Journal of Modern Optics

ISSN: 0950-0340 (Print) 1362-3044 (Online) Journal homepage: http://www.tandfonline.com/loi/tmop20

Vacuum friction

Stephen M. Barnett & Matthias Sonnleitner

To cite this article: Stephen M. Barnett & Matthias Sonnleitner (2018) Vacuum friction, Journal of Modern Optics, 65:1, 23-29, DOI: 10.1080/09500340.2017.1374482

To link to this article: http://dx.doi.org/10.1080/09500340.2017.1374482

Published online: 14 Sep 2017.

JOURNAL OF MODERN OPTICS, 2018
VOL. 65, NO. 1, 23–29
https://doi.org/10.1080/09500340.2017.1374482

Vacuum friction

Stephen M. Barnett and Matthias Sonnleitner

School of Physics and Astronomy, University of Glasgow, Glasgow, UK

ABSTRACT
We know that in empty space there is no preferred state of rest. This is true both in special relativity but also in Newtonian mechanics with its associated Galilean relativity. It comes as something of a surprise, therefore, to discover the existence a friction force associated with spontaneous emission. The resolution of this paradox relies on a central idea from special relativity even though our derivation of it is non-relativistic. We examine the possibility that the physics underlying this effect might be explored in an ion trap, via the observation of a superposition of different mass states.

ARTICLE HISTORY
Received 2 June 2017
Accepted 25 August 2017

KEYWORDS
Optical forces; spontaneous emission; Doppler shifts; trapped ions

> The term 'holistic' refers to my conviction that what we are concerned with here is the fundamental interconnectedness of all things. I do not concern myself with such petty things as fingerprint powder, telltale pieces of pocket fluff and inane footprints. I see the solution to each problem as being detectable in the pattern and web of the whole. The connections between causes and effects are often much more subtle and complex than we with our rough and ready understanding of the physical world might naturally suppose. (Douglas Adams (*1*))

1. Introduction

It has long been appreciated that the optical Doppler shift could be used to cool a gas of atoms (*2*) or a trapped ion (*3*, *4*). The essential idea is that a narrow-line laser tuned below a resonant transition frequency for the atom will be absorbed, preferentially if the atom is moving towards the light source because of the Doppler shift and hence be slowed down so as to accommodate the momentum of the photon (*5*). If the absorption of a laser photon is followed by spontaneous emission a further photon can be absorbed and, after a number of cycles, the average velocity of the atom is reduced. If the single laser beam is supplemented by five more then Doppler cooling can be achieved in three dimensions (*6*) and an atom feels an average frictional force $\mathbf{F} = -\alpha\mathbf{v}$, where $\mathbf{v}$ is the velocity of the atom. These ideas were the seed from which the field of laser cooling and trapping, and much else besides, has flowered (*7*).

Hidden within the combination of the optical Doppler shift and the interaction between light and atoms is a paradox, which we have identified recently (*8*). The point is simply made: an excited atom in an otherwise empty region of space can return to its ground state by the spontaneous emission of a photon. In doing so it receives a recoil so as to conserve momentum; if the emitted photon has momentum $\hbar\mathbf{k}$ then the momentum of the atom changes, correspondingly, by $-\hbar\mathbf{k}$. If the atom is stationary then the essentially isotropic nature of spontaneous emission means that there the net or *average* change in the momentum is zero. If the atom is moving, however, then a photon emitted in the direction of motion of the atom will have, by virtue of the Doppler shift, a higher frequency and hence a higher momentum than one emitted in the opposite direction, as depicted in Figure 1. If we take the average over these events we are led to a net reduction in the momentum of the atom in the emission process. This reduction, moreover, is proportional to the velocity of the atom. In short, we have a friction force associated with the spontaneous emission event. Yet the existence of a force in one frame that does not exist in another seems to be at odds with both the Galilean and Einsteinian principles of relativity. Hence, we have a paradox.

