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CRANFIELD

HUMAN FACTORS SPECIFICATION FOR A MACHINE TOOL SCALE READER

by

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Human factors specification for a machine tool scale reader

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SUMMARY

The length measuring devices on many machine tools must usually cover a large range while maintaining high accuracy for small incremental movements. Many machines use the operator as part of the measuring system, where he must assign numerals and carry out visual interpolation. This report attempts to define the preferrred characteristics of such indication systems in so far as they are influenced by the capabilities and limitations of the human operator. While no radical innovations are proposed, the principles underlying the design of scale readers have been examined as objectively as possible and recommendations substantiated by reference to relevant experiments.

Contents

In September 1963 the Director of the Machine Tool Industry Research Association arranged a meeting between representatives of the association and the Society of Instrument Manufacturers, to examine the requirements for optical scale readers for machine tools. Mr. Easterby of the Ergonomics Laboratory was invited to this meeting and subsequently requested to examine the human factors requirements of optical scale readers, for incorporation in a specification for a revised range of instruments which the machine tool industry feels it requires.

This report is the result of this study and the author gratefully acknowledges the value of discussions with members of the Production Department at Cranfield and in particular the stimulation provided by Mr. W.T. Singleton of the Ergonomics Laboratory.

1. Introduction

The measuring devices used on many large machine tools such as boring, milling and planing machines must usually cover a large range while maintaining high accuracy for small incremental movements. The basic measuring system may be required to measure distances of up to 10 ft. and incremental distances to an accuracy of ±.001", i.e. a system with a resolution of 1 part in 10^5 is often required.

No matter how carefully the mechanical characteristics of the machine tool and its associated measuring system are controlled, the ultimate determinate of the machining process is the performance of the operator. Thus it is essential that the precision of the machine, the measuring elements and the operator be properly matched.

For large, expensive machines the permissible cost of the measuring system can be quite high, allowing complex hardware to be employed to obtain satisfactory resolution. However, for smaller machines the system must be less ambitious, the cost per co-ordinate axis being limited to approximately 1% to 2% of the total machine cost. This limitation rules out the possibility of systems using diffraction gratings, servo numerical indicators, etc. and instead the operator's abilities must be exploited.

This report therefore attempts to define the preferred characteristics of such an indication system, in so far as they are influenced by the capabilities and limitations of the human operator. While no radical innovations are proposed, the principles underlying the design of scale readers have been examined as objectively as possible and recommendations substantiated by reference to relevant experiments.

2. Functions of the measuring system

2.1 Task Analysis

A functional diagram of a machine tool (Fig. 1) shows the relationships between the measuring system, the operator and the other machine tool elements. An essential feature of a functional diagram is the separation of the control and display elements from the machine tool proper. The controls and displays on a machine are not simple extensions of the mechanism, but links between the operator and his machine. This functional distinction ensures that the proper emphasis is given to the requirement of both the operator and the machine in designing these control and display links.

Any control movements initiated by the operator are based on information about the relationship between the machine elements and the workpiece. The tool is moved along a particular co-ordinate axis by referring to information derived from two distinct sources; either by direct observation of the tool and workpiece (i.e. from a real world display) or by observation of indications provided by the measuring system (i.e. from an artificial display). Functionally these two displays are put to different uses.

The real world display of the tool and workpiece may be used to establish a reference point by bringing the tool into contact with the workpiece. The information loop (see Fig. 1) is used as a coincidence detector. The operator, functioning as a feedback element, closes the loop which reduces the error between the actual output and the desired reference input. The reference input for this loop is zero (i.e. zero tool workpiece separation) and the error signal is the tool/workpiece separation.

The artificial display is used in a different mode. Its function is to establish the dimensional relationships between successive tool positions required for the machining process. Here the information loop (Fig. 1 again) is used as a positioning loop to locate the tool at a point which may or may not coincide with the surface of the workpiece. The reference for this functional loop is a dimension based on some arbitrary datum derived from the engineering drawing, and the error signal is the difference between this value and the reading on the display.

In the same way as for an electrical or mechanical position-servo-mechanism, accuracy and speed of location of the required position are facilitated by additional feedback of the rate of change of position. This allows the operator to predict the behaviour of the machine when it is in motion. Thus, where speed and accuracy are important for operation of the machine tool, it is essential for the operator to be able to derive this rate information from the display.

3. Allocation of functions

3.1 Coding functions

The essential function of the display is to assign numerals to the physical variable of length over 5 decades, (viz. .001 to 99" or .01 to 999 mm.) (Fig. 2). A secondary function is the provision of information on the rate of change of position of the tool relative to the workpiece. These two requirements raise two distinct problems -

a) The coding of the display, i.e. what is the best form of the display for the operator to interpret?

b) The dynamics of the system, i.e. what is the effect of the movements of the machine controls on the display?

These dynamic problems are considered in detail in sections 3.5 et seq.

The coding of the display can take one of two forms which are commonly known as analogue or digital displays. An analogue display in this context presents the operator with a scale which corresponds to or is an analogue of length. Superimposed on this display would be numerals to define each value of the display. The analogue display can provide some numerical information, but it usually relies on the operator to discriminate and assign numbers to some of the markers, and in some instances to points between markers. It has the advantage of indicating, in an acceptable form, the rate of change of position.

The digital display is simply a set of numerals defining one value of the length. This is excellent for rapid and unambiguous interpretation of the numerical information but is unacceptable for indication of rate of change of position.

