

**The Study, Modelling and Implications of Realised Volatility for  
Chinese Stock Index Futures and Spot Markets**

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Chinese Stock Index Futures and Spot Markets**

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The thesis submitted in partial fulfillment of the requirements of the  
Portsmouth University of the Degree of Doctor of Philosophy

## **Confirmation**

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## Table of Contents

1.1. Background.....	11
1.2. Motivations.....	16
1.3. Aims and Structure.....	17
1.4. Contributions.....	21
Chapter Two: Volatility Models.....	24
2.1. Conditional Volatility Model.....	25
2.2. Realised Volatility Model.....	34
2.3. Chapter Summary.....	48
Chapter Three: Realised Volatility Characteristics.....	49
3.1. Original Research Data.....	50
3.2. Microstructure Noise Revise Technique.....	53
3.3. Daily Volatility Jumps.....	66
3.4. Intraday Volatility Jumps and Periodicity.....	77
3.5. Realised Volatility Long Memory Characteristics.....	88
3.6. Chapter Summary.....	102
Chapter Four: Volatility Spillover and Forecast.....	103
4.1. Realised Volatility Transmission.....	104
4.2. Markov Regime-switching Model.....	123
4.3. Realised Volatility Model and Forecast.....	148
4.4. Chapter Summary.....	171
Chapter Five: Realised Volatility Implications.....	173
5.1. Implication on Hedge Ratio.....	174
5.2. Implication on Value-at-Risk.....	190
5.3. Chapter Summary.....	206
Chapter Six: Concluding Remarks.....	208
6.1. Key Takeaways.....	208
6.2. Limitation.....	210
6.3. Further Research Recommendations.....	210
6.4. Implications.....	211
References:.....	213

## Figures

Figure 1. Classification of Volatility Models.....	11
Figure 2. 5 mins Data Correlogram.....	64
Figure 3. Jump Test Results.....	72
Figure 4. Estimated Results.....	85
Figure 5. Statistic Descriptions.....	95
Figure 6. Autocorrelation Function.....	98
Figure 7. Conditional Covariance Graphs for the Two Models.....	116
Figure 8. Cointegration Residual.....	118
Figure 9. Two Types Estimated Realised Volatilities.....	138
Figure 10. Dynamic Markove Regime-switching Models.....	140
Figure 11. Out-of-sample ARFIMA Forecast Results.....	159
Figure 12. In-sample Forecast Results.....	168
Figure 13. VaR Graph Results.....	200

## Tables

Table 1. Description of CSI 300 Futures Contract.....	51
Table 2. Original Logarithm Data Description.....	51
Table 3. Data Description of Logarithm Returns.....	53
Table 4. Microstructure Noise Revise Technique.....	55
Table 5. Calculated Values for Each Interval.....	63
Table 6. Data Description.....	73
Table 7. Total Number of Jumps Detected under <b>H0</b> (No jumps) versus <b>H1</b> (Jumps exist)..	75
Table 8. Summary of Key Concepts.....	78
Table 9. Intraday Jumps Test under <b>H0</b> (No jumps) versus <b>H1</b> (Jumps exist).....	87
Table 10. ADF and KPSS Statistics.....	96
Table 11. GPH Test Results.....	99
Table 12. ADF and KPSS Statistics.....	99
Table 13. GPH Test Results.....	100
Table 14. Intraday Conditional Volatility Results.....	114
Table 15. Realised Volatility Results.....	117
Table 16. Daily Conditional Volatility Results.....	120
Table 17. Regime Classification Details.....	142
Table 18. Estimated Coefficients.....	144
Table 19. Realised Volatility Models.....	150
Table 20. Model Selection for ARFIMA.....	156
Table 21. Estimated ARFIMA Results.....	157
Table 22. Out-of-sample Forecast Standard Errors.....	160
Table 23. HAR Model Estimation Results.....	161
Table 24. HAR-J-MS Model.....	164
Table 25. Daily Logarithm Returns' Data Description.....	186
Table 26. Estimated Results.....	187
Table 27. Calculated Hedge Efficiency Value.....	188
Table 28. Kupiec LR Test Results.....	204

## Abbreviations

- ARCH: Autoregressive Conditional Heteroskedasticity
- ARFIMA: Autoregressive Fractionally Integrated Moving Average
- ARMA: Autoregressive Moving Average
- EMH: Efficient Market Hypothesis
- EWMA: Exponentially Weighted Moving Average Method
- GARCH: Generalised Autoregressive Conditional Heteroskedasticity
- HAR: Heterogeneous Autoregressive
- HAR-J-MS: Heterogeneous Autoregressive-Jumps- Markov Regime-switching
- IV: Integrated Volatility
- MLF: Maximum Likelihood Function
- MS: Markov Regime-switching Model
- RV: Realised Volatility
- SHSE/SZSE-Shanghai Stock Exchange/Shenzhen Stock Exchange
- VAR: Vector Autoregression
- VaR: Value-at-Risk
- VECM: Vector Error Correction Model



## Abstract

Realised volatility is a recently developed measure (Andersen et al., 2001), and it has attracted the attention of numerous economic researchers. This thesis aims to explore how realised volatility can be applied in Chinese capital markets in the area of shares and stock index futures, to investigate the applicability of an optimal realised volatility forecast model and to examine the implications of realised volatility on optimal hedge ratio and Value-at-Risk (VaR) performance.

The empirical results indicate four important realised volatility characteristics of the selected markets. First, the optimum data frequency intervals for applying realised volatility models are equal to five minutes. Second, there is empirical support of the hypothesis that daily volatility jumps exist, and that there are significant intraday volatility jumps and periodicity effects, where logarithm realised volatility shows resilient long memory characteristics. Two important issues (volatility transmission and Markov regime-switching) are examined before modelling realised volatility. Third, based on these results, this thesis proposes, for the first time in economic history, a Heterogeneous Autoregressive-Jumps- Markov Regime-switching (HAR-J-MS) model, which combines the daily volatility jump components and regime-switching effects. The empirical results indicate the superior forecasting power of this new proposed model. Fourth, the empirical results suggest realised volatility performs better on optimal hedge ratio and VaR compared to other models.

This thesis contributes to the current literature in four respects. First, it provides fresh and timely evidence on the features of both the spot and futures financial markets in the largest emerging economy: China. Second, this thesis not only investigates China's financial markets from the traditional perspective of conditional volatility, but also from the relative new perspective of realised volatility (Andersen et al., 2001). Third, it investigates volatility spillover by using both intraday and daily data. Fourth, in terms of methodology, this thesis proposes, for the first time, a HAR-J-MS model to combine the influence of daily volatility jumps and a Markov regime-switching based on a HAR framework, which constitutes a methodological innovation.

Overall, this thesis is a comprehensive research paper on realised volatility. To the best of the author's knowledge, there are few studies that apply realised volatility on Chinese stock index futures and spot markets. This thesis fills this gap in the literature.

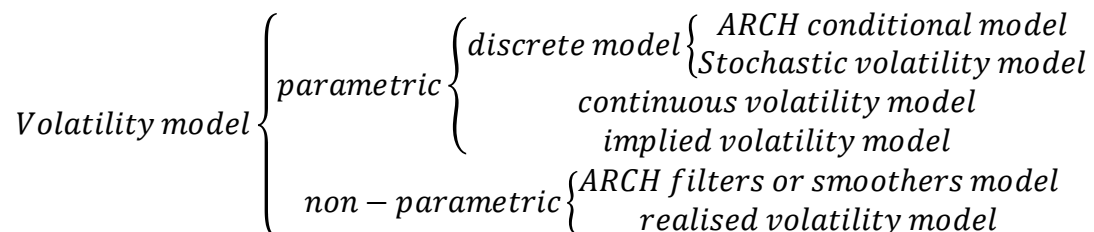
## Chapter One: Introduction

### 1.1. Background

Since the seminal paper of Black and Scholes (1973), volatility has gained an important position in current financial research, especially in the areas of hedging, option pricing, and risk management. However, the Black and Scholes model assumes a constant volatility, which does not have enough empirical support. In real financial markets, volatility does not only change among different time periods, but also shows numerous special characteristics, such as long memory, clustering effect, asymmetric effect, etc. (Ghysels et al., 1996; Ghysels and Sinko, 2011). In recent years, volatility has become one of the most investigated topics in empirical financial and time series econometric analysis areas.

From an intuitive point of view, volatility refers to the shake range of one stochastic process. However, when comparing volatility with asset return, volatility is an unobservable latent variable. Until now, there has been no widely accepted definition of volatility, and only some relative measures of volatility exist. Two wide categories are used to divide volatility estimation methods: parametric method and non-parametric method. The specific classifications of volatility estimation methods are represented in Figure 1.

Figure 1. Classification of Volatility Models.



Currently, most volatility studies focus on parametric estimation methods. These parametric methods treat volatility as an unobservable latent variable, and build an ex-ante econometric volatility forecast model. That is to say, most of the existing papers focus on building conditional volatility models. According to different time interval  $h$ , these parametric models can be divided into discrete models ( $h > 0$ ) and continuous models ( $h \rightarrow 0$ ). There are two representative discrete models: the ARCH model only assumes the recent period of volatility is unobservable, but the SV model assumes the whole historical conditional volatility is unobservable.

Continuous models are normally written in stochastic differential equation format, which then transforms an instantaneous volatility model into continuous diffusion process and diffusion coefficients. The method used to estimate coefficients will be the maximum likelihood estimation method, whereas the stochastic volatility mainly focuses on instantaneous volatility estimation. Departing from these models, there is another special parametric volatility estimation method, which is called implied volatility. The implied volatility is estimated using Black and Scholes' (1973) option pricing model, and assumes asset price follows a random walk continuous process.

There are four general problems with the parametric volatility estimation method: First, the ex-ante volatility estimation relies highly on the underlying econometric model, and the econometric model assumptions are not always the same as the real market situation. Second, the parametric estimation method cannot adequately explain the empirical results of asset return. For example, empirical evidence shows asset returns do not follow a normal distribution, and have a fat tail feature. Third, estimating the coefficients using the parametric method is very complicated, especially for the SV model. Fourth, it is difficult to expand the

parametric method into multiple dimensions. In some cases, it is almost impossible for the parametric volatility estimation method to expand into multiple dimensions.

Based on these drawbacks of the parametric volatility model, much research now focuses on non-parametric volatility estimation models. These non-parametric models are ex-post nominal volatility based on asset real return, and can directly provide accurate volatility measurement without any special functional form. According to the different time interval of  $h$ , the non-parametric estimation models can be divided into two broad categories: ARCH filters and smoothers model and realised volatility model. The first category measures the instantaneous volatility ( $h \rightarrow 0$ ), and it relies heavily on the assumption that asset price follows continuous sample paths. The ARCH filters models only apply the past returns' information, but the smoothers model applies both past and futures returns' information. All these instantaneous volatility models assume there are numerous observations during a small time interval. Obviously, the real life data cannot satisfy such a strong assumption. Hence, the ARCH filters and smoothers model will suffer numerous biases in ex-post volatility estimation.

Comparing the ARCH filters and smoothers model, realised volatility makes full use of intraday information, and can adequately estimate the nominal latent volatility at time interval  $[t - h, t]$ . In the normal situation  $h = 1$ , which measures daily nominal latent volatility. In fact, the realised volatility is the sum of high-frequency asset returns' square root at a given interval  $[t - h, t]$ . The concept of realised volatility can be traced back to Merton's (1980) innovative work. He pointed out that the sum of intraday high-frequency returns' square can infinitely approach Brownian motion integrated volatility. He believed that the

data sample frequency is irrelative to returns' mean value, but will significantly influence return variance estimation.

Due to technique problems on the storage and collection of high-frequency data, the research on realised volatility has been developed very slowly in previous years. Realised volatility had been only recently applied, as a result of developments in high-frequency data collection and storage techniques. Andersen et al. (2001, 2003), Barndorff-Nielsen and Shephard (2002a, b), interalia, proposed realised volatility estimation method based on Merton's (1980) work, and proved that realised volatility has numerous desirable statistical characteristics. First, the ex-post realised volatility (RV) can be an unbiased estimator of daily volatility when the mean value of return equals zero which nearly holds true for the long period stochastic process of return, requiring also sample frequency to be high enough. Second, realised volatility is the consistent non-parametric estimation of daily nominal volatility. Third, the logarithm realised volatility approximates normal distribution. Overall, it is not only easy to calculate, but also does not rely on any econometric models. Moreover, realised volatility is observable and researchers can directly assess its characteristics. It is also easy to expand realised volatility models into multiple dimensions. These characteristics demonstrate that realised volatility is a better estimation method compared to parametric methods.

Theoretically, Andersen et al. (2001) has already justified the use of realised volatility if the data sample frequency is high enough. However, the microstructure noise effect will increase along with higher data frequency, wherein the asset price will not follow a continuous diffusion process. In the tick- by-tick data, real data does not follow the classical assumption of realised volatility: the asset price should be a no-friction continuous process. In the realised volatility estimation, it is good to apply more high-frequency data to get more market

information on the one hand. On the other hand, higher frequency will lead to an increase in microstructure noise effect, and consequently increase bias in realised volatility estimation. These two sides constitute the famous bias-variance dilemma. Due to the effect of microstructure noise, there is no simple estimation method for realised volatility.

Currently, the most popular method for solving microstructure noise is the optimal sampling frequency method proposed by Andersen et al. (2005), and the key concept of this method is to select the highest data frequency combined with the lowest microstructure noise effect. However, this optimal sampling method has three obvious drawbacks. First, this method will undervalue realised volatility, and this undervaluing will increase along with data frequency increases. Second, different optimal data sample frequency exists for different markets, different periods, and different assets. Third, optimal sampling frequency will lose numerous data, and lead to the loss of numerous statistical information. For example, Andersen et al. (2000) analysed the high-frequency data for the foreign exchange market. They pointed out that a 5 minute interval is the optimal sampling frequency. Hence, only 268 observations were used to calculate daily realised volatility, and most of high-frequency data were wasted.

One of the key purposes of studying realised volatility is to build an accurate volatility forecast model. Based on Andersen et al. (2000), Corsi (2009) proposed a Heterogeneous Autoregressive Model (HAR) based on Muller et al.'s (1997) heterogeneous market hypothesis. The advantages of the HAR model are the simple model structure and the financial theory supporting the econometric method (e.g., Huang et al., 2016). However, the HAR model only considers the differences among different types of investors, but ignores the other influential factors, such as volatility jumps, volatility transmission between futures and spot markets, and structure change effect. To make a more accurate realised volatility

forecast model, this thesis proposes a model called the Heterogeneous Autoregressive-Volatility-Jumps-Markov Regime-switching (HAR-J-MS) model. The empirical results indicate this new proposed model has superior performance compared to other volatility models.

## **1.2. Motivations**

The current study uses two Chinese financial markets for several reasons.

First, China is currently one of the largest economies in the world and one of the main emerging markets having the biggest population in the world (around 1.3 billion), which is more or less one fifth of the entire global population. In 2014, the International Monetary Fund (IMF) ranked China's so-called “socialist market economy” as the largest economy by purchasing power parity, and the second largest economy by nominal Gross Domestic Product (GDP) in the world, in terms of GDP. However, China is also the largest country in terms of population, and of vital importance to the global economy. As the largest emerging market, China has been the engine of the world’s economy over the past three decades, with a GDP growth rate above 8%. According to the IMF, China is the largest manufacturing economy and the global hub for manufacturing in the world. We can see “made in China” all over the shelves in most of the western world supermarkets. China is not only the largest exporter of goods in the world, but also the second largest importer of goods, and the fastest growing consumer market.

Second, China is also in the process of economic transition and industrialisation, which provides a great sample for conducting research on a transitional economy. Since the economic reform and opening-up policy introduced by Xiaoping Deng in 1978, China has seen a surge in the private sector and a steady decline in state-owned enterprises, in terms of



the share of the national economy. Over the same period, the industry sector, together with the service sector, has gradually dwarfed the traditional sectors, such as the agricultural sector. In such a special case, there is a possibility that the economic mechanisms are different from the traditional academic research objects, such as the western developed markets, and the research on China might complement most of the existing literature based on western developed markets.

Finally, despite its size, Chinese market is under-researched whilst at the same time China's financial market is characterised by high volatility and growth. The Shanghai Stock Exchange Composite Index, for example, increased from 100 points in 1990 to about 5000 points up to the 26th May, 2015, giving a 5000% profit, but almost halved itself within six months in 2015, which is uncommon in other countries.

### **1.3. Aims and Structure**

There are three main aims of this thesis: Firstly, to explore the realised volatility's characteristics of Chinese stock index Futures and spot markets. It includes multiple questions: For instance, what is the optimal sampling frequency for the high-frequency data from Chinese stock index Futures and spot markets? Are there daily, intraday volatility jumps and periodicity in Chinese stock index Futures and spot markets? Does the realised volatility have long memory characteristic? Chapter 3 answers these questions with high-frequency data together with multiple conventional econometric tools. Secondly, to investigate the optimal realised volatility forecast model for both two markets. Are there volatility transmission and regime-switching effect for the two markets? Chapter 4 answers this question with ARFIMA long memory model and HAR model. Lastly, to examine the implications of realised volatility on optimal hedge ratio and Value-at-Risk performance of

both two markets. In other words, is realised volatility a good instrument to calculate the optimal hedge ratio and Value-at-Risk? Chapter 5 answers these questions with three econometrics models are applied, including VECM model, BEKK-GARCH model and realised volatility. From the results, it shows that Chinese stock index Futures and spot markets are highly connected. The hedge performance of these three models is compared with the minimum risk measure method. The results indicate realised volatility estimation provides the highest efficiency, and BEKK-GARCH provides the lowest efficiency. Regarding VaR, three econometric models (GARCH, APARCH and RV) are compared, and one VaR performance evaluation method proposed by Kupiec (1995) is also explored.

Chapter two discusses the theoretical backgrounds of two most widely used volatility models, which are conditional volatility model and realised volatility model. There are following two reasons to discuss conditional volatility model in details: Firstly, conditional volatility model has a long history and important position in financial market. Secondly, this thesis will use conditional volatility as a reference model to compare with realised volatility model. The discussions of conditional volatility starts from a standard ARCH model, and then expands to and then expands to GARCH, GARCH-in-Mean, asymmetric GARCH and other three GARCH types models.

The sub-chapter of realised volatility starts with a simple discrete model, and then moves to continuous model without microstructure noise. Consequently, three microstructure noises assumptions are discussed. After this, basic realised volatility characteristics and a revised realised volatility model proposed by Zhang et al. (2005) are investigated. Finally, the literature about realised volatility forecast models is discussed as well.

Chapter three mainly examines the realised volatility characteristics of Chinese stock index Futures and spot markets. This is the first aim as stated in above. This chapter begins from the basic data descriptions of the original 1mins high-frequency data. Then four important issues in realised volatility estimation are examined in details. The first issue is the effect of microstructure noise. By applying optimal sampling frequency method and filter technique, the optimal data sample is selected which is the 5 mins interval for both two markets.

The second issue is the daily volatility jumps. The empirical results suggest applying bi-power realised volatility instead of standard realised volatility estimation. The third issue is the intraday volatility jumps and periodicity. One important result is concluded, that is intraday volatility jumps are highly linked to macroeconomic news release. The last issue is the long memory characteristics of realised volatility. The empirical results suggest both two markets' realised volatility and logarithm realised volatility have long memory characteristic. The results of this chapter will contribute to the realised volatility modelling and forecast.

Chapter four investigates the optimal realised volatility forecast model for selected two markets. Before setting up realised volatility model, two important effects are investigated, which are dynamic volatility transmission and Markov regime-switching effect for selected two markets. The empirical results indicate the following: Firstly, there exists no realised volatility transmission between spot and futures market. This means two markets' realised volatility cannot influence each other's' realised volatility value. Secondly, there exists a strong structure change effect in logarithm realised volatility estimation for both two markets. Consequently, two important types of realised volatility estimation methods are investigated: ARFIMA long memory model and HAR model. This thesis first time proposes HAR-J-MS model to forecast realised volatility for Chinese stock index Futures and spot markets. The

HAR-J-MS model combines the daily volatility jumps and Markov regime-switching effect based on HAR estimation framework. The results indicate HAR-J-MS provides a very good forecasting power compared to other realised volatility forecast models.

Chapter five examines the last aim of this thesis, which is to investigate two important implications of realised volatility. These two implications are optimal hedge ratio and Value-at-Risk performance. The optimal hedge ratio is the key factor in hedge performance by using Futures contract. This chapter compares optimal hedge ratio by using realised volatility and conditional volatility. Three econometrics models are applied, including VECM model, BEKK-GARCH model and realised volatility. From the results, it shows that Chinese stock index Futures and spot markets are highly connected. The hedge performance of these three models is compared with the minimum risk measure method. The results indicate realised volatility estimation provides the highest efficiency, and BEKK-GARCH provides the lowest efficiency.

Another important implication of realised volatility is to estimate VaR value. VaR is currently most widely used risk management method in bank sector and other financial sectors. Three econometric models (GARCH, APARCH and RV) are compared, and one VaR performance evaluation method proposed by Kupiec (1995) is also explored. The empirical results suggest that realised volatility can provide better VaR estimation comparing to other two models. Both of two results indicate realised volatility provide better estimation values for optimal hedge ratio and VaR performance.

On overall, this thesis mainly focuses on three aims. The first one is to investigate realised volatility's characteristics of selected two markets. The empirical results indicate four

important features: 5 mins interval is optimal data frequency; there exists daily volatility jumps indeed; there are significant intraday volatility jumps and periodicity effects; logarithm realised volatility shows strong long memory characteristics.

The second aim is to explore the optimal realised volatility forecast model. Two important issues are examined before modelling realised volatility. The empirical results indicate no realised volatility transmission between Futures and spot markets, and there are strong regime change effects in realised volatility estimation periods. Based on chapter three and chapter four's results, this thesis first time propose an HAR-J-MS model, which combines the daily volatility jumps components and regime-switching effects. The empirical results indicate a superior forecasting power of this new proposed model for both two markets.

The third aim is to investigate two important implications of realised volatility. These two implications are optimal hedge ratio and Value-at-Risk performance. The mainly compared models are based on conditional volatility estimation method. The empirical results suggest realised volatility performs better on optimal hedge ratio and VaR compared to other models. Hence, this thesis suggests realised volatility has more accuracy to estimate daily real volatility compared to conditional volatility methods.

#### **1.4. Contributions**

To the best of our knowledge, this study is the first to examine whether volatility transmission exists between Chinese stock index future and spot markets by using both conditional volatility and realized volatility framework and contribute to the literature in the following ways.

1. The main contribution of this paper is to investigate volatility spillover by using both intraday and daily data. In previous literatures, volatility spillover is either discussed in daily data level (e.g. Feng and Jiang, 2013), or is examined in intraday high frequency data level (e.g. Zhou et al., 2014). In this paper, both two types' data are applied. Nowadays, an investigation into volatility spillover on a daily level cannot capture the dynamic misconstruction volatility influence, while intraday high frequency data will provide an inside view of these two markets' volatility spillover process.
2. The results of this study may be used to compare and contrast these two estimation methods, and explore the research topic deeper. Conditional volatility can be taken as intraday volatility, and realized volatility can be taken as daily aggregated volatility. Meanwhile, intraday high frequency data can be treated as micro-structure data, and daily aggregated data can be treated as macro-structure data, which is new and barely mentioned in the literature.
3. The study applies BEKK-GARCH to investigate volatility spillovers and adopts the VAR approach as a robustness test. Volatility spillovers reflect direction of information flows. We examine volatility spillover by using both daily and intraday data level can distinguish direction of information flow from micro and macro market structure. This study is the first to investigate the dynamic linkage of volatility between these two markets. The study results will shed light on the volatility relationship between these two stock markets and provide risk management guidelines for the two markets' investors.
4. To the best of our knowledge, this thesis is the first to examine whether volatility transmission exists between Chinese stock index futures and spot markets by using both conditional and realised volatility frameworks. In particular, his thesis investigates volatility spillover by using both intraday and daily data, so it enables to compare and contrast these two estimation methods, and explore the research topic deeper. In terms of methodology, this thesis proposes a

HAR-J-MS model for the first time to combine the influence of daily volatility jumps and Markov regime-switching modelling based on a HAR framework, which constitutes a methodological innovation. The empirical results indicate the new proposed HAR-J-MS model can provide superior forecasting power for both futures and spot markets. Other than that, this thesis also employs various other advanced methods, such as BEKK-GARCH, for comparison and robustness purposes.

The rest of this thesis is organized as follows. Chapter 2 reviews the relevant literature on the volatility models. Chapter 3 provides a comprehensive evaluation of the realised volatility characteristics. In Chapter 4, I develop a model to analyze the volatility spillover and forecast problems. In Chapter 5, I examine the implications of realised volatility on the hedge ratios and Value-at-risk. Chapter 6 concludes the thesis.

## Chapter Two: Volatility Models

This thesis investigates Chinese stock index futures and spot markets' realised volatility models and implications. Volatility, as one of the most important factors in financial markets, has been widely studied and developed. In order to understand realised volatility deeply, it is important to appreciate other volatility models, and explore the commonalities and differences between them. The most famous volatility models include the conditional volatility model, implied volatility model, stochastic volatility model and realised volatility model.

This chapter discusses the two most used volatility models in detail: conditional volatility and realised volatility. This thesis is based on the realised volatility framework, and discusses the theoretical background of realised volatility in detail. There are two reasons for discussing conditional volatility model in detail. First, the conditional volatility model has a long history and important position in financial markets. Since the seminal paper of Engle (1982) introduced the Auto Regressive Conditional Heteroskedasticity (ARCH) model, a variety of conditional volatility models have arisen. Currently, there is very wide use of the conditional volatility model in banking, hedge fund and other financial investment companies. Much current research still works on the development of conditional volatility models. Second, this thesis will use conditional volatility as a reference model to compare the realised volatility model to show the advantages and disadvantages of realised volatility. Based on these two reasons, conditional volatility is discussed in detail in this thesis.



## **2.1. Conditional Volatility Model**

The original idea of volatility is to measure the range of price change, or to measure risk. In the early stages, papers such as Black and Scholes (1973) typically assumed a constant volatility. Engle (1982) first built an ARCH model to track volatility, and let current volatility depend on past error terms of price equation. Furthermore, Bollerslev (1986) extent the ARCH model to allow variance to depend on lags and lags of squared error, so his extension allows conditional variance to follow an ARMA process. Both ARCH and GARCH models investigate the intertemporal relationship between risk and expected return. These types of volatility are called conditional volatility models. GARCH models show two desirable characteristics to model a financial market's volatility.

First, numerous empirical results indicate that financial market volatility has a volatility clustering effect (Mandelbrot, 1963). That is, a large volatility will generally lead to another large volatility, and a low volatility tends to be followed by another low volatility. These characteristics can also be described as existing positive autocorrelation coefficients in squared return, and are well captured in the GARCH models. Second, financial returns' unconditional probability distributions generally have Leptokurtic characteristics. That is, the distribution has fatter tails and more values close to centre value compared to normal distribution (Mandelbrot, 1963). Hence, it is unrealistic to assume returns are independently and identically distributed (i.i.d.). Another advantage of the GARCH model is to assume returns are not independently distributed.

### 2.1.1. ARCH Model

The ARCH (1) model can be represented as the following:

$$y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \quad (2.1)$$

$$\sigma_t^2 = E(u_t^2 | u_{t-1}) = a_0 + a_1 u_{t-1}^2 \quad (2.2)$$

where  $y_t$  refers to dependent variable and  $x_t'$  refers to independent variable.  $u_t$  is an uncorrelated error term that follows a normal distribution. Equation 2.1 is the mean equation and 2.2 refers to the variance equation. The ARCH effect is mainly reflected by equation 2.2.

The ARCH model can also be written in the following format:

$$\begin{cases} u_t = v_t \sqrt{h_t} \\ h_t = a_0 + \alpha u_{t-1}^2 \\ v_t \sim N(0, 1) \end{cases} \quad (2.3)$$

The above equation 2.3 is the same as equation 2.1 and 2.2, but with a different written format. Consequently, the above ARCH (1) model can be developed as a general ARCH (p) model as in the following:

$$y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \quad (2.4)$$

$$\sigma_t^2 = E(u_t^2 | u_{t-1}, u_{t-2}, \dots) = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + \dots + a_p u_{t-p}^2 \quad (2.5)$$

In equation 2.4 and 2.5, it still assumes that  $u_t$  does not have autocorrelation effect, i.e.  $E(u_t u_s) = 0, s \neq t$ . Meanwhile, the unconditional expectation and conditional expectation value of  $u_t$  is assumed to equal zero, that is  $E(u_t) = E(u_t | u_{t-1}) = 0$ .

Under a traditional regression analysis framework, it shows  $E(u_t^2) = \sigma_t^2$  from equation 2.4. That is to say  $\sigma_t^2$  is an expectation value of  $u_t^2$ , or predicted value. Hence, at any time,  $t$ ,  $\sigma_t^2$  is not exactly equal to  $u_t$ . And one bias value exists, which can be expressed as:

$$u_t^2 - \sigma_t^2 = \varepsilon_t \quad (2.6)$$

Hence, the equation 2.5 can be rewritten as:

$$u_t^2 = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + \dots + a_p u_{t-p}^2 + \varepsilon_t \quad (2.7)$$

The stationary condition for the ARCH model is the following:

$$1 - a_1 z + a_2 z^2 + \dots + a_p z^p = 0 \quad (2.8)$$

All the solutions are located outside the unit circle. The unconditional expectation of  $u_t^2$  is:

$$E(u_t^2) = \sigma_t^2 = \frac{a_0}{1 - a_1 - a_2 - \dots - a_p} \quad (2.9)$$

In equation 2.9, if  $\sum_{i=1}^p a_i < 0$ , then AR (p) is a stationary model.

### 2.1.2. GARCH Model

The key difference between ARCH and GARCH is that GARCH lets current volatility depend on its own past volatility. A standard GARCH (1, 1) can be written as:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{cases} \quad (2.10)$$

In equation 2.10, the first equation is the mean equation and the second equation represents a variance equation. A GARCH (1, 1) model can be written as an infinite ARCH ( $\infty$ ) model:

$$\begin{aligned} \sigma_t^2 &= a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ \Rightarrow (1 - \beta_1 L) \sigma_t^2 &= a_0 + a_1 u_{t-1}^2 \\ \Rightarrow \sigma_t^2 &= (1 - \beta_1 L)^{-1} (a_0 + a_1 u_{t-1}^2) \quad (2.11) \\ \Rightarrow \sigma_t^2 &= \frac{a_0}{1 - \beta_1} + a_1 \sum_{j=1}^{\infty} \beta_1^{j-1} u_{t-j}^2 \end{aligned}$$

Consequently, GARCH (q, p) can be achieved by developing equation 2.10, and it can be written as:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = a_0 + \sum_{i=1}^p a_i u_{t-1}^2 + \sum_{j=1}^q \beta_j \sigma_{t-1}^2 \end{cases} \quad (2.12)$$

By using a causal operator, the variation equation in 2.12 can be written as:

$$\begin{aligned} a(L) &= a_1 L + a_2 L^2 + \dots + a_p L^p \\ \beta(L) &= 1 - \beta_1 L - \beta_2 L^2 - \dots - \beta_q L^q \quad (2.13) \\ \sigma_t^2 &= \beta(L)^{-1} a_0 + \beta(L)^{-1} a(L) u_t^2 \end{aligned}$$

The GARCH model is widely used in various aspects, especially in financial areas. For example, the GARCH model can be used to track and forecast stock index volatility. The stock index volatility generally shows volatility cluster phenomena, and GARCH can capture this characteristic well.

The stationary condition for the GARCH model can be examined as in the following. When  $u_t^2 - \sigma_t^2 = \varepsilon_t$ , is put into the equation 2.12, it gives:

$$u_t^2 - \varepsilon_t = a_0 + \sum_{i=1}^p a_i u_{t-1}^2 + \sum_{j=1}^q \beta_j (u_{t-1}^2 - \varepsilon_{t-1}) \quad (2.14)$$

Rearranging the above equation, it provides:

$$u_t^2 = a_0 + \sum_{i=1}^m (a_i + \beta_i) u_{t-1}^2 + \varepsilon_t - \sum_{j=1}^q \beta_j \varepsilon_{t-1} \quad (2.15)$$

where  $m = \max(q, p)$ , if  $q > p$ , then:

$$a_{p+1} = \dots = a_q = 0$$

If  $q < p$ , then:

$$\beta_{q+1} = \dots = \beta_p = 0$$

The necessary condition to ensure equation 2.15 is stationary can be the following:

$$1 - \sum_{i=1}^m (a_i + \beta_i)Z^i = 0 \quad (2.16)$$

If all the solutions in above the equation are located outside the unit circle, then the GARCH model is stationary. The above equation is the same as the following:

$$1 - (a_1 + \beta_1)z - (a_2 + \beta_2)z^2 - \dots - (a_m + \beta_m)z^m = 0 \quad (2.17)$$

The unconditional expectation value for  $u_t^2$  equals to:

$$E(u_t^2) = \sigma_t^2 = \frac{a_0}{1 - \sum_{i=1}^m (a_i + \beta_i)} \quad (2.18)$$

From the above equations, it shows that the necessary conditions to estimate GARCH are  $\sum_{i=1}^m (a_i + \beta_i) < 1$ ,  $a_0 > 0$ ,  $a_i$  and  $\beta_i$  are non-negative value.

### **2.1.3. GARCH-in-Mean Model**

In the above GARCH model, it assumes the conditional mean is constant during the estimation period. However, in real life data, this assumption cannot always hold true. At the same time, the conditional mean depends on the conditional volatility. The above GARCH does not consider this situation. Engle et al. (1987) first created a model called GARCH-in-Mean, and this model considers that the conditional mean depends on the conditional volatility. The GARCH (1, 1)-in-Mean model can be written as:

$$\begin{cases} y_t = x_t' \phi + \gamma \sigma_t^2 + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = a_0 + \sum_{i=1}^p a_i u_{t-1}^2 + \sum_{j=1}^q \beta_j \sigma_{t-1}^2 \end{cases} \quad (2.19)$$

In the mean equation, the variable refers to risk, which can be variance ( $\sigma_t^2$ ), standard derivation ( $\sigma_t$ ) or logarithm variance ( $\log(\sigma_t^2)$ ). In equation 2.19, the risk is variance.

#### 2.1.4. Asymmetric GARCH Model

In the above GARCH models, it assumes the positive shock and negative shock have the same effect on current conditional variance. This assumption cannot adopt real life situations adequately. In financial markets, a negative return in general leads to a large volatility compared to a positive return. This phenomenon is called asymmetric effect, or leverage effect. Two widely used models are developed in order to capture this leverage effect.

Glosten et al (1993) proposed GJR-GARCH, which is a type of E-GARCH but can be described as a TARARCH model. On the contrary, TGARCH has been employed from Rabemananjara and Zakoian (1993), as an extension of TARARCH models employed first by Zakoian (1991) and then by Glosten et al. (1993) to capture this leverage effect in volatility. The key idea is to use a dummy variable to set up a threshold, in order to divide the positive and negative shock effects on volatility. A standard dummy variable in TGARCH can be set as following:

$$\begin{cases} I_{t-1} = 0, u_{t-1} \geq 0 \\ I_{t-1} = 1, u_{t-1} \leq 0 \end{cases} \quad (2.20)$$

Then the mean and variance equations can be written as:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = a_0 + a_1 u_{t-1}^2 + a_1' u_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1}^2 \end{cases} \quad (2.21)$$

In equation 2.21, it considers the differences in effect between positive shock and negative shock. The variance equation in 2.21 can be written in detail in the following:

$$\begin{cases} \sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2, u_{t-1} \geq 0 \\ \sigma_t^2 = a_0 + (a_1 + a_1') u_{t-1}^2 + \beta_1 \sigma_{t-1}^2, u_{t-1} < 0 \end{cases} \quad (2.22)$$

The above is a standard TGARCH (1, 1) model, which can be extended to TGARCH (q, p) as in the following:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = a_0 + \sum_{i=1}^p a_i u_{t-p}^2 + \sum_{j=1}^q \beta_j \sigma_{t-q}^2 + \sum_{k=1}^r a_k' u_{t-1}^2 I_{t-1} \end{cases} \quad (2.23)$$

where  $r$  refers to the number of threshold. And if  $u_t < 0$ ,  $I_t = 1$ ;  $u_t \geq 0$ ,  $I_t = 0$ .

Nelson (1991) provided another model to capture this leverage effect, and the model is called as EGARCH. The letter E refers to exponential. In the EGARCH model, the variance in equation is not  $\sigma_t^2$ , but the logarithm variance ( $\ln(\sigma_t^2)$ ). A standard EGARCH (1, 1) can be modelled as:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \ln(\sigma_t^2) = a_0 + \theta \frac{u_{t-1}}{\sigma_{t-1}} + a_1 \frac{|u_{t-1}|}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2 \end{cases} \quad (2.24)$$

In the above equation,  $\ln(\sigma_t^2)$  can be either positive or negative, but it can ensure variance to be positive. That is,  $\sigma_t^2 = \exp[\ln(\sigma_t^2)]$ , to ensure positive value. The advantage of EGARCH is no restriction condition on the model in order to ensure positive variance. The term  $\beta_1 \ln \sigma_{t-1}^2$  is used to capture the volatility cluster effect.  $\theta \frac{u_{t-1}}{\sigma_{t-1}}$  and  $a_1 \frac{|u_{t-1}|}{\sigma_{t-1}}$  are used to capture the asymmetric effect. It is easy to know that these two terms ( $\theta \frac{u_{t-1}}{\sigma_{t-1}}$  and  $a_1 \frac{|u_{t-1}|}{\sigma_{t-1}}$ ) are a standard format of error term. If assuming  $u_t \sim N(0, \sigma_t^2)$ , then  $\frac{u_{t-1}}{\sigma_{t-1}} \sim N(0, 1)$ .

From equation 2.24, the asymmetric characteristics can be described in the following format:

$$\begin{cases} \ln(\sigma_t^2) = a_0 + (a_1 + \theta) \frac{|u_{t-1}|}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2, u_{t-1} > 0 \\ \ln(\sigma_t^2) = a_0 + (a_1 - \theta) \frac{|u_{t-1}|}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2, u_{t-1} \leq 0 \end{cases} \quad (2.25)$$

The term  $\theta \frac{u_{t-1}}{\sigma_{t-1}}$  decides whether there is an asymmetric effect. If  $\theta = 0$ , the variance should be symmetric, and there will be no leverage effect. That is, the volatility will be consistent

with both positive shock and negative shock. If  $\theta \neq 0$  and  $u_{t-1} < 0$ , the volatility will be expected to increase, as in the following:

$$\theta \frac{u_{t-1}}{\sigma_{t-1}} > 0, u_{t-1} < 0$$

If  $\theta \neq 0$  and  $u_{t-1} > 0$ , the volatility will be expected to decrease:

$$\theta \frac{u_{t-1}}{\sigma_{t-1}} < 0, u_{t-1} > 0$$

Hence, the key aspect to examine is indeed whether an asymmetric effect exists:

$$H_0: \theta = 0$$

$$H_1: \theta < 0$$

The EGARCH can also be expanded into a general EGARCH (q, p) model, as follows:

$$\left\{ \begin{array}{l} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \ln(\sigma_t^2) = a_0 + \sum_{k=1}^r \theta_k \frac{u_{t-1}}{\sigma_{t-1}} + \sum_{i=1}^p a_i \frac{|u_{t-1}|}{\sigma_{t-1}} + \sum_{j=1}^q \beta_j \ln \sigma_{t-1}^2 \end{array} \right. \quad (2.26)$$

### **2.1.5 Other GARCH Models**

GARCH family models have numerous important implications in financial areas. Various versions of the GARCH model have been created in the latest decades. The three other most widely used versions of the GARCH model will be discussed here: PGARCH, CGARCH and QGARCH models.

The PGARCH model has been mainly contributed by Taylor (1986), Schwert (1989), Ding, Granger and Engle (1993), Degiannakis and Xekalaki, (2010) and Bauwens et al. (2012). The standard PGARCH (1, 1) can be described as:



$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^h = a_0 + a_1(|u_{t-1}| - \gamma_1 u_{t-1})^h + \beta_1 \ln(\sigma_{t-1}^h) \end{cases} \quad (2.27)$$

where  $h$  refers to value of power, the asymmetric effect is captured by  $\gamma_1$ . If  $\gamma_1 = 0$ , then there will be no asymmetric effect in the model. A generalised PGARCH (q, p) model can be written as:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^h = a_0 + \sum_{i=1}^p a_i (|u_{t-1}| - \gamma_i u_{t-1})^h + \sum_{j=1}^q \beta_j \ln(\sigma_{t-1}^h) \end{cases} \quad (2.28)$$

where  $i = 1, 2, \dots, r$ ,  $|\gamma_i| \leq 1$ ; other situations  $\gamma_i = 0$ ; the number of thresholds cannot exceed  $p$ , that is  $r < p$ .

The key difference between CGARCH (aka Component Garch) and the standard GARCH is the assumption of the constant value of long term volatility (see., e.g., Engle and Lee, 1999). That is,  $E(u_t^2) = E(\sigma_t^2) = \bar{Q}$ . In the CGARCH model, the mean value of volatility will change among different time periods. A standard CGARCH (1, 1) can be drawn as the following:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ Q_t = \bar{Q} + \rho(Q_{t-1} - \bar{Q}) + \phi(u_{t-1}^2 - Q_{t-1}) \\ \sigma_t^2 = Q_t + a_1(u_{t-1}^2 - \bar{Q}_{t-1}) + \beta_1(\sigma_{t-1}^2 - \bar{Q}_{t-1}) \end{cases} \quad (2.29)$$

where  $\bar{Q}$  refers to expected mean value of variance and  $Q$  represents the change value of long term volatility, or the expected value of conditional variance.

QGARCH refers to the quadratic GARCH model, developed by Engle (1990), Campbell and Hentschel (1992) and Sentana (1995). The most recent application of this model was used by So and Wong (2012). The standard QGARCH (1, 1) can be described as:

$$\begin{cases} y_t = x_t' \phi + u_t, u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \phi u_{t-1} \end{cases} \quad (2.30)$$

where  $\phi u_{t-1}$  is applied to capture asymmetric effect. Sentana (1995) proved that QGARCH has similar characteristics with the standard GARCH model. That is, both models have constant mean and variance.

In summary, this sub-chapter comprehensively introduces the conditional volatility models, including early ARCH, GARCH and GARCH-in-Mean. Two asymmetric models, TGARCH and EGARCH are discussed in detail. Three other types of models are also introduced, including PGARCH, CGARCH and QGARCH. Conditional volatility plays an important role in financial market volatility. Recent developments include Andersen et al. (2003), Audrino and Trojani (2007), etc. Since conditional volatility has such a long history of development along with an important position in the current financial industry, it is important to examine this type of volatility model in detail to understand realised volatility deeply.

## **2.2. Realised Volatility Model**

Realised volatility is a recently developed measure, first mentioned by Merton (1980). However, due to high-frequency storage technique limitations, the theory did not undergo any significant development in the years 1980 to 2000. Since the development of the storage technique, interval day high-frequency data becomes possible to achieve, which provides the fundamental condition for developing Merton's concept. Andersen and Bollerslev (1998) first outstandingly proved that the summarisation of all intraday high-frequency squared returns was an unbiased estimator of daily integrated volatility. Based on Andersen and Bollerslev (1998), realised volatility has seen significant progress during recent years.

This sub-chapter starts with a simple discrete realised volatility model, and then moves to a continuous model without microstructure effect. Consequently, different microstructure noises are discussed. Some revised methods to eliminate microstructure noise are introduced. Lastly, some realised volatility forecast models are discussed. This sub-chapter focuses on a theoretical discussion of realised volatility, and will provide a background for the next chapters in this thesis.

### 2.2.1. Simple Discrete Model

Consider a simple discrete model, where the asset daily return can be described as:

$$r_t = h_t^{1/2} \eta_t \quad (2.31)$$

where  $\{\eta_t\}_{t=1}^T$  is a white noise with mean value equal to 0, and variance equal to 1. That is  $\eta_t \sim N(0,1)$ . Assume at trading day  $t$ , the logarithm asset price is the high-frequency history tick data. Considering the following segmentation  $\Lambda_t = \{\tau_0, \tau_1, \dots, \tau_{n_t}\}$ , which includes all the observation values, let  $p_{t,i}$ ,  $i = 1, 2, \dots, n_t$  refer to the trading price at day  $t$  and observation  $i$ , where  $n_t$  refers to the total observation values within one trading day. Further assume:

$$r_{t,i} = h_{t,i}^{1/2} \eta_{t,i} \quad (2.32)$$

where  $\eta_{t,i} \sim N(0, n_t^{-1})$ ,  $r_{t,i} = p_{t,i} - p_{t,i-1}$  refers to the return at trading day  $t$  and observation  $i$ .

Then the following equation holds truth:

$$r_t = \sum_{i=0}^{n_t} r_{t,i}, h_t = \frac{1}{n_t} \sum_{i=0}^{n_t} h_{t,i} \quad (2.33)$$

Define information set  $\mathfrak{F}_{t,i} \equiv \mathfrak{F}\{p_{a,b}\}_{a=-\infty, b=0}^{a=t, b=i}$ , which is constituted by all the history trading information before the trading day  $t$  and time  $i$ . Hence,  $\mathfrak{F}_{t,0}$  represents all the trading information before trading day  $t$ . Furthermore, it gives:

$$E(r_t^2 | \mathfrak{F}_{t,0}) = h_t, V(r_t^2 | \mathfrak{F}_{t,0}) = 2h_t^2 \quad (2.34)$$

The realised variance can be calculated by summarising all the intraday squared returns, and the realised volatility is the squared root of realised variance. That is:

$$Rv_t^{(all)} = \sum_{i=0}^{n_t} r_{t,i}^2, Rvol_t^{(all)} = \sqrt{Rv_t^{(all)}} \quad (2.35)$$

The daily squared return can be written as:

$$r_t^2 = \left( \sum_{i=0}^{n_t} r_{t,i} \right)^2 = \sum_{i=0}^{n_t} r_{t,i}^2 + 2 \sum_{i=0}^{n_t-1} \sum_{j=i+1}^{n_t} r_{t,i} r_{t,j} \quad (2.36)$$

where:

$$\begin{aligned} E(r_t^2 | \mathfrak{F}_{t,0}) &= E \left( \sum_{i=0}^{n_t} r_{t,i}^2 \middle| \mathfrak{F}_{t,0} \right) + 2E \left( \sum_{i=0}^{n_t-1} \sum_{j=i+1}^{n_t} r_{t,i} r_{t,j} \middle| \mathfrak{F}_{t,0} \right) \\ &= E \left( Rv_t^{(all)} \middle| \mathfrak{F}_{t,0} \right) + 2E \left( \sum_{i=0}^{n_t-1} \sum_{j=i+1}^{n_t} r_{t,i} r_{t,j} \middle| \mathfrak{F}_{t,0} \right) \end{aligned} \quad (2.37)$$

If intraday returns are independently distributed, then:

$$E(r_t^2 | \mathfrak{F}_{t,0}) = E \left( Rv_t^{(all)} \middle| \mathfrak{F}_{t,0} \right) = h$$

Hence the daily squared return and realised variance are both unbiased estimators of daily return variance, and realised variance is more efficient than daily squared return. Furthermore, if the observation frequency can arrive at any high level, then the realised variance can approach daily return variance at any accuracy. That is:

$$\lim_{n \rightarrow \infty} V(Rv_t^{(all)} | \mathfrak{F}_{t,0}) = 0 \quad (2.38)$$

### **2.2.2. Continuous Model Without Microstructure Noise**

The theory of realised volatility is based on price decomposition theory and quadratic variation theory. Assume  $p_t$  is a  $N \times 1$  logarithm price vector and it obeys the following model:

$$dp_t = u_t dt + \Omega_t dW_t \quad (2.39)$$

In the above equation,  $W_t$  refers to a standard Brownian motion,  $\Omega_t$  refers to a  $N$  by  $N$  matrix with strict stationarity. Under above assumption, continuous compound return at the period  $[t, t + h]$  can be represented as the following:

$$r_{t+h,h} = p_{t+h} - p_t \quad (2.40)$$

$r_{t+h,h}$  follows a Gaussian distribution:

$$r_{t+h,h} \left| \sigma\{u_{t+\tau}, \Omega_{t+\tau}\}_{\tau=0}^h \sim N\left(\int_0^h u_{t+\tau} d\tau, \int_0^h \Omega_{t+\tau} d\tau\right) \quad (2.41)$$

Combined with quadratic variation theory, if  $\Delta \rightarrow 0$ , it can get:

$$\sum_{j=1}^{h/\Delta} (r_{t+j\Delta, \Delta} * r_{t+j\Delta, \Delta}^T) - \int_0^h \Omega_{t+\tau} d\tau \rightarrow 0 \quad (2.42)$$

In the above equations,  $h$  refers to number of interval periods. For example, if  $h = 1$ , it refers to one unit of interval period (such as 1 day or 1 month). Meanwhile,  $\Delta = 1/\text{sample number}$ .

