

Flow field visualization based motion segmentation for crowd counting

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Abstract

It is evident that crowd counting is one of bottlenecks for crowd related computer vision theory and applications such as surveillance. Since accuracy of estimating crowd size dominantly depends on the performance of motion detection of pedestrians, this paper attacks the challenging problem mainly by proposing a motion segmentation method based on flow field visualization. Firstly, the motion crowd and background are represented as different texture images by employing line integral convolution. Then information entropy is introduced to quantify the textures as different values so that the texture images can be segmented, further an optimal threshold is obtained via Otsu method to segment the binaryzation entropy image. Finally the area of motion foreground pixels is calculated for each image in a crowd motion video. The size of the crowd is estimated by least squares fitting using abundant datum of foreground pixels' area and the number of individuals in a crowd. Experimental results demonstrate that the proposed crowd counting method outperforms background subtraction and Gaussian Mixture Model in terms of Mean Absolute Error and Mean Relative Error.

Keywords : Image understanding, Crowd counting, Flow field visualization, Line integral convolution, Information entropy, Otsu segmentation, Least squares fitting.

1. Introduction

The term crowd is defined as “Two or more persons engaged in one or more behaviors judged common or concerted in one or more dimensions” [1]. The social influences of a crowd are heavier than individuals. In order to improve the effectiveness of control and management of crowd safety, crowd simulation, surveillance and related analytics have been attracting increasing attention in the field of computer science [2]. Crowd surveillance usually includes crowd detection, counting and analysis [3] [4], [5]. There are two main categories of approaches in solving the problem of crowd detection and counting. 1) Micro based methods: a crowd is considered as a collection of individuals so that it is required to detect [6], [7] and track every individual [8], [9], [10] and use their movement trajectories and postures [11] to estimate the size of crowd [12], [13]. This kind of method is suitable for deal with the small-scale crowds.

However, it seems hardly to detect and track the individuals accurately in dense crowds due to the occlusions among pedestrians. 2) Macro based methods: this kind of approaches considers a large-scale crowd as a global entity. Many contributions have been produced based on the global analysis of a crowd, including the extraction of suited scale-invariant feature points to estimate the people contained [14], Bernoulli model models to detect and estimate the size of crowd by the foreground shapes [15], an potential energy-based model for estimating the number of people in a crowd [16], and a Bayesian regression method to estimate the size of inhomogeneous crowds with a series of low level features [17]. Though this category of methods has capacity of performing well in large-scale crowds, its accuracy of people counting is rather lower than those of individual-based methods. Generally speaking, the segmentation of crowd motion is crucial to achieve a robust and accurate crowd counting. Therefore, this paper is focused on the

moving pedestrians' detection based people counting in crowd scenes.

Due to the necessity of moving pedestrians detection in a crowd, it is vital to propose a strategy that has the capability of distinguishing the foreground targets and background pixels. Traditional target detecting methods such as background subtraction and Gaussian mixture model are limited by phenomena of the inside holes once the appearances of target of interest and its background are similar. The reason for this drawback is that these methods only use the intensity information of every isolated pixel in the current image frame. The idea that treats the motion of a crowd as a flow field is useful for integrating more information of several particles or one particle's motion in a long time period [18]. Inspired by this idea, many approaches have been proposed for crowd analysis, such as using the chaotic modelling in a Lagrangian particle dynamics system to detect the anomaly behavior of a crowd [19] and representing crowd and traffic flow using streak line based on fluid dynamics [20]. Take the state of the art into account, a motion segmentation method is proposed in this paper based on the technology of flow field visualization which can represent a flow field as a texture image. To represent a crowd as a fluid, optical flow method and its variants are used as the first step to deal with this problem [21]. Optical flow method is a representation of the instantaneous motion of intensity points in two consecutive frames. Though many approaches have been proposed to improve the efficiency and performance of optical flow [22], [23], [24], it still can only be used to represent the information of two consecutive images of the crowds' motion. It is necessary to represent the flow field and reveal more information of it. Flow visualization has an advantage of representing a flow field in details. The flow field visualization techniques can be classified to two categories [25]: particle traces based method and texture synthesis techniques [26]. The former computes the particles traces of a flow field, such as streamline, pathline and streakline. Particle tracing techniques depend critically on the placement of the insertion points, which leads to the fact that many details in the data field are missed. On the other hand, texture synthesis technique [26] produces a texture by generating a set of spots on the spatial domain. LIC (i.e., Line Integral Convolution) technique is a typical method to visualize vector fields. LIC method

employs a low-pass filter to convolve an input noise texture along pixel-centered symmetrically bi-directional streamlines to exploit spatial correlation in the flow direction, which provides a global dense representation of a flow [27], [28]. After the processing of flow field visualization the moving pedestrians and background can be represented as different textures. Therefore, the task of pedestrian detection can be transformed to texture segmentation.