Our earlier publication on this problem was necessarily somewhat formal and rigorous (*8*). Our aim here is to present the key ideas in a more physical fashion. We shall find that the resolution of the paradox lies in a key idea from special relativity, but the remarkable feature of our analysis is that it requires ideas familiar only from *non-relativistic* physics. We consider also the possibility that the central idea involved in this resolution might be observable in an ion trap experiment.

CONTACT Stephen M. Barnett stephen.barnett@glasgow.ac.uk

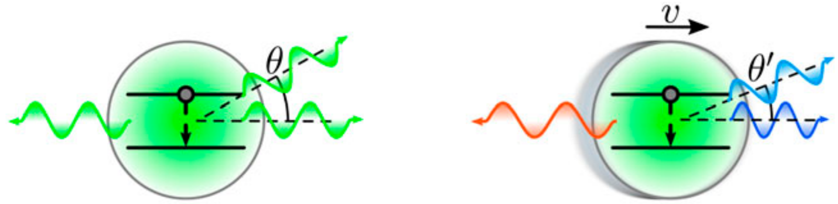

Figure 1. Illustration of the (spontaneous) emission process of an atom at rest (left figure) or moving at speed v (right). An atom at rest emits photons of the frequency ω_0 in all directions θ so that the average recoil is zero. A moving atom emits photons of frequency $\omega_0' \approx \omega_0(1 + \frac{v}{c}\cos\theta')$ in direction $\cos\theta' \approx \cos\theta + \frac{v}{c}(1-\cos^2\theta)$. Integrating the recoil over all directions results in a non-zero change of the atom's momentum.

2. Vacuum friction: a physical derivation

Let us consider the spontaneous emission by an atom via an electric dipole transition. A simple calculation based on Fermi's golden rule gives the spontaneous emission rate (*9*)

$$\Gamma = \frac{\omega_0^3|\mathbf{d}|^2}{3\pi\epsilon_0\hbar c^3}, \tag{1}$$

where $\mathbf{d}$ is the transition dipole matrix element and ω_0 is the atomic transition frequency. If the atom is moving then there will be a change to this decay rate but only that required by the time dilation of special relativity, as may be confirmed by direct calculation (*10*). This effect is of second order in the velocity and will not be of concern to us in this paper; we are interested in those effects that arise at first order in v/c.

We can apply physical reasoning to arrive at least at an approximate form for the paradoxical friction force and it is instructive to follow this approach. To this end, we introduce the idea of a spontaneous emission rate into an infinitesimal solid angle,

$$\mathrm{d}\Gamma = \Gamma\frac{\mathrm{d}\Omega}{4\pi}, \tag{2}$$

which is simply the spontaneous emission rate multiplied by the probability of emission into the solid angle $\mathrm{d}\Omega$. If we integrate this over all directions of emission then we recover the full spontaneous emission rate, $\int \mathrm{d}\Gamma = \Gamma$.

If the atom is stationary then the emitted photon will carry away an amount of momentum given by $\hbar k_0 = \hbar\omega_0/c$. The net or average recoil force on our atom is simply

$$\begin{aligned}\mathbf{F} &= -\int \hbar\mathbf{k}_0 \mathrm{d}\Gamma P_e(t) \\ &= -\int \hbar\mathbf{k}_0 \mathrm{d}\Gamma e^{-\Gamma t} \\ &= 0,\end{aligned} \tag{3}$$

where $P_e(t)$ is the probability that the atom is still in the excited state at time t. The zero value of this net force is a direct consequence of the lack of a preferred direction for the spontaneous emission which is, itself, a consequence of the isotropy of empty space.