Under above situations, the covariance matrix can be written as:

$$Cov(t) = \sum_{j=1}^{1/\Delta} (r_{t+j\Delta, \Delta} * r_{t+j\Delta, \Delta}^T) \quad (2.43)$$

For stock  $j$ , its covariance can be represented as  $V_{jt}^2 = \{Cov(t)\}_{jj}$ , and logarithm standard deviation of stock  $j$  can be described as  $l_{jt} = \log V_{jt}$ . The correlation coefficient between stock  $i$  and stock  $j$  can be described as  $\rho_{ijt} = \{Cov(t)\}_{ij} / (V_{it} * V_{jt})$ . As Andersen et al. (2001) points out, realised volatility is an unbiased and highly efficient estimator of return volatility. Much empirical evidence (Corsi, 2009 and Meddahi, 2002) shows that realised volatility performs better than conditional volatility (e.g. GARCH Based Models).



### 2.2.3. Characteristics of Realised Volatility

Andersen et al. (2001) pointed out the following, under the framework of semi-martingales theory, the summarisation of all intraday squared return is is:

$$RV_t \xrightarrow{p} IV_t$$

According to Jacod and Protter (1998), Barndorff-Nielsen and Shephard (2002a, b) asymptotic distribution of realised volatility:

$$n_t^{\frac{1}{2}} \frac{1}{\sqrt{2Iq_t}} (RV_t - IV_t) \xrightarrow{d} N(0,1) \quad (2.44)$$

where  $Iq_t$  is defined as:

$$Iq_t = \int_0^1 \sigma^4(t + \tau - 1) d\tau \quad (2.45)$$

Furthermore, without the microstructure noise, Barndorff-Nielsen and Shephard (2002a, b) provide quartic realised volatility as the following:

$$RQ = \frac{n_t}{3} \sum_{i=0}^{n_t} r_{t,i}^4 \quad (2.46)$$

where

$$n_t^{\frac{1}{2}} \frac{1}{\sqrt{\frac{2}{3} RQ}} (RQ - Iq_t) \xrightarrow{d} N(0,1) \quad (2.47)$$

Barndorff-Nielsen and Shephard (2002a, b), Goncalves and Meddahi (2009) and Nielsen and Frederiksen (2008) studied equation 2.3. They found equation 2.3 has poor finite sample properties, but the logarithm of realised volatility has desirable sample properties. That is:

$$n_t^{\frac{1}{2}} \frac{1}{\sqrt{\frac{2}{3} \frac{RQ}{(RQ)^2}}} [(\log(RQ) - \log(Iq_t))] \xrightarrow{d} N(0,1) \quad (2.48)$$

### 2.2.4. Microstructure Noise

This sub-chapter discusses the estimation of integrated volatility under microstructure noise. The microstructure noise in financial markets can come from variety of sources, including discrete asset price (Harris, 1990), bid–ask spread, trading mechanism (O'Hara, 1995) and so on.

In the previous sub-chapter, information segmentation set  $\Lambda_t = \{\tau_0, \tau_1, \dots, \tau_{n_t}\}$  was considered. Now let  $p_{t,i} \equiv p(t + \tau_i)$ , and assume the logarithm asset price is influenced by microstructure noise:

$$p_{t,i} = p_{t,i}^* + \varepsilon_{t,i} \quad (2.49)$$

where  $p_{t,i}^*$  refers to the real efficient asset price, and  $\varepsilon_{t,i}$  represents to the microstructure noise.

It gives:

$$r_{t,i} = r_{t,i}^* + \varepsilon_{t,i} - \varepsilon_{t,i-1} = r_{t,i}^* + v_{t,i} \quad (2.50)$$

where  $r_{t,i}^* = p_{t,i}^* - p_{t,i-1}^*$ , which refers to the asset real return. Obviously  $r_{t,i}$  satisfy the AR (1) process, hence realised volatility is a bias estimator of real daily return variance. Furthermore:

$$RV = \sum_{i=1}^{n_t} (r_{t,i}^*)^2 + 2 \sum_{i=1}^{n_t} r_{t,i}^* v_{t,i} + \sum_{i=1}^{n_t} v_{t,i}^2 \quad (2.51)$$

The expectation value of realised volatility conditional on real return will be:

$$E(RV | r^*) = RV + 2n_t E(\varepsilon_{t,i}^2) \quad (2.52)$$

From above equation, it shows that realised volatility is a bias estimator of integrated volatility.

Bandi and Russell (2008) studied the following three types of microstructure noise:



Assumption 1 (Microstructure effect):

1. Microstructure noise  $\varepsilon_{t,i}$  has a mean value equal to zero, and stationary covariance with stochastic effect.
2.  $v_{t,i} = \varepsilon_{t,i} - \varepsilon_{t,i-1}$ , and variance of  $v_{t,i}$  is  $O(1)$ .

Under this assumption, it gives:

$$RV \xrightarrow{a.s} \infty, \text{ where } n_t \rightarrow \infty$$

Assumption 2 (Independent microstructure effect):

1. Microstructure noise  $\varepsilon_{t,i}$  has a mean value equal to zero, with independent stochastic distribution.
2. Microstructure noise is independent of price process.
3.  $v_{t,i} = \varepsilon_{t,i} - \varepsilon_{t,i-1}$ , and variance of  $v_{t,i}$  is  $O(1)$ .

Under this assumption, Zhang et al. (2005) provide:

$$n_t^{\frac{1}{2}} \left( RV_t - IV_t - 2n_t E(\varepsilon_{t,i}^2) \right) \xrightarrow{d} 2(E(\varepsilon_{t,i}^4))^{1/2} N(0,1)$$

In real life application, no matter how high the data frequency is, the total number of price observations is limited. That is to say, discrete price characteristics will lead to bias in estimation. This bias satisfies the following:

$$RV \approx IV + 2n_t E(\varepsilon_{t,i}^2) + [4n_t E(\varepsilon_{t,i}^4) + \frac{2}{n_t} \int_0^1 \sigma_t^4 dt]^{\frac{1}{2}} N(0,1) \quad (2.53)$$

Assumption 3 (Dependent microstructure effect):

1. Microstructure noise  $\varepsilon_{t,i}$  has a mean value equal to zero with stationary covariance.

Meanwhile, for any  $k > 0$ , it has  $E(\varepsilon_{t,i}^{4+k}) < \infty$ .

2. Microstructure noise is independent of price process.

3.  $v_{t,i} = \varepsilon_{t,i} - \varepsilon_{t,i-1}$ , and variance of  $v_{t,i}$  is  $O(1)$ .

Under this assumption, Zhang (2006) and Ait-Sahalia et al. (2011) provided:

$$RV \approx IV + 2n_t E(\varepsilon_{t,i}^2) + [4n_t \Omega + \frac{2}{n_t} \int_0^1 \sigma_t^4 dt]^{1/2} N(0,1)$$

where

$$\Omega = V[(\varepsilon_{t,1} - \varepsilon_{t,0})^2] + 2 \sum_{i=1}^{\infty} COV[(\varepsilon_{t,1} - \varepsilon_{t,0})^2, (\varepsilon_{t,i+1} - \varepsilon_{t,i})^2]$$

From the above result, for a large value of  $n_t$ , the realised volatility is independent with real return in equation 2.35. Realised volatility converges to infinite with the ratio  $n_t$ . Bandi and Russel (2008) and Zhang et al. (2005) got the unbiased estimator of microstructure effect:

$$\frac{1}{2n_t} RV \xrightarrow{p} E(\varepsilon_{t,i}^2)$$

Andersen et al. (2003) believed that taking lower frequency data can solve the microstructure effect. For example, lower the data frequency from 5 minute intervals to 10 minute intervals. However, Zhang et al (2005) believed this was not the appropriate solution to eliminate the microstructure effect. They recommended to first define a new information segmentation set  $\Lambda_t^{(calendar)}$ , and then it can get equally spaced sparse observations  $n_t^{(calendar)}$ . Obviously,  $\Lambda_t^{(calendar)}$  is a sub-segmentation of  $\Lambda_t$ :

$$RV^{(calendar)} = \sum_{i=0}^{n_t^{(calendar)}} r_{t,i}^2$$

Based on Rootzen (1980) and Jacod and Protter (1998) results, Barndorff-Nielsen and Shephard (2002a, b), Mykland and Zhang (2006), Ait-Sahalia et al. (2011) and Zhang et al. (2005) proved that the microstructure effect equals to  $2n_t^{(calendar)} E(\varepsilon_{t,i}^2)$ , under assumption 2 and 3, the following holds true:

$$RV_t^{(calendar)} \approx IV + 2n_t^{(calendar)} E(\varepsilon_{t,i}^2) + [4n_t^{(calendar)} E(\varepsilon_{t,i}^4) + \frac{1}{n_t^{(calendar)}} \int_0^1 \sigma_t^4 dt]^{1/2} N(0,1)$$

In the above equation the discrete variance increases, which is known as the bias-variance problem.

### **2.2.5. Revised Realised Volatility**

Under the assumption 2 above, Bandi and Russell (2006) and Zhang et al. (2005) provided an optimal calendar sampling method, which can lead to the minimise mean squared error (MSE):

$$MSE\left(n_t^{(calendar)}\right) = 2n_t^{(calendar)}E\left(\varepsilon_{t,i}^2\right) + 4n_t^{(calendar)}E\left(\varepsilon_{t,i}^4\right) + \left[8RV_t^{(calendar)}E\left(\varepsilon_{t,i}^2\right) - 2V\left(\varepsilon_{t,i}^2\right)\right] + \frac{2}{n_t^{(calendar)}}IQ_t^{(calendar)}$$

Hence, the optimal calendar sample frequency should equal:

$$n_t \approx \left\{ \frac{IQ_t}{4[E\left(\varepsilon_{t,i}^2\right)]^2} \right\}^{1/3}$$

Zhang et al. (2005) proposed an unbiased estimator of integrated variance under the existing microstructure effect, which is called two scale estimators. The method applies the calendar sampling method, and also takes advantage of high-frequency data. For example, the data time by using 10 minute intervals will be 9:30-9:40, 9:40-9:50..... 9:31-9:41, 9:41-9:51..... More generally, the full segmentation can be written as  $\Lambda_t = \{\tau_0, \dots, \tau_{n_t}\}$ , in which  $\Lambda_t$  is divided as  $k$  sub-segmentations denoted as  $\Lambda_t^{(k)}$ ,  $k = 1, \dots, K$  satisfying:

$$\Lambda_t = \cup_{k=1}^k \Lambda_t^{(k)}, \text{ when } k \neq j, \Lambda_t^{(k)} \cap \Lambda_t^{(j)} = \varphi$$

Let  $n_t^{(k)}$  refer to the number of observations in each sub-segmentation, and define the realised volatility in sub-segmentation  $k$  as:

$$RV_t^{(k)} = \sum_{i=0}^{n_t^{(k)}} r_{t,i}^2$$

Zhang et al. (2005) proposed the following daily realised volatility estimation:

$$RV_t^Z = \frac{1}{K} \sum_{k=1}^K RV_t^{(k)} - \frac{\bar{n}_t}{n_t} RV_t^{(all)} \quad (2.54)$$

where  $n_t$  refers to the number of sub-segmentations:

$$\bar{n}_t = \frac{1}{K} \sum_{k=1}^K n_t^{(k)} = \frac{n_t - K + 1}{K}$$

The equation 2.54 is known as two scale estimators. Under the assumption 2, Zhang et al. (2005) provided an asymptotic distribution of this estimation method:

$$n_t^{-\frac{1}{6}} (RV_t^Z - IV_t) \xrightarrow{d} \left[ 8c^{-2} E(\varepsilon_{t,i}^4) + c \frac{4}{3} IV_t \right]^{\frac{1}{2}} N(0,1)$$

Ait-Sahalia et al. (2011) provided a revised model for equation 2.54, and the final estimation model is:

$$RV_t^{(A-Z)} = \left(1 - \frac{\bar{n}_t}{n_t}\right)^{-1} RV_t^Z \quad (2.55)$$

Both equation 2.55 and 2.54 are estimated under the assumption 2. In order to add the possible correlation effect, Zhang (2006) and Ait-Sahalia et al. (2011) provided another estimation model by using the two scale estimators method. First, the average lag  $J$  realised variance is defined as the following:

$$RV_t^{(AL)} = \frac{1}{J} \sum_{i=0}^{n_t-J} (r_{t,i+J} - r_{t,i})^2$$

Consequently, they expand the two scale estimator method, and provide a general multiple scale estimators method:

$$RV_t^{(AMZ)} = RV_{t,K}^{(AL)} - \frac{\overline{n_t^K}}{n_t^J} RV_{t,J}^{(AL)}$$

The sub-segmentation estimation method is given as:

$$RV_t^{(A-AMZ)} = \left(1 - \frac{\overline{n_t^K}}{n_t^J}\right)^{-1} RV_t^{AMZ}$$

Zhang (2006) and Ait-Sahalia et al. (2011) pointed out:

$$RV_t^{(A-AMZ)} \approx IV_t + \frac{1}{n_t^6} \left[ \frac{1}{c^2} \xi^2 + c \frac{4}{3} \int_0^1 \sigma_t^4 dt \right]^{\frac{1}{2}} N(0,1)$$

where  $c$  is a constant,  $\xi^2 = 16V(\varepsilon_{t,i})^2 + 32 \sum_{i=1}^{\infty} COV(\varepsilon_{t,0}, \varepsilon_{t,i})^2$ .

The above is a revised realised volatility model provided by Zhang et al. (2005). However, other researchers propose further methods to eliminate the microstructure effect before modelling, by using a variety of filters. The key idea of these techniques is to use filters to eliminate spurious correlation, and then use revised data to estimate realised volatility. For example, Bollen and Inder (2002) applied an autoregressive filter, and Ebend (1999) adopted a move-average filter. Hansen and Lunde (2006) pointed out that a move-average filter has a better effect to revise data if the microstructure effects are independently distributed, and by using a move-average filter, the realised volatility is an unbiased estimator of integrated volatility.

### **2.2.6. Realised Volatility Forecast Model**

In financial econometrics analysis, the standard return does not follow a normal distribution, but follows heave-tailed distribution. This factor has led numerous researchers to focus on heave-tail distribution. However, Andersen et al. (2000) pointed out that by using realised

volatility, the standard exchange ratio follows normal distribution. In the stock market, Andersen et al. (2001) arrived at the same conclusion. They also pointed out logarithm of realised volatility follows a normal distribution. By investigating the dynamic characteristics of logarithm realised volatility, this thesis shows that logarithm realised volatility is almost stationary.

Meanwhile, realised volatility shows a significant long memory process. Andersen et al. (2003) applied the ARFIMA (p, d, q) model to analyse realised volatility's long memory characteristics. Based on Muller et al.'s (1997) heterogeneous ARCH model, Corsi (2009) proposed a Heterogeneous Autoregressive Realised Volatility model (HAR-RV). The model considers that the current volatility is constituted by the combined effect of heterogeneous markets' volatility. Based on the HAR-RV model, McAleer and Medeiros (2008) investigated the multiple regime-switching issue, and proposed a smooth transition model to capture the nonlinear and long memory characteristics of realised volatility.

As Corsi (2009) pointed out, financial time series data may have a fake long memory characteristic. It is important to analyse the driven force to lead this fake long memory characteristic. He pointed out that sometimes a short memory model may track and forecast the long memory characteristics at a high degree of accuracy. Granger et al. (2009) discussed the possible driven forces to lead financial long memory characteristics. They concluded that a set of nonlinear short memory models, especially without structure change effect, can create long memory behaviour. These nonlinear models include Granger and Hyung's (2013) structure change model, Engle and Lee's (1999) volatility component model, Hamilton and Susmel's (1994) regime-switching model and Medeiros and Veiga's (2009) multiple regime-switching model. Meanwhile, Hillebrand (2005) discussed the volatility jump characteristics

in realised volatility estimation. Scharth and Medeiros (2009) applied a multiple regime-switching model to describe the dynamic characteristics of realised volatility. They used the past accumulative returns as main sources to lead a regime-switching. The past accumulative returns were found to be highly significant, and can be used to explain the long memory characteristics in the model. Consequently, they pointed out that the nonlinear model has significantly higher forecasting power compared to the ARFIMA and HAR-RV models, especially in the case of a high volatility period.

Martens (2001, 2002) proposed a model combined with long memory and nonlinear characteristics. The model can also be used to capture leverage effect in volatility. Deo et al. (2006) considered the long memory SV model. Koopman et al. (2005) proposed a model combining long memory characteristics with unobserved components. Hillebrand et al. (2006) provided different types nonlinear models with long memory characteristics. In their models, volatility can directly influence returns. Currently, numerous researchers propose a variety of different models to analyse realised volatility's long memory. It is still a further research area to investigate whether a combination of long memory and nonlinear characteristics will significantly increase realised volatility forecast accuracy (e.g., Wei, 2012; Wang et al., 2016).

Lieberman and Phillips (2008) provided an explanation of realised volatility's long memory characteristics. They believed long memory is caused by past accumulative realised volatility, and they also provided a method to estimate the parametric  $d$  in ARFIMA ( $p, d, q$ ) model. Ait-Sahalia and Mancini (2008) compared a variety of long memory models' forecasting power for out-of-sample period data. Corsi et al. (2008) investigated the volatility of realised volatility. Andersen et al. (2005) provided a general model adjustment procedure to calculate unbiased volatility loss function.

### **2.3. Chapter Summary**

This chapter investigates the theoretical frameworks of the conditional volatility and realised volatility models. The aim of this thesis is to investigate Chinese stock index futures and spot markets' realised volatility. The conditional volatility sub-chapter starts with a standard ARCH model, and then expands to the GARCH, GARCH-in-Mean, asymmetric GARCH and three other GARCH models. Conditional volatility has a long history and still has an important position in current financial markets. Currently, numerous researchers still work in this field and the conditional volatility theory will continue to develop in the future.

The realised volatility sub-chapter starts with a simple discrete model, and then moves to a continuous model without microstructure noise. Consequently, three microstructure noise assumptions are discussed. After this, basic realised volatility characteristics and a revised realised volatility model proposed by Zhang et al. (2005) are investigated. Finally, some studies on realised volatility forecast models are discussed.



### **Chapter Three: Realised Volatility Characteristics**

This chapter examines the realised volatility characteristics of Chinese stock index futures and spot markets. In chapter two, the theoretical background theories of realised volatility is discussed in detail, and this chapter will empirically test these theories.

This chapter starts with basic statistic descriptions of original data. The original data is 1 minute high-frequency data from the period 19/04/2012 to 19/04/2013. After this, the microstructure noise effect is discussed and examined. By using an optimal sampling frequency method and filter technique, the optimal data sample is selected. This chapter suggests daily volatility jumps exist in realised volatility estimation, and suggests the use of bi-power realised volatility to revise realised volatility. Furthermore, intraday volatility jumps and periodicity are examined. The results strongly suggest that the intraday volatility jumps are linked to Chinese macroeconomic news releases. Lastly, the long memory characteristics of realised volatility are investigated. Both markets' realised volatility and logarithm realised volatility show strong long memory characteristics, and the results suggest applying long memory models (e.g. HAR-RV and ARFIMA) to track and forecast realised volatility for Chinese stock index futures and spot markets.

### **3.1. Original Research Data**

The original data used for this research is the 1 minute intraday high-frequency data. The sample period is from 19/04/2012 to 19/04/2013, which is a one year period. The total observations are 59,048 for both spot and futures market data. The data were downloaded from the Bloomberg database. Different opening hours exist between futures and spot markets. The futures market opens 15 minutes before the spot market and closes 15 minutes later than the spot market. Specifically, the futures market opening hours for each weekday are from 09:15 to 11:30 and 13:00 to 15:15, but the spot market opening hours are from 09:30 to 11:30, and then from 13:00 to 15:00.

This chapter matches the same number of daily observations for these two markets, that is from 09:30 to 11:30, and then from 13:00 to 15:00. Also requiring mention is that the futures contract data are the closest month activated contract, since the closest month activated contracts generally provide the best liquidity. The following table summarises the basic descriptions of CSI 300 futures contract.

**Table 1. Description of CSI 300 Futures Contract.**

<b>Contract name: CSI 300 Futures contract</b>	
<b>Contract value: RMB 300* Index points</b>	<b>Minimum Changing: 0.2 points</b>
<b>Delivery method: Cash</b>	<b>Daily price maxi movement: 10%</b>
<b>Initial margin: 12% of contract value</b>	<b>Multiplier: RMB 300</b>
<b>Last trading day: Every third Friday of delivery month</b>	
<b>Trading time: Monday -- Friday: 09:15 -11:30 and 13:00-15:15</b>	
<b>Delivery Months: Normally four contracts are available. The most close next two months and two closest quarterly months. So, if in February, the contract months will available in March, April, June and September.</b>	

The following table shows the logarithm of original data characteristics.

**Table 2. Original Logarithm Data Description.**

	<b>Mean</b>	<b>Std. Dev.</b>	<b>Maximum</b>	<b>Minimum</b>
<b>Futures</b>	7.799387	0.066962	7.936088	7.655864
<b>Spot</b>	7.797249	0.067993	7.933965	7.651191
	<b>Skewness</b>	<b>Kurtosis</b>	<b>Jarque-Bera</b>	<b><math>Q^2(12)</math></b>
<b>Future</b>	-0.124322	1.942585	2926.029	713640
<b>Spot</b>	-0.125893	1.882124	3256.073	713659
<b>Unit Root Test</b>				
		<b>Futures</b>		<b>Spot</b>
<b>ADF Test</b>		-1.471867		-1.462559
<b>PP Test</b>		-1.450659		-1.422452

Unit Root Test for First difference		
	Futures	Spot
<b>ADF Test</b>	-240.4528	-102.3388
<b>PP Test</b>	-240.4278	-172.9957

Notes: The Jarque-Bera test is to test the normality distribution of return. Ljung-Box statistics,  $Q^2(12)$  test the series autocorrelation up to 12 lags. The 1% critical value for ADF and PP test is 3.435. The confidence level to test null hypothesis is 1%.

This table shows that both log values have a mean close to 7.79, and standard deviation close to 0.06. Both series have a similar maximum and minimum value, with the futures market having a slightly higher value in both statistics. The Skewness and Kurtosis statistics indicate that both series have a negative Skewness and Leptokurtic distribution. The Jarque-Bera statistics demonstrate that both of these series do not follow a normal distribution. With the unit root test results, both ADF test and PP test results indicate that these two series are non-stationary processes. However, by using the first difference, these two series are stationary processes. Hence, both series are I (1) processes.

Both realised volatility and conditional volatility are based on market returns. Hence, it is necessary to further investigate the data returns characteristics. Here, the logarithm returns will be adopted. The original data is 1 minute high-frequency intraday data, and the total interval returns for 1 day are 240 (four hours). For each interval, the return is calculated as  $R_t = \log\left(\frac{R_t}{R_{t-1}}\right)$ . There are two reasons to use logarithm return rather than original return: first, logarithm return has a more desirable characteristic, such as closer to normal distribution, and this characteristic can satisfy the basic assumption of realised volatility. Second, logarithm return refers to continuous compounding return. The following table is the basic data description of logarithm return for both futures and spot markets.

**Table 3. Data Description of Logarithm Returns.**

	Mean	Std. Dev.	Maximum	Minimum
<b>Futures</b>	-5.41e-07	0.000718	0.015025	-0.017549
<b>Spot</b>	-4.07e-07	0.000545	0.013055	-0.018691
	Skewness	Kurtosis	Jarque-Bera	$Q^2(12)$
<b>Future</b>	-0.340217	43.12972	3994523	47.617
<b>Spot</b>	-1.735214	103.3285	2490569	7766.8
<b>Unit Root Test</b>				
	Futures		Spot	
<b>ADF Test</b>	-240.4528		-102.3388	
<b>PP Test</b>	-240.4278		-172.9957	

Notes: The Jarque-Bera test is to test the normality distribution of return. Ljung-Box statistics,  $Q^2(12)$  test the series autocorrelation up to 12 lags. The 1% critical value for ADF and PP test is 3.435. The confidence level to test null hypothesis is 1%.

The above table shows that both logarithm returns have a mean close to 0, and standard deviation near 0.0006. Both series have a similar maximum and minimum value, with the futures market having a slightly higher value in both statistics. The Skewness and Kurtosis statistics indicate that both series have a negative Skewness and significant Leptokurtic distribution. The Jarque-Bera statistics demonstrate that both of these series do not follow a normal distribution. With the unit root test results, both ADF test and PP test results indicate that these two logarithm return series are stationary processes.

### **3.2. Microstructure Noise Revise Technique**

As Andersen et al. (2005) pointed out, along with data frequency increase, realised volatility estimation infinitely approaches integrated volatility. However, higher frequency data will also increase the microstructure noise effect. On the other hand, lower frequency can reduce microstructure noise but increase estimation bias. Hence, there is no simple estimation method for realised volatility. The fundamental theory backgrounds and three basic assumptions of microstructure noise are examined in detail in the previous sub-chapter. Meanwhile, one important revised realised volatility model (two scale estimator method), proposed by Zhang et al. (2005), is also discussed in detail.

There are a variety of revise techniques to solve microstructure problems. These techniques can generally be divided into two broad categories: data-revise technique and model-revise technique. This sub-chapter introduces and discusses three important microstructure noise revise techniques to achieve the optimal one for further studies. One technique is based on model-revise technique, which is a kernel-based estimator. The other two are data-revise techniques, including filter technique and optimal sampling technique.

### **3.2.1. Literature Review**

A total of nine recent papers are investigated in detail in this sub-chapter. All these nine papers provide a variety of revise techniques in order to eliminate microstructure noise effect. The key concepts of these nine papers can be summarised in the table below.

**Table 4. Microstructure Noise Revise Technique.**

<b>Author(s)</b>	<b>Revise Technique</b>
<b>Ait-Sahalia (2005)</b>	Compare optimal sampling method and other revise techniques
<b>Ubykata and Oya (2009)</b>	Propose new test statistics for the dependence, cross and auto covariance estimators of bivariate noise processes
<b>Christensen and Podolskij (2007, 2012), Christensen et al. (2009)</b>	Analysis impact of microstructure noise on the realised range-based variance, and propose a bias correction to the range-statistic
<b>Ait-Sahalia et al. (2011)</b>	Two-time-scales method and multiple-time-scale method
<b>Ghysels and Sinko (2011)</b>	Mixed data sampling regression framework

The above table is a general summary of the key concepts of these nine recent papers. Different papers provide different approaches to revise microstructure noise effect. The details of these nine papers are discussed as follows:

Ait-Sahalia (2005) compared the optimal sampling method and other revise techniques for microstructure noise adjustment in realised volatility estimation.

Ubykata and Oya (2009) proposed new test statistics for the dependence, cross and auto covariance estimators of bivariate noise processes. It derived their asymptotic distributions and provided additional tests for the statistical significance of covariance estimators. Monte Carlo simulation showed that the covariance estimators and test statistics perform better in a finite sample. Further evidence from empirical illustration suggested that the covariance estimators and proposed test statistics are capable of capturing various dependence patterns in market microstructure noise. These results can shed more light on the sign of noise

autocorrelation in the presence of market microstructure frictions, such as bid–ask bounces and the clustering of order flow.

Christensen and Podolskij (2007, 2012) and Christensen et al. (2009) analysed the impact of microstructure noise on the realised range-based variance and proposed a bias correction to the range-statistic. They provided a new estimator, which is shown to be consistent for the integrated variance and asymptotically mixed Gaussian under simple forms of microstructure noise. They suggested selecting an optimal partition of the high-frequency data in order to minimise its asymptotic conditional variance. The finite sample properties of their estimator were studied with Monte Carlo simulations, and they implemented it using Microsoft high-frequency data from TAQ. They found that a bias-corrected range-statistic often leads to much smaller confidence intervals for the integrated variance compared to the realised variance.

### **3.2.2. Microstructure Noise Revise Technique**

A variety of revise techniques exist, as discussed in the literature review. Currently, there is no standard framework to solve the microstructure noise problem in realised volatility estimation. Here, three widely used revised techniques will be discussed in detail. These three methods include kernel-based estimator, filter method and optimal sampling method.

#### **3.2.2.1. Kernel-based Estimator**

Hansen and Lunde (2006) considered the following simple kernel-based estimator:



$$RV^{(K)} = RV + 2 \sum_{h=1}^H \frac{n_t}{n_t - h} \widehat{\gamma}_h \quad (3.1)$$

where

$$\widehat{\gamma}_h = \frac{n_t}{n_t - h} \sum_{j=1}^{n_t - h} r_{t,j} r_{j+h}$$

Zhou (1996) first considered the kernel-based estimator to deal with the microstructure noise in high-frequency data. He pointed out that, under the independent effect structure,  $H$  should equal to one for this special situation. Hansen and Lunde (2006) expanded Zhou's result, and pointed out that the kernel-based estimator is an unbiased estimator of realised volatility, but is not a consistent estimator under the following assumptions:

1. Microstructure noise  $\varepsilon_{t,i}$  has a mean value equal to zero, with independent stochastic distribution.
2. Microstructure noise is independent of price process.
3.  $v_{t,i} = \varepsilon_{t,i} - \varepsilon_{t,i-1}$ , and variance of  $v_{t,i}$  is  $O(1)$ .

They also pointed out this inconsistency cannot be solved by an increase in  $H$  value.

Therefore, Hansen and Lunde (2006) considered the Bartlett kernel estimator as the following:

$$RV^{(BK)} = RV + 2 \sum_{h=1}^H \left(1 - \frac{h}{H+1}\right) \widehat{\gamma}_h \quad (3.2)$$

However, equation 3.2 is not a consistent estimator of realised volatility either. Barndorff-

Nielsen et al. (2006) proposed a flat kernel estimator:

$$RV^{(FK)} = RV + 2 \sum_{h=1}^H k\left(\frac{h-1}{H}\right) (\widehat{\gamma}_h - \widehat{\gamma}_{-h})$$

where  $k(x)$  is a deterministic weight function which is defined in the  $[0,1]$  area, and satisfies  $k(0) = 1, k(1) = 0$ . Nielsen et al. (2006) compared three different kinds of kernels, summarised as the following:

1) Bartlett kernel  $k(x) = 1 - x$

2) Kernel of the 2nd order  $k(x) = 1 - 2x + x^2$

3) Epanechnikov kernel  $k(x) = 1 - x^2$

They found that Bartlett kernel and Zhang et al.'s (2005) two time-scale methods have the same asymptotic distribution. The Bartlett kernel has a higher efficiency compared to the Epanechnikov kernel, but a lower efficiency compared to a second order kernel. They also found that a flat kernel estimator is robust to endogenous effects and irregular time observation.

### **3.2.2.2. Filter Method**

Campbell and Shiller (1987) pointed out that non-synchronous trading will lead to first order pseudo autocorrelation in asset returns:

$$\hat{r}_{t,n} = \varphi_t \hat{r}_{t,n-1} + \varepsilon_{t,n} \quad (3.3)$$

where  $\varphi_t$  refers to the coefficient of first order autocorrelation,  $\varepsilon_{t,n} \sim i.i.d.(0, \sigma_{\varepsilon,t}^2)$ . Hasbrouck (1996) proposed an asset price model as:

$$\begin{aligned} r_{t,n} &= p_{t,n} - p_{t,n-1} \quad (3.4) \\ &= (\hat{p}_{t,n} + \eta_t \omega_{t,n}) - (\hat{p}_{t,n-1} + \eta_t \omega_{t,n-1}) \\ &= (\hat{p}_{t,n} - \hat{p}_{t,n-1}) + \eta_t (\omega_{t,n} - \omega_{t,n-1}) \end{aligned}$$

$$\begin{aligned}
&= \hat{r}_{t,n} + \eta_t(\omega_{t,n} - \omega_{t,n-1}) \\
&= \sigma_t \hat{\varepsilon}_{t,n} + \eta_t(\omega_{t,n} - \omega_{t,n-1})
\end{aligned}$$

where  $\hat{\varepsilon}_{t,n}$  refers to the unobservable information component in efficient price, and obeys a mean value equal to zero, variance equal to 1's white noise process with independent distribution. Hence, the daily high-frequency returns are a MA (1) process with the following autocovariance function:

$$E(r_{t,n} r_{t,n-h}) = \begin{cases} \sigma_t^2 + 2\eta_t^2 & h = 0 \\ -\eta_t^2 & h = 1 \\ 0 & h \geq 2 \end{cases} \quad (3.5)$$

where  $\sigma_t^2$  refers to real price variance, and  $\eta_t^2$  represents the extra variance caused by microstructure noise. Under these assumptions, the integrated variance at day  $t$  equals  $IV = N_t \sigma_t^2$ , and  $N_t$  is the daily number of observations.

Based on equation 3.4, French and Roll (1986), Harris (1990) and Zhou (1996) adopted first order serial covariance revise technique to eliminate the daily return's autocorrelation caused by microstructure noise. Oomen (2005, 2006) and Hansen and Lunde (2006) applied this method to re-estimate realised volatility. The key of this method is to use adjacent daily returns' cross product to revise the microstructure noise component ( $2\eta_t^2$ ). However, this method does not have satisfactory robustness. When poor fluidity exists, the estimated volatility can be negative in value. Hence, this covariance revise method has an important drawback.

Corsi et al. (2001) proposed a simple Exponentially Weighted Moving Average Method (EWMA) to revise the microstructure noise. This method can be described as the following:

Assume return  $r_{t,n}$  has a MA (1) structure:

$$r_{t,n} = \omega_{t,n} - \theta_t \omega_{t,n-1} = (1 - \theta_t L) \omega_{t,n} \quad (3.6)$$

where  $\omega_{t,n} = i.i.d.(0, \Omega^2(\sigma_t, \eta_t))$ ,  $\theta_t = f(\sigma_t, \eta_t)$ . The equation 3.6 inverse form is:

$$\omega_{t,n} = (1 - \theta_t L)^{-1} r_{t,n}$$

The EWMA filter can be defined by using recursive equation as in the following:

$$EWMA[\theta, r]_{t,n} = \theta_t EWMA[\theta, r]_{t,n-1} + (1 - \theta) r_{t,n} \quad (3.7)$$

Iterate equation 3.7 can get the relationship between  $(1 - \theta_t L)^{-1}$  and EWMA filter as:

$$\begin{aligned} EWMA[\theta, r]_{t,n} &= (1 - \theta) \{ r_{t,n} + \theta_t r_{t,n-1} + \theta_t^2 r_{t,n-2} + \dots \} \\ &= \{ (1 - \theta) (1 - \theta_t L)^{-1} r \}_{t,n} \end{aligned}$$

The microstructure noise in equation 3.6 can be re-estimated by using an EWMA filter such as:

$$\omega_{t,n} = (1 - \theta)^{-1} EWMA[\theta, r]_{t,n}$$

Furthermore, since

$$E(EWMA[\theta, r]_{t,n}^2) = (1 - \theta_t)^2 E[\omega_{t,n}^2] = \sigma_t^2$$

the EWMA filter does not change the return volatility. Parametric  $\theta_t$  can be estimated by using high-frequency return's first order autocorrelation coefficient. From equation 3.6, it shows the first order autocorrelation coefficient is:

$$\rho_1(1) = \frac{-\theta_t}{1 + \theta_t^2}$$

$$\theta_t = -\frac{1}{2\rho_t(1)} (1 - \sqrt{1 - 4\rho_t(1)^2})$$

The revised high-frequency price can be achieved by using the EWMA filter as:

$$F(x_t) = EWMA[\theta_t, x_t]$$

### **3.2.2.3. Optimal Sampling Method**

As Anderson et al. (2005) pointed out, along with an increase in data frequency, the realised volatility estimation approaches integrated volatility at infinity. However, higher frequency data also increases the microstructure noise effect. On the other hand, a lower frequency can reduce microstructure noise but increases estimation bias. While there is no simple estimation method for realised volatility, a variety of revised techniques solve this problem. This chapter applies Andersen's (2005) proposed volatility signature plot method.

The volatility signature plot provides a graphical representation of the average realised volatility against the sampling frequency. The accuracy improves as the sampling frequency increases, though at a high sampling frequency, market friction is a source of additional noise in the volatility estimate. The daily variance can be written as:

$$(\log P_{t,n} - \log P_{t,1})^2$$

which can be decomposed into realised volatility and intraday autocovariance, such as in:

$$(\log P_{t,n} - \log P_{t,1})^2 = RV + 2 \sum_{j=1}^{n-1} \sum_{i=j+1}^{n-1} (\log P_{t,j+1} - \log P_{t,j})(\log P_{t,j-i+1} - \log P_{t,j-i})$$

Further, the highest frequency for which the autocovariance bias term minimises represents the optimal sampling frequency.

### **3.2.3. Revise to Microstructure Noise**

This sub-chapter discusses the specific microstructure noise revise methods. The basic data description of original 1 minute intraday high-frequency price and return are represented in the previous sub-chapter. As mentioned before, the above discussed revise techniques can be divided into model-revise technique and data-revise technique. For example, the kernel-

based technique (Hansen and Lunde, 2006) and two time-scale technique (Zhang et al., 2005) belong to model-revise technique, whereas the filter technique and optimal sampling frequency technique can be categorised into data-revise technique. This thesis will only apply data-revise technique due to the following two reasons:

First, in order to examine the realised volatility jumps and periodicity characteristics, if the revised model is applied, it will lead to a realised volatility jump detection problem. That is, all the available jumps tests are based on the standard realised volatility model. The test statistics follows:  $(RV - BV)$  obeys a  $Z$  statistics distribution proposed by Barndorff-Nielsen and Shephard (2004). The revised realised volatility model will change the distribution of  $Z$  statistics and lead to the incorrect of jump test results. Second, the data-revise technique can significantly reduce microstructure noise effect. If we apply both data-revise technique and model-revise technique, it may lead to an over revise problem, which may consequently lead to incorrect realised volatility estimations. Hence, this chapter will only apply data-revise technique to eliminate microstructure noise effect.

Two data-revise techniques will be applied, including optimal sampling and filter techniques. The first step is to investigate the optimal sampling frequency by using the volatility signature plot proposed by Andersen et al (2005). The sub-chapter examines five intraday intervals, namely 1 min, 5 mins, 10 mins, 15 mins and 30 mins. Table 5 shows the calculated average daily squared log return, average intraday realised volatility and average intraday autocovariance for each interval.

**Table 5. Calculated Values for Each Interval.**

Futures market					
	average daily squared	Average	intraday	average	intraday
	log return	realised volatility		autocovariance	
1 min interval	0.000196	1.9848E-05		0.000176152	
<b>5 mins interval</b>	<b>0.000196</b>	<b>2.11467E-05</b>		<b>0.000174856</b>	
10 mins interval	0.000196	2.03713E-05		0.000175629	
15 mins interval	0.000196	2.07573E-05		0.000175243	
30 mins interval	0.000196	1.99636E-05		0.000176036	

Spot Market					
	average daily squared	Average	intraday	average	intraday
	log return	realised volatility		autocovariance	
1 min interval	0.000111	1.03932E-05		1.01E-04	
<b>5 mins interval</b>	<b>0.000111</b>	<b>1.91459E-05</b>		<b>9.19E-05</b>	
10 mins interval	0.000111	1.8367E-05		9.26E-05	
15 mins interval	0.000111	1.87169E-05		9.23E-05	
30 mins interval	0.000111	1.99636E-05		9.10E-05	

The above table shows that the optimal sampling frequency for both the futures and spot markets is the 5 minute interval, as it provides the highest average intraday realised volatility statistics and lowest average intraday autocovariance statistics for both markets. The average intraday autocovariance is the indicator reflecting the level of microstructure noise. At the five different intervals, the futures market has very close statistics for average intraday autocovariance, while the spot market shows significantly different average intraday

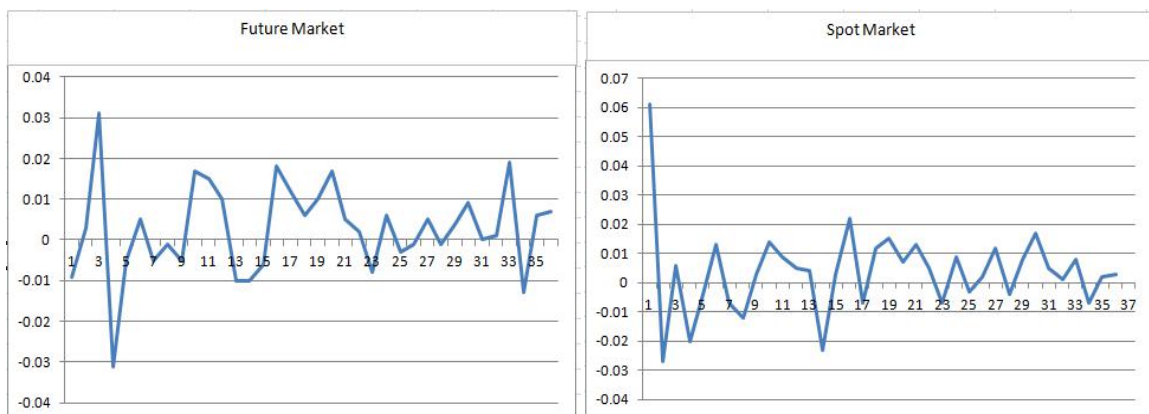
autocovariance. Therefore, the spot market contains higher levels of microstructure noise compared to the futures market.

Once the optimal sampling frequency is selected, the next step is to apply the filter technique discussed in 3.3.2.2 to revise autocorrelation in the data. The autocorrelation can be examined by using autocorrelation function (ACF), which can be defined as:

$$\rho_j = \frac{Cov(y_t, y_{t-1})}{\sqrt{Var(y_t)}\sqrt{Var(y_{t-1})}}, j = 0, \pm 1, \pm 2...$$

Normally, the ACF can be drawn as correlogram to show the relationship between  $\rho_j$  and its lag term at  $j$ . The correlogram of both futures and spot markets' 5 min interval data are represented in Figure 2.

**Figure 2. 5 mins Data Correlogram.**



These two correlograms show that both futures and spot markets' 5 minute interval data does not show a high degree of autocorrelation. Specifically, the maximum autocorrelation coefficient of futures market equals 0.03, which is shown in the first lag. After the first lag, all the other lag coefficients are located within the area 0.02. For the spot market, the first lag coefficient equals 0.06. After the first lag, all the other lag coefficients do not exceed 0.02. Hence, both futures and spot markets do not show a high degree of autocorrelation characteristics. On the other hand, adopting the EWMA filter technique proposed by Corsi et al. (2001), will also increase bias in the realised volatility estimation. Meanwhile, from a



statistics theory point of view, original data can show the true characteristics of the markets compared to revised data. Based on the above two reasons, this chapter will not apply the EWMA filter to revise microstructure noise effect. Hence, the final data will be the 5 minute interval high-frequency data for both futures and spot markets.

#### **3.2.4. Discussion**

So far, we examined the effect of microstructure noise on realised volatility estimation. Past studies provide a variety of revise techniques for microstructure noise. This sub-chapter divides these methods into two categories: model-revised techniques and data-revised techniques. Consequently, three widely used revise techniques are discussed in detail. These are kernel-based technique (Hansen and Lunde, 2006), EWMA filter technique (Corsi et al., 2001) and optimal sampling frequency technique (Andersen et al., 2005). This sub-chapter provides two reasons why data-revised techniques outperform model-revised techniques for further research.

In the data-revise sub-chapter, the first step is to use the volatility signature plot technique to examine the optimal data frequency. Both futures and spot markets conclude that 5 minute intervals are the optimal sampling frequency for realised volatility estimation. Furthermore, the two markets' correlograms are examined, and the results do not show a significant autocorrelation relationship in both markets' data. Hence, this chapter argues that it is better to use original 5 minute interval data rather than adopting the EWMA filter revise technique. In summary, the final data that will be used is the 5 minute interval high-frequency data for both markets.

### **3.3. Daily Volatility Jumps**

Volatility estimation and forecast plays an important role in asset pricing, portfolio selection and financial derivative model design. Normally, the asset price (especially stock price) follows a continuous diffusion process, and the return shows stationary characteristics. However, among the availability of more intraday high-frequency data, numerous researchers have shown that asset returns may experience a large range of price changes during a very small interval period (Eaker et al., 2003; Becker et al., 2009; Barndorff-Nielsen and Shephard, 2004; Lee and Mykland, 2008). This is called a jump process. A jump process has an important position in a financial market's volatility estimation and forecast. Although the magnitude of jumps is very small, jumps will cause large shocks in the stock market and other financial markets. Hence, a study on daily volatility jumps has an important influence on financial market risk management.

Currently, some researchers have investigated mature financial markets' volatility jumps characteristics. For example, Barndorff-Nielsen and Shephard (2004) investigated the volatility jumps, and pointed out that volatility jumps will significantly influence the accuracy of realised volatility estimation. Consequently, they developed a model called bi-power realised volatility to investigate volatility jumps. Based on Barndorff-Nielsen and Shephard's work, Lee and Mykland (2008) developed a method to detect the intraday volatility jumps. Numerous other researchers have built other kinds of models to detect volatility jumps effects (e.g. Corsi et al., 2010 and Todorov, 2011).

This sub-chapter aims to investigate whether Chinese stock index futures and spot markets' daily realised volatility has jump characteristics by using a recently developed daily jumps

test (Barndorff-Nielsen and Shephard, 2004). The data is one year 5 minute interval high-frequency data from the period 19/04/2012 to 19/04/2013, which is selected in the previous sub-chapter. The main contributions of this sub-chapter to the current literature are the following three points: first, there is no current literature examining this topic for Chinese futures and spot markets. The volatility jumps are an important factor influencing the estimation accuracy of realised volatility. Hence, this chapter contributes to further research in Chinese financial markets' realised volatility estimation area, such as the realised volatility forecast model (e.g. HAR model). Second, the Chinese financial market will introduce the futures option in the next few years. The option price is highly relative to the underlying asset's price volatility. This sub-chapter may further contribute to the Chinese option market's pricing. Third, volatility jumps have an extremely important position in financial markets' risk management. This chapter may also contribute to Chinese financial companies' overall risk management.

### **3.3.1. Literature Review**

The jumps characteristics of stock return based on realised volatility estimation is a recent topic, and most of these studies start after the year 2003. A total of eight recent papers are reviewed. These articles include empirical evidence of existing of volatility jumps (e.g. Eaker et al., 2003; Becker et al., 2009), and econometrics method to detect and measure volatility jumps (Barndorff-Nielsen and Shephard, 2004; Lee and Mykland, 2008). All of these studies indicate the fact that some degrees of volatility jumps exist in most financial markets' realised volatility. Furthermore, some jump diffusion models are estimated and tested. The most famous one is provided by Barndorff-Nielsen and Shephard (2004). In their paper, the authors argued that the quadratic variation of the jump component can be estimated

by the difference between realised variance and realised bi-power variation. Besides this, Corsi et al. (2010) developed a threshold bi-power variation model, and Todorov (2011) introduced a new jump-driven SV model to track volatility jumps. The details of the above papers are discussed here:

Eaker et al. (2003) examined continuous-time SV models using the data period for S&P 500 is from 2/01/1980 to 31/12/1999, and for NASDAQ is from 24/09/1985 to 31/12/1999 respectively. Excluding weekends and holidays, this chapter has 5,054 daily observations for the S&P and 3,594 observations for the NASDAQ. Using formal and informal diagnostics, this paper shows strong evidence for jumps in volatility and jumps in returns for the test period. This chapter also investigates how these factors and estimation risk impact option pricing.