3.2 Use of an analogue display

In general the use of an analogue display imposes the following constraints.

a) The maximum acceptable number of unnumbered points between numbered division points is 10. Above this figure - serious interpretation errors occur. (Murrell 1, 1958).

b) The maximum acceptable number of scale units not covered by division markers is 10. For readings to an accuracy of 1 scale unit this preferred maximum is reduced to 5 (i.e. interpolation by fifths) or better still to 2 (interpolation by halves). (Morgan et al, 2. 1962).

c) At least 3 numbered division markers must be within the field of view.

d) With multi-decade scales, to avoid gross interpretation errors the numerical value of all the decades corresponding to a particular point should be within the field of view.

e) For reliable interpretation (e.g. accurate reading 98% of the time) the minimum separation between the scale markers for accurate discrimination is 2' arc subtended at the operator's eye (i.e. at 15" viewing distance approximately 0.009"). (Murrell 1, 1958).

It is apparent that to cover a 5 decade scale to an accuracy of 1 in 10^5, it is impossible to fulfil all the requirements using only an analogue scale. Any attempt to improve the discrimination using simple magnifiers, restricts the length of scale in the field of view, with consequent difficulties in identifying the numerical values of the higher decades. Increasing the number of division markers introduces serious technical limitations. Reducing the number of markers leads to inaccuracies of interpolation.

3.3 Use of a digital display

In general the use of a digital display imposes the following constraints -

a) The figures must be unambiguously displayed. There must be no confusion as to the identity of the appropriate numeral. Particularly important is the stability of indication of the final digit.

b) If rate of change of position is important, some auxiliary display must be provided. (Grether, 4, 1949).

c) Every scale unit must be defined by a digit.

It would appear that apart from the limitations of indication of rate information, the digital display is acceptable, but it would require 10^5 separate combinations of 5 numerals to define every point in the range.

3.4 Use of a combined analogue/digital display

As a compromise solution, a combined analogue/digital display can be used; the digital display can provide the numerical information and the analogue display provides the rate of change of position information. Taking an extreme case, every point on the scale could have an engraved line and an engraved numeral, but this solution is obviously technically unacceptable. However, provided that the rate of movement of the display in response to control movement is still discernible, the analogue display need not necessarily be used to display the lower decades. The larger and more easily discernible decades can be covered by an analogue display with numerical indications at the engraved points. For the lower decades, where the significant figures cannot be reliably resolved by visual interpolation, some device is required which gives in direct digital form those significant figures not specified by the analogue display. The most important point to be borne in mind is the method of integrating the two sources of display information.

The possible schema for arranging the display codings are shown in Fig. 3. The five decades required can each be defined by either a numeral (shown as N in the schema) or an engraved marking (shown as a solid square in the schema). Thus, the example shown illustrates a five decade scale with numerals for the three highest decades and in addition engravings for the four highest decades. This requires the operator to assign numerals to the last two decades, the ultimate numeral having to be achieved by interpolation. Alongside each of the possible schema is a note defining the technical and human factors limitations of each type. The technical notes are obvious but the human factors limitations require a further elaboration.

When using the reader, the operator must, at some stage, memorise the

5 figure number. The ability to memorise and reproduce 5 digit numbers has been quite thoroughly studied (Conrad, 5, 1951: Pollack, 7, 1954). The accuracy of recall of the digits comprising a number varies with the relative location of the individual digits. The first and last digits are most accurately remembered, the penultimate digit is subject to slightly more errors while the intervening digits suffer the most errors of memory and recall. These findings clearly imply that close attention must be given to the display of the 2nd and 3rd digits, particularly as it is likely that the 3rd digit would be the choice for the overlap between the analogue display and the digital interpolator. It is thus extremely important that all the analogue engravings be supplemented by numerical indications. This will greatly facilitate the operator's task in interpreting the display since he will not be required to change his interpretative frame of reference when reading the 2nd and 3rd digits, which happen to be the most susceptible to forgetting errors.

The display should therefore provide, within the field of view, and preferably in line, the numerals for each decade of the measurement, supplemented by some form of analogue display. The display should not rely only on the analogue display (i.e. an unnumbered engraved marker) to define any of the digits. This proviso applies particularly to the display of the 2nd and 3rd digits.

The decision as to the appropriate number of decades to be indicated in analogue form is largely dependent on the magnification used. This in turn is a function of the accuracy required of the reader and the distance of the operator from the reader. In broad terms it is obviously undesirable to have the field of view less than the distance between 3 analogue markers (to fulfil requirement c in section 3.2) and the field of view of the operator should not exceed the field of precise vision, i.e. 2°.

The design of the display is therefore critically influenced by the selection of the analogue scale characteristics and the accuracy of positioning of the digital interpolator on the analogue reference markers.

3.5 Dynamics - reference setting display

The setting of the scale reader to a particular reference marker requires the accuracy of setting to be of the same order of magnitude as the accuracy of the measuring-system elements. Any marked disparity between the orders of accuracy of calibration and accuracy of setting and observation will result in the system errors being dominated by the larger of the two errors. Chapanis, 7, 1951, illustrates this point well and Appendix I of the report calculates, using his principles, the appropriate setting and observation accuracies required for a 5 decade scale reader. Thus, in order to read a 5 decade scale 99% of the time to 1 part in 10^5 the observation errors must have a standard deviation equal to or less than $\frac{1}{6} \times 10^{-5}$. This accuracy figure now requires translation into a form that is meaningful in terms of the characteristics of the human eye.

The size of an object is best defined in terms of the angle it subtends at the observer's eye. This results in a uniform method of specification of an object regardless of its actual size, its distance from the observer or the magnification system in use.

The most accurate method of setting relies on the inherently better performance of the human operator in making relative judgements rather than absolute judgements, i.e. a much more consistent performance is made in setting operations when using a bisection display rather than a simple pointer (Kissam, 8, 1962). The geometrical form of the bisection type display may vary but it would appear that the preferred type is the paired-line target (Kissam, 8, 1962) as shown in Fig. 4.

Using a paired-line target, an optimum geometrical form can be defined, the setting accuracy deteriorating if separation is too wide or too narrow. The curves shown in Fig. 5 illustrate this point, the data being based on experiments by Hick, Bates, 9, 1952 and Kissam, 8, 1962.