If the atom is moving then this motion selects for us a direction in space. Let us take this direction to define the z-axis so that the atomic velocity is $\mathbf{v} = v\hat{\mathbf{z}}$. Because of this motion the Doppler effect means that the frequency of the emitted photon will depend on the direction in which it is emitted. If the angle between the direction of the emitted photon and the z axis is θ then the observed frequency of the photon will be

$$\omega_0' = \omega_0\left(1 + \frac{v}{c}\cos\theta\right). \tag{4}$$

If we insert this Doppler-shifted frequency into our expression for the net force, we find a first tentative expression for the force due to spontaneous emission on a moving atom:

$$\begin{aligned}\mathbf{F}_1 &= -e^{-\Gamma t}\int \hbar\frac{\omega_0}{c}\left(1 + \frac{v}{c}\cos\theta\right)\cos\theta \mathrm{d}\Gamma\,\hat{\mathbf{z}} \\ &= -\frac{1}{3}e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma\frac{v}{c}\,\hat{\mathbf{z}} \\ &= -\frac{1}{3}e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma\,\mathbf{v},\end{aligned} \tag{5}$$

which has the characteristic form of a friction force. Here we have exploited the rotational symmetry about the z-axis to infer directly that the force must be parallel to this axis.[1]

Is there anything we have left out and, in particular, is there any non-relativistic effect that is missing? The answer, of course, is yes and the additional feature is the aberration of light due to motion. This is a familiar element of special relativity where the constancy of the speed of light leads to a modification of the angles measured in different frames and this requires us to replace the cosines and sines by (*11*)

$$\cos\theta \to \cos\theta' = \frac{\cos\theta + v/c}{1+(v/c)\cos\theta}$$
$$\sin\theta \to \sin\theta' = \frac{\sin\theta}{\gamma(1+(v/c)\cos\theta)}, \quad (6)$$

where γ is the usual Lorentz factor, $\gamma = (1-v^2/c^2)^{-1/2}$. It is important to note, however, that this idea is much older than relativity. Indeed, it was first noted by Bradley in 1729 (*12*) in the response to the appearance of consistent discrepancies in the measurement of stellar parallaxes. Bradley's explanation for this was a modification in the perceived angles due to the finite value of the speed of light:

> And in all Cases, the Sine of the Difference between the real and visible Place of the Object, will be to the Sine of the visible Inclination of the Object to the Line in which the Eye is moving, as the Velocity of the Eye to the Velocity of Light. (*12*)

Putting these words into mathematical form we arrive at

$$\sin(\theta - \theta') = \frac{v}{c}\sin\theta', \quad (7)$$

which is readily recovered from the relativistic expression (6) in the limit of small velocity. Rather than work with Bradley's expression, it is more transparent (although strictly equivalent) to work with the relativistic formula and restrict ourselves to low velocities by working to first order in v/c. The aberration means that the angle between the z-axis and the direction of emission of the photon, the term $\cos\theta$ in our expression for the force should be replaced by $\cos\theta'$. If we make this substitution then we arrive at the correct expression for the net force:

$$\begin{aligned}\mathbf{F} &= -e^{-\Gamma t}\int \hbar\frac{\omega_0}{c}\left(1+\frac{v}{c}\cos\theta\right)\cos\theta'\mathrm{d}\Gamma\,\hat{\mathbf{z}} \\ &= -e^{-\Gamma t}\int \hbar\frac{\omega_0}{c}\left(\cos\theta+\frac{v}{c}\right)\mathrm{d}\Gamma\,\hat{\mathbf{z}} \\ &= -e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma\mathbf{v}. \quad (8)\end{aligned}$$

As a check of this idea we can work with the primed angle, θ', and note that this involves a change in the integration variable

$$\begin{aligned}d(\cos\theta') &= \frac{d(\cos\theta)}{(1+(v/c)\cos\theta)^2} \\ \Rightarrow \mathrm{d}\Omega &= \mathrm{d}\Omega'(1+(v/c)\cos\theta)^2. \quad (9)\end{aligned}$$