Barndorff-Nielsen and Shephard (2004) showed that realised power variation and its extension, realised bi-power variation, are somewhat robust to rare jumps. They demonstrated that in special cases, realised bi-power variation estimates integrated variance in SV models. The difference between realised variance and realised bi-power variation estimates is the quadratic variation of the jump component. This paper, for the first time, provided a method that can separate quadratic variation into its continuous and jump components.

Todorov (2011) introduced a newly jump-driven SV model, in which the volatility is a moving average of past jumps. Two particular semi-parametric classes of jump-driven stochastic volatility models were discussed in detail. In the first one, the price has a continuous component with time-varying volatility and time-homogeneous jumps. For the second one, the jump-driven SV model has only jumps in the price, which has time-varying

size. In the empirical application sub-chapter, the best performance model is the jump-driven SV model containing a continuous component in the price. It outperforms a standard two-factor affine jump diffusion model, and the pure-jump jump-driven SV model with the particular jump specification.

### **3.3.2. Methodology**

Assume  $p_{t,i}$  refers to logarithm stock price at day  $t$  and time  $i$ ,  $i = 1, 2, 3 \dots n_t$ . The  $n_t$  refers to total number of intraday equal intervals. Let  $r_{t,i} = p_{t,i} - p_{t,i-1}$ , which refers to the logarithm intraday return. Then the realised volatility can be estimated by using:

$$RV_t = \sum_{i=1}^{n_t} r_{t,i}^2$$

However, the above estimation ignores the market micro effect. The above estimation is an unbiased estimator of integrated volatility if ignoring the market micro noise. Hansen and Lunde (2006) built a model to adjust this micro structure noise, and this model is:

$$RV_t = \sum_{i=1}^{n_t} r_{t,i}^2 + 2 \sum_{h=1}^q \left(1 - \frac{h}{q+1}\right) \sum_{i=1}^{n_t-h} r_{t,i} r_{t,i+h}$$

where  $q$  is a very small non-negative integer, and  $h$  is also a non-negative integer with  $h \leq q$ . Barndorff- Nielsen and Shephard (2004) provided a continuous jump diffusion framework to deal with volatility jumps issues, and it can be represented as the following:

$$dp(t) = u(t)dt + \sigma(t)dW(t) + k(t)dq(t), 0 \leq t \leq T$$

where  $p(t)$  refers to logarithm asset price at time  $t$ ;  $u_t$  is a continuous with locally bounded variance process;  $\sigma(t)$  represents strictly positive stochastic volatility process, and it obeys right continuous and left limits;  $W(t)$  is a standard Brownian motion;  $dq(t)$  is a counting process with  $dq(t) = 1$  corresponding to a jump at time  $t$  and  $dq(t) = 0$  otherwise. The jump intensity is  $\lambda(t)$ , which means  $P[dq(t) = 1] = \lambda(t)dt$ ;  $k(t)$  is the size of the corresponding jump.

The accumulation return  $r_t \equiv p(t) - p(0)$ , and then the quadratic variance should equal:

$$[r,r]_t = \int_0^t \sigma^2(s)ds + \sum_{0 < s \leq t} k^2(s)$$

Let  $r_{t,\Delta} = p(t) - p(t - \Delta)$ ,  $r_{t,\Delta}$  refers to the discrete return at  $\Delta$  period. If  $\Delta = 1$  (refers to 1 interval per day), then the daily return  $r_{t+1} \equiv r_{t+1,1}$ . Then the daily realised volatility equals:

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} r_{t+j,\Delta}^2$$

When  $\Delta \rightarrow 0$ , it gives:

$$RV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s)ds + \sum_{t < s \leq t+1} k^2(s) \quad (1)$$

Consequently, the bi-power realised volatility is defined as:

$$BV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{1/\Delta} |r_{t+j,\Delta}| |r_{t+(j-1),\Delta}|$$

where  $\mu_1 = \sqrt{\frac{2}{\pi}} = E(|Z|)$ , represents the expected value of the absolute value of  $Z$ , and  $Z$  obeys a normal distribution. When  $\Delta \rightarrow 0$ , it gives:

$$BV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s) ds \quad (2)$$

Combining the equation (1) and (2), the jump component in quadratic variation can be calculated as follows when  $\Delta \rightarrow 0$ :

$$RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \rightarrow \sum_{t < s \leq t+1} k^2(s)$$

However, it cannot ensure  $RV_{t+1}(\Delta) \geq BV_{t+1}(\Delta)$  for all  $\Delta$ , but the jump value must be non-negative. The above equation is adjusted as following:

$$J_{t+1} \equiv \max [RV_{t+1}(\Delta) - BV_{t+1}(\Delta), 0] \quad (3)$$

The above equation (3) provides a framework to calculate the jump component in quadratic variation. However, in a real data set, it cannot achieve  $\Delta \rightarrow 0$ . This will lead to measure error. In order to judge whether the above calculation belongs to jumps component or measure error, the following statistical test is developed. Define the following statistics:

$$TQ_{t+1}(\Delta) \equiv \Delta^{-1} \mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} |r_{t+j,\Delta,\Delta}|^{4/3} |r_{t+(j-1),\Delta,\Delta}|^{4/3} |r_{t+(j-2),\Delta,\Delta}|^{4/3}$$

where  $\mu_{4/3} = 2^{2/3} \Gamma(7/6) \Gamma(1/2)^{-1} = E(|Z|^{4/3})$ . The significant jump component can be tested by  $Z_{t+1}(\Delta)$  statistics:

$$Z_{t+1}(\Delta) \equiv \Delta^{-\frac{1}{2}} \frac{[RV_{t+1}(\Delta) - BV_{t+1}(\Delta)] * RV_{t+1}(\Delta)^{-1}}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5) * \text{maximum}\{1, TQ_{t+1}(\Delta) * BV_{t+1}(\Delta)^{-2}\}]^{1/2}} \quad (4)$$

From the above equation, the estimated value of significant discrete jumps can be:

$$J_{t+1,\alpha}(\Delta) \equiv I[Z_{t+1}(\Delta) > \Phi_\alpha] \cdot [RV_{t+1}(\Delta) - BV_{t+1}(\Delta)]$$

where  $I[\cdot]$  refers to indicator function;  $\alpha$  represents confidence level and  $\Phi_\alpha$  is critical value.

Considering the bid–ask spread and price discrete influences, the asset price will not follow the semi-martingale assumption, which will lead to bias on bi-power realised volatility estimation. In order to solve this issue, the adjusted bi-power realised volatility is:

$$BV_{1,t+1}(\Delta) = \mu_1^{-2}(1 - 2\Delta)^{-1} \sum_{j=3}^{1/\Delta} |r_{t+j,\Delta}| |r_{t+(j-2),\Delta}|$$

The adjusted  $TQ_{t+1}(\Delta)$  statistics are:

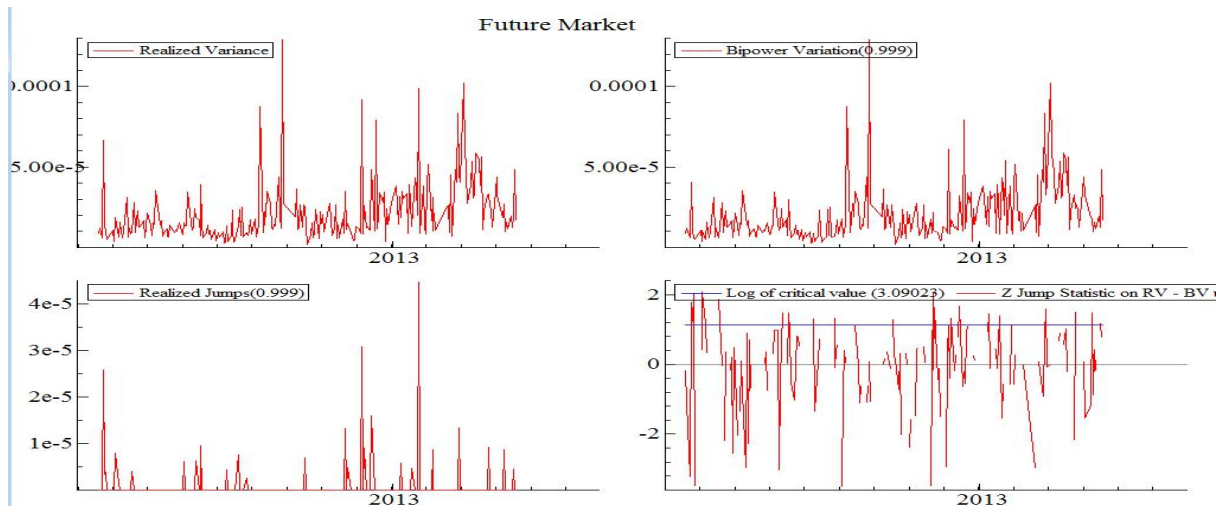
$$TQ_{t+1}(\Delta) \equiv \Delta^{-1} \mu_{4/3}^{-3} (1 - 4\Delta)^{-1} \sum_{j=3}^{1/\Delta} |r_{t+j,\Delta}|^{4/3} |r_{t+(j-2),\Delta}|^{4/3} |r_{t+(j-4),\Delta}|^{4/3}$$

The adjusted  $Z_{1,t+1}(\Delta)$  statistics can be achieved by using adjusted  $BV_{1,t+1}(\Delta)$  and  $TQ_{1,t+1}(\Delta)$  instead of  $BV_{t+1}(\Delta)$  and  $TQ_{t+1}(\Delta)$  in equation (4). When  $\Delta \rightarrow 0$ ,  $Z_{1,t+1}(\Delta)$  follows a standard normal distribution.

### 3.3.3. Empirical Results

The daily realised volatility, bi-power realised volatility and volatility jumps are calculated and graphed by using the method discussed in the methodology sub-chapter. The tested data are 5 minutes high-frequency intraday data, and the total interval for 1 day equals 48. That is, the intraday trading period is from 02:30-04:30 and 06:00-08:00 (London time), four hours in total. The following figure draws the estimated results.

**Figure 3. Jump Test Results.**





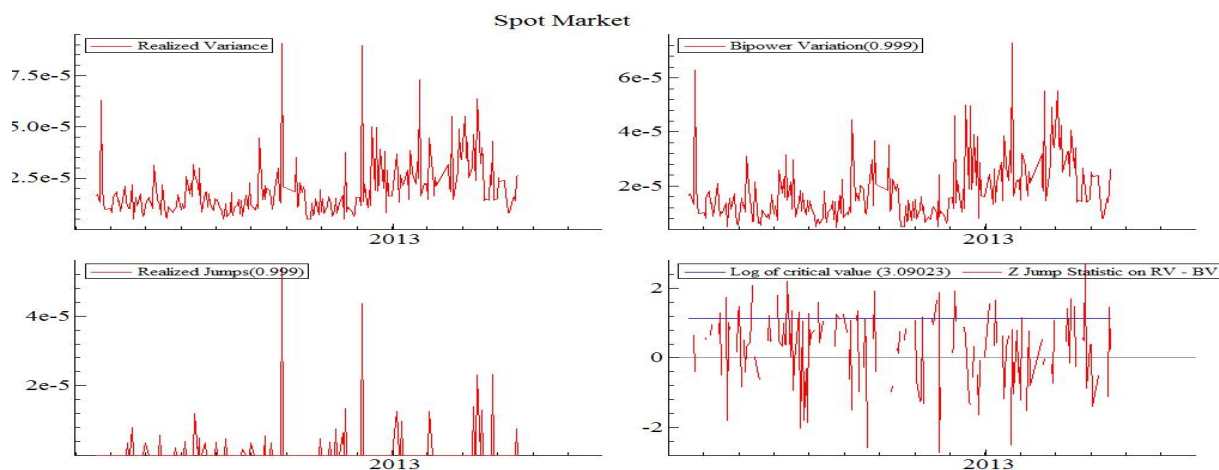


Table 6 provides the basic statistical description of realised volatility, bi-power realised volatility and jumps.

**Table 6. Data Description.**

Futures market				
	RV	Log(RV)	$J_t$	Log( $J_t + 1$ )
Mean	2.11e-05	-11.04736	1.08e-06	1.08e-06
St. Dev.	1.84e-05	0.6743953	4.42e-06	4.42e-06
Max	0.000129	-8.953094	4.47e-05	4.47e-05
Min	1.62e-06	-13.33436	0.000000	0.000000
Skewness	2.468397	0.143873	6.428122	6.428052
Kurtosis	11.21600	2.951705	52.80863	52.80755
J-B	934.0584	0.865493	26902.86	26901.73
$Q^{12}$	147.05	242.91	5.7567	5.7568
ADF	-5.236039	-4.879518	-16.10383	-16.10384
PP	-14.87458	-14.21806	-16.09928	-16.09929

Notes: The Jarque-Bera test is to test the normality distribution of return. Ljung-Box statistics,  $Q^2(12)$  is to test the series autocorrelation up to 12 lags. The 1% critical value for ADF and PP test is 3.435. The confidence level to test null hypothesis is 1%.

Spot Market				
	RV	Log(RV)	$J_t$	Log( $J_t + 1$ )
Mean	1.91e-05	-11.03605	1.32e-06	1.32e-06
St. Dev.	1.31e-05	0.568267	5.39e-06	5.39e-06
Max	9.08e-05	-9.307013	5.41e-05	5.41e-05
Min	4.93e-06	-12.22010	0.000000	0.000000
Skewness	2.381618	0.392824	6.694292	6.694207
Kurtosis	10.87497	3.062458	56.39271	56.39141
J-B	861.1529	6.314959	30805.37	30803.91
$Q^{12}$	189.66	366.98	3.1915	3.1916
ADF	-5.075574	-4.288500	-16.48167	-16.48169
PP	-14.50867	-12.43372	-16.48045	-16.48046

Notes: The Jarque-Bera test is to test the normality distribution of return.

Ljung-Box statistics,  $Q^2(12)$  is to test the series autocorrelation up to 12 lags.

The 1% critical value for ADF and PP test is 3.435.

The confidence level to test null hypothesis is 1%.

The realised volatility of both the futures and spot markets shows strong autocorrelation characteristics by the Q-statistics and at the log (RV) value. Comparing the logarithmic realised volatility to the original realised volatility, the logarithmic realised volatility closely matches the normal distribution of both markets, a desirable characteristic. The ADF and PP test results indicate that both markets' realised volatilities (and logarithms) are stationary processes. Hence, it is better to use logarithmic realised volatility to model and forecast asset volatility rather than original realised volatility model.

In the jumps series, this paper uses (jumps+1) in the logarithm to avoid the zero value. The jumps series for both markets show low autocorrelation characteristics compared to the

realised volatility series. The spot market has higher volatility jumps compared to the futures market in general, and both markets show jump cluster effects. That is, a jump occurring will more likely lead to another jump in a short period of time. Lastly, comparing realised volatility and bi-power realised volatility, the jump component does not show a high percentage in the realised volatility estimation. This conclusion can be supported by the total number of jumps detected, as presented in table 7.

Table 7. Total Number of Jumps Detected under  $H_0$  (No jumps) versus  $H_1$  (Jumps exist).

	Futures market	Spot market
Expected number of jumps under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	0.244	0.244
Number of jumps detected under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	26	31
Proportion of detected jumps under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	0.106557	0.127049
Critical level under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	0.001	0.001
Critical value under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	3.09023	3.09023

From the above table, both futures and spot markets show a low percentage of jumps characteristics in realised volatility estimation. On average, we reject our null hypothesis ( $H_0$ ) that there is no jump against  $H_1$  (Jumps exist) and conclude that one significant jump will be expected every 10 days. The results also indicate that the spot market has a slightly higher

chance for jumps compared to the futures market. This shows that the spot market reacts to new information more rapidly, and new information may arrive at the spot market first, and then flow to the futures market. From the above conclusions, we can infer that the spot market dominates the futures market to some degree. This result is consistent with Yang et al. (2012). Given the fact that China is an emerging market with substantial volatility and the futures markets have only established for a very short period, people should not be surprising to see that these methods fail to achieve a 99% level, in my opinion.

#### **3.3.4. Discussion**

This sub-chapter examines the daily volatility jumps in the Chinese stock index and spot market by using a recently developed technique. This sub-chapter applies for the first time the Barndorff-Nielsen and Shephard (2004) method to test Chinese financial markets. Some interesting realised volatility characterises of the Chinese stock index futures and spot markets are found. Firstly, the logarithm of realised volatility shows a high degree of autocorrelation and is almost subject to a normal distribution, which leads to a good forecasting power of logarithm realised volatility. Hence, this sub-chapter suggests that subsequent volatility forecast models be based on logarithm realised volatility.

In the daily volatility jumps test sub-chapter, a low degree of autocorrelation is shown, but with high cluster effect. That is, it is not easy to model and forecast the volatility jumps, but one volatility jump will generally lead to another volatility jump. In general, the results also conclude that volatility jumps have a low percentage in realised volatility estimation. On average, a significant jump will occur every ten trading days. Meanwhile, the spot market shows a slightly higher percentage of jumps than the futures market, and the spot market's

jumps are more rapid. This suggests that, the spot market generally dominates the futures market. This result is consistent with Yang et al., (2012).

### **3.4. Intraday Volatility Jumps and Periodicity**

Intraday volatility jumps and periodicity are another two recent interesting topics existing in realised volatility estimation. This section expands on the daily volatility jumps to intraday volatility jumps. Intraday volatility jumps means potential high risk for financial companies at intraday frequency level. Hence, capturing and understanding intraday volatility jumps plays an important role in daily risk management. Numerous researchers provide a variety of test models to detect intraday volatility jumps (e.g. Lee and Mykland, 2008; Boudt et al., 2010). However, Laurent et al. (2011) pointed out that ignoring intraday periodicity will lead incorrect estimation of intraday volatility jumps. On the other hand, the intraday volatility jumps will also shift the value of the traditional intraday periodicity factor estimator. Hence, Laurent et al. (2011) suggested a dynamic relationship between intraday volatility jumps and periodicity. Also, they provided estimation methods to model these two characteristics in realised volatility estimation.

In this sub-chapter, we will examine intraday volatility jumps and periodicity characteristics for Chinese stock index futures and spot markets. The empirical data is 5 minutes high-frequency data. The results may contribute to current literature by the following three points: first, this is the first time examining Chinese stock index futures and spot markets' intraday volatility jumps and periodicity using the dynamic estimation method proposed by Laurent et al. (2011). Second, intraday volatility jumps and periodicity play an important role in realised volatility estimation. The results may help readers to understand the realised volatility characteristics more deeply. Third, intraday volatility jumps play an important role in daily

risk management. A good understanding of these volatility characteristics will benefit daily risk management.

### **3.4.1. Literature Review**

There are four papers that discuss the intraday volatility periodicity issue, another four papers that investigate the intraday volatility jumps characteristics, and one more paper that examines the dynamic effect between intraday volatility periodicity and intraday volatility jumps. The key concepts of these nine papers can be summarised, as in table 8

**Table 8. Summary of Key Concepts.**

<b>Intraday Volatility Periodicity</b>	
McMillan and Speight (2004)	Point out ignoring intraday volatility periodicity will lead to misleading conditional volatility model estimation.
Vradhyula and Ergan (2004)	Find a U-sharp intraday periodicity in volatility.
Wu (2012)	Propose a test to detect periodicity based on point process theory.
Intraday Volatility Jumps	
Lee and Mykland (2008)	Introduce a nonparametric test to detect intraday volatility jumps.
Gilder (2009)	Examines the empirical properties of intraday jumps and co-jumps in 72 US equities by using the ABD test (Andersen et al., 2007).
Duyvesteyn et al. (2011)	Links the intraday volatility jumps to macroeconomic announcements.

The above is the summary of the key ideas in these nine papers, and the specific details can also be found here:

McMillan and Speight (2004) aimed to investigate the intraday periodicity of FTSE-100 index futures market. The nature of periodicity was first examined. Subsequent empirical results concerning the temporal aggregation of GARCH models show that the uses of returns, which are not adjusted for such periodicity, are misleading. However, adjustment using a sine-cosine wave method or standardisation by mean-absolute-returns provides more consistent results, and the latter method dominates in out-of-sample forecasting of the volatility of successive individual futures contracts. The potential time to maturity effects of single contracts are also considered, but are statistically rejected for both forms of periodicity-adjusted data.

Vradhyula and Ergan (2004) used 5-minute data to examine market volatility in the Dow Jones Industrial Average with the presence of trading collars. A polynomial specification was used for capturing intraday seasonality. Results indicated that market volatility is 3.4% higher in declining markets when trading collars are in effect. Results also supported a U-shaped intraday periodicity in volatility.

Gilder (2009) examined the empirical properties of jumps and co-jumps in 72 U.S. equities by using the intraday jump test (ABD) of Andersen et al. (2007), correcting for the intraday volatility pattern. The intraday nature of the ABD test allows an examination of both the statistical (the frequency, size and direction) and economic (association with systematic news) properties of jumps and co-jumps. Including a proxy for the S&P 500 market index allows two co-jump categories to be analysed: systematic and idiosyncratic co-jumps. Jumps were

found to be frequent and sizes were largely symmetric. No evidence of a link between the release of macroeconomic news at 10:00:00 EST time and the arrival of jumps/co-jumps was found. In excess of 60% of jumps were found to be involved in a co-jump with the majority of these being idiosyncratic in nature. However, idiosyncratic co-jumps and singular jumps (those jumps not involved in a co-jump) are easily diversified away by holding portfolios of moderate size. No significant differences were found to exist between the properties of systematic, idiosyncratic and singular co-jump/jumps; all appeared to be symmetrically distributed, showed no deterministic intraday pattern and did not significantly differ in their size. The results were recently carried to China in Wang et al. (2015).

Duyvesteyn et al. (2011) expanded on the work of Wright and Zhou (2009), who demonstrated that the average jump mean in bond prices can predict excess bond returns, capturing the countercyclical behaviour of risk premia. They showed that these jumps often take place at 8:30 and 10:00, directly linking them to specific macroeconomic news announcements. Mean reversion, which looks at the total return over the past period rather than just the part related to jumps, has no predictive ability. Hence, it is important to consider excess returns that are related to macroeconomic announcements that matter to market participants, and jumps are a good market proxy for what investors believe is important news.

Wu (2012) carried out a simulation and showed that the performance of the cubic spline procedure (proposed by Engle and Russell, 1998) is not entirely satisfactory. He further defined periodicity point processes rigorously and proved a time change theorem. A new intraday periodic adjustment procedure was then proposed, and the effectiveness of this procedure was demonstrated in the simulation example. The new approach was easy to



implement and well supported by the point process theory. It provides an attractive alternative to the cubic spline procedure.

### **3.4.2. Methodology**

There are a variety of methodologies to examine and model intraday volatility jumps and periodicity. In this sub-chapter, the methodologies used to investigate intraday volatility jumps and periodicity are the non-parametric estimation methods proposed by Laurent et al. (2011).

#### **3.4.2.1. Non-Parametric Estimation of Periodicity**

Without microstructure noise, the periodicity factor refers to the standard deviation of the standardised returns as:

$$\bar{r}_{i,\Delta} = \frac{r_{i,\Delta}}{\left(\frac{\widehat{IV}}{M_\Delta}\right)^{1/2}}$$

where  $\widehat{IV}$  is an estimator of the integrated variance and  $M_\Delta = \frac{1}{\Delta}$ . Taylor and Xu (1997) proposed the following estimation procedure for  $f_i$ . First, collect all the standardised returns having standard deviation  $f_i$ . Denote these  $\bar{r}_{1,i}, \dots, \bar{r}_{n_i,i}$ , with  $n_i$  being the number of standardised returns having standard deviation  $f_i$ . Then compute their standard deviations, that is:

$$SD_{i,\Delta} = \sqrt{\frac{1}{n_i} \sum_{j=1}^{n_i} \bar{r}_{j,i}^2}$$

Finally, their estimator is a standardised version of these periodicity estimates:

$$\hat{f}_{i,\Delta}^{SD} = \frac{SD_{i,\Delta}}{\sqrt{\frac{1}{M_\Delta} \sum_{j=1}^{M_\Delta} SD_{j,\Delta}^2}}$$

Andersen and Bollerslev (1997) showed that more efficient estimates can be obtained if the whole time series dimension of the data is used for the estimation of the periodicity process. They considered the regression equation:

$$\text{Log}|\bar{r}_{i,\Delta}| - c = \text{log}f_{i,\Delta} + \varepsilon_{i,\Delta}$$

where the error term  $\varepsilon_{i,\Delta}$  is *i.i.d.* distributed with mean zero and having the density function of the centred absolute value of the log of a standard normal random variable, and  $c = -0.63518$ .

In absence of jumps, the standard deviation is efficient if the standardised returns are normally distributed. In the presence of jumps, the above estimator is inefficient. Hence, Boudt et al. (2008) proposed a robust non-parametric estimator to replace the standard deviation. This method is called median absolute deviation (MAD). The MAD of a sequence of observations  $y_1, \dots, y_n$  is defined as:

$$1.486 \times \text{median}_i |y_i - \text{median}_i y_j|$$

where 1.486 is a correction factor to guarantee that the MAD is a consistent scale estimator at the normal distribution. The MAD estimator for the periodicity factor equals:

$$\hat{f}_{t,i}^{MAD} = \frac{MAD_{t,i}}{\sqrt{\frac{1}{M_\Delta} \sum_{j=1}^{M_\Delta} MAD_{t,j}^2}}$$

### **3.4.2.2. Intraday Jumps Test**

Without intraday volatility periodicity and microstructure noise, the intraday jumps test statistics can be represented as (Lee and Mykland, 2008):

$$I_{i,\Delta} = \frac{|r_{i,\Delta}|}{\sqrt{M_{\Delta}^{-1}BPV_{\Delta}}}$$

where

$$\sqrt{M_{\Delta}^{-1}BPV_{\Delta}} = \int_{(i-1),\Delta}^{i,\Delta} \sigma_s^2 ds + 2\sigma_{\varepsilon X}^2$$

However, the above jump test statistics do not consider intraday periodicity and will lead to incorrect jump detection. These statistics will over-detect jumps when volatility periodicity is high and vice versa. Hence, Laurent et al. (2011) proposed the following adjustment:

$$J_{i,\Delta} = \frac{|r_{i,\Delta}|}{\hat{f}_{i,\Delta} \sqrt{M_{\Delta}^{-1}BPV_{\Delta}}}$$

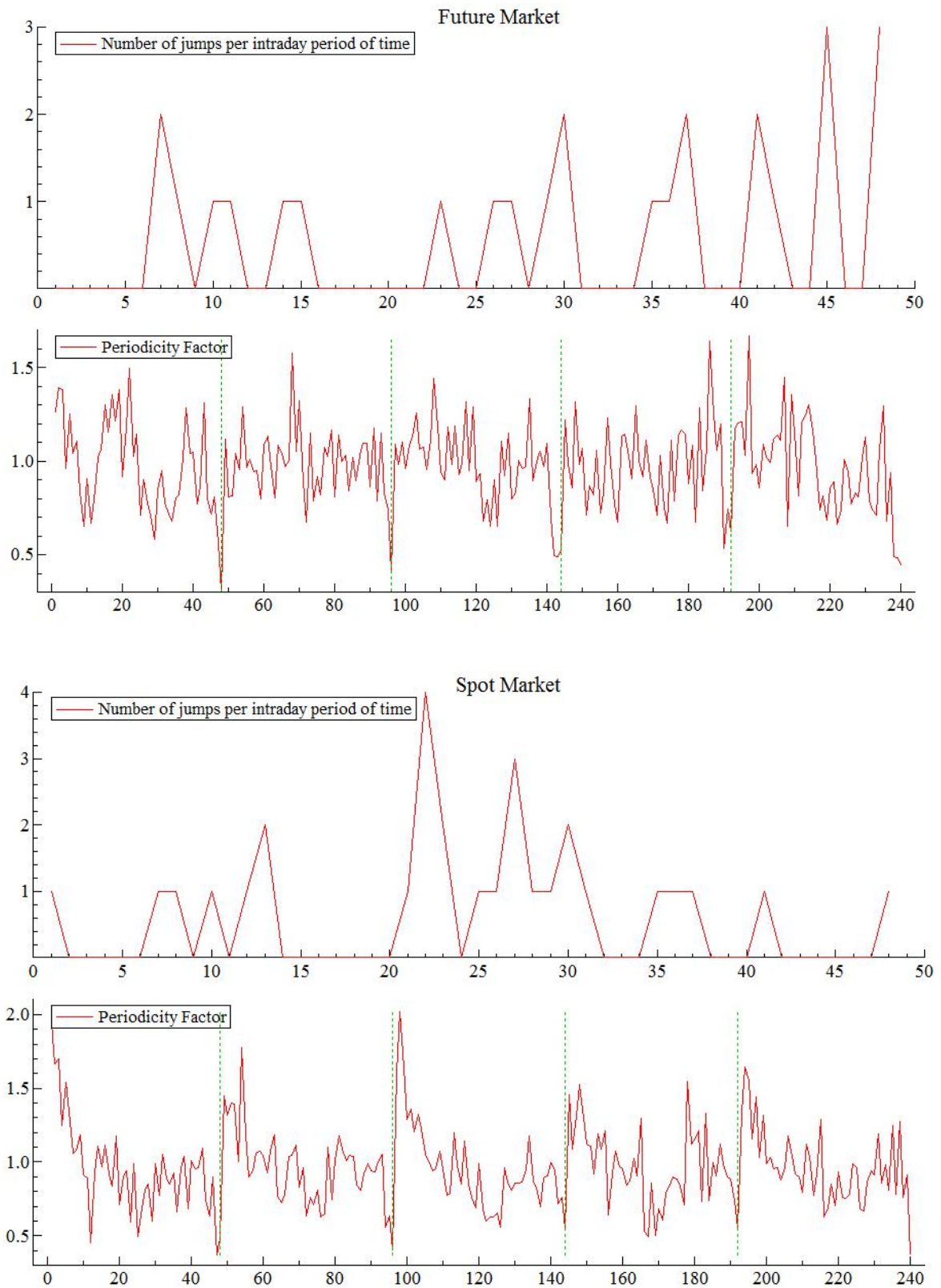
This test statistic assumes that the microstructure noise follows the same periodic pattern as the spot volatility of the underlying efficient price process. This is inconsistent with the assumption that the variance of the microstructure noise is constant over the day. Hence, a further Z statistic is conducted as in the following:

$$Z_{i,\Delta} = \frac{|r_{i,\Delta}|}{\sqrt{\hat{f}_{i,\Delta}^2 M_{\Delta}^{-1}BPV_{\Delta} + 2\sigma_{\varepsilon X}^2}}$$

### **3.4.3. Empirical Results**

The 5 minutes interval data are used here. The intraday periodicity is calculated by using MAD, and the intraday volatility jump is tested by using  $J$  statistics, as mentioned in the methodology sub-chapter. The estimated results are represented in the figure blow.

**Figure 4. Estimated Results.**



From the above graphed results, the key characteristics of intraday volatility jumps and periodicity can be summarised as the following:

For the futures market, the intraday jumps most likely happen at 10:00, 13:30, 13:45 and 15:00 (Chinese Local Time). 10:00 is the time when Chinese daily macroeconomic news is released. This result implies that the daily macroeconomic news is one of the most important factors causing intraday volatility jumps. This result is consistent with the Duyvesteyn et al. (2011) and Evan (2011) conclusion. Comparing the morning market to the afternoon market, the afternoon market generally has more intraday volatility jumps. At the same time, the most likely time to have an intraday volatility jump period is 14:00–15:00, which is the last trading hour of the futures market. Meanwhile, intraday volatility shows a cluster effect. Three broad jumps cluster periods are found, i.e., 10:00–11:00, 13:00–13:40 and 14:00–15:00. Lastly, the most intense intraday volatility jumps happen at the time between 14:45 to 15:00, which is the last 15 minutes of one trading day. The intraday periodicity factor shows a strong cycle characteristic. The general shape of the intraday periodicity factor can be described as the *M* shape.

For the spot market, the most likely periods to have intraday volatility jumps are 10:00, 11:25, 13:15 and 13:30 (Chinese local time). Using the same argument as the in the futures market, 10:00 is the daily macroeconomic news release time in China. Hence, the spot market's intraday volatility jumps are also highly linked to the macroeconomic news release. The largest intraday volatility jump happens at 11:25, which is 5 minutes before the close of the morning market. In general, the afternoon market has more intraday volatility jumps compared to the morning market. Meanwhile, the spot market's intraday volatility also shows a volatility cluster effect. Two broad volatility cluster periods can be concluded, which are 9:30–10:30 and 11:25–13:30. The intraday periodicity factor shows similar characteristics to the futures market. These include a strong cycle characteristics, and an *M* shape in general.

Consequently, we test the null hypothesis:  $H_0$  (No jumps) versus  $H_1$  (Jumps exist) and the total numbers of intraday jumps for both markets are reported in Table 9.

**Table 9. Intraday Jumps Test under  $H_0$  (No jumps) versus  $H_1$  (Jumps exist).**

	Futures market	Spot market
Number of jumps detected under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	26	29
Proportion of detected jumps under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	0.00221995	0.00247609
Number of periods with at least one significant jump under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	26	28
Proportion of periods with at least one significant jump under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	0.106557	0.114754
Critical value under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	4.84597	4.81597
Expected number of jumps under $H_0$ (No jumps) versus $H_1$ (Jumps exist)	0.244	0.244

From this table, it shows that in general, we can hardly reject the null hypothesis:  $H_0$  (No jumps) versus  $H_1$  (Jumps exist) and both futures and spot markets show a low percentage of intraday volatility. The proportion of detected intraday jumps is only near 0.2%. On the other hand, the results also suggest that, on average, there will be significant volatility jumps within ten trading days for both futures and spot markets in China. Interestingly, we find that our results are in consistency with the conclusions in Duyvesteyn et al. (2011) and Evan (2011).

#### **3.4.4. Discussion**

This sub-chapter examines the intraday volatility jumps and periodicity for Chinese stock index futures and spot markets. The data used is 5 minute high-frequency intraday data. The methodologies are the recent dynamic estimation methods proposed by Laurent et al. (2011). The sub-chapter shows five important results:

First, there are significant intraday volatility jumps at 10:00 for both futures and spot markets. China releases its daily macroeconomic news at 10:00. Hence, the results suggest that the intraday volatility jump is highly linked to macroeconomic news release. This result is consistent with the Duyvesteyn et al. (2011) and Evan (2011) conclusions. Second, intraday volatility shows a volatility cluster effect. The futures market can conclude three high intraday volatility jump cluster periods, and the spot market can conclude two high intraday volatility jump cluster periods. Third, the afternoon market has a higher percentage of intraday volatility jumps compared to the morning market for both futures and spot markets. Fourth, the most rapid intraday volatility jump happens at 15 minutes before the futures market closing time, and 5 minutes before the spot market closing time. Fifth, both futures and spot markets' intraday periodicity factors show a strong cycle characteristic, and can be described as an *M* shape in general.

#### **3.5. Realised Volatility Long Memory Characteristics**



Another interesting question of realised volatility is its long memory characteristics. Long memory characteristics mean time series decay very slowly, which cannot be explained by a finite number of stationary auto regress and move-average formats. It is usually explained by a fractional differential process. The mathematical definition of long memory can also be found in the methodology section. As Andersen et al. (2011) pointed out, realised volatility has significant long memory characteristics or fractionally integrated characteristics. If realised volatility has long memory characteristics, then it can be modelled and forecasted accurately by using long memory models such as the HAR-RV and ARFIMA models. Much empirical evidence shows that long memory characteristics exist in asset return and volatility (e.g. Andersen et al., 2001; Isa et al., 2007; Corsi, 2009). Meanwhile, some other studies concluded that long memory volatility models can perform better than short memory volatility models (Lima et al., 1998; Lu et al., 2006; Granger et al., 2009).

The long memory characteristics of asset prices can be linked to the market efficiency hypothesis (e.g. Andersen et al, 2000). The efficient market hypothesis is to assume the asset price follows the Markov process, which means the futures asset price only depends on current price and cannot be influenced by its past value. However, as discussed above, long memory process means past values have significant influence on futures value. From an asset price aspect, the current major opinion is that asset price follows the Markov process. But from an asset volatility point of view, the current major opinion is that volatility has long memory characteristics.

This sub-chapter will examine the realised volatility's long memory characteristics for both Chinese markets. This sub-chapter may contribute to the current literature in the following way: first, this is the first time investigating realised volatility's long memory characteristics

for Chinese stock index futures and spot markets. Second, if both two markets' realised volatility shows strong long memory characteristics, then the long memory model will be more suitable to model and forecast these two markets' realised volatility. This conclusion may contribute to the further research on modelling Chinese financial markets' realised volatility.

### **3.5.1. Literature Review**

A total of seven papers are examined in detail. There are three papers to compare the long memory volatility model and short memory volatility model (Lima et al., 1998; Lu et al., 2006; Granger et al., 2009). All of these three papers concluded that long memory volatility can generally outperform a short memory volatility model. Andersen et al. (2001) first pointed out that realised volatility has a long memory characteristic. Based on Andersen et al. (2001) result, Corsi (2009) proposed a Heterogeneous Autoregressive Model to forecast Realised Volatility (HAR-RV). Corsi showed that the HAR-RV model can track realised volatility's long memory characteristics, and can provide very good forecasting power. Isa et al., (2007) empirically examined the Malaysian stock exchange market. The empirical results showed that Malaysian stock volatility has long memory characteristics. Lastly, the relationship between long memory volatility model and structural breaks is discussed (Zivot et al., 2010, Yang et al., 2015). The details of these seven papers can also be found here:

Isa et al. (2007) investigated the asymmetry and long memory volatility behaviour of the Malaysian Stock Exchange daily data over a period from 1991–2005. The long-spanning data set enabled them to examine piecewise before, during and after the economic crisis encountered in the Malaysian stock market. The daily index returns were adjusted for infrequent trading effect and the estimated Hurst's parametric allowed them to rank the market efficiency across the periods. The leverage effect, clustering volatility and long memory behaviour of the volatility were fitted by the asymmetry GARCH models and GARCH with the inclusion of realised volatility at the final period.

Corsi (2009) proposed an additive cascade model of volatility components defined over different time periods. This volatility cascade led to a simple AR-type model in realised volatility with the feature of considering different volatility components realised over different time horizons and is thus termed the Heterogeneous Autoregressive model of Realised Volatility (HAR-RV). In spite of the simplicity of its structure and the absence of true long memory properties, simulation results showed that the HAR-RV model successfully achieved the purpose of reproducing the main empirical features of financial returns (long memory, fat tails, and self-similarity) in a very tractable and parsimonious way. Moreover, empirical results showed remarkably good forecasting performance.

Granger et al. (2009) compared the out-of-sample forecasting performance of three long memory volatility models (fractionally integrated, break and regime-switching) against three short memory models (GARCH, GJR and volatility component). Using S&P 500 returns, they found that structural break models produced the best out-of-sample forecasts, when futures volatility breaks are known. Without knowing the futures breaks, GJR models produced the best short horizon forecasts and FI models dominated for volatility forecasts of 10 days and beyond. The results suggested that S&P 500 volatility is non-stationary, at least in some time periods.

Yang et al. (2015) investigated the realised volatility forecast of stock indices under the structural breaks. They utilised a pure multiple mean break model to identify the possibility of structural breaks in the daily realised volatility series. They employed the intraday high-frequency data for the period 4 January 2000 to 30 December 2011. Yang et al. (2015) examine the effects of structural breaks on the performance of ARFIMAX-FIGARCH models

for the realised volatility forecast, by utilising a variety of estimation window sizes designed to accommodate potential structural breaks.

### **3.5.2. Methodology**

Long memory is also known for its long dependence characteristics. It is usually defined as having one kind of persistent autocorrelation characteristic in a time series. That is, the coefficient of autocorrelation decreases at the hyperbolic ratio. Under the long memory characteristic, some degree of autocorrelation exists in all observations, and the history value will significantly influence futures value. The opposite of the long memory process is the random walk process, which refers to a futures value that does not depend on past value. There are varieties of mathematical definitions of long memory. Two widely used definitions will be discussed.

According to the Brockwell and Davis (1991) definition, assume  $p(k)$  is the  $k$  – order ACF of a stationary time series  $\{X_t\}$ , and  $p(k)$  decreases at hyperbolic ratio among the order of lag  $k$  increase, then this time series  $\{X_t\}$  has long memory characteristics if  $p(k)$  satisfy:

$$p(k) \sim ck^{2d-1} (k \rightarrow \infty)$$

In the above equation,  $c$  is a constant and  $d$  refers to the long memory parametric. Granger and Ding (1996) provide another definition of long memory process:

A time series  $\{X_t\}$  is a long memory process, if its spectral density function  $f(w)$  has the following two characteristics:

$$f(w) \rightarrow \infty, \text{ when } w \rightarrow \infty$$

*After removes at most finite  $w$  vaule,  $f(w)$  has upper bound.*

There is a variety of long memory test methods based on different definitions of long memory processes. Two widely used methods will be examined, including the  $R/S$  analysis method and the Geweke and Porter-Hudak method (GPH).

The  $R/S$  analysis method is also called the rescaled range analysis method. The nonparametric method is widely used to test the time series long memory characteristics based on the definition of Brockwell and Davis (1991). This method can be used to test the time series with large Skewness and Kurtosis under a non-Gaussian distribution. The first step of  $R/S$  analysis is to estimate Hurst value ( $H$  value), and then use  $d = H - 0.5$  to estimate the long memory parametric  $d$ , and finally calculate the following regression:

$$\log (R/S)_n = \log (c) + H*\log (n)$$

The  $R/S$  analysis uses  $H$  value to judge whether one time series has a long memory characteristic. If  $0.5 < H < 1$ , the time series has a long memory or long dependence characteristic.

The GPH (1983) method is a semi-parametric method to test the long memory characteristics based on the second definition as mentioned above. The GPH method can be described as the following:

For a time series  $\{X_t\}$  that has a sample length equal to  $T$ , define periodogram as:

$$I(\lambda_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^T X_t \exp(it\lambda_j) \right|^2$$

where  $\lambda_j = \frac{2\pi j}{T}$  is the frequency, and the  $f(\lambda_j)$  is the spectral density function at frequency  $\lambda_j$ .

According to the GPH estimation method, the long memory parametric  $d$  can be achieved by:

$$\log(I(\lambda_j)) = C - d \log \left[ 4 \sin^2 \left( \frac{\lambda_j}{2} \right) \right] + \varepsilon_j, j = 1, \dots, m$$

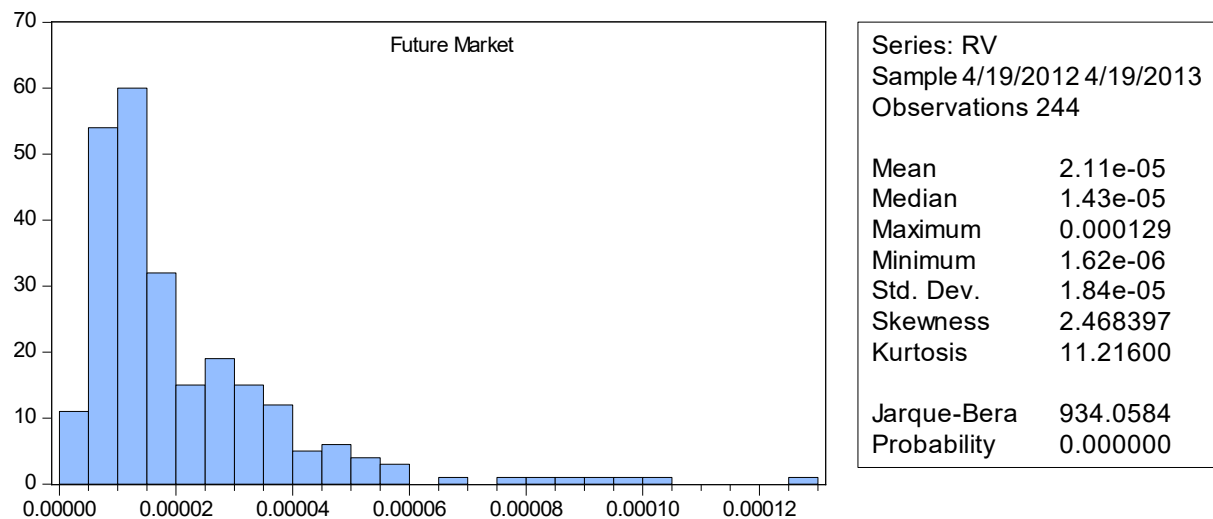
The most widely used  $m$  value in the above equation equals  $T^{0.5}$ .

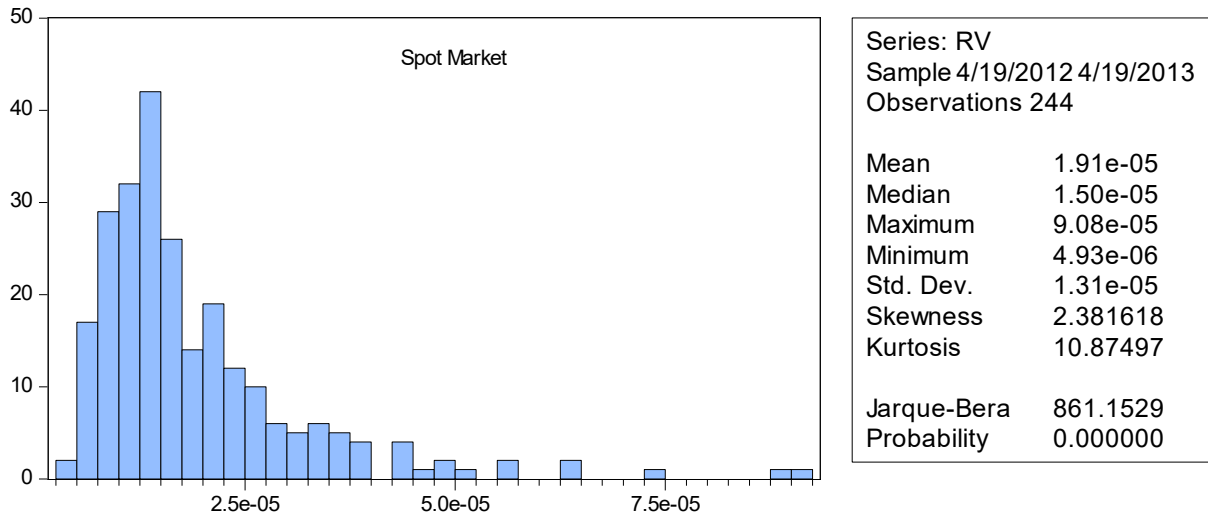
### 3.5.3. Empirical Results

The data used is 5 minutes high-frequency intraday data. The realised volatility is estimated and used in this analysis. The following figure contains the estimated statistical descriptions for both futures and spot markets' realised volatility.

From Realized Volatility's density distribution of both two markets, it shows significant positive skewness and leptokurtosis characteristics. From the Jarque-Bera statistics, it is clearly to reject the null hypothesis ( $H_0$  (No jumps) versus  $H_1$  (Jumps exist)) for both two markets, and conclude that both two Realized Volatility series are not normal distributed.

**Figure 5. Statistic Descriptions.**





Consequently, the ADF test ( $H_0$  (there is a unit root in time series) versus  $H_1$  (there is no unit root in time series)) and KPSS ( $H_0$  (the time series is stationary) versus  $H_1$  (the time series is not stationary)) test are conducted, and the test results are represented in table 10.

