# Sraffa's reproduction prices versus prices of production: probability and convergence.

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**Abstract**

The paper argues that the reproduction prices introduced in the first chapter of Sraffa's book are not necessarily compatible with the profit equalising prices that form the substance of the book.

It uses probabilistic arguments about how probable it is that reproduction prices will approximate to profit equalising prices. By use of random matrix techniques it shows that the solutions space associated with prices of production is similar to that associated with classical labour values.

In the latter part of the paper, a random sample of reproduction schemes is simulated over time, under assumptions of capital movement, to see whether

such systems dynamically converge on profit equalising prices. It is found that some converge, and some fail to converge.

## 1 Reproduction prices

In the first pages of (Sraffa, 1960) a simple two product economy producing corn and iron is introduced with the following structure:

| | | | | |
|---|---|---|---|---|
| 280 wheat | + | 12 iron | $\rightarrow$ | 400 wheat |
| 120 wheat | + | 8 iron | $\rightarrow$ | 20 iron |

In volume I of Capital(Marx, 1887) Marx used a price theory in which commodity prices are taken to be proportional to the labour required to make the commodity. He used this price theory to argue for a theory of surplus value, according to which workers were exploited of the full value created by their work. In Chapter 20 of the second volume(Marx, 1974) he introduces reproduction schemes, matrices of inter-sectional flows of commodities that had to occur if the economy was to reproduce itself.

The matrices have 4 column vectors:

C constant capital, his term for expenditure on capital goods and raw materials.

V varible capita, his term for expenditure on wages.

S surplus or profit.

O output

We give an example in Table 1 of a 2 x 4 matrix, with the row labeled I representing the production of capital goods and raw materials, and row II the production of

| | C | V | S | O |
|---|---|---|---|---|
| I | £100 | £50 | £50 | £200 |
| II | £100 | £150 | £150 | £400 |

Table 1: A stationary state specified in money

consumer goods. In Marx's tables all quantities are in terms of money rather than in terms of use values[1]. For accounting reasons the relation $O = C + V + S$ must hold.

Further $\sum C = O_1$that is to say consumption of capital goods equals their production, and $\sum(V + S) = O_2$. Together this implies that sector I of the economy must trade $O_1 - C_1$ in capital goods for $C_2$worth of consumer goods produced in sector II. So we have an equilibrium equation

$$C_2 = V_1 + S_1$$

This is the equilibrium condition of an economy in a stationary state where it simply reproduces itself neither growing nor shrinking.The basic analysis in this paper will assume this stationary state. Real economies may grow or shrink, but the rate at which they do this is typically quite small. A developed industrial economy like the US can go long periods in which the rate of growth averages only 3% a year or less, so analysis of price systems in a stationary state is a reasonable first approximation.

Although it is not done by Marx, one can in principle construct a dual table like Table 2in tons or consumer goods (corn) and tons of capital goods(coal). In this

[1]They thus differ from the technology matrices of Morishima(Morishima, 1973), though, as we shall see there is an underlying relationship between the two.

| | Coal | Corn wage | Corn Profit | Ooutput | |
|---|---|---|---|---|---|
| I | 10 | 20 | 20 | 20 ton | Coal |
| II | 10 | 60 | 60 | 160 ton | Corn |
| Total | 20 ton | 80 ton | 80 ton | | |

Table 2: A stationary state specified in tons matter

case the first column represents the coal used up productively by the two industries, and next come the consumer goods (corn) consumed by the workers and employers in the two sectors. Again we have the requirement that the total consumption and total production of each good must balance, 160 tons of corn, and 20 tons of coal.

It is clear from this table that the coal industry must sell 10 tons of coal to the corn industry and get back in return 40 tons of corn, which in turn implies that the relative price of a ton of coal must be 4 times the price of a ton of corn. Refering back to the first table and comparing it with the second we see that indeed the price of a ton of coal was £4 and but a ton of corn cost only £2.50. The important point here is that given the physical table, the relative prices necessarily follow.

The example is artificial in that in practice sectors I and II would each produce a whole vector of outputs, but given the constants of proportionality between the elements of these two vectors, the exchange relation between them establishes relative sectoral prices.

Marx later extends the scheme to 3 sectors, by dividing consumer goods into necessities(IIa) which are assumed to be bought out of wage incomes and luxuries(IIb) which are bought out of property incomes. If we retain the label II for necessities and use III for luxuries, we have the three way trade between sectors in Fig 3.1.

The tables are given in money terms, much as modern national accounts are, but the assumption explicitly remained that these quantities of money are proportional to quantities of labour((Marx, 1974) Chapter 21, Section 7). But in principle other pricing structures are possible so long as they allow the trade pattern in Fig 3.1. In what follows we will show that the reproduction schemes themselves imply a disctinct set of price configurations and that these price configurations only partially overlap with those presupposed by either labour values or prices of production.