Evaluating the net force in this manner and again working to first order in v/c gives

$$\begin{aligned}\mathbf{F} &= -e^{-\Gamma t}\int \hbar\frac{\omega_0}{c}\left(1+\frac{v}{c}\cos\theta\right)\cos\theta'\mathrm{d}\Gamma' \\ &\quad\times\left(1+\frac{v}{c}\cos\theta\right)^2\hat{\mathbf{z}} \\ &= -e^{-\Gamma t}\int \hbar\frac{\omega_0}{c}\left(1+3\frac{v}{c}\cos\theta'\right)\cos\theta'\mathrm{d}\Gamma'\,\hat{\mathbf{z}} \\ &= -e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma\mathbf{v}, \quad (10)\end{aligned}$$

where we have again worked to first order in v/c. We have arrived at the same expression using two sets of angles, θ and θ'. In the first we take account of the aberration by transforming the angle of emission, but in the second it is the solid angle into which the emission occurs that is transformed.

The above analysis considered isotropic emission and one might wonder if our simple result changes for a more general emission pattern. A general emission pattern is defined by a quantity $\gamma(\mathbf{k})$, the rate at which light with wavevector $\mathbf{k}$ is emitted, if the atom is in the excited state. The total decay rate is thus

$$\Gamma = \int \gamma(\mathbf{k})d^3\mathbf{k}, \quad (11)$$

where we integrate over all directions and frequencies, $d^3\mathbf{k} = \omega^2\mathrm{d}\omega\mathrm{d}\Omega$. As before we express the wavevector in spherical coordinates, $\mathbf{k} = \omega/c(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)^T$. In the rest frame of the emitter, we get a recoil force

$$\mathbf{F} = -e^{-\Gamma t}\int d^3\mathbf{k}\hbar\mathbf{k}\gamma(\mathbf{k}). \quad (12)$$

For a spontaneously decaying atom $\gamma(-\mathbf{k}) = \gamma(\mathbf{k})$ and this net recoil force in the rest frame is, of course, zero.

In a frame where the emitter is moving at a velocity $\mathbf{v} = v\hat{\mathbf{z}}$, we express $\mathbf{k}'$ using the Doppler shift and aberration to first order,

$$\begin{aligned}\mathbf{k}' &\simeq \frac{\omega}{c}\left(1+\frac{v}{c}\cos\theta\right)\begin{pmatrix}\sin\theta'\cos\phi \\ \sin\theta'\sin\phi \\ \cos\theta'\end{pmatrix}, \\ &= \mathbf{k} + \frac{\omega\mathbf{v}}{c^2}, \quad (13)\end{aligned}$$

where we used the aberration formula, equation (6), to arrive at the last line. We could also have used Bardley's non-relativistic expression as we are working only to first order in v/c.

It is not necessary to give an explicit transformation for the general emission pattern $\gamma(\mathbf{k})$, as we know that the amount of radiation emitted into each volume element must be invariant, so that

$$\gamma(\mathbf{k})d^3\mathbf{k} = \gamma'(\mathbf{k}')d^3\mathbf{k}'. \tag{14}$$

As the decay rate Γ only changes with second order in velocity, we can express the force in the moving frame in terms of the unprimed quantities,

$$\begin{aligned}\mathbf{F} &= -e^{-\Gamma t}\int d^3\mathbf{k}'\hbar\mathbf{k}'\gamma'(\mathbf{k}'),\\ &= -e^{-\Gamma t}\hbar\int d^3\mathbf{k}\gamma(\mathbf{k})\left(\mathbf{k}+\omega\mathbf{v}/c^2\right),\\ &= -e^{-\Gamma t}\frac{\hbar\mathbf{v}}{c^2}\int d^3\mathbf{k}\gamma(\mathbf{k})\omega\end{aligned} \tag{15}$$

and we again find a friction force proportional to the velocity of the atom.

The spontaneous decay rate depends on the density of modes at the transition frequency ω_0 and so we write $\gamma(\mathbf{k}) = \delta(\omega-\omega_0)\widetilde{\gamma}(\Omega)$ where we can set $\int d\Omega\widetilde{\gamma}(\Omega) = \Gamma/\omega_0^2$ such that (11) is satisfied. It follows that the average or net force is

$$\begin{aligned}\mathbf{F} &= -e^{-\Gamma t}\frac{\hbar\mathbf{v}}{c^2}\int d\omega\omega^3\delta(\omega-\omega_0)\int d\Omega\widetilde{\gamma}(\Omega)\\ &= -e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma\mathbf{v}\end{aligned} \tag{16}$$

in agreement with our earlier expression (8).