**Table 10. ADF and KPSS Statistics.**

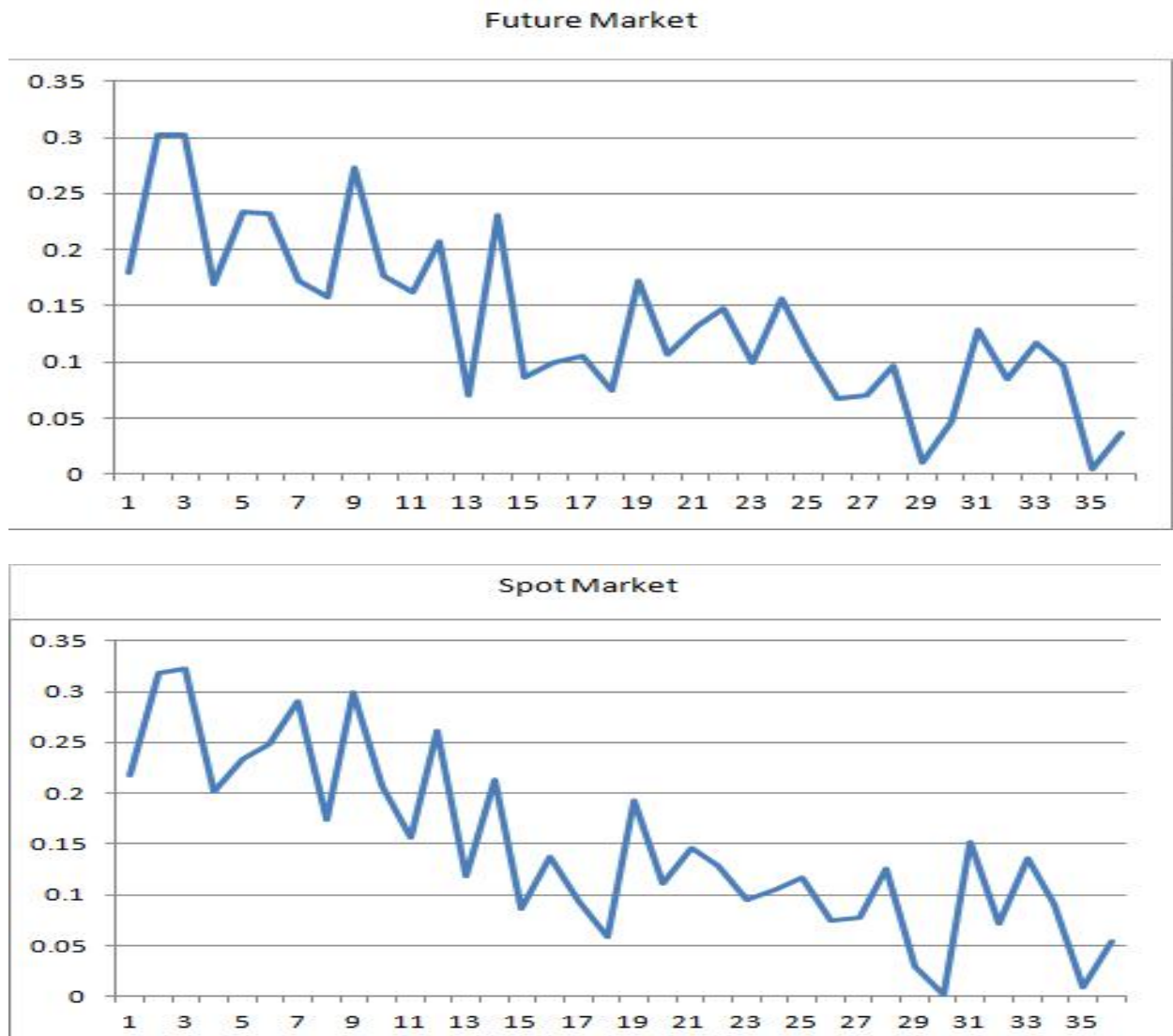
<b>Futures market</b>			
	<b>ADF Test</b>		<b>KPSS Test</b>
<b>t-statistics</b>	-5.236039	<b>t-statistics</b>	1.233739
<b>1%</b>	-3.457400	<b>1%</b>	0.739000
<b>5%</b>	-2.873339	<b>5%</b>	0.463000
<b>10%</b>	-2.573133	<b>10%</b>	0.347000
<b>Spot Market</b>			
	<b>ADF Test</b>		<b>KPSS Test</b>
<b>t-statistics</b>	-5.075574	<b>t-statistics</b>	1.058455
<b>1%</b>	-3.457400	<b>1%</b>	0.739000



<b>5%</b>	-2.873339	5%	0.463000
<b>10%</b>	-2.573133	10%	0.347000

The above table results show that both futures and spot markets have long memory characteristics due to the following reason: the null hypothesis for the ADF test is that there is a unit root in time series; the null hypothesis for the KPSS test is that the time series is stationary. There are following situations for the test results: If it rejects ADF and accepts KPSS, then the time series is a I (0) process; if it rejects KPSS and accepts ADF, then the time series is a I (1) process. If it rejects both tests, then the time series does not belong to neither the I (0) process nor the I (1) process. Then the time series should be an I (d) process with  $0 < d < 1$ . This obeys the definition of long memory process. From table 10, it shows that both ADF and KPSS tests are rejected at 1%, 5% and 10% confidence level. Hence it can be concluded that both futures and spot markets' realised volatility have a long memory characteristic. Meanwhile, these long memory characteristics can also be reflected by using the ACF as presented in the figure below.

Figure 6. Autocorrelation Function.



The above figure shows that the realised volatility's ACF decays slowly for both futures and spot markets. Even after 36 lags, the ACF value still does not equal zero for both markets. This also supports that the two markets' realised volatility has a long memory characteristic. Furthermore, the GPH method is applied to confirm the above conclusion. The estimated GPH method results are represented in table 11.

**Table 11. GPH Test Results.**

<b>Futures market</b>			
<b>m</b>	<b>d</b>	<b>t</b>	<b>p</b>
$m=T^{0.5}$	0.18035	0.0452679	0.0001
<b>Spot Market</b>			
<b>m</b>	<b>d</b>	<b>t</b>	<b>P</b>
$m=T^{0.5}$	0.195978	0.0452679	0.0000

From the above table, it shows that, both Futures and spot markets reject the null hypothesis which  $d = 0$  with very low p-value. The futures market's  $d = 0.18035$ , and the spot market's  $d = 0.195978$ . Both two  $d$  values range between 0 and 0.5. Hence, this GPH method also confirms that both Futures and spot markets' Realized Volatility have long-memory characteristics.

The above discussion confirms that both markets' realised volatility have long memory characteristics. Section 3.4 concludes that logarithm realised volatility has more desirable characteristics compared to original realised volatility, such as closer to normal distribution. Hence, it is interesting to examine whether the two markets' logarithm realised volatility has long memory characteristics. The following table is the ADF and KPSS test results for both markets' logarithm realised volatility.

**Table 12. ADF and KPSS Statistics.**

<b>Futures market</b>			
<b>ADF Test</b>		<b>KPSS Test</b>	
<b>t-statistics</b>	-4.879519	<b>t-statistics</b>	1.205496

<b>1%</b>	-3.457400	1%	0.739000
<b>5%</b>	-2.873339	5%	0.463000
<b>10%</b>	-2.573133	10%	0.347000
<b>Spot Market</b>			
<b>ADF Test</b>		<b>KPSS Test</b>	
<b>t-statistics</b>	-4.288502	<b>t-statistics</b>	1.032732
<b>1%</b>	-3.457400	1%	0.739000
<b>5%</b>	-2.873339	5%	0.463000
<b>10%</b>	-2.573133	10%	0.347000

Table 12 results show that both ADF and KPSS tests are rejected at three different confidence levels. Hence, it can be concluded that both markets' logarithm realised volatility also has long memory characteristics. Furthermore, the GPH test is also applied to confirm this conclusion. The GPH test statistics are summarised below.

**Table 13. GPH Test Results.**

<b>Futures market</b>			
<b>m</b>	<b>d</b>	<b>t</b>	<b>p</b>
<b><math>m=T^{0.5}</math></b>	0.25593	0.069525	0.0002
<b>Spot Market</b>			
<b>m</b>	<b>d</b>	<b>t</b>	<b>P</b>
<b><math>m=T^{0.5}</math></b>	0.36116	0.069525	0.0000

Table 13 confirms that both markets' logarithm realised volatility have long memory characteristics as well.

#### **3.5.4. Discussion**

This sub-chapter aims to examine the realised volatility's long memory characteristics of Chinese stock index futures and spot markets. The 5 minutes high-frequency intraday data is applied here. From the empirical results, it can be concluded that both markets' realised volatility and logarithm realised volatility have long memory characteristics.

This sub-chapter first applies ADF and KPSS tests to examine the stationarity of both markets' realised volatility. Both ADF and KPSS tests reject the null hypothesis at different confidence levels. This sub-chapter analyses the above result in detail, and concludes that both two series are I (d) processes where  $0 < d < 1$ . This result implies both series have long memory characteristics. Furthermore, this chapter also applies the GPH method to confirm the above result. The GPH results also suggest that both series have long memory characteristics. Consequently, this chapter examines both markets long memory characteristics in logarithm realised volatility. Long memory characteristics mean the series can apply the long memory model to track and forecast its value. This conclusion will provide the fundamental theory support of realised volatility forecast in futures studies.

### **3.6. Chapter Summary**

This chapter examines the realised volatility characteristics of Chinese stock index futures and spot markets. This chapter begins from the basic data descriptions of the original 1 minute high-frequency data. Next, four important issues in realised volatility estimation are examined in detail.

The first issue is the effect of microstructure noise. By applying the optimal sampling frequency method and filter technique, the optimal data sample is selected. The second issue is the daily volatility jumps. The empirical results suggest applying bi-power realised volatility instead of standard realised volatility estimation. The third issue is the intraday volatility jumps and periodicity. One important result is concluded, and that is that intraday volatility jumps are highly linked to macroeconomic news release. The last issue is the long memory characteristics of realised volatility. The empirical results suggest both markets' realised volatility and logarithm realised volatility have long memory characteristics.

This chapter discusses some important realised volatility characteristics of Chinese stock index futures and spot markets. The results of this chapter provide a strong theoretical foundation for further realised volatility models and forecasts in later chapters.

## Chapter Four: Volatility Spillover and Forecast

This chapter explores the optimal realised volatility forecast model for Chinese stock index futures and spot markets. In chapter three, some important realised volatility characteristics for situation-specific Chinese markets are discussed in detail. These include the optimal data sample frequency to estimate realised volatility, the daily volatility jumps, intraday volatility jumps and periodicity and long memory characteristic.

Based on chapter three's results, chapter four will consider the following two issues before estimating the realised volatility forecast model. First, whether a dynamic volatility spillover process exists between spot and futures markets' realised volatility. That is to say, will one market's realised volatility significantly influence the other market's realised volatility? Sub-chapter 4.1 examines the dynamic volatility transmission between the Chinese stock index futures and spot markets using conditional volatility and realised volatility. Under the conditional volatility framework, Diagonal VECM and Diagonal BEKK will be used. Under the realised volatility framework, the Vector Error Correction Model (VECM) will be used.

Second, sub-chapter 4.2 examines whether structure breaking exists in the estimation period. If there are structure breakpoints, then a nonlinear model will be applied. After exploring these two issues, in sub-chapter 4.3, two widely used realised volatility estimation models will be introduced: ARFIMA long memory and HAR models. The details of these two models can be found in later studies. This thesis, for the first time, proposes a HAR-J-MS model, and the empirical results suggest that HAR-J-MS has very good forecasting power for logarithm realised volatility.

#### **4.1. Realised Volatility Transmission**

Volatility has an important position in current financial studies, especially in the areas of hedging, option pricing, and risk management. Some recent studies have investigated mature western financial markets' volatility features (Corsi, 2009; Patton, 2011; Bollerslev et al., 2012 and Watcher, 2013), but few papers have examined volatility related to emerging financial markets (Liu and An, 2011 and Yang et al., 2012). After the economic revolution in 1979, China's economy has undergone significant development and is currently the second largest economy in the world, according to IMF's 2014 GDP data. China established its stock index futures market in April 2010; it is interesting and important to investigate the influence of this new financial market.

Current studies indicate that a dynamic volatility spillover effect exists between two linked financial markets (So and Tse, 2004; Chen et al., 2009; Johansson and Ljungwall, 2009, Liao, 2013), commonly called volatility spillover or the transmission process. One important reason to explore this dynamic volatility process is to determine the direction of new information flow.

According to Fama's (1970) efficient market hypothesis, in an efficient market, all price movements are caused by new information. That is, if the Chinese stock index futures and spot markets are efficient, then bi-directional (or no) volatility transmission will be expected, as all new information should be reflected in both markets simultaneously.



Chan, et al. (1991) pointed out that: in an inefficient market, if volatility transmits from the futures to the spot market, it indicates that the futures market acquires new information faster than the spot market and vice versa.

Some recent studies (Anderson et al., 2005; Barndorff-Nielsen and Shephard, 2002; Goncalves and Meddahi, 2009) show that realised volatility provides more accurate volatility estimation because it uses intraday high-frequency data. To the best of our knowledge, this study is the first to examine whether volatility transmission exists between Chinese stock index futures and spot markets by using both a conditional volatility and realised volatility framework. The main contribution of this paper is to investigate volatility spillover by using both intraday and daily data.

In previous literature, volatility spillover is either discussed at daily data level only, or is examined at intraday high-frequency data level only. In this paper, both types of data are applied. The results of this study may be used to compare and contrast these two estimation methods, and explore the research topic deeper. Conditional volatility can be taken as intraday volatility, and realised volatility can be taken as daily aggregated volatility. Meanwhile, intraday high-frequency data can be treated as microstructure data, and daily aggregated data can be treated as macrostructure data. Volatility spillover reflects the direction of information flow. Examining volatility spillover using both daily and intraday data levels can distinguish the direction of information flow from micro and macro market structures.

The empirical results indicate strong bi-directional volatility spillover effects under conditional volatility, but no evidence under realised volatility. Two different robustness tests confirm the results. Based on the empirical results, this paper concludes that there is strong bi-directional intraday volatility spillover effect, but the expected intraday spillover effect should equal zero under a daily aggregated level. That is, from a microstructure point of review, both markets reflect new information simultaneously. However, from a macro aggregated level point of review, there is not a strong information linkage between these two markets. This paper concludes that volatility spillover depends on data frequency; different data structure (micro or macro) will provide a different answer on volatility spillover in the same two markets.

#### **4.1.1. Literature Review**

The volatility spillover effect can be divided into two categories: 1) domestic market spillover effects and 2) international market spillover effects. Within the domestic market, Kang et al. (2013) examined the volatility spillover effect between the Korean stock index futures and spot markets, and Zhong et al. (2004) tested Mexico's stock index futures and spot markets. Both studies found a volatility spillover effect the between domestic futures and spot markets. Specifically, Zhong et al. (2004) indicated that volatility transmits from the futures market to the spot market, and the futures market leads to an increase in volatility for the spot market. These two papers demonstrate that the Korean financial market has more mature market characteristics than Mexico's financial market, according to the efficient market hypothesis.

Focusing on Chinese financial markets, Zhou et al. (2014) adopted both the VAR and TVP-VAR model to examine volatility spillover effects between the futures and spot markets in China, finding strong bi-directional volatility spillovers between these markets, and that the change in the futures' market volatility decreased the change in the spot market's volatility. Yang et al. (2012) investigated intraday price discovery and volatility transmission between the Chinese stock index and the newly established stock index futures markets. The results indicated that the cash market plays a more dominant role in the price discovery process, and there was no strong evidence of a volatility transmissions effect between the futures and spot markets.

Meanwhile, other studies have focused on links in international market volatility. For example, Johansson and Ljungwall (2009) explored volatility links among the different stock markets in the Greater China Area, including mainland China, Hong Kong, and Taiwan. Their empirical findings showed that there is no long-run relationship among the markets. However, they found short-run spillover effects in both returns and volatility in the region, and both China and Hong Kong were affected by mean spillover effects from Taiwan. Liu and An (2011) investigated volatility spillover between the U.S. and Chinese markets, showing a bi-directional relationship with a stronger effect from the U.S. to Chinese markets than in the opposite direction. Hwang (2012) examined stock market volatility links within the Asia-Pacific region, and found strong correlations among the stock markets during the 2008 financial crisis. All of these studies support the existence of international finance markets' volatility transmission.

Researchers use various models to test volatility spillover, including VECM, EGARCH, Cointegration analysis, BEKK-GARCH, VECH-GARCH, and CCC-GARCH models.

Comparing VECM-GARCH and BEKK-GARCH, BEKK requires fewer parameters to estimate and ensure positive definiteness of conditional covariance matrices, an advantage over VECM, and the most important factor in the estimation of the multivariable GARCH models (Iltuzer and Tas, 2012). Caporin and McAleer (2012, 2013) compared two multivariate conditional volatility models, BEKK and DCC, concluding that BEKK possess asymptotic properties under untestable moment conditions, whereas the asymptotic properties of DCC are simply stated under a set of untestable regularity conditions.

#### **4.1.2. Methodology**

This sub-chapter examines the dynamic volatility transmission between the Chinese stock index futures and spot markets using conditional volatility and realised volatility. Under the conditional volatility framework, Diagonal VECM and Diagonal BEKK will be used. Under the realised volatility framework, the VECM will be used. The test data is 5 minutes high-frequency data. Different estimation methods will lead to different test methods as discussed in the following:

The *conditional volatility spillover test models* are based on bivariate VAR (1) as follows:

$$R_{i,t} = u_i + \varphi_i R_{i,t-1} + \varepsilon_{i,t}$$

where  $R_{i,t}$  is a [2x1] vector which refers to the spot and futures markets' return at the time  $t$ ;  $u_i$  is a [2x1] vector which represents the long term coefficients drift;  $\varepsilon_{i,t}$  is also a [2x1] vector which refers to the random error terms of these two markets at the time  $t$ . Here, the equation defines  $H_t$  as the [2x2] conditional variance-covariance matrix of the  $\varepsilon_{i,t}$ , and  $\varepsilon_{i,t}|\Psi_{t-1} \sim N(0, H_t)$  with  $\Psi_{t-1}$  represents the information set at time  $t - 1$ .

The fundamental GARCH (1, 1) model is written as:

$$R_{i,t} = u_i + \varphi_i R_{i,t-1} + \varepsilon_{i,t}$$

$$H_{i,t} = \alpha_i + \beta_i \varepsilon_{i,t-1}^2 + \gamma_i H_{i,t-1}$$

The GARCH model assumes that the conditional variance value depends on its own lag value and the lag value of the residual in the main equation. Consequently, two commonly used methods exist to estimate  $H_t$ , namely the Diagonal VECH and Diagonal BEKK estimations:

Diagonal VECH:  $\mathbf{H}_t = \mathbf{C} + \mathbf{A}\varepsilon_{t-1}\varepsilon_{t-1}' + \mathbf{B}\mathbf{H}_{t-1}$

Diagonal BEKK:  $\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B}$

In the above models,  $H_t$  is a [2x2] matrix of the conditional variance-covariance of  $\varepsilon_t$  at time  $t$ ; C is a [2x2] lower triangular matrix; A is a [2x2] diagonal matrix which represents the degree of  $H_t$  relative to the past error term in the mean equation; B is a [2x2] diagonal matrix. A disadvantage of the Diagonal VECH model is that there is no guarantee of a positive semi-definite covariance matrix. A variance-covariance matrix must always be positive semi-definite. The BEKK model (Engle and Kroner, 1995) addresses this problem, ensuring that the  $H_t$  matrix is always positive and definite.

The maximum likelihood estimation method estimates the models, optimised with the Berndt, Hall, Hall, and Hausman (BHHH) algorithm. According to Brooks (2002), the following represents the likelihood function ( $\theta$ ) :

$$L(\theta) = -\frac{TN}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T (\log |H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t)$$

Where  $\theta$  denotes all unknown parameters for estimation;  $N$  is the number of assets, and  $T$  is the number of observations. Meanwhile,  $\theta$  in the maximum likelihood estimation is asymptotically related to the normal distribution.

Specifically, the following represents an alternative expanded conditional variance-covariance matrix  $H_t$ :

Diagonal VECH:

$$H_t = M + \begin{bmatrix} A1(1,1) & 0 \\ A1(1,2) & A1(2,2) \end{bmatrix} \varepsilon_{t-1} \varepsilon_{t-1}' + \begin{bmatrix} B1(1,1) & 0 \\ B1(1,2) & B1(2,2) \end{bmatrix} H_{t-1}$$

Diagonal BEKK:  $H_t = M'M +$

$$\begin{bmatrix} A1(1,1) & 0 \\ A1(1,1) * A1(2,2) & A1(2,2) \end{bmatrix} \varepsilon_{t-1} \varepsilon_{t-1}' \begin{bmatrix} A1(1,1) & 0 \\ A1(1,1) * A1(2,2) & A1(2,2) \end{bmatrix} + \begin{bmatrix} B1(1,1) & 0 \\ B1(1,1) * B1(2,2) & B1(2,2) \end{bmatrix} H_{t-1} \begin{bmatrix} B1(1,1) & 0 \\ B1(1,1) * B1(2,2) & B1(2,2) \end{bmatrix}$$

The following defines the null and alternative hypothesis for each model, respectively:

Diagonal VECH:  $H_0 : A1(1,2) = B1(1,2) = 0$

$$H_1 : A1(1,2) \neq 0 \text{ or } B1(1,2) \neq 0$$

Diagonal BEKK:  $H_0 : A1(1,1) * A1(2,2) = 0 \text{ and } B1(1,1) * B1(2,2) = 0$

$$H_1 : A1(1,1) * A1(2,2) \neq 0 \text{ or } B1(1,1) * B1(2,2) \neq 0$$

In the above, the null hypothesis for each model states that there are no volatility spillover effects, and the alternative hypothesis states that there are volatility spillover effects across these two markets.

The *dynamics logarithm realised volatility* transmission can be studied by using a bivariate VECM in the following form:

$$\Delta s_t = \beta_{s,0} + \gamma_s ec_{t-1} + \sum_{j=1}^p \beta_{ss,j} \Delta s_{t-j} + \sum_{j=1}^q \beta_{sf,j} \Delta f_{t-j} + \varepsilon_{s,t}$$

$$\Delta f_t = \beta_{f,0} + \gamma_f ec_{t-1} + \sum_{j=1}^p \beta_{fs,j} \Delta s_{t-j} + \sum_{j=1}^q \beta_{ff,j} \Delta f_{t-j} + \varepsilon_{f,t}$$

where  $ec_{t-1} = f_t - a_0 - a_1 s_t$  is the estimated error correction term. The error correction coefficients  $\gamma_f$  and  $\gamma_s$  represent the speed of adjustment in response to deviations from the log-run equilibrium. The coefficients  $\beta_{sf,i}$  and  $\beta_{fs,i}$  capture the short term predictive power of one variable for the other. The coefficients measuring the reaction of spot and futures returns to their own lagged values ( $\beta_{ss,j}, \beta_{ff,j}$ ) indicate the degree of mean-reverting behaviour of both time series.

Under the above VECM framework, one can interpret the volatility spillover effect as whether one market's realised volatility has a long-run relationship with the other market's realised volatility, and whether one market's realised volatility can create Granger causality in the other market's realised volatility. In the above VECM model,  $\Delta s_t$  and  $\Delta f_t$  refer to the first difference in realised volatility.

Testing the Granger causality and cointegration relationship for realised volatility is the first step. The following tests the Granger causality relationship within the VECM model mentioned above:

$$H_0: \beta_{fs,1} = \beta_{fs,2} = \dots = \beta_{fs,p} = 0$$

$H_1$ : At least one not equal to zero.

This model tests whether the futures market's volatility transmits to the spot market. To test volatility transmission in the opposite direction:

$$H_0: \beta_{sf,1} = \beta_{sf,2} = \dots = \beta_{sf,p} = 0$$

$H_1$ : At least one not equal to zero.

Similarly, the following tests the long-run (cointegration) relationship:

$$H_0: \gamma_f = 0 \text{ and } \gamma_s = 0$$

$H_1$ : At least one not equal to zero.

The above tests can be conducted by performing  $t$ -tests on error correction coefficients and  $F$ -tests on the joint significant sums of the lags of each variable. Meanwhile, this lead-lag relationship can also be tested by using the Likelihood Ratio test as follows:

The above VECM model can also be written in matrix format (unrestricted model):

$$\begin{bmatrix} \Delta s_t \\ \Delta f_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \sum_{i=1}^p \begin{bmatrix} \phi_{11}^{(i)} & \phi_{12}^{(i)} \\ \phi_{21}^{(i)} & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} \Delta s_{t,t-i} \\ \Delta f_{t,t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

The  $H_0$  for the Granger causality test is:

$$H_0: \phi_{12}^{(i)} = 0, \text{ where } i = 1, 2, \dots, p.$$

Hence, it can give the restricted model as:

$$\begin{bmatrix} \Delta s_t \\ \Delta f_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \sum_{i=1}^p \begin{bmatrix} \phi_{11}^{(i)} & 0 \\ 0 & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} \Delta s_{t,t-i} \\ \Delta f_{t,t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$



Respectively, the variance - covariance matrix of residual for unrestricted and restricted models can be marked as  $\widehat{\Omega}_u$  and  $\widehat{\Omega}_r$ . Hence, the Likelihood Ratio (LR) statistic can be written as:

$$(T - \kappa)(\ln|\widehat{\Omega}_r| - \ln|\widehat{\Omega}_u|)$$

$\kappa$  represents the number of parametric in the unrestricted model, which equals  $p$  in this case. The LR statistic follows a  $\chi^2$  distribution with the freedom of  $p$ .

The second step is to use the common factor weights method (Schwarz and Szakmary, 1994) to quantity measure the degree of volatility spillover by each market. The common factor weights of futures and spot markets can be written as:

$$\theta_f = \frac{\gamma_s}{\gamma_s + |\gamma_f|} \text{ and } \theta_s = 1 - \theta_f$$

In the above equation, if  $\theta_f = 0$ , it means the futures market totally leads the spot market's volatility transmission, and vice versa. According to Bohl et al. (2011, 2015), this common factor weights method is similar to the other common factor weights method (Gonzalo and Granger, 1995) and information shares developed by Hasbrouck (1995). The Schwarz and Szakmary (1994) measure can be derived from the Gonzalo-Granger Framework and conclusions are qualitatively similar to those based on information shares.

### **4.1.3. Empirical Results**

This sub-chapter tests the volatility spillover based on both the conditional volatility and realised volatility frameworks. Under the conditional volatility framework, the test models are Diagonal BEKK and VECM. Under the realised volatility framework, the test model is based on VECM as discussed in above section.

### 4.1.3.1. Conditional Volatility Results

Table 14 shows the estimated results for Diagonal BEKK and Diagonal VECH.:

Table 14. Intraday Conditional Volatility Results

#### Estimation results of mean equations

	Diagonal VECH	Diagonal BEKK
$u_1$	-1.12E-05 (1.23E-05)	-1.05E-05 (1.21E-05)
$\varphi_1$	-0.171594* (0.008159)	-0.160960* (0.007906)
$u_2$	-2.37E-05 (0.00123)	-2.07E-05 (0.00124)
$\varphi_2$	-0.050953* (0.008828)	-0.070198* (0.007814)

#### Estimation results of variance-covariance equations

	Diagonal VECH	Diagonal BEKK
$A1(1,1)$	0.047142* (0.001685)	0.227099* (0.003530)
$A1(2,2)$	0.046282* (0.002125)	0.205641* (0.004936)
$A1(1,2)$	0.035109* (0.001699)	= $A1(1,1) * A1(2,2)$
$B1(1,1)$	0.902980* (0.003205)	0.951583* (0.001395)
$B1(2,2)$	0.864003* (0.005426)	0.932603* (0.002883)
$B1(1,2)$	0.894599* (0.004446)	= $B1(1,1) * B1(2,2)$

#### Wald test on the null hypotheses that all diagonal parameters jointly equals to zero.

<b>Diagonal VECH</b>	$H_0: A1(1,2) = B1(1,2) = 0$	Chi-square value = 115245 Probability = 0.0000
<b>Diagonal BEKK</b>	$H_0: A1(1,1) * A1(2,2) = 0$ <i>and</i> $B1(1,1) * B1(2,2) = 0$	Chi-square value = 194340 Probability = 0.0000

The statistics in the table are coefficients and bootstrapped standard errors (in parentheses). Normal standard errors are similar.

\* represents significance at 1% level.

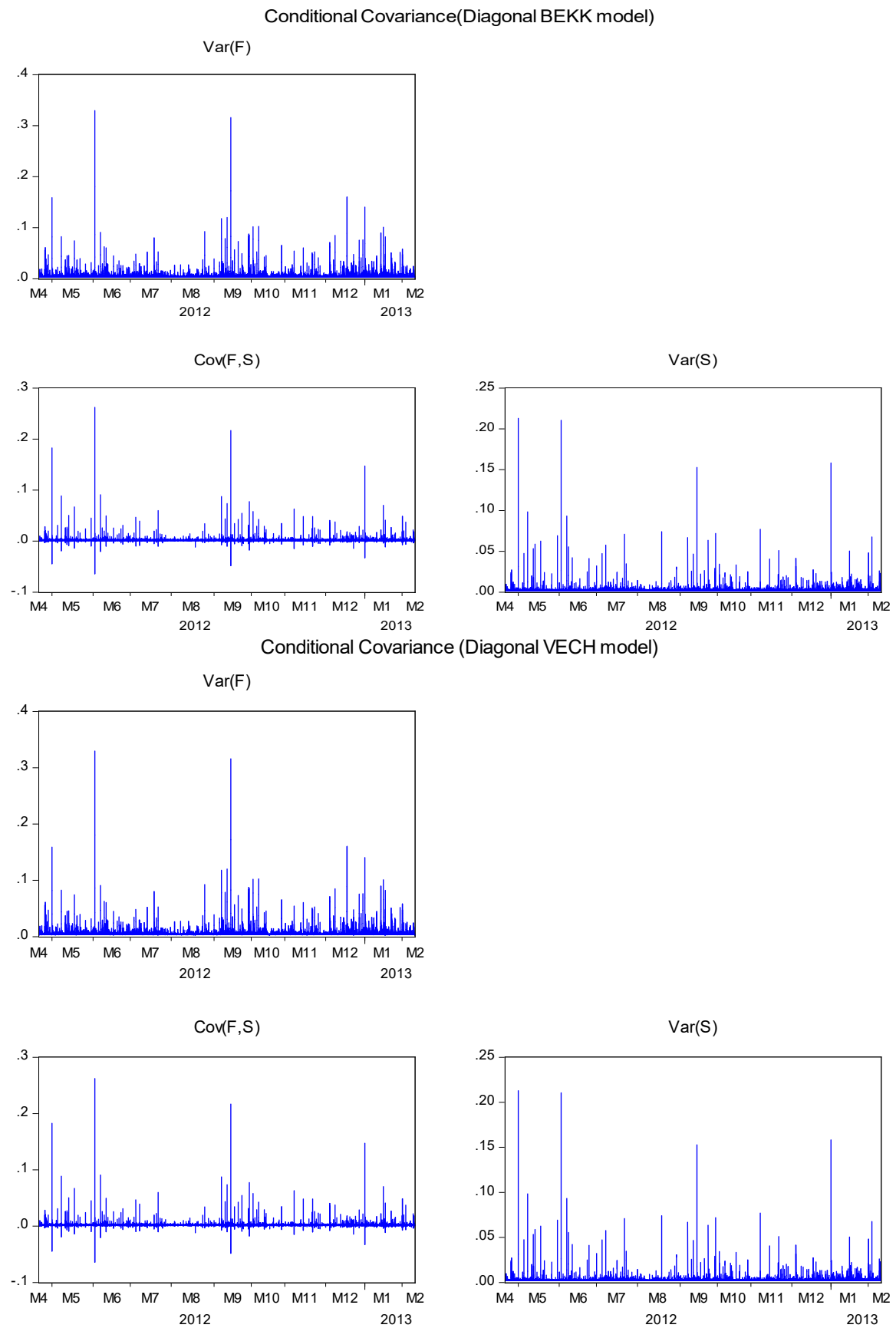
The results indicate significant volatility spillover between the futures and spot markets as all elements in matrix  $A$  and  $B$  are highly significant at the 1% level. Specifically, the Wald

Coefficient test returning Chi-squared values = 115245 and 194340, respectively, reject the null hypothesis in both models, indicating the existence of a volatility spillover effect. In the mean equations, both models indicate a highly significant of AR (1) process as  $\varphi_i$  is significantly different from zero at the 1% level.

In the variance-covariance equation, all elements are significant at the 1% level. Specifically, for both models, the diagonal parameters ( $A1(1,1)$ ,  $A1(2,2)$ ), which measure the past shock effects of each market on current volatility, are significant at the 1% level. This implies that there are strong ARCH effects. Meanwhile, the diagonal parameters ( $B1(1,1)$ ,  $B1(2,2)$ ) in both models, which measure the past volatility effects of each market on current volatility, are also significant at the 1% level. This implies there are strong GARCH effects. All diagonal parameters indicate that the current conditional variance-covariance of both intraday 1-minute returns is strongly influenced by its own shocks and volatility.

In the Diagonal VECM model, the off-diagonal parameters ( $A1(1,2)$  and  $B1(1,2)$ ) measure the cross-market impacts of the markets. Respectively,  $A1(1,2)$  indicates a positive cross effect running from past shocks to current volatility between the markets. Meanwhile,  $B1(1,2)$  indicates a positive cross effect running from past volatility to current volatility. The above conclusions are supported by the Diagonal BEKK model because  $A1(1,1) * A1(2,2)$  and  $B1(1,1) * B1(2,2)$  are different from zero at the 1% level. The figure below illustrates the above conclusions using conditional covariance graphs.

Figure 7. Conditional Covariance Graphs for the Two Models.



### 4.1.3.2. Realised Volatility Results

Let  $p = 2$  and  $q = 2$  for lag terms because the optimal lag terms should be long enough to investigate the necessary lead-lag relationship. However, if the lag terms are too long, they will generate more bias in the system equation and create unnecessary complexity for equation analysis. Table 15 shows the estimated VECM results investigating volatility spillover.

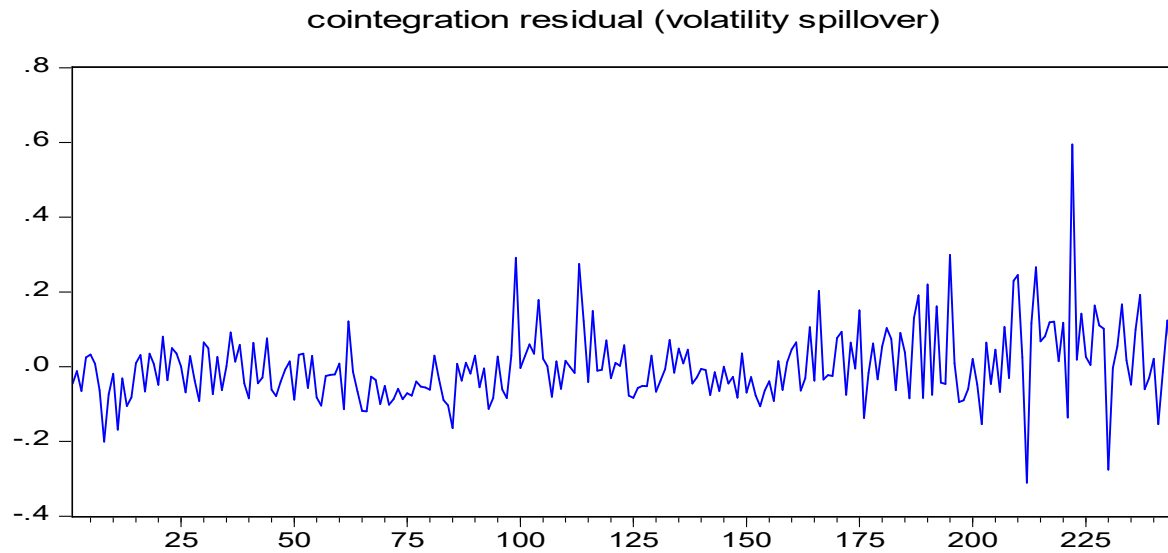
**Table 15. Realised Volatility Results.**

System Equation for Realised Volatility(Volatility Spillover)						
Equation	$\Delta s_t = \beta_{s,0} + \gamma_s ec_{t-1} + \sum_{j=1}^2 \beta_{ss,j} \Delta s_{t-j} + \sum_{j=1}^2 \beta_{sf,j} \Delta f_{t-j} + \varepsilon_{s,t}$					
	$\Delta f_t = \beta_{f,0} + \gamma_f ec_{t-1} + \sum_{j=1}^2 \beta_{fs,j} \Delta s_{t-j} + \sum_{j=1}^2 \beta_{ff,j} \Delta f_{t-j} + \varepsilon_{f,t}$					
	$\beta_{s,0}$	$\gamma_s$	$\beta_{ss,1}$	$\beta_{ss,2}$	$\beta_{sf,1}$	$\beta_{sf,2}$
$\Delta s_t$	0.001179	0.238444	-0.624585	0.379274	0.114027	-0.006014
	(-0.155490)	(-3.028557)*	(-6.766822)*	(-4.097610)*	(-1.640275)	(-0.086316)
	$\beta_{f,0}$	$\gamma_f$	$\beta_{fs,1}$	$\beta_{fs,2}$	$\beta_{ff,1}$	$\beta_{ff,2}$
$\Delta f_t$	0.001723	0.603299	0.059433	-0.045604	-0.024538	-0.333732
	(0.175149)	(5.908642)*	(0.496513)	(-0.379914)	(-9.145592)*	(-3.693674)*
<b>Restriction Test</b>						
$\beta_{sf,1} = \beta_{sf,2} = 0$ [ $\chi^2(2)$ ]		$\beta_{fs,1} = \beta_{fs,2} = 0$ [ $\chi^2(2)$ ]		$\gamma_s = \gamma_f$ [ $\chi^2(1)$ ]		
3.688583		0.751633		42.62088*		
Common Factor Weight Result				$\theta_f = 0.28$		

Note: \* represent statistical significance at the 1% level. The statistics are coefficients and  $t$ -statistics calculated from bootstrapped standard errors (in parentheses). The  $t$ -statistics from normal standard errors are similar.

Meanwhile, the estimated cointegration residual is represented in the following figure.

**Figure 8. Cointegration Residual.**



The realised volatility VECM model shows that all variables are significant at the 1% level, except the constant and  $\beta_{sf,1}$  terms in the  $\Delta s_t$  regression. In the  $\Delta f_t$  equation, only the  $\gamma_f$ ,  $\beta_{ff,1}$  and  $\beta_{ff,2}$  terms are significant at the 1% level. The coefficient on the lagged spot index changes in the spot equations ( $\beta_{ss,1}$  and  $\beta_{ss,2}$ ) and are significant at the 1% level, with the first lag having a negative sign. The corresponding coefficients in the futures equations, ( $\beta_{ff,1}$  and  $\beta_{ff,2}$ ) are also significant, and both have negative signs. The above statistics demonstrate that both the spot and futures markets exhibit a mean reversion characteristic.

The error correction terms  $\gamma_s$  and  $\gamma_f$  do not equal zero, indicating a long-run relationship between the two volatility series. From the test statistics, the futures market has a faster adjustment speed compared to the spot market, consistent with the price discovery's finding.

All terms, including  $\beta_{sf,1}$ ,  $\beta_{sf,2}$ ,  $\beta_{fs,1}$ ,  $\beta_{fs,2}$ , cannot reject the null hypothesis at the 1% level, which indicates that there is no volatility spillover effect between the futures and spot markets' realised volatility.

Meanwhile, both joint hypotheses ( $\beta_{sf,1} = \beta_{sf,2} = 0$  and  $\beta_{fs,1} = \beta_{fs,2} = 0$ ) are accepted, supporting the above conclusion. This conclusion differs significantly from the above conditional volatility results, as well as from Yang et al.'s [2012] results. Hence, this study concludes that there is no volatility spillover effect under the realised volatility framework, but a significant volatility spillover effect under the conditional volatility framework. Since there is no volatility spillover effect, this does not explain the  $\theta_f$  statistics in the common factor weight analysis. Overall, the realised volatility results show that a long-run relationship exists between the spot and futures markets. However, there is no evidence to support a short-run dynamic relationship between these two variables. This paper concludes that there is no volatility spillover effect in the spot and futures' realised volatility.

Compared to conditional volatility test results, the realised volatility provides a different answer regarding the volatility spillover within the same markets. The key difference between the conditional volatility model and the realised volatility model lies in the fact that, in this study, realised volatility is an aggregate daily volatility, but conditional volatility is a 1-minute interval volatility. A bi-directional intraday volatility spillover effect exists between Chinese stock index futures and spot markets, but the bi-directional effect is random, with the expected aggregate effect equal to zero, and this explains the differing results. Under this type of volatility spillover, the intraday conditional volatility will show some degree of bi-directional spillover effect, but the effect will disappear in a realised volatility framework.

The volatility spillover links to new information flow. Similar to volatility, within the intraday level, new information flows to the futures and spot markets simultaneously. The expected aggregate daily new information effect equals zero, which means that new information provides an equal effect for both markets at a daily level.

For robustness, the conditional volatility is re-estimated at the daily level, and volatility spillover is additionally tested using a daily BEKK-GARCH model, compared with the intraday high-frequency conditional volatility results. Realised volatility spillover is tested using the VAR approach, compared with the first difference from the VECM model. Table 16 presents the robustness test results.

**Table 16. Daily Conditional Volatility Results**

Daily BEKK-GARCH conditional volatility model results				
Coefficient Test	A1(1,1)	A1(2,2)	B1(1,1)	B1(2,2)
z-statistics	1.4455	3.1104	1.6186	3.7016
p-value	(0.1483)	(0.0019)	(0.1055)	(0.0002)
Wald Test	A1(1,1)*A1(2,2)=0		B1(1,1)*B1(2,2)=0	
Chi-square value	1.0481		1.3942	
p-value	0.3059		0.2377	
Realised volatility spillover using VAR approach				
	Futures market		Spot market	
Futures (-1)	0.2006 (0.1173)		0.0973 (0.0878)	
Spot (-1)	-0.0885 (0.15774)		0.094987 (0.1180)	



The daily BEKK-GARCH results indicate that the null hypothesis holds true, and there is no volatility spillover effect at the daily aggregate level. The robustness test results match the previous results.

#### **4.1.4. Discussion**

This sub-chapter examines volatility transmission between the futures and spot markets under two volatility estimation frameworks: conditional volatility and realised volatility. Under the conditional volatility framework, the Diagonal VECM model and Diagonal BEKK model are applied to examine the volatility spillover effect. The results from both models indicate that the null hypothesis holds true at 1% level, indicating that there is significant volatility transmission between these two markets. Specifically, the results reveal that both past shocks and volatility will significantly influence current volatility through the cross-market effects. This volatility transmission also indicates that there is a strong bi-directional causality, meaning that spot market volatility can influence the futures market, and vice versa.

The conditional volatility results offer two important implications. First, the mean equations from both models indicate that there is a strong AR (1) process, in other words, the current spot and futures prices are significantly influenced by the last 5-minute price. Meanwhile, the negative coefficients indicate a strong mean-reverting effect for both the futures and spot markets. Second, there is a strong bi-directional volatility transmission across the market. If there is high volatility in the futures market, then the spot market tends to show high volatility in recent short periods, and vice versa. This result also indicates that new information flows into both markets simultaneously, with no lead-lag volatility relationship within the 5-minute interval.

This conclusion differs significantly from Yang et al. (2012), who concluded that spot market volatility leads to volatility in the futures market, and new information flows from the spot to the futures market. There are a number of reasons for the different results. First, the period under investigation is different. Specifically, the data period from Yang et al. (2012) is between 16/04/2010 and 30/07/2010, when Chinese stock index futures contracts were initially introduced. Because the market was newly established, it did not have as many investors compared to the mature spot market, so the spot market will generally respond to new information faster. However, our study data covers the period between 19/04/2012 and 19/04/2013, where the futures market had an increasing number of investors and two years of development. The increase in investors and volumes improves efficiency in the futures market.

Under the realised volatility framework, the VECM model and two-step method are applied to investigate the volatility spillover effect. This study concludes that there is no evidence to support short term volatility spillover using a realised volatility estimation. However, there is a long-run relationship between these two markets' realised volatility. Comparing realised volatility to conditional volatility, the conditional volatility results suggest a strong bi-directional volatility spillover effect, but the realised volatility results suggest no volatility spillover effect. Both the bi-directional volatility spillover and no spillover effect imply that the futures and spot markets are efficient, since new information flows to these two markets simultaneously. One of the other key explanations for these different conclusions may be that conditional volatility is the 5-minute intraday volatility, but realised volatility is a daily volatility estimation. This may imply a volatility spillover in intraday frequency, but no spillover in the daily interval.

The two alternative robustness test approaches confirm this study's conclusions. In the realised volatility estimation, a number of studies suggest that there are intraday volatility jumps and intraday periodicity, which will significantly influence the realised volatility estimation (Lee and Mykland, 2008). Futures research into realised volatility transmission should consider these two effects to examine this topic more deeply.

#### **4.2. Markov Regime-switching Model**

Currently, numerous researchers have found that stock markets' returns and volatility can be modelled and forecasted more accurately by using a nonlinear model compared to a linear model (Lee and Yoder, 2007; Liu et al., 2012; Moore and Wang, 2007; Miao et al., 2011; Baba and Sakurai, 2011). This sub-chapter will examine whether the nonlinear model performs better than the linear model to capture Chinese stock index futures and spot markets' realised volatility behaviours.

There are a variety of nonlinear model estimation methods, but this chapter will focus on one important and widely used model: the Markov regime-switching model. Hamilton (1989) first proposed the Markov switch model to capture the effects of dramatic political and economic events on the properties of financial and economic time series. Under the regime-switching model, the risk or return characteristics of asset classes will be different under distinct regimes, and the regime variable is auto correlated rather than independent. Specifically, the model's parameters are not constant through the sample period, but rather change with structural shifts, which divide the period into distinct regimes with different parametric values. Moreover, the change in regime is considered a random variable instead of a perfectly

foreseeable deterministic event, and switching between regimes occurs stochastically and endogenously following a Markov process.

As mentioned above, numerous papers have contributed to this topic. Alizedeh and Nomikos (2004) applied a Markov regime-switching model to capture FTSE 100 stock index future's volatility. The results indicated that the Markov regime-switching model performs better than conditional volatility models. This consequently leads a more efficient hedge ratio when using futures contracts. The same conclusion was also found in Nomikos et al. (2008) and Salvador and Arago (2013). Meanwhile, numerous other researchers have applied a Markov regime-switching model to capture stock index movement behaviour (Moore and Wang, 2007; Miao et al., 2011; Baba and Sakurai, 2011). From the above examples, it demonstrates that a Markov regime-switching model has an important role in current financial markets.

This sub-chapter has three specific aims. The first one is to examine whether a Markov regime-switching can be used to capture the Chinese stock index futures and spot markets' realised volatility behaviours accurately. Second, this chapter will compare original realised volatility and logarithm realised volatility estimation under a Markov regime-switching model. And find out which estimation method is better. Third, if this chapter suggests strong structure change effect, then dummy variables will be introduced to capture the structure change effect.

This sub-chapter contributes to the current literature in the following aspects. First, this is the first time applying a Markov regime-switching model to capture the behaviours of Chinese stock index futures and spot market' realised volatility. Most of the previous studies focused on mature markets, but this sub-chapter focuses on emerging markets, as such very recent

studies such as Ma et al. (2015), Xu and Wan (2015), Alan et al. (2016) and Yarovaya et al. (2016) underscore the importance of emerging markets in this area. Second, under the author's knowledge, all the past papers applied a Markov regime-switching model under a conditional volatility framework, but this sub-chapter for the first time applies the Markov regime-switching model under a realised volatility framework. Third, the results may contribute to the further realised volatility forecast model for Chinese stock index futures and spot markets. That is, if the results suggest that the Markov regime-switching model can capture the target markets' realised volatility well, then it implies that a nonlinear model will be more suitable to model and forecast the target markets' realised volatility compared to a linear model.

#### **4.2.1. Literature Review**

A total of nine recent papers are examined in detail in this sub-chapter. Three papers examine optimal hedge ratio estimation by comparing the Markov regime-switching framework to other estimation methods, i.e., Alizedeh and Nomikos, 2004; Nomikos et al., 2008; Salvador and Arago, 2013. All of these three papers concluded that the Markov regime-switching framework generally performs better than other methods in optimal hedge ratio estimation. There is one recent paper examining the optimal VaR estimation by comparing the Markov regime-switching model with the GARCH model (Chang, 2011). The results indicate that the Markov regime-switching model performs better than the GARCH model. Two papers expand on the standard Markov regime-switching model (Lee and Yoder, 2007; Liu et al., 2012). Specifically, Lee and Yoder (2007) developed a bivariate Markov regime-switching BEKK-GARCH model, and Liu et al. (2012) proposed a different two-state Markov regime-switching model.