## 2 Prices of production

In Volume III of Capital(Marx, 1894b) Chapter 9, Marx introduces a distinct model of prices, which he terms production prices. He points out that the prior assumption of a constant ratio $\frac{s}{v}$ accross industries whilst the ratio $\frac{c}{v}$ varied between industries would lead to the profit rate $\frac{s}{c+v}$ to vary between industries. He hypothesised that this would be unstable and that as a result of capital movements into high profit industries, a uniform average rate of profit would be attained. The resultant production prices would preserve an equal rate of profit but the side effect would be that prices would diverge from labour values, being systematically higher in industries with a high organic composition of capital. The relatively higher prices in high organic composition industries would then enable them to earn the average profit rate.

Whilst some commentators argued that prices of production tended to undermine his prior arguments about labour values and exploitation (Hilferding, 1951; Samuelson, 1973; Steedman, 1981), the basic hypothesis of a law of an equal rate of profit was accepted until the publication of the pioneering econophysics work Laws of Chaos(Farjoun and Machover, 1983). This argued on probabalistic grounds that the distribution of prices was more likely to follow a simple labour value model than a price of production model. More recently Greenblatt(Greenblatt, 2014) has also proposed a stochastic model in which labour values appear as an emegent property along with a spread of profit rates.

Multiple empirical studies (Cockshott and Cottrell, 1997, 1998; Fröhlich, 2013; Sánchezc and Montibeler, 2015; Shaikh, 1998; Zachariah, 2006) have indicated that production prices are not systematically better at predicting actual market prices than simple labour values. It has also been shown that the Marx's basic assumption that the rate of profit is the same in high and low organic compostion industries is not born out empirically today(Zachariah, 2006, 2009), whatever may have been the case in the 19th century. However this empirical work does not help us to say whether the observed relationship between labour values/production prices and market prices is 'close'. The studies reproduced in Table 5 show mean absolute errors of the order of 10% between labour values/production prices and observed prices. But is 10% close or distant?

We can only say that if we have some a-priori estimate of just how close we should expect the market price vector to be labour values/production prices in the absence of the operation of a law of value, or Marx's law of the equalisation of profit rates.

In what follows a new probabalistic technique using reproduction schemes is developed to evaluate these empirical results. The basic intuition is that one can systematically count what fraction of possible reproduction schemes are consistent with prices of production or labour values.

As such they enable us to numerically evaluate the a-priori likelyhood of labour values/prices of production being within, for example, 10% of market prices.

## 3 Constraints on reproduction schemes

In Tables 1 and 2 we showed that from the physical flow between sectors one could work out the relative sectoral prices. The aim here is to show how one can start out from a physical flow pattern for a 3 sector economy and deduce the relative sectoral prices that must correspond to it.

We will use $G$, for Goods, to stand for our 3x3 matrix of flows of goods in kind such that the first column corresponds to the in-kind flows of capital goods that Marx denotes by his C column vector, the second column to the in-kind flows of wage goods corresponding to the column vector V, and the last column to the flows of luxuries denoted by the column vector S.

We stipulate that all elements are positive non-zero and that each column of $G$ adds to 1, ie, the elements of each column in $G$ are expressed as fractions of the total output of the coresponding sector. In other words we *normalise* the columns. A concrete example is given in Table 3.

We denote the elements of $G$ as $g_{i,j}$ for $i, j$ : 1..3.

If **p** is a 3 element price vector for capital, wage and luxury goods then

In order to have only 3 prices when in fact each sector makes a wide variety of goods, we assume that the prices are index prices defined over bundles of capital, wage and luxury goods. Given the actual physical flows in $G$ then the trade pattern

| | | Physical flow table | | | |
|---|---|---|---|---|---|
| | Coal | Corn | Caviar | outputs | |
| I | 16047 | 2801 | 14151 | 20004 | ton coal |
| II | 464 | 11898 | 3573 | 20017 | ton corn |
| III | 3493 | 5318 | 2286 | 20011 | ton caviar |
| Totals | 20004 | 20017 | 20010 | | |

the equivalent G matrix

| | | |
|---|---|---|
| 0.80 | 0.14 | 0.71 |
| 0.02 | 0.59 | 0.18 |
| 0.17 | 0.27 | 0.11 |

Table 3: For the purposes of studying the relation between physical flows on sectoral prices it is convenient to to express the flow elements as numbers between 0 and 1. We can do this by normalising a physical flow table, dividing each column element by the total of the column.

in Fig 3.1 establishes price constraints:

$$\begin{aligned} p_1 g_{3,1} &= p_3 g_{1,3} \\ p_3 g_{2,3} &= p_2 g_{3,2} \\ p_2 g_{1,2} &= p_1 g_{2,1} \end{aligned} \tag{3.1}$$

Where $P$ is a 3 element price vector whose elements are written $p_i$. For example given the $G$ matrix in Table 3 we can use the above equations to solve for the relative prices deriving :

$$P = [2.123, 0.352, 0.524]$$

from which we can derive the corresponding monetary relations given in Table 4.

