	Alt3>Alt2>Alt1>Alt4	Consistent
P' (100%)	0.7817	0.8052	0.8628	0.7147	Alt3>Alt2>Alt1>Alt4	Consistent
P' (150%)	0.7287	0.7429	0.7864	0.6772	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (50%)	0.6332	0.6663	0.7119	0.6007	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (100%)	0.5854	0.6168	0.6542	0.5703	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (150%)	0.5588	0.5892	0.6220	0.5534	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (50%)	0.6812	0.7073	0.7460	0.6369	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (100%)	0.6392	0.6602	0.6870	0.6078	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (150%)	0.6153	0.6334	0.6535	0.5913	Alt3>Alt2>Alt1>Alt4	Consistent
Level of consistency						100%

The situation illustrates that the weights of ES, P and S-C might influence the final preference in selection process which of them play important role in evaluating or selecting new staff in company or institution. Therefore, these criteria can be considered as critical criteria and the most sensitivity criteria in the model. Meanwhile, care should be given to the weighting of these sensitivity criteria, since this step may affect the final ranking. Aforementioned, if the ranking is highly sensitive to small changes in the parameter values, a careful review of those parameters is recommended. In addition, this analysis may also assist the company to improve their selecting staff by taking into account these critical criteria in order to meet the company requirement.

<u>Fuzzy AHP – fuzzy TOPSIS (Vinodh et al., 2014)</u>

Fig. 6.3 and Table 6.30 show the sensitivity analysis results for established fuzzy AHP – fuzzy TOPSIS model proposed by (Vinodh et al., 2014). The pattern of changes of weights for all criteria in Fig. 6.76 are slightly similar

to proposed hybrid fuzzy MCDM model. Two points depict that the ranking are affected, which are oration, (O) criterion with 150% changes and past experience, PE with 150% as well. Table 6.30 represents the consistency of correct ranking order based on original rank of (Vinodh et al., 2014) model presents 86.67% level of consistency. As discussed in point before, when criterion O and PE are increased 150%, the ranking order are changed to Alt3>Alt1>Alt2>Alt4 and Alt3>Alt2>Alt4>Alt1 respectively. This is depicted that the hybrid fuzzy MCDM model proposed by (Vinodh et al., 2014) is less robust and less stable than the proposed model.

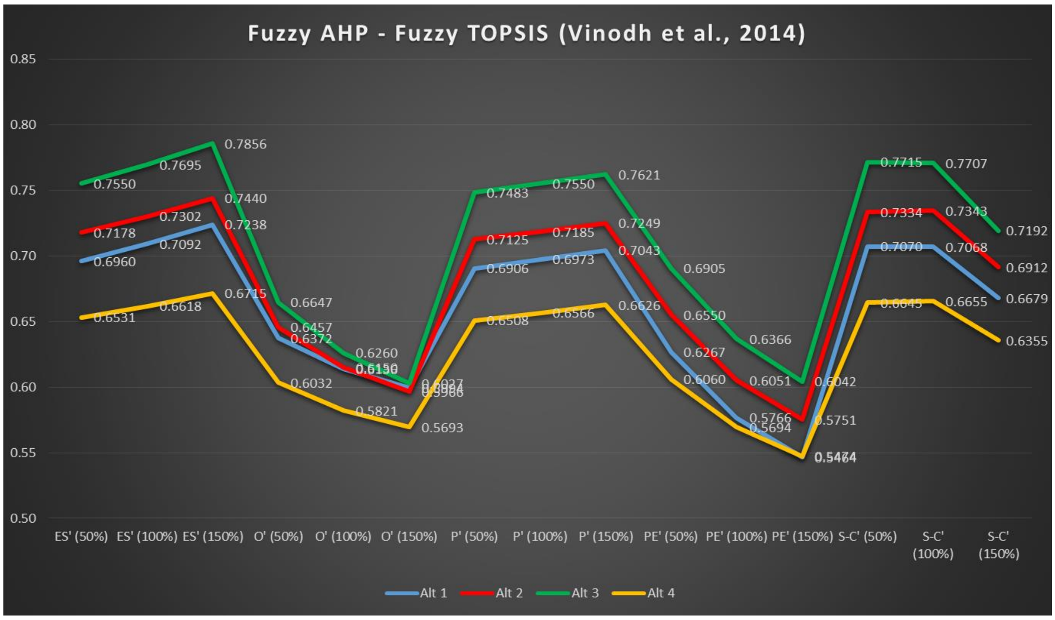


Fig. 6. 3. Sensitivity analysis results caused by varying the weights of the criteria by fuzzy AHP – fuzzy TOPSIS model (Vinodh et al., 2014) for type-1 fuzzy sets

Table 6. 30. Sensitivity analysis results of fuzzy AHP – fuzzy TOPSIS model (Vinodh et al., 2014) for type-1 fuzzy sets

Changes of criteria (%)	Alt1	Alt2	Alt3	Alt4	Ranking results	Consistency based on original result
ES' (50%)	0.6960	0.7178	0.7550	0.6531	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (100%)	0.7092	0.7302	0.7695	0.6618	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (150%)	0.7238	0.7440	0.7856	0.6715	Alt3>Alt2>Alt1>Alt4	Consistent
O' (50%)	0.6372	0.6457	0.6647	0.6032	Alt3>Alt2>Alt1>Alt4	Consistent
O' (100%)	0.6136	0.6150	0.6260	0.5821	Alt3>Alt2>Alt1>Alt4	Consistent
O' (150%)	0.5994	0.5966	0.6027	0.5693	Alt3>Alt1>Alt2>Alt4	Inconsistent
P' (50%)	0.6906	0.7125	0.7483	0.6508	Alt3>Alt2>Alt1>Alt4	Consistent
P' (100%)	0.6973	0.7185	0.7550	0.6566	Alt3>Alt2>Alt1>Alt4	Consistent
P' (150%)	0.7043	0.7249	0.7621	0.6626	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (50%)	0.6267	0.6550	0.6905	0.6060	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (100%)	0.5766	0.6051	0.6366	0.5694	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (150%)	0.5464	0.5751	0.6042	0.5474	Alt3>Alt2>Alt4>Alt1	Inconsistent
S-C' (50%)	0.7070	0.7334	0.7715	0.6645	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (100%)	0.7068	0.7343	0.7707	0.6655	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (150%)	0.6679	0.6912	0.7192	0.6355	Alt3>Alt2>Alt1>Alt4	Consistent
Level of consistency						86.67%

Fuzzy AHP – fuzzy VIKOR (Rezaie et al., 2014)

As can be seen in Fig. 6.4 and Table 6.31, both present the sensitivity analysis results for established fuzzy AHP – fuzzy VIKOR model proposed by (Rezaie et al., 2014). The pattern of changes of weights for all criteria in Fig. 6.77 are different from proposed model and fuzzy AHP – fuzzy TOPSIS model (Vinodh et al., 2014). Considering that, the final ranking evaluation that used is VIKOR method, not TOPSIS. In the context of VIKOR method, final ranking are started from the small to large values from 0 to 1 range. The smallest the better rank. Several points' show the ranking are affected which are two from oration, (O) criterion with 100% and 150% changes and past experience, (PE) with 150% as well. Table 6.31 shows the consistency of correct ranking order based on original rank of (Rezaie et al., 2014) model presents 80% level of consistency. While the weights of criterion O increases 100% and 150%, the ranking order are changed to Alt3>Alt1>Alt2>Alt4 and Alt3>Alt2>Alt2>Alt4 respectively. When criterion PE is increased 150%, then the ranking order is changed to Alt3>Alt2>Alt4>Alt1. The hybrid fuzzy MCDM model proposed by (Rezaie et al., 2014) is less robust and less stable than the proposed model and (Vinodh et al., 2014).