This simple expression for the net force **F**, which can be obtained more rigorously (*8*), has confirmed our initial instincts that there is indeed a vacuum friction force, however counterintuitive this conclusion may be. Our next task is to resolve this paradox.

3. Resolution of the paradox

If you ask a class of physics students to state Newton's second law of motion, it is likely that the answer you will get (apart from those smart enough to spot a trap) will be $\mathbf{F} = m\mathbf{a}$. If we apply this to the force, we have just derived then we find

$$\dot{\mathbf{v}} = -e^{-\Gamma t}\frac{\hbar\omega_0}{mc^2}\Gamma\mathbf{v}, \tag{17}$$

where m is the mass of the atom. This effect is certainly small: it is proportional to the ratio of the photon energy to the rest mass energy of the atom and this ratio is typically of the order 10^{-10}. There is an important point of principle, however, in that if the deceleration exists, whatever its value, then we have a conflict with relativity, both of the Einsteinian and Galilean forms. To emphasize this point we can integrate this equation to find the net change in the velocity of the atom:

$$\mathbf{v}(\infty) = \exp\left(-\frac{\hbar\omega_0}{mc^2}\right)\mathbf{v}(0) \approx \left(1-\frac{\hbar\omega_0}{mc^2}\right)\mathbf{v}(0). \tag{18}$$

So, if true, this says that the observed speed of the atom is reduced by a simple factor and that this is the case, moreover, irrespective of the speed of the atom. So if we see an excited atom in motion then, on average, following the emission the speed will be reduced but if we are co-moving with the atom then there is no corresponding average change in the speed. This cannot be true.

The resolution of the paradox comes from a surprising place in that it embodies an intrinsically relativistic notion, indeed perhaps the most famous idea in relativity – the equivalence of energy and mass or inertia. In the physics class mentioned above, you might find a student who pauses to spot the catch and says, in answer to the question, 'force equals rate of change of momentum',[2] $\mathbf{F} = \dot{\mathbf{p}}$. They might recall, for example, the classic problem of the motion of a space rocket, that burns fuel and, in the process, reduces its mass (*13*). If this is the resolution of the paradox then we can only infer that the emission of the photon corresponds to a loss of mass by the atom. Let us see where this leads us. If we allow for the possibility that the emission process changes the mass of the atom then Newton's second law gives us:

$$\dot{m}\mathbf{v} + m\dot{\mathbf{v}} = -e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma\mathbf{v}. \tag{19}$$

A change in the average velocity is, as we have seen, paradoxical and suggests that we should set $\dot{\mathbf{v}} = 0$. If we do this we are left with a simple equation for the rate of change of the mass of the atom:

$$\dot{m} = -e^{-\Gamma t}\frac{\hbar\omega_0}{c^2}\Gamma. \tag{20}$$

which leads directly to the suggestion that the atom is lighter after emitting the photon than when it was prepared initially in its excited state:

$$m(\infty) - m(0) = -\frac{\hbar\omega_0}{c^2}. \tag{21}$$

From the viewpoint of those schooled in special relativity, this makes perfect sense: the transition has lowered the total energy of the atom by $\hbar\omega_0$ and using the most famous equation in physics, $E = mc^2$, we are led to conclude that the ground state atom is indeed lighter than the excited state atom by precisely $\hbar\omega_0/c^2$. Indeed very early in the development of relativity, Einstein established this relationship between a change in internal energy and a change in inertia (*14*). From the viewpoint of special relativity this is entirely unsurprising, of course, given the close relationship between momentum and energy, which combine as a four-vector.