Note that since the first equation fixes the ratio $p_1/p_3$and the next fixes $p_2/p_3$

| | Const capital C | Wages V | Profits S | O |
|---|---|---|---|---|
| I | £34067 | £986 | £7415 | £42468 |
| II | £986 | £4188 | £1872 | £7046 |
| III | £7415 | £1872 | £1198 | £10485 |
| total | £42468 | £7046 | £10485 | £60000 |

Table 4: An example of a 3 sector economy in a stationary state. The sector II now produces wage goods and sector III luxuries. This should be read in conjuction with Figure 3.1. Note the symmetry of the table around the diagonal corresponding to the trade pattern in the figure. This monetary table is derived from the Table 3 by solving equation set 3.1.

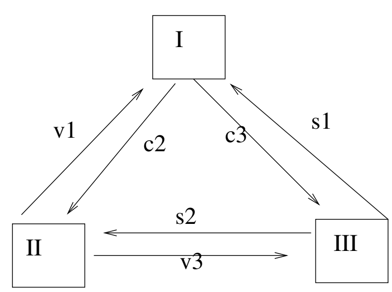


Figure 3.1: Three way inter sector trade. So for example sector II sells sector I wage goods worth v1 and buys back in return means of production c2.

then this implies $p_1/p_2$ is also fixed, so we have to interpret the last of the three equalities as a constraint on what kind of physical flow matrix is compatible with inter sector trade. The price constraints set by the $G$ matrix define market clearing prices for a system in which all sectors are self financing, that is to say, there is no credit provided by one sector to another. This was an implicit assumption of Marx's analysis in Volume II of Capital. But these reproduction constraints themselves impose restrictions on the structure of the G matrix. Not all normalised $G$ matrices are compatible with self financed simple reproduction.

Vol I, Vol II and Vol III of Capital actually provide three distinct price models which partially overlap. In Figure 3.2 we illustrate the volumes of configuration space that we are interested in. Reproduction schemes define, by equation 3.1, a set of market clearing price configurations - the large circle. Smaller circles denote the volumes of configuration space compatible with prices of production and labour values. Not all configurations that are compatible with labour values or prices of production are compatible with simple reproduction. By being compatible with prices of production we mean that the prices derived from Equation 3.1 result in rates of profit that are equal, or very nearly equal, in all sectors. By being compatible with labour values we mean that the prices from Equation 3.1 lead to nearly equal ratios of wages to profits in each sector.

The experiment reported in section 4 allows us to estimate the degree of overlap between these sets.

## 3.1 Derivation of $g_{2,1}$

Let us first examine how the structure of the G matrix is constrained by reproduction.

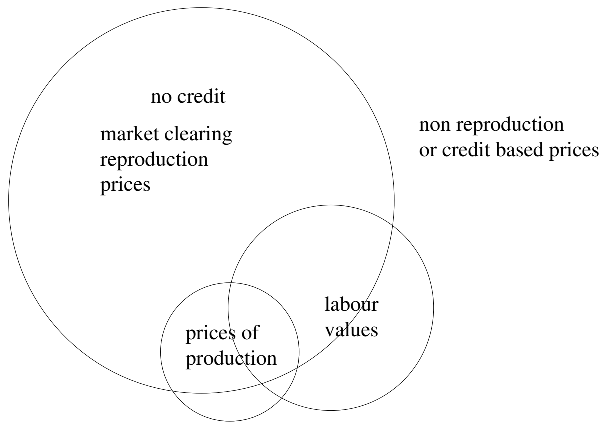


Figure 3.2: The set of price systems, this study is restricted to market clearing reproduction prices without credit.

Given $g_{3,1}, g_{1,3}, g_{2,3}, g_{3,2}, g_{1,2}$ we can derive $g_{2,1}$as follows:

$$p_1/p_3 = g_{1,3}/g_{3,1}$$

$$p_3/p_2 = g_{3,2}/g_{2,3}$$

$$p_1/p_2 = \frac{p_1}{p_3} \times \frac{p_3}{p_2} = \frac{g_{1,3}}{g_{3,1}} \times \frac{g_{3,2}}{g_{2,3}}$$

but from the original trade relation we have

$$p_1/p_2 = g_{1,2}/g_{2,1}$$

so

$$g_{1,2}/g_{2,1} = \frac{g_{1,3}}{g_{3,1}} \times \frac{g_{3,2}}{g_{2,3}}$$

and

$$g_{2,1} = \frac{g_{1,2}}{\frac{g_{1,3}}{g_{3,1}} \times \frac{g_{3,2}}{g_{2,3}}} \tag{3.2}$$

Alternatively the constraint can be expressed in terms of elements of the other two columns:

$$g_{2,3} = g_{2,1} \times \frac{g_{1,3}}{g_{3,1}} \times \frac{g_{3,2}}{g_{1,2}} \tag{3.3}$$

or

$$g_{1,2} = g_{2,1} \times \frac{g_{1,3}}{g_{3,1}} \times \frac{g_{3,2}}{g_{2,3}} \tag{3.4}$$

Taken along with our constraint that the columns of $G$ sum to 1, we have a 4 constraints on the 9 elements of the matrix leaving only 5 degrees of freedom to the configuration space of reproduction schemes. That is to say that simple reproduction schemes are samples drawn from an underlying 5 dimensional vector space. Given such a space we can systematically sample it.