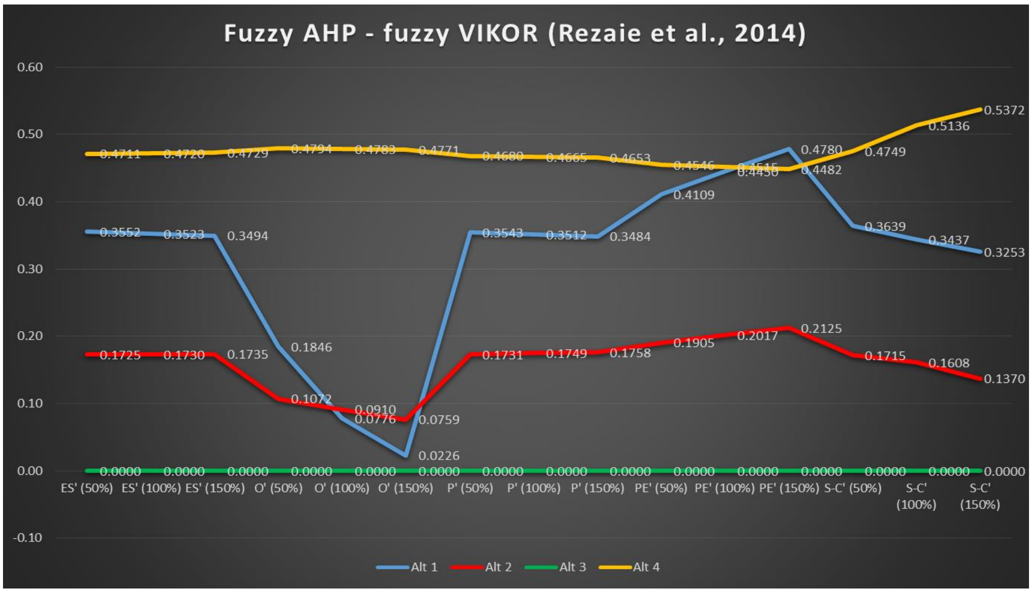


Fig. 6. 4. Sensitivity analysis results caused by varying the weights of the criteria by fuzzy AHP – fuzzy VIKOR model (Rezaie et al., 2014) for type-1 fuzzy sets

Table 6. 31. Sensitivity analysis results of fuzzy AHP – fuzzy VIKOR model (Rezaie et al., 2014) for type-1 fuzzy sets

Changes of criteria (%)	Alt1	Alt2	Alt3	Alt4	Ranking results	Consistency based on original result
ES' (50%)	0.3552	0.1725	0.0000	0.4711	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (100%)	0.3523	0.1730	0.0000	0.4720	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (150%)	0.3494	0.1735	0.0000	0.4729	Alt3>Alt2>Alt1>Alt4	Consistent
O' (50%)	0.1846	0.1072	0.0000	0.4794	Alt3>Alt2>Alt1>Alt4	Consistent
O' (100%)	0.0776	0.0910	0.0000	0.4783	Alt3>Alt1>Alt2>Alt4	Inconsistent
O' (150%)	0.0226	0.0759	0.0000	0.4771	Alt3>Alt1>Alt2>Alt4	Inconsistent
P' (50%)	0.3543	0.1731	0.0000	0.4680	Alt3>Alt2>Alt1>Alt4	Consistent
P' (100%)	0.3512	0.1749	0.0000	0.4665	Alt3>Alt2>Alt1>Alt4	Consistent
P' (150%)	0.3484	0.1758	0.0000	0.4653	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (50%)	0.4109	0.1905	0.0000	0.4546	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (100%)	0.4450	0.2017	0.0000	0.4515	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (150%)	0.4780	0.2125	0.0000	0.4482	Alt3>Alt2>Alt4>Alt1	Inconsistent
S-C' (50%)	0.3639	0.1715	0.0000	0.4749	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (100%)	0.3437	0.1608	0.0000	0.5136	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (150%)	0.3253	0.1370	0.0000	0.5372	Alt3>Alt2>Alt1>Alt4	Consistent
Level of consistency						80%

Due to these results, there are several related points to consider; 1) the useful of AHP method, 2) improper centroid defuzzification used and 3) improper normalization process used. According to (Zhü, 2014), fuzzy AHP violates the main logic of fuzzy sets and fuzzy judgement of fuzzy AHP are less effective than the 1-9 scale of the non-fuzzy AHP. Centroid defuzzification plays important role in getting defuzzification values to make sure the final results are compatible with human judgment. At the same time, the generality of fuzzy events are not lost. Some normalization process methods are not suitable to deal with fuzzy numbers, the proper way of normalization process method is concerned in order to make sure the range that we use would give appropriate results.

<u>Extension of consistent fuzzy preference relations – fuzzy TOPSIS for type-2 fuzzy sets (proposed)</u>

The proposed hybrid fuzzy MCDM model for type-1 fuzzy sets is extended for type-2 fuzzy sets in order to utilise in many different situation under fuzzy environment especially uncertainty problems. The extension of hybrid fuzzy MCDM model is proposed as discussed in Section 6.4.1. Fig. 6.5 and Table 6.32 illustrate the analysis results of changing the criteria weights for proposed fuzzy hybrid fuzzy MCDM model for type-2 fuzzy sets. This figure presents that when the weights of the criteria change, the values of the CC_i vary slightly. According to sensitivity analysis results here, alternative 3 or candidate 3, Alt_3 is determined to be the most appropriate to be selected as a potential staff to be selected, because he or she always has a maximum CC_3 value after the weight changes are applied. Alternative or candidate 3 has the highest CC_3 value of 0.8832 when criterion emotional steadiness, (ES), is increased by 100%, whereas it has its lowest value of 0.6224 when criterion past experience is raised by 150%. Moreover, it can be observed that the CC_3 values of candidate 4 show upward tendency when the weight of emotional steadiness, (ES) and personality, (P) are increased by 50% to 100%. Though, when criterion oration, (O), past experience, (PE), and self-confidence, (S-C), are increased by 50%, 100% and 150%, values of the CC_3 shows downward tendency.

For the second rank best ranking is candidate 2, Alt_2, with the maximum CC_2 value of 0.8291, obtained when criterion P is increased by 150%. It has the smallest CC_2 value of 0.5892 when criterion PE is raised by 150%. Moreover, the CC_2 values of candidate 2, Alt_2 depict slight upward trend when the weights of criterion ES and P are increased by 50% and 100%. Though, when criterion O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_2 shows downward tendency. Candidate 1, Alt_1 has the third ranking with the maximum CC_1 value, 0.7946, when criterion P is increased by 100%. While, the lowest value, 0.5570, when criterion PE is increased by 150%. Furthermore, it can be noted that when criterion ES and P are increased by 50% and 100%, values of the CC_1 show upward tendency. Yet, when criterion O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_1 shows downward tendency. The last ranking of the case study is candidate 4, Alt_4 with the maximum CC_4 value, 0.7441, when criterion P is increased by 100%. While the lowest CC_4 value is 0.5548, when criterion PE is increased by 150%. It can be concluded that the CC_4 values are upward tendency when criterion ES and P are increased by 50% and 100%. Also, when criterion O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_4 shows downward tendency.