The theory of LIC addresses the fact that the flow field image is convoluted by a noise texture, the background regions are represented as white noise which has a random distribution of gray values of pixels, while, the motion regions of the LIC image have more uniform distribution of gray values of pixels, the foreground and background regions of a crowd image can be distinguished by the distribution of intensities. Entropy is an effective tool to measure the uncertainty of random variables according to the information theory [29]. The distribution of the intensities of pixels in a LIC image can be described by a histogram. For a background region the distribution of the intensity values is disorderly which indicates higher entropy will be got in this region. On the contrary, the distribution of the intensities in a motion region is orderly, it is to say, lower entropy is to be obtained accordingly. Hence, a LIC image can be represented as two categories of regions, i.e., the high entropy and low entropy. To segment moving pedestrians is to find a threshold for distinguishing the different values of entropy. Among the automatic threshold selecting techniques, Otsu method is proved as a convincing one which selects the threshold by maximizing the between-class variances of the histogram [30]. In this paper, the Otsu method is employed to segment the motion foreground and background regions. After image binarization, the foreground pixels and the true number of the pedestrians are calculated for training using least squares fitting. Any other videos captured from the same scene can be used for the purpose of testing of crowd counting. The framework of the proposed method is shown in Fig. 1.

The remainder of the paper is organized as the follows: Section 2 develops the flow field visualization technology for describing the crowd motion. Section 3 presents a motion segmentation method based on information entropy and Otsu thresholding technique. In Section 4, least square fitting is presented for the training of pedestrian

counting. Section 5 describes the evaluation on the proposed approach using a set of videos of crowd motion and demonstrates the effectiveness of the proposed method. Section 6 concludes this paper with future directions.

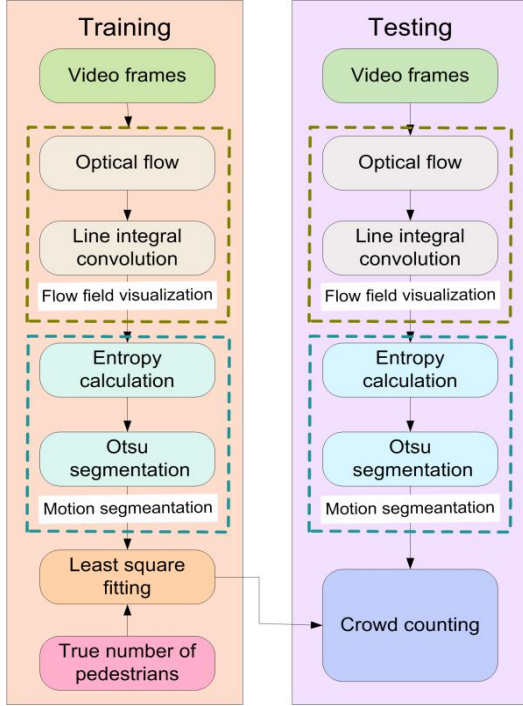


Fig. 1 Framework of the proposed method

2. Crowd motion flow field visualization

Since a crowd is regarded as moving fluid in this paper, the techniques of fluid kinematics are used to represent and detect crowd motion. To achieve this target, the velocity of each pixels of the image is calculated to gain the vectors of the fluid field which can be computed by optical flow method. Furthermore, the LIC algorithm, an effective method of fluid field visualization, is employed to reveal more details of the crowd motion flow field.

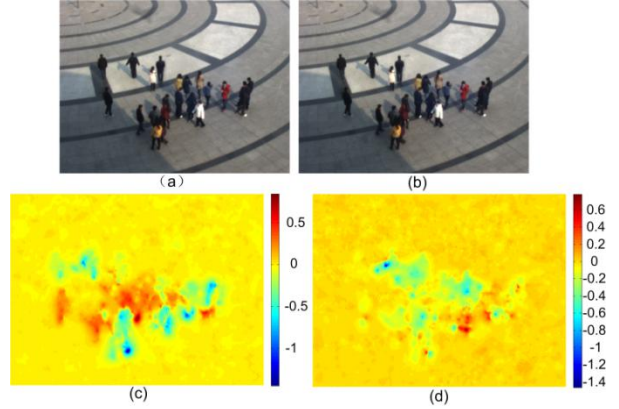


Fig. 2. Optical flow of a moving crowd, (a) frame 111, (b) frame 112, (c) optical flow of x direction, (d) optical flow of y direction