Another three papers apply the Markov regime-switching model empirically (Moore and Wang, 2007; Miao et al., 2011; Baba and Sakurai, 2011). Moore and Wang applied the Markov regime-switching model to investigate the volatility in the stock market for the new European Union (EU) member states, and concluded that the stock market's volatility can be reduced by joining the EU. Miao et al. (2010) examined S&P 500 and Nikkei 225 by using the Markov regime-switching log-normal model, and the results showed that the change-points can match the big events well. Lastly, Baba and Sakurai (2011) applied the Markov regime approach on a VIX index, and indicated three broad regimes in the VIX index, including tranquil regime with low volatility, turmoil regime with high volatility and crisis regime with extremely high volatility. The specific details of these nine papers can also be found here:

Nomikos et al. (2008) estimated constant and dynamic hedge ratios in the New York Mercantile Exchange oil futures markets and examined their hedging performance. They also introduced a Markov Regime-switching VECM with GARCH error structure. This specification linked the concept of disequilibrium with that of uncertainty across high and low volatility regimes. Overall, in and out-of-sample tests indicated that state dependent hedge ratios are able to provide significant reduction in portfolio risk.

Miao et al. (2011) evaluated the ability of a Markov Regime-switching log-normal (RSLN) model to capture the time-varying features of stock return and volatility. The model displays a better ability to depict a fat tail distribution compared with using a log-normal model, which means that the RSLN model can describe observed market behaviour better. The major objective was to explore the capability of the model to capture stock market behaviour over

time. By analysing the behaviour of calibrated regime-switching parameters over different lengths of time intervals, the change-point concept was introduced and an algorithm was proposed for identifying the change-points in the series corresponding to the times when there are changes in parametric estimates. This algorithm for identifying change-points was tested on the Standard and Poor's 500 monthly index data from 1971 to 2008, and the Nikkei 225 monthly index data from 1984 to 2008. It is evident that the change-points they identify match the big events observed in the U.S. stock market and the Japanese stock market (e.g., the October 1987 stock market crash), and that the segmentations of stock index series, which are defined as the periods between change-points, match the observed bear–bull market phases.

Baba and Sakurai (2011) investigated the role of U.S. macroeconomic variables as leading indicators of regime shifts in the VIX index using a regime-switching approach. They found that there were three distinct regimes in the VIX index during the 1990 to 2010 period: tranquil regime with low volatility, turmoil regime with high volatility and crisis regime with extremely high volatility. They also showed that the regime shift from the tranquil to the turmoil regime is significantly predicted by lower term spreads.

Studies in Chinese market include Kogan et al. (2006) that use a static Arrow-Debreu economy example as well as a parsimonious general equilibrium model (similar to Black and Scholes (1973)) and find that: 1) Irrational traders can have a significant impact on asset prices regardless of their long-run survival, i.e. they can influence asset prices with a negligible share of wealth over a long time. 2) Irrational traders' portfolio policies can deviate from their limits long after the price process approaches its long-run limit. They show that survival and price impact are related but independent and distinct concepts. The assumption that survival is a necessary condition for long-run price impact is false.

Using a sample of 76 Chinese firms with both A-shares and B-shares from Jan. 2000 to Nov. 2001 and measures of information asymmetry (Price Impact measure (PI), Adverse Selection Component of the Bid-Ask Spread (AS), Probability of Informed Trading (PIN)), Chan et al. (2008) run regressions of foreign share discount on information asymmetry and find that: 1) On an univariate basis, PI, AS and PIN explain a significant proportion of cross-sectional variations of the B-share discount, 44%, 46% and 8% respectively. 2) On a multivariate basis, PI and AS tend to be more significant than other control variables. 3) When B-share market is opened to domestic investors, B-share discounts become smaller because of less information disadvantage.

Similarly, Andrade et al. (2012) analyse 623 Shanghai A-share stocks that traded in at least 90% of the trading days during the six-month period from November 29, 2006 to May 29, 2007 (Source: RESSET) using 1) measures of bubble intensity: Cumulative return, P/E ratio, Announcement return, Composite bubble measure, China-HK premium as Dependent Variables; 2) Analyst Coverage: the number of security analysts following a stock as Independent Variable. Regardless of the bubble intensity measure, smaller bubbles in stocks match with greater analyst coverage, which is not driven by a positive correlation between analyst coverage and firm size. Moreover, the results are robust to including a battery of additional control variables, as well as addressing concerns about analyst coverage being an endogenous regressor. Stocks with greater analyst coverage display lower turnover, the abating effect of analyst coverage on turnover is weaker when there is more disagreement among analysts. Their paper suggests that policy makers concerned with mitigating asset price bubbles should encourage public information dissemination.



As a supplement for previous literature which provides controversial results with partial equilibrium models, Li and Yang (2012) use a theoretical general equilibrium model to examine implications of prospect theory for disposition effect, asset prices, and trading volume. The model explains a wide range of financial phenomena and suggests new testable predictions. They find three conclusions: 1) Diminishing sensitivity causes the prospect theory investor's effective risk aversion to be positively related to stock returns, leading to a disposition effect, price momentum, a reduced return volatility, and a positive return-volume correlation. 2) Loss aversion predicts the same as diminishing sensitivity when the stock has a very negatively skewed dividend process. However, it makes opposite ones for stocks with nonskewed or positively skewed dividends. 3) In calibrated economies, there is a nontrivial range of preference parameters for prospect theory to simultaneously explain the disposition, the momentum effect, and the equity premium puzzle.

Using Variance decomposition and Panel Regressions with a data set covering about 10490 identical twins and 24486 fraternal twins who participate in stock market (Source: Swedish Twin Registry (STR) ) as well as portfolio data between 1999 and 2007 for each individual, Cronqvist et al. (2015) find that there are several factors explain an individual investor's style: 1) Genetic, or biological factor ingrained in an investor from birth. 2) More human capital, labour income which is more correlated with GDP growth, more behavioural biases (preference for speculative assets )-----more growth stock holdings 3) Life course theory matters. Investors with adverse macroeconomic experiences have stronger preferences for value investing later in life. Their paper suggests that value premium could reflect both risk-based compensation and mispricing due to investors' behavioural biases. The overall composition of investor population, with respect to genetic make-up, age, and life

experiences, can affect the relative demand for value versus growth stocks and in the end potentially the value premium.

Qian et al. (2015) investigate the impact of incentives and communication costs on information production and use using the evidence from bank lending in China. Their data cover borrowers located in 33 cities across China over 2000 to 2006. (Source: a large bank that is ultimately owned by the state) and use the decentralization reform after 2001.12 (China enter WTO) as a break point to divide their sample into two parts: Pre-reform period: 2000.1-2002.6, and Post-reform period: 2004.1-2006.12. They show that better incentives and lower communications costs improve information production and use, which, in turn, expands the supply of credit and improves (lending) outcomes. They find three conclusions: 1) Decentralization reform led to production and use of high-quality information. 2) Banks place more weight on their internally generated credit ratings after reform when the loan officer and branch president work together longer. 3) Both credit rating and loan interest rate better predict loan performance after reform as communication costs fall. Their results are robust in terms of a battery of alternative methods: 1) OLS regressions and of internal credit ratings on firm characteristics for both periods. 2) Probit regressions of promotion on loan officers' past performance. 3). OLS regressions of Ex Ante Loan Terms on Credit Rating: Pre-Reform vs. Post-Reform. 4). OLS regressions of Ex Ante Loan Terms on Credit Rating: The Effect of Communication Costs. 5). Probit regressions of Loan Performance on Credit Rating. 6). Probit regressions of Ex Post Loan Outcome on Ex Ante Interest Rate

Li et al. (2017) investigate the trust and stock price crash risk, using empirical evidence from China. They have obtained a very comprehensive data set, which includes: 1) 2,0272 firm year observations representing 2408 Chinese A share listed companies. 2) Sample period: For

social trust measures and controls 2001-2014; for crash risk measures 2002-2015. 3) Proxy of social trust (TRUST1) from 2002 and 2000 surveys. 4) TRUST2 (voluntary blood donation in 2000 from Chinese Society of Blood Donation). 5) TRUST3 (citizen's trustworthiness from Annual Report on Urban Competitiveness). They measure firm-specific crash risk and social trust, while using Crash Risk as dependent variable and social trust as independent variable. They find firms in provinces of high social trust tend to have a smaller firm-specific stock price crash risk, which is robust to a battery of sensitivity tests and is more prominent for state-owned enterprises, for firms with weak monitoring, and for firms with higher risk-taking. Social trust is positively related to accounting conservatism and negatively related to the likelihood of financial restatements which leads to fewer firm-specific crash risks in the future, consistent with the explanation by agency theory of crash risk. In other words, they discover social trust as an important predictor of stock price crash, which is omitted in previous literature. Their paper suggests that future research could consider other managerial behaviours and whether they are affected by regional social trust.

Most of the recent literature conduct qualitative analysis and provide possible reasons for the 2015 Chinese stock crash, but the reason is still unclear. It is hard to find any consensus from the literature as both risk explanations and behaviour reasons have been presented. Ma (2015) and Yin (2015), for example, suggest that the government policy and the development of financial innovation are important reasons for the bull market from 2014 to mid of 2015. Specifically, high leveraged investments, including umbrella trust, OTC financing, securities margin trading, etc. are important forms of the latter. It is also stated that the implementation of the de-leveraging policy of the government is the direct cause of the stock market crash. Besides, factors such as the large scale of IPO is also one of the reasons for the stock market crash. However, Ba (2016) suggest that the lack of efficient supervision on OTC financing from relevant departments is an important reason for the stock market fluctuation. Besides, as

relevant departments underestimate the quantity of the money from OTC financing, their actions to clear the money from OTC financing lead to property prices drop. Furthermore, the innovation in respect of financial business and financial products, and the development of internet finance make the border between different financial institutions be more obscure, which doesn't match the separate supervision system. From another perspective, Yi (2015) and Cui and Yi (2016) suggest that the stock market boom and crash in 2015 is related to the supporting development strategy of the stock market of the government and the corresponding loose money policy. Besides, it is also closely related to other factors, such as the quality of Chinese stock market investors, the lag of government supervision, the push of mainstream media, the immaturity of Chinese stock market. Other approaches that attempt to provide a rational explanation about the increasing volatility in Chinese market after 2015, also exist.

Guan (2016) analyzes the problems of Chinese stock market from the macroscopic level, middle level, and microcosmic level. He suggests that these problems make the stock market to be more volatile. Huang and Qi (2017) analyze the use of leverage money, including on exchange financing and OTC financing, in Chinese stock market. The results indicate that leverage transaction contribute to the increase or decrease of the index, complying with the cycle. Chen et al (2015) use behavioral financial theory to analyze the appear and burst of the irrational bubble in Chinese stock market from 2014 to 2015. They suggest that the stock market boom can be contributed to the dominant role of personal investors in Chinese stock markets that use leverage money to invest in the stock markets. The de-leverage action of relevant departments make investors sell their stocks, and high leverage makes the stock index drop quickly. Nevertheless, some literature uses the stock market crash as a sample to conduct relevant studies. For example, by using VAR model and GARCH model, Lin (2016) conduct empirical analysis utilizing data from the Investor Confidence Index, Shanghai

Composite Index and CSI 300 between April 2008 and October 2015. He finds that Investor Confident Index and the trend of the stock index are closely related. Utilizing the data from October 2004 to October 2015, Luo and Ren (2016) investigate the influence of short-sale and margin-purchase on stock price crash risk in the Chinese markets at the individual stock level. They find that stocks' future price crash risk and short-sale are significantly negatively related, while it is significantly positively related to margin-purchase. Lleo and Ziemba (2015) use the data from 1990 to 2015 to test whether several indicators, including the price-to-earnings ratio (P/E) based on current earnings, the Bond-Stocks Earnings Yield Differential model (BSEYD) and the Cyclically Adjusted Price-to-Earnings ratio (CAPE), accurately predicts the downturns of the Shanghai Stock Exchange Composite Index. The results indicate that crash prediction models apply to the Chinese stock market. Specifically, the logarithm of the P/E can predict crashes over the entire sample from 1990 to 2015. In comparison, in the subsample from 2006 to 2015, the evidence of the predictive ability of the BSEYD models and CAPE are mixed. Han and Liang (2016) use the 2015 Chinese stock market crash as a sample to examine the effect of index futures trading on spot market quality. A difference-in-difference approach is adopted to identify the net effect. The results indicate that there is a significantly negative effect on the spot market quality after the index futures trading is forbidden.

However, none of the papers on the Chinese stock market mentioned above distinguished between realised volatility and parametric volatilities especially in a high-frequency context, which leaves some leeway for this thesis to contribute. This work adds to the above literature by borrowing the realised volatility measure from the theoretical literature and applying to the high-frequency data from Chinese stock market. By doing so, this work extends to discussion of Chinese stock bubbles to high-frequency level and provides a unique perspective from the realised volatility measure.

### 4.2.2. Methodology

Under the Markov switch approach, one time series can be divided into  $m$  states, denoted  $s_i$ ,  $i = 1, \dots, m$ , representing  $m$  regimes. In the later discussion, it assumes that  $m$  can only = 1 or 2, which refers to a bivariate Markov regime-switching model. Movements of the state variable between regimes are governed by a Markov process, this is:

$$P[a < y_t \leq b | y_1, y_2, \dots, y_{t-1}] = P[a < y_t \leq b | y_{t-1}]$$

The above equation states that the probability distribution of the state at any time  $t$  depends only on the state at time  $t - 1$  and not on the states that were passed through at time  $t - 2, t - 3, \dots$

The basic form of Hamilton's model comprises an unobserved state variable, denoted  $z_t$ , which is postulated to evaluate according to a first order Markov process:

$$Prob[s_t = 1 | s_{t-1} = 1] = p_{11}$$

$$Prob[s_t = 2 | s_{t-1} = 1] = 1 - p_{11} = p_{12} \quad (4.3.1)$$

$$Prob[s_t = 2 | s_{t-1} = 2] = p_{22}$$

$$Prob[s_t = 1 | s_{t-1} = 2] = 1 - p_{22} = p_{21}$$

where  $p_{11}$  denotes the probability of being in regime one, given that the system was in regime one during the previous period, and  $p_{21}$  denotes the probability of being in regime two, given that the system was in regime two during the previous period.

The simplest Markov regime-switching model is the regime-switching autoregressive model with 1 order (MS-AR (1)), which can be written as the following:

$$\begin{cases} y_t = c_0 + \phi_0 y_t + \varepsilon_t, \varepsilon_t \sim iid(0, \sigma_0^2), s_t = 0 \\ y_t = c_1 + \phi_1 y_t + \varepsilon_t, \varepsilon_t \sim iid(0, \sigma_1^2), s_t = 1 \end{cases}$$

The regime duration is used to measure the time length to stay in one regime. From the definition 4.3.1, it shows the following: for  $s_t = 1$ , if  $p$  value is high, then the probability of transforming from current state 1 to next state 0 is low.

Assume state 1 begins from  $t+1$ , and ends with  $t+i$ . State 2 begins from  $t+i+1$  as:

$$s_{t+1} = s_{t+2} = \dots = s_{t+i} = 1, s_{t+i+1} = 0$$

Hence, the expectation duration value of state 1 can be written as:

$$\begin{aligned} E(\text{duration}) &= \sum_{i=1}^{\infty} i \times \Pr(s_{t+1} = 1, \dots, s_{t+i} = 1, s_{t+i+1} = 0) \\ &= 1 \times (1-p) + 2 \times p(1-p) + 3 \times p^2(1-p) + \dots \\ &= (1-p)\{1 + 2p + 3p^2 + \dots\} \\ &= (1-p) \left\{ \frac{1}{1-p} + \frac{p}{1-p} + \frac{p^2}{1-p} + \dots \right\} \\ &= 1 + p + p^2 + \dots \\ &= \frac{1}{1-p} \end{aligned}$$

With the same argument as above, under the assumption:

$$s_{t+1} = s_{t+2} = \dots = s_{t+i} = 0, s_{t+i+1} = 1$$

The expectation duration value for state 0 is:

$$\begin{aligned} E(\text{duration}) &= \sum_{j=1}^{\infty} j \times \Pr(s_{t+j} = 0, s_{t+j+1} = 1) \\ &= \frac{1}{1-q} \end{aligned}$$

For the Markov regime-switching model, there is a conditional and unconditional expectation

value of regime. The conditional expectation value of regime will be discussed first. Under a bivariate regime-switching model, the regime value can either be 0 or 1. Hence, if  $s_{t-1} = 1$ , under the model 4.3.1, the probability of  $s_t = 1$  is  $p$  and the probability of  $s_t = 0$  is  $1 - p$ . The conditional expectation value of state  $s_t$  can be written as:

$$\begin{aligned}
 E(s_t | s_{t-1} = 1) &= \sum_{s_t=0}^1 s_t \times \Pr [s_t | s_{t-1} = 1] \quad (4.3.2) \\
 &= 0 \times (1 - p) + 1 \times p \\
 &= p \\
 &= \Pr (s_t = 1 | s_{t-1} = 1)
 \end{aligned}$$

On the other hand, if  $s_{t-1} = 0$ , then the probability of  $s_t = 0$  equals to  $q$  and the probability of  $s_t = 1$  equals to  $1 - q$ . Hence:

$$\begin{aligned}
 E(s_t | s_{t-1} = 0) &= \sum_{s_t=0}^1 s_t \times \Pr [s_t | s_{t-1} = 0] \quad (4.3.3) \\
 &= 0 \times q + 1 \times (1 - q) \\
 &= 1 - q \\
 &= \Pr (s_t = 1 | s_{t-1} = 0)
 \end{aligned}$$

The above discussion is based on a one dimension situation. Sometimes, the matrix can be used to represent a regime-switching. Defined in the following matrix:

$$s_t = \begin{bmatrix} s_t \\ 1 - s_t \end{bmatrix} \quad (4.3.4)$$

If

$$s_t = 1, \text{ then } s_t = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$s_t = 0, \text{ then } s_t = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Combined with model 4.3.1, it can get that, when  $s_{t-1} = 1$ :



$$E(s_t|s_{t-1}) = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p \\ 1-p \end{bmatrix}$$

Combined with model 4.3.4, it can get:

$$s_{t-1} = \begin{bmatrix} s_{t-1} \\ 1 - s_{t-1} \end{bmatrix}$$

Further combined with model 4.3.2 and 4.3.3, it can get:

$$E(S_t|S_{t-1}) = PS_{t-1} \quad (4.3.5)$$

Define random error terms  $V_t$ , and  $V_t$  as:

$$V_t = S_t - E(S_t|S_{t-1}) \quad (4.3.6)$$

Rewrite model 4.3.5 as a VAR (1) format:

$$S_t = PS_{t-1} + V_t \quad (4.3.7)$$

When

$$s_{t-1} = 1, S_{t-1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Rewriting 4.3.7 gives:

$$\begin{bmatrix} s_{1t} \\ s_{2t} \end{bmatrix} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} V_{1t} \\ V_{2t} \end{bmatrix}$$

Meanwhile, from model 4.3.6, it shows:

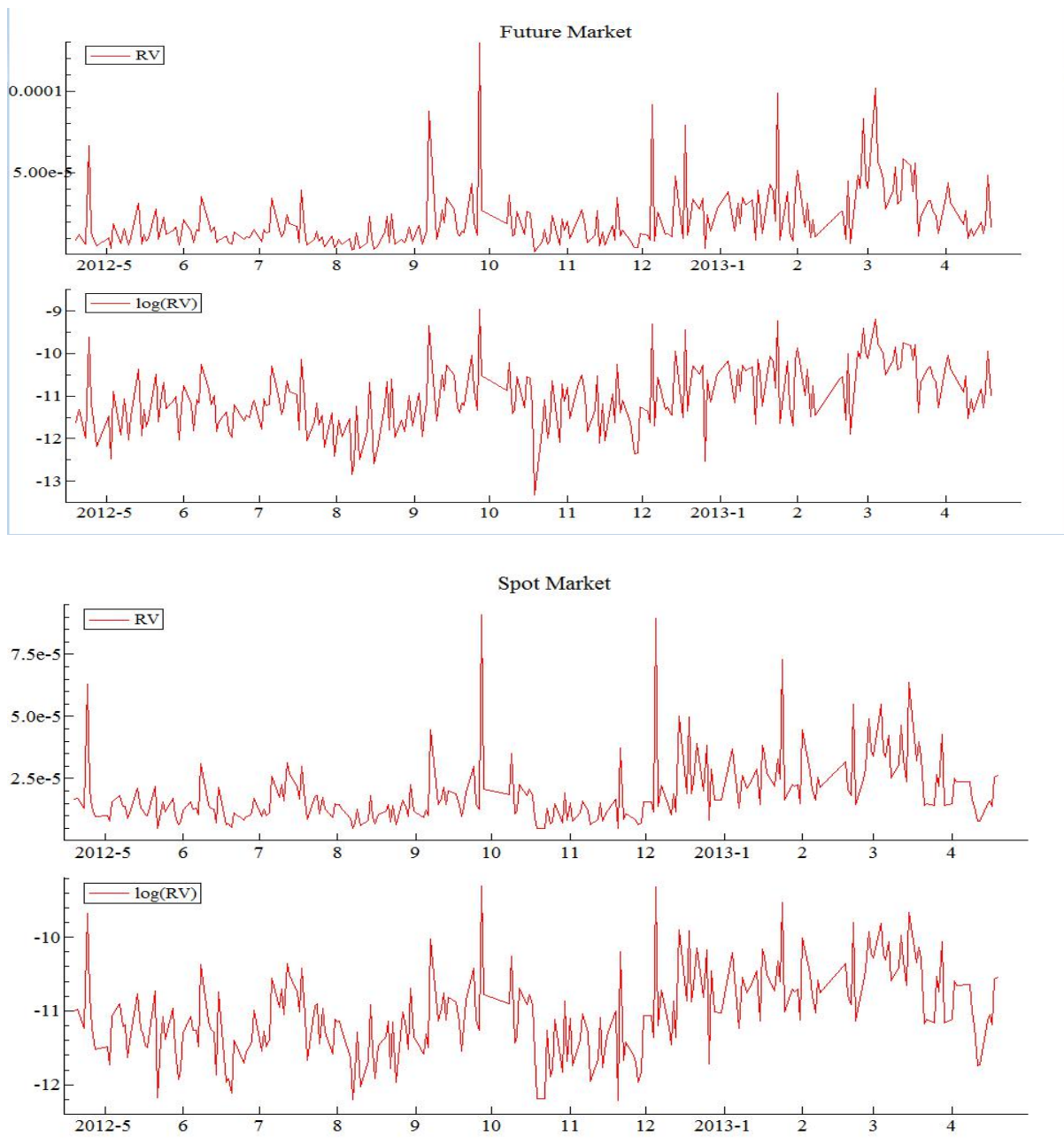
$$\begin{aligned} E(V_t|S_{t-1}) &= E\{[S_t - E(S_t|S_{t-1})]|S_{t-1}\} \\ &= E(S_t|S_{t-1}) - E[E(S_t|S_{t-1})] \\ &= E(S_t|S_{t-1}) - E(S_t|S_{t-1}) \\ &= 0 \end{aligned}$$

### **4.2.3. Empirical Results**

Three specific research aims are explained in detail at the beginning. Two types of realised volatility are estimated, including original realised volatility and logarithm realised volatility.

The empirical data are 5 minutes high-frequency data. The basic shape of these two realised volatility estimations can be represented in the following figure:

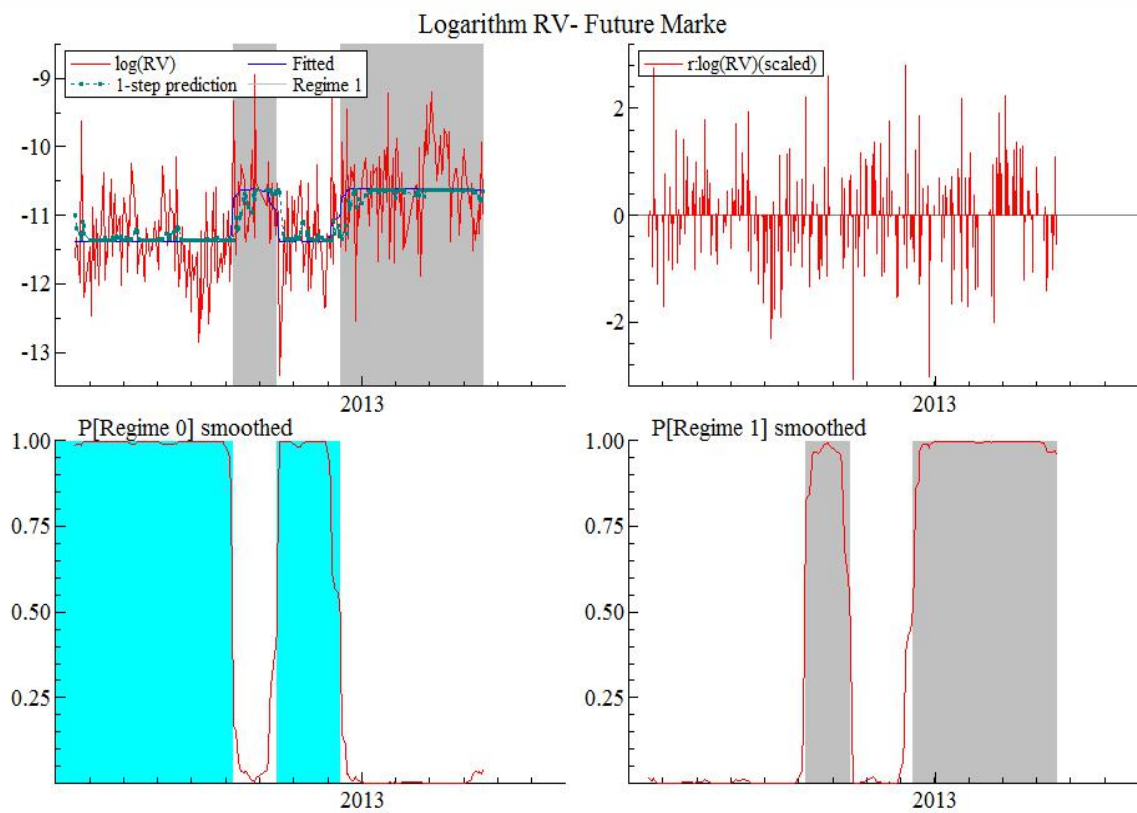
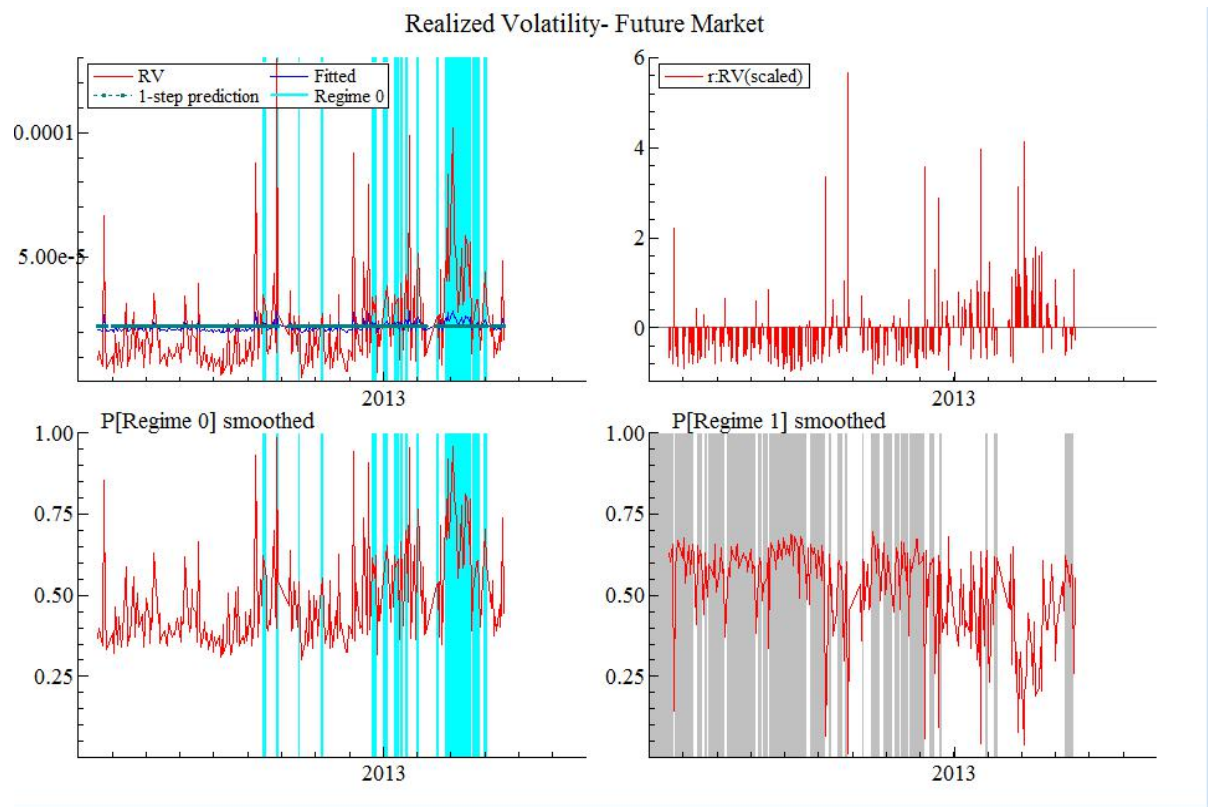
**Figure 9. Two Types Estimated Realised Volatilities.**



Consequently, the dynamic Markov regime-switching models are estimated based on the methodology discussed before. The estimated model results can be graphed and presented in the following figure:



**Figure 10. Dynamic Markove Regime-switching Models.**



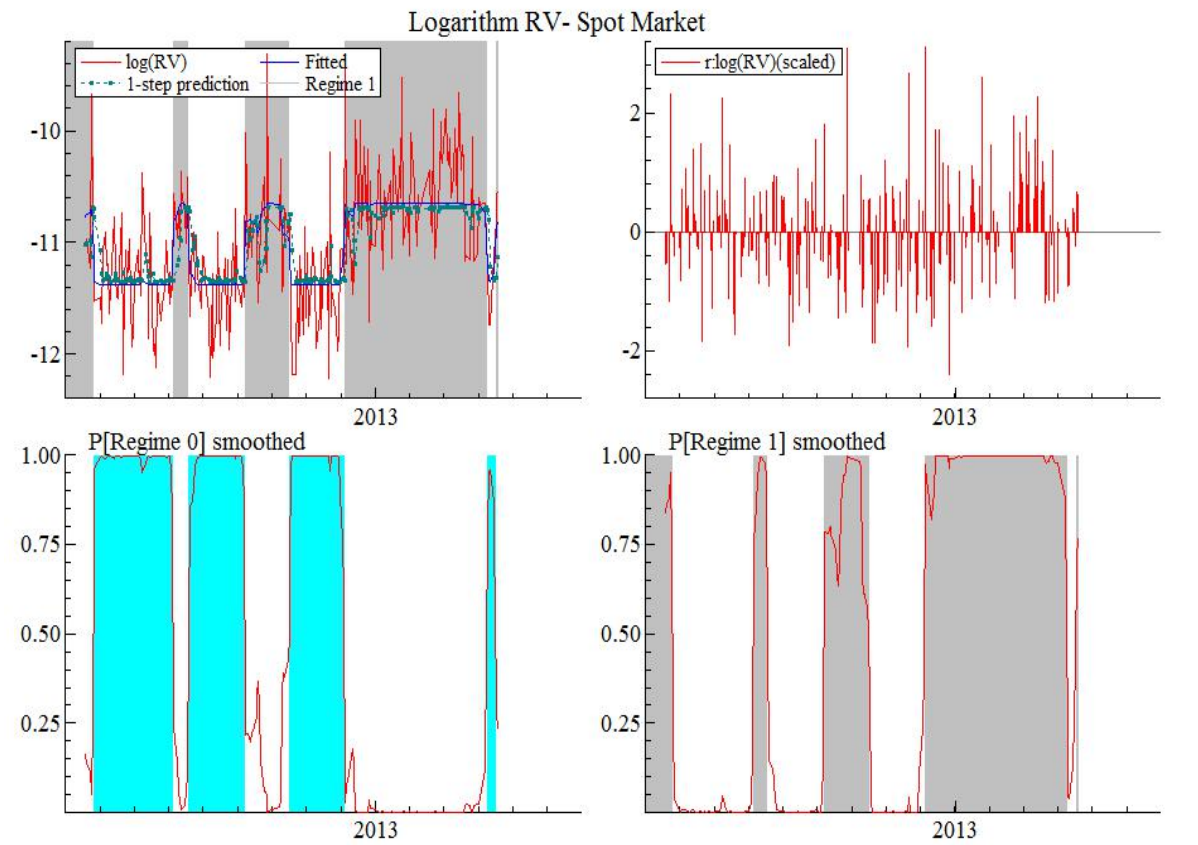
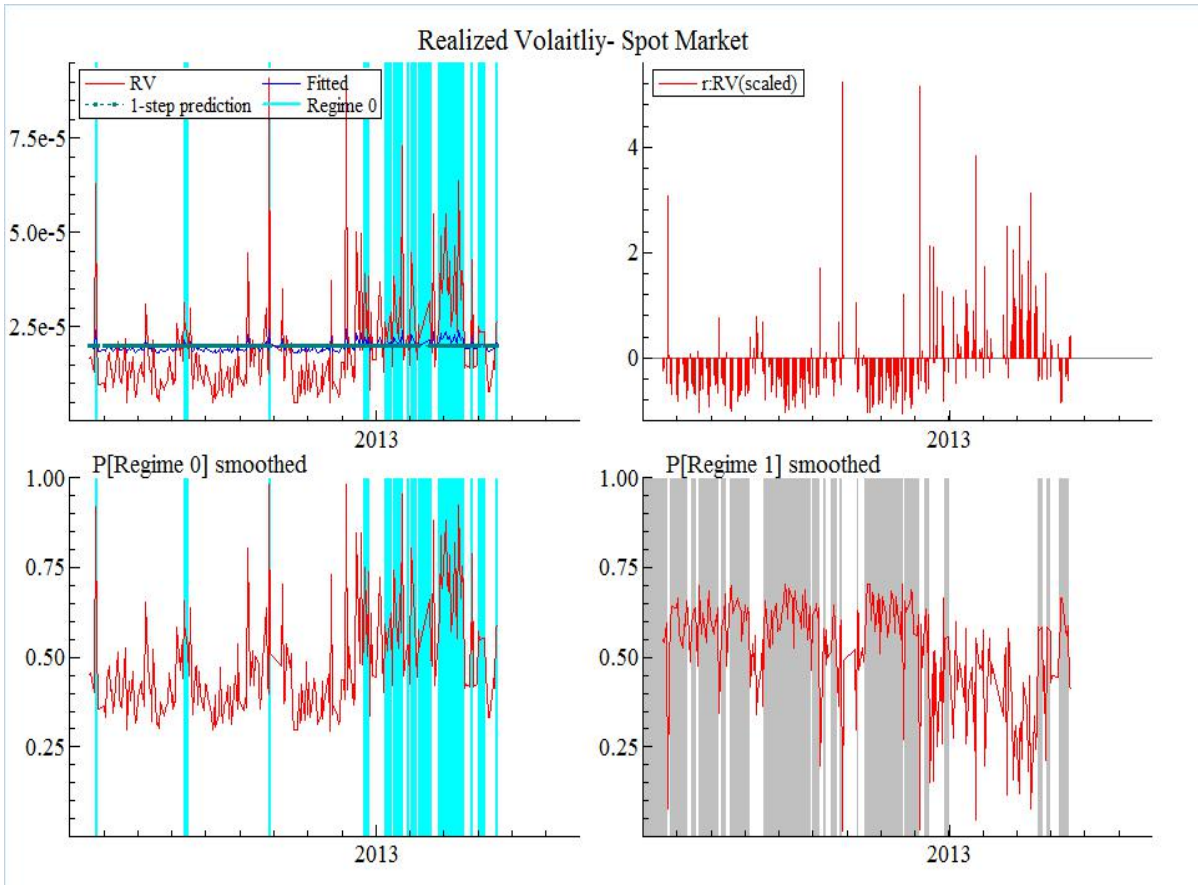


Figure 7 shows that logarithm realised volatility has better stability compared to original realised volatility for both markets. There are numerous regime breakpoints in original realised volatility estimation values, but there are only countable numbers of breakpoints in logarithm realised volatility. Hence, from the stability point of view, logarithm realised volatility performs better than original realised volatility under the Markov switch model.

In this chapter, two regimes (states) are defined as the following: regime 1 refers to a high volatility period and regime 0 refers to a low volatility period. The specific smoothed probability value of each regime can also be found in figure 7. Consequently, the regime classifications are calculated and the details can be found below.

**Table 17. Regime Classification Details.**

<b>Logarithm RV for Futures market</b>	
<b>State 0</b> <b>(low volatility)</b>	1) 19/04/2012 to 06/09/2012 (98 days). 2) 17/10/2012 to 11/12/2012 (40 days).
<b>State 1</b> <b>(High Volatility)</b>	1) 07/09/2012 to 16/10/2012 (23 days). 2) 12/12/2012 to 19/04/2013 (83 days).
Percentage of state 0: 56.6%	Percentage of state 1: 43.3%
<b>Logarithm RV for Spot Market</b>	
<b>State 0</b>	1) 26/04/2012 to 05/07/2012 (48 days). 2) 19/07/2012 to 06/09/2012 (36 days). 3) 17/10/2012 to 04/12/2012 (35 days). 4) 10/04/2013 to 17/04/2013 (6 days).
<b>State 1</b>	1) 19/04/2012 to 25/04/2012 (5 days). 2) 06/07/2012 to 18/07/2019 (9 days). 3) 07/09/2012 to 16/10/2012 (23 days).

4) 05/12/2012 to 09/04/2013 (80 days).	
5) 18/04/2013 to 19/04/2013 (2 days).	
Percentage of state 0: 51.23%	Percentage of state 1: 48.77%

The results show that the futures market is nearly 14% higher in the low volatility periods comparing to high volatility period. However, the spot market shows a slightly higher percentage in low volatility periods. Meanwhile, the spot market has much more regime breakpoints compared to the futures market. The above results reflect one fact: the futures market is more stable than the spot market.

Comparing these regime breakpoints, there are two common breakpoints for both the futures and spot markets. These two points are 06/09/2012 and 16/10/2012. Meanwhile, these two points indicate the same direction of a regime-switching. That is, the point 06/09/2012 indicates regime-switching from state 0 (low volatility) to state 1 (high volatility) for both markets. And the point 16/10/2012 indicates regime-switching from state 1 to state 0 for both markets.

On 06/09/2012, the Chinese government pronounced a large investment schedule on the real estate market. This good news led to the fact that at least 20 listed building corporations' stocks increased about 10%. After this day, the market kept rising and this led to a period of higher volatility. This fact is clearly captured by the dynamic Markov regime-switching model. The Markov regime model suggests that that day was the regime breakpoint to lead a market switch from a low volatility period to a high volatility period.

After a long period of increased value of the stock index caused by the effects of good news released on 06/09/2012, the stock index had a sharp decrease in value during the period 13/10/2012 to 16/10/2012. This period was three days before another common regime breakpoint for both markets. During these three days, a large decrease in the value of the two markets was observed, and the stock index kept a low value for quite a long period, which led the regime-switching from a high volatility period to a low volatility period. This specific market information is also well captured by the dynamic Markov regime-switching model.

Consequently, the estimated coefficients of the dynamic Markov regime-switching models can be represented below.

**Table 18. Estimated Coefficients.**

	Futures Market	
	Realised Volatility	Logarithm Realised Volatility
Constant (0)	2.87226e-005* (4.32)	-11.3837* (-193)
Constant (1)	1.59146e-005 (2.39)	-10.6088* (-160)
Sigma	1.78269e-005* (2.70)	0.635370* (21.7)
$p(0 0)$	0.499511* (3.28)	0.984431* (89.0)
$p(0 1)$	0.499511* (2.93)	0.011702 (0.996)
Log likelihood Value:	2312.91975	-245.933684
Mean(RV)	2.11437e-005	-11.0474
Var(RV)	3.36119e-010	0.551198
Probability Matrix	$P = \begin{bmatrix} 0.49951 & 0.49951 \\ 0.50049 & 0.50049 \end{bmatrix}$	$P = \begin{bmatrix} 0.98443 & 0.011702 \\ 0.015569 & 0.98830 \end{bmatrix}$
Linearity LR test ( $\chi^2$ )	N/A	55.233*
	Spot Market	
	Realised Volatility	Logarithm Realised Volatility
Constant (0)	2.46384e-005* (4.55)	-11.3796* (-240)
Constant (1)	1.54458e-005* (2.86)	-10.6480* (-210)
Sigma	1.26510e-005 (2.08)	0.433942* (20.2)



$p(0 0)$	0.499511* (3.31)	0.963536* (49.6)
$p(0 1)$	0.499511* (3.10)	0.0415815 (1.89)
Log likelihood Value:	2395.50736	-165.49913
Mean(RV)	1.91459e-005	-11.036
Var(RV)	1.70786e-010	0.321604
Probability Matrix	$P = \begin{bmatrix} 0.49951 & 0.49951 \\ 0.50049 & 0.50049 \end{bmatrix}$	$P = \begin{bmatrix} 0.96354 & 0.041581 \\ 0.036464 & 0.95842 \end{bmatrix}$
Linearity LR test ( $\chi^2$ )	N/A	84.642*

\* refers to significant at 1% confidence

One interesting point from the above table is that both linearity LR tests for both markets' original realised volatility are not available. The reason may be that numerous optimal regime breakpoints exist in a small interval period, and consequently leads the quasi-maximum likelihood estimation to fail. From the linearity LR test point of view, the logarithm realised volatility performs better than original realised volatility.

Specific to the futures market, the coefficients of  $p(0|0)$  and  $p(0|1)$  are close to 0.5 and significant at 1% confidence level under original realised volatility estimation. This implies the chance of switching from regime 0 (low volatility period) to regime 1 (high volatility period) nearly equals 0.5, which is a random guess value. This proves that the Markov dynamic regime-switching model does not work very well for original realised volatility.

However, in the logarithm realised volatility value, the term  $(0|0)$  is highly significant and the coefficient equals 0.98. Meanwhile, the term  $p(0|1)$  is not significant and the coefficient nearly equals 0.01. These statistics suggest that the chance to move from regime 0 (low volatility period) to regime 0 is very high under logarithm realised volatility estimation, nearly 98%. However, the chance of switching from regime 1 to regime 0 is almost impossible, which only has 1% chance. This statistic suggests that the futures markets'

logarithm realised volatility has very good stability. If the current state is a low volatility period, then the chance to stay in low volatility is very high. The above statistic also suggests that logarithm realised volatility is a better proxy for daily volatility compared to original realised volatility estimation.

For the spot market, the coefficient of  $p(0|0)$  and  $p(0|1)$  are also close to 0.5 and significant at 1% confidence level under original realised volatility estimation. With the similar argument as above, the  $p(0|1)$  near equals to 0.5 means that the chance to switch from regime 1 to regime 0 nearly equals random guess. This proves that the Markov dynamic regime-switching model does not work very well for original realised volatility in the spot market. In the terms of logarithm realised volatility, the term of  $p(0|0)$  is significant at 1% confidence level, and the coefficient equals 0.96. This means the logarithm realised volatility estimation has good stability for spot the market, and the chance to stay in a low volatility period is very high if the past period is also a low volatility period. Meanwhile, the term  $p(0|1)$  is not significant at 1% level, which indicates that it is almost impossible to switch from a high volatility period to a low volatility period. From the above conclusions, it shows that logarithm realised volatility performs much better than original realised estimation under a dynamic Markov regime-switching framework.

To sum up, the logarithm realised volatility is more suitable to apply the dynamic Markov regime-switching model for both futures and spot markets. Under original realised volatility estimation, the Markov regime-switching model makes it hard to judge the structure breakpoints. However, the structure breakpoints are clearly identified under logarithm value. Meanwhile, the linearity LR test also indicates that logarithm realised volatility estimation

shows clear regime breakpoints. Comparing the futures and spot markets, the futures market shows more stability than the spot market.

From the above results, this chapter suggests applying a nonlinear model to model and forecast logarithm realised volatility in later studies. Although there are various ways to set up a nonlinear econometrics model, one simple method is to model these nonlinear characteristics via dummy variables. For example, the following dummy can be defined:

$$\begin{aligned} \text{Dummy} &= 1, \text{ if } \text{state} = 1 \\ &= 0, \text{ otherwise} \end{aligned}$$

Combined with specific regime classifications, two dummy variables are set for both futures and spot markets to capture the structure change effect.

#### **4.2.4. Discussion**

This sub-chapter examines three specific questions. First, it aims to test whether the Markov regime-switching model can be used to capture realised volatility behaviours of Chinese stock index futures and spot markets. The empirical results indicate the dynamic Markov regime-switching model can capture the specific two markets' structure change information well. Two common regime breakpoints are selected and discussed in detail. Both these two breakpoints match the real market situation well.

Second, this chapter compares original realised volatility and logarithm realised volatility. The empirical results indicate that the logarithm realised volatility estimation method performs much better than the original realised volatility under the Markov regime-switching framework. The logarithm realised volatility shows clear regime breakpoints but original

realised volatility cannot. Third, a dummy variable is introduced to model structure change under logarithm realised volatility for both markets. This dummy variable will be used in the later sub-chapter for realised volatility modelling and forecasting.

### **4.3. Realised Volatility Model and Forecast**

In chapter three, this thesis discussed the characteristics of Chinese stock index futures and spot markets' realised volatility. These included select optimal frequency to estimate realised volatility, detect daily volatility jumps, intraday volatility jumps and periodicity and explore realised volatility's long memory features. The results indicate the 5 minute interval is the optimal frequency; daily volatility jumps, intraday volatility jumps and periodicity effect exist; and realised volatility shows strong long memory features. These results provide strong theoretical foundations for realised volatility modelling and forecast.

The existing papers provide a variety of realised volatility forecasting models (Andersen et al., 2003; Engle and Gallo, 2006; Lu, 2006; Refenes et al., 2012; Medeiros et al., 2012). Obviously, a standard framework does not exist to model and forecast realised volatility. Different models have their own advantages and disadvantages. The optimal model is totally dependent on the specific market situations. However, there are two most important and widely used models, which are the Autoregressive Fractionally Integrated Moving Average Volatility (ARFIMA) model (Andersen et al., 2001) and Heterogeneous Autoregressive Realised Volatility (HAR-RV) model (Corsi, 2009). The ARFIMA model is widely used based on realised volatility's long memory feature. Andersen et al. (2001) showed that the traditional Autoregressive Moving Average (ARMA) cannot be used to model realised

volatility, due to the long memory characteristics. They suggested that fractionally integrated methods may have better modelling accuracy than traditional ARMA models.

Another important realised volatility model is the Heterogeneous Autoregressive Realised Volatility (HAR-RV) model proposed by Corsi (2009). Corsi proposed this model, which is based on the Muller et al. (1997) heterogeneous market hypothesis. The heterogeneous market hypothesis states that a financial market is constituted of heterogeneous investors. Muller et al. divided all investors into three types: short term investors, middle term investors and long term investors. For example, short term investors mainly care about daily returns and long term investors are more focused on long term investment strategies. Meanwhile, heterogeneous market hypothesis also assumes only new information will lead to price changes, based on Fama's (1970) efficient market hypothesis. Furthermore, Muller et al. assumed that different types of investors react to new information at different speeds. The short term investors react to new information the quickest and long term investors react the slowest. Due to the above reason, Corsi (2009) proposed a realised volatility model based on heterogeneous market hypothesis, and showed that the HAR-RV model can perform very well when modelling and forecasting realised volatility.

In this sub-chapter, the key aim is to select the optimal realised volatility estimation and forecast model for Chinese stock index futures and spot markets. Two widely used models (ARFIMA and HAR-RV) will be discussed in detail. Meanwhile, all the previous studies' results will be considered, including optimal sampling, long memory, daily jumps, intraday jumps and periodicity, volatility transmission and Markov regime-switching effects.