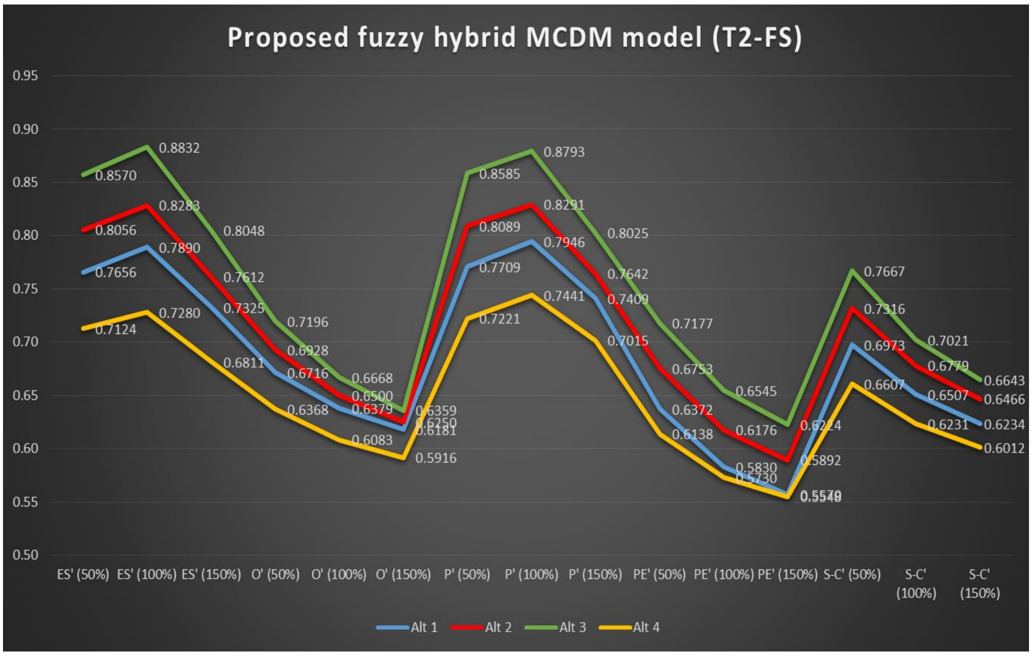


Fig. 6. 5. Sensitivity analysis results caused by varying the weights of the criteria by proposed hybrid fuzzy MCDM model for type-2 fuzzy sets

As a consequence, the proposed hybrid fuzzy MCDM model for type-2 fuzzy sets is robust and stable, since changes in the criteria weights do not significantly affect the final ranking order of the alternatives candidates. As related before, referring to the Table 6.32, the consistency of correct ranking order based on original rank presents 100% accurate. Even the ranking values are changed, but the ranking order are significantly consistent with the original ranking. In the context of sensitivity analysis evaluation, it presents that the proposed hybrid fuzzy MCDM model for type-2 fuzzy sets is definitely consistent even the weights of criteria are changed. With the latest development of type-2 fuzzy sets and the concept of interval type-2 fuzzy sets, causal relationship in the hybrid fuzzy MCDM model deserves to receive more comprehensive evaluation to the flexibility of spaces representing uncertainties than type-1 fuzzy sets. This is because, type-2 fuzzy sets are characterised by fuzzy membership functions, as each element on this set is a fuzzy set in [0,1], unlike type-1 fuzzy sets where the membership grade is in a crisp number in [0,1] (Karnik & Mendel, 2001a). Representing both, the proposed and the extension of hybrid fuzzy MCDM model for type-1 fuzzy sets and type-2 fuzzy sets respectively, the patterns depict have same trend. The different are the CC_is' values. Definitely, it has been noticed that both proposed hybrid fuzzy MCDM models or type-1 and type-2 fuzzy sets produce 100% level of consistency. But, model for type-2 is more recommended based on approval status table proposed by (Luukka, 2011) for CC_i values acceptance. As a consequence, the proposed hybrid fuzzy MCDM model for type-2 fuzzy sets are better than model for type-1 fuzzy sets.

Table 6. 7. Approval status table (Luukka, 2011)

***CCi* value**	**Assessment status**
$CC_i \in [0,0.2)$	Do not recommend
$CC_i \in [0.2,0.4)$	Recommend with high risk
$CC_i \in [0.4,0.6)$	Recommend with low risk
$CC_i \in [0.6,0.8)$	Approved
$CC_i \in [0.8,1]$	Approved and preferred

Table 6. 32. Sensitivity analysis results of proposed hybrid fuzzy MCDM model for type-2 fuzzy sets

Changes of criteria (%)	**Alt1**	**Alt2**	**Alt3**	**Alt4**	**Ranking results**	**Consistency based on original result**
ES' (50%)	0.7656	0.8056	0.8570	0.7124	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (100%)	0.7890	0.8283	0.8832	0.7280	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (150%)	0.7325	0.7612	0.8048	0.6811	Alt3>Alt2>Alt1>Alt4	Consistent
O' (50%)	0.6716	0.6928	0.7196	0.6368	Alt3>Alt2>Alt1>Alt4	Consistent
O' (100%)	0.6379	0.6500	0.6668	0.6083	Alt3>Alt2>Alt1>Alt4	Consistent
O' (150%)	0.6181	0.6250	0.6359	0.5916	Alt3>Alt2>Alt1>Alt4	Consistent
P' (50%)	0.7709	0.8089	0.8585	0.7221	Alt3>Alt2>Alt1>Alt4	Consistent
P' (100%)	0.7946	0.8291	0.8793	0.7441	Alt3>Alt2>Alt1>Alt4	Consistent
P' (150%)	0.7409	0.7642	0.8025	0.7015	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (50%)	0.6372	0.6753	0.7177	0.6138	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (100%)	0.5830	0.6176	0.6545	0.5730	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (150%)	0.5570	0.5892	0.6224	0.5548	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (50%)	0.6973	0.7316	0.7667	0.6607	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (100%)	0.6507	0.6779	0.7021	0.6231	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (150%)	0.6234	0.6466	0.6643	0.6012	Alt3>Alt2>Alt1>Alt4	Consistent
Level of consistency						100%

<u>Fuzzy AHP – fuzzy TOPSIS (Kiliç & Kaya, 2015)</u>

Fig. 6.6 and Table 6.33 show the sensitivity analysis results for established fuzzy AHP – fuzzy TOPSIS model proposed by (Kiliç & Kaya, 2015). The pattern of changes of weights for all criteria in Fig. 6.6 are different to proposed hybrid fuzzy MCDM model. One point show the ranking is affected, which is 'past experience', *PE* with 150%. As can be seen from Fig. 6.6, the values and patterns of changes of CC_i are too small compare to the proposed model. The ranking values between alternative to other alternative are too small. That is mean that the gap are small to represent the assessment status of acceptance. Referring approval status from Table 6.7, based on ranking results from original results and sensitivity analysis from (Kiliç & Kaya, 2015) model, the assessment status of all changes are 'recommend with low risk'. Even, this model gives same ranking to proposed model, but the CC_i values are quite low

and the gap are too small. Table 6.33 shows the consistency of correct ranking order based on original rank of (Kiliç & Kaya, 2015) model presents 93.33% level of consistency. As discussed in point before, when criterion PE is increased 150%, the ranking order are changed to Alt3>Alt2>Alt4>Alt1. This is depicted that the hybrid fuzzy MCDM model proposed by (Kiliç & Kaya, 2015) is good in robustness but lesser than the proposed model for type-2 fuzzy sets.