## 4 First Experiment

A programme was developed that created successive random samples of the configuration space of reproduction schemes. First the elements of $G$ were assigned random values $> 0$ and $< 1$ such that the totals on columns 2 were each 1, and the expected value of each element was $\frac{1}{3}$. Then with equal probability one of the equations 3.2 to 3.4 was used to over-ride the previous random variable assignment to one of the elements. This constraint however is not guaranteed to satisfy the condition that the column must sum to 1, but that is achieved by subsequently altering the diagonal elements of the matrix to ensure that all columns sum to 1.

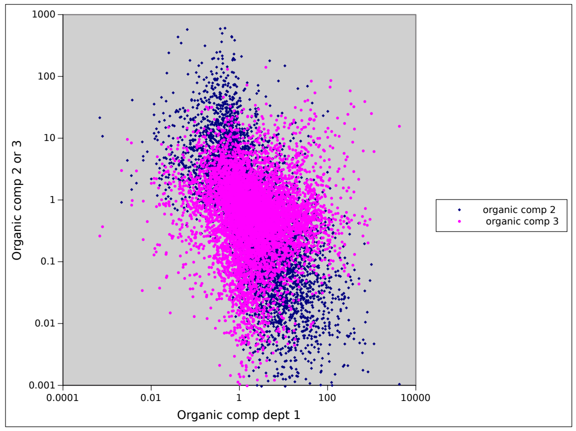


Figure 4.1: Spread of relative organic compositions over the entire sample set with the subsampling technique.

The diagonal elements do not enter into inter-sector trade and hence can be altered without disturbing the relations established in equations 3.2 to 3.4.

The mean of $G$ over 120,000 samples to two decimal places was

| | | |
|---|---|---|
| 0.40 | 0.28 | 0.32 |
| 0.25 | 0.40 | 0.28 |
| 0.35 | 0.32 | 0.40 |

This implies that the expected values for the organic compositions of capital, for reproduction schemes meeting equation 3.2 will differ between departments. This means we are not encountering a simple situation of uniform expected organic compositions. This can be seen in the distribution of relative organic compositions in Figure 4.1.

For each reproduction scheme configuration the market price vector was set by constraint 3.1.

Labour values were computed as follows

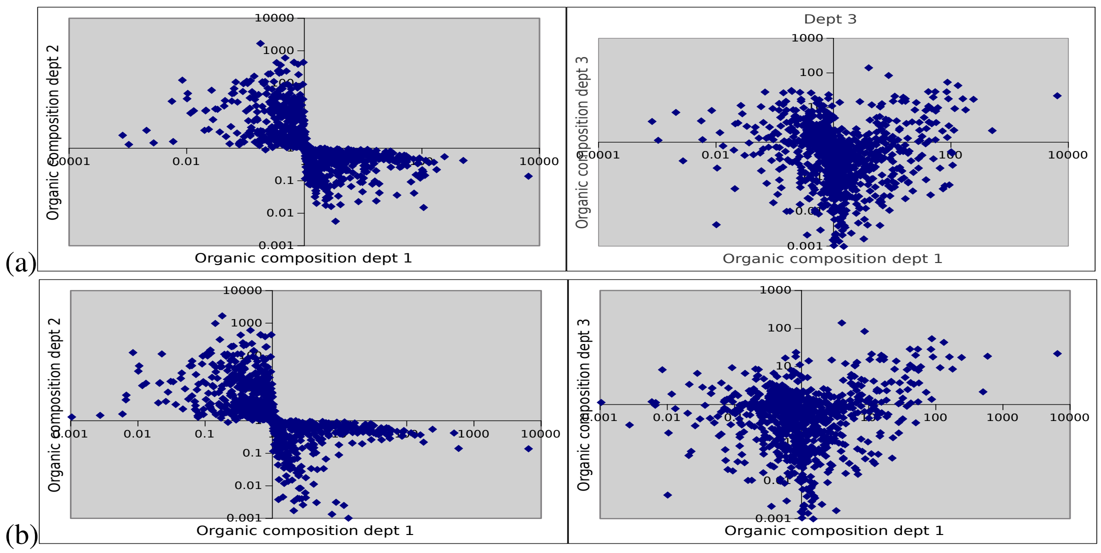


Figure 4.2: Plot of the relative departmental organic compositions of reproduction schemes in which market clearing prices were within 10% of : (a) labour values; (b) prices of production. Note that the characteristic ’bow tie’ configuration for the left plots also appears in the overall sample in Figure 4.1, but that whilst the first and third quadrants are empty here, samples were present in these quadrants in Figure 4.1. This indicates that these quadrants of configuration space are incompatible with either prices of production or labour values.