The financial market's volatility holds an extremely important position in all financial organisations. Numerous financial activities are totally dependent on volatility, such as calculating optimal hedge ratio, fair option price and risk management. This chapter also contributes to the current literature in the following two aspects: first, to the best of the author's knowledge, this is the first time modelling and forecasting Chinese stock index futures and spot markets using a realised volatility framework. This situation is due to the following two reasons: first, most of the existing papers focus on mature markets and ignore emerging markets, such as the Chinese stock index futures and spot markets, which were only established in 2010. Second, this sub-chapter will combine numerous realised volatility features (discussed in chapters 3 and 4) into realised volatility modelling and forecasting. The optimal forecast model selected from this chapter may contribute to the further research in Chinese stock index futures and spot markets realised volatility modelling areas.

#### **4.3.1. Literature Review**

In this literature review sub-chapter, a total of eleven papers on realised volatility modelling are reviewed. All of these eleven papers provide a variety of realised volatility models. The specific models suggested by these authors can be summarised in the following table.

**Table 19. Realised Volatility Models.**

<b>Authors</b>	<b>Realised Volatility Model</b>
<b>Engle and Gallo (2006)</b>	Multiplicative Error Model which is consistent and asymptotically normal under a wide range of specifications for the error density function.
<b>Liu and Maheu (2009)</b>	Bayesian Model Averaging Approach.

The above table's summary shows that different researchers provide different models to model and forecast realised volatility. Obviously, one model does not exist that is significantly better than any other model. One model can only be more suitable depending on specific market situations. In this sub-chapter, the key aim is to find the optimal realised volatility forecast model for Chinese stock index futures and spot markets. It is necessary to review the relevant literature. The specific details of these eleven papers can also be found here:

Engle and Gallo (2006) proposed to jointly consider absolute daily returns, daily high–low range and daily realised volatility to develop a forecasting model based on their conditional dynamics. As all are a non-negative series, they developed a multiplicative error model that is consistent and asymptotically normal under a wide range of specifications for the error density function. The estimation results showed significant interactions between the indicators. They also showed that one-month-ahead forecasts adequately match (both in and out-of-sample) the market-based volatility measure provided by the VIX index, as redefined by the CBOE.

Lu (2006) studied the modelling of large data sets of high-frequency returns using a LMSV model. A new method of de-seasonalising the volatility in high-frequency data was proposed, which allows for slowly varying seasonality. Using both simulated and real data, they compared the forecasting performance of the LMSV model for forecasting realised volatility (RV) to that of a linear long memory model fit to the log RV. The performance of the new seasonal adjustment was also compared to a recently proposed procedure using real data.

Liu and Maheu (2009) used a Bayesian model averaging approach to forecast realised volatility. Candidate models included autoregressive and heterogeneous autoregressive specifications based on the logarithm of realised volatility, realised power variation, realised bi-power variation, a jump and an asymmetric term. Applied to equity and exchange rate volatility over several forecast horizons, Bayesian model averaging provided very competitive density forecasts and modest improvements in point forecasts compared to benchmark models. They discussed the reasons for this, including the importance of using realised power variation as a predictor. Bayesian model averaging provides further improvements to density forecasts when they moved away from linear models and average over specifications that allow for GARCH effects in the innovations to log-volatility.

Refenes et al. (2012) have proposed the Heterogeneous Autoregressive Realised Volatility model, which was extended in order to account for asymmetric responses to negative and positive shocks occurring at distinct frequencies, as well as for the long range dependence in the heteroskedastic variance of the residuals. Compared with established HAR and ARFIMA realised volatility models, the proposed model exhibited superior performance in-sample fitting, as well as in-sample volatility forecasting performance.

#### **4.3.2. Methodology**

Two important realised volatility forecast models will be discussed, including the ARFIMA and HAR-RV models. The ARFIMA model is based on realised volatility's long memory feature, and the HAR-RV model is based on Muller et al.'s (1997) heterogeneous market hypothesis. The basic methodologies of these two models are represented as follows:



### 4.3.2.1. ARFIMA Model

Realised volatility has strong long memory features for both stock index futures and spot markets. It suggests applying the long memory model to track and forecast realised volatility, which is consistent with Andersen et al.'s (2001) suggestion. Andersen et al. (2001) found out that the logarithm realised volatility can be described by the dynamic Gaussian process, and that logarithm realised volatility shows a strong long memory feature. That is, among the lag orders increase, the speed of autocorrelation coefficient decays slower than the exponential format. Hence, Andersen et al. (2001) suggested applying the ARFIMA model to capture this dynamic Gaussian process.

The standard ARFIMA (p, d, q) model is usually as below:

$$\Phi(L) * (1 - L)^d * (Y - \mu) = \Theta(L) * \varepsilon_t$$

where  $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p$ ,  $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q$ .

$\Phi(L)$  refers to p-order causal operator for autoregressive and  $\Theta(L)$  represents q-order causal operator for moving average.  $L$  refers to causal operator,  $(1 - L)^d$  is fractional difference operator and  $\mu$  represents to mean value of  $Y$ ,  $\varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$ .

The ARFIMA (p, d, q) model coefficient are estimated by the maximum likelihood method, which is optimised by the BHHH algorithm. The likelihood function  $L(\theta)$  can be represented as the following (Brooks, 2002):

$$L(\theta) = -\frac{TN}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^T (\log |H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t)$$

where  $\theta$  denotes all the unknown parameters to be estimated;  $N$  is the number of assets, and  $T$  is the number of observations. Meanwhile, the  $\theta$  in the maximum likelihood estimation is asymptotic to normal distribution.

#### **4.3.2.2. HAR-RV Model**

Muller et al. (1997) proposed the heterogeneous market hypothesis. This hypothesis assumes the market constitutes heterogeneous investors, and these investors have different risk preferences and speeds to achieve new information and rationality. Hence, these different investors will react differently to the same new market information. Under this heterogeneous market hypothesis, Corsi (2009) proposed the HAR-RV model to forecast realised volatility. The model captures these heterogeneous behaviours through the autoregressive process as following:

$$RV_{t+H}^d = \beta_0 + \beta_d RV_t^d + \beta_w RV_t^w + \beta_M RV_t^M + \varepsilon_{t+H} \quad (4.4.1)$$

where  $RV_t^d$  refers to daily realised volatility;  $RV_t^w$  represents weekly realised volatility, and

$$RV_t^w = \frac{1}{5} (RV_{t-5}^d + RV_{t-4}^d + \dots + RV_{t-1}^d)$$

$RV_t^M$  is the monthly realised volatility, which equals:

$$RV_t^M = \frac{1}{20} (RV_{t-20}^d + RV_{t-19}^d + \dots + RV_{t-1}^d)$$

The above model mainly infers that one market's realised volatility constitutes a different period's volatility. That is, the current realised volatility is influenced by long term, middle term and short term investors together. The coefficients refer to the degree of different investors' influence on current realised volatility. At the same time, the HAR-RV can capture the long memory feature of realised volatility as well.

The previous results suggest that the current realised volatility is significantly influenced by volatility jumps, and also significantly influenced by the structure change effects. The pervious results also indicate volatility transmission does not exist between the futures and spot markets. Hence, this chapter proposes for the first time a HAR-J-MS model to track and forecast Chinese stock index futures and spot markets' logarithm realised volatility.

The first step is to add jumps into the HAR-RV model to make the HAR-J model as follows:

$$RV_{t+H}^d = \beta_0 + \beta_d RV_t^d + \beta_w RV_t^w + \beta_M RV_t^M + \gamma J + \varepsilon_{t+H}$$

where  $\gamma J$  refers to daily volatility jump influences. The second step is to add the dummy variable to capture the structure change effects. Two dummy variables are defined as follows:

$$\begin{aligned} D_f &= 1, \text{ if state period} = 1 \text{ (future market)} \\ &= 0, \text{ other wise} \end{aligned}$$

and

$$\begin{aligned} D_s &= 1, \text{ if state period} = 1 \text{ (spot market)} \\ &= 0, \text{ other wise} \end{aligned}$$

The final realised volatility forecast model based on HAR can be written as equation 4.4.2:

$$RV_{t+H}^d = \beta_0 + \beta_d RV_t^d + \beta_w RV_t^w + \beta_M RV_t^M + \gamma J + \delta D_f + \varepsilon_{t+H} \text{ (future)} \quad (4.4.2)$$

$$RV_{t+H}^d = \beta_0 + \beta_d RV_t^d + \beta_w RV_t^w + \beta_M RV_t^M + \gamma J + \delta D_s + \varepsilon_{t+H} \text{ (spot)}$$

The coefficients in the HAR-J-MS model are estimated by using the OLS estimation method.

### **4.3.3. Empirical Results**

The data used here is 5 minutes high-frequency data, selected in the previous chapter. Two criteria are applied to the data selection. The first one is the volatility signature plot proposed by Andersen et al. (2005) in order to minimise microstructure noise. The results indicate that

5 minutes interval is better than any other frequency. It also applies the filter technique proposed by Corsi et al. (2001) to eliminate autocorrelation in the data. However, it seems that 5 minutes high-frequency data has extremely low autocorrelation in the data. Consequently, the 5 minutes high-frequency is selected as optimal.

Once the optimal data sample is selected, the realised volatility and logarithm realised volatility are estimated for both markets. This sub-chapter finally selects logarithm realised volatility rather than original realised volatility to estimate the forecast model, and this is due to following two reasons: first, logarithm realised volatility has a stronger long memory feature compared to original realised volatility, and logarithm realised volatility follows near perfect normal distribution. Both of these two features indicate logarithm realised volatility has better forecasting power than original realised volatility. Second, the logarithm realised volatility shows clear structure breakpoints, but original realised volatility makes it hard to judge these structure breakpoints. Meanwhile, realised volatility is more stable than original realised volatility for both markets. Based on the above two reasons, the logarithm realised volatility estimation method will be applied.

#### **4.3.3.1. ARFIMA Long Memory Results**

The first step is to select the optimal lags length in the ARFIMA (p, d, q) model. A total of nine models are estimated by letting p= 1, 2, 3 and q=1, 2, 3. The parametric d is estimated based on AIC statistics for each model. The calculated AIC statistics are represented below.

**Table 20. Model Selection for ARFIMA.**

	<b>Spot Market</b>	<b>Futures Market</b>
<b>ARFIMA (1, d, 1)</b>	1.3830	<b>2.0158</b>
<b>ARFIMA (1, d, 2)</b>	<b>1.3782</b>	2.0252

ARFIMA (1, d, 3)	1.3883	2.0315
ARFIMA (2, d, 1)	1.3883	2.0217
ARFIMA (2, d, 2)	1.3860	2.0296
ARFIMA (2, d, 3)	1.3950	2.0182
ARFIMA (3, d, 1)	1.3878	2.0297
ARFIMA (3, d, 2)	1.3982	2.0380
ARFIMA (3, d, 3)	1.4016	2.0441

Based on the above result, the optimal model for the spot market is ARFIMA (1, d, 2), and for the futures market is ARFIMA (1, d, 1). Once the optimal lags length is selected, the optimal models are estimated, and the estimated models are represented as below.

**Table 21. Estimated ARFIMA Results.**

	Spot Market	Futures Market
<b>constant</b>	-11.0228* (-95.9)	-11.0341* (-25.7)
<b>d parametric</b>	-0.494361* (-3.08)	-0.418447* (-5.87)
<b>AR_1</b>	-0.987864* (65.0)	-0.222821 (-1.28)
<b>MA_1</b>	-0.366402* (-2.28)	-0.152778 (-0.720)
<b>MA_2</b>	0.0978799 (1.20)	N/A
<b>Log likelihood value</b>	-162.142403	-240.939148

The statistics in table are coefficients and t-statistics (in parentheses)

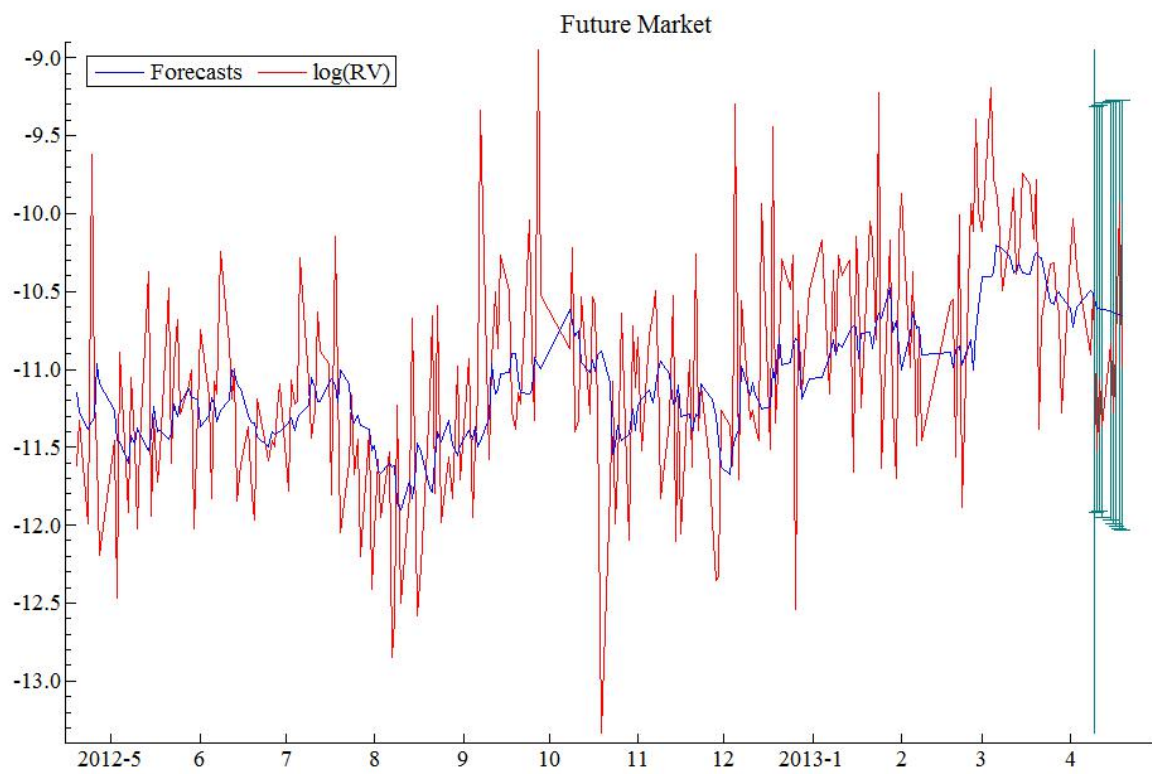
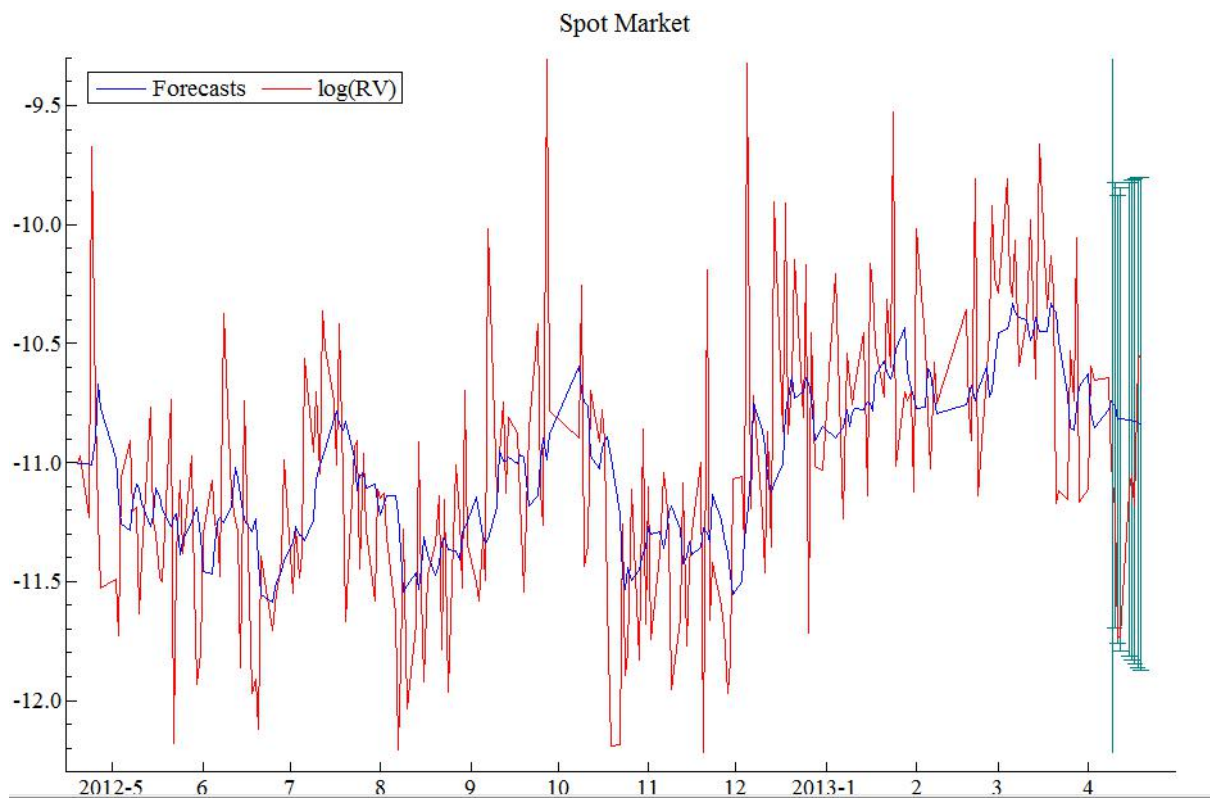
\* represents significance at 1% level.

The above results indicate both two markets' parameters  $d$  are nearly equal to -0.4 and are highly significant at 1% level. This indicates both series have long memory characteristics. Meanwhile, the MA\_2 term is not significant at 1% confidence level for the spot market. For the futures market, both AR\_1 and MA\_1 terms are not significant at 1% level. These statistics indicate that ARFIMA (p, d, q) models have some degree of accuracy to model and

forecast logarithm realised volatility, but the selected models are not totally satisfactory at high criteria levels.

Consequently, the forecasts are conducted for both markets. The in-sample data span is from 19/04/2012 to 09/04/2013, and the out-of-sample forecast period is from 10/04/2013 to 19/04/2013, which represents eight total trading values. The forecast results are represented in the figure below.

**Figure 11. Out-of-sample ARFIMA Forecast Results.**



The above figure shows that both logarithm realised volatility have good forecasting power based on the selected models. Furthermore, the forecast standard errors for eight forecast trading days are calculated and presented below.

**Table 22. Out-of-sample Forecast Standard Errors.**

	<b>Spot Market</b>	<b>Futures Market</b>
<b>Forecasted Day 1</b>	0.468601	0.650127
<b>Forecasted Day 2</b>	0.471108	0.650593
<b>Forecasted Day 3</b>	0.486793	0.665776
<b>Forecasted Day 4</b>	0.496493	0.672502
<b>Forecasted Day 5</b>	0.503762	0.678287
<b>Forecasted Day 6</b>	0.509512	0.682788
<b>Forecasted Day 7</b>	0.514207	0.686537
<b>Forecasted Day 8</b>	0.518128	0.689717
<b>Average standard errors</b>	0.496075	0.672041

The above results show that, on average, the spot market performs better than the futures market. In the previous ARFIMA model estimation results, both AR\_1 and MA\_1 terms are not significant for the futures market, and these insignificant results can also be reflected by a higher average forecast standard error compared to the spot market. Overall, both of these logarithm series show strong forecasting power by the low value of average standard error, and the spot market generally outperforms the futures market under the ARFIMA model estimation.



### 4.3.3.2. HAR-J-MS Model Results

This sub-chapter will estimate the HAR-J-MS model in tracking Chinese stock index futures and spot markets' realised volatility. As discussed above, the 5 minutes interval will be applied as well as logarithm realised volatility estimation. The first step is to estimate the basic HAR model and examine the model characteristics. Based on the equation 4.4.1, 5 types of  $RV_{t+H}^d$  are estimated with  $H = 1, 5, 10, 15, 20$ , which respectively represent one day, one week, two weeks, three weeks and one month forward average realised volatility. The coefficients are estimated by using the OLS method and the results are represented below.

**Table 23. HAR Model Estimation Results.**

Spot Market					
Variables	1 day	1 week	2 weeks	3 weeks	1 month
$\beta_0$	-0.080676 [0.788804]	1.204854 [0.909950]	-2.503292 [0.008249]	-4.048072* [1.009280]	-5.661540* [1.215969]
$\beta_d$	-0.217478* [0.065054]	-0.009236 [0.075268]	0.008249 [0.080674]	-0.099038 [0.084399]	-0.174760 [0.092523]
$\beta_w$	1.109966* [0.132753]	0.141802 [0.153470]	-0.033983 [0.165028]	-0.341195 [0.172948]	-0.178414 [0.190724]
$\beta_M$	0.099590 [0.129007]	0.756311* [0.148843]	0.796113* [0.159793]	1.068899* [0.168734]	0.832208* [0.191962]
<b>AIC</b>	1.042690	1.327796	1.443161	1.512210	1.641723
<b>R-square</b>	0.484234	0.412333	0.432155	0.456377	0.462634

<b>Adj R-square</b>	0.445645	0.254536	0.256456	0.265414	0.269874
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<b>Futures market</b>					
<b>Variables</b>	1 day	1 week	2 weeks	3 weeks	1 month
$\beta_0$	-0.056507 [0.947232]	-0.851690 [1.116550]	-2.704447 [1.179826]	-3.737436* [1.235156]	-4.404002* [1.575491]
$\beta_d$	-0.311388* [0.063115]	0.018763 [0.074629]	0.016233 [0.078840]	-0.081977 [0.081048]	-0.075728 [0.087220]
$\beta_w$	1.262742* [0.144158]	0.134187 [0.169580]	0.070194 [0.180014]	-0.192907 [0.186886]	-0.015990 [0.204201]
$\beta_M$	0.042654 [0.154668]	0.768960* [0.181789]	0.666319* [0.193965]	0.932339* [0.204629]	0.684640* [0.238477]
<b>AIC</b>	1.641425	1.963849	2.065475	2.112821	2.221645
<b>R-square</b>	0.505251	0.305625	0.276513	0.284656	0.298465
<b>Adj R-square</b>	0.445645	0.254536	0.256456	0.265414	0.269874

\* refers to significant at 1% level.  
[ ] represents standard error.

The above table shows the following: for the spot market, only  $\beta_d$  and  $\beta_w$  are significant at 1% level in 1 day forward realised volatility estimation. This indicates the 1 day forward realised volatility mainly constitutes short term and middle term investors. The long term investors do not significantly influence 1 day advanced realised volatility. The above results support that the Chinese stock index spot market has heterogeneous characteristics. This empirical result can be used as one case to support Muller et al.'s (1997) heterogeneous

market hypothesis. Comparing 1 day forward to other periods, the results based on other periods indicate only  $\beta_M$  is significant at 1% level. This means only long term investors will significantly influence the middle term and long term's logarithm realised volatility. This result supports that Chinese investors become more rational in financial markets and care more about long term investment rather than short term speculation.

Meanwhile, the AIC statistics show that the 1 day forward HAR model provides the optimal model estimation.

For the index futures market, very similar characteristics are shown when compared to the spot market.  $\beta_d$  and  $\beta_w$  are significant at 1% level for 1 day forward realised volatility estimation, which indicates the short term realised volatility is mainly dominated by short-term and middle term investors. In the other frequency periods, only  $\beta_M$  is significant at 1% level, which indicates the middle term and long term's realised volatility are only significantly influenced by long term investors. This result also supports that Chinese stock index futures market becomes more mature. Similar to the spot market, the 1 day forward HAR gives the best model estimation.

Based on the HAR model, this chapter adds for the first time intraday volatility jumps and structure change effects into the standard HAR model for modelling Chinese stock index futures and spot markets' logarithm realised volatility. The proposed model is the HAR-J-MS model, which is discussed in detail in the above methodology sub-chapter. The models are estimated according to equation 4.4.2, and three different periods are applied. These periods include  $H = 1, 10, 20$ , which respectively refers to 1 day, 2 weeks and 1 month forward average logarithm realised volatility. The estimated results are represented below.

**Table 24. HAR-J-MS Model.**

<b>Spot Market</b>			
<b>Variable</b>	<b>1 Day</b>	<b>10 Day</b>	<b>30 Day</b>
$\beta_0$	-5.922273** [1.247080]	-8.632058** [0.994773]	-8.405514** [0.876978]
$\beta_d$	-0.193928** [0.058990]	-0.030644 [0.062907]	-0.102025 [0.065218]
$\beta_w$	0.807260** [0.122642]	-0.031872 [0.128172]	0.210058 [0.137757]
$\beta_M$	-0.128233 [0.122926]	0.309273* [0.138418]	0.163226 [0.143707]
$\gamma$	28240.55** [4358.034]	34166.22** [4656.396]	32073.33** [4647.254]
$\delta$	0.412124** [0.081267]	0.585050** [0.066178]	0.690794** [0.056343]
<b>AIC</b>	0.790872	0.944632	0.933121
<b>R-square</b>	0.619415	0.556898	0.574970
<b>Adj R-square</b>	0.616252	0.512356	0.565438

<b>Futures market</b>			
<b>Variable</b>	<b>1 Day</b>	<b>10 Day</b>	<b>30 Day</b>
$\beta_0$	-3.468536* [1.648649]	-10.64071** [1.694038]	-8.208792** [1.376832]

$\beta_d$	-0.275032** [0.059934]	-0.017546 [0.071204]	-0.049334 [0.072729]
$\beta_w$	1.162848** [0.136651]	0.152507 [0.161816]	0.044896 [0.170217]
$\beta_M$	-0.187674 [0.179383]	-0.066152 [0.213737]	0.292190 [0.203204]
$\gamma$	411172.2** [7527.684]	47022.96** [9545.672]	46829.75** [9551.133]
$\delta$	0.273315* [0.119800]	0.718850** [0.123876]	0.725208** [0.086231]
<b>AIC</b>	1.524549	1.856204	1.862899
<b>R-square</b>	0.536402	0.345723	0.369263
<b>Adj R-square</b>	0.516252	0.312356	0.365438

\* refer to significant at 5% level.

\*\* refer to significant at 1% level.

[ ] refers to standard error.

The above table shows 5 variables are significant at 1% confidence level for the 1 day forward forecast model of the spot market. These 5 variables are  $\beta_0$ ,  $\beta_d$ ,  $\beta_w$ ,  $\gamma$  and  $\delta$ . These statistical results indicate that the 1 day forward realised volatility is significantly influenced by short term investors, middle term investors, daily realised volatility jumps and regime period effects. However, the long term investor does not significantly influence 1 day forward realised volatility. Compared to the basic HAR model, the AIC R-squared and adjusted R-squared statistics are significantly improved by adding jumps and regime period components. This indicates that jumps and regime period will significantly influence the 1 day forward logarithm realised volatility estimation. Comparing 1 day forward to other frequency periods, the 1 day forward model provides the lowest AIC statistic, which indicates

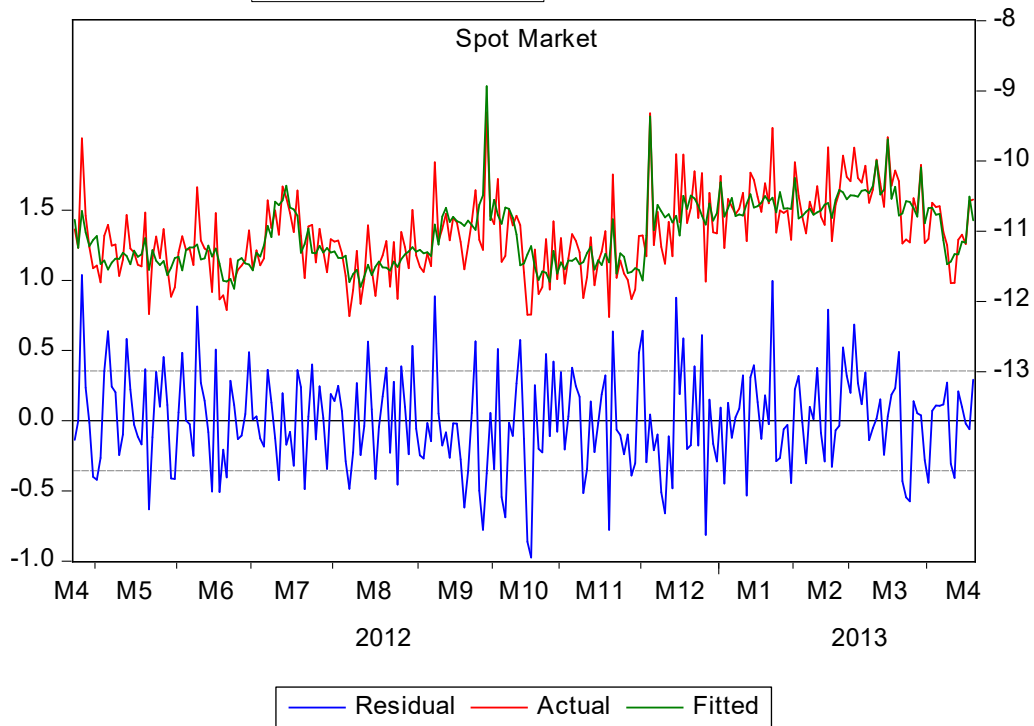
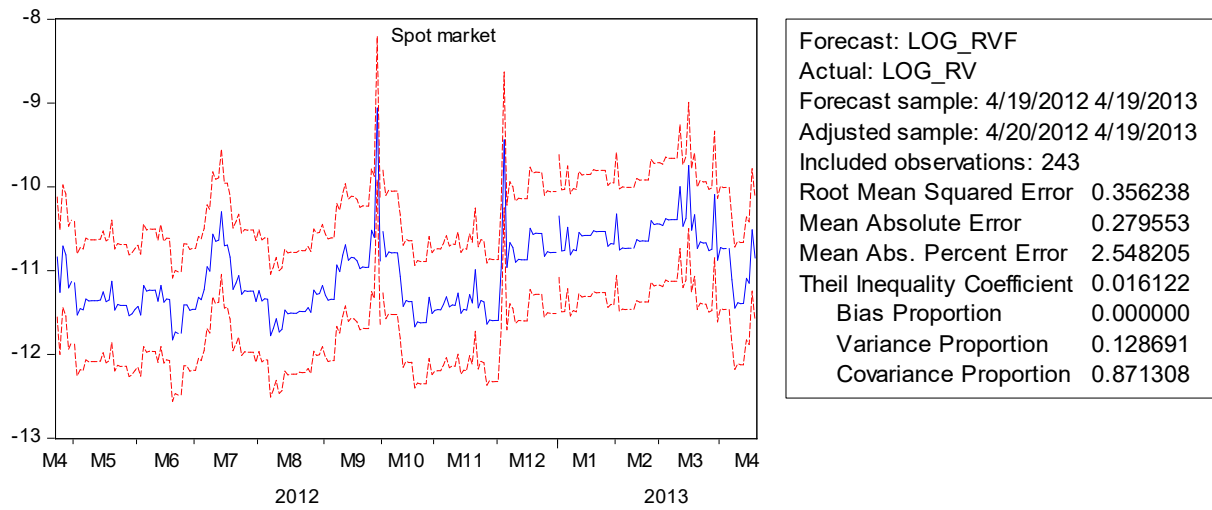
the 1 day forward model is the optimal model among these three models. In these three models, all jumps and regime dummy variables show highly significant results. This indicates jumps and regime period effects.

For the futures market, three variables ( $\beta_d$ ,  $\beta_w$  and  $\gamma$ ) are significant at 1% confidence level and two variables ( $\beta_0$  and  $\delta$ ) are significant at 5% level for the 1 day forward estimation model. This indicates that short term realised volatility is driven by short term and middle-term investors, and daily volatility jumps can significantly influence short term realised volatility. By adding jumps and regime dummy components, the 1 day forward model decreases its AIC statistic. Comparing the 1 day forward model to other period models, the 1 day forward model provides the lowest AIC statistic, which indicates it is the optimal estimation model for the futures market. All the jumps and regime dummy variables are highly significant in all three models, indicating that the HAR-J-MS model performs better than the original HAR model.

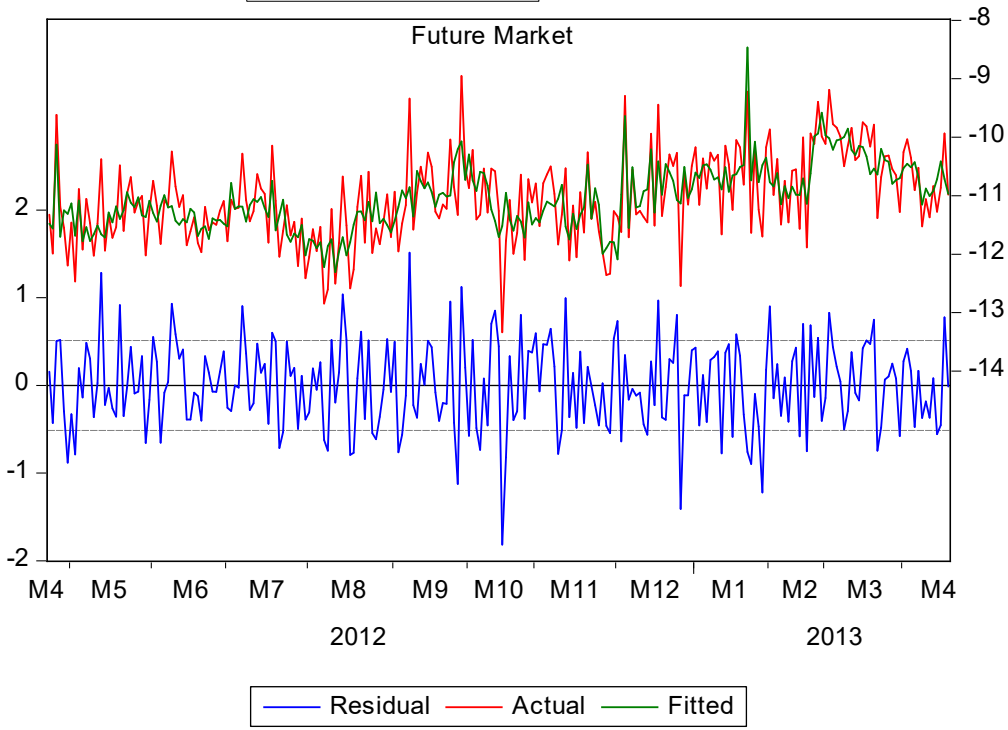
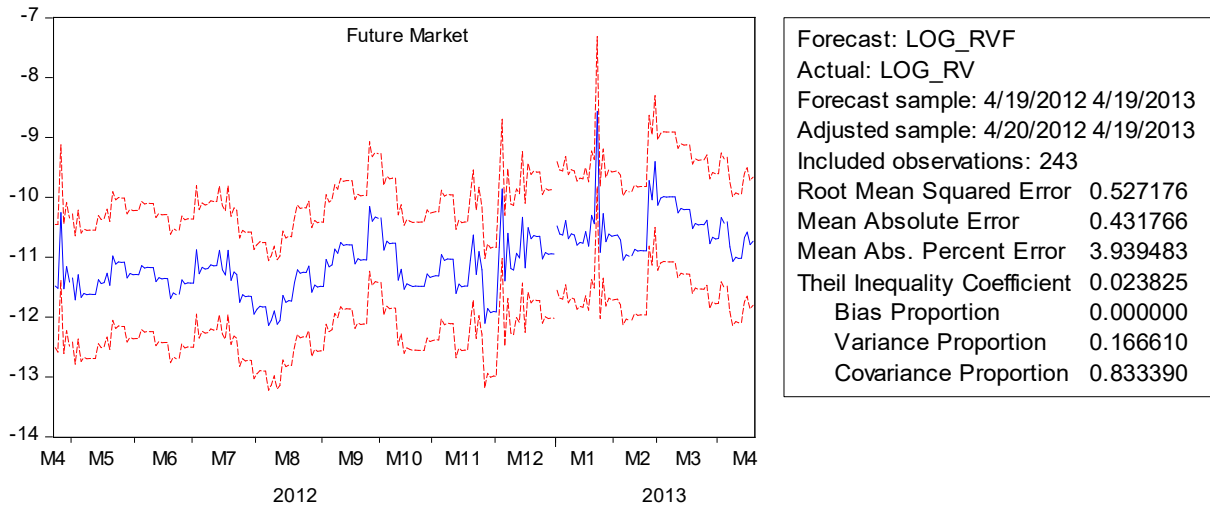
Overall, the HAR-J-MS model provides a better model estimation compared to the basic HAR model. Specifically, adding jumps and regime dummies in both markets leads to an AIC statistics decrease, and all the models indicate jumps and regime dummy variables are highly significant. Comparing the 1 day forward model to other models, the 1 day forward model outperforms other types of models in both futures and spot markets. Comparing futures to spot markets, the spot market is more suitable for applying the HAR-J-MS model, rather than the futures market. Especially, the R-squared value equals 61.94% for the 1 day forward model for the spot market. Hence, this chapter concludes with the following two points: first, the 1 day forward model performs better than other models. Second, the HAR-J-MS model performs better than the basic HAR model.

Consequently, in-sample forecast tests are conducted for both markets. The selected test models are the 1 day forward HAR-J-MS models for both markets, and the forecast results are graphed and can be found in the figure below.

**Figure 12. In-sample Forecast Results.**







The above figure indicates that HAR-J-MS models have strong forecasting power for logarithm realised volatility. Both markets have a low value on MSE, Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). Meanwhile, both markets have very low Theil Inequality Coefficient values, which are 0.01 for the spot market and 0.02 for the futures market. The main bias comes from covariance for both markets. From the residual plots, the estimation model's residual can fit the actual values well. This also proves the

HAR-J-MS is a good estimation model to forecast logarithm realised volatility for both markets.

Comparing the futures market to the spot market, the HAR-J-MS model shows better forecasting power for the spot market when it has lower values in MSE, MAE, MAPE and TIC statistics. Lastly, comparing the HAR-J-MS to the ARFIMA model, the ARFIMA models' average MAE statistics are 0.496075 for the spot market and 0.672041 for the futures market. The HAR-J-MS models show much lower MAE values for both markets, which are 0.279553 for spot the market and 0.431766 for the futures market. Hence, this chapter concludes the HAR-J-MS forecasting power performs better than the ARFIMA long memory model for Chinese stock index futures and spot markets' logarithm realised volatility.

#### **4.3.4. Discussion**

This sub-chapter attempts to discover the optimal logarithm realised volatility forecast model for Chinese stock index futures and spot markets. This chapter is based on chapters' two and three results. First, the optimal data is selected, and the final data is the 5 minutes high-frequency intraday data for both markets. Based on previous results, this sub-chapter points out that logarithm realised volatility is more suitable to set up a forecast model rather than the original realised volatility for two reasons: logarithm realised volatility follows near perfect normal distribution and has a stronger long memory feature.

Two realised volatility models are discussed in detail, including the ARFIMA long memory model and the HAR-J-MS models. The optimal ARFIMA model lag length for the spot market is ARFIMA (1, d, 2), and for the futures market is ARFIMA (1, d, 1). Furthermore,

in-sample forecast tests are conducted; both these models show acceptable forecasting power. Consequently, this chapter examines the newly proposed HAR-J-MS model. The basic HAR models are first estimated with 5 interval periods, which are 1 day forward, 1 week forward, 2 weeks forward, 3 weeks forward and 5 weeks forward. The results indicate that 1 day forward provides the optimal model estimation and forecasting power for both markets. The jumps and regime dummy variables are added into the basic HAR model. The empirical results show that both of these new variables are highly significant for both markets, and can increase the models' forecasting power. Meanwhile, 3 different interval periods are tested for the HAR-J-MS model, which are 1 day forward, 10 days forward and 30 days forward. The results show 1 day forward provides the best forecasting power. Comparing the HAR-J-MS model to the ARFIMA model, the HAR-J-MS model performs better than ARFIMA long memory by providing much lower MAE values for both markets.

#### **4.4. Chapter Summary**

This chapter discusses the realised volatility estimation and forecast model for Chinese stock index futures and spot markets. This chapter is based on chapter three's results: 5 minutes interval is the optimal frequency to estimate realised volatility; logarithm realised volatility has more desirable features compared to original realised volatility; daily volatility jumps will significantly influence realised volatility estimation; and realised volatility has strong long memory characteristics.

Based on the above results, this chapter examines two other important issues before setting up a realised volatility forecast model. These two issues are realised volatility transmission between futures and spot markets, and structure breakpoints existing in the model estimation

period. The empirical results indicate the following: first, realised volatility transmission does not exist between spot and futures markets. This means the two markets' realised volatility cannot influence the others' realised volatility value. Second, a strong structure change effect exists in logarithm realised volatility estimation for both markets.

Consequently, this chapter explores two important types of realised volatility estimation methods: ARFIMA long memory and HAR models. This thesis for the first time proposes the HAR-J-MS model to forecast logarithm realised volatility for Chinese stock index futures and spot markets. The results indicate that HAR-J-MS provides very good forecasting power compared to the ARFIMA long memory model.

## Chapter Five: Realised Volatility Implications

This chapter explores two important implications of realised volatility for Chinese stock index futures and spot markets. Chapter four discusses the optimal realised volatility forecast model. This includes the realised volatility transmission between two markets, the Markov regime-switching model, the ARFIMA long memory model and the newly proposed HAR-J-MS realised volatility forecast model.

Based on chapter four's results, this chapter will consider the following two important implications of the realised volatility forecast model. These two implications are: first, whether realised volatility has better performance on the optimal hedge ratio compared to conditional volatility estimation. Second, whether realised volatility has better performance on VaR. Three models will be included when evaluating VaR performance, including GARCH, APARCH with student distribution, and realised volatility.

This chapter explores these two important topics for the first time in relation to Chinese stock index futures and spot markets based on a realised volatility framework. This sub-chapter contributes to Chinese financial companies' hedge performance, which is the key function of the futures market. Meanwhile, this chapter may also contribute to Chinese financial companies' risk management performance.

## **5.1. Implication on Hedge Ratio**

In the previous sub-chapters, realised volatility's characteristics and forecast abilities have been discussed in detail. The results generally support the fact that realised volatility has more desirable features and higher forecasting power compared to other volatility estimations. This sub-chapter discusses one important implication of realised volatility on optimal hedge ratio. The optimal hedge ratio refers to variance minimum hedge ratio.

The key purpose of the futures market is to hedge the underlying market's risk. Currently, numerous banks, hedge funds and other financial institutions use stock index futures to hedge the stock market's overall risk. Hence, hedge is an important topic in modern finance. Numerous researchers have investigated this topic under the conditional volatility framework (Shen and Harris, 2003; Lien and Yang, 2006; Kenourgios et al., 2008; Lien, 2009), and other researchers have provided a variety of revised conditional volatility models to increase hedge performance (Myers and Moschini, 2002; Alizedeh and Nomikos, 2004; Nomikos et al., 2008; Salvador and Arago, 2013).

Among the increase in availability of intraday high-frequency data in recent years, numerous researchers have moved their eyes to realised volatility to estimate optimal hedge ratio (Sato and Kunitomo, 2009; Shen and Lai, 2009; Stoja et al., 2010; Lai, 2012). The majority of these papers found that realised volatility has better performance than conditional volatility on optimal hedge ratio estimation. The aim of this chapter is to investigate whether realised volatility can provide better performance than conditional volatility for Chinese stock index futures and spot markets. Three models will be examined, including the VECM, BEKK-GARCH model and realised volatility model.

This sub-chapter contributes to the current literature in the following two ways: first, to the best of the author's knowledge, this is the first time investigating the optimal hedge performance for Chinese stock index futures and spot markets based on a realised volatility framework. Previous studies focused on mature markets, and this chapter examines the emerging markets. Second, this chapter will compare the hedge performance between conditional volatility and realised volatility. The results may contribute to the further studies on Chinese stock index futures and spot markets' optimal hedge ratio.

### **5.1.1. Hedge by Using Futures Contract**

This sub-chapter introduces the fundamental theory of hedge before comparing hedge performance by using different volatility models. One of the important proposals for the futures market is to hedge the underlying market's risk. There are two types of hedges: short hedge and long hedge. Short hedge refers to selling assets at a future point in time and at a certain price. Long hedge refers to having no existing assets, but you are willing to buy them at a future point, at a certain price. The key idea of hedge is to lock in asset price and reduce the potential risk.

In real life, some people are willing to hedge their potential company risk, but others are against taking hedge active due to the following two reasons: first, the stockholders can hedge their own investment risk by diversification, and they do not wish the company to take hedge active to hedge for them. However, comparing the hedge cost between company and individual, the company has a much lower hedge cost. Hence, it will be cheaper to hedge at a company level. Second, some people argue that hedge may lead to a worse situation. For

example, if one company locks its production materials' price by using hedge, the company will benefit if the materials' price goes up; however, the company will suffer losses if the materials' price goes down.

Even though there are two main counter points against hedge activities, numerous financial institutions still use large amount of futures contracts to hedge the potential risk. Hedge activities are the main components of the futures market, and the key proposal of hedge is to reduce the potential risk. One key issue in hedge is the following: there are only limited futures contracts types trading in the futures market, and the underlying assets that need to be hedged may not be listed in the futures markets. Hence, the difference between futures contracts and underlying assets will lead to cross hedging. For example, if one airline company wants to hedge the aircraft fuel price, this company may use the heating oil futures contracts to hedge aircraft fuel price risk.

A hedge ratio compares the value of a position protected via hedge with the size of the entire position itself. If the asset needing to be hedged is the same as the futures contract listed in the futures market, then the hedge ratio should be 1. However, in the cross hedging situation, the hedge ratio 1 is not always optimal. The optimal number of hedging contracts is given as the following:

$$N = \frac{h * V_A}{V_F} = \frac{\rho * \sigma_S * V_A}{\sigma_F * V_F}$$

The above equation explains that the optimal number of hedging contracts depends on the coefficient of correlation between the changing futures and spot markets ( $\rho$ ), the volatility of the futures and spot markets ( $\sigma_F$  and  $\sigma_S$ ), the value of one futures contract ( $V_F$ ) and the value of the position being hedged ( $V_A$ ). The value of  $\rho$ ,  $V_F$  and  $V_A$  can be calculated using standard market information, but the key parts are the estimation methods of  $\sigma_F$  and  $\sigma_S$ .



One of direct implications of the optimal hedge ratio (minimise variance ratio) is this: if you are good at selecting stocks which perform better than the average market; if you have a stock or stock portfolio; if you are not sure how the market will perform in the next few months, but you are sure the selected stocks will outperform than average market, then you can sell the amount  $h \frac{V_A}{V_F}$  of stock index futures contracts as stated in the above equation for arbitrage purposes.

As an example, let's consider one investor who has 20,000 IBM stock in April, with each stock price equalling 100 dollars. The investor believes the market will become very volatile in next 3 months but that IBM stock will outperform the average market. The investor aims to use August S&P 500 stock index futures contracts to hedge the market risk. Assume IBM has  $\beta$  (same as  $h$  in above equation) equal to 1.1, the August S&P contract price is 900, and one futures contract equals 250 dollar times the stock index price, then  $V_A = 20,000 \times 100 = 2,000,000$  and  $V_F = 900 \times 250 = 225,000$ , and the optimal hedge contract numbers should equal:

$$1.1 \times \frac{2,000,000}{225,000} = 9.78$$

Given this, the investor should go short for 10 futures contracts and close the position in July. Now assume IBM stock price goes down to 90 dollars and the S&P 500 index goes down to 750. The investor's loss on IBM stock equals  $20,000 \times (100 - 90) = 200,000$  dollars, but the gain from the futures contracts equals  $10 \times 250 \times (900 - 750) = 375,000$  dollars. Therefore, the total revenue will be 175,000 dollar. This example shows how investors can use futures contracts to hedge the uncertain risk, and gain arbitrage profits from the market.