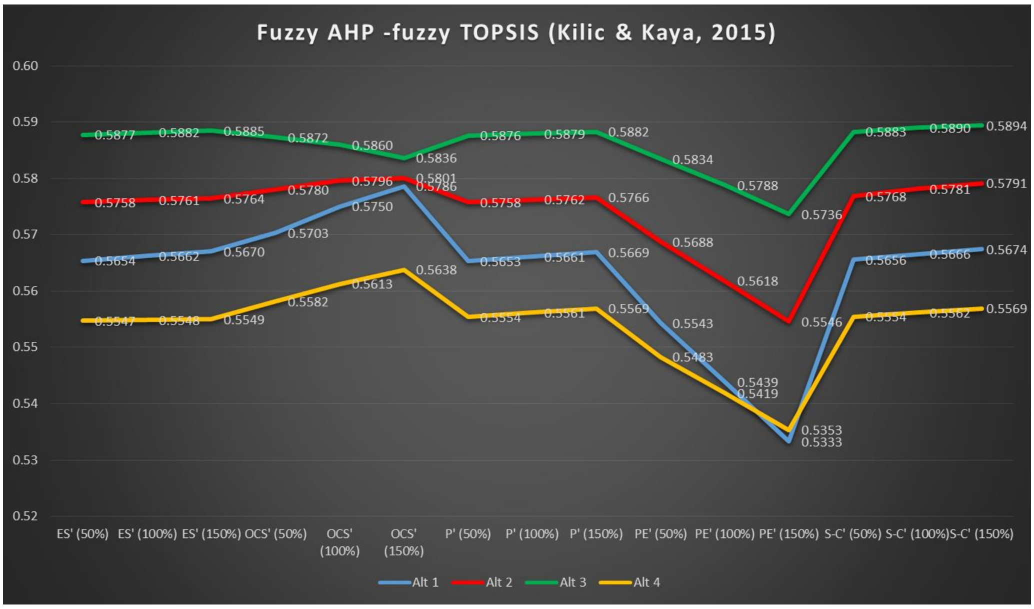


Fig. 6. 6. Sensitivity analysis results caused by varying the weights of the criteria by fuzzy AHP - fuzzy TOPSIS model (Kiliç & Kaya, 2015) for type-2 fuzzy sets

Table 6. 33. Sensitivity analysis results of proposed hybrid fuzzy MCDM model for type-2 fuzzy sets

Changes of criteria (%)	Alt1	Alt2	Alt3	Alt4	Ranking results	Consistency based on original result
ES' (50%)	0.5654	0.5758	0.5877	0.5547	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (100%)	0.5662	0.5761	0.5882	0.5548	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (150%)	0.5670	0.5764	0.5885	0.5549	Alt3>Alt2>Alt1>Alt4	Consistent
O' (50%)	0.5703	0.5780	0.5872	0.5582	Alt3>Alt2>Alt1>Alt4	Consistent
O' (100%)	0.5750	0.5796	0.5860	0.5613	Alt3>Alt2>Alt1>Alt4	Consistent
O' (150%)	0.5786	0.5801	0.5836	0.5638	Alt3>Alt2>Alt1>Alt4	Consistent
P' (50%)	0.5653	0.5758	0.5876	0.5554	Alt3>Alt2>Alt1>Alt4	Consistent
P' (100%)	0.5661	0.5762	0.5879	0.5561	Alt3>Alt2>Alt1>Alt4	Consistent
P' (150%)	0.5669	0.5766	0.5882	0.5569	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (50%)	0.5543	0.5688	0.5834	0.5483	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (100%)	0.5439	0.5618	0.5788	0.5419	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (150%)	0.5333	0.5546	0.5736	0.5353	Alt3>Alt2>Alt4>Alt1	Inconsistent
S-C' (50%)	0.5656	0.5768	0.5883	0.5554	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (100%)	0.5666	0.5781	0.5890	0.5562	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (150%)	0.5674	0.5791	0.5894	0.5569	Alt3>Alt2>Alt1>Alt4	Consistent
Level of consistency						93.33%

<u>Extension of consistent fuzzy preference relations – fuzzy TOPSIS for z-numbers (proposed)</u>

This part discusses the analysis results of changing the criteria weights for proposed fuzzy hybrid fuzzy MCDM model for z-numbers. Fig. 6.7 shows that when the weights of the criteria change, the values of the CC_i vary slightly. According to sensitivity analysis results here, alternative 3 or candidate 3, Alt_3 is determined to be the most appropriate to be selected as a potential staff, because he or she always has a maximum CC_3 value after the weight changes are applied. Alternative or candidate 3 has the highest CC_3 value of 0.8750 when criterion emotional steadiness, (ES), is increased by 100%, whereas it has its lowest value of 0.6233 when criterion past experience, PE is raised by 150%. In addition, it can be observed that the CC_3 values of candidate 3 show upward tendency when the weight of emotional steadiness, (ES) and personality, (P) are increased by 50% to 100%. However, when criterion oration, (O), past experience (PE), and self-confidence, (S-C), are increased by 50%, 100% and 150%, values of the CC_3 shows downward tendency.

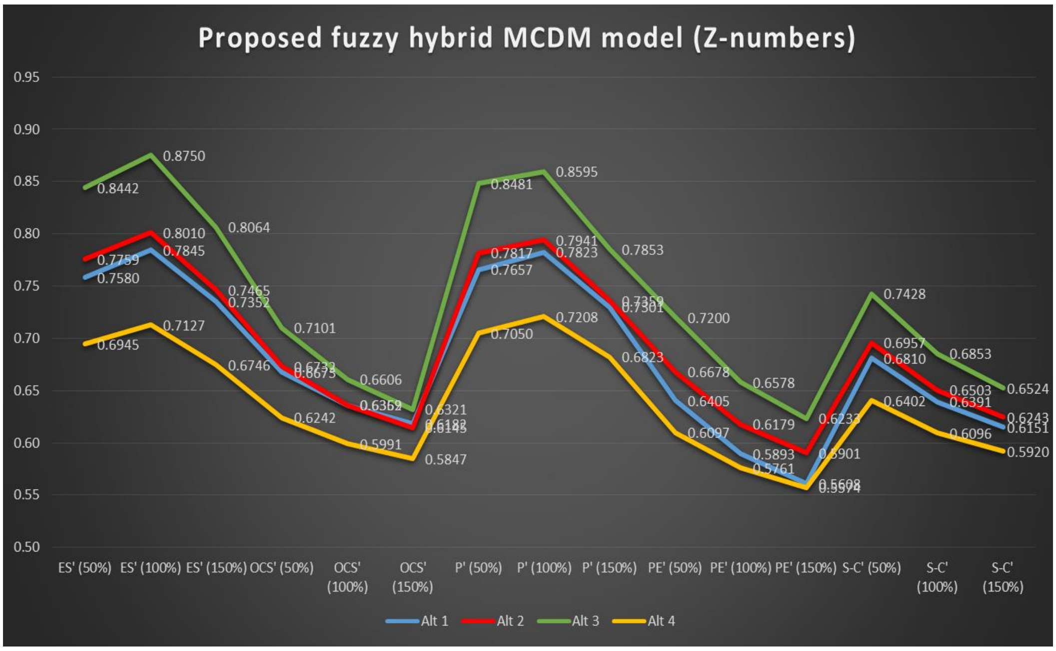


Fig. 6. 7. Sensitivity analysis results caused by varying the weights of the criteria by proposed hybrid fuzzy MCDM model for z-numbers

The second best candidate is candidate 2, Alt_2, with the maximum CC_2 value of 0.8010, obtained when criterion ES is increased by 100%. It has the smallest CC_2 value of 0.5901 when criterion PE is raised by 150%. Moreover, the CC_2 values of candidate 2, Alt_2 depict slight upward trend when the weights of criterion ES and P are increased by 50% and 100%. Nevertheless, when criteria O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_2 shows downward tendency. Candidate 1, Alt_1 has the third ranking with the maximum CC_1 value, 0.7845, when criterion ES is increased by 100%. While, the lowest value, 0.5608, when criterion PE is increased by 150%. Additionally, it can be noted that when criterion ES and P are increased by 50% and 100%, values of the CC_1 show upward tendency. Yet, when criteria O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_1 shows downward tendency. The last ranking of the case study is candidate 4, Alt_4 with the maximum CC_4 value, 0.7208, when criterion P is increased by 100%. While the lowest CC_4 value is 0.5574, when criterion PE is increased by 150%. It can be concluded that the CC_4 values are upward tendency when criterion ES and P are increased by 50% and 100%. Moreover, when criterion O, PE, and S-C, are increased by 50%, 100% and 150%, values of the CC_4 shows downward tendency.