$$v_1 = g_{1,2}/(1 - g_{1,1})$$

that is to say divide the real wage in dept I by its net output. The assumption made is that the labour used in each sector is proportional to the flow of wage goods consumed.

$$v_2 = g_{2,2} + v_1 g_{2,1}$$

$$v_3 = g_{3,2} + v_1 g_{3,1}$$

At the end of this we have $v$ as vertically integrated labour coefficients, derived from wages.

$$\sum v = 3$$

The last is a normalisation condition used to ensure that under all prices models, the sum of prices is the same. If we do not apply this, we would have a sum of values $< 3$ in effect ignoring surplus value. But we are free to apply this linear rescaling to $v$ since the assumption Marx makes is that the rate of surplus value is the same everywhere. The implicit assumption here is that the real wage is the same in all sectors.

An iterative estimation is used for prices of production. We first set all prices to 1. Then we repeatedly perform the following steps.

Set $r$ to one plus the rate of profit.

$$r \leftarrow 1 + \frac{p_3}{p_2 + p_1} \tag{4.1}$$

This works because the physical output of each industry is unity by virtue of using a normalised $G$. Next set a new estimate of the price vector $np$.

$$np \leftarrow (\ r \times (p_2\ g_{\iota,2} + p_1\ g_{\iota,1})) \tag{4.2}$$

Finally we normalise the sum of prices to be 3, the same as before.

$$p \leftarrow \frac{3 \times np}{\sum np} \tag{4.3}$$

Runs were made with many reproduction schemes. The cummulative total number of resulting reproduction schemes was recorded along with the number of schemes that conformed either to labour values or to prices of production. Results are shown in Tab 5.

Conformance of either labour values or prices of production was determined by measuring whether their mean absolute deviation (MAD) from market prices were below a specified threshold. The experiment used a 10% threshold as that is of the right order for the best that is observed empirically. In addition the program records the mean of the MADs between exchange values and the two pricing theories.

## 4.1 Results

From the random sampling of reproduction schemes it appears that the mean spread of prices of production from market clearing prices is smaller than the mean spread of labour values. This is incompatible with a number of empirical studies as shown lower in the table where the relationship is the reverse. However the observed spreads of both values and production prices from market clearing prices are lower

Table 5: Relative frequencies and spreads of prices of production and labour values as computed from sample of 100000 reproduction schema. and compared with empirical studies.

| versus | Labour value market price | Production price market price | Production price labour value |
|---|---|---|---|
| Fraction of schemes with MAD <10% | 0.44% | 0.44% | 6.84% |
| Mean MAD | 28% | 23.7% | 14.8% |
| Empirical MAD | | | |
| China(Sánchezc and Montibeler, 2015) | 14.2% | 16.5% | 12.0% |
| USA(Ochoa, 1989) | 10.3% | 12.6% | 16.9% |
| Spain(Sanchez and Nieto Ferrandez, 2010) | 12.2% | 18.8% | 19.0% |
| Germany (1978)(Zachariah, 2006) | 16.0% | 22.6% | |
| France (1980) (Zachariah, 2006) | 12.0% | 18.2% | |

than found in our a-priori estimation. This may be a result of the empirical data typically using longer price vectors with 30 to 60 elements rather than just 3, with resultant reversion to a mean. This was essentially Farjoun's argument(Farjoun, 1984) for why empirical dispersions of prices to market values would be smaller than those obtained in toy examples using reproduction schemes.

# 5 Discussion

Sraffa (Sraffa, 1960) showed that given:

1. An assumption of an dqual rate of profit.
2. A technology matrix.
3. A specification of the real wage,

it was possible to deduce a price system that would reproduce both the material conditions of production and the class distribution of income. This paper shows that the $G$ matrix, a use value dual of Marx's reproduction schemes can also define a price system that will reproduce the material conditions of production and the class distribution of income.

The $G$ matrix plays both the role of Sraffa's technology matrix and his real wage, but Marxian reproduction schemes do not necessitate a uniform rate of profit, nor do they require that prices are proportional to labour values. Reproduction schemes can exist with these properties, but Table 5 shows that both labour value conforming schemes and price of production conformant schemes make up a small portion of the possible schemes. Even with a very lax definition of *conforming*, being within 10% of, less than $\frac{1}{200}$th of all schemes meet this criterion. It would appear that labour value conforming reproduction schemes are as common as price of production conforming ones. If one looks at Figure 4.2 showing where the conforming instances occur in the planes of relative surplus value between sectors, the pattern is almost identical in both cases, with many of the same data points appearing in both rows.

From Sweezy onwards it has been conventional for Marxian economists to present individual example reproduction schemes that either have prices proportional to labour values or prices given by an equal rate of profit. The statistical analysis here shows that in doing so, economists have been using what are, on a-priori grounds, rare exceptions to prove rules.