Taking this view, however, misses our point. We have employed an entirely non-relativistic analysis to arrive at a paradox the only resolution of which seems to imply the necessity of a central feature of special relativity. Perhaps Adams's eponymous hero Dirk Gently was indeed correct and that there is a fundamental interconnectedness in the physical world and the need for a holistic approach.

4. Possibility of an ion trap experiment

Is there an experiment to be done in order to test these ideas? As stated, the lack of a change in the average velocity of a radiating atom would be a difficult, and perhaps not very satisfying, result to establish. It is indicative of the scale of the challenge that we would be seeking to find no net change as opposed to a change of perhaps one part in 10^{10}. There is an interesting experimental challenge, however, the demonstration of which would verify the resolution presented above. This is to show that an atom prepared in an electronically excited state has a smaller mass than one in the ground state and that this difference is given by the transition energy divided by the square of the speed of light.

To show that an excited state is more massive than a ground state requires a mass measurement with a precision of $\hbar\omega_0/mc^2$ which, for visible light and an atomic mass will be of the order of one part in 10^{10}. It is reasonable to start with the observation that such a measurement, if it is possible, will require a significant period of time to perform and this means, necessarily that we need to identify an atom with a very long-lived excited state. We note that ion traps have been used to measure atomic masses with a precision of $1-10\,\text{keV}$ (*15*). Although this is roughly three orders of magnitude larger than the typical optical transition energy we are seeking to measure it is at least encouraging. Excited ionic states with very long lifetimes are known, moreover, and some of these have been considered as candidates for frequency standards. For these reasons, it is natural to consider an ion trap experiment as the way to test this idea. We shall not propose a specific implementation here but rather present a 'back of the envelope' assessment of what might be possible with currently available technology.

Let us consider a single ion, prepared in its ground state and trapped in a harmonic trap with trap angular frequency $\Omega = \sqrt{k/m}$, where k is the trap stiffness and m denotes the mass of the ion. Using suitable optical Pulses, we can transfer the ion into a superposition of the initial ground state and a suitably chosen metastable excited state, separated from the ground state by the energy $\hbar\omega_0$. If this is performed using a two-photon transition, then this can be done without affecting the motional state of the ion, which will continue to undergo harmonic oscillations in the trap. The ground and excited states, having different masses, will oscillate at different frequencies in the trap, as depicted in Figure 2. The frequency of oscillations for the excited state, Ω^*, and for the ground state, Ω, are related by

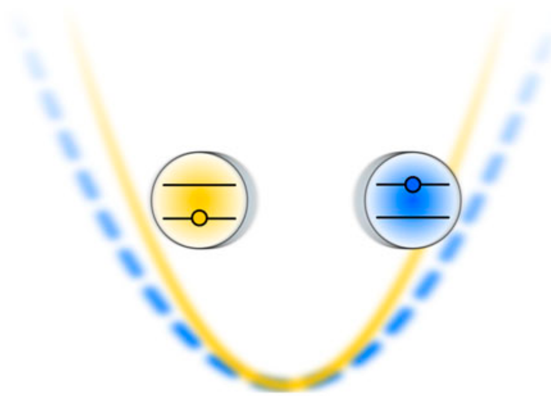

Figure 2. As the oscillation frequency of a trapped ion depends on its mass, $\Omega \sim (m)^{-1/2}$, an ion in an excited internal state oscillates at a different frequency, $\Omega^* \sim (m+\hbar\omega_0/c^2)^{-1/2}$.

$$\Omega^* = \sqrt{\frac{k}{m+\hbar\omega_0/c^2}} \approx \Omega\left(1-\frac{\hbar\omega_0}{2mc^2}\right). \quad (22)$$

It follows that even if the motional state is unchanged in the creation of the superposition, the subsequent motion will become entangled with the internal ionic state, as if the two states were associated with different potentials. The time taken for the motions of the ground and excited states to first become maximally separated is

$$T = \frac{\pi}{\Omega-\Omega^*} \approx \frac{2\pi}{\Omega}\frac{mc^2}{\hbar\omega_0}, \quad (23)$$

which corresponds to $mc^2/(\hbar\omega_0)$ periods of the ion's motion in the trap.