Table 6. 34. Sensitivity analysis results of proposed hybrid fuzzy MCDM model for z-numbers

Changes of criteria (%)	Alt1	Alt2	Alt3	Alt4	Ranking results	Consistency based on original result
ES' (50%)	0.7580	0.7759	0.8442	0.6945	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (100%)	0.7845	0.8010	0.8750	0.7127	Alt3>Alt2>Alt1>Alt4	Consistent
ES' (150%)	0.7352	0.7465	0.8064	0.6746	Alt3>Alt2>Alt1>Alt4	Consistent
O' (50%)	0.6673	0.6732	0.7101	0.6242	Alt3>Alt2>Alt1>Alt4	Consistent
O' (100%)	0.6362	0.6359	0.6606	0.5991	Alt3>Alt1>Alt2>Alt4	Inconsistent
O' (150%)	0.6182	0.6145	0.6321	0.5847	Alt3>Alt1>Alt2>Alt4	Inconsistent
P' (50%)	0.7657	0.7817	0.8481	0.7050	Alt3>Alt2>Alt1>Alt4	Consistent
P' (100%)	0.7823	0.7941	0.8595	0.7208	Alt3>Alt2>Alt1>Alt4	Consistent
P' (150%)	0.7301	0.7359	0.7853	0.6823	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (50%)	0.6405	0.6678	0.7200	0.6097	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (100%)	0.5893	0.6179	0.6578	0.5761	Alt3>Alt2>Alt1>Alt4	Consistent
PE' (150%)	0.5608	0.5901	0.6233	0.5574	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (50%)	0.6810	0.6957	0.7428	0.6402	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (100%)	0.6391	0.6503	0.6853	0.6096	Alt3>Alt2>Alt1>Alt4	Consistent
S-C' (150%)	0.6151	0.6243	0.6524	0.5920	Alt3>Alt2>Alt1>Alt4	Consistent
Level of consistency						86.67%

Table 6.34 summaries the sensitivity analysis results of proposed hybrid fuzzy MCDM model for z-numbers. It presents that the proposed hybrid fuzzy MCDM model for z-numbers is quite robust and stable, since changes in the criteria weights are slightly affected the final ranking order of the alternatives candidates. As related before, referring to the Table 6.35 below, the consistency of correct ranking order based on original rank presents 86.67% level of consistency. The ranking order are significantly consistent with original ranking. However, when criterion O are increased by 100% and 150%, the ranking order are changed to Alt3>Alt1>Alt2>Alt4 both of them. In the context of sensitivity analysis evaluation, it presents that the proposed hybrid fuzzy MCDM model for z-numbers is consistent even the weights of criteria are changed. Since there is no found hybrid fuzzy MCDM model for z-numbers in literature so far, the proposed hybrid fuzzy MCDM model for z-numbers can be considered as pioneer for integrating MCDM methods to deal with z-numbers. As a consequence, there no comparative study for z-numbers evaluation.

Table 6. 35. Sensitivity analysis comparative validation results

Hybrid fuzzy MCDM model	Level of Consistency
Type-1 Fuzzy Sets	
Fuzzy AHP – Fuzzy TOPSIS (Vinodh et al., 2014)	86.67%
Fuzzy AHP – Fuzzy VIKOR (Rezaie et al., 2014)	80%
Consistent Fuzzy Preference Relations – Fuzzy TOPSIS (Proposed Model)	100%
Type-2 Fuzzy Sets	
Fuzzy AHP – Fuzzy TOPSIS (Kiliç & Kaya, 2015)	93.33%
Consistent Fuzzy Preference Relations – Fuzzy TOPSIS (Proposed Model)	100%
Z-numbers	
Consistent Fuzzy Preference Relations – Fuzzy TOPSIS (Proposed Model)	86.67%

Table 6.35 summarises the sensitivity analysis for all comparative studies in this research work. Representing all models above have good results in level of consistency which are above 80%. As can be seen here, the proposed model for type-1 fuzzy sets and the extension of proposed model for type- 2 fuzzy sets achieve 100% level of consistency. These two models are recommended and suggested to solve other case studies since the level of consistency of ranking is better than others. This followed by fuzzy AHP – fuzzy TOPSIS (Kiliç & Kaya, 2015) with 93.33%. The proposed model for z-numbers and fuzzy AHP – Fuzzy TOPSIS (Vinodh et al., 2014) share same level with 86.67%. The fuzzy AHP – fuzzy VIKOR proposed by (Rezaie et al., 2014) achieves 80% level of consistency. This is depicted that the proposed hybrid fuzzy MCDM models for type-1 and type-2 fuzzy sets are more robust than the other models in this study. For the proposed model for z-numbers, it still a good model because it gives consistent results with other established model in literature.

6.7 Summary of the Chapter

In this chapter, the applicability of proposed hybrid fuzzy MCDM model that is incorporated with intuitive multiple centroid in solving respective case study with different fuzzy sets are presented. The proposed model and its extension are applied to a staff recruitment problems in Saprudin, Idris & Co Company, in Malaysia. The candidates was evaluated based on several criteria by decision makers. All of them are evaluated by using proposed model and two established hybrid fuzzy models in literature in order to find out the ranking of the candidates. Also, each hybrid fuzzy MCDM models including proposed model are validated using sensitivity analysis with regard to find out the robustness of the model. In Chapter 7, the thesis concludes the whole research work.

CHAPTER 7

CONCLUSION

7.1 Introduction

This chapter is devoted to summarise the contributions of this study, the concluding remarks, limitations and recommendations for future works. It illustrates a summary of all the works contributed to knowledge in every chapter of the thesis and suggests some significant recommendations towards improving the knowledge of fuzzy sets and MCDM. In terms of study, there are three main points to discuss and emphasize in these contributions of this study, concluding remarks, limitations and recommendations for future works which are; literature review, methodology and case study. Therefore, with no loss of generality of all chapters in the thesis, details on those aforementioned points are intensively discussed in sections of this chapter.

7.2 Contributions

These contributions are underpinned by all publications [List of Publication (Page 247)], which indicate the novelty and durability of the study in improving and enhancing the theory of fuzzy sets and MCDM techniques.

The primary contribution of this study towards literature of fuzzy sets is the development of a novel intuitive multiple centroid defuzzification method and the development of novel hybrid fuzzy MCDM model that consist of consistent fuzzy preference relations and fuzzy TOPSIS. In developing the intuitive multiple centroid defuzzification method, a novel direction of computing the centre point of fuzzy numbers is proposed where it is calculated based on the median point of separated parts of fuzzy numbers representation. This kind of centroid method for fuzzy numbers is suggested in this study because it enhances the capability of the proposed method to give correct centre points in all possible fuzzy sets including type-1 fuzzy sets, type-2 fuzzy sets and z-numbers as highlighted in Chapter 4 of the thesis. Several theoretical properties of the novel centroid method are introduced and there are several established defuzzification properties proposed by (Roychowdhury & Pedrycz, 2001) are fulfilled in this study to strengthen the capability of the method of centroid fuzzy numbers appropriately. Then, the novel hybrid fuzzy MCDM model is developed that incorporated with the novel intuitive multiple centroid method. Along with this contribution, this study suggests the sensitivity analysis technique in order to validate the proposed hybrid fuzzy MCDM model with others established hybrid fuzzy MCDM models to evaluate the robustness and consistency of the models.