Overall, hedge plays an important role in the futures market. Numerous current financial institutions, such as banks, hedge funds and stock investment companies, apply futures contracts to hedge potential risk. The key issue in hedge is that most of time the hedge types belong to cross hedging due to the limit of available listed futures contracts. The cross hedging will lead to calculating the optimal hedge ratio issue. The key factor in optimal hedge ratio is the volatility of the two hedge assets. Hence, investigating and forecasting volatility are extremely important for numerous financial companies. Meanwhile, this chapter also shows one example on how to apply optimal hedge ratio to arbitrage from the market.

### **5.1.2. Literature Review**

In this sub-chapter, 11 papers in total on optimal hedge ratio are discussed in detail. One early paper discusses whether optimal hedge ratio is constant over time or not (Myers and Moschini, 2002). The empirical results suggest the optimal hedge ratio should be a dynamic process, which will change during different time periods. Three papers discuss the relationship between structure change and optimal hedge ratio (Alizedeh and Nomikos, 2004; Nomikos et al., 2008; Salvador and Arago, 2013). All these three papers suggest the optimal hedge ratio should depend on different structure periods.

Another three papers discuss the revise technique to more accurately estimate optimal hedge ratio under the conditional volatility framework (Shen and Harris, 2003; Lien and Yang, 2006; Kenourgios et al., 2008; Lien, 2009). Specifically, Shen and Harris (2003) proposed an alternative to the standard approach to optimal hedge ratio estimation that is robust to the leptokurtosis of returns. Lien (2009) pointed out that the conventional hedge ratio provides the best performance in large sample case.

Meanwhile, four papers relative to realised volatility to estimate optimal hedge ratio are discussed (Sato and Kunitomo, 2009; Shen and Lai, 2009; Stoja et al., 2010; Lai, 2012). Specifically, Sato and Kunitomo (2009) applied the Separating Information Maximum Likelihood Method to solve the estimation problem of the realised volatility and hedging coefficient. Sheu and Lai (2009) and Stoja et al. (2010) compared realised volatility estimation and conditional volatility estimation on optimal hedge ratio. Both of these two papers concluded that realised volatility outperforms conditional volatility in calculating optimal hedge ratio. Lastly, Lai (2012) discussed the influence of intraday data frequency on optimal hedge ratio under realised volatility framework. He concluded 10 minute fixed sampling seems to work best in hedging practice. The specific details of these eleven papers can also be found here:

Myers and Moschini (2002) developed a multivariate generalised ARCH parametricisation suitable for testing the hypothesis that the optimal futures hedge ratio is constant over time, given that the joint distribution of cash and futures prices is characterised by autoregressive conditional heteroskedasticity. The advantage of the new parametricisation is that it allows for a flexible form of time-varying volatility, even under the null of a constant hedge ratio. The model was estimated using weekly corn prices. Statistical tests rejected the null hypothesis of a constant hedge ratio and also rejected the null that time variation in optimal hedge ratios can be explained solely by deterministic seasonality and time to maturity effects.

Shen and Harris (2003) proposed an alternative to the standard approach to the estimation of the optimal hedge ratio that is robust to the leptokurtosis of returns. When using derivative instruments such as futures to hedge a portfolio of risky assets, the primary objective is to

estimate the optimal hedge ratio. When agents have mean-variance utility and the futures price follows a martingale, the optimal hedge ratio is equivalent to the minimum variance hedge ratio, which can be estimated by regressing the spot market return on the futures market return using ordinary least squares. To accommodate time-varying volatility in asset returns, estimators based on rolling windows, GARCH, or EWMA models are commonly employed. However, all of these approaches are based on the sample variance and covariance estimators of returns. In particular, when the distribution of the data is leptokurtic, as is commonly found for short horizon asset returns, these estimators will attach too much weight to extreme observations. Shen and Harris used the robust optimal hedge ratio to construct a dynamic hedging strategy for daily returns on the FTSE100 index using index futures. They estimated the robust optimal hedge ratio using both the rolling window approach and the EWMA approach, and compared their results to those based on the standard rolling window and EWMA estimators. It was shown that the robust optimal hedge ratio yields a hedged portfolio variance that is marginally lower than that based on the standard estimator. Moreover, the variance of the robust optimal hedge ratio was as much as 70% lower than the variance of the standard optimal hedge ratio, substantially reducing the transaction costs that are associated with dynamic hedging strategies.

Nomikos et al. (2008) estimated constant and dynamic hedge ratios in the New York Mercantile Exchange oil futures markets and examined their hedging performance. They also introduced a Markov Regime-switching VECM with a GARCH error structure. This specification linked the concept of disequilibrium with that of uncertainty across high and low volatility regimes. Overall, in and out-of-sample tests indicated that state dependent hedge ratios are able to provide significant reduction in portfolio risk.

Lien (2009) compared the hedging effectiveness of the conventional hedge ratio and time-varying conditional hedge ratios. It was shown that, in large sample cases, the conventional hedge ratio provides the best performance. For small sample cases, a sufficiently large variation in the conditional variance of the futures return is required to produce the opposite result. The result is due to the fact that the hedging effectiveness measure is based upon the unconditional variance; meanwhile, the conventional hedge ratio minimises the unconditional variance and the conditional hedge ratio aims at minimising the conditional variance.

Sato and Kunitomo (2009) used the Separating Information Maximum Likelihood Method to solve the estimation problem of the realised volatility and hedging coefficient, by using high-frequency data with possibly micromarket noise. By analysing the Nikkei-225 futures data, they found that the estimates of realised volatility and the hedging coefficients have significant bias when using the traditional historical method, which should be corrected. The SIML method can handle the bias problem in the estimation by removing the possible micromarket noise in multivariate high-frequency data. They showed that the SIML method has asymptotic robustness under non-Gaussian cases, even when the market noises are autocorrelated and endogenous with the efficient market price or the signal term.

Sheu and Lai (2009) proposed a new class of multivariate volatility models encompassing realised volatility estimates to estimate the risk-minimising hedge ratio, and compared the hedging performance of the proposed models with those generated by return-based models. In an out-of-sample context with a daily rebalancing approach, based on an extensive set of statistical and economic performance measures, the empirical results showed that improvement can be substantial when switching from daily to intraday. This comes from the advantage that the intraday-based RV can potentially provide more accurate daily covariance

matrix estimates than RV utilising daily prices. It also analysed the effect of hedge horizon on hedge ratio and hedging effectiveness for both the in-sample and the out-of-sample data.

Stoja et al. (2010) compared the estimated minimum variance hedge ratios from a range of conditional hedging models with the realised minimum variance hedge ratio constructed using intraday data. They showed that the reduction in conditionally hedged portfolio variance falls far short of the ex-post maximal reduction in variance obtained using the realised minimum variance hedge ratio. While this is partly due to systematic bias, correcting for this bias does little to improve hedging effectiveness. The poor performance of conditional hedging models is therefore more likely to be attributable to the unpredictability of the integrated hedge ratio.

Lai (2012) focused on determining the efficient sampling frequency for RV calculation in the context of a futures hedge. It was documented that realised variance sampled at ultra-high frequency is unreliable when observed prices are contaminated by market microstructure noise. Accordingly, in practice, it is common to choose moderate frequency ranges from 5 to 40 minutes, instead of at every tick, for balancing a bias/variance trade-off. Since the degree of microstructure noise varies across cash and futures markets, it was argued that finding the efficient frequency for the two assets should have important implications for effective hedging. Using NASDAQ 100 transaction prices, the performance with fixed and optimal time-varying sampling schemes were examined by both risk minimisation and utility maximisation criteria. The results showed that employing a 10-minute fixed sampling seems to work well in hedging practice.

### **5.1.3. Methodology**

This chapter will examine three different estimations to calculate optimal hedge ratio. The optimal hedge ratio refers to minimise variance ratio, and it is calculated as daily frequency. These three estimation methods include the Vector Error Correction (VECM) model, BEKK-GARCH model and ex-post realised volatility (RV) models.

The first model framework used to calculate optimal hedge ratio is the VECM model. A bivariate VECM can be written in the following form:

$$\begin{aligned}\Delta s_t &= \beta_{s,0} + \gamma_s ec_{t-1} + \sum_{j=1}^p \beta_{ss,j} \Delta s_{t-j} + \sum_{j=1}^q \beta_{sf,j} \Delta f_{t-j} + \varepsilon_{s,t} \\ \Delta f_t &= \beta_{f,0} + \gamma_f ec_{t-1} + \sum_{j=1}^p \beta_{fs,j} \Delta s_{t-j} + \sum_{j=1}^q \beta_{ff,j} \Delta f_{t-j} + \varepsilon_{f,t}\end{aligned}$$

where  $\Delta s_t$  refers to logarithm daily return for the spot market,  $\Delta f_t$  refers to logarithm daily return for the futures market.  $ec_{t-1} = f_t - a_0 - a_1 s_t$  is the estimated error correction term. The error correction coefficients  $\gamma_f$  and  $\gamma_s$  represent the speed of adjustment in response to deviations from the log-run equilibrium. Meanwhile, the short term predictive power of one variable for the other is captured by coefficients  $\beta_{sf,i}$  and  $\beta_{fs,i}$ . The coefficients measuring the reaction of spot and futures returns to their own lagged values ( $\beta_{ss,j}; \beta_{ff,j}$ ) indicate the degree of mean-reverting behaviour of both time series.

The optimal hedge ratio under the VECM framework can be calculated as:

$$h = \frac{Cov(\varepsilon_{s,t}, \varepsilon_{f,t})}{Var(\varepsilon_{f,t})}$$

There are two widely used dynamic conditional volatility models, which are the VECGARCH model and BEKK-GARCH model. A disadvantage of the Diagonal VECGARCH model is that there is no guarantee of a positive semi-definite covariance matrix. A variance-covariance matrix must always be positive semi-definite. This problem is addressed by the BEKK model (Engle and Kroner, 1995) which ensures the  $H_t$  matrix is always positive definite. Hence, this chapter only applies the BEKK-GARCH model to calculate optimal hedge ratio. The BEKK-GARCH model is based on a bivariate VAR (1) model as follows:

$$R_{i,t} = u_i + \varphi_i R_{i,t-1} + \varepsilon_{i,t}$$

where  $R_{i,t}$  is a [2x1] vector that refers to the spot and futures markets' return at the time  $t$ ;  $u_i$  is a [2x1] vector that represents the long term coefficients drift;  $\varepsilon_{i,t}$  is also a [2x1] vector that refers to the random error terms of these two markets at the time  $t$ . Here, the equation defines  $H_t$  as the [2x2] conditional variance-covariance matrix of the  $\varepsilon_{i,t}$ , and  $\varepsilon_{i,t}|\Psi_{t-1} \sim N(0, H_t)$  with  $\Psi_{t-1}$  represents the information set at time  $t - 1$ .

The standard BEKK-GARCH is:

$$H_t = C' C + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B$$

$H_t$  is a [2x2] matrix of conditional variance-covariance of  $\varepsilon_t$  at time  $t$ ;  $C$  is a [2x2] lower triangular matrix;  $A$  is a [2x2] diagonal matrix which represents the degree of  $H_t$  relative to the past error term in the mean equation;  $B$  is a [2x2] diagonal matrix that refers to the relationship between current conditional variance and the past conditional variance.

Specifically, the conditional variance -covariance matrix  $H_t$  can be alternatively represented as an expanded form as following:



$$\text{Diagonal BEKK: } H_t = M'M + \begin{bmatrix} A1(1,1) & 0 \\ A1(1,1) * A1(2,2) & A1(2,2) \end{bmatrix}' \varepsilon_{t-1} \varepsilon_{t-1}' \begin{bmatrix} A1(1,1) & 0 \\ A1(1,1) * A1(2,2) & A1(2,2) \end{bmatrix} +$$

$$\begin{bmatrix} B1(1,1) & 0 \\ B1(1,1) * B2(2,2) & B1(2,2) \end{bmatrix}' H_{t-1} \begin{bmatrix} B1(1,1) & 0 \\ B1(1,1) * B1(2,2) & B1(2,2) \end{bmatrix}$$

Then the optimal hedge ratio under the BEKK-GARCH framework can be calculated as:

$$h = \frac{B1(1,1) \times B2(2,2)}{[B1(1,1)]^2}$$

Realised volatility is an ex-post volatility estimate. According to quadratic variation theory, the realised volatility equals to integrated volatility when  $\Delta \rightarrow 0$ . That is:

$$\sum_{j=1}^{h/\Delta} (r_{t+j\Delta, \Delta} * r_{t+j\Delta, \Delta}^T) - \int_0^h \Omega_{t+\tau} d\tau \rightarrow 0$$

The realised volatility can be calculated as:

$$RV_t = \sum_{i=0}^{n_j} r_{t,i}^2$$

where  $j$  refers to total intraday frequency. Then the optimal hedge ratio under the RV framework can be written as:

$$h = \frac{Cov(RV_S, RV_F)}{Var(RV_F)}$$

#### **5.1.4. Empirical Results**

The data used to estimate the VECM model and BEKK-GARCH model is daily logarithm return, which estimates realised volatility using 5 minute intraday high-frequency data. In the previous chapter, intraday high-frequency data characteristics are examined, but the daily

logarithm return's basic statistics have not been explored. The basic statistic descriptions of daily logarithm return for both futures and spot markets can be represented below.

**Table 25. Daily Logarithm Returns' Data Description.**

	Mean	Std. Dev.	Maximum	Minimum
<b>Futures</b>	-0.000132	0.004489	0.019844	-0.012993
<b>Spot</b>	0.000403	0.004822	0.020314	-0.011780
	Skewness	Kurtosis	Jarque-Bera	$Q^2(12)$
<b>Future</b>	1.086320	5.984045	138.5197	33.016
<b>Spot</b>	6.878300	4.916245	68.70264	24.117
<b>Unit Root Test</b>				
	Futures	Spot		
<b>ADF Test</b>	-15.52047	-15.56618		
<b>PP Test</b>	-15.52974	-15.58320		

Notes: The Jarque-Bera test is to test the normality distribution of return.  
Ljung-Box statistics,  $Q^2(12)$  is to test the series autocorrelation up to 12 lags.  
The 1% critical value for ADF and PP test is 3.435.  
The confidence level to test null hypothesis is 1%.

This table shows that both daily logarithm return values have a mean close to 0, and standard deviation close to 0.004. Both series have the similar maximum and minimum value, with the spot market having slightly higher in both statistics. The Skewness and Kurtosis statistics indicate that both series have a positive Skewness and Leptokurtic distribution. The Jarque-Bera statistics demonstrate that both of these series do not follow a normal distribution. With the unit root test results, both ADF test and PP test results indicate that these two series are stationary processes. Three test models are estimated according to the methodology discussed in the above section. The estimated results can be represented below.

**Table 26. Estimated Results.**

<b>VECM Model Results (p=1, q=1)</b>					
$\beta_{s,0}$	$\gamma_s$	$\beta_{ss}$	$\beta_{sf}$	$\beta_{f,0}$	$\gamma_f$
0.000346	8.22E-06	0.099259	-0.130536	-0.000205	-2.66E-05
[0.000324]	[4.49E-05]	[0.170324]	[0.182440]	[0.000302]	[4.18E-05]
$\beta_{fs}$	$\beta_{ff}$	$Cov(\varepsilon_{s,t}, \varepsilon_{f,t})$	$Var(\varepsilon_{f,t})$	$h$	
0.142610	-0.146645	2.00E-05	2.01E-05	0.995025	
[0.158363]	[0.169639]				

<b>BEKK-GARCH</b>				
M(1,1)	M(1,2)	M(2,2)	A1(1,1)	A1(2,2)
1.64E-06	2.79E-06	4.00E-06	0.139852	0.126761
[3.71E-07]	[5.61E-07]	[7.79E-07]	[0.037058]	[0.076219]
B(1,1)	B(2,2)	B1(1,1)×B2(2,2)	B1(1,1)×B1(1,1)	$h$
0.956230	0.886084	0.8473	0.914376	0.926643
[0.010522]	[0.026457]			

<b>Realised Volatility</b>		
$Cov(RV_s, RV_F)$	$Var(RV_F)$	$h$
0.321604	0.347154	0.956402

Currently, there are three widely used methods to test hedge effect. These three methods are: Minimum Risk Measure Method, Sharpe Ratio Model Measure Method and Utility

Maximisation Measure Method. In this chapter, the Minimum Risk Measure Method will be applied.

Ederington (1979) believed the purpose of hedge is to reduce risk, and proposed the Minimum Risk Measure Method to examine hedge performance. The risk can be represented by the variance of hedged portfolio, compared to the variance of an un-hedged portfolio, the hedge efficiency equals the percentage change between the variance of the hedged portfolio and un-hedged portfolio. Let  $Var(U_t)$  represent the variance of the un-hedged portfolio and  $Var(H_t)$  refer to the hedged portfolio. The hedge efficiency can be measured as follows:

$$H_t = \frac{Var(U_t) - Var(H_t)}{Var(U_t)}$$

and

$$Var(U_t) = Var(\Delta S_t)$$

$$Var(H_t) = Var(\Delta S_t) + h^2 Var(\Delta F_t) - 2hCov(\Delta S_t, \Delta F_t)$$

The value of  $H_t$  is between -1 and 1. If  $H_t$  is close to 1, then it reflects a better hedge effect.

The calculated  $H_t$  values for the tested three models can be represented below.

**Table 27. Calculated Hedge Efficiency Value.**

	VECM	BEKK-GARCH	RV
$H_t$	0.913746	0.851631	0.929095

The above table shows that the best hedge performance model is the realised volatility estimation, the second is VECM and the worst is BEKK-GARCH. However, all of these three models provide very good hedge performance, even BEKK-GARCH provides an  $H_t$  over 0.85. The above results are consistent with the Shen and Lai (2009) and Stoja et al. (2010) conclusion. One key reason leading to this conclusion is that realised volatility is an

ex-post measurement of daily true volatility, which can generally perform better than conditional volatility to model and forecast markets' true volatility. Hence, realised volatility generally gives better output on the variance minimum hedge ratio. On the other hand, all of these three models provide such high hedge performance supporting the fact that Chinese stock index futures and spot markets are highly connected. That is, the correlation between the futures and spot markets is very high.

#### **5.1.5. Discussion**

This sub-chapter discusses the important implication of realised volatility on optimal hedge ratio. In the previous studies, the characteristics of realised volatility have been examined, and results indicated realised volatility can perform well in daily volatility modelling and forecasting.

This sub-chapter first introduces the basic theory of hedge, and then points out that hedging has an important position in the current finance arena. The definition of variance minimum optimal hedge ratio is given. Meanwhile, this chapter uses an example to explain how to apply a futures contract to hedge the market's overall risk, and earn low-risk profits from the financial market. A total of eleven papers are examined in the literature review sub-chapter, of these, there are seven papers examining optimal hedge ratio under a conditional volatility framework, and another four papers comparing the performance of conditional volatility and realised volatility. These four papers generally concluded that realised volatility can outperform conditional volatility.

Three econometrics models are applied here, including the VECM model, BEKK-GARCH model and realised volatility. As a follow-up to the previous chapter, this chapter starts with investigating daily return's basic characteristics. Consequently, the optimal hedge ratio for these three models is calculated. From the results, it shows that Chinese stock index futures and spot markets are highly connected. Then the hedge performance of these three models is examined by using a minimum risk measure method. The results indicate realised volatility estimation provides the highest efficiency, and BEKK-GARCH provides the lowest efficiency. Hence, based on above results, this chapter suggests using realised volatility estimation to calculate optimal hedge ratio for Chinese stock index futures and spot markets.

## **5.2. Implication on Value-at-Risk**

The previous sub-chapter examined the implication of realised volatility on optimal hedge ratio. Beside estimation of optimal hedge ratio, there is another important issue in the financial market area, which relates to risk control and management. Since the financial crash in 1987, numerous financial regulators now require financial organisations to enhance their risk management system. One most used risk management method is called Risk at Value (VaR), and this method is currently required for most banks and other financial companies.

The VaR method is used to answer for the maximum loss in an extremely bad market situation. This is the question that greatly concerns numerous risk managers. Due to the importance of the VaR measurement, numerous researchers have made contributions in this area (Giot and Laurent, 2004; Kuester et al., 2005; Angelidis and Degiannakis, 2008; Brownlees and Gallo, 2009; Louzis et al., 2011). Most of these papers have tried to improve

the VaR performance by using variety econometrics models. In the early stages, the most popular model to estimate VaR was the GARCH model. However, much empirical evidence shows returns have fat tails distribution (Giot and Laurent, 2004; Kuester et al., 2005). Hence the APARCH model based on student distribution has become more popular in later VaR estimation. Among the developments of realised volatility theory, more and more researchers believe realised volatility can generate better VaR performance than other models (Angelidis and Degiannakis, 2008; Aloui and Mabrouk, 2010 Louzis et al., 2011).

This sub-chapter aims to examine whether realised volatility can generate a better VaR performance for Chinese stock index futures and spot markets. The compared models are GARCH and APARCH with student distribution models. This sub-chapter contributes to current literature in the following ways: first, this is the first time in examining whether realised volatility can generate a better VaR estimation for Chinese stock index futures and spot markets. Applying realised volatility estimation on VaR is a timely question, and most of the existing papers examine mature markets, but this sub-chapter examines this advanced topic for an emerging market. Second, risk management plays an extremely important role in current financial research. The results of this sub-chapter may contribute to further research on risk management for Chinese financial companies.

### **5.2.1. Value-at-Risk**

The key concept of VaR can be written as the following:

*Within X% confidence level, the maximum loss of next N day will not exceed V*

The value of V in the above statement refers to VaR value for a portfolio. There are two parameters in VaR: Time horizon (N days) and confidence level (X %). In the above

statement, the probability of loss exceeds V equals (100-X) in next N days. Currently, the bank is required to calculate VaR with N=10 and X=99 by regulators.

The concept of VaR is easy to accept, and this concept has attracted much attention. When applying VaR, it actually asks what the worst situation will possibly be. This is an important question that is highly considered by numerous financial managers. In real practices, risk managers normally consider the N=1 situation first, and the N days VaR can be calculated as:

$$N \text{ days VaR} = 1 \text{ days VaR} \times \sqrt{N}$$

There are two widely used VaR calculation methods: the Historical Simulation Method and Model-building method. Assume the 501 days historical simulation method is applied to calculate 1 day ahead 99% VaR of one company. The historical simulation method directly uses historical data to forecast the possible different situations. The first step of the historical simulation method is to select the factors that can influence the selected portfolio; these factors normally include exchange rate, stock price and interest rate. The next step is to collect the recent 501 days' data. These data provide the 500 possible change situations. The first day's data can be marked as day 0, the second day's can be marked as day 1, and so on. Define scenario 1 as the percentage change between day 0 and day 1; scenario 2 equals the percentage change between day 1 and day 2, and so on.

For each scenario, the portfolio value change can be calculated from one day to the next. The probability distribution of 1 day ahead portfolio value change can also be calculated. The estimated 99% VaR value equals the loss at first percentile quantile of these 500 scenarios. Assume the past 500 days data has good forecasting power for the next 1 day ahead, and then



this company has 99% confidence to conclude the following: the maximum loss in this portfolio will not exceed the estimated VaR value.

In real life, one financial company's investment portfolio may contain thousands of contract positions, such as forward contracts, options, futures and other derivatives. In the VaR estimation, the financial company assumes the next trading day's investment portfolio will be the same as the previous day's portfolio. However, the actual change of portfolio will either increase (or decrease) portfolio risk. Under this situation, 10 days 99% VaR value will increase (or decrease) compared to previous days' results.

Before introducing the second method to calculate VaR, the model-building method, this sub-chapter discusses the relationship between yearly volatility and daily volatility. For option pricing, volatility is normally measured as yearly volatility. When estimating VaR, volatility is measured as daily volatility. Assume one year has 252 trading days, the relationship between yearly and daily volatility can be written as the following:

$$\sigma_{year} = \sigma_{day} \sqrt{252}$$

The above equation shows that daily volatility equals 16% of yearly volatility. The single asset situation will be first considered. Consider that a portfolio only contains one single Microsoft stock which is worth 10 million dollars. The selected parametric values to calculate VaR are: N=10 days and X=99%.

Assume Microsoft stock daily volatility equals 2% (that is 32% yearly), since the total value of stock equals 10 million dollars, then the standard derivation of the daily portfolio value change equals  $10 \text{ million} \times 2\% = 200,000$  dollars. The model-building method normally assumes the expectation stock price change equals zero and follows a normal distribution.

Since  $N(-2.33) = 0.01$ , the change of price goes down  $(2.33 \times \sigma_{day})$  value equals 1%. That is, under a 99% confidence level, the price will not go down more than  $(2.33 \times \sigma_{day})$  value. Hence the 1 day ahead 99% VaR for this situation equals:

$$2.33 \times 200000 = 466,000$$

Assume the daily stock price change is independent between different days. Then N days VaR equals 1 day ahead VaR times  $\sqrt{N}$ . Hence, 10 days 99% VaR should be:

$$466,000 \times \sqrt{10} = 1,473,621$$

Further consider one portfolio only contains 5 million AT&T stocks. Assuming the daily volatility change for this stock equals 1% (16% yearly), with the similar calculation method as above, the 1 day ahead standard derivation of portfolio value change equals:

$$5 \text{ million} \times 0.01 = 50,000$$

Assume the stock price follows a normal distribution, then 1 day ahead 99% VaR equals:

$$50,000 \times 2.33 = 116,500$$

10 days 99% VaR should equal:

$$116,500 \times \sqrt{10} = 368,405$$

Consider one portfolio that contains 10 million Microsoft stocks and 5 million AT&T stocks. Assume the Microsoft and AT&T stock prices follow a bivariate normal distribution, the distribution correlation equals 0.3. If  $\sigma_X$  and  $\sigma_Y$  refers to the standard derivation of variable X and Y, and the correlation between X and Y equals  $\rho$ , then the standard derivation of  $(X+Y)$  should equal:

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

Let X refer to the daily price change of Microsoft stock and Y refer to the daily price change of AT&T stock, then:

$$\sigma_X = 200,000 \text{ and } \sigma_Y = 50,000$$

The standard derivation of this portfolio should be:

$$\sqrt{200,000^2 + 50,000^2 + 2 \times 0.3 \times 200,000 \times 50,000} = 220,227$$

Assume the stock price change follows a normal distribution, and the mean value equals zero.

Then the 1 day ahead 99% VaR should be:

$$220,227 \times 2.33 = 513,129$$

10 days 99% VaR equals  $\sqrt{10}$  times 1 day ahead 99% VaR, which equals 1,622,657.

The above cases show one important feature of risk diversification:

- 1) The 99% level Value at Risk for a single Microsoft stock over a 10 day period equals to 1,437,621.
- 2) The 99% level Value at Risk for single AT&T stock over a 10 day period equals to 368,405.
- 3) The 99% level Value at Risk for portfolio, which contains both Microsoft stock and AT&T stocks over a 10 day period, equals to 1,622,657. That is:

$$(1,437,621 + 368,405) - 1,622,657 = 183,369$$

which demonstrates the benefits from risk diversification. If the correlation between Microsoft stock and AT&T equals 1, then the VaR value of combination Microsoft and AT&T should equal the single Microsoft VaR plus AT&T VaR values. If the correlation is smaller than 1, then some part of risk will be diversified away.

### **5.2.2. Literature Review**

In this sub-chapter, a total of nine papers are investigated in detail on the VaR topic. All of these nine papers discuss the same issue, which is to select the optimal estimation and forecast model for VaR. Specifically, Giot and Laurent (2004) compared ARCH-type models and realised volatility, and they found no significant difference between these two models on the calculation of VaR estimation. Angelidis and Degiannakis (2008) suggested using different models for bank index and general index at the Athens Stock Exchange.

Clement al. (2008) suggested that the Heterogeneous Autoregressive Model provides the most accurate VaR estimation. Hung et al. (2008) suggested the GARCH-t model has good accuracy compared to other models. Brownlees and Gallo (2009) investigated different realised volatility estimation models to calculate VaR, and found the realised kernels model provides the best performance.

Chang (2011) discussed the Markov regime-switching in VaR estimation, and pointed out that considering the regime-switching effect will boost VaR accuracy.

Aloui and Mabrouk (2010) evaluated the VaR and the expected shortfalls for some major crude oil and gas commodities for both short and long trading positions. Classical VaR estimations and other extensions with regards to long memory, asymmetry and fat tail in energy markets volatility were conducted. They computed the VaR for three ARCH/GARCH-type models including FIGARCH, FIAPARCH and HYGARCH. These models were estimated in the presence of three alternative innovation's distributions: normal, student and skewed student. Their results showed that accounting for long memory, fat tails and asymmetry performs better in predicting a 1-day-ahead VaR for both short and long trading positions. Moreover, the FIAPARCH model outperformed the other models in the

VaR's prediction. These results presented several potential implications for energy markets risk quantifications and hedging strategies.

### **5.2.3. Methodology**

In this sub-chapter, three models are used to test the VaR performance for Chinese stock index futures and spot markets. These three models are the GARCH, APARCH with student distribution, and realised volatility estimation models.

The GARCH model can be written as:

$$r_t = \mu_t + \varepsilon_t, \varepsilon_t = h_t^{1/2} e_t, e_t \sim t(v),$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

$r_t$  refers to the daily return at day  $t$ ,  $\mu_t$  represents the mean value of daily return conditional on time  $t - 1$ ,  $\varepsilon_t$  is the error term for return,  $h_t$  is the variance of return conditional on time  $t - 1$ . Assume error term follows a  $t$  distribution at degree of freedom  $v$ . Then the  $t$ -days VaR can be written as:

$$VaR_q^t = h_t^{1/2} e_q$$

where  $h_t^{1/2}$  is the forecasted conditional volatility at day  $t$ , and  $e_q$  refers to the  $q$  quantile at distribution of  $\varepsilon_t$ , and it is independent on time  $t$ .

The normal APARCH (Ding, Granger and Engle, 1993) is an extension of the GARCH model. It is a very flexible ARCH-type model as it nests at least seven GARCH specifications.

The APARCH (1, 1) can be written as:

$$\sigma_t^\delta = \omega + \alpha_1 (|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta$$

where  $\omega$ ,  $\alpha_1$ ,  $\gamma_1$ ,  $\beta_1$  and  $\delta$  are the parameters to be estimated. Pervious numerous empirical results indicate returns have fat tail characteristics. To solve this problem, the student APARCH (ST APARCH) is introduced as the following:

$$\varepsilon_t = \sigma_t Z_t$$

where  $z_t$  is *i.i.d.* with  $t(0, 1, U)$ .

The theory of realised volatility is based on price decomposition theory and quadratic variation theory. Assume  $p_{t,i}$  refers to logarithm stock price at day  $t$  and time  $i$ ,  $i = 1, 2, 3 \dots n_t$ , where  $n_t$  refers to total number of intraday equally intervals. Let  $r_{t,i} = p_{t,i} - p_{t,i-1}$ , which refers to the logarithm intraday return. Then the realised volatility can be estimated by using:

$$RV_t = \sum_{i=1}^{n_t} r_{t,i}^2$$

The VaR model based on realised volatility can be written as:

$$\varepsilon_t = r_t / RV_t$$

$$\varepsilon_t = c + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3} + Z_t$$

$$Z_t \sim N(\mu_t, \sigma_t^2)$$

where  $\varepsilon_t$  is the ratio of daily return at  $t$  and the daily realised volatility. In order to eliminate the autocorrelation in the  $\varepsilon_t$  series, this sub-chapter apply an AR (3) regression. The residuals  $Z_t$  in the AR (3) model follow a normal distribution which mean value equals  $\mu_t$ , standard deviation equals  $\sigma_t$ .

Based on the  $t - 1$  day's information, the  $t$  day VaR can be represented as:

$$VaR_q^t = RV_t Z_q$$

$RV_t$  refers to the forecast value of daily realised volatility at day  $t$ .  $Z_q$  refers to the  $q$  quantile at distribution of  $Z_t$ , and it is independent on time  $t$ . The realised volatility forecast model is the new HAR-J-MS model:

$$RV_t^d = \beta_0 + \beta_d RV_{t-1}^d + \beta_w RV_{t-1}^w + \beta_M RV_{t-1}^M + \gamma J + \delta D_f + \varepsilon_{t+H} \text{ (future)}$$

$$RV_t^d = \beta_0 + \beta_d RV_{t-1}^d + \beta_w RV_{t-1}^w + \beta_M RV_{t-1}^M + \gamma J + \delta D_s + \varepsilon_{t+H} \text{ (spot)}$$

where  $RV_t^d$  refers to daily realised volatility;  $RV_t^w$  represents weekly realised volatility, and  $RV_t^M$  is the monthly realised volatility.  $\gamma J$  refers to daily volatility jumps influences and two dummy variables are defined as the following:

$$D_f = 1, \text{ if state period} = 1 \text{ (future market)}$$

$$= 0, \text{ other wise}$$

and

$$D_s = 1, \text{ if state period} = 1 \text{ (spot market)}$$

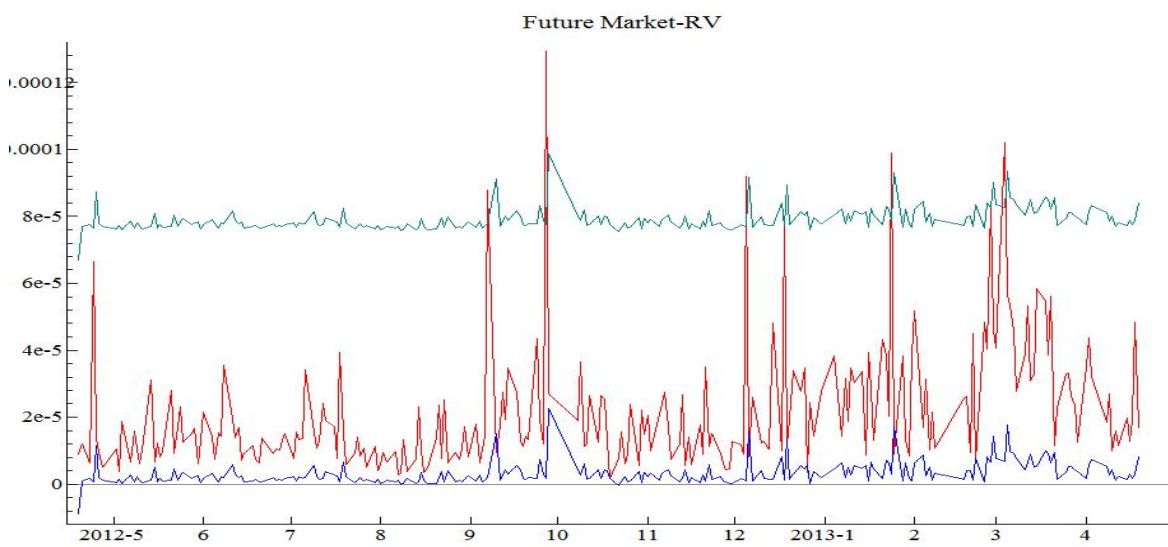
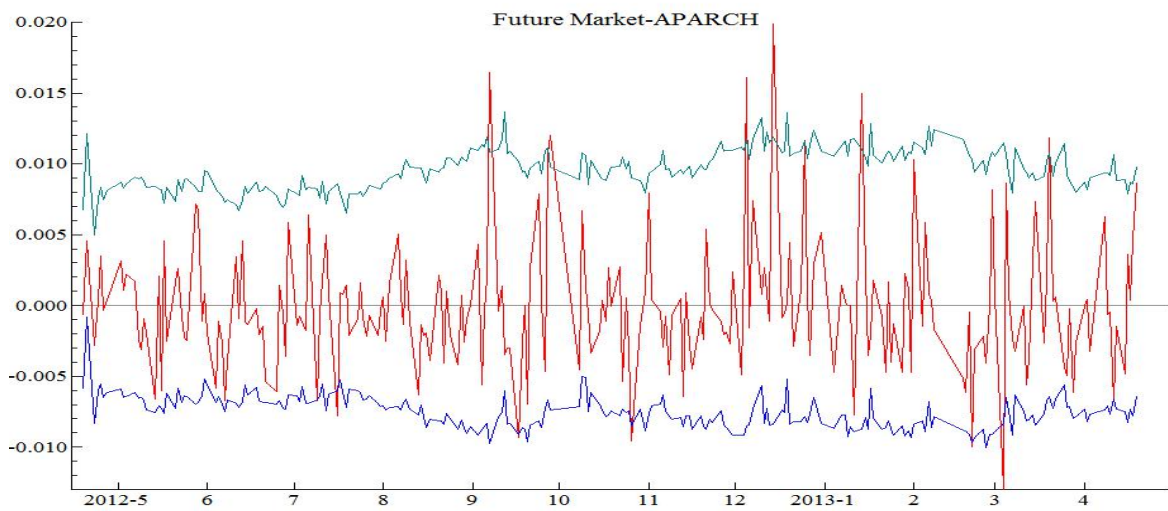
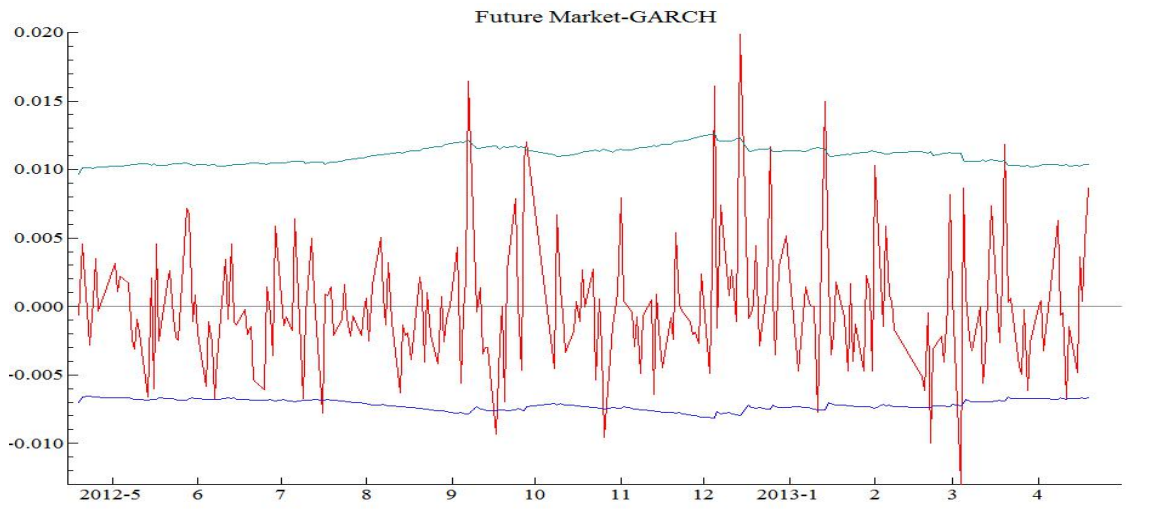
$$= 0, \text{ other wise}$$

#### **5.2.4. Empirical Results**

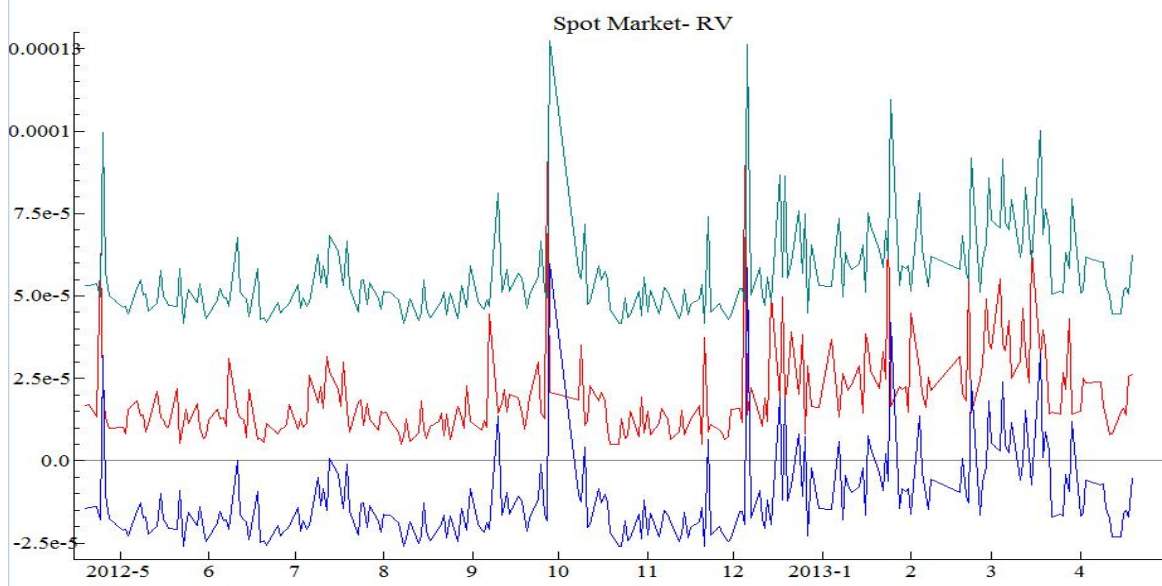
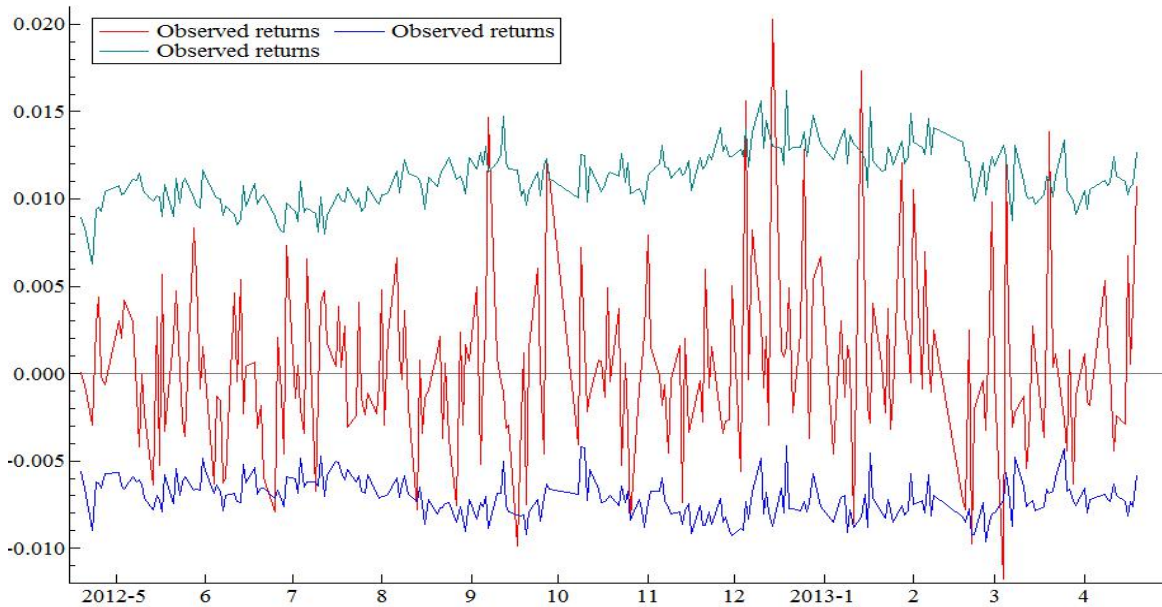
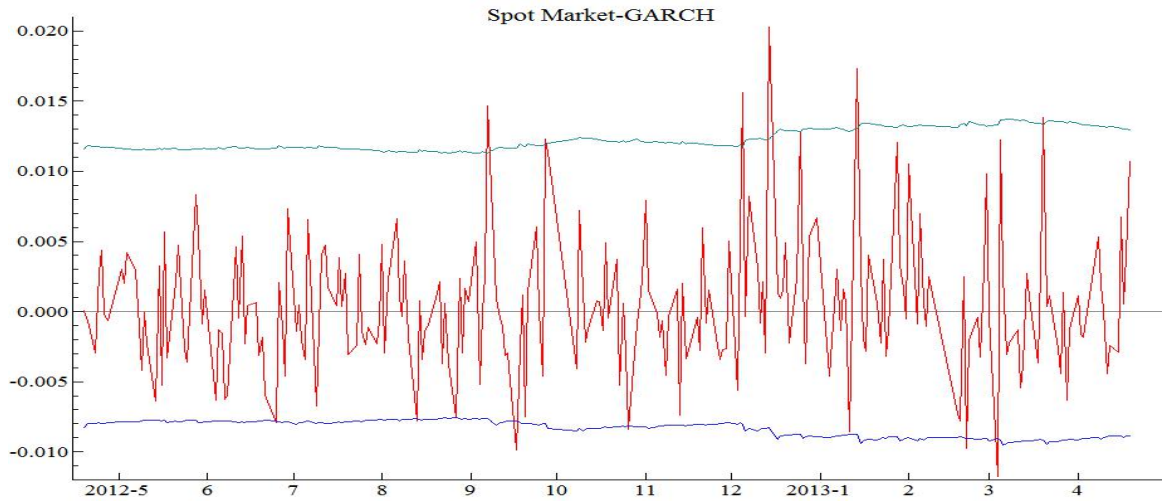
The daily VaR estimation can be divided into long side and short side. The long side of the daily VaR is defined as the VaR level for traders having long positions, and this is the traditional VaR where traders incur losses when negative returns are observed. Correspondingly, the short side of the daily VaR is the VaR level for traders having short positions, where traders incur losses when stock prices increase. This sub-chapter calculates long position and short position VaR separately.

Three models will be applied in this empirical sub-chapter, and these models are discussed in detail in the methodology sub-chapter. The estimated results can be graphed in the figure below.

**Figure 13. VaR Graph Results.**







In the above figure, the upper line refers to the long position VaR value, the middle line represents the estimated values of conditional volatility or realised volatility, and the lower line is the short position VaR value. The expected situation will be the middle line will not move above (down) than the upper line (lower line). From the above figures, it can generally be known that the GARCH model performs worst and realised volatility performs best.

A statistic test proposed by Kupiec (1995) can be used to compare the estimation accuracy of these three VaRs.

The following hypothesis can be tested:

$$H_0: f = a$$

$$H_1: f \neq a$$

where  $f$  refers to failure rate, and  $a$  refers to VaR level. At 5% confidence level, a confidence interval for  $f$  is given by:

$$\left[ f - 1.96 \sqrt{\frac{f(1-f)}{T}}, f + 1.96 \sqrt{\frac{f(1-f)}{T}} \right]$$

The above test is the Kupiec LR test, and the hypothesis is tested using a Likelihood Ratio test. The LR statistic is given as:

$$LR = -2 \log \left( \frac{a^N (1-a)^{T-N}}{f^N (1-f)^{T-N}} \right)$$

where  $N$  is the numeral of VaR violations,  $T$  is the total number of observations and  $a$  is the theoretical failure rate. The LR test statistic is asymptotically distributed as a  $\chi^2(1)$ . The empirical Kupiec LR test results for these three estimation models can be represented below.

For the futures market, the realised volatility provides the best VaR estimation. With 95% confidence level, the number of success cases approximately equals  $244 \times 0.95 \approx 232$ , and

only 12 cases locate outside the confidence level. Meanwhile, the p-value suggests it cannot reject null hypothesis, which indicates this VaR estimation can satisfy the VaR requirement. At 99% confidence level, only  $244 \times 0.025 \approx 6$  cases locate outside the confidence interval, and the p-value also cannot reject null hypothesis, which indicates the realised volatility satisfies the VaR requirement at 99% level. Consequently, the short positions' VaR are all located in the confidence interval, which indicates a superior VaR performance.

Realised volatility outperforms the two models examined for both long and short positions. Nevertheless, the APARCH model provides the best VaR performance among the two alternative GARCH models examined. The performance of the APARCH and GARCH models can be accepted by 95% VaR level. However, the null hypotheses for long position are rejected at higher (99% VaR) levels, while not rejected when the position becomes short. These statistics suggest the performance of APARCH and GARCH cannot be accepted with 99% VaR for long position, but can be accepted for short position. I find that realized volatility is able to provide the right measure of VaR for the futures (spot) market in 232 (230) out of 244 cases at 95% level, and in 238 (238) out of 244 cases at 99% level. In terms of economic significance of measuring the VaR, realized volatility has a better performance in futures markets than in spot markets. This results keep consistence with current major results (Angelidis and Degiannakis, 2008; Aloui and Mabrouk, 2010 Louzis et al., 2011).