To observe this separation of the ground and excited states would certainly be technically demanding but is not a hopeless task. The first requirement is for a long-lived excited state so that the experiment can be performed before the metastable state decays. Ions studied for possible frequency standards have been shown to have very long-lived metastable states. These include the $^{171}\text{Yb}^+$ ion; its $^2\text{F}_{7/2}$-state is remarkably stable with a lifetime of several years (*16*, *17*). The transition frequency between this state and the ground state is $\omega_0/(2\pi) = 642\,\text{THz}$ and this means that $\hbar\omega_0/(mc^2) \approx 1\cdot 36\times 10^{-11}$. Hence, the maximum spatial separation between the ground and excited states will occur after about $7\cdot 4\times 10^{10}$ motional periods. For a trap frequency of $1\cdot 3\,\text{MHz}$ (*17*), this corresponds to a time of $5\cdot 7\times 10^4\,\text{s}$ or 15 h, which is substantially less than the excited state lifetime.

One would, of course, have to keep the trap stable and avoid interruptions to the ion, such as collisions with background gas atoms.

Although challenging, there is a subtle and highly unusual feature of the proposed experiment that makes it worthy of further consideration. This is the fact that we are exploring a superposition of *different mass* states rather than energy states as the spatial separation of the wave-packets for the ground and excited states depends on the difference in rest masses for these states. It is interesting to note that there are good reasons arising from Gallilean invariance why such a superposition is not possible in non-relativistic quantum theory, although this restriction does not extend to the relativistic domain (*18*).

5. Conclusion

We have seen how a simple application of ideas from non-relativistic physics leads to a paradox, the existence of a vacuum friction force. If we take this force to be a damping of the velocity then we run into a problem, the corresponding necessity of a preferred frame of absolute rest. The existence of such a state of absolute rest would be in direct conflict with special relativity but also with Newton's mechanics and, in particular, with his first law of motion.

The quantitive resolution of the paradox is, as we have seen, that the exited atom loses mass in undergoing spontaneous decay and that the amount of mass lost is precisely $\hbar\omega_0/c^2$. When we combine this idea with energy quantization we are led directly to $E = mc^2$, a key consequence of special relativity. The remarkable feature of this, however, is no explicitly relativistic ideas were used to derive it; we needed to use only non-relativistic quantum theory, the first-order Doppler shift and Bradley's 1729 notion of aberration due to motion. We may ponder the point at which relativity sneaked into our analysis or simply marvel at the way in which in physics seems to take care of itself and has no regard for our attempts to classify parts of it as classical or quantum, or as relativistic or non-relativistic.

Endnote (SMB)

Danny Segal was a lovely man, a generous and caring teacher, and a talented, enthusiastic and imaginative physicist. He enjoyed, perhaps especially, the absurdities that our chosen discipline throws up from time to time and I would very much loved to have had the chance to show to him the one presented above, to benefit from his wisdom and to see him smile.

Notes

1. We could include, explicitly, the dipole radiation pattern but, for simplicity, consider a spatially averaged dipole.
2. Newton's statement was: 'Mutationem motus proportionalem esse vi motrici impressae & fieri secundum lineam rectam qua vis illa imprimitur.' (*19*), rendered by Ball as 'The change of momentum [per unit time] is always proportional to the moving force impressed, and takes place in the direction in which the force is impressed.' (*20*)

Acknowledgements

We are grateful to Nils Trautmann, Mohamed Babiker, Jim Cresser and Helmut Ritsch with whom we have enjoyed many interesting discussions on this topic.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by a Royal Society Research Professorship [RP150122]; the Austrian Science Fund FWF [grant number J 3703-N27].