All these experiments indicate an optimum value of α_b of the order of 2' arc, where α_b is the angle subtended at the eye by the distance between one edge of the graduation and one of the paired lines of target marker, when the marker is centred. The width of the graduation marker itself does not appear to be critical, but it obviously must be well above the threshold level of absolute discrimination. An acceptable value is an angle of $1\frac{1}{2}'$ arc at the observer's eye.

The existence of an optimum value of α_b can, however, be slightly misleading. If the scale reader were always used with a fixed relationship between the scale, the reader and the operator, then the optimum value of α_b would in fact give minimum setting error. However, any departure either way from the designed viewing distance would result in an increase in the setting error expressed as an angular error at the operator's eye. We are, however, concerned with linear and not angular error and this linear error varies inversely with the viewing distance. It is thus more appropriate to replot these curves in terms of the actual linear accuracy of setting shown as a function of the linear dimension b and the viewing distance, all these distances being the actual dimensions and not the apparent size at the observer's eye. A further refinement is to introduce the magnification of the scale reader, since this essentially reduces the apparent distance of the scale from the operator. The data sheet in figure 6 therefore shows the value of the required system tolerance against the ratio of viewing distance divided by the magnification, with dimension b as a parameter. The derivation of the curves is detailed in Appendix II. With these curves it should be possible to select the preferred values of b corresponding to a limiting measuring-system tolerance and possible ranges of viewing distance and magnifications for differing optical designs. Use of the preferred value of b will ensure that the errors introduced by setting of the interpolator to the reference marker will be insignificant compared with the accuracy of the measuring system.

3.6 Dynamics - reference setting control

The previous discussion on the optimum characteristics of the paired-line display assumes a reasonably optimal control/display configuration. An important parameter here is the sensitivity of movement of the display to control movements which vary the location of the interpolator relative to the parent scale. An excellent parallel to this problem has been studied in detail by Craik and Vince (10, 1945) where they recommend knob sizes for controls with the axis parallel to the operator's body and with the axis at right angles to the operator's body. With the axis parallel to the body, the preferred mode of operation is to roll the knob between the forefinger and thumb, using the area from the most extreme joint to the end of the finger or thumb. With the axis at right angles to the body a completely different mode is used, the wrist being rotated, keeping the position of the fingers constant. These differing anatomical limitations give rise to two ranges of optimum diameters and control/display movement ratios, depending on the orientation of the axis of the control knob.

The exact derivation of these optimum values is detailed in Appendix III and two data sheets are shown which enable the ranges of control/display ratios (expressed as movement of the display/revolution of the control knob) to be defined. (Figs. 7, 8). Since the magnification of the optical system effects this ratio, the curves are shown with $^e/k$ as the independent variable. The knob diameters and the control/display ratios are not critical. The data sheets therefore show the limiting knob sizes, and the ranges of corresponding preferred and acceptable control/display ratios. This method of specification indicates the latitude which is available in the geometrical design of the scale reader and its controls.

The other important feature of the reference setting control is the direction of motion stereotype, i.e. the preferred arrangement between the direction of rotation of the control and changes in the scale display. Based on the work of Bradley (11, 1954), there are three basic conventions which could be considered -

i) a clockwise rotation of the control should result in the scale markers moving from left to right (or bottom to top for vertical and transverse readers).

ii) scale numbers should increase from left to right (or bottom to top for vertical and transverse readers).

iii) clockwise rotation of the control should increase the setting value.

However, it is impossible to fulfil simultaneously, all these requirements but for this type of application Bradley's (11, 1954) experiments indicate that where terminal errors are critical, condition (ii) is the most important to adhere to. The control of the scale reader should therefore be based on scale values increasing from left to right (or bottom to top) and clockwise rotation therefore decreasing the setting value.

4. Psychophysics

4.1 Numeral and letter sizes

The tolerances on numeral and letter sizes for optimum legibility are fairly wide, but confusion of identification can be avoided by careful selection of the type style. The use of serifs can often lead to interpretation errors and particular attention should be given to relative shapes of threes and eights, twos and fives, to avoid reading one for the other. McCormick (13) details some of the recommended styles which have been shown by experiment to minimise the effects of reading errors.

The proportions of the numerals also influence their legibility and again, from some of the studies reported, a ratio of 0.6 to 0.8 for the overall width to overall height is preferable. For black figures on a white ground the stroke width/height ratio should be .016. This ratio should not be exceeded, especially under limiting conditions of contrast (see section 4.3), but it is acceptable to reduce this ratio to .013 and still maintain reasonable legibility. Two satisfactory styles of type available commercially are shown in Fig. 9.

The preferred overall height of the numerals is influenced by the contrast values available, and as it is likely that the contrast will not be exceptionally high, the minimum apparent size of numeral should lie between 0.10″ and 0.20″. These values apply to apparent size of the numeral as seen by the operator and allowance can be made for any magnification system when considering the engraved numerals on the scale itself.

4.2 Marker sizes

From considerations of analogue and digital displays outlined in section 3, the actual scale markers should not be used for direct visual interpolation. Consequently the marker sizes are not as critical as they otherwise might be, but a suitable structure can provide the operator with additional information. The apparent size of the markers should be at least as large as the following dimensions:

Major markers (1st and 2nd decades)	.012″ wide, .100″ high
Minor markers (3rd decade)	.008″ wide, .050″ high
Intermediate marker (every fifth division in the 3rd decade)	.010″ wide, .075″ high

4.3 Contrast, brightness and specular reflections

The contrast between the image of the engraved lines, targets and numerals and the background can affect the performance of the operator. Equally, the brightness of the background can also affect the performance if the brightness falls below a limiting value. The general illumination level of the surrounding areas can create difficulties if the background is not bright enough since the operator's eyes adapt to this higher brightness and, as a result, are not able to make fine discriminations at the lower brightness levels of the scale reader display.