2.1. Calculation of optical flow field

Many methods have been proposed to improve the performance for calculating optical flow for different applications. However, the flow field visualization is the focus in this paper, a more advanced optical flow calculating method is not necessary here. So, we just use a traditional Horn-Schunck method to get the optical flow field of crowd motion. The experimental results have certified that our method can play well in our crowd counting system, which can be seen in Section 5.

For an image, the intensity of the pixel (x, y) at the time t can be expressed by $E(x, y, t)$. According to the approach of Horn-Schunck optical [21], the optical flow vector (u^{k+1}, v^{k+1}) of the $(k+1)$ th frame can be calculated by formula (1) and (2) [23]:

$$u^{k+1} = \bar{u}^k - \frac{E_x \cdot \bar{u}^k + E_y \cdot \bar{v}^k + E_t}{\alpha^2 + E_x^2 + E_y^2} \cdot E_x \quad (1)$$

$$v^{k+1} = \bar{v}^k - \frac{E_x \cdot \bar{u}^k + E_y \cdot \bar{v}^k + E_t}{\alpha^2 + E_x^2 + E_y^2} \cdot E_y \quad (2)$$

where E_x , E_y , and E_t are the derivatives of the image at the position (x, y, t) in the corresponding directions, i.e., $E_x = \partial E / \partial x$, $E_y = \partial E / \partial y$, $E_t = \partial E / \partial t$, \bar{u}^k and \bar{v}^k are the estimated local average optical flow velocities, $u = \partial x / \partial t$ and $v = \partial y / \partial t$ are the velocity in x and y direction of $E(x, y, t)$, α^2 is a weighting factor, which plays an important role in preventing haphazard adjustments to the estimated flow velocity corrupted by noise [21]. A larger value of α results in a smoother flow. Fig. 2 shows an optical image

computed from two consecutive frames of a crowd sequence.

2.2. Line integral convolution for crowd flow field visualization

The motion of a crowd can be regarded as fluid motion. One motion image can be gained between two consequence images from the surveillance video of crowd scene. In comparison with optical flow, the techniques of fluid field visualization can reveal more details of a flow field. LIC is a well-known method that applies texture synthesis technique to visualize a vector field. LIC employs a low-pass filter to convolve an input noise texture image along streamlines defined by an input vector field to generate an output image to exploit spatial correlation in the flow direction. There are two key steps in LIC algorithm: calculating streamlines in the vector field and convoluting the texture image along the streamlines.

In a vector field $\mathbf{v}(x, t)$, the general definition of a streamline is the tangent to the streamline should be equal to the velocity at a given instant in time t :

$$\frac{d\mathbf{x}(\tau)}{d\tau} = \mathbf{v}(\mathbf{x}(\tau), t), \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (3)$$

where, τ is the parameter that follows the streamline, and t is the physical time. Since the streamlines are independent of time, all the parameters describing a streamline are at a given instant of the physical time. Then a streamline through the point is formed, further the process of the convolution can be expressed as [27]:

$$I_{out}(x, y) = \frac{\sum_{i=0}^l I_{in}(P_i) h_i + \sum_{i=0}^{l'} I_{in}(P'_i) h'_i}{\sum_{i=0}^l h_i + \sum_{i=0}^{l'} h'_i} \quad (4)$$

where $h_i = \int_{s_i}^{s_i + \Delta s_i} k(\omega) d\omega$ is the weight, $I_{out}(x, y)$ is the value of output pixel at point (x, y) , $I_{in}(P_i)$ is the value of input pixel P_i , l and l' are the convolution distances along the positive and negative directions respectively, P_i is the i th pixels along the streamline in the positive direction, and P'_i represents a step in the negative direction, $k(\omega)$ is the low pass filter used for the LIC, Δs_i is the arc length between the point s_i and s_{i+1} along the streamline. The entire procedure of the calculation of LIC is shown in Fig. 3.