**Table 28. Kupiec LR Test Results.**

Long Position				Short Position			
Futures market-GARCH							
Quantile	Success rate	Kupiec LRT	P-value	Quantile	Failure rate	Kupiec LRT	P-value
0.95	0.92623	2.5480	0.11043	0.05	0.032787	1.7237	0.18921
0.99	0.96721	8.0081	0.00465	0.01	0.00409	1.1046	0.29327
Futures market-APARCH							
0.95	0.93852	0.63239	0.42648	0.05	0.01639	7.7656	0.0053
0.99	0.96721	8.0081	0.0046	0.01	0.00409	1.1046	0.29327
Futures market-RV							
0.95	0.95082	0.0034	0.95303	0.05	0.00	N/A	0.00
0.99	0.97541	3.7299	0.05344	0.01	0.00	N/A	0.00
Long Position				Short Position			
Spot Market-GARCH							
Quantile	Success rate	Kupiec LRT	P-value	Quantile	Failure rate	Kupiec LRT	P-value
0.95	0.93852	0.63239	0.42648	0.05	0.03278	1.7237	0.18921
0.99	0.96311	10.554	0.0011	0.01	0.00409	1.1046	0.2932
Spot Market-APARCH							
0.95	0.94262	0.26741	0.60507	0.05	0.01229	10.343	0.00129
0.99	0.97131	5.7214	0.01675	0.01	0.00	N/A	0.00
Spot Market-RV							
0.95	0.94262	0.26741	0.60507	0.05	0.00	N/A	0.00
0.99	0.97541	3.7299	0.05344	0.01	0.00	N/A	0.00

For the futures market, the realised volatility provides the best VaR estimation. With 95% confidence level, the number of success cases approximately equals  $244 \times 0.94 \approx 230$ , and only 14 cases locate outside the confidence level. Meanwhile, the p-value suggests it cannot reject null hypothesis, which indicates this VaR estimation can satisfy the requirement at 95% confidence level. At 99% confidence level, only  $244 \times 0.025 \approx 6$  cases locate outside the confidence interval, and the p-value also cannot reject null hypothesis, which indicates the realised volatility satisfy VaR requirement at 99% level. Consequently, the short positions' VaR are all located in the confidence interval, which indicates a superior short position VaR performance for the spot market.

Comparing the realised volatility estimation to the other two models, these provide lower satisfaction statistics for both long and short positions. However, comparing the APARCH model with the GARCH model, the APARCH model provides slightly better VaR performance, the same as the futures market. Meanwhile, with 95% confidence level, the null hypothesis cannot be rejected for both markets. This indicates the performance of APARCH and GARCH can be accepted at 95% VaR level for the spot market. However, the statistical results reject null hypotheses for long position with 99% VaR, but accept short position. This statistic suggests the performance of APARCH and GARCH cannot be accepted with 99% VaR for long position, but can be accepted for short position.

Overall, the realised volatility provides the best VaR estimation for both markets. The APARCH and GARCH provide similar lower performance. Comparing VaR in long position and short position, short position VaR outperforms long position VaR in all cases. The futures market shows better VaR estimation for all three models than the spot market. This

results keep consistence with current major results (Angelidis and Degiannakis, 2008; Aloui and Mabrouk, 2010 Louzis et al., 2011).

### **5.2.5. Discussion**

This sub-chapter discusses whether realised volatility can generate a better VaR performance for Chinese stock index futures and spot market. This chapter begins with the introduction of the VaR theoretical background, and discusses the importance of risk management in the current financial arena. Consequently, a total of nine recent papers are examined. All of these nine papers discuss the optimal estimation methods to generate VaR value. From these papers, the estimation methods can be divided into three stages. The first stage is to use the GARCH model, the second stage is to use the revise GARCH model (e.g. APARCH with student distribution), and recent studies use realised volatility estimation.

In the methodology sub-chapter, three test models are discussed in detail (GARCH, APARCH and RV), and one VaR performance evaluation method proposed by Kupiec (1995) is also explored. The empirical results suggest that realised volatility can provide better VaR estimation compared to other two models for Chinese stock index futures and spot markets. This results is consistent with current major results (Angelidis and Degiannakis, 2008; Aloui and Mabrouk, 2010 Louzis et al., 2011). Meanwhile, the results also indicate that the estimated VaR values based on realised volatility can be accepted at 95% and 99% level for both markets.

### **5.3. Chapter Summary**

This chapter discusses the two important implications of realised volatility. These two implications are optimal hedge ratio and VaR performance. The optimal hedge ratio is the key factor in hedge performance when using futures contracts. This chapter compares optimal hedge ratio by using realised volatility and conditional volatility. Three econometrics models are applied here, including the VECM model, BEKK-GARCH model and realised volatility. The results show that Chinese stock index futures and spot markets are highly connected. The hedge performance of these three models is examined by using the minimum risk measure method. The results indicate realised volatility estimation provides the highest efficiency, and BEKK-GARCH provides the lowest efficiency.

Another important implication of realised volatility is in estimating VaR value. VaR is currently most widely used risk management method in the banking and other financial sectors. Three econometric models (GARCH, APARCH and RV) are compared, and one VaR performance evaluation method proposed by Kupiec (1995) is also explored. The empirical results suggest that realised volatility can provide better VaR estimation compared to the other two models.

Overall, realised volatility provides better estimation values for optimal hedge ratio and VaR. The results suggest that realised volatility is a more accurate estimator for daily real volatility for Chinese stock index futures and spot markets.

## Chapter Six: Concluding Remarks

### 6.1. Key Takeaways

Realised volatility is a recent measure, and has attracted much research attention. Numerous empirical results indicate that realised volatility has more desirable characteristics and more estimation accuracy than other volatility models. However, the majority of existing studies focus on mature markets, and few papers apply realised volatility on emerging markets. On the other hand, the Chinese stock index futures market is also a newly introduced financial derivatives market, established in April 2010. Hence, there is almost no literature discussing realised volatility for Chinese stock index futures and spot markets. This thesis comprehensively studies the realised volatility in Chinese financial markets.

The study had three main aims: first, to explore realised volatility's characteristics in Chinese stock index futures and spot markets. Second, to investigate the optimal realised volatility forecast model for both markets. Third, to examine the implications of realised volatility on the optimal hedge ratio and VaR performance of both markets.

The empirical results revealed four important realised volatility characteristics of the examined markets. First, 5-minute intervals were found to be the optimal data frequency based on two reasons: Five-minute intervals provide the lowest average intraday covariance, and the autocorrelation for 5 minute intervals is extremely low. Second, daily volatility jumps effect does not exist in realised volatility estimation. On average, a significant jump will occur every 10 trading days for both markets, and the spot market shows a slightly higher percentage of jumps than the futures market, while Evans (2011) find a significant jump every 4-10 trading days on average. Third, there are significant intraday volatility jumps and periodicity effects, and intraday volatility jumps are highly linked to macroeconomic news



release. Fourth, both original and logarithm realised volatility show strong long memory characteristics. These results are consistent with Yang et al. (2012), Duyvesteyn et al. (2011) and Evan (2011).

The second aim was to explore the optimal realised volatility forecast model. Two important issues had been examined before modelling realised volatility. The empirical results indicate no realised volatility transmission between futures and spot markets. This conclusion differs significantly from Yang et al. (2012), who concluded that spot market volatility leads to volatility in the futures market, and new information flows from the spot to the futures market. There are a number of reasons for the different results. First, the period under investigation is different. Specifically, the data period from Yang et al. (2012) is between 16/04/2010 and 30/07/2010, when Chinese stock index futures contracts were initially introduced. Because the market was newly established, it did not have as many investors compared to the mature spot market, so the spot market will generally respond to new information faster. However, our study data covers the period between 19/04/2012 and 19/04/2013, where the futures market had an increasing number of investors and two years of development. The increase in investors and volumes improved efficiency in the futures market.

There had been regime change effects in realised volatility estimation periods. Based on chapters' three and four results, this thesis for the first time proposes a HAR-J-MS model, which combines the daily volatility jumps components and regime-switching effects. The empirical results indicate superior forecasting power of this new proposed model for both markets.

The last aim of this thesis was to investigate two important implications of realised volatility, namely optimal hedge ratio and VaR performance. The main models were based on the conditional volatility estimation method. The empirical results suggest realised volatility performs better on optimal hedge ratio and VaR compared to other models. Hence, this thesis

suggests realised volatility is more accurate in estimating daily real volatility compared to conditional volatility methods. The above results are partly consistent with, Shen and Lai (2009) and Stoja et al. (2010) conclusion. For instance, it is found that BEKK-GARCH provides an  $H_t$  over 0.85, while Stoja et al. (2010) find that the average reduction across the five models is 60.3 percent, 27.3 percent and 29.3 percent, respectively, for GBP-EUR, GBP-JPY for EUR-JPY. Despite the above mentioned findings and contribution, also some limitations apply in our study findings.

## **6.2. Limitation**

This thesis bears one limitation, which stems from the measure of realised volatility. Realised volatility estimation is based on the assumption of a continuous diffusion process, which is not exactly the same as real asset price behaviour. Hence, there is still bias in realised volatility estimation. Although realised volatility is a more accurate estimator of daily integrated volatility compared to previous models, it may still have some problems if realised volatility is directly treated as an observable variable, and directly used as the benchmark for other volatility models. So far, there is no widely accepted way to solve it and all the studies on this topic suffer from the same problem.

## **6.3. Further Research Recommendations**

Some researchers have developed realised volatility measures in recent years, and have used it to solve empirical or theoretical problems. This thesis falls into this category. However, there are still numerous unsolved problems that need further investigation:

1. The first and foremost important futures research direction is to solve the limitation mentioned above, which may shape the future of this area.
  
2. This thesis mainly focuses on a one dimension situation and does not consider the multiple dimension one. Sometimes, many empirical finances are based on multiple dimensions. For example, Andersen et al. (2001) applied multiple dimension vectors to model and forecast the German Mark/American Dollar, Japanese Yen/American Dollar two exchange rate series. Hence, how to expand the realised volatility estimation method from one dimension to multiple dimensions may be a further research area for this topic.
  
3. The HAR model only considers the differences among different types of investors. The investors are divided into short term, middle term and long term investors. However, this classification can be further divided. Different investors have different purposes in financial markets. For example, some investors are purely speculators, and some others are purely hedgers. By considering this difference, the HAR model can be further developed, making forecasting more accurate.
  
4. The HAR model does not consider the asymmetric effect of volatility. That is, numerous empirical results show that negative news will generate higher reactions from investors compared to positive news. By adding this asymmetric effect, it may further increase the HAR model's forecasting power.

#### **6.4. Methodological Implications**

Overall, this thesis provides evidence in favour of the wider use of realised volatility measure for risk management in the areas of optimal hedge ratio and VaR performance. The optimal hedge ratio is the key factor in hedge performance when using futures contracts. This thesis compares optimal hedge ratio using realised volatility and conditional volatility and three econometrics models (VECM model, BEKK-GARCH model and realised volatility). The results indicate that realised volatility estimation provides the highest efficiency, and BEKK-GARCH provides the lowest efficiency. Finally, we used realised volatility to estimate VaR values. VaR is currently the most widely used risk management method in the banking and other financial sectors. The empirical results suggest that realised volatility can provide better VaR estimation than the GARCH and APARCH models.

## References:

- Aït-Sahalia, Y., and Mancini, L. 2008. *Out of Sample Forecasts of Quadratic Variation*. Journal of Econometrics, 147(1), 17-33.
- Ait-Sahalia, Y., Mykland, P. A. and Zhang. L., 2005, *How Often to Sample a Continuous-time Process in the Presence of Market Microstructure Noise*, *Review of Financial Studies*, 18, 351-416.
- Ait-Sahalia, Y., Mykland, P. A. and Zhang. L., 2011, *Ultra High Frequency Volatility Estimation with Dependent Microstructure Noise*, *Journal of Econometrics*, 160, 160-170.
- Alan, N. S., A. K. Karagozoglu, and Korkmaz, S., 2016. *Growing Pains: The Evolution of New Stock Index Futures in Emerging Markets*, *Research in International Business and Finance* 37, 1-16.
- Alizedeh, A. and Nomikos, N., 2004, *A Markov Regime-switching Approach for Hedging Stock Indices*, *Journal of Futures markets*, 24, 679-674.
- Aloui, C. and Mabrouk, S., 2010, *Value at Risk Estimations of Energy Commodities Via Long Memory, Asymmetry and Fat-tailed GARCH Models*, *Energy Policy*, 38, 2326-2339.
- Andersen, T. and Bollerslev, T., 1997, *Intraday Periodicity and Volatility Persistence in Financial Markets*, *Journal of Empirical Finance*, 4, 115-158.
- Andersen, T. and Bollerslev, T., 1998, *Answering the Skeptics: Standard Volatility Models do Provide Accurate Forecasts*, *International Economic Review*, 39, 885-905.
- Andersen, T. G., Bollerslev, T., Diebold, F. X. and Labys, P., 2000, *Exchange Rate Returns Standardized by Realised Volatility Are Gaussian*, *Multinational Finance Journal*, 4, 159-179.
- Andersen, T. G., Bollerslev, T., Diebold, F. X. and Ebens, H., 2001, *The Distribution of Realised Stock Return Volatility*, *Journal of Financial Economics*, 61, 43-76.
- Andersen, T. G., Bollerslev, T., Diebold, F. X. and Labys, P., 2003, *Modelling and Forecasting Realised Volatility*, *Econometrica*, 77, 529-626.
- Andersen, T. G., Bollerslev, T. and Huang, X., 2011, *A Reduced from Framework for Modelling Volatility of Speculative Prices Based on Realised Variation Measures*, *Journal of Econometrics*, 160, 176-189.
- Andersen, T. G., Bollerslev, T. and Meddahi, N., 2005, *Correcting the Errors: Volatility Forecasting Evaluation Using High Frequency Data and Realised Volatility*, *Econometrica* 73, 279-296.
- Andersen, T.G., Bollerslev, T. and Dobrev, D., 2007. *No-arbitrage semi-martingale restrictions for continuous-time volatility models subject to leverage effects, jumps and iid noise: Theory and testable distributional implications*. *Journal of Econometrics*, 138(1), 125-180.
- Andrade, S.C., Bian, J. and Burch, T.R., 2013. *Analyst coverage, information, and bubbles*. *Journal of Financial and Quantitative Analysis*, 48(05), 1573-1605.
- Angelidis, T. and Degiannakis, S., 2008, *Forecasting One Day Ahead VaR and Intraday Realised Volatility in the Athens Stock Exchange Market*, *Managerial Finance*, 34, 489-497.
- Audrino, F. and Trojani, F., 2007. *Accurate Short Term Yield Curve Forecasting Using Functional Gradient Descent*. *Journal of Financial Econometrics*, 5(4), 591-623.
- Ba. SS. (2016) A Simple Analysis on the Abnormal Fluctuations in the Chinese Stock Market and Relevant Reasons (in Chinese), working paper. 巴曙松. (2016).关于股市异常波动原因及救市措施简析. <http://www.cf40.org.cn/plus/view.php?aid=10847>

- Baba, N. and Sakurai, Y., 2011. *Predicting Regime-switching In The VIX Index With Macroeconomic Variables*. Applied Economics Letters, 18(15), 1415-1419.
- Banid, M. F. and Russell, R. J., 2006, *Separating Microstructure Noise from Volatility*, Journal of Financial Economics, 79, 655-692.
- Bandi, F. M. and Russell, J. R., 2008, *Microstructure Noise, Realised Variance, and Optimal Sampling*. Review of Economic Studies, 75(2), 339-369.
- Banid, M. F. and Russell, R. J., 2011, *Market Microstructure Noise, Integrated Variance Estimators and the Accuracy of Asymptotic Approximates*, Journal of Econometrics, 160, 145-159.
- Barndorff-Nielsen OE., and Shephard N. 2002a. *Econometric Analysis of Realised Volatility and its Use in Estimating Stochastic Volatility Models*. Journal of the Royal Statistical Society, Series B 64: 253–280.
- Barndorff-Nielsen OE., and Shephard N. 2002b. *Estimating Quadratic Variation Using Realised Variance*. Journal of Applied Econometrics 17: 457–477.
- Barndorff-Nielsen, O. E. and Shephard, N., 2004, *Power and Bipower Variation With Stochastic Volatility and Jumps*, Journal of Financial Econometrics, 2, 1-37.
- Barndorff-Nielsen, O. E., Hansen, P. H., Lunde, A. and Shephard, N., 2011, *Subsampling Realised Kernels*. Journal of Econometrics, 160(1), 204-219
- Bauwens, L., Hafner, C.M. and Laurent, S., 2012. *Handbook of Volatility Models and Their Applications* (Vol. 3). John Wiley and Sons.
- Becker, R., Clements, A. E. and McClelland, A., 2009, *The Jump Component of S&P 500 Volatility and The VIX Index*, Journal of Banking and Finance, 33, 1033-1038.
- Black, F. and Scholes, M., 1973, *The Pricing of Options and Corporate Liabilities*, Journal of Political Economy, 81, 637-654.
- Bohl, M. T., Diesteldorf, J., and Siklos, P. L., 2015. *The Effect of Index Futures Trading on Volatility: Three Markets for Chinese Stocks*, China Economic Review 34, 207-224.
- Bohl, M. T., Salm, C. A. and Schuppli, M., 2011, *Price Discovery and Investor Structure in Stock Index Futures*, Journal of Futures market, 3, 282-306.
- Bollen, B. and Inder, B., 2002, *Estimating Daily Volatility in Financial Markets Utilizing Intraday Data*, Journal of Empirical Finance, 9, 551-562.
- Bollerslev, T., 1986, *Generalized Autoregressive Conditional Heteroskedasticity*, Journal of Econometrics, 3, 307-327.
- Bollerslev, T., Sizova, N. and Tauchen, G., 2012, *Volatility in Equilibrium Asymmetries and Dynamic Dependencies*; Review of Finance, 16, 31-80.
- Boudt, K., Peterson, G. B. and Croux, C., 2008, *Estimation and Decomposition of Downside Risk for Portfolios with Non-Normal Returns*, Journal of Risk, 2, 79-103.
- Boudt, K. and Christophe, C., 2010, *Robust M-Estimation of Multivariate GARCH Models*, Computational Statistics and Data Analysis, 54, 2459-2469.
- Brockwell, P. J. and Davis, R. A., 1991, *Time Series: Theory and Methods*, New York: Springer-Verlag Press, 506-552.
- Brooks, C., 2002, *Introductory Econometrics for Finance*, Cambridge University Press, Cambridge.
- Brooks, C. and Hinich, J. M., 2006, *Detecting Intraday Periodicities with Application to High Frequency Exchange Rate*, Journal of Royal Statistical Society, 55, 241-259.
- Brownlees, C. and Gallo, G. M., 2009, *Comparison of Volatility Measures: A Risk Management Perspective*, Journal of Financial Econometrics, 8, 29-56.
- Campbell, J. Y. and Hentschel, L., 1992, *No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns*, Finance Economy, 31, 281-318.

- Campbell, J. Y. and Shiller, R. J., 1987, *Cointegration and Tests of Present Value Models*, Journal of Political Economy, 95, 1062-1088.
- Caporin, M. and M. McAleer, 2012, *Do We Really Need Both BEKK and DCC? A Tale of Two Multivariate GARCH Models*, Journal of Economic Surveys, 26(4), 736-751.
- Caporin, M. and McAleer, M. 2013, *Ten Things You Should Know about the Dynamic Conditional Correlation Representation*, Econometrics, 1(1), 115-126.
- Chan, K., Menkveld, A.J. and Yang, Z., 2008. *Information asymmetry and asset prices: Evidence from the China foreign share discount*. *The Journal of Finance*, 63(1), 159-196.
- Chan, K., Chan, K. C. and Karolyi, G. A., 1991, *Intraday Volatility in the Stock Index and Stock Index Futures Markets*, Review of Financial Studies, 4, 657-684.
- Chang, K., 2011, *The Optimal Value-at-Risk hedging Strategy Under Bivariate Regime-switching ARCH Framework*, Applied Economics, 43, 2627-2640.
- Chen, T., Cai, J. and Ho, R., 2009, *Intraday Information Efficiency on the Chinese Equity Market*, China Economy Review, 20, 527-541.
- Chen B., He M., Li KQ., and Peng QL. (2015) An analysis of the Irrational Bubbles on Chinese Stock Markets Based on Behavioral Finance: empirical evidence from 2014-2015, *Financial Economics*(18), 114-116(in Chinese). 陈波, 何淼, 李康琪, & 彭秋林. (2015). 基于行为金融理论的中国股市非理性泡沫研究——来自 2014-2015 的经验证据. *金融经济*(18), 114-116.
- Christensen K, Podolskij M, and Vetter M., 2009, *Bias-Correcting the Realised Range-Based Variance in the Presence of Market Microstructure Noise*. *Finance and Stochastics*; 13: 239-268.
- Christensen K., and Podolskij M., 2007, *Realised Range-Based Estimation of Integrated Variance*. *Journal of Econometrics* 141: 323-349.
- Christensen, K., and Podolskij, M., 2012. *Asymptotic Theory of Range-Based Multipower Variation*. *Journal of Financial Econometrics*, 10(3), 417-456.
- Corsi, F., Zumbach, G., Muller, U. and Dacorogna, M., 2001, *Consistent High Precision Volatility from High Frequency Data*, *Economic Notes*, 30, 183-204.
- Corsi, F., Mittnik, S., Pigorsch, C. and Pigorsch, U., 2008, *The Volatility of Realised Volatility*, *Econometric Reviews*, 27, 46-78.
- Corsi, F., 2009, *A Simple Approximate Long Memory Model of Realised Volatility*, *Journal of Financial Econometrics*, 7, 174-196.
- Corsi, F., Pirino, D. and Reno, R., 2010, *Threshold Bipower Variation and the Impact of Jumps on Volatility Forecasting*, *Journal of Econometrics*, 159, 276-288.
- Cronqvist, H., Siegel, S. and Yu, F., 2015. *Value versus growth investing: Why do different investors have different styles?*. *Journal of Financial Economics*, 117(2), pp.333-349.
- Degiannakis, S. and Xekalaki, E., 2010, *ARCH Models for Financial Applications*, John Wiley and Sons.
- Deo, R., Hurvich, C. and Lu, Y., 2006, *Forecasting Realised Volatility Using a Long Memory Stochastic Volatility Model: Estimation, Prediction and Seasonal Adjustment*, *Journal of Econometrics*, 131, 29-58.
- Dijk, C., Pooter, D. M. and Martens, M., 2004, *Modelling and Forecasting S&P 500 Volatility: Long Memory, Structure Breaks and Nonlinearity*, Tinbergen Institute Discussion Paper. Available at SSRN: <http://ssrn.com/abstract=557746> .

- Ding, Z., Granger, C. W. J. and Engle, R. F., 1993, *A Long Memory Property of Stock Market Returns and a New Model*, Journal of Empirical Finance, 1, 83-106.
- Ding, Z. and Granger, C. W. J., 1996, *Modelling Volatility Persistence of Speculative Returns: A New Approach*, Journal of Econometrics, 50, 987-1007.
- Duyvesteyn, J. G., Martens, M. and Nic, S., 2011, *Forecasting Bond Returns Using Jumps in Intraday Prices*, Journal of Fixed Income, 20(4), 80-90.
- Ebend, H., 1999, *Realised Stock Index Volatility*, Johns Hopkins University, Baltimore, Working paper, 420. Available at Repec: <https://ideas.repec.org/p/jhu/papers/420.html>
- Ederington, L.H., 1979. *The Hedging Performance of the New Futures Markets*. Journal of Finance, 34(1), 157-170.
- Engle, R. F., 1982, *Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation*, Journal of Econometric Society, 50, 987-1007.
- Engle, R. F., 1990, *Discussion of Stock Market Volatility and the Crash of 87*, Review of Financial Studies, 3, 103-106.
- Engle, R.F. and Gallo, G.M., 2006. *A Multiple Indicators Model for Volatility Using Intra-Daily Data*. Journal of Econometrics, 131(1), 3-27.
- Engle, R.F. and Kroner, K.F., 1995, *Multivariate Simultaneous Generalised GARCH*, Econometric Theory, 11, 125-50.
- Engle, R. F. and Lee, G. L., 1999, *A Long-run and Short-Run Component Model of Stock Return Volatility*, in *Cointegration, Causality and Forecasting*, Oxford University Press, 475-497.
- Engle, R. F., Lillian, D. and Robins, R., 1987, *Estimating Time Varying Risk Premia in the Term Structure: the GARCH-M Model*, Econometrica, 55, 391-407.
- Engle, R.F. and Russell, J.R., 1998. *Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data*. Econometrica, 66(5), 1127-1162.
- Eraker, B., Johannes, M. and Polson, N., 2003. *The Impact of Jumps in Volatility and Returns*. Journal of Finance, 58(3), 1269-1300.
- Evans, K. P., 2011, *Intraday Jumps and US Macroeconomic News Announcements*, Journal of Banking and Finance, 35, 2511-2527.
- Fama, E. F., 1970, *Efficient Capital Market: A review of Theory and Empirical Work*, Journal of Finance, 25, 383-471.
- Floros, C., 2009, *Price Discovery in South African Stock Index Futures Market*, International Research of Finance and Economics, 34, 149-159.
- French, K.R. and Roll, R., 1986. *Stock Return Variances: The Arrival of Information and the Reaction of Traders*. Journal of Financial Economics, 17(1), 5-26.
- Geweke, J. and Porter-Hudak S., 1983, *The Estimation and Application of Long Memory Time Series Models*, Journal of Time Series Analysis, 4, 221-238.
- Ghysels, E., Harvey, A.C. and Renault, E., 1996. *5 Stochastic Volatility*. In *Handbook of statistics*, 14, 119-191.
- Ghysels, E. and Sinko, A., 2011, *Volatility Forecasting and Microstructure Noise*, Journal of Econometrics, 160,257-271.
- Gilder, D., 2009, *An Empirical Investigation of Intraday Jumps and Cojumps in US Equities*, Working Paper, Aston University. Available at SSRN: <http://ssrn.com/abstract=1343779> .
- Giot, P. and Laurent, S., 2004. *Modelling Daily Value-At-Risk Using Realised Volatility and ARCH Type Models*. Journal of Empirical Finance, 3(11), 379-398.



- Giot, P., Laurent, S. and Petitjean, M., 2010, *Trading Activity, Realised Volatility and Jumps*, Journal of Empirical Finance, 17, 168-175.
- Glosten, L. R., Jagannathan, R. and Runkle, D., 1993, *Relationship between the Expected Value and the Volatility of the Nominal Excess Return on Stocks*, Journal of Finance, 48, 1779-1801.
- Goncalves, S. and Meddahi, N., 2009, *Bootstrapping Realised Volatility*, Econometrica, 77(1), 283-306.
- Granger, C. W. J., 1969, *Investigating Causal Relationship by Econometric and Cross-sectional Method*, Econometrica, 37, 424-438.
- Granger, C. W. J. and Ding Z., 1996, *Varieties of Long Memory Models*, Journal of Econometrics, 73, 61-78.
- Granger, C. W. and Hyung, N., 2013, *Occasional Structural Breaks and Long Memory*, Annals of Economics and Finance, 14(2), 739-764.
- Granger, C. W. J., Poon, S. and Hyung, N., 2009, *Frontiers of Economics and Globalization, Chapter 9: A Source of Long Memory in Volatility*, Emerald Group Publishing Limited.
- Guan T. (2016) A Reflection of the Abnormal Fluctuations in the Chinese Stock Market in 2015 (in Chinese), working paper. 管涛. (2016) .反思 2015 中国股市异动 | 积极发展多层次的资本市场 . [http://mp.weixin.qq.com/s?\\_\\_biz=MjM5NjgyNDk4NA==&mid=2685956043&idx=2&sn=3c04b2008e32b12d71e6da8a65c05af6&scene=21#wechat\\_redirect](http://mp.weixin.qq.com/s?__biz=MjM5NjgyNDk4NA==&mid=2685956043&idx=2&sn=3c04b2008e32b12d71e6da8a65c05af6&scene=21#wechat_redirect)
- Hamilton, J. D., 1989, *Rational Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates*, Journal of Economic Dynamics and Control, 12,385-423.
- Hamilton, J. D. and Susmel, R., 1994, *Autoregressive Conditional Heteroskedasticity and Change in Regime*, Journal of Econometrics, 64, 307-333.
- Han, Q., & Liang, J. (2016). Index Futures Trading Restrictions and Spot Market Quality: Evidence from the Recent Chinese Stock Market Crash. Journal of Futures Markets.
- Hansen, P. R. and Lunde, A., 2006, *Realised Variance and Market Microstructure Noise*, Journal of Business and Economic Statistics, 24, 127-161.
- Hansen, P. R., Reinhard, P., Asyer, L. and Valeri, V., 2014, *Realised Beta GARCH: Multivariate GARCH Model with Realised Measures of Volatility and Covolatility*, Journal of Applied Econometrics, 29(5), 774-799.
- Harris, L., 1990, *Estimation of Stock Variance and Serial Covariance from Discrete Observations*, Journal of Financial and Quantitative Analysis, 25, 291-306.
- Hasbrouck, J. 1995. *One Security, Many Markets: Determining the Contributions to Price Discovery*. Journal of Finance, 1175-1199.
- Hasbrouck, J., 1996. *Order Characteristics and Stock Price Evolution: An Application to Program Trading*. Journal of Financial Economics, 41(1), 129-149.
- Hillebrand, E., 2005, *Neglecting Parametric Changes in GARCH Models*, Journal of Econometrics, 139, 121-138.
- Hillebrand, E., Medeiros, M. C., Rapach, D. and Wohar, M., 2006, *Forecasting in the Presence of Structural Breaks and Uncertainty*, Frontiers of Economics and Globalization, Elsevier.
- Hou, Y., and Li, S., 2016. *Information Transmission between U.S. and China Index Futures Markets: An Asymmetric DCC GARCH Approach*, Economic Modelling 52, Part B, 884-897.

- Huang, Z., H. Liu, and Wang, T., 2016. Modelling Long Memory Volatility Using Realised Measures of Volatility: A Realised HAR GARCH Model, *Economic Modelling* 52, Part B, 812-821.
- Huang, YC and Qi Q. (2017) The Leverage in Chinese Stock Market Fluctuations. *Chinese Finance*(1), 55-57 (in Chinese). 黄运成, & 漆琴. (2017). 中国股市波动中的杠杆作用. *中国金融*(1), 55-57.
- Hung, J., Lee, M. and Liu, H., 2008 *Estimation of Value at Risk for Energy Commodities Via Fat-tailed GARCH Model*, *Energy Economics*, 30, 1173-1191.
- Hwang, J., 2012, *Dynamic Correlation Analysis of Asian Stock Market*, *International Advances in Economics Research*, 18, 227-237.
- Iltuzer, Z. and Tas, O., 2012, *The Analysis of Bidirectional Causality Between Stock Market Volatility and Macroeconomic Volatility*, *International Journal of Business and Social Science*, 3, 12-29.
- Isa, Z., Nor, A. S. and Cheong, C. W., 2007, *Asymmetry and Long Memory Volatility: Some Empirical Evidence Using GARCH*, *Physical A: Statistical Mechanics and Its Applications*, 373, 651-664.
- Jacod, J., Mykland, P., Podolskij, M. and Vetter, M., 2009 *Microstructure Noise in the Continuous Case: the Pre-Averaging Approach*, *Stochastic processes and their applications* 119, 2249-2276.
- Jacod, J. and Protter, P., 1998, *Asymptotic Error Distributions for the Euler Method for Stochastic Differential Equations*, *Annals of Probability*, 26, 267-307.
- Johansson, A. C. and Ljungwall, C., 2009, *Spillover Effects among the Greater China Stock Markets*, *Law Finance and Economic Growth in China*, 37, 839-851.
- Kang, S.H., Cheong, C. and Yoon, S., 2013, *Intraday Volatility Spillovers between Spot and Futures Indices: Evidence from the Korean Stock Market*, *Physical A*, 392, 1795-1802.
- Kenourgios, D., Samitas, A. and Drosos, P., 2008, *Hedge Ratio Estimation and Hedging Effectiveness: The Case of the S&P 500 Stock Index Futures Contract*, *International Journal of Risk Assessment and Management*, 9(1-2), 121-134.
- Kogan, L., Ross, S.A., Wang, J. and Westerfield, M.M., 2006. *The price impact and survival of irrational traders*. *Journal of Finance*, 61(1), 195-229.
- Koopman, S. J., Jungbacker, B. and Hol, E., 2005, *Forecasting Daily Variability of the S&P 100 Stock Index Using Historical, Realised and Implied Volatility Measurement*, *Journal of Empirical Finance*, 12, 445-475.
- Kuester, K., Mittnik, S. and Paolella, M. S., 2005, *Value-at-Risk Prediction: A Comparison of Alternative Strategies*, *Journal of Financial Econometrics*, 4, 53-89.
- Kupiec, P.H., 1995. *Techniques for Verifying the Accuracy of Risk Measurement Models*. *Journal of Derivatives*, 3(2), 73-84.
- Lai, Y., 2012, Realised Variance, *Efficient Sampling and Futures Hedging*, Working Paper, National Chi Nan University. Available at SSRN: <http://ssrn.com/abstract=2017578> .
- Laurent, S., Boudt, K. and Croux, C., 2011, *Robust Estimation of Intra-week Periodicity in Volatility and Jump Detection*, *Journal of Empirical Finance*, 18,353-367.
- Lee, H. and Yoder, J., 2007, *A Bivariate Markov Regime-switching GARCH Approach to Estimate Time Varying Minimum Variance Hedge Ratios*, *Applied Economics*, 39(10), 1253-1265.
- Lee, S. and Mykland, A. P., 2008, *Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics*, *Review of Financial Studies*, 21, 2535-2563.

- Li, X., Wang, S.S. and Wang, X., 2017. *Trust and stock price crash risk: Evidence from China*. Journal of Banking & Finance, 76, 74-91.
- Li, Y. and Yang, L., 2013. *Prospect theory, the disposition effect, and asset prices*. Journal of Financial Economics, 107(3), pp.715-739.
- Liao, Y., 2013. *The Benefit of Modelling Jumps in Realised Volatility for Risk Prediction: Evidence from Chinese Mainland Stocks*, Pacific-Basin Finance Journal 23, 25-48.
- Lieberman, O. and Phillips, P. C., 2008, *Refined Inference on Long Memory in Realised Volatility*, Econometric Reviews, 27, 254-267.
- Lien, D. and Yang, L., 2006, *Sport-Futures Spread, Time-Varying Correlation, and Hedging With Currency Futures*, Journal of Futures market, 26, 1019-1038.
- Lien, D., 2009, *A Note on the Hedging Effectiveness of GARCH Models*, International Review of Economics and Finance, 18, 110-122.
- Lima, P. D., Crato, N. and Breidt, F. J., 1998, *The Detection and Estimation of Long Memory in Stochastic Volatility*, Journal of Econometrics, 83, 325-348.
- Lin, Y. (2016) *The Change of Investor Confidence And Stock Fluctuations* Doctoral Dissertation, Beijing Jiaotong University (in Chinese). 林扬. (2016). *投资者信心变化与股价波动关系研究*. (Doctoral dissertation, 北京交通大学).
- Liu, Q. and An, Y., 2011, *Information Transmission in Informationally Linked Markets: Evidence from US and Chinese Commodity Futures Markets*, Journal of International Money and Finance, 30, 778-795.
- Liu, C. and Maheu, J.M., 2009. *Forecasting Realised Volatility: A Bayesian Model-Averaging Approach*. Journal of Applied Econometrics, 24(5), 709-733.
- Liu, X., Margaritis, D. and Wang, P., 2012, *Stock Market Volatility and Equity Returns: Evidence From a Two-State Markov Switching Model with Regressor*, Journal of Empirical Finance, 19, 483-496.
- Lleo, S., & Ziemba, W. T. (2015). *Predicting Chinese Stock Market Crashes*. SSRN working paper.
- Louzis, D., Spyros, X. and Refenes, A., 2011, *Are Realised Volatility Models Good Candidates for Alternative Value at Risk Prediction Strategies*, SSRN Working Paper. Available at SSRN: <http://ssrn.com/abstract=1814171> .
- Lu, Y., Hurvich, C. and Deo, R., 2006, *Forecasting Realised Volatility Using a Long Memory Stochastic Volatility Model: Estimation, Prediction and Seasonal Adjustment*, Journal of Econometrics, 131, 29-58.
- Luo, Y., & Ren, J. (2016). *Short Sale, Margin Purchase, and Stock Price Crash Risk*, SSRN working paper.
- Ma, CP. (2015) *A Reflection of the Reasons Of The 2015 Crash In Chinese Stock Market*. Global Market Information Newspaper (43), 29-29(in Chinese). 马承平. (2015). *2015年年中股灾历史回顾及原因*. 环球市场信息导报(43), 29-29.
- Ma, F., Liu, L., Liu, Z. and Wei, Y., 2015. *Forecasting Realised Range Volatility: A Regime-switching Approach*. Applied Economics Letters, 22(17), 1361-1365.
- Maheu, J. M. and McCurdy, T. H., 2004, *News Arrival, Jump Dynamics, and Volatility Components for Individual Stock Returns*, Journal of Finance, 59, 755-793.
- Mandelbort, B., 1963, *The Variation of Certain Speculative Prices*, Journal of Business, 36, 394-419.
- Martens, M., 2001, *Forecasting Daily Exchange Rate Volatility Using Intraday Returns*, Journal of International Money and Finance, 20, 1-23.

- Martens, M., 2002, *Measuring and Forecasting S&P 500 Index Futures Volatility Using High Frequency Data*, Journal of Futures markets, 22, 497-518.
- McAleer, M. and Medeiros, M.C., 2008. *A Multiple Regime Smooth Transition Heterogeneous Autoregressive Model for Long Memory and Asymmetries*. Journal of Econometrics, 147(1), 104-119.
- McMillan, G. D. and Speight, A. E. H., 2004, *Intraday Periodicity, Temper Aggregation and Time to Maturity in FTSE-100 Index Futures Volatility*, Applied Financial Economics, 14, 253-263.
- Meddahi, N., 2002, *A Theoretical Comparison between Integrated and Realised Volatility*, Journal of Applied Econometrics, 17, 479-508.
- Medeiros, M. C. and Veiga, A., 2009, *Modeling Multiple Regimes in Financial Volatility with a Flexible Coefficient GARCH (1, 1) Model*, Econometric Theory, 25(01), 117-161.
- Medeiros, M. C., Mealeer, M. and Asai, M., 2012, *Asymmetry and Long Memory in Volatility Modelling*, Journal of Financial Econometrics, 10, 495-512.
- Merton, R., 1980, *On Estimating the Expected Return on the Market: An Exploratory Investigation*, Journal of Financial Economics, 8, 323-361.
- Miao, B., Xie, H., Wu, Y. and Guo, B., 2011, *A Segmented Regime-switching Model with Its Application to Stock Market Indices*, Journal of Applied Statistics, 38(10), 2241-2252.
- Moore, T. and Wang, P., 2007, *Volatility in Stock Returns for New EU Member States: Markov Regime-switching Model*, International Review of Financial Analysis, 16, 282-292.
- Muller U., Dacorogna M., Dav R., Olsen R., Pictet O., and von Weizsacker J. 1997, *Volatilities of Different Time Resolutions – Analysing the Dynamics of Market Components*. Journal of Empirical Finance 4: 213-239.
- Myers, R. and Moschini, G., 2002, *Testing for Constant Hedge Ratios in Commodity Markets: A Multivariate GARCH Approach*, Journal of Empirical Finance, 9, 589-603.
- Mykland, P.A. and Zhang, L., 2006. ANOVA for Diffusions and Itô Processes. Annals of Statistics, 34(4), 1931-1963.
- Nelson, D., 1991, *Conditional Heteroskedasticity in Asset Returns: A new Approach*, Econometrica, 59, 347-370.
- Nielson, M. and Frederiksen, P. H., 2008, *Finite Sample Accuracy and Choice of Sampling Frequency in Integrated Volatility Estimation*, Journal of Empirical Finance, 15(2), 265-286.
- Nomikos, N., Alizadeh, A. and Poulialis, P., 2008, *A Markov Regime-switching Approach for Hedging Energy Commodities*, Journal of Banking and Finance, 32, 1970-1983.
- O'Hara, M., 1995, *Market Microstructure Theory*, Blackwell, Oxford.
- Oomen, R. C. A., 2005, *Properties of Bias-Corrected Realised Variance Under Alternative Sampling Schemes*, Journal of Financial Econometrics, 3, 555-557.
- Oomen, R. C. A., 2006, *Properties of Bias-Corrected Realised Variance Under Alternative Sampling Schemes*, Journal of Business and Economic statistics, 24, 219-237.
- Patton, J. A., 2011, *Volatility Forecast Comparison Using Imperfect Volatility Proxies*, Journal of Econometrics, 160, 246-256.

- Qian, J.Q., Strahan, P.E. and Yang, Z., 2015. *The impact of incentives and communication costs on information production and use: Evidence from bank lending*. *Journal of Finance*, 70(4), 1457-1493.
- Rabemananjara, R. and Zakoian, J. M. 1993. *Threshold ARCH models and asymmetries in volatility*, *Journal of Applied Econometrics* 8: 31-49.
- Refenes, P. A., Spyros, X. and Louzis, D., 2012, *Stock Index Realised Volatility Forecasting in the Presence of Heterogeneous Leverage Effects and Long Range Dependence in the Volatility of Realised Volatility*, *Applied Economics*, 44, 3533-3550.
- Rootzen, H., 1980, *Limit Distributions for the Error in Approximations of Stochastic Integrals*, *Annals of probability*, 8, 241-251.
- Salvador, A. and Arago, V., 2013, *Measuring Hedging Effectiveness of Index Futures Contracts: Do Dynamic Models Outperform Static Models? A Regime-switching Approach*, *Journal of Futures market*, 34(4), 374-398.
- Sato, S. and Kunitomo, N., 2009, *The SIML Estimation of Realised Volatility of the Nikkei-225 Futures and Hedging Coefficient With Micro-Market Noise*, *Mathematics and Computers in Simulation*, 1272-1289.
- Scharth, M. and Medeiros, M.C., 2009. *Asymmetric Effects and Long Memory in the Volatility of Dow Jones Stocks*. *International Journal of Forecasting*, 25(2), 304-327.
- Schwert, W., 1989, *Stock Volatility and the Crash of 87*, *Review of Financial Studies*, 3, 77-102.
- Sentana, E., 1995, *Quadratic ARCH Models*, *Review of Economic Studies*, 62, 639-661.
- Shen, J. and Harris, R., 2003, *Robust Estimation of the Optimal Hedge Ratio*, *Journal of Futures Markets*, 23, 799-816.
- Sheu, H. and Lai, Y., 2009, *The Incremental Value of a Futures Hedge Using Realised Volatility*, *Journal of Futures Markets*, 30, 874-896.
- So, R. and Tse, Y., 2004, *Price Discovery in the Hang Seng Index Market: Index, Futures and the Tracker Fund*, *Journal of Futures market*, 24, 887-907.
- So, M.K. and Wong, C.M., 2012. *Estimation of Multiple-Period Expected Shortfall and Median Shortfall for Risk Management*. *Quantitative Finance*, 12(5), 739-754.
- Stoja, S., Shen, J. and Harris, R., 2010, *The Limits to Minimum Variance Hedging*, *Journal of Business Finance and Accounting*, 37, 737 -761.
- Takahashi, M., Watanabe, T., and Omori, Y., 2016. *Volatility and Quantile Forecasts by Realised Stochastic Volatility Models with Generalized Hyperbolic Distribution*, *International Journal of Forecasting* 32, 437-457.
- Taylor, S., 1986, *Modelling Financial Time Series*, New York, John Wiley and Sons.
- Taylor, S. J. and Xu, X., 1997, *The Incremental Volatility Information in One Million Foreign Exchange Quotations*, *Journal of Empirical Finance*, 4, 317-340.
- Todorov, V., 2011, *Econometric Analysis of Jump-Driven Stochastic Volatility Models*, *Journal of Econometrics*, 16, 12-21.
- Todorova, N., A. Worthington, and M. Souček, 2014. *Realised Volatility Spillovers in the Non-Ferrous Metal Futures Market*, *Resources Policy* 39, 21-31.
- Ubukata, M. and Oya, K., 2009, *Estimation and Testing for Dependence in Market Microstructure Noise*, *Journal of Financial Econometrics*, 7, 106-151.
- Vradhyula, U. S. and Ergun, A. F., 2004, *Trading Collar, Intraday Periodicity and Stock Market Volatility*, *Applied Financial Economics*, 14, 909-913.

- Wang, H., M. Yue, and Zhao, H., 2015. *Cojumps in China's Spot and Stock Index Futures Markets*, Pacific-Basin Finance Journal 35, Part B, 541-557.
- Wang, Y., F. Ma, Y. Wei, and Wu, C., 2016. *Forecasting Realised Volatility in A Changing World: A Dynamic Model Averaging Approach*, Journal of Banking and Finance 64, 136-149.
- Watcher, J. A., 2013, *Can Time-Varying Risk of Rare of Disasters Explain Aggregate Stock Market Volatility*, Journal of Finance, 68, 987-1035.
- Wei, Y., 2012. *Forecasting Volatility of Fuel Oil Futures in China: GARCH-Type, SV or Realised Volatility Models?* Physica A: Statistical Mechanics and its Applications 391, 5546-5556.
- Wright, J.H. and Zhou, H., 2009. *Bond Risk Premia and Realised Jump Risk*. Journal of Banking and Finance, 33(12).2007-2022.
- Wu, Z., 2012, *On the Intraday Periodicity Duration Adjustment of High-Frequency Data*, Journal of Empirical Finance, 19, 282-291.
- Xie YJ and Jiang Z. (2016) An Explanation of the Stock Market Fluctuations, Business 2), 165-165(in Chinese). 谢喻江, & 蒋智. (2016). 反身性理论对股市波动的解释——以 2015 年中国股市波动为例. 商(2), 165-165.
- Xu, F., and Wan, D., 2015. *The Impacts of Institutional and Individual Investors on the Price Discovery in Stock Index Futures Market: Evidence from China*, Finance Research Letters 15, 221-231.
- Yang, J., Yang, Z. and Zhou, Y., 2012, *Intraday Price Discovery and Volatility Transmission in Stock Index and Stock Index Futures Markets: Evidence from China*, Journal of Futures Market, 32, 99-121.
- Yang, K., Chen, L. and Tian, F., 2015. *Realised Volatility Forecast of Stock Index Under Structural Breaks*. Journal of Forecasting, 34(1), 57-82.
- Yarovaya, L., Brzezczynski, J., and Lau, C. K. M. 2016. *Intra- And Inter-Regional Return and Volatility Spillovers across Emerging and Developed Markets: Evidence from Stock Indices and Stock Index Futures*, International Review of Financial Analysis 43, 96-114.
- Yin ZL.. (2015) The Reasons of the Fluctuations in Chinese Stock Market. Chinese Finance(16), 77-79. (in Chinese). 尹中立. (2015). 股市缘何暴涨暴跌. 中国金融(16), 77-79.
- Zakoian, M. 1991, *Threshold heteroscedasticity models*. Journal of Economic Dynamics and Control, 18(5), 931-955.
- Zhang, L., 2006, *Efficient Estimation of Stochastic Volatility Using Noisy Observations: A Multi-scale Approach*, Bernoulli, 12, 1019-1043.
- Zhang, L., Mykland, P. A. and Ait-Sahalia, Y., 2005, *A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High Frequency Data*, Journal of the American Statistical Association, 100, 1394-1411.
- Zhong, M., Darrat, A.F. and Otero, R., 2004, *Price Discovery and Volatility Spillovers in Index Futures Markets: Some Evidence from Mexico*, Journal of Banking and Finance, 28, 3037-3054.
- Zhou, B., 1996, *High Frequency Data and Volatility in Foreign-exchange Rate*, Journal of Business and Economic Statistics, 14, 45-52.
- Zivot, E., Yu, W. and Choi, K., 2010, *Long Memory versus Structural Breaks in Modelling and Forecasting Realised Volatility*, Journal of International Money and Finance, 29, 857-875.