For displays involving the bisection technique for setting (paired-line), the background brightness should be at least 10 foot lamberts to ensure that observation errors are independent of the background brightness. With the contrast values obtainable with engraved scales (approx. 0.4) this will ensure satisfactory levels for absolute and vernier acuity.

A critical factor is the amount of light falling on the display which is then reflected from the surface of the lens or protective glass cover (specular reflection). This stray illumination effectively increases the background brightness without an associated change in the brightness of the object, which makes discrimination more difficult. A detailed examination of this topic is not possible here because of the difficulty of quantifying the illumination levels, but from examination of the threshold curve for brightness contrast discrimination (Blackwell, 14) it is obvious that careful attention to the geometry of the reader can minimise these effects.

Two useful techniques have been used in an attempt to combat reflected light from the surfaces of the display. One is to use a commercially available non-reflecting glass which diffuses the reflected light. It must, however, be close to the display (within $^1/_4$") otherwise it also diffuses the display image. It also requires a flat display surface and this is, of course, difficult if the outer surface of the viewing lens is, in fact, the cover to the scale reader. Another technique is by suitable choice of the angle of the glass cover face. The reflection which appears in the field of view can be made to correspond to a dark area of the immediate environment - for example, the operator's clothing rather than bright parts of adjacent machines or overhead lighting. It may also be possible to make marginal improvements using a tinted glass which does not transmit the colour of the immediate surround. However, these suggestions are qualitative only - there does not appear to be a great deal of theoretical or experimental evidence to support these ideas, and they must be considered in this light.

5. Summary

The ideal characteristics of the scale reader for achieving the best performance from the operator should be as follows:

a) Display presentation

The reader should present an in-line numerical display, each numeral defining the value for each decade of the measurement.

In addition, an analogue display of the actual engraved scale, which should cover the 3 highest decades should be presented showing at least two engraved markings within a 2° field of view of the operator. The scale should preferably have the graduations increasing from left to right.

b) Setting

The preferred arrangement of setting the reader to one of the engraved markings is a paired-line target, with the dimensions defined by the optimum curves included in this report.

For readers where the co-ordinate axis is vertical or at right-angles to the operator's body (vertical or transverse co-ordinates), the diameter of the reference-setting control knob should lie between 0.8″ and 2.0″ with the axis parallel to the operator's body. Clockwise rotation of the control should decrease the scale reading (provided the graduations increase from bottom to top in the display). The ratio of movement of the reader per revolution of the control knob should be within the limits in the data sheet in this report.

For readers where the co-ordinate axis is parallel to the operator's body, the diameter of the control knob should lie between 1″ and 3.5″ with the axis at right angles to the operator's body. Clockwise rotation of the control should decrease the scale reading (provided the graduations increase from left to right in the display).

c) Numerals and markers

The apparent size of numerals should be between .10″ and .20″, the numerals being a sans-serif form with a width/height ratio of 0.6 to 0.8 and a stroke width/height ratio between 0.13 and 0.16.

Three marker sizes should be used, one size (.012″ wide and .100″ high) to define the two highest decades, a smaller size (.008″ wide and .050″ high) to define the 3rd decade and a third intermediate size (.010″ wide and .075″ high) to define the mid points of the third decade. These are the desirable minimum apparent sizes.

d) Brightness and contrast

The background brightness of the display should not fall below 10 ft. lamberts and the contrast of the numerals, engravings and graduation line display with this background should be at least 0.4.

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Appendix I

(a) Overall errors

A measuring system (i.e. scale graduation) tolerance of $\pm e_s$ corresponds to approximately one engraving in 300 exceeding this limit. This value e_s will be referred to as the measuring system tolerance.

i.e. the distribution of engraving errors may be assumed to be normally distributed with mean of zero and standard deviation of $e_s/3$.

i.e. $\sigma_s = \frac{e_s}{3}$.

For an observer reading the scale it is also possible to assume that the reading errors will be distributed normally with a standard deviation of, say, σ_o.

The total variance of the measuring system and the operator is

$$\sigma^2 = \sigma_o^2 + \sigma_s^2$$

Thus, for optimum performance the display must be designed to keep the contribution of the operator to the total variance as small as possible in relation to the variance due to limitation of the measuring system itself. It is reasonable to assume the operator observation error should not increase the measuring system tolerance by more than about 10%.

i.e. $\sigma = 1.1\ \sigma_s$

i.e. $\sigma^2 = 1.21\ \sigma_s$

$\therefore\ \sigma^2 = \sigma_s^2 + 0.21\ \sigma_s^2$

But since $\sigma^2 = \sigma_s^2 + \sigma_o^2$

$\sigma_o^2 = .21\ \sigma_s^2$

or $\sigma_o = .46\ \sigma_s$

i.e. $\sigma_o \simeq \frac{\sigma_s}{2}$

(b) Tolerable observation error

From Appendix 1(a) above, if the standard deviation of the operator's error is one half of the standard deviation of the measuring system error, then the standard deviation of the overall system error = 1.12 measuring system error.

Accepting that the tolerance of the measuring system is equivalent to 3 standard deviations of the error of the measuring system, i.e.

$$e_s = 3\,\sigma_s$$

$$\therefore \qquad \sigma_s = \frac{e_s}{3}$$

$$\text{Now} \qquad \sigma_o = \frac{\sigma_s}{2}$$

$$\therefore \qquad \sigma_o = \frac{e_s}{6}$$

i.e. to maintain the overall accuracy within 1.12 times the measuring system tolerance the standard deviation of the observer's error must be, at most, 1/6 of the measuring system tolerance.

Appendix II

Derivation of data sheet -

Optimum paired-line target

Let e_o = tolerable observation error (equals 3 times the standard deviation of the observer's error)

b = width of paired target (refer figures 4, 5, 6) ins.

k = magnification

d = viewing distance (ins.)