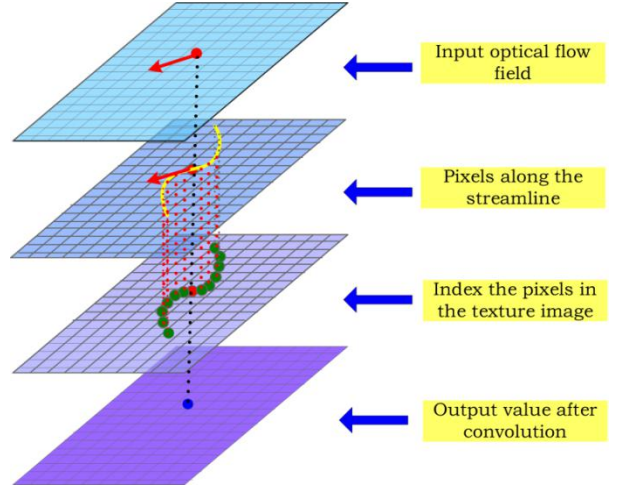


Fig. 3. The implementation of LIC

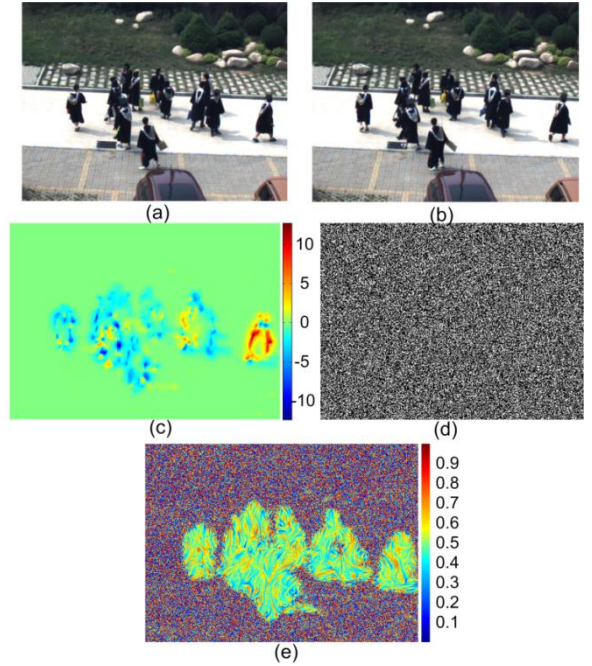


Fig.4. LIC based representation of a crowd motion flow field, (a) and (b) are two consecutive frames of a crowd video, (c) the optical flow image, (d) the noise texture, (e) the result of LIC.

Fig. 4 presents the representations of LIC and optical flow for two consecutive images of a crowd motion. The texture image used in this paper is random white noise as shown in Fig. 4. (d). For a gray image, it means that every pixel is described by a random number between 0 to 255. It can be seen from Fig. 4 that the motion regions are described

using LIC clearly stands out of that produced from optical flow method

3. Moving pedestrians segmentation

Pedestrian motions are represented as textures after visualizing crowd field, the problem of motion detection is hence transferred to that of texture segmentation. It can be seen, as shown in Fig. 4 (e), that the LIC method produces clear difference between the motion of interest and its background region. For a texture image of a crowd motion region, the intensity distribution of this texture image is uniform relatively. On the contrary, the intensity distribution of background region is rough because they are constituted by noises. Since the entropy is an efficient tool for measuring the degree of complex of a signal, it is used here to distinguish the difference of the motion texture. Finally, an image segmentation method called OTSU is used to segment pedestrian motion.

3.1. Entropy measurement

The crowd motion is represented as a gray texture image. The distributions of intensities of motion and background regions are different. In background regions, the intensity distributions are more complex than motion regions in that the background regions are composed by noises. Since information entropy is an effective method to quantify the minimum descriptive complexity of a random variable, Shannon entropy is employed here to quantify the complexity of intensity distributions in an image region [32]. The higher the entropy the more disorderly distribution of intensity is included in this image region. The intensity distribution of an image region can be represented by its gray histogram.

For an image region, the histogram of this region can be presented mathematically in equation (5) as follow:

$$p(i) = \sum_{(m,n) \in I} \delta(i - I_{m,n}) \quad (5)$$

where δ is the Kronecker delta function. $i \in (0, \dots, M-1)$ is the index of intensity. Further, the entropy of an image region is obtained by

$$H(p) = - \sum_{i=0}^{M-1} p(i) \log [p(i)] \quad (6)$$

Therefore, for a video of moving crowd the entropies of the motion regions are low, and that of the

background regions are high, which can be seen in Fig. 5(c), where the entropies are calculated for every 15×15 pixels region.