Aforementioned in Chapter 2, there are three kinds of fuzzy numbers found in the literature of fuzzy sets; they are type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. It is worth noting here again that the intuitive multiple centroid method is developed for type-1 fuzzy sets and extended to type-2 fuzzy sets and z-numbers as well. Also, the development of hybrid fuzzy MCDM model is proposed in order to solve human based decision making problems under fuzzy environment. Chapter 4 presents the first contribution in methodology in detail the process of development of intuitive multiple centroid defuzzification method for fuzzy sets that consist type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Reviewing the advantages and limitations of the established centroid defuzzification methods for fuzzy sets was very useful task to investigate the proposed intuitive multiple centroid method that should be adopted in this research work and used in proposed hybrid fuzzy MCDM model. A novel intuitive multiple centroid for fuzzy sets is developed in this chapter that covers type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Furthermore, the theoretical and empirical validations are broadly discussed in this chapter. All relevant properties are considered on differentiating fuzzy numbers for justifying the applicability of centroid for them.

In Chapter 5, the novel hybrid fuzzy MCDM model is developed that is incorporated with the novel intuitive multiple centroid and the extension versions. In the analysis, the proposed hybrid fuzzy MCDM model contributes significant benchmarking examples of type-1 fuzzy sets where it extends for type-2 fuzzy sets and z-numbers. This chapter discusses the development of hybrid fuzzy MCDM model based on the extended method of consistent fuzzy preference relations that is used to derive the weight of criteria while the extended of fuzzy TOPSIS is used to rank the alternatives. These proposed models are capable to apply for all possible fuzzy sets as the linguistic terms. Computation and description details of results and sensitivity analysis are discussed in Chapter 6.

Contributions cover under this section is described in detail by Chapter 6 of the thesis. In Chapter 6, the hybrid fuzzy MCDM model that consist of extended of consistent fuzzy preference relations and extended of fuzzy TOPSIS based on proposed intuitive multiple centroid defuzzification method is applied to staff selection in MESSRS Saprudin, Idris & Co, Malaysia. It has to be noted here that, this case study is considered all possible types of fuzzy sets consist of type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Type-1 fuzzy sets are concerning the imprecision, while type-2 fuzzy sets and z-numbers concerning regarding the footprint of uncertainties and uncertain environment (reliability) respectively. Consideration of this case study in this thesis reflects the capability of the proposed methodology to not only solve the problems correctly such that the results are consistent with human intuition but also solving any related with different fuzzy sets involving type-1 fuzzy sets, type-2 fuzzy sets and z-number effectively.

Generally, contributions to knowledge by this study are described in detail by this section. It is worth pointed out here that some contributions are prepared for knowledge enhancement while some are done for decision making purposes. In the following section, the concluding remarks of this study are provided.

7.3 Concluding Remarks

This section discusses the concluding remarks of this study. These concluding remarks summarised all works done in chapters provided in the thesis. The concluding remark for literature review covers with descriptions of established works on centroid defuzzification methods and hybrid fuzzy MCDM models.

In the literature review chapter, gaps of established for centroid methods are identified where these centroid methods have incapability to give appropriate centre values while dealing with several different conditions in fuzzy numbers such as non-normal cases, asymmetry cases and singleton cases. These aforementioned gaps by established centroid methods are analysed and solved by the first objective of this study. This indicates that the first objective of this study is successfully accomplished where it caters off all limitations of the established works on centroid defuzzification methods by developing a new centroid methodology for type-1 fuzzy sets also the extensions for type-2 fuzzy sets and z-numbers. While in the literature, several gaps have been identified in applying hybrid fuzzy MCDM model in human based decision making problems where most of the researchers or practitioners prefer to use triangular fuzzy numbers instead of trapezoidal fuzzy numbers. To gain an intuitive insight into this solution, the trapezoidal fuzzy numbers is more complete to represent human perception under fuzzy environment. Consequently, the implementation of trapezoidal fuzzy numbers have been raised lately in literature. Another gap regarding fuzzy MCDM techniques here is, most of the researchers in literature prefer to use type-1 fuzzy sets rather than type-2 fuzzy sets and z-numbers. This is because the implementation of type-2 fuzzy sets and z-numbers are more complicated compared to type-1 fuzzy sets,

This concluding remark for methodology covers description on the development of the hybrid fuzzy MCDM model that is incorporated with intuitive multiple centroid defuzzification method. In Chapter 4, the methodology for intuitive multiple centroid defuzzification method is developed where it consist of intuitive multiple centroid for type-1 fuzzy sets, extension of intuitive multiple centroid for interval type-2 fuzzy sets and extension of intuitive multiple centroid for z-numbers. Along with these proposed methods development, theoretical and empirical validations are outlined in this chapter. The theoretical validation considers relevant established and new properties for centroid defuzzification purposed while the empirical validation takes into account the comparative studies for consistency against several established centroid methods for type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. Based on these descriptions, the third and sixth objectives of this study are achieved. Besides, the proposed hybrid fuzzy MCDM model outperforms other established model for all possible fuzzy sets.

This concluding remarks covers description on the case study of the thesis. In Chapter 6, staff selection case study is considered and evaluated using the proposed hybrid fuzzy MCDM methodology developed in this study. This case study is considered all possible cases of fuzzy sets that covers type-1 fuzzy sets, type-2 fuzzy sets and z-numbers. All of these fuzzy sets are utilised which are type-1 fuzzy sets are concerning the imprecision, while type-2 fuzzy sets and z-numbers concerning regarding the footprint of uncertainties and uncertain environment (reliability) respectively. The proposed methodology developed in this study produces consistent and efficient ranking results for the case study examined. This implies that the last objective is accomplished.

Overall, the concluding remarks of this study are described in detail by this section where this reflects by the successfulness in accomplishing all objectives set up by this study. In the following section, limitations in this study are discussed.

7.4 Limitations and Recommendation for Future Works

This section discusses some limitations and recommendation for future works of this study where they are figured out from the proposed of intuitive multiple centroid and the proposed of hybrid fuzzy MCDM model. Thus, in this respect, the limitations are highlighted through literature review, methodology and case study as mentioned earlier.

Citing prior research study from the basis of literature review and help lay the foundation for understanding the research problems' investigating. Frankly speaking, discovering the limitations in literature can be served as an important opportunity to identify new gaps in the literature and to describe the need for further research. In this research work, there is a shortage of studies on the combination of two or more decision making techniques, especially fuzzy

MCDM since it is new and developing field. Thus, there is a limited numbers of studies that developments and applications of two or more fuzzy MCDM techniques in solving decision making problems. Moreover, so far there is no hybrid fuzzy MCDM model proposed for z-numbers application since it can be categorized as the latest product of fuzzy numbers.

The limitations for methodology, firstly covers description on the proposed of intuitive multiple centroid defuzzification method that is not capable in dealing with non-linear fuzzy numbers. This is due to the fact that the proposed of intuitive multiple centroid method considers only linear fuzzy numbers as they are easy to deal with as compared to non – linear fuzzy numbers. In addition, majority of established centroid methods consider only linear type of fuzzy numbers in their studies. Thus, consideration of non – fuzzy linear numbers cases are neglected in this case. Secondly, with respect to the proposed hybrid fuzzy MCDM model, this study suggests to analyse the dependency of criteria and the evaluation for decision makers. In literature, the researchers state that when a decision is to be made, there is a need to look at all the potential relationships or dependencies among the decision elements. The good problem structuring for MCDM would seek to study the dependence between the decision criteria. As a consequence, in recent years, investigating dependency in MCDM problems has become more important. Another important phase in decision making process is the evaluation towards decision makers. This is because they are human, human makes mistakes. Different people would give different judgement. (Santos & Camargo, 2013) proposed influence degree in multi criteria group in order to get the evaluation for decision makers based on their experience works.