Three sector reproduction schemes, however, capture something additional that is missing in Sraffa, the fact that different social classes have different consumption patterns. Marx dealt with the more general case where the capitalists divide their expenditure in some fixed proportion between necessities and luxuries, what would in modern terms be called a Leontief demand function. The analysis here has taken the simpler assumption that capitalist expenditure is exclusively on luxuries. Similarly we neglect that some commodities, for instance coal, may have been a means of production, a wage good, and have been bought by capitalists to heat their houses.

The simplification is arguably valid, since one could in principle divide the coal industry into 3 sub industries, one supplying factories, one supplying workers cottages, and one supplying mansions. These sub industries would then be statistically aggregated into sectors I, II or III. But the inter-sectoral constraints may have implications for the feasibility of attaining prices of production.

Reproduction prices represent a static macro-economic equilibrium condition. So long as there is no growth in production and no change in technology and no movement of capital between sectors, reproduction prices will keep the economy in an equilibrium. They are market clearing prices given the technology and income distribution. On the other hand, the alternative concept of equilibrium present in Volume III of Capital (Marx, 1894a) and further developed in Production of Commodities (Sraffa, 1960) assumes capital mobility between sectors. Borkiewicz's criticism(Hilferding, 1951) of Volume III was based on arguing that the procedure presented for transforming labour values to prices of production was statically incompatible with reproduction prices. But the dynamic question remains open. If you start of in a macroeconomic equilibrium with reproduction prices operating, but with divergent profit rates as shown in Table 4, then can capital movements produce a new equilibrium with a price structure that both achieves reproduction and profit rate equalisation?

On the one hand the structure of reproduction is so finely balanced, with such intricate interdependence between the elements of the reproduction table that perhaps any movement in capital would throw the whole system into a catastrophic crisis. Alternatively one may argue that even if one keeps technology and labour supply, and money capital constant, the system has still got some degrees of freedom left in terms of the relative sizes of three sectors.

One can see that capital movement is very likely to result in a change in the class distribution of income. A movement of capital in or out of sector II means a bigger or smaller real wage, and in consequence reduces or increases the real quantity of luxuries being consumed by employers. So a movement into row 2 of the table must go along with balancing changes in columns 2 and 3, but whether these will be dynamically achievable is harder to say. It may depend both on the adjustment process and on the initial starting structure of the table.

## 6 Second Experiment

In order to investigate the dynamic process of capital movement from initial reproduction states, a second experiment was carried out. Like the first experiment it used a sample of reproduction schemes, prepared in the same way as in the previous experiment. It combined these with rules for capital mobility, for price adjustment, possible buffer stocks and adjustment of sectoral outputs. The time evolution of the economies represented by the initial reproduction schemes was then evaluated for 150 time steps.

**Initialisation** A $G$ matrix is prepared as in the first experiment. An initial price vector is derived and a resulting initial monetary reproduction scheme is derived. From an assumed money wage of £2 a initial vector of labour allocation $\lambda$ is derived. In conjunction with the labour vector the $G$ matrix is used to derive a linear production function for each sector. Each sector is allocated sufficient cash to pay wages and buy means of production at current prices and the current scale of production.

Simulation cycles start at the point where production has just finished, so the firms in each sector have a stock equal to what was produced, plus any unsold stock from the previous period. Stocks of goods held are recorded in the $A$, for available, matrix.

**Capital allocation rule** Let $s$ be the sector with the highest rate of profit. For each sector $x \neq s$ if the rate of profit in $x$ is more than 1% below the rate in $s$ then sector $x$ will transfer 1% of its money capital to sector $x$. Each sector divides its money capital into constant and variable capital in the same ratio as its final allocation in the previous period. We thus get new column vectors $V_t, C_t$ for variable and constant capital for time $t$.

**Wage and labour rule** Wage rates are then set such that

$$w_t \leftarrow \frac{\sum V_t}{\sum \lambda_{t-1}}$$

and the new wage rate and new is used to reallocate labour so that

$$\lambda_t \leftarrow \frac{V_t}{w_t}$$

.

**Prices sectors I and II** The total requirement for means of production for each sector, given $\lambda_t$ is then determined using the production functions. If this exceeds the total stocks of means of production held by all sectors then we have a sellers market in means of production whose prices rise to a market clearing level.

$$p_1 \leftarrow \frac{\sum C_t}{\sum_\iota A_{\iota,1}}$$

otherwise if stocks exceed requirements, we have a buyers market and the price of means of production is reduced by 3%. The price of wage goods is then set as

$$p_2 \leftarrow \frac{w \sum \lambda}{\sum_\iota A_{\iota,2}}$$

Sectors then pay wages and workers spend their wages on the output of sector II at the current $p_2$. Each sector then purchases its requirement of means of production from sector I at price $p_1$.

**Demand for luxuries** For sectors I and II we now know their total sales and their total cost of production. By subtracting purchases from sales we get their profits which are assumed to be entirely spent on luxuries. For the capitalists of sector III we have the odd situation that as Marx points out, their profits are self financing. Whatever they spend on luxuries will return to them as additional profit. The simulation thus adopts the parsimonious assumption that their expenditure on luxuries will remain constant in money terms. The price of luxuries is then set to clear the market given the physical stocks available.

**Production** Production takes place constrained either by the available labour in each sector or the available means of production, as per the linear production function. If labour is the limiting factor this may result in some unused stock of means of production which are carried over to the next period.

## 6.1 Results

Figure 6.1 shows the results of the simulation in terms of the initial and final standard deviations of the rate of profit. A simulation run is represented as a point whose x position is given by the starting spread of its profit rate and its y position by its terminating profit rate spread. A point on the 45° diagonal represents a system that has undergone no profit rate convergence during the simulation. A point close to the x axis indicates a system that has undergone convergence.

One can clearly see that the simulated systems fall into two distinct clusters - one just below the 45° line, and one close to or below the 1% line. Provided that profit rates are within 1% they are taken to have converged, since only discrepancies bigger than this are assumed to trigger capital flows.

Detailed examination of the final sectoral output figures for the simulations run showed that many simulated economies had undergone a drastic contraction in terms of physical output. Since the amount of money circulating does not change during the simulation, rises in prices obscure this effect if one looks only at the figures for output in money terms. We define an economy to be healthy under capital movement if the final value of output measured in the prices operating at time $t_0$ are $>$ 98% of the starting value of output. We define an economy as having collapsed if output is less than 50% of its starting value. One can see in Figure 6.1 that there is no particular relationship between the economy being healthy and its profit rate

| Sector | I | II | III |
|---|---|---|---|
| Collapsing | 2.16 | 0.59 | 1.18 |
| Healthy | 1.23 | 0.65 | 1.88 |
| Converging | 1.60 | 1.31 | 2.21 |
| Non converging | 1.96 | 0.47 | 1.15 |

Table 6: Geometric mean of initial organic compositions by sector and group for the economies simulated in Figure 6.1.

converging. Some of the economies whose profit rates equalise are healthy and some are collapsing. Conversely some healthy economies retain dispersed profit rates even in the presence of capital movements that, according to accepted theory, should result in an equalisation of the rate of profit.

Table 6 does show however that the collapsing economies tend to be characterised by greater sectoral disparities in organic composition, and higher organic compositions in sector I. Systems that do not converge their rates of profit are characterised by particularly low organic compositions in sector II.

## 7 Discussion

The first experiment shows that only a very small fraction of possible self reproducing capitalist economies are characterised by equal rates of profit. Similarly only a very small fraction of possible reproduction schemes have price structures close to labour values. The existing literature on the *transformation problem* relates to either logical or temporal transition between the small subset of the value conformant reproduction schemes and the small subset of price of production conformant schemes.

The second experiment indicates that one can not simply assume that the mech-

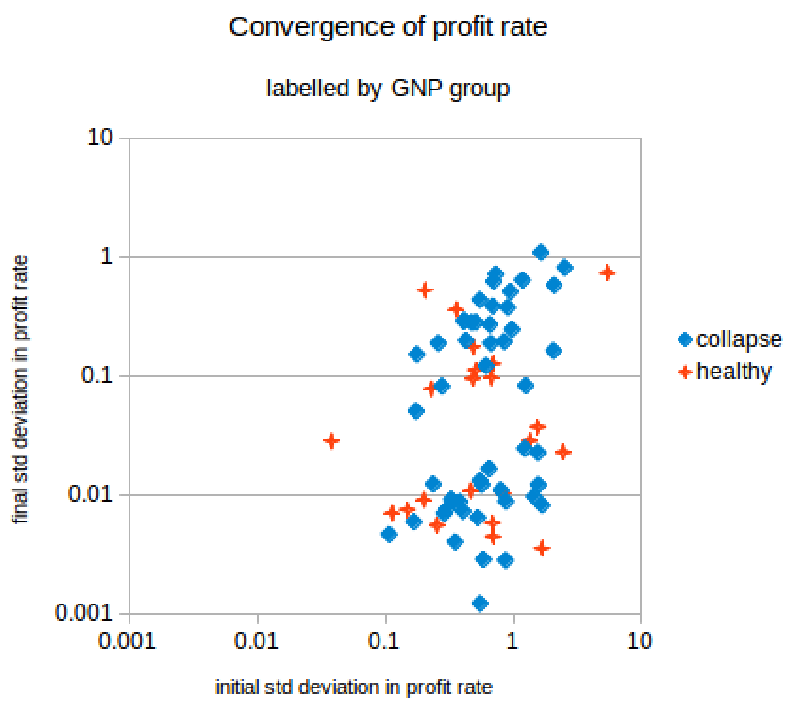


Figure 6.1: When simulated over time, some reproduction schemes can converge towards an equal rate of profit. However the population of schemes forms two distinct clusters, one capable of converging and one which does not converge. Schemes which show no convergence over time would lie on a line at 45° going through the origin of this plot. Healthy models are those in which GNP remains constant or grows, collapse models are those whose GNP has fallen by more than 50% at the end of the simulation.

anism that is supposed to bring about an equal rate of profit will, in general, work. For some starting points, combinations of technology and distributions of income, the hypothesised convergence mechanism fails. In these cases the system either remains healthy with a continuing spread of profit rates, or the economy shrinks catastrophically.

The exact nature of the dynamics that produce this result are at present unclear, but it appears that in the cases of catastrophic contraction, the problem arises due to insufficient means of production being produced, which acts as a constraint on all subsequent output. If the economy moves to a labour distribution where more means of production would be used by the current distribution of the labour force than it can produce, then clearly it must undergo contracted reproduction.

In the case of simulated economies that fail converge on a uniform rate of profit, one hypothesis is that if sector II has a particularly low organic composition of capital, then a movement of capital into sector II leads to a net increase in the demand for labour power. This raises wages and increases demand for sector II, so rather than the price of necessities falling consequent on inward capital movement, wage goods may rise in price. Another possibility is that the distribution of profit rates may undergo oscillations. Further investigation into detailed trajectories of prices and profit rates of individual sectors would be required to test these hypotheses.

## 8 Model and reality

We know that real capitalist economies do not often go into catastrophic collapse due to inadequate production of means of production, though the collapse of industrial production in the former USSR after conversion to capitalism may be an

Table 7: Mean price and value vectors.

| | Capital goods | Wage goods | Luxuries |
|---|---|---|---|
| Mean labour values | 0.95 | 1.11 | 0.93 |
| Mean production price | 1.08 | 1.02 | 0.90 |
| Mean market clearing price | 1.00 | 0.92 | 1.07 |

instance of this. Why is this?

It may be that some version of the Anthropic Principle (Barrow et al., 1988) is in operation. We do not see these collapses because the collapses are history sensitive, and the economies starting out in technological and income configurations that would result in collapse are eliminated. That may apply to the former socialist economies suddenly exposed to a profit maximising principle, they contracted until the technical structure of the economy changed. The end result would be that at any given time, the population of capitalist economies would have been purged of those with technical structures that would lead them to collapse under free capital movement.

Alternatively the basic market clearing price mechanism that is used in the model may not be realistic. The model basically assumes unit elasticity, a 1% fall in output, other things being equal, raises prices by 1%. Perhaps capitalist economies are only stable against collapse given non-linear price responses.

Instead of looking at the problem of collapse, consider that a substantial fraction of healthy models fail to attain an equal rate of profit. This is less of a problem since it accords with what we observe in reality. We know that typical capitalist economies have a dispersion of profit rates(Fröhlich, 2013).

All reproduction schemes meeting the constraints described in Section 3 define a set of market clearing prices for economies with no credit operations. Real economies have credit and therefore the set of actual market prices we observe will be less constrained than is implied by reproduction schemes. However, reproduction schemes do have the virtue that they allow us to generate a large sample of simple economies and associated price structures sans any assumptions about the underlying price mechanism of the economy. They allow us to explore the space of possible self reproducing economies and the price structures associated with them.

The input output tables used in empirical studies are approximations to systems of simple reproduction. They are only approximations, since they depict economies that are typically growing, but the growth rate is typically small, and the conventions associated with the construction of input output tables impose similar balance constraints to those seen in reproduction schemes. The existence of credit transfers between industries in the IO tables, will however introduce a complication absent in the simple marxian schemes.

Using unbiased samples from the space of reproduction schemes we can determine the a-priori probability of different pricing theories. That is to say, the probability that such pricing theories would be true if real economies were distributed with equal probability over all possible positions in configuration space. We are assuming, in effect, that if economies undergo a random walk through configuration space, the probability of their transiting from one macro-state to another is proportional to the volume occupied by these macro-states.

The macro-state defined by market prices being within 10% of labour values has a similar volume to the macro state with market prices with 10% of prices of production. A priori, we should expect a reproducing economy to be this close to a labour value conformant configuration as to a price of production conformant one.

If, on the other hand, there is some bias in the random walk, so that economies end up closer to either of these pricing systems than one would a-priori expect, then this is analogous to evolution in a space with a potential defined over it. The discrepancy between observed and a-priori probability distributions should then enable one, to estimate, via some appropriate negative exponential law, the depth of potential wells. Conversely one could say how strong the potential field would have to be to produce a world in which a either labour values or production prices were the operational laws. Even without a deeper analysis though, it is appears from these results that the assumption of prices of production as an operational law implies a weaker potential well favouring it than need be assumed for labour values. The expected a-priori dispersions of labour values are wider than those for prices of production. The fact that this is not what is empirically observed implies that the potential well associated with prices of production is weaker than that associated with labour values. Possibly this an effect of labour being more mobile than capital. It is easier for steel workers to move into catering jobs than to convert steel mills into restaurants. Alternatively, the obstacles to profit rate equalisation shown in the second experiment may act as a frustrating factor effectively reducing the potential well around prices of production.

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