3.2. Otsu segmentation

After computing the entropy of a texture image, it is necessary to find a threshold to segment the moving crowd and background. Otsu method is an effective method for finding the optimal threshold [33]. In Otsu method, the image is partitioned into two classes C_0 and C_1 at grey-level t , i.e., $C_0 = \{0, 1, 2, \dots, t\}$ and $C_1 = \{t+1, t+2, \dots, M-1\}$, where M is the total of the gray levels of the image. Let q_0 and q_1 represent the estimate of class probabilities defined as:

$$q_0(t) = \sum_{i=0}^t p(i), \quad q_1(t) = \sum_{i=t+1}^{M-1} p(i) \quad (7)$$

where $p(i)$ represents the probability that gray level i occur in the image. Therefore, the class means can be defined as:

$$\mu_0(t) = \sum_{i=0}^t \frac{ip(i)}{q_0(t)}, \quad \mu_1(t) = \sum_{i=t+1}^{M-1} \frac{ip(i)}{q_1(t)} \quad (8)$$

The individual class variances can be defined as:

$$\sigma_0^2(t) = \sum_{i=0}^t [i - \mu_0(t)]^2 \frac{p(i)}{q_0(t)}, \quad \sigma_1^2(t) = \sum_{i=t+1}^{M-1} [i - \mu_1(t)]^2 \frac{p(i)}{q_1(t)} \quad (9)$$

Therefore, the class variance can be gained as a difference of total variance and within class variance:

$$\sigma_b^2(t) = \sigma^2 - \sigma_w^2(t) = q_0(t)[1 - q_0(t)][\mu_1(t) - \mu_0(t)]^2 \quad (10)$$

The threshold $T = \text{ArgMax}(\sigma_b^2(t))$ can be gained by maximizing the class variance $\sigma_b^2(t)$. Then the motion and background regions can be segmented by threshold T , the output binary image $I_{out}(m, n)$ can be segmented from input image $I_{in}(m, n)$ as follows:

$$I_{out}(m, n) = \begin{cases} 1 & \text{if } I_{in}(m, n) \geq T \\ 0 & \text{if } I_{in}(m, n) < T \end{cases} \quad (11)$$

Fig. 5 (d) shows the result of segmentation for an entropy image by Otsu method. The moving pedestrians and background are partitioned into two classes clearly.

4. Pedestrian counting using least squares fitting

Many contributions have proved that the density of a crowd could be distinguished by the relationship between the pedestrian area and background area [17],

[34]. The accuracy of the estimation of crowd density depends on the quality of motion foreground segmentation. In this paper, we focus on the motion segmentation of a crowd scene. A compact method of least squares fitting is used here to describe the relationship of crowd's size and the area of foreground area.

Suppose the data points are (x_i, y_i) , where x is the foreground area and y is the density of crowd. The fitting curve $f(x)$ has the deviation $d_i = y_i - f(x_i)$ from each data point. The summed square of residuals is given by $S = \sum_{i=1}^n d_i^2$, where n is the number of the data points. According to the least squares method, the best fitting line has the property that gains the minimum for S . The task of training is to establish the relationship between the area of foreground regions and the number of pedestrians in the training video. Therefore, we can estimate the number of pedestrians in test videos captured from the same scene with the training video. Fig. 6 shows the fitting line of the data points of foreground pixels' number and the crowd size of the video sequence S1.L1.13-59 view3 of PETS2009 dataset.

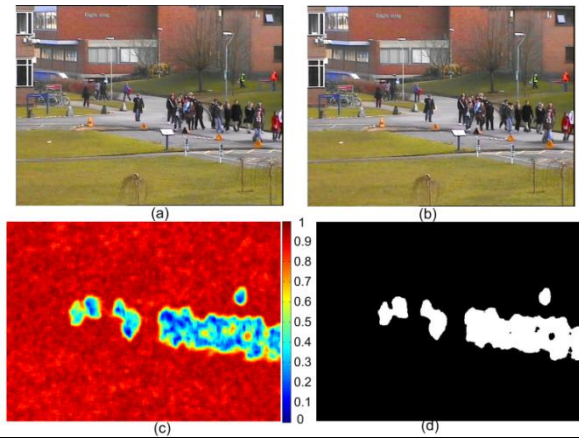


Fig. 5. Entropy representation and motion segmentation. (a) frame179, (b) frame180, (c) entropy, (d) Otsu segmentation.

5. Experiments and results

The proposed approach is evaluated through a number of experiments of motion region segmentation and pedestrian counting. A public dataset (PETS 2009) [35] and a set of videos captured in the campus of Yanshan University (YSU) are

employed for the purpose of evaluation.

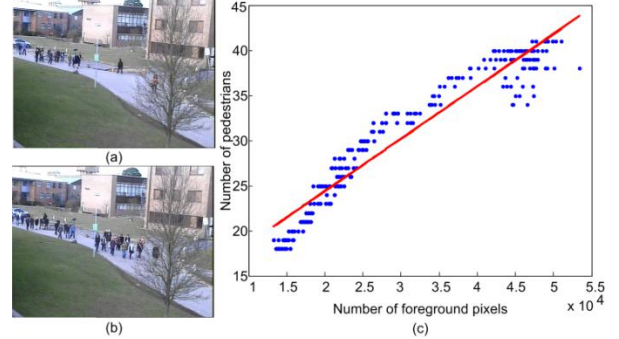


Fig. 6. (a) frame37, (b) frame136, (c) Least squares fitting of the crowd sequence.

5.1. Motion region segmentation

In order to evaluate the performance of motion object detection of the proposed method, four images are chosen from the crowd motion videos to compare the results of the proposed method with background subtraction and Gaussian mixture model. Fig. 7 shows four groups of results of motion segmentation. The images of top two rows are obtained from the PETS 2009. The densities of moving pedestrians are relatively higher. The bottom two rows show the images from YSU datasets, and the number of moving targets is relatively lower. It can be seen that, from Fig. 7, background subtraction method can partially detect moving target correctly, however some targets may be lost and some null holes will occur if the background and foreground are similar; for Gaussian mixture model, there are many null holes which are harmful for calculating the area of the motion regions. As for the proposed method, all of the moving pedestrians can be detected correctly. The area of motion region gained by the proposed method is larger than the size of target because our method represents the motion by LIC which depends on the streamlines of the flow field. Fortunately, these slight dilations are contributed to every motion region. Therefore, it does not affect the performance of crowd counting.

5.2. Pedestrian Counting

To count pedestrian, one image sequence is selected for training, i.e., getting the relationship between the area of motion foreground and the

number of moving pedestrian using least squares fitting. In each scene at least two sequences are needed in our experiments, one is used for training and the others are for testing. In our experiments the areas of foreground motion regions are computed by three moving detection methods. Furthermore, the numbers of pedestrian are estimated by least squares fitting.

In case I, three videos are selected from PETS 2009 dataset. In this scene, the number of pedestrian changed from 3 to 41, the sequence S1.L1.13-57 view1 which includes more individuals is chosen for training, other two sequences S1.L1.13-59 view1 and S1.L2.14-31 view1 are used for testing. Fig. 8 shows the results of pedestrian counting based on the three methods. Fig. 8 (b) and Fig. 8(c) show the number of pedestrian estimated by the proposed method are closer to the ground truth in comparison with background subtraction and Gaussian mixture model. Fig. 8 shows that the more number of pedestrians the lower the accuracy of pedestrians counting, since there are more occlusions existing more pedestrians in the scenes.

In case II, two groups of video are used to evaluate the performance of our method for low density crowd. In these scenes, the number of pedestrians is less than 14. In each scene, image sequences YSU.101 view1 and YSU.201 view2 are chosen for training, and other two sequences (YSU.102 view1 and YSU.202 view2) for test respectively. Using the same strategy of case I, we also compare the result of the three methods. From the evaluation curve of Fig. 9 we can see, the results of proposed method are similar to the ground truth, and the other two methods are poorer than the proposed approach. It indicates that the proposed method is also suitable for estimating the number of pedestrians for low-density crowd.

Due to the fact that the ground truth and estimated value of the number of moving pedestrians of the video, it is better to analyze the performance from the global perspective. Mean Absolute Error (MAE) and Mean Relative Error (MRE) can be used to evaluate the error of pedestrian counting which can be calculated as [36]:

$$MAE = \frac{1}{N} \sum_{i=1}^N |E(i) - T(i)|, MRE = \frac{1}{N} \sum_{i=1}^N \frac{|E(i) - T(i)|}{T(i)} \quad (12)$$

where N is the number of frames of the test sequence, $E(i)$ and $T(i)$ are the estimated and the true number of moving pedestrians in the i_{th} frame, respectively.

From formula (12) it indicates that, the lower values of MAE and MRE the more accurate of the pedestrian counting methods. The $MAEs$ and $MREs$ of the 4 videos of different methods (background subtraction, Gaussian mixture model and the proposed method) are calculated and compared, which can be seen in Fig. 10. It is obvious that the proposed method achieves the lowest MAE and MRE . For instance, for S1.L1.13-59 view1, the proposed method obtains the MRE value 13.23%, while Gaussian mixture model achieves 30.27%, about 2.29 times that of the proposed method. The estimation of the number of pedestrians based on background subtraction gets the largest MRE , which is 47.59%, more than 3 times of the MRE gained by the proposed method. As for the sequence YSU.102 view1, the value of MAE outputted by the proposed method is 0.81, while the method based on Gaussian mixture model and background subtraction get MAE 1.56 and 1.31 respectively.

6. Conclusions and future work

In this paper, we propose a motion segmentation method based on the technology of flow field visualization, which is focused on the application of crowd counting. Firstly, line integral convolution is used to represent the motion of a crowd. The background and motion regions are represented as different textures after the processing of line integral convolution. Secondly, entropies of each sub-region are calculated to represent the textures of motion and background regions as lower and higher values. Furthermore, Otsu method is employed to segment the motion crowd. Finally, the relationship between the area of foreground regions and the number of pedestrians is obtained by least squares fitting. This relationship is used to estimate the number of pedestrians in the same scene scenario. Experimental results show, in comparison with background subtraction and Gaussian mixture models, the proposed method are able to detect moving pedestrian without inside holes. The numbers of pedestrians estimated by our method are more accurate than its counterpart based on background subtraction and Gaussian mixture models.

Future work have been targeted on improvement of the ability of line integral convolution to represent

the crowd motion in details, such as distinguish the pedestrians moving with different velocities and directions. It also challenges identifying more

effective features of a crowd to improve the accuracy of crowd counting.

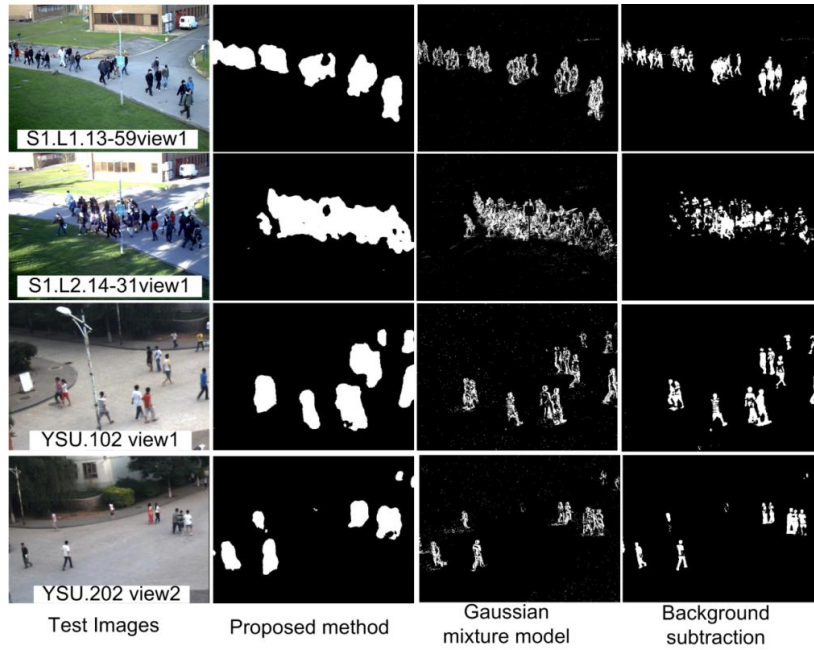


Fig. 7. Motion region segmentation using the proposed method, Gaussian mixture model and background subtraction.

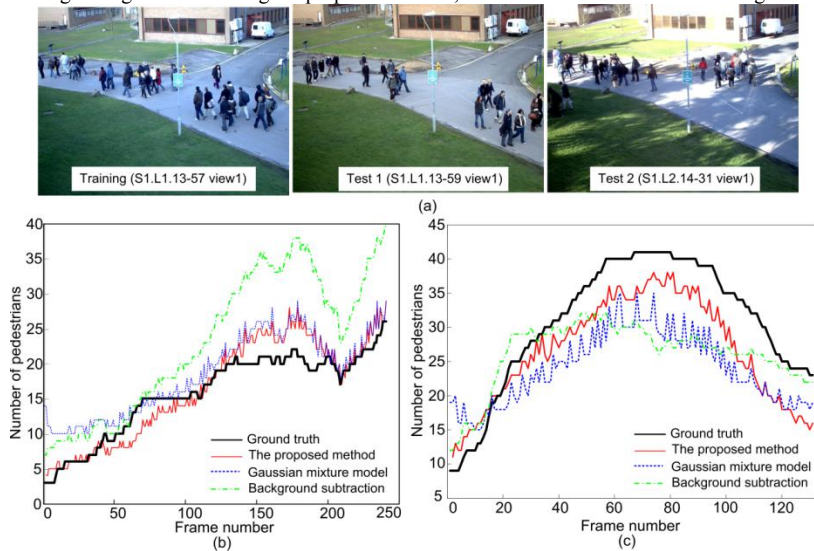


Fig. 8. Pedestrian counting of case I. (b) represents the result of test 1, (c) represents the result of test 2.

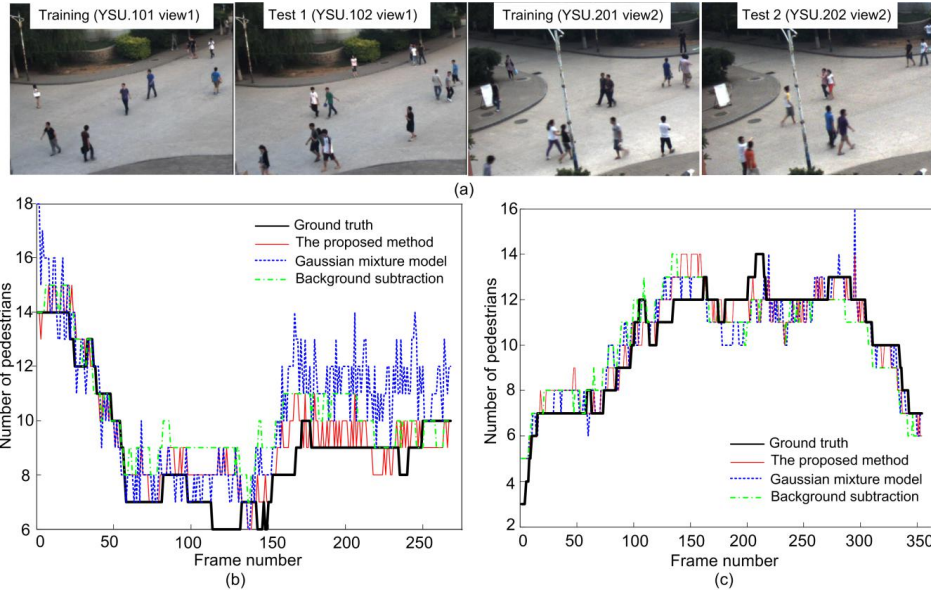


Fig. 9. Pedestrian counting of case II. (b) represents the result of test 1, (c) represents the result of test 2.

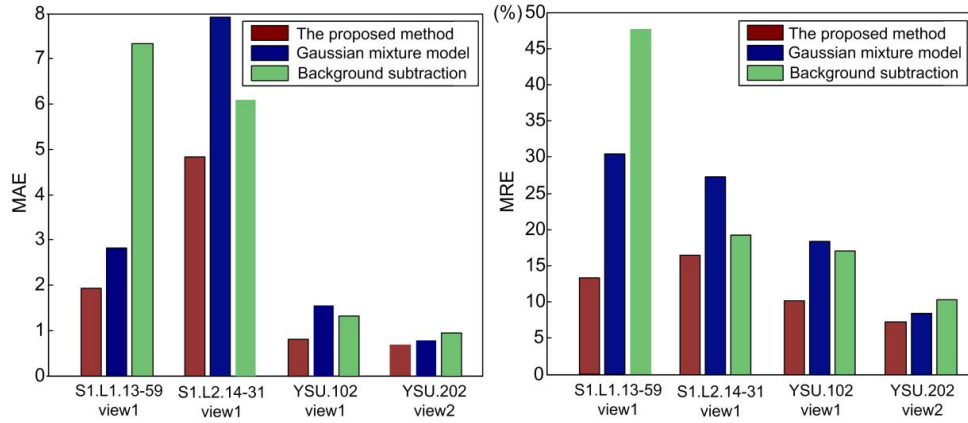


Fig.10. Comparison of MAE (left) and MRE (right) of different pedestrian counting methods.

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