It is important to restrict the discussion to limitations related to the case study under investigation. The limitations for case study can be seen in discussion in sections from Chapter 6, where the proposed MCDM model for z-numbers produces results for sensitivity analysis evaluation lesser that type-1 and type-2 fuzzy sets. This shows that the proposed hybrid fuzzy MCDM model for z-numbers is less robust and less stable than type-1 and type-2 fuzzy sets. Here, this study suggest to develop proper reliability linguistic scales in order to give better interaction with fuzzy sets. As discussed in previous point in methodology, the consideration of decision makers' evaluation is one of the important aspect to get better model. This evaluation would study the tendency of decision makers in evaluating either the criteria or alternatives for any case study. Defining the list of the selection criteria and generating their important weights are based on previous study. Hence, it is limited in that it cannot be generalised straightaway to the decision makers' viewpoints. More research is required to investigate the feasibility of achieving research finding decision makers' perspectives.

Generally, the limitations of this study are described in detail by this section where this reflects by the successfulness in accomplishing all objectives set up by this study. In the following section, recommendations for future works are discussed.

The recommendations focus on improvising the theoretical and empirical qualities in the theory of fuzzy sets and MCDM. With this respect, recommendations for future work of this study are pointed out through literature review, methodology and case study.

In this research study, a new hybrid fuzzy MCDM model that based on intuitive multiple centroid defuzzification method is developed. There are two novelty development here which are; 1) the development of intuitive multiple centroid defuzzification method and; 2) the development of hybrid fuzzy MCDM model that is incorporated with intuitive multiple centroid method. Even though the proposed methodology gives good theoretical and empirical results, it is recommended for future work that other methods that capable to effectively capture human intuition are thoroughly explored. This recommendation is purposely suggested by this study because when more detailed investigations on fuzzy sets are made, more complex cases of fuzzy sets are figured out, thus indicates that a more commanding hybrid fuzzy MCDM model is required in this case. Therefore, exploring for suitable methods in the literature or real decision making case study are necessary as this is crucial for human based decision making purposes. Another recommendation by this study is on the utilisation of other types of fuzzy set apart from linear. As far as researches on fuzzy MCDM methods are concerned, majority of them use linear type of fuzzy sets in their analysis. Thus, consideration of non-linear fuzzy sets in the future works suggest the representations of fuzzy sets is more generic and practical as not all cases are well represented by linear type of fuzzy sets.

Methodology is crucial for any research study because an unreliable method produces unreliable results. As a consequence, it undermines the value of interpretations of findings. In order to make sure the research has originality and novelty, the methodology must reliable enough to get good results. In this study, supposedly, we need to add two more related phases in order to get reliable methodology. As mentioned in Section 7.4, the dependency of each criteria and the evaluation for decision makers should be added in the proposed hybrid fuzzy MCDM model (refer Section 5.2.2). Therefore, other two MCDM techniques are needed in order to evaluate the dependency of criteria and evaluation for decision makers. The chronological evidence suggest that z-numbers are not yet established in the literature of fuzzy sets as compared to type-1 and interval type-2 fuzzy sets. This study recommends both theoretical and empirical frameworks of z-numbers are extensively explored. This is due to z-numbers is more practical than type-1 and interval type-2 fuzzy sets in terms of representation. Thus, finding the suitable ways to deal with z-numbers is necessary. With respect to centroid defuzzification methodology, the only way to defuzzify the z-numbers is to reduce them into type-1 fuzzy sets. This implies that z-numbers are not effectively dealt as this affect the representation of z-numbers. So, this study recommends for future works that methods that are capable to simultaneously defuzzify the z-numbers is developed and solve various human based decision making problems.

The recommendation for future plan covers description on the case study of the thesis. Chapter 6 discusses the staff selection case study where from the observation, this case study is quite direct or straight forward. This study recommends for future work that the proposed hybrid fuzzy MCDM model should be applied in bigger or complex case studies. Due to the analysis results, the proposed models give better results than other established in literature. Therefore, application in bigger case studies will stimulate the improvement in human based decision making problems. Also, this study recommends the improvement for methodology as previous section to add two more stages for MCDM techniques, then the applicability of the improvising proposed model would give better contribution to human based decision making problems under fuzzy environment appropriately.

Overall, the recommendations for future works of this study are described in detail by this section where this reflects by the successfulness in accomplishing all objectives set up by this study. The recommendations are provided in order to improvise this research study for human based decision making problems.

7.5 Summary of the Chapter

In conclusion, the contributions, the concluding remarks, limitations and recommendations for future works by this study are highlighted. Therefore, the thesis ends its discussion by citing all references used throughout the thesis which are provided next after this chapter.

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List of Publications

K. M. N. Ku Khalif and A. Gegov, "Hybrid fuzzy MCDM model for z-numbers using intuitive vectorial centroid," Journal of Intelligent and Fuzzy Systems, 2017 (In print).

K. M. N. Ku Khalif, A. Gegov and A. S. A. Bakar, "Z-TOPSIS using Fuzzy Similarity for Performance Assessment," *in proceeding on IEEE International Conference on Fuzzy Systems*, Naples, Italy, 2017. (Accepted).

K. M. N. Ku Khalif and A. Gegov, "Implementing adaptive multiple centroid in Bayesian logistic regression for interval type-2 fuzzy sets," *International Joint Conference, IJCCI 2015, Lisbon, Portugal, November 12-14, 2015,* Revised Selected Papers, Computational Intelligence, Springer.

K. M. N. Ku Khalif and A. Gegov, "Generalised fuzzy Bayesian network with adaptive Multiple Centroid,"*International Fuzzy System Association, European Society for Fuzzy Logic and Technology,* page 757-764, Gijon, Spain, 2015.

K. M. N. Ku Khalif and A. Gegov, "Bayesian logistic regression using Multiple Centroid for interval type-2 fuzzy sets," *in proceeding on 7th International Conference Fuzzy Computation Theory and Application,* page 69-79, Lisbon, Portugal, 2015.

A. S. A. Bakar, **K. M. N. Ku Khalif** and A. Gegov, "Ranking of interval type-2 fuzzy numbers based on centroid point and spread," *in proceeding on 7th International Conference Fuzzy Computation Theory and Application,* page 131-140, Lisbon, Portugal, 2015.

A. M. Yaakob, **K. M. N. Ku Khalif** and A. Gegov, S. F. A. Rahman,"Interval type 2-Fuzzy Rule based system approach for selection of alternatives using TOPSIS," *in proceeding on 7th International Conference Fuzzy Computation Theory and Application,* page 112-120, Lisbon, Portugal, 2015.

Appendix A - Questionnaire

University of Portsmouth

SCHOOL OF COMPUTING

Decision Maker no:

Candidate no:

This questionnaire is mainly address to decision maker in MESSRS Sapruddin, Idris & Co in Malaysia. It is used to see the decision maker's evaluation representing the scoring of potential candidates with respect to some criteria when selecting of the most appropriate potential employee from several potential candidates.

Thank you for agreeing to provide information regarding your thoughts for selecting the group of potential candidates.

The following questions should not take more than 10 minutes:

1) For pair matrix table given below, think about which criterion has a great influence (is more important) with respect to another given criterion, by suing the scale shown in the table below:

1	Equally important
2	Intermediate value
3	Moderately more important
4	Intermediate value
5	Strongly more important
6	Intermediate value
7	Very strong more important
8	Intermediate value
9	Extremely more important

With regard to the weightage of criterion, which of the pair of criteria, given below matrix table is more important? and how much more? (Tick in the appropriate box)

Criterion	Emotional Steadiness	Oration	Past experience	Personality	Self-confidence
Emotional Steadiness	1				
Oration		1			
Past experience			1		
Personality				1	
Self-confidence					1

Question 2 and 3 are referred to the same question with different scales. Both scales are used for study purposes.