ORCID

Stephen M. Barnett http://orcid.org/0000-0003-0733-4524
Matthias Sonnleitner http://orcid.org/0000-0003-1474-787X

References

(1) Adams, D. *Dirk Gently's Holistic Detective Agency*; Heinemann: London, 1987.
(2) Hänsch, T.W.; Schawlow, A.L. Cooling of Gases by Laser Radiation. *Opt. Commun.* **1975,** *13,* 68–69.
(3) Wineland, D.; Dehmelt, H. Proposed $10^{14}\Delta\nu < \nu$ Laser Fluoresence Spectroscopy on Tl^{+} Mono-Ion Oscillator III. *Bull. Am. Phys. Soc.* **1975,** *20,* 637–637.
(4) Mavadia, S.; Stutter, G.; Goodwin, J.F.; Crick, D.R.; Thompson, R.C.; Segal, D.M. Optical Sideband Spectroscopy of a Single Ion in a Penning Trap. *Phys. Rev. A* **2014,** *89,* 032502.
(5) Adams, C.S.; Riis, E. Laser Cooling and Trapping of Neutral Atoms. *Prog. Quant. Electr.* **1997,** *21,* 1–79.
(6) Chu, S.; Hollberg, L.; Bjorkholm, J.E.; Cable, A.; Ashkin, A. Three-Dimensional Viscous Confinement and Cooling of Atoms by Resonance Radiation Pressure. *Phys. Rev. Lett.* **1985,** *55,* 48–51.
(7) Ashkin, A. *Optical Trapping and Manipulation of Neutral Particles Using Lasers*; World Scientific: Hackensack, NJ, 2006.
(8) Sonnleitner, M.; Trautmann, N.; Barnett, S.M. Will a Decaying Atom Feel a Friction Force? *Phys. Rev. Lett.* **2017,** *118,* 053601.

(9) Loudon, R. *The Quantum Theory of Light*, 3rd ed.; Clarendon: Oxford, 2000.
(10) Cresser, J.D.; Barnett, S.M. The Rate of Spontaneous Emission of a Moving Atom. *J. Phys. B: At. Mol. Opt.* **2003,** *36,* 1755–1759.
(11) Rindler, R. *Relativity Special, General and Cosmological*, 2nd ed.; Oxford University Press: Oxford, 2006.
(12) Bradley, J. Letter from the Reverend Mr. James Bradley Savilian Professor of Astronomy at Oxford, and F. R. S. to Dr. Edmund Halley Astronom. Reg. &c. Giving an Account of a New Discovered Motion of the Fix'd Stars. *Phil. Trans.* **1729,** *35,* 637-661.
(13) Fowles, G.R.; Cassidy, G.L. *Analytical Mechanics*, 5th ed.; Harcourt Brace: Fort Worth, 1993.
(14) Einstein, A. Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig? *Ann. Phys. (Berlin)* **1905,** *323,* 639–641.
(15) Blaum, K.; Dilling, J.; Nörtershäuser, W. Precision Atomic Physics Techniques for Nuclear Physics with Radioactive Beams. *Phys. Scripta* **2013,** *T152,* 014017.
(16) Roberts, M.; Taylor, P.; Barwood, G.P.; Gill, P.; Klein, H.A.; Rowley, W.R.C. Observation of an Electric Octupole Transition in a Single Ion. *Phys. Rev. Lett.* **1997,** *78,* 1876–1879.
(17) Roberts, M.; Taylor, P.; Barwood, G.P.; Rowley, W.R.C.; Gill, P. Observation of the $^2S_{1/2}$-$^2F_{7/2}$ Electric Octupole Transition in a Single $^{171}Yb^+$ ion. *Phys. Rev. A* **2000,** *62,* 020501.
(18) Weinberg, S. *The Quantum Theory of Fields*; Cambridge University Press: Cambridge, 1995; Vol. 1.
(19) Newton, I. *Philosophae Naturalis Principia Mathematica*; Royal Society: London, 1686.
(20) Ball, W.W.R. *An Essay on Newton's "Principia"*; Macmillan: New York, 1893.