αe_o = angle subtended at the eye by tolerable observation error (minutes arc)

α_b = angle subtended at the eye by dimension b (minutes arc)

$$\alpha e_o = \frac{e_o}{cd} \; ; \quad \alpha_b = \frac{b}{cd}$$

$(c = \text{constant} = 2.91 \times 10^{-4})$

If a magnification system is used, the angle subtended at the eye by the image of the object is increased by k

$$\therefore \quad \alpha e_o = \frac{ke_o}{cd} \quad ; \quad \alpha_b = \frac{kb}{cd}$$

Now using Kissam's data (8, 1962) αe_o and α_b are related by the function plotted in the curve in Fig. 5. This shows 3 standard deviations of the operator's observation error (αe_o) as α_b is varied.

$$\text{i.e. } \alpha e_o = f[\alpha_b]$$

$$\therefore \quad \frac{1}{c} \; e_o \frac{k}{d} = f[\frac{kb}{cd}]$$

$$\therefore \quad e_o = c \cdot \frac{d}{k} f\left[b \cdot \frac{1}{c \cdot \frac{d}{k}}\right]$$

But from Appendix I, $e_o = \frac{e_s}{2}$ where e_s is to tolerance of the measuring system

$$\therefore \quad e_s = 2c \frac{d}{k} f\left[b \cdot \frac{1}{c \cdot d/k}\right]$$

These curves are plotted with $^d/k$ as the independent variable and b as a parameter in figure 6.

Appendix III

Gain of reference setting control

Craik and Vince (10, 1945) describe experiments where they define the optimum circumferential or angular movement of the control for a range of preferred knob diameters. These sizes and ranges are dependent on:

D - diameter of knob

m - optimum circumferential movement (applies to transverse and vertical co-ordinates)

θ - optimum angular movement - degrees (applies to lateral co-ordinates)

G - gain of control - movement of reader relative to scale/per revolution of knob (inches/rev.)

e_s - measuring system tolerance (3 standard deviations of the measuring system error)

e_o - tolerable observation error (3 standard deviations of the observation error)

j - just noticeable misalignment of display

k - display magnification

Relation of j to e

Craik defines the error at the optimum as 0.2 of the just-noticeable difference.

$$\therefore \quad j = \frac{5e_o}{k}$$

$$\text{But } e_o = \frac{e_s}{2}$$

$$\therefore \quad j = \frac{5e_s}{2k}$$

Data sheet for transverse and vertical co-ordinate displays

$$\frac{Gm}{\pi d} = j$$

$$\text{But} \quad j = \frac{5e}{2k}$$

$$\therefore \frac{Gm}{\pi d} = \frac{5e}{2k}$$

$$\therefore Gm = \frac{5\pi}{2} \cdot \frac{e}{k} \cdot d$$

The left quadrant of the data sheet plots Gm against $^e/k$ with d as a parameter ranging over the preferred range of 0.8" to 2.0".

The right quadrant relates Gm to G for the preferred and acceptable ranges of m.

Preferred -	.45" - .65"		
Acceptable -	.3" - .45"	OR	.65" - 8"
Unacceptable - Less than .3"		OR	Greater than .8"

<u>Data sheet for lateral co-ordinate displays</u>

$$\frac{\theta G}{360} = j$$

But $j = \frac{5e}{2k}$

$$\therefore \frac{\theta G}{360} = \frac{5e}{2k}$$

$$\therefore G\theta = 900 \cdot \frac{e}{k}$$

The left quadrant of the data sheet plots Gθ against $^e/k$ and since this expression is independent of D, Gm is the same for all D between 1.0" and 3.5", the preferred range of diameters.

The right quadrant relates Gθ to G for preferred and acceptable range of θ.

Preferred -	27°-42°		
Acceptable	20°-27°	OR	42°-50°
Unacceptable - Less than 20°			Greater than 50°

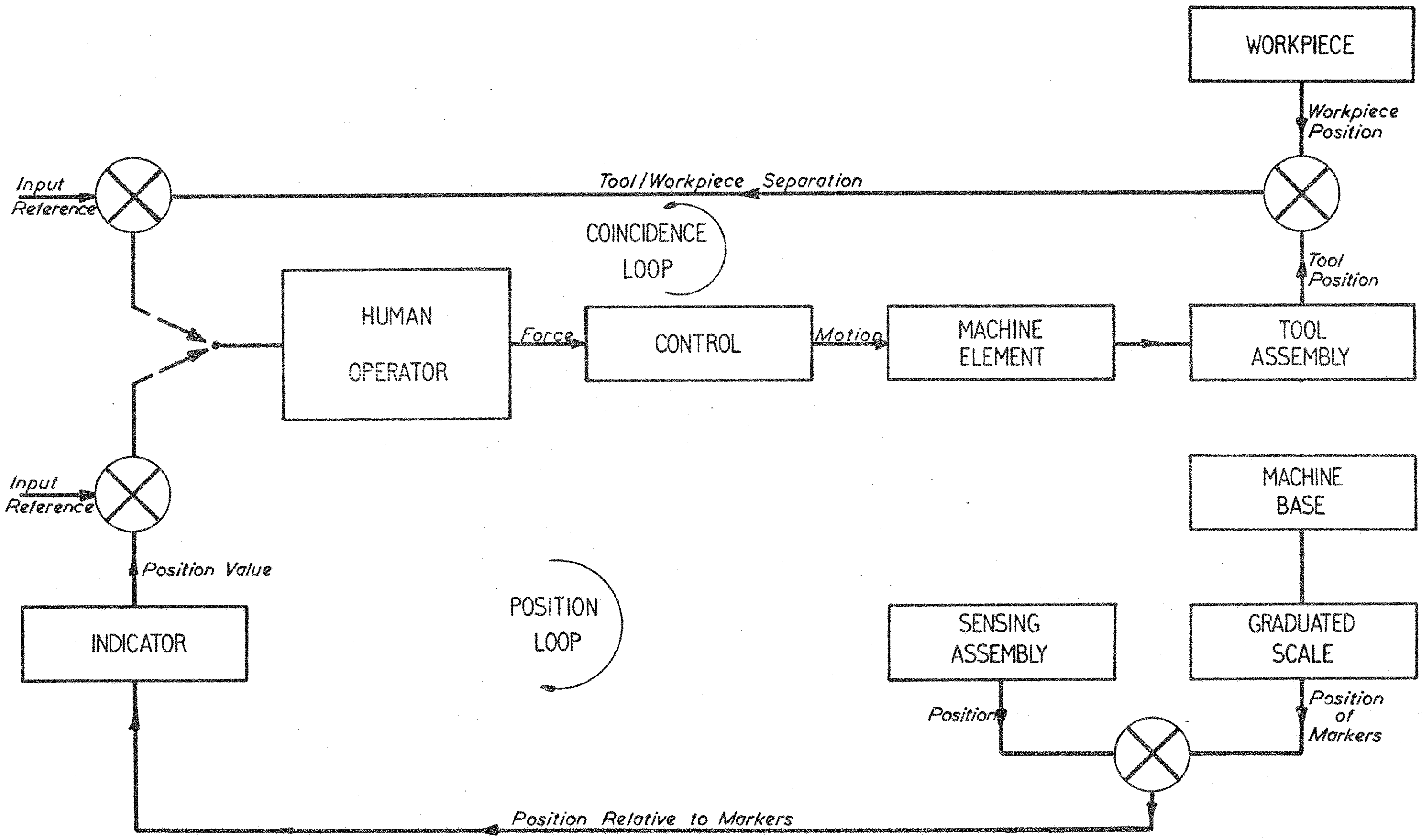


FIG. I. FUNCTIONAL DIAGRAM OF MACHINE TOOL INDICATING SYSTEM

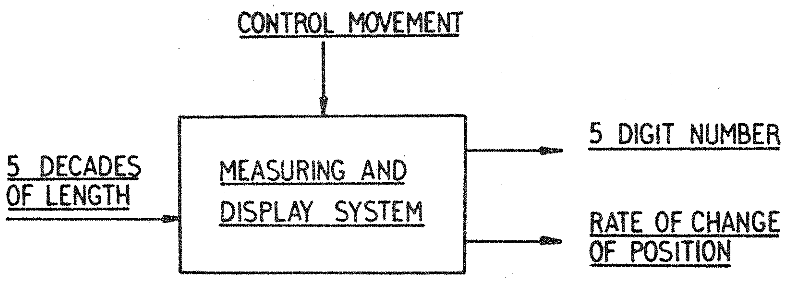


FIG. 2. FUNCTIONAL DIAGRAM OF MEASURING SYSTEM

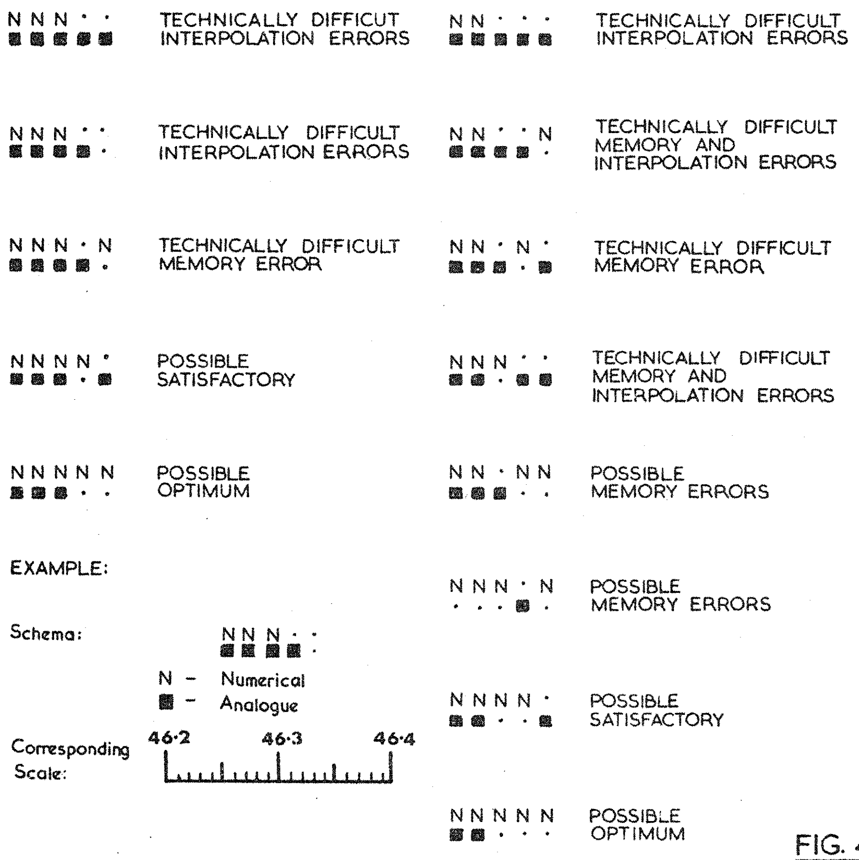
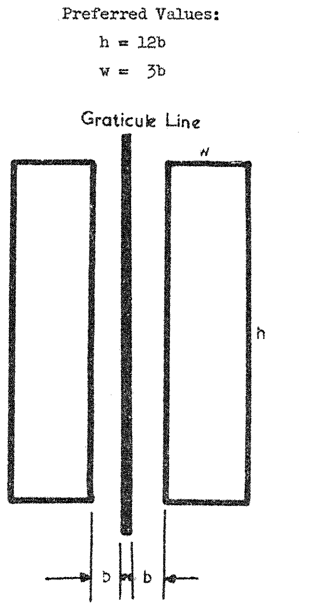


FIG 3 SCHEMA OF POSSIBLE DISPLAY CODINGS



Based on Kissam (1962)

FIG. 4 PAIRED-LINE TARGET

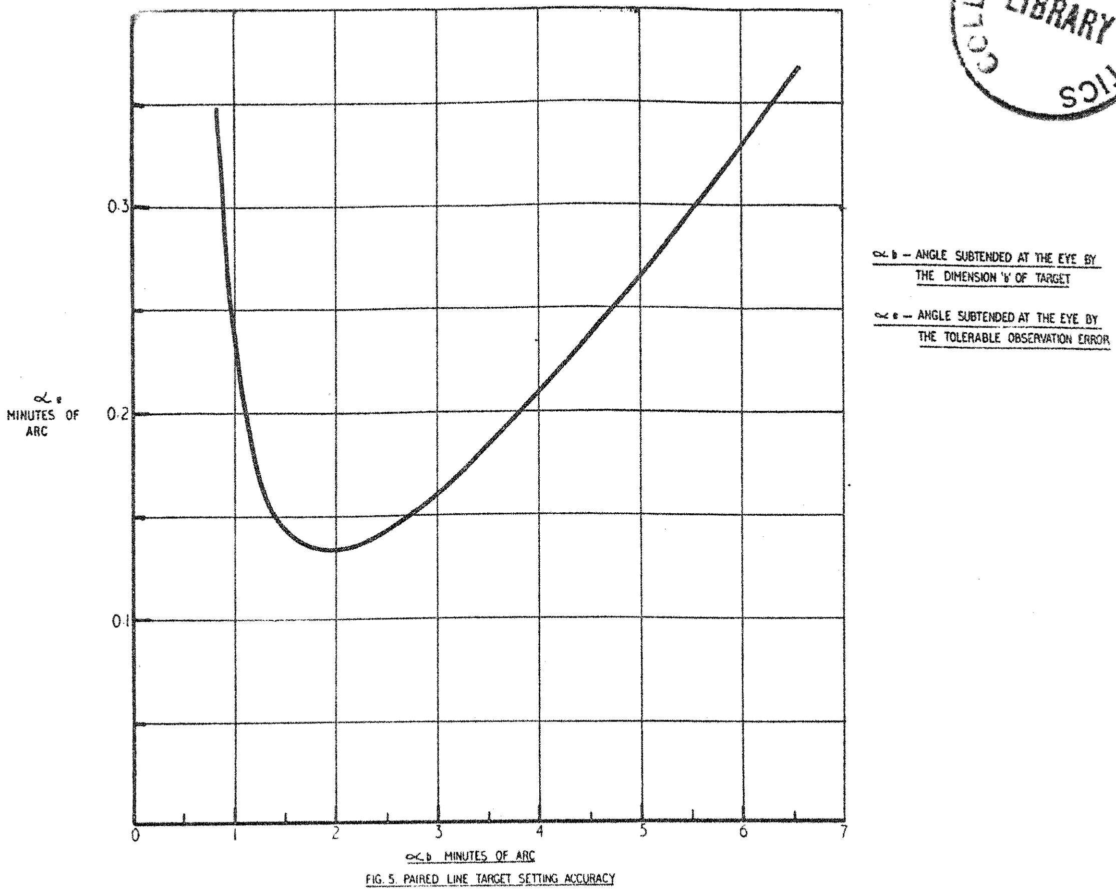


FIG 5. ACCURACY OF SETTING PAIRED-LINE TARGET

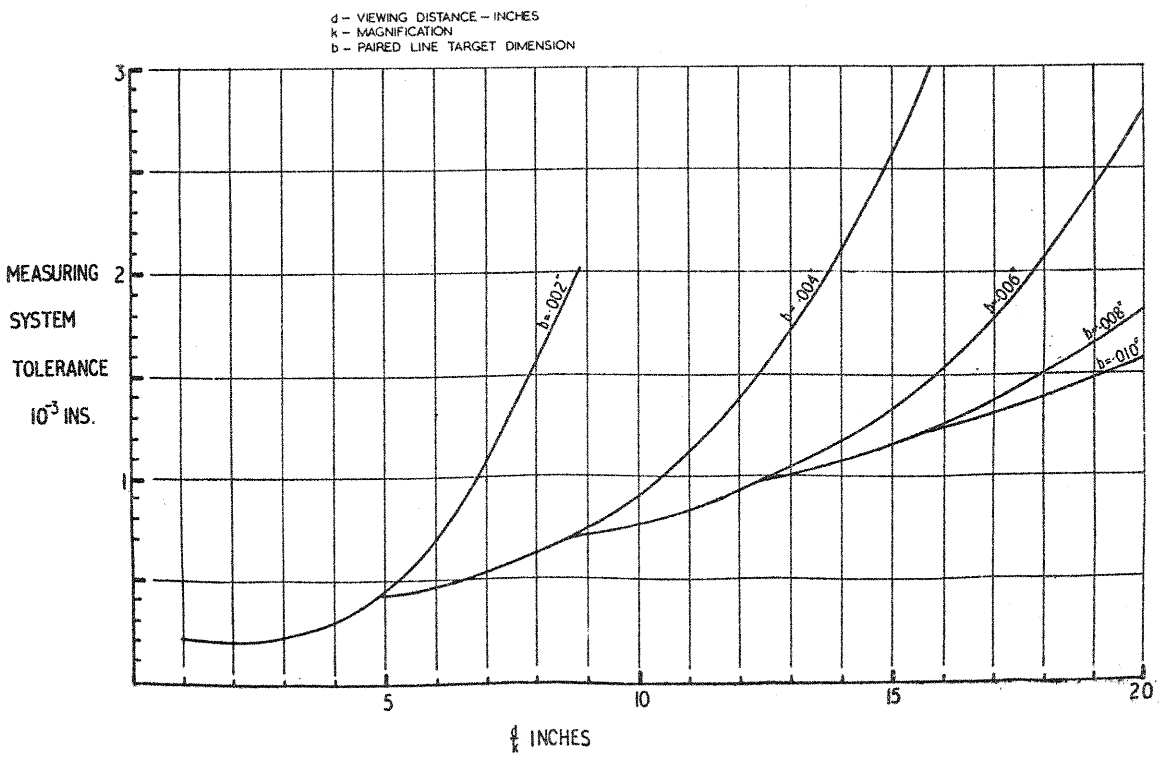


FIG 6. DATA SHEET–OPTIMUM PAIRED-LINE TARGET

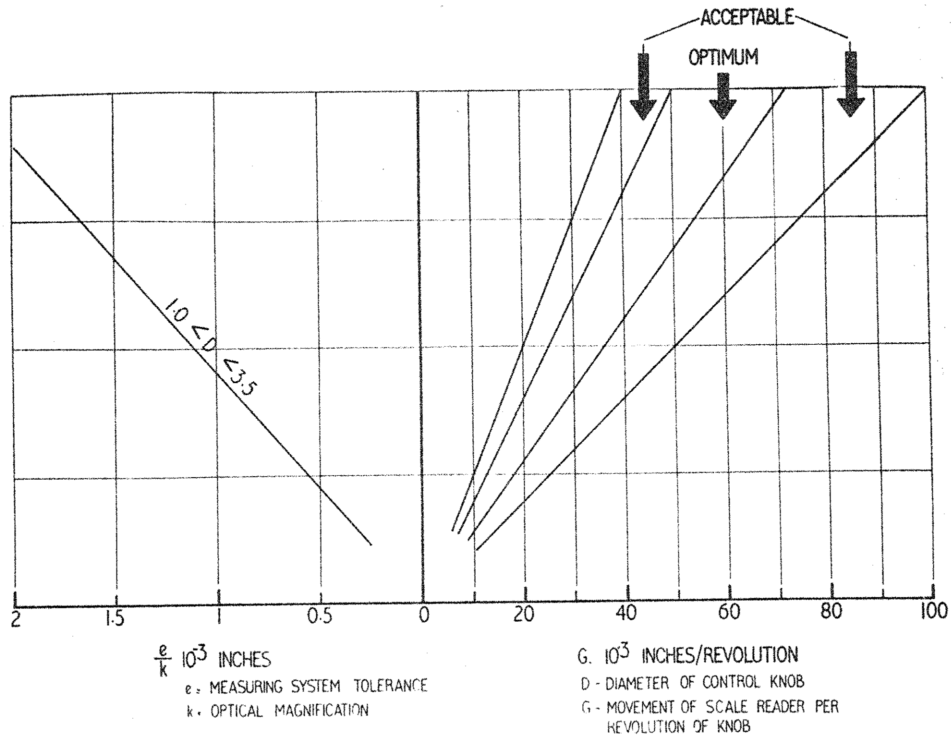


FIG 7 DATA SHEET—OPTIMUM CONTROL GAIN FOR LATERAL CO-ORDINATE DISPLAY

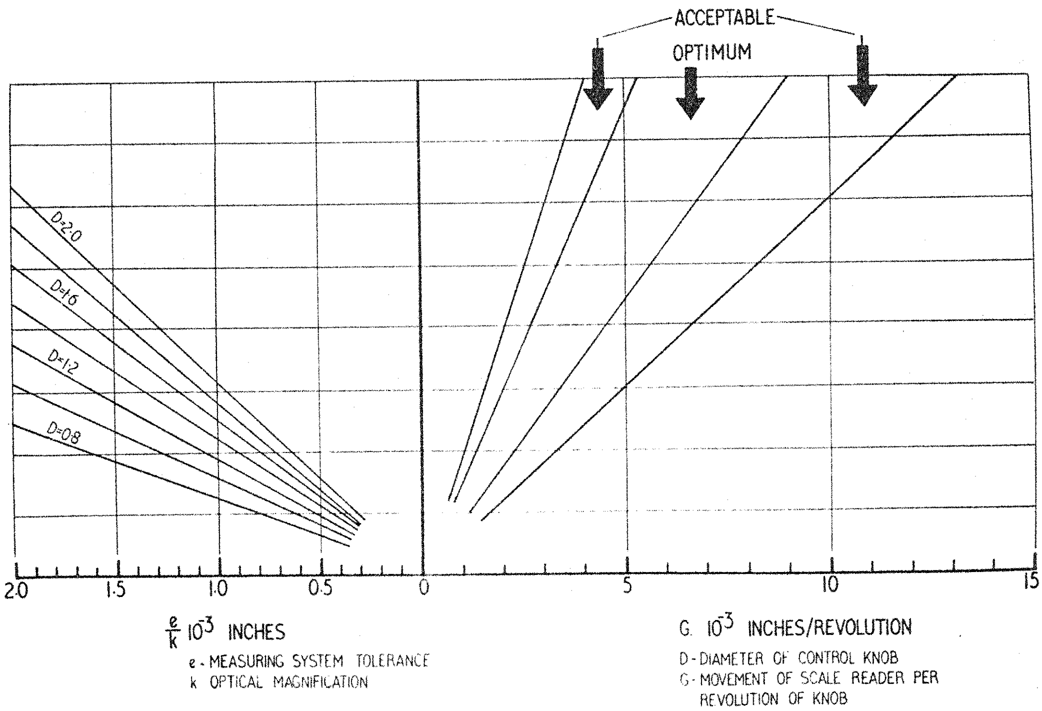


FIG. 8 DATA SHEET—OPTIMUM CONTROL GAIN FOR TRANSVERSE AND VERTICAL COORDINATE DISPLAY

1 2 3 4 5 6 7 8 9 0

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FIG. 9 - EXAMPLES OF PREFERRED TYPES OF NUMERALS