2) For candidate's evaluation, what scores do you assign to each candidate given below with respect to? by using the scale shown in the table below:

1	Absolutely-low (AL)
2	Very-low (VL)
3	Low (L)
4	Fairly-low (FL)
5	Medium (M)
6	Fairly-high (FH)
7	High (H)
8	Very-high (VH)
9	Absolutely-high (AH)

a. Emotional steadiness

☐	☐	☐	☐	☐	☐	☐	☐	☐
AL	VL	L	FL	M	FH	H	VH	AH

b. Oration

☐	☐	☐	☐	☐	☐	☐	☐	☐
AL	VL	L	FL	M	FH	H	VH	AH

c. Past experience

AL VL L FL M FH H VH AH

d. Personality

AL VL L FL M FH H VH AH

e. Self-confidence

AL VL L FL M FH H VH AH

3) For candidate's evaluation, what scores do you assign to each candidate given below with respect to? by using the scale shown in the table below:

1	Very-low (VL)
2	Low (L)
3	Medium-low (ML)
4	Medium (M)
5	Medium-high (MH)
6	High (H)
7	Very-high (VH)

f. Emotional steadiness

VL L ML M MH H VH

g. Oration

VL L ML M MH H VH

h. Past experience

VL L ML M MH H VH

i. Personality

VL L ML M MH H VH

j. Self-confidence

VL L ML M MH H VH

4) Reliability of scores given from question 1, 2 and 3.

1	Very-low (VL)
2	Low (L)
3	Medium (M)
4	High (H)
5	Very-high (VH)

a. Emotional steadiness

VL L M H VH

b. Oration

VL L M H VH

c. Past experience

VL L M H VH

d. Personality

VL L M H VH

e. Self-confidence

VL L M H VH

Appendix B - Confirmation Letter for PhD Research Placement

SAPRUDIN, IDRIS & CO

ADVOCATES & SOLICITORS • PEGUAMBELA & PEGUAMCARA

Partners :
IDRIS BIN MD. ZAIN
LL.B (Hons) (UiTM),
DSLP (IIUM), MCL (IIUM)

SARAEZA HJ. ABAS
LL.B (Hons) IIUM

No. 42B (2nd Floor)
Jalan SS22/21, Damansara Jaya
47400 Petaling Jaya
Selangor Darul Ehsan

Tel : 03 - 7725 2429 / 7726 2429
Fax : 03 - 7726 9249

Email: idrismdzain@gmail.com

29th September 2015

To whom it may concern,

CONFIRMATION LETTER FOR PHD RESEARCH PLACEMENT

With all due respect, the above is referred.

This letter is to confirm that Ku Muhammad Naim KU KHALIF (Malaysia ID: 860908-23-6137) has been employed for his PhD research placement with MESSRS SAPRUDIN, IDRIS & CO since 06 July 2015 until 26 September 2015.

During his tenure of employment, he has displayed a unique ability to identify and solving problems. He has been instrumental in streamlining in analysing new staff selection. He has been assigned to do research regarding selecting and hiring a capable and dedicated staff with lowest risk of resigning. He used the application of fuzzy logic and Multi-Criteria Decision Making (MCDM) in dealing with uncertain environment. Currently, he's 3rd year student doing PhD at University of Portsmouth, UK. The details of him are shown as follows:

Name:	Ku Muhammad Naim Ku Khalif
Student ID:	682796
University:	University of Portsmouth, UK
Department:	Computational Intelligence, School of Computing
Course:	PhD in Computing
Research Placement Project:	Integrated fuzzy AHP-TOPSIS based on defuzzification Vectorial Centroid
Supervisor:	Dr Alexander Gegov (+447876628291)

I hope this information proves useful.

Yours faithfully,
MESSRS. SAPRUDIN, IDRIS & CO

Idris Bin Md Zain
Advocate & Solicitor
Partner

THIS FILE IS HANDLED BY OUR PETALING JAYA, SELANGOR OFFICE
(PLEASE QUOTE REFERENCE WHEN REPLYING)

Seremban office : No. 161 & 162, 2nd Floor, Wisma Dato' Rashid, Jalan Za'aba, 70100 Seremban, Negeri Sembilan Darul Khusus,
Tel : 06 -762 7095, 962 9851 & 767 9572 Fax : 06 - 762 0873 Email: *sapco19@yahoo.com*

Appendix C - Certificate of Ethics Review

Certificate of Ethics Review

Project Title:	Intuitive Defuzzification Method of Fuzzy Sets for Machine Learning and Multi-Criteria Decision Making (MCDM) Techniques under Uncertain Environment
User ID:	682796
Name:	Ku Muhammad Naim Ku Khalif
Application Date:	21/09/2015 07:15:15

You must download your referral certificate, print a copy and keep it as a record of this review.

The FEC representative for the School of Computing is Carl Adams

It is your responsibility to follow the University Code of Practice on Ethical Standards and any Department/School or professional guidelines in the conduct of your study including relevant guidelines regarding health and safety of researchers including the following:

- University Policy
- Safety on Geological Fieldwork

All projects involving human participants need to offer sufficient information to potential participants to enable them to make a decision. Template participant information sheets are available from the:

http://bit.ly/UoPEthics • Univeristy's Ethics Site (Participant information template).

It is also your responsibility to follow University guidance on Data Protection Policy:

- General guidance for all data protection issues
- University Data Protection Policy

SchoolOrDepartment: SOC
PrimaryRole: PostgraduateStudent
SupervisorName: Dr Alexander Gegov
HumanParticipants: Yes
ParticipationBeyondAnsweringQuestionsOrInterviews: Yes
ParticipantInformationSheets: I did my research placement at legal firm in Malaysia, which at Saprudin, Idris & Co, Damansara Jaya, Malaysia. I've spoke and made discussion with senior executive there regarding this placement before and he agreed to take me to do my research. I was supervised by the senior executive Mr Ilham Abadi Idris. I've assigned to do research regarding selecting and hiring a capable and dedicated staff with the lowest risk of him/ her resigning. Work stress, inexperienced worker or unable to adapt with the environment might be the cause of resignation. This firm experienced at least two staffs resigned within three years operation. Once a staff resigned, recruitment new staff is not only time consuming but also involves financial implication. So, to tackle this problem, the application of fuzzy and several other

techniques in machine learning will be used there.

Certificate Code: C8FB-5298-31A2-1DDA-B2C6-8093-BE52-6035 Page 1

ParticipantConfidentiality: They didn't give the real name or details for the decision makers and potentials staff to be selected in the firm because that is confidential for them. So, I don't have the confidential data. What I did are collecting the evaluation data from the decision makers in evaluating potentials staff using fuzzy scales that I constructed. Then, I have to analyse the evaluation data collected from the decision makers by using replacement names (example: Decision maker 1, Decision maker 2...Alternative 1, Alternative 2..etc)
InvolvesNHSPatientsOrStaff: No
NoConsentOrDeception: No
CollectingOrAnalysingPersonalInfoWithoutConsent: No **InvolvesUninformedOrDependents:** No
DrugsPlacebosOrOtherSubstances: No
BloodOrTissueSamples: No
PainOrMildDiscomfort: No
PsychologicalStressOrAnxiety: No
ProlongedOrRepetitiveTesting: No
FinancialInducements: No
PhysicalEcologicalDamage: No
HistoricalOrCulturalDamage: No
HarmToAnimal: No **HarmfulToThirdParties:** No **OutputsPotentiallyAdaptedAndMisused:** No
Confirmation-ConsideredDataUse: Confirmed **Confirmation-ConsideredImpactAndMitigationOfPontentialMisuse:** Confirmed
Confirmation-ActingEthicallyAndHonestly: Confirmed

Supervisor Review

As supervisor, I will ensure that this work will be conducted in an ethical manner in line with the University Ethics Policy.

Supervisor signature:

Date: