Debt Stabilization in a Non-Ricardian Economy*

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This version: May 2017

Abstract

In models with a representative infinitely lived household, tax smoothing implies that the steady state of government debt should follow a random walk. This is unlikely to be the case in OLG economies, where the equilibrium interest rate may differ from the policy maker's rate of time preference. It may therefore be optimal to reduce debt today to reduce distortionary taxation in the future. In addition, the level of the capital stock in these economies is likely to be sub-optimally low, and reducing government debt will crowd in additional capital. Using a version of the model of perpetual youth developed by Blanchard (1985) and Yaari (1965), with both public and private capital, we show that it is optimal in steady state for the government to hold assets. However, we also show how and why this level of government assets can fall short of both the level of debt that achieves the optimal capital stock and the level that eliminates income taxes. Finally we compute the optimal adjustment path to this steady state.

JEL Codes: E21, E32, E63

Keywords: Non-Ricardian consumers, macroeconomic stability, distortionary taxes.

1 Introduction

The problems caused by excessive levels of public debt do not need enumerating. As governments around the world try to bring deficits under control, and subsequently to reduce levels of debt in relation to GDP, a natural question to ask is how far debt levels should be reduced, and how quickly, once any immediate crisis resulting from large default risk premia has diminished. In other words, what should be the ultimate target for the debt-to-GDP ratio, and how quickly should we get there? Until now, most analysis of this question has

*Our thanks to Charles Brendon, Alfred Greiner, Tom Holden, Eric Leeper, Pei-Ju Liao, Patrick Minford, Balazs Parkanyi, Matteo Salto, Mathias Trabandt, Mike Wickens, and participants at the ASSA/AEA meetings in San Diego and seminars in Brussels, Cardiff and Oxford for helpful comments, but all responsibility remains ours. Leith and Wren-Lewis and grateful for financial support from the ESRC (Award No. RES-062-23-1436).

been undertaken using models in which consumers in effect live forever, by appropriately internalizing the utility of their children. This tends to have the implication that the optimal level of debt depends upon the initial level of debt as policy makers seek to minimize the costs of distortionary taxation going forwards (see Barro (1979) and Chamley (1985,1986) for example). The implications of the benchmark result in such models is striking: once fears of default have receded, the optimum level of debt is closely tied to the historic debt level. This martingale process for debt has also re-emerged in New Keynesian style DSGE models (see, for example, Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004a)), where policy makers also care about the costs of inflation in a sticky-price environment as well as minimizing the costs of tax distortions. These applications of tax smoothing all suggest that attempts to reduce the extent of distortionary taxation in the long run will require short run increases in these taxes whose cost outweighs the eventual gain.

However, within this literature there have been attempts to analyze the optimal quantity of debt by introducing additional costs or benefits associated with the level of government debt. For example, in Aiyagari et al. (2002) implicit risk premia in an economy with incomplete financial markets may encourage the government to accumulate sufficient assets to pay for (exogenously determined, but stochastic) government spending after eliminating distortionary taxation, although introducing ad-hoc limits on the levels of assets held by the government will ensure policy is more akin to that described in the original tax smoothing result of Barro (1979). Aiyagari and McGratten (1998) allow for a role for government debt in that it can help alleviate households' borrowing constraints, while Shin (2006) allows for household heterogeneity and idiosyncratic income shocks to provide a role for government debt in facilitating precautionary saving. However, with the exception of Aiyagari and McGratten (1998), where the policy maker trades off the use of government debt to facilitate household self-insurance and the crowding out of private capital, these papers do not allow for the latter phenomenon.[1]

In overlapping generations economies, where agents do not care about their children (or do not care about them enough), this effect is central to the desirability of stabilizing debt. There are, in fact, two reasons why the random walk steady-state debt result no longer holds in these Non-Ricardian economies. First, if the economy is not dynamically inefficient, then the real interest rate is likely to exceed the rate of time preference, which means that, from a Ramsey planner's point of view, it may be worth sacrificing some current utility in order to achieve a steady state where distortionary taxes are lower than they currently are (even if the current generation may lose out as a result). Second, as noted above, the level of the capital stock (and therefore output and consumption) in these economies is likely to be sub-optimally low, and reducing government debt will 'crowd in' additional capital.

This raises an immediate question: will the implicit debt target in such models be the debt level that eliminates the need for distortionary taxes or will it be the level that achieves the optimal capital stock? This is one of the issues we examine in this paper. Using an

[1] In Aiyagari and McGratten(1998) this trade-off is finely balanced under their benchmark calibration such that historically observed debt levels are close to optimal.

elaborate version of the model of perpetual youth developed by Blanchard (1985) and Yaari (1965), which allows us to vary the extent of Non-Ricardian behavior parametrically, we derive the optimal steady-state level of government assets. We show how and why this level of government assets can fall short of both the level of debt that achieves the optimal capital stock and the level that eliminates income taxes. In other words, although optimal policy leads to an accumulation of government assets, this falls well short of both the 'war chest' level needed to fund government consumption/investment, eliminate distortionary taxes and offset the monopolistic competition distortion and the level needed to crowd-in the first best level of private capital.

Another issue we explore is whether or not variations in public capital can counteract any crowding out of private capital. We find little use of public capital in this way in steady state. However, when we explore the non-linear path the policy maker follows in moving the economy from its current position to the desired long-run solution of the Ramsey problem, there is a role for public investment. The dynamics under optimal policy imply that the pace of debt stabilization should be very slow, but that during the transition a sell-off of public capital can optimally reduce debt service costs when debt levels are sub-optimally high.

Finally, we attempt to reconcile our results, which suggest it is optimal for the Ramsey policy maker to accumulate assets, with the observation that very few governments have more than paid off their debt stocks. We find that a very modest degree of policy maker myopia (as a simple means of capturing political frictions in fiscal policy making) is sufficient to support a positive public debt stock in steady state. However the welfare costs of this are very high, suggesting that improving policy institutions to remove short-sightedness should be a policy priority

Section 2 outlines an extended model of perpetual youth, which features exogenous growth, distortionary taxation, government consumption and public and private physical capital accumulation. In section 3, we discuss social welfare, the model's calibration, and our numerical results for both the steady state of the Ramsey problem and the non-linear Ramsey dynamics. A final section concludes.

2 The Model

In this section we outline our model. Our economy is populated by overlapping generations of consumers who face a constant probability of death, such that, even if taxes were lump sum, Ricardian Equivalence would not hold in our model.[2] These consumers supply labor to imperfectly competitive firms, who combine this labor with capital rented from a representative capital rental firm and public capital accumulated by the government, to produce a differentiated product. Consumers' labor income is taxed and they hold financial wealth in the form of bonds and equities, as well as life-insurance contracts. We introduce public capital to allow for the possibility that the policy maker may build up the stock of such

[2] For a recent analysis that investigates further the short term role that fiscal policy can play in this class of models, see Devereau (2010).

capital to offset the crowding-out of private sector capital due to government debt.

2.1 Consumers' Behavior

Here we introduce the main departure from the canonical Neo-Classical representative agent model. As we note below, for the random walk in steady-state debt result to hold, the real rate of interest has to be exactly equal to the rate of time preference. One reason why this might not be is that agents fail to act as if they internalize the utility of their children, either because they are selfish, or because of distortions like estate taxes. A benchmark model that examines economies where agents do not internalize the welfare of their children is the model of perpetual youth developed by Blanchard (1985) and Yaari (1965). In this model, to induce finitely-lived households to hold a positive stock of government debt, real interest rates rise above the households' rate of time preference.

In the perpetual-youth model, households face a constant probability of death $(1-\gamma)$. As this is a constant exogenous probability, and there is a continuum of households, there is no aggregate uncertainty in our economy. This implies that a consumer born at time i, who is still alive at time t, receives utility from consuming a basket of consumer goods at time t,

$$c_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

They also derive utility from consumption of publicly provided goods, g_t, and suffer disutility from supplying labor to imperfectly competitive firms, l_t^i.[3] We can write this household's expected utility function as,

$$\sum_{t=0}^{\infty} (\beta\gamma)^t \left[\ln c_t^i + \vartheta \ln g_t + \varkappa \ln(1-l_t^i) \right] ,$$

where ϑ and $\varkappa$ are the relative weights in utility from public goods consumption and leisure, respectively. Also, by reducing the household's discount factor β by the survival probability γ, we are implicitly conditioning on the survival of this particular household (otherwise there would be double-counting of the probability of death).

Due to the difficulties in conceptualizing complete financial contracts amongst markets participants some of whom are as yet unborn, we assume that financial markets are incomplete, but in an economy without aggregate uncertainty. Accordingly, we assume that households can hold risk-free real one-period government bonds b_t^i, which pay a gross real interest rate of r_t regardless of the state of nature (including the survival of the bond holder). Households also buy shares v_t^i, for a real price q_t^v, in the capital rental firm which pays out its net cash flows as dividends, d_t.[4] They can also enter into survival-contingent contracts

[3] In the context of our model economy, the very wealthy individuals may have a downward sloping labour supply, as pointed out by Ascari and Rankin (2007). While this is an important aspect, in this paper we focus the analysis on aggregate dynamics and abstract from distributional issues. And, on the aggregate, the labour supply has the standard shape.

[4] By assuming firms accumulate capital rather than households doing so directly, we ensure that the capital

with other households, which pay an agreed sum to other households in the event of the individual's death, but entitle the individual to similar payments from deceased households should the individual survive. The individual will construct a portfolio of bonds, equities and survival-contingent contracts such that the payoff from that portfolio should the individual die is zero. However, if household i is lucky enough to survive their combined return from risk-free bonds, equities and survival-contingent contracts written against those bonds and shares will be $\frac{b_{t-1}^i r_{t-1}}{\gamma}$ and $\frac{(q_t^v+d_t)v_{t-1}^i}{\gamma}$, respectively. This is simply an alternative means of capturing the insurance contracts usually undertaken within the Blanchard-Yaari set-up.

Consumers seek to maximize utility subject to the demand schedule for their labor services and their budget constraint, which can be written as

$$b_t^i + q_t^v v_t^i + c_t^i = (1-\tau_t)w_t l_t^i + \frac{r_{t-1}b_{t-1}^i}{\gamma} + \frac{(q_t^v + d_t)v_{t-1}^i}{\gamma} + (1-\gamma)\int_0^1 \Omega_{jt}dj$$

where all variables are real. Here consumers earn after-tax income from their labor services, $(1-\tau_t)w_t l_t^i$, and receive their share of the profits of final goods producers, $(1-\gamma)\int_0^1 \Omega_{jt}dj$.

Let us define

$$H_t^i \equiv \left[(1-\tau_t)w_t l_t^i + (1-\gamma)\int_0^1 \Omega_{jt}dj\right]$$

and

$$W_t^i \equiv \frac{r_{t-1}}{\gamma}b_{t-1}^i + \left(\frac{q_t^v + d_t}{\gamma}\right)v_{t-1}^i$$

as the non-financial and financial income of generation i households in period t. Then, the budget constraint can be written as

$$Q_{t,t+1}W_{t+1}^i + c_t^i = H_t^i + W_t^i.$$

W_t^i represents the real payoff from the household's portfolio in all states of nature, but conditional on the household surviving, and $Q_{t,t+1} = \gamma r_t^{-1}$ is the price of receiving one unit of that payoff. Note that, should the household not survive, the payoff from the portfolio is zero, such that the expected payoff from one unit of the portfolio across all states of nature, including the survival/non-survival of the household, is the risk free real rate of interest r_t.

Maximizing household utility subject to the budget constraint yields the consumption Euler equation,

$$Q_{t,t+1} = \gamma\beta\left(\frac{c_t^i}{c_{t+1}^i}\right)$$

or equivalently,

$$1 = \beta r_t\left(\frac{c_t^i}{c_{t+1}^i}\right),$$

the labor supply condition,

$$(1-\tau_t)w_t(1-l_t^i) = \varkappa c_t^i,$$

accumulation decision is undertaken by an infinitely-lived entity, such that it is comparable to standard analyses.

and the no-arbitrage condition for equities,

$$q_t^v = r_t^{-1} \left(q_{t+1}^v + d_{t+1} \right).$$

Using the household budget constraint, together with the Euler equation, and the no-arbitrage condition for equities, we obtain the consumer's consumption function

$$c_t^i = (1 - \gamma\beta) \left[W_t^i + \sum_{s=0}^{\infty} (\gamma)^s \left(\prod_{i=0}^{s-1} r_{t+i}^{-1} \right) H_{t+s}^i \right]$$

where the household discounts future labor and profit income more heavily than its straight rate of time preference, as it will not receive that income should it die, but expectations are taken over all states of nature, other than the survival/non-survival of the household. We can further write this as

$$c_t^i = (1 - \gamma\beta) \left[W_t^i + lw_t^i \right]$$

where lw_t^i represents generation i's human wealth, given as the discounted value of labor income and profits, where the effective discount factor accounts for the probability of survival,

$$lw_t^i \equiv H_t^i + \sum_{s=1}^{\infty} (\gamma)^s \left(\prod_{i=0}^{s-1} r_{t+i}^{-1} \right) H_{t+s}^i \ = H_t^i + \left(\frac{\gamma}{r_t} \right) lw_{t+1}^i.$$

2.2 Aggregating across Consumers and Consumption Dynamics.

If the size of each cohort when born is 1, then the size of a cohort i at time t is given by γ^{t-i} and the total size of the population is then given by $\sum_{i=-\infty}^{t} \gamma^{t-i} = \frac{1}{1-\gamma}$. Aggregate (per capita) variables are defined as, $x_t = (1-\gamma) \sum_{i=-\infty}^{t} \gamma^{t-i} x_t^i$. Aggregating the consumers' labor supply yields, in per-capita terms,

$$\varkappa c_t = (1 - \tau_t) w_t (1 - l_t).$$

It is similarly possible to aggregate across consumers from different generations to obtain an aggregate consumption function,

$$c_t = (1 - \gamma\beta) \left[W_t + lw_t \right].$$

Aggregate human wealth lw_t is given by

$$lw_t = H_t + \gamma \frac{lw_{t+1}}{r_t},$$

where period-t non-financial income H_t is defined as

$$H_t \equiv \left[(1 - \tau_t) w_t l_t + (1 - \gamma) \int_0^1 \Omega_{jt} dj \right].$$

The aggregate of financial wealth is

$$W_t = r_{t-1}b_{t-1} + (q_t^v + d_t)v_{t-1}$$

and it takes account of the fact that not all households will have survived from last period into the current one. The households' aggregate (per-capita) budget constraint is then given by

$$b_t + q_t^v v_t + c_t = (1-\tau_t)w_t l_t + r_{t-1}b_{t-1} + (q_t^v + d_t)v_{t-1} + (1-\gamma)\int_0^1 \Omega_{jt}dj.$$

2.3 The Capital Rental Firm's Behavior

We assume that there is a single representative firm accumulating private capital for rental to the final goods producing firms. This firm seeks to maximize the discounted value of its cash flows. This objective function is consistent with maximizing the value of the households' equity. Therefore the firm's objective function is to maximize the following expression,

$$\left(q_t^v + d_t\right)v_{t-1} = p_t^k k_{t-1} - e_t + \sum_{z=1}^{\infty}\left(\prod_{i=0}^{z-1} r_{t+i}^{-1}\right)\left[p_{t+z}^k k_{t+z-1} - e_{t+z}\right],$$

where p_t^k is the real rental cost of capital, k_{t-1} is the private capital stock used in production at time t, and e_t is real investment expenditure. Assuming the capital stock depreciates at rate δ, the equation of motion of the capital stock is then

$$k_t = e_t + (1-\delta)k_{t-1}.$$

The first order condition for investment is given by,

$$\lambda_t^k = 1,$$

where λ_t^k is the Lagrange multiplier associated with the equation of motion for the capital stock. Given the homogeneity of our profit function, this is equivalent to Tobin's q so that, in the absence of capital adjustment costs, Tobin's q is one. Also, differentiating the Lagrangian with respect to k_t gives the equation of motion for Tobin's q,

$$1 = r_t^{-1}\left(p_{t+1}^k + 1 - \delta\right).$$

The capital accumulated by this sector is then rented out to the imperfectly competitive firms producing final goods for consumers, as described below.

This marginal q can be related to average q (and therefore the value of households' equity) as

$$\lambda_t^k k_t + p_t^k k_{t-1} - e_t = (q_t^v + d_t)v_{t-1},$$

so we can re-define non-human wealth as,

$$W_t = r_{t-1}b_{t-1} + (p_t^k + 1 - \delta)k_{t-1}.$$

2.4 Capital and Labor Demand: Cost Minimization of Final Goods Firms

We assume there is a continuum of monopolistically competitive firms, indexed by j, which produce the final goods that enter the CES aggregate consumption basket. The optimal combination of capital and labor, employed in the production of these goods, is obtained from the cost minimization problem of the firm, given the production function it faces,

$$y_{jt} = k_{jt-1}^{\alpha_1}(A_t^l l_{jt})^{\alpha_2}(k_{t-1}^p)^{\alpha_3}$$

where k_{jt-1} is the private capital employed by the firm, l_{jt} is the labor employed by the firm, A_t^l is labor embodied technical progress, and k_{t-1}^p is the public stock of capital which is a public good accumulated by the government. We assume that this production function exhibits constant returns to scale in its arguments, so that the firm faces diminishing returns in its private factors. Accordingly, the economy can experience exogenous growth through labor-embodied technical progress, which occurs at a gross quarterly rate of ω, such that $A_{t+1}^l = \omega A_t^l$.

This implies the following cost minimizing combination of labor and capital which, since all final goods firms are identical, can be written in terms of aggregate variables as

$$\frac{l_t}{k_{t-1}} = \frac{\alpha_2}{\alpha_1}\frac{p_t^k}{w_t}$$

where w_t is the real wage rate and p_t^k the real rental price of capital. The real marginal cost is then defined as,

$$mc_t = (y_t)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} (p_t^k)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} (w_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} (A_t^l)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} \left(k_{t-1}^p\right)^{-\frac{\alpha_3}{\alpha_1+\alpha_2}}$$

while total output is given by,

$$y_t = k_{t-1}^{\alpha_1}(A_t^l l_t)^{\alpha_2}(k_{t-1}^p)^{\alpha_3}.$$

2.5 Price Setting of Final Goods Firms

Given the demand curve for each individual good j, $y_t\left(j\right) = \left(p_t\left(j\right)/P_t\right)^{-\varepsilon} y_t$, firms set prices at a constant markup over marginal cost and, in a symmetric equilibrium where $p_t\left(j\right) = P_t$, we have

$$(1-\varepsilon) + \varepsilon mc_t = 0.$$

Equilibrium real (per capita) profits of all final goods producers are then given as,

$$(1-\gamma)\int_0^1 \Omega_{jt}dj = y_t - \left(w_t l_t + p_t^k k_{t-1}\right).$$

2.6 The Government

The government faces the following flow budget constraint,

$$g_t + e_t^p = \tau_t w_t l_t + b_t - r_{t-1} b_{t-1},$$

where it finances public consumption g_t and investment e_t^p by taxing labor income at rate τ_t and issuing real one-period bonds b_t, which pay a gross real rate of interest r_t. The government owns a stock of public capital k_{t-1}^p, which evolves as

$$k_t^p = e_t^p + (1-\delta^p)\, k_{t-1}^p,$$

where we allow the depreciation rate of public capital δ^p to differ from that of private capital, δ.

In the Ramsey policy we consider below, the government generally has access to three instruments, namely public consumption and investment, as well as the labor income tax. Therefore, although we occasionally allow for lump sum taxes as a diagnostic tool, in our benchmark model the government only has access to distortionary taxation.

This completes the derivation of the model, which is summarized in Appendix A.1. Since our model features exogenous growth, we further render the equilibrium stationary by detrending the relevant variables by the level of labor-embodied technical progress – where all detrended variables are denoted by a tilde and defined as $\widetilde{x}_t = x_t / A_t^l$ (see details in Appendix A.2).

3 Social Welfare

In Appendix A.4 we derive the social welfare metric we employ in the paper. In doing so we follow Calvo and Obstfeld (1988) in distinguishing between the intertemporal and distributional aspects of welfare. We choose to focus on the former, such that we can rewrite the objective function in terms of detrended variables as,

$$U_0 = \sum_{t=0}^{\infty} \beta^t \ln(u_t)$$

where $\ln(u_t) = \ln(\widetilde{c}_t) + \vartheta \ln(\widetilde{g}_t) + \varkappa \ln(1-l_t) + (1+\vartheta)\sum_{s=0}^{t-1} \ln(\omega) + (1+\vartheta)\ln A_0^l$. This implies we can obtain an exact expression for discounted lifetime welfare in terms of stationary variables,

$$U_t = \beta U_{t+1} + \ln(\widetilde{c}_t) + \vartheta \ln(\widetilde{g}_t) + \varkappa \ln(1-l_t) + (1+\vartheta)\left[\frac{\beta}{1-\beta}\ln(\omega)\right] + (1+\vartheta)\ln A_0^l.$$

Note that, in the benchmark analysis, we assume that the policy maker discounts the future at the same rate as households do, but without accounting for the probability of death. However, in the sensitivity analysis below, we shall allow the policy maker to possess a discount factor $\rho < \beta$ as a means of capturing the myopia implied by the various political frictions that can give rise to a deficit bias problem - see Alesina and Passalacqua (2017).

3.1 Optimal Fiscal Policy

Given the social welfare function, the optimal policy problem can be set up in terms of a Lagrangian as,

$$L_0 = \max_{\mathbf{y}_t} \sum_{t=0}^{\infty} \beta^t [U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) - \boldsymbol{\lambda}_t f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t)]$$

where $\mathbf{y}_t$ and $\mathbf{x}_t$ are vectors of the model's endogenous and exogenous variables, respectively, $U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t) = \ln \widetilde{c}_t + \vartheta \ln \widetilde{g}_t + \varkappa \ln(1-l_t) + tip$, where tip refers to terms in productivity growth which are independent of policy, $f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{x}_t) = 0$ are the model's equilibrium conditions, and $\boldsymbol{\lambda}_t$ is a vector of Lagrange multipliers associated with these constraints.

The optimization implies the following first order conditions,

$$\left[\frac{\partial U(.)}{\partial \mathbf{y}_t} + \beta F \frac{\partial U(.)}{\partial \mathbf{y}_{t-1}} + \beta^{-1}\boldsymbol{\lambda}_{t-1}F^{-1}\frac{\partial f(.)}{\partial \mathbf{y}_{t+1}} + \boldsymbol{\lambda}_t \frac{\partial f(.)}{\partial \mathbf{y}_t} + \beta \boldsymbol{\lambda}_{t+1} F \frac{\partial f(.)}{\partial \mathbf{y}_{t-1}}\right] = 0 \qquad (1)$$

where F is the lead operator, such that F^{-1} is a one-period lag. We can then solve these first order conditions in combination with the non-linear equilibrium conditions of the model, $f(\mathbf{y}_{s+1}, \mathbf{y}_s, \mathbf{y}_{s-1}, \mathbf{x}_s) = 0$. We do this fully non-linearly to obtain the steady-state of the policy maker's problem. Since this is a perfect foresight economy, we can also solve for the non-linear transition dynamics using standard techniques, and we discuss those dynamic paths below.

Social Planner's Allocation

In exploring optimal policy, it is helpful to contrast the decentralized equilibrium with the allocation that would be achieved by a social planner who simply implemented the first-best solution. The social planner's problem, in stationary form, is given by,

$$L_0 = \sum_{t=0}^{\infty} \beta^t [\ln \widetilde{c}_t + \vartheta \ln \widetilde{g}_t + \varkappa \ln(1-l_t)] + tip$$

subject to,

$$\widetilde{y}_t = \left(\widetilde{k}_{t-1}/\omega\right)^{\alpha_1} l_t^{\alpha_2} \left(\widetilde{k}_{t-1}^p/\omega\right)^{\alpha_3} \tag{2}$$

$$\widetilde{k}_t = \widetilde{e}_t + (1-\delta)\,\widetilde{k}_{t-1}/\omega \tag{3}$$

and

$$\widetilde{k}_t^p = \widetilde{e}_t^p + (1-\delta^p)\,\widetilde{k}_{t-1}^p/\omega \tag{4}$$

$$\widetilde{y}_t = \widetilde{c}_t + \widetilde{g}_t + \widetilde{e}_t + \widetilde{e}_t^p \tag{5}$$

Note that government debt does not exist in the social planner's problem, so the constraints involved in inheriting a positive debt level disappear. Deriving the FOCs and eliminating the associated Lagrange multipliers gives us the optimal relationship between government spending and consumption,

$$\widetilde{g}_t = \vartheta \widetilde{c}_t \tag{6}$$

while the labor allocation is given by

$$\varkappa \widetilde{c}_t = \left(\alpha_2 \frac{\widetilde{y}_t}{l_t}\right)(1-l_t)\,. \tag{7}$$

The intertemporal consumption/saving decision, which is the modified Golden rule, is

$$\omega \widetilde{c}_t^{-1} = \beta \widetilde{c}_{t+1}^{-1} \left(\alpha_1 \frac{\widetilde{y}_{t+1}}{\widetilde{k}_t/\omega} + 1 - \delta\right) \tag{8}$$

and the balance between public and private forms of capital is given by

$$\alpha_1 \left(\widetilde{k}_{t-1}/\omega\right)^{-1} = \alpha_3 \left(\widetilde{k}_{t-1}^p/\omega\right)^{-1} + (\delta - \delta^p)\,(\widetilde{y}_t)^{-1}\,. \tag{9}$$

Simultaneously solving equations (2)-(9) then yields the social planner's allocation.

In the Ramsey problem the policy maker chooses tax rates, public consumption and investment to maximize social welfare subject to the constraints implied by the decentralized equilibrium. In order to develop intuition for the outcome in this case, it is helpful to contrast the social planner's FOCs to the equivalent conditions obtained as part of the decentralized equilibrium. Firstly, we can write the labor allocation under the decentralized equilibrium (the details of the derivations are included in Appendix A.3) as:

$$\varkappa \widetilde{c}_t = \left[(1-\tau_t)\frac{\varepsilon - 1}{\varepsilon}\right]\left(\alpha_2 \frac{\widetilde{y}_t}{l_t}\right)(1-l_t)\,. \tag{10}$$

This condition is in the same form as (7), except for the wedge due to the tax and monopolistic competition distortions, $(1-\tau_t)\frac{\varepsilon-1}{\varepsilon}$, which imply that the use of labor in production is sub-optimally low. Accordingly, an ability to offer a subsidy equivalent to a negative income tax of $\tau_t = 1 - \frac{\varepsilon}{\varepsilon-1}$ would eliminate this distortion.

Similarly, we obtain the aggregate consumption Euler equation in the decentralized equilibrium (details also in the Appendix) as

$$\widetilde{c}_t = \widetilde{c}_{t+1}\omega\beta^{-1}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\alpha_1\frac{\widetilde{y}_{t+1}}{\widetilde{k}_t/\omega}+1-\delta\right]^{-1}+\frac{(1-\gamma\beta)(1-\gamma)}{\gamma\beta}\left(\widetilde{b}_t+\widetilde{k}_t\right). \qquad (11)$$

Relative to the social planner's allocation in (8), the presence of monopolistic competition in the decentralized economy distorts the intertemporal savings allocation, while the presence of finite lives gives rise to an additional term in the aggregate consumption dynamics which the social planner's allocation does not feature. This last term in (11) captures the fact that, in order to induce finitely lived households to hold non-human wealth, the (growth-adjusted) returns to that wealth need to exceed their rate of time preference.

This is the first important implication of allowing for finite lives with no bequests: the real rate of interest can differ from the rate of time preference even in steady state (the implications of this point are discussed in Erosa and Gervais (2001)).[5] The second important difference an OLG model makes is that government debt can crowd out capital. In steady state, if consumption and real interest rates were unchanged, government debt would crowd out private capital one for one. In fact consumption is likely to fall if capital falls, increasing the extent of crowding out. However, a reduction in the capital stock will also raise real interest rates, which for given consumption levels will raise the overall level of aggregate assets, which moderates the degree of crowding out of capital. (In the infinite life case, which we approach as γ tends to one, any increase in government debt leads to an equal increase in savings, so there is no crowding out.)

Just as government debt crowds out capital, if the government holds assets ($\widetilde{b}_t < 0$), capital will be crowded in. If, when $\widetilde{b}_t = 0$, capital is sub-optimally low, then accumulating government assets can be used to move towards the optimal level of capital. We could define the level of government assets that achieve this optimum capital stock as the 'optimum capital' level of assets, or A^K. Unless the economy with $A_t = -\widetilde{b}_t = 0$ is dynamically inefficient[6], such a move would not represent a Pareto improvement, because the higher taxes that the government would require to accumulate assets would hit the current generation. However, as any debt policy is almost certain to disadvantage some generation, this should not prevent us considering using debt as a means of moving towards A^K.

To correct both distortions, the optimal path for government debt would follow

$$-\frac{1}{\varepsilon}\left(\alpha_1\frac{\widetilde{y}_{t+1}}{\widetilde{k}_t/\omega}\right)\widetilde{c}_t = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma\beta}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\alpha_1\frac{\widetilde{y}_{t+1}}{\widetilde{k}_t/\omega}+1-\delta\right]\left(\widetilde{b}_t+\widetilde{k}_t\right). \qquad (12)$$

In the absence of the monopolistic competition distortion ($\varepsilon \to \infty$) this would simply imply

[5] Erosa and Gervais (2001) stress the fact that even in the aggregate steady-state the demographic turnover observed in OLG models implies that standard results from representative agent models may not apply. Therefore, for example, the time variation in tax elasticities over the life-cycle can imply that it is optimal to have non-zero capital tax rates even in steady-state.

[6] In this model of perpetual youth, $r > \omega/\beta$, so the economy is never dynamically inefficient. However introducing either government assets, or allowing income to decline with age, can allow the possibility that $r < \omega/\beta$, as we note below.

$b_t = -k_t$, while with the monopolistic competition still in place the policy maker would wish to accumulate assets in excess of the private capital stock, $b_t < -k_t$, in order to reduce interest rates below the households' rate of time preference and encourage the use of capital. Accordingly a benevolent policy maker armed with a lump sum tax would levy that tax to finance a subsidy for labor in order to offset the distortions due to the under-utilization of labor implied by distortionary taxation and monopolistic competition. The policy maker would then pursue a path for debt which would negate both the monopolistic competition and finite lives distortions on the intertemporal savings decision in the decentralized economy. Of course, without access to a lump sum tax it is generally not possible to simultaneously achieve both goals unless the level of debt implied by (12) also happened to be at a level which finances public consumption and investment in line with the social planner's allocation - (6) and (9), respectively - as well as the labor market subsidy, $\tau_t = 1 - \frac{\varepsilon}{\varepsilon - 1}$.

We now turn to explore optimal policy in the absence of the lump sum taxation required to simultaneous offset the distortions to both the intratemporal allocation of labor and the intertemporal savings decisions. Interestingly, we shall see that the optimal policy need not imply that we drive debt to a (negative) level which lies between that implied by the need to eliminate the distortions to intertemporal savings behavior, (12), and the 'war chest' level needed to support desired levels of public consumption, investment, and offset the monopolistic competition distortion by turning the income tax into a subsidy $\tau_t = 1 - \frac{\varepsilon}{\varepsilon - 1}$.

3.2 Calibration

In order to analyze the main implications of our model, we first calibrate the model based on empirically observed levels of real GDP growth, public and private capital, government consumption, labor income shares and government debt in the U.S. Between 1980 and 2008, the average annualized growth rate was 2.88%, private and public capital to GDP ratios were 2.3 and 0.6 respectively, government consumption was 16% of GDP, the labor income share was around 54% and government debt averaged 55.6% percent of GDP.[7] Table 1 summarizes the values of the calibrated baseline parameters and Table 2 summarizes the resultant steady state.

The elasticity of demand with respect to price ε is set to 11, consistent with a steady-state mark-up, $\varepsilon/(\varepsilon - 1)$, equal to 1.1. Parameter $\varkappa$, measuring the weight on leisure in utility, was set to 1.19 such that households in our model economy allocate about a third of their time to market activities (which is broadly in line with the empirical evidence). The weight given to government consumption in utility, $\vartheta = 0.24$, implies that the policy maker would ensure that government consumption as a share of private consumption is similar to the patterns found in the US data. With a quarterly discount factor β of 0.9938 and a survival probability of $\gamma = 0.995$, implying an expected adult working life of 50 years[8], our model can

[7] The debt to GDP ratio was obtained from the Public Debt Reports of the U.S. Department of the Treasury, while the rest of the data values were obtained from the U.S. Bureau of Economic Analysis, the National Income and Product Accounts.

[8] We focus on economically active individuals (from 15 to 64 years old). 50 years is then a compromise

β	γ	ϑ	$\varkappa$	α_1	α_2	δ	δ^p	ε
0.9938	0.995	0.24	1.19	0.35	0.59	0.021	0.0071	11.0

Table 1: Calibration of baseline model - Parameters

b/y	ω	g/y	k/y	k^p/y	wl/y	r	τ
55.6%	2.88%	0.16	2.27	0.64	0.54	6%	0.39

Table 2: Calibration of baseline model - Initial Steady State

match these steady-state ratios with elasticities of output with respect to private capital and labor of $\alpha_1 = 0.35$ and $\alpha_2 = 0.59$, respectively. This, in turn, implies a coefficient on public capital in production of 0.06, which is very close to the 0.05 adopted in Baxter and King (1993) and well within the range of estimates considered in the meta-analysis of Bom and Ligthart (2014). The depreciation rate on private capital, δ, is equal to 0.021, as estimated by Christiano and Eichenbaum (1992). The depreciation rate of public capital, $\delta^p = 0.0071$, was obtained from averaging the depreciation rates implied by the data on the public sector capital stock and its depreciation, over the sample period considered.

It should be noted that this calibration is not based on the steady state of the Ramsey problem, but the steady state of the structural model equations given the levels of government consumption, investment, and taxes needed to support observed data levels of government spending, public capital and government debt as a proportion of GDP, as well as labor income shares, growth rates, and private capital/output ratios. In particular, government consumption and investment are such that their ratios to output match the data values, while the tax rate τ is set to ensure the government budget constraint is satisfied. We shall see that, when fiscal variables are chosen optimally, the economy will move a long way from this starting point. For this reason, we do not employ any approximation techniques in solving the model, such that steady-state solutions and dynamics of the model are all obtained as fully non-linear solutions to the Ramsey policy problem described above.

3.3 The Optimal Debt Target

In this section, we examine the optimal level of steady-state government assets implied by our model, using the calibration set out above. This is the solution to the Ramsey policy maker's problem, obtained by solving the non-linear equations of the model together with the first order conditions (1). In Table 3, the first column of numbers details the steady state implied by our calibration, which is taken to be the starting point prior to the various optimal policy exercises being undertaken. The remaining columns describe the steady state that emerges under various forms of optimal policy.[9]

To understand clearly the economic processes involved, it is easiest to begin, in the second column of numbers of Table 3, with the allocation chosen by the social planner. This

between the years that Europeans are active, which is the reference variable for labour, and life expectancy which is probably a more relevant variable for consumption. We also set "economic" life expectancy equal to 50 years as a way of having a lower discount rate and, therefore, higher non-Ricardian effects. Nevertheless,

Variable	Calibration	Soc. Planner	mc = 1, L.S.	mc = 1	mc<1 (Benchmark)	Zero Debt	Debt > 0
b/y	0.56	n.a.	-2.58	-2.49	-2.82	0	0.56
k/y	2.27	2.58	2.58	2.58	2.36	2.29	2.28
k^p/y	0.64	0.65	0.65	0.64	0.65	0.63	0.62
r	6%	n.a.	5.47%	5.48%	5.39%	5.85%	5.93%
c/y	0.55	0.54	0.54	0.55	0.57	0.58	0.59
g/y	0.16	0.13	0.13	0.13	0.13	0.13	0.12
τ	0.39	n.a.	n.a.	0.17	0.18	0.29	0.32
l	0.33	0.48	0.48	0.43	0.39	0.35	0.34
w	2.17	n.a.	2.58	2.57	2.22	2.17	2.16
Welfare Cost		n.a.	0	6.1%	18.8%	34.6%	39.1%

Table 3: Steady State of Ramsey Problem

allocation chooses the capital stock (public and private), consumption (public and private) and hours worked to maximize welfare, without the need to raise taxes or government debt. The third column of numbers then looks at a decentralized economy in which taxes are lump sum and there is no monopoly power,[10] but the government does choose the optimal level of debt, as well as continuing to choose the optimum level of public consumption and capital. It chooses to hold a level of government assets exactly equal to the size of the capital stock. In effect the government lends to the private sector who use the funds to purchase private capital, such that the private sector holds no net assets.

To see why this has to be the case, recall that our social welfare function assumes a discount rate equal to the rate of time preference. Without government debt, the Blanchard/Yaari model will imply a growth corrected real interest rate that exceeds this discount rate and, as a result, consumers will start accumulating assets from the moment they are born. Consequently, the capital stock will be below the level that would occur if all consumers were infinitely lived. To correct this underinvestment, the government has to reduce the growth corrected real interest rate to the social discount rate, which would be the real interest rate a decentralized equilibrium with infinitely lived consumers would achieve, but at that interest rate Blanchard/Yaari consumers would no longer wish to accumulate assets.

Column 4 replaces lump-sum taxes by distortionary labor income taxes, but still assumes there is no monopolistic competition distortion. As we would expect, this reduces hours worked and there is a decline in welfare.[11] We now have a second motive for accumulating government assets, which is to eliminate these distortionary taxes. If government assets were

in sensitivity analysis, we also consider the consequences of adopting a different probability of death.

[9] In the next sub-section, we present the transition from the calibrated steady state to the steady state implied by three of these optimal policies - namely, the social planner's allocation, and the solutions to the Ramsey policy maker's problems with and without access to a lump-sum tax instrument.

[10] Equivalently, we could allow the policy maker to have access to a lump-sum tax and a subsidy instrument (such as a negative income tax) with which to offset the monopolistic competition distortion.

[11] The welfare cost measures in the table give the percent decrease in welfare in each scenario, relative to the first best.

large enough, we might imagine that interest receipts on these assets could fund the optimal level of public consumption and maintain the optimal level of public capital. As we can see from Table 3, column 4, taxes are still positive. More interesting is that the optimal level of government financial assets is now lower than in the previous column. This is somewhat counter-intuitive. As distortionary taxes are not eliminated when government assets exactly equal the capital stock, we might have expected the optimal level of government assets to lie between this level and the level that eliminated all distortionary taxation. The reason we do not get this result is because of the endogeneity of interest rates.

To formalize this slightly, consider a highly simplified characterization of the trade-offs facing the policy maker in our model in which social welfare could be represented as follows:[12]

$$W_t = -\sum_{i=0}^{\infty} \beta^i \left[T_{t+i}^2 + \alpha \left(A_{t+i} - A^K \right)^2 \right]$$

where T represents the revenues raised by distortionary taxes and A^K is the level of government assets that, in the absence of distortionary taxes, would maximize social welfare. If finite lives were the only distortion in the economy, then the government can mimic the social planner's allocation by lending to households to enable them to accumulate capital without holding any net assets, $A^K = k$. But with monopolistic competition as an additional distortion $A^K > k$ and the equilibrium real interest rate depends on actual government assets such that when $A < (>)k$ then $r > (<)1/\beta$. However, if we assume that there are no lump sum taxes then there is an additional incentive to accumulate assets to finance government expenditure and reduce distortionary taxes to zero.

We begin by assuming that there is no monopolistic competition distortion, then $A^K = k$. This model therefore contains the two 'distortions' present in Table 3, column 4: distortionary taxes and that the decentralized economy without government assets will accumulate too little capital. In our overlapping generations economy the equilibrium interest rate is not simply equal to household preferences but depends on the stock of government debt/assets, which we give a general form, $r_t = r\left(A_t - k\right)$, where $r(0) = 1/\beta$, and $r'(.) < 0$ i.e. as the government accumulates financial assets the equilibrium real interest rate falls, cet. par., falling below the households' rate of time preference once $A > k$.

The government chooses taxes and government assets to maximize social welfare subject to its budget constraint

$$A_t = r_{t-1} A_{t-1} + T_t - G$$

where, for simplicity, we treat government spending as fixed.

The Lagrangian for this problem can be written as

$$L = \sum_{i=0}^{\infty} \beta^i \left[T_{t+i}^2 + \alpha \left(A_{t+i} - A^K \right)^2 + 2\lambda_{t+i}(A_{t+i} - r_{t+i-1} A_{t+i-1} - T_{t+i} + G) \right]$$

[12] While this is a simple and ad-hoc representation, it helps provide intuition for the results we obtain.

with first order conditions

$$T_{t+i} - \lambda_{t+i} = 0$$

and

$$\alpha\left(A_{t+i} - A^K\right) + \lambda_{t+i} - \beta\lambda_{t+i+1}\left[r_{t+i} + \beta r'_{t+i}(.)A_{t+i}\right] = 0.$$

In steady state these can be simplified to

$$\alpha(A - A^K) = \left[\beta r(.) - 1\right]T + \beta r'(.)AT. \tag{13}$$

When $A = A^K = k$, $\beta r = 1$, and $r\prime(0) < 0$, this equation cannot hold when taxes are positive. Instead, the only feasible steady state with positive taxes is where $A < A^K = k$ and $\beta r(.) > 1$. The reason is straightforward. If we tried to increase government assets above A^K to eliminate more distortionary taxes, this would reduce real interest rates, and therefore the return on these assets. This lower return would offset the benefits of lower taxes, so we would fail to eliminate additional distortionary taxes. In fact, in this simple characterization of our model, the optimal level of government assets that maximizes debt interest receipts is below A^K. Column 4 in Table 3 suggests that this is also true in our microfounded model.

In Column 5 of Table 3, we add in the final distortion in our model, monopoly power. This is like a tax on profits and the returns to capital, and therefore significantly reduces the level of capital relative to output (and, of course, the level of output itself). The government attempts to compensate for this to some extent by increasing government financial assets compared to the level in the previous column, and the growth corrected real interest rate now falls below the rate of time preference. However the extent to which it can mitigate this monopoly distortion is small, because by reducing real interest rates it is lowering receipts from these assets (note the income tax rate rises). This can be seen from equation (13) in the simple representation of the trade-offs facing the policy maker. In the presence of the monopolistic competition distortion $A^K > k$, and A must fall short of $A^K > k$ once interest rates fall below the households' rate of time preference and given that further accumulation of financial assets on the part of the government will depress interest rates further $r'(.) < 0$.[13]

Column 6 of Table 3 does not compute optimal government financial assets, but instead sets them to zero, while column 7 sets them to more realistic values, as given by the data average $b/y = 0.56$. (The level of public consumption and physical capital continue to be chosen optimally.) These columns indicate the welfare losses implied by not having the government hold financial assets, or issuing government debt. Welfare decreases substantially, partly because the capital stock falls, but also because the extent of distortionary taxation increases. The size of these welfare losses indicates the extent of the costs of positive government debt in this type of economy.[14] Before considering the question of how quickly the

[13] Conditional on the steady-state values reported in this benchmark case, the 'target' debt stock implied by equation (12) is an annualised debt to gdp ratio of -16.4 suggesting that the optimal policy falls well short of the value of A^k that would apply in the case of a monopolistic competition distortion.

[14] We also considered a range of empirically relevant values for the debt to GDP ratio. A b/y value of 0.4, for example, implies a welfare loss of 17.29% relative to the socially optimal outcome, which increases by a

government might attempt to transform financial debt into financial assets to achieve these welfare gains, we consider some robustness exercises.

Sensitivity Analysis We undertake a sensitivity analysis of the optimal steady-state debt-to-GDP ratio with respect to a few parameters. These are chosen to reflect the possibility that recent trends in ratios used in the calibration may differ from historical averages, as well as providing additional insight into the mechanisms driving our main results. Specifically, we consider variations in the preference for public goods in utility ϑ, the productivity growth rate ω, the survival probability rate γ, the elasticity of output with respect to labor α_2 (relative to that of private capital) and the elasticity of output with respect to public capital α_3 (relative to that of private capital), and finally the policy maker's rate of time preference ρ. For the first four parameters ($\vartheta,\ \omega,\ \gamma,$ and α_2), we consider alternative values that aim to capture recent data trends and expected future values as reflected in economic forecasts. The alternative value for α_3 spans the range of empirical estimates. While for the policy maker's rate of time preference parameter ρ, we choose a range of values that describe different degrees of myopia on the part of the government, who discounts the future more heavily than households.

The results are reported in Table 4, where column 1 repeats Column 5 of Table 3 and gives the optimal level of government assets in the presence of the monopoly and tax distortions (the case we consider as the benchmark scenario). Column 2 in Table 4 reduces the preference for public goods consumption, by reducing the weight on government consumption in utility from $\vartheta = 0.24$ to $\vartheta = 0.15$. This parameter reflects the desirable ratio of government consumption to private consumption. The lower value we choose captures a recent downward trend in the 'g/c' ratio in the data and is at the lower end of the range of observed values over the post-WW II period. The optimal policy now slightly raises the stock of government assets relative to the benchmark case - essentially, there is less need to maintain high interest rate income on government assets, given that the stream of government consumption requiring financing is now reduced. Accordingly, more government assets are accumulated in order to reduce interest rates and encourage private capital accumulation, in a manner which offsets both the finite lives distortion and that due to imperfect competition.

Column 3 reduces the annualized exogenous growth rate ω from the historical average of 2.88% to a lower expected rate of 1.84%, which is an average of OECD projections over the next 50 years (OECD Economic Outlook, May 2013). We notice that the lower rate of productivity growth raises the stock of government assets and, at the same time, there is a marked increase in the ratios of both public and private investment to GDP. This reflects the policy maker's attempt to mimic a similar increase in the capital-to-GDP ratios that arises in the social planner's allocation. Essentially, more capital accumulation is required to help compensate for the reduced labor productivity, while maximizing welfare. In the decentralized economy with finite-lives, a corresponding accumulation of government assets

further 1.6% when $b/y = 0.8$. Generally, the higher are the government debt levels, the further is the economy away from its optimal setting and the larger are the associated welfare losses.

Variable	Benchmark	$\vartheta = 0.15$	$\omega = 1.84\%$	$\gamma = 0.9958$	$\alpha_2 = 0.57$	$\alpha_3 = 0.106$	ρ= 0.99
b/y	-2.82	-3.19	-3.07	-2.82	-3.00	-2.16	3.04
k/y	2.36	2.37	2.55	2.36	2.49	2.02	2.21
k^p/y	0.65	0.66	0.74	0.65	0.65	1.20	0.48
r	5.39%	5.34%	4.32%	5.41%	5.38%	5.45%	6.31%
c/y	0.57	0.61	0.57	0.57	0.56	0.58	0.6
g/y	0.13	0.09	0.13	0.13	0.13	0.13	0.12
τ	0.18	0.09	0.17	0.18	0.18	0.26	0.46
l	0.39	0.40	0.40	0.39	0.39	0.37	0.29
w	2.22	2.23	2.36	2.21	2.53	2.05	2.08

Table 4: Steady State of Ramsey Problem - Sensitivity Analysis

is needed in order to reduce interest rates and facilitate the crowding in of private sector capital, which is what we observe in Column 3.

Column 4 raises the survival probability from $\gamma = 0.995$ to $\gamma = 0.9958$, reflecting an increase in the retirement age and longer working lives, as observed in the U.S. and other developed economies. The consumption planning horizon of the individuals in our economy is now 60 years. The higher γ reduces the impact of government debt/assets on the real interest rate. Accordingly, the policy maker needs to accumulate slightly higher levels of assets to achieve a given reduction in the real interest rate and the desired crowding in of private sector capital. However, the observed differences are very small, only to a third decimal point for most variables, suggesting that longer working lives in the range considered have almost no bearing on optimal debt levels.

In column 5, we consider an alternative parameterization of the production function, with a lower elasticity of output with respect to labor (lower α_2) and a higher elasticity with respect to private capital (higher α_1), reflecting a recent downward trend in labor income shares in the data and the capital-labor substitution hypothesis (as discussed in Karabarbounis and Neiman (2014) and Elsby et al. (2013)). Relative to the benchmark calibration, α_2 is now reduced by 0.2 to 0.57, while α_1 rises to 0.37.[15] There is, in this case, an optimal re-allocation of resources (as given by the social planner), that sees more accumulation of private capital and an increase in the private capital to GDP ratio. The policy maker would try to achieve a similar allocation, by accumulating more assets and reducing interest rates, thus inducing more crowding in of private capital, which is what we observe in the results of column 5.

Column 6 then investigates the relative importance of public capital in production, by considering a higher elasticity of output with respect to public capital $\alpha_3 = 0.106$ (that matches the average estimate in Bom and Ligthart (2014)), and a correspondingly lower elasticity with respect to private capital, α_1. As public capital is now relatively more important, we observe an increase in the ratio of public to private capital. As such, the policy maker accumulates relatively fewer assets, the interest rate is higher and the level of private

[15]The new value of α_2 is set to match a lower labour income share of 0.52 (as average over the 2012-2014 period) and the assumed benchmark markup of 1.1.

capital lower than under the benchmark case. At the same time, a higher tax rate is needed to finance the increased public investment expenditures, which in turn discourages labor supply.

Policy Maker Myopia

The final column of Table 4 drops the assumption that policy makers share the same rate of time preference as individual households. Instead, we assume that the policy maker discounts the future more heavily than an infinitely-lived household would, $\rho < \beta$. This is intended to act as a short-cut means of capturing the numerous political frictions that give rise to a deficit bias problem (see, Alesina and Passalacqua (2017) for an extensive survey), where the policy maker essentially attaches more weight to the short-run cost of deficit reduction relative to the longer-run benefits of lower debt. Specifically, in the final column of Table 4, we allow policy makers to be slightly more myopic than households and discount the future more heavily, such that its discount factor is lower, $\rho = 0.99 < \beta = 0.9938$. This myopia turns the desired debt to GDP ratio positive (at a rate of 304% of GDP) with an associated rise in the tax rate. There is a significant rise in the real interest rate and crowding out of private sector capital. The impact of myopia is so striking that it might be thought that this amounts to an extreme degree of the policy maker's short-sightedness. We can assess this in different ways. The annualized increase in discounting of the future is given by $(\rho^{-4} - \beta^{-4}) * 100 = 1.58\%$, which is not obviously outrageous. Alternatively, we can imagine the policy maker faces a probability of electoral death which reduces their time horizon. The extra discounting assumed in this experiment implies that the policy maker still has an effective time horizon of over 65 years.

We explore this issue further in Figure 1 which plots both the steady-state debt to GDP levels and the welfare costs of varying the myopia of the policy maker. This shows that the steady-state debt level turns positive with increased discounting of 0.75% (an effective time horizon of 137 years) and a welfare cost of 35.4% relative to the first best. While with myopia of an increased discounting of around 1% p.a. the desired debt to GDP ratio has risen to 100% with a welfare cost of 50%. This gives a measure of the scale of the costs associated with even relatively modest political frictions. The results reported in Figure 1 suggest that the welfare costs of myopia are most pronounced at very high debt levels. Therefore, it appears that creating fiscal institutions which credibly allow for fiscal consolidations is likely to be welfare improving in the long run.

We could extend the robustness checks further by changing the functional forms of the utility and production functions, allowing for different degrees of substitutability between public and private consumption and capital. However, this is not dissimilar to the changes in weights attached to government consumption in utility and public/private capital in production considered above and is unlikely to change the fact that the optimal policy mainly seeks to drive interest rates close to their modified golden rule level in the long run.

3.4 Transition Paths

In this section, we present a brief analysis of the optimal transition path to the Ramsey steady state, using a simulation of the full non-linear Ramsey policy.[16] Our simulation begins at the calibrated steady state which features public and private capital to GDP ratios of 0.64 and 2.27, respectively, alongside a debt to GDP ratio of just over 50%. Starting from that initial position, the Ramsey policy will move us towards the steady state labelled 'benchmark' in Table 4, where the long-run capital to GDP ratios for public and private capital are 0.65 and 2.36, respectively, and the government debt to GDP ratio has fallen to -2.82.

We look at the transition between this initial state to the Ramsey steady state in two ways. The first year impact of adopting the optimal fiscal policy is shown in Figures 2 and 3, where the solid line details the paths followed by key variables in the initial year of the optimal policy. The full transition path is shown in Figures 4 and 5. The most striking aspect of the early response to the switch to optimal policy is that it is desirable to reduce debt by undertaking a very large sale of public capital. Although the initial stock of public capital is close to its optimal steady state value, the optimal transition path involves cutting this stock (relative to GDP) by almost half and then gradually rebuilding it.

Releasing this substantial quantity of goods leads to a sharp fall in the real interest rate, which increases both consumption and the stock of private capital. There is also a sharp fall in public consumption, but as Table 3 shows (particularly the final column) this is not so much a temporary deviation from the steady state as a correction from a sub-optimal allocation in the initial calibration.

Although there are clear welfare advantages to moving towards the steady state level of debt quickly, standard smoothing arguments mean that it is not optimal to sharply increase taxes or to reduce government consumption beyond its optimal level. It is interesting that these arguments do not apply to public capital in our model. One reason for this is that, as public capital can be costlessly transformed into private capital, any substitution between the two will mean output falls only because we move away from the optimal factor mix. However, it is important to remember that there are no investment adjustment costs in our model, so such large movements in capital are not going to be realistic. However, given the very long time scales over which the stock of public capital is rebuilt during the transition adding capital adjustment costs would simply lengthen the period over which the public capital is reduced, prior to being rebuilt.

Once we move beyond the initial period, the remainder of the adjustment is far smoother as Figures 3 and 4 show. Although a significant part of the debt reduction is achieved very quickly by selling public capital, it takes over 100 years to achieve the first 50% of the adjustment, and complete adjustment takes around 500 years. This very long adjustment period is not surprising for two reasons. First, while complete tax smoothing no longer applies, the Blanchard-Yaari framework with realistic values for the probability of death gives

[16] We solve the non-linear FOCs in Dynare under perfect foresight (since our model does not contain any aggregate uncertainty and idiosyncratic risk is insured through the household's purchase of death-contingent annuities) using the Newton algorithm.

only quantitatively minor deviations from Ricardian Equivalence, and so a large smoothing element is retained. Second, earlier analysis using models of this type suggest very long drawn out dynamics (e.g. Leith and Wren-Lewis (2000)). The result that debt adjustment should be very slow appears fairly robust (see Marcet and Scott (2008) and Leeper and Leith (2017), for example).

Following the reduction in public capital, the transition is relatively smooth, although the policy mix changes as the transition progresses. The observed evolution in the policy mix reflects the non-linearities associated with different levels of government debt/assets. At large debt levels, it is desirable to reduce debt interest costs by encouraging saving - this is achieved by the reduction in public capital and by committing to raise taxes and government spending in the future. However, as government debt levels fall the impact of interest rates on debt dynamics are far less pronounced and taxes must rise while government spending is reduced to sustain the accumulation of government assets. Finally, as the stock of government assets is increased, higher interest rates actually facilitate the transition and the optimal policy gradually rebuilds the stock of public capital and reduces tax rates. Accordingly, we observe a substantial reduction in real interest rates in the early stages of the transition when debt levels are high, followed by a gradual rise and eventual over-shooting of real interest rates when debt levels turn negative.

One interesting feature of the adjustment path is the behavior of consumption. The idea that a reduction in debt requires consumption to initially decline before increasing to a higher steady state value is familiar and is, of course, one reason why reductions in debt are so difficult to achieve politically. However comparing Figures 2 and 4 shows that throughout the adjustment path consumption is always above its initial level, because initially consumption jumps up following the sale of public capital. This suggests that, thanks to the existence of public capital that can be transformed into private capital, its sale can reduce the costs of debt reduction for the current generations.

Despite the fact that the speed of adjustment is very slow, the size of adjustment required from current levels of debt is also very large. As a result, the implications for debt reduction today will still be significant. We should also note, however, that our starting point for adjustment does not involve interest rates at the zero lower bound and a large recession, so our analysis has no immediate implications for the 'stimulus versus austerity' debate. However, we can contrast the transition paths for identical economies starting from different initial levels of public debt. Here, we can see that since all the economies will tend to the same steady-state level of government assets in the long run, any initial shock to government debt will only be eliminated very slowly, with clear differences across the transition paths for at least 150 years. This implies that, even if it may be optimal to substantially reduce government debt in the long run, the fact that the recent financial crisis has raised government debt levels does not imply that that fiscal correction need be noticeably more rapid.

The Figures also contrast the Ramsey policy implemented through variations in distortionary taxation (along with optimal values of government consumption and investment), with the policies that would be pursued by a policy maker enjoying an ability to levy lump-sum taxes (dashed green line) and the allocation that would be chosen by the social planner (dotted blue line). In the initial periods both the policy maker with access to lump sum taxation and the social planner would temporarily reduce the stock of public capital, however not to the same extent as our benchmark policy maker. The ability to levy lump sum taxes is highly beneficial as it allows the policy maker to dramatically accumulate government assets, reduce real interest rates and crowd in private capital in a manner which is close to mimicking the social planner's allocation. The only key difference is that the social planner is not faced with the monopolistic competition distortion which is still a feature of the economy with lump-sum taxes. In contrast, the sustained increase in distortionary taxation in the benchmark economy, depresses hours worked and consumption for a prolonged period and greatly slows the transition period relative to the path chosen by the social planner and approximated by the policy maker who possesses lump-sum taxes as a fiscal instrument. Finally, it should be noted that the case where the policy maker had access to both lump sum taxes and an instrument with which to subsidize production (such as a negative labor income tax) would perfectly mimic the dynamic path chosen by the social planner.

4 Conclusions

In models without default where agents are effectively infinitely lived, there is no optimal debt target because the costs of reducing debt are always higher than the cost of accommodating the existing level of debt. In OLG models this is no longer true for two reasons. First, the real rate of interest is likely to be above the rate of time preference, so the benefits, in terms of lower taxes, of future reductions in debt now outweigh the current costs of achieving lower debt. Second, the level of the capital stock is likely to be below the socially optimal level, and reductions in debt will crowd in capital.

In this paper we examine the optimal level of debt in one particular OLG model, the model of perpetual youth. We show that the optimal debt target in a calibrated version of this model involves positive government assets (i.e. a negative debt target), but these assets are below both the level required to eliminate distortionary taxes, and the level required to achieve the optimum capital stock. This is because, when the economy is distorted by monopolistic competition and income taxes, as debt declines the real rate of interest falls below the rate of time preference before the economy reaches the optimal capital stock. The optimal transition path towards this steady state is very drawn out, involving hundreds of years, but as the steady state involves historically unprecedented levels of government assets, the implications for debt adjustment in the short term may still be quantitatively significant.

Finally, we found that introducing policy maker myopia, as a proxy for the kinds of political frictions that lead policy makers to prioritize avoiding the short-term costs of fiscal consolidation over the longer-term benefits, had a significant impact on the steady-state debt level, turning it positive for relatively modest degrees of policy maker short-sightedness. This suggests that enhancing fiscal policy institutions, to allow policy makers to undertake very gradual fiscal consolidations which credibly reduce debt levels in the longer term, is likely to be significantly welfare improving.

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A Appendix

A.1 Summary of Aggregate Model

The aggregate consumption function is

$$c_t = (1-\gamma\beta)\left[W_t + lw_t\right] \tag{14}$$

where all variables are in per capita terms.

The aggregate financial wealth in real terms is

$$W_t = r_{t-1}b_{t-1} + (p_t^k + 1 - \delta)k_{t-1} \tag{15}$$

while aggregate human wealth is

$$lw_t = H_t + \gamma\frac{lw_{t+1}}{r_t} \tag{16}$$

and the period-t non-financial income is given by

$$H_t \equiv (1-\tau_t)w_t l_t + (1-\gamma)\int_0^1 \Omega_{jt}dj. \tag{17}$$

The definition of profits is

$$(1-\gamma)\int_0^1 \Omega_{jt}dj = y_t - \left(w_t l_t + p_t^k k_{t-1}\right). \tag{18}$$

The government budget constraint is given by

$$g_t + e_t^p = \tau_t w_t l_t + b_t - r_{t-1}b_{t-1}. \tag{19}$$

Combine the households' aggregate resource constraint with the government budget constraint and the definition of profits to obtain the aggregate resource constraint

$$g_t + e_t^p + c_t + e_t = y_t. \tag{20}$$

Labor supply satisfies the condition

$$(1-\tau_t)w_t(1-l_t) = \varkappa c_t. \tag{21}$$

The equations of motion of the private and public capital stocks are

$$k_t = e_t + (1-\delta)k_{t-1} \tag{22}$$

and

$$k_t^p = e_t^p + (1-\delta^p)k_{t-1}^p \tag{23}$$

while the first order condition for investment is given by

$$1 = r_t^{-1}\left(p_{t+1}^k + 1 - \delta\right). \tag{24}$$

Monopolistic competition implies

$$(1-\varepsilon) + \varepsilon mc_t = 0. \tag{25}$$

The firms' cost minimisation gives the factors' share

$$\frac{l_t}{k_{t-1}} = \frac{\alpha_2}{\alpha_1}\frac{p_t^k}{w_t} \tag{26}$$

while the real marginal cost is given by

$$mc_t = (y_t)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} (p_t^k)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} (w_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} (A_t^l)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} \left(k_{t-1}^p\right)^{-\frac{\alpha_3}{\alpha_1+\alpha_2}}. \tag{27}$$

The aggregate output function is

$$y_t = k_{t-1}^{\alpha_1}(A_t^l l_t)^{\alpha_2}\left(k_{t-1}^p\right)^{\alpha_3}. \tag{28}$$

A.2 Stationary Model

With an exogenous growth rate in labor-embodied technical progress of ω, such that $A_{t+1}^l = \omega A_t^l$, we can render the equilibrium stationary by deflating the following variables $\{y_t,\ c_t,\ g_t,\ w_t,\ k_t,\ k_t^p,\ e_t,\ e_t^p,\ b_t,\ W_t,\ lw_t\}$ by the level of labor-embodied technical progress, such that $\widetilde{x}_t = x_t/A_t^l$.

The aggregate consumption function:

$$\widetilde{c}_t = (1-\gamma\beta)\left[\widetilde{W}_t + \widetilde{lw}_t\right] \tag{29}$$

Aggregate financial wealth:

$$\widetilde{W}_t = \frac{r_{t-1}}{\omega}\widetilde{b}_{t-1} + \frac{\left(p_t^k + 1 - \delta\right)}{\omega}\widetilde{k}_{t-1} \tag{30}$$

Aggregate human wealth:

$$\widetilde{lw}_t = \widetilde{H}_t + \gamma\omega\frac{\widetilde{lw}_{t+1}}{r_t} \tag{31}$$

Period-t non-financial income:

$$H_t \equiv (1-\tau_t)\widetilde{w}_t l_t + (1-\gamma)\int_0^1 \widetilde{\Omega}_{jt} dj \tag{32}$$

The definition of profits:

$$(1-\gamma)\int_0^1 \widetilde{\Omega}_{jt}dj = \widetilde{y}_t - \left(\widetilde{w}_t l_t + p_t^k \widetilde{k}_{t-1}/\omega\right) \tag{33}$$

The government budget constraint:

$$\widetilde{g}_t + \widetilde{e}_t^p = \tau_t \widetilde{w}_t l_t + \widetilde{b}_t - \frac{r_{t-1}}{\omega}\widetilde{b}_{t-1} \tag{34}$$

The aggregate resource constraint:

$$\widetilde{g}_t + \widetilde{e}_t^p + \widetilde{c}_t + \widetilde{e}_t = \widetilde{y}_t \tag{35}$$

The labor supply:

$$(1-\tau_t)\widetilde{w}_t\,(1-l_t) = \varkappa \widetilde{c}_t \tag{36}$$

The equations of motion of the private and public capital stocks:

$$\widetilde{k}_t = \widetilde{e}_t + (1-\delta)\,\widetilde{k}_{t-1}/\omega \tag{37}$$

and

$$\widetilde{k}_t^p = \widetilde{e}_t^p + (1-\delta^p)\,\widetilde{k}_{t-1}^p/\omega \tag{38}$$

The first order condition for investment:

$$1 = r_t^{-1}\left(p_{t+1}^k + 1 - \delta\right) \tag{39}$$

Price-setting implies,

$$(1-\varepsilon) + \varepsilon mc_t = 0 \tag{40}$$

Factors' share:

$$\frac{l_t}{\widetilde{k}_{t-1}/\omega} = \frac{\alpha_2}{\alpha_1}\frac{p_t^k}{\widetilde{w}_t} \tag{41}$$

The real marginal cost:

$$mc_t = (\widetilde{y}_t)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}}\alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}}\alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}}\left(p_t^k\right)^{\frac{\alpha_1}{\alpha_1+\alpha_2}}(\widetilde{w}_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}}\left(\widetilde{k}_{t-1}^p/\omega\right)^{-\frac{\alpha_3}{\alpha_1+\alpha_2}} \tag{42}$$

The production function:

$$\widetilde{y}_t = \left(\widetilde{k}_{t-1}/\omega\right)^{\alpha_1} l_t^{\alpha_2}\left(\widetilde{k}_{t-1}^p/\omega\right)^{\alpha_3}. \tag{43}$$

A.3 Further Derivations

In section 3.1 in the paper, we contrast the social planner's first order condition for labor and the aggregate consumption Euler equation with equivalent conditions obtained in the decentralized equilibrium. In this section of the Appendix, we show how we derived the latter (equations (10) and (11) in the main text).

Firstly, we combine the labor supply condition (36) with the demand for labor, $w_t = mc_t\left(\alpha_2 \frac{y_t}{l_t}\right)$, and the firms' pricing decision (40) that defines the real marginal cost $mc_t = \frac{\varepsilon-1}{\varepsilon}$, to obtain the labor allocation under the decentralized equilibrium,

$$\varkappa \widetilde{c}_t = \left[(1-\tau_t)\frac{\varepsilon-1}{\varepsilon}\right]\left(\alpha_2 \frac{\widetilde{y}_t}{l_t}\right)(1-l_t)\,, \tag{44}$$

This is equation (10) in the main text.

Secondly, combining the consumption function (29) with the evolution of human wealth (31) and non-human wealth (30) yields the aggregate consumption Euler equation in the decentralized equilibrium,

$$\widetilde{c}_t = \frac{\omega \widetilde{c}_{t+1}}{\beta r_t} + \frac{(1-\gamma\beta)(1-\gamma)}{\gamma\beta}\left(\widetilde{b}_t + \widetilde{k}_t\right).$$

Furthermore, using the FOC for investment (39), together with the demand for capital in production, $p_t^k = mc_t\left(\alpha_1 \frac{y_t}{k_{t-1}}\right)$, and the real marginal cost relationship $mc_t = \frac{\varepsilon-1}{\varepsilon}$, the above expression can be re-written as,

$$\widetilde{c}_t = \widetilde{c}_{t+1}\omega\beta^{-1}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\alpha_1 \frac{\widetilde{y}_{t+1}}{\widetilde{k}_t/\omega} + 1 - \delta\right]^{-1} + \frac{(1-\gamma\beta)(1-\gamma)}{\gamma\beta}\left(\widetilde{b}_t + \widetilde{k}_t\right), \tag{45}$$

which is equation (11) in the main text.

A.4 Welfare Metric

Defining what is optimal in an OLG model involves deciding how to compare different generations. Since we are interested in formulating optimal policy for our economy populated with overlapping generations of finitely lived consumers we must face the tricky issue of constructing a welfare metric. Calvo and Obstfeld (1988) define the social welfare function at time 0 as,

$$U_0 = \sum_{s=0}^{\infty}\left[\sum_{t=s}^{\infty} u(s,t)(\gamma\beta)^{t-s}\right]\rho^s + \sum_{s=-\infty}^{0}\left[\sum_{t=0}^{\infty} u(s,t)(\gamma\beta)^{t-s}\right]\rho^s$$

where $u(s,t) = \ln c_t^s + \vartheta \ln g_t + \varkappa \ln(1-l_t^s)$ is the utility at time t of a household born at time s. The first summation is the utility of representative agents of generations yet to be born, discounted at the policy maker's discount factor, ρ. The second is the expected utility of households currently alive. These utilities are discounted back to the birth date of the currently living generations, rather than the current period. Calvo and Obstfeld (1988) note that this is necessary to avoid the time inconsistency in preferences that would otherwise emerge by treating generations asymmetrically. In other words, if the policy maker did not discount utilities back to birth dates, then she would wish to change the consumption plans she put in place for currently unborn generations the moment they are born.

By changing the order of summation the welfare function can be rewritten as,

$$U_0 = \sum_{t=0}^{\infty} \left[\sum_{s=-\infty}^{t} u(s,t) \left(\frac{\gamma\beta}{\rho}\right)^{t-s} \right] \rho^t$$

so that the instantaneous flow utility to the policy maker is given by the summation over generations of their instantaneous utility discounted by the private discount factor and adjusted by the public discount factor. These are then discounted over time using the policy maker's discount factor, ρ. This can be further rewritten as,

$$U_0 = \sum_{t=0}^{\infty} \left[\sum_{z=0}^{\infty} u(t-z,t) \left(\frac{\gamma\beta}{\rho}\right)^{z} \right] \rho^t$$

which allows us to decompose the policy-maker's problem into two parts. The first part involves the policy maker's optimal allocation of consumption and labor supply across households. The second relates to the intertemporal aspects of the problem. Since we are only interested in the macroeconomic effects of fiscal adjustment in an environment where government debt can potentially crowd-out private capital, we abstract from the intratemporal intergenerational problem and focus on the intertemporal problem, such that the social welfare function is given by,

$$U_0 = \sum_{t=0}^{\infty} \rho^t [\ln c_t + \vartheta \ln g_t + \varkappa \ln(1-l_t)]$$

Finally, in our benchmark analysis we assume $\rho = \beta$ such that the policy maker discounts the future at the same rate as households, but without accounting for the probability of death. However, there is no necessary reason for us to do this and, in sensitivity analysis, we also look at an alternative with more discounting. In solving its intertemporal problem the policy maker ignores the distribution of variables across generations at a given point in time by focusing on per-capita variables.[17] This is the welfare metric we employ after rewriting it in terms of stationary variables as given in the main text.

[17] Allowing aggregate policy to consider distributional issues when implementing macro policy would require us to track the distribution of financial wealth across generations, which is generally intractable due to the impact of the birth of new generations on that distribution. At the same time, it should be noted that, at least in principle, the government could implement a lump-sum intratemporal redistribution scheme to maximise social welfare. However, such a policy would effectively offset the differential tax treatment of different generations that the perpetual youth model relies on to break from Ricardian Equivalence.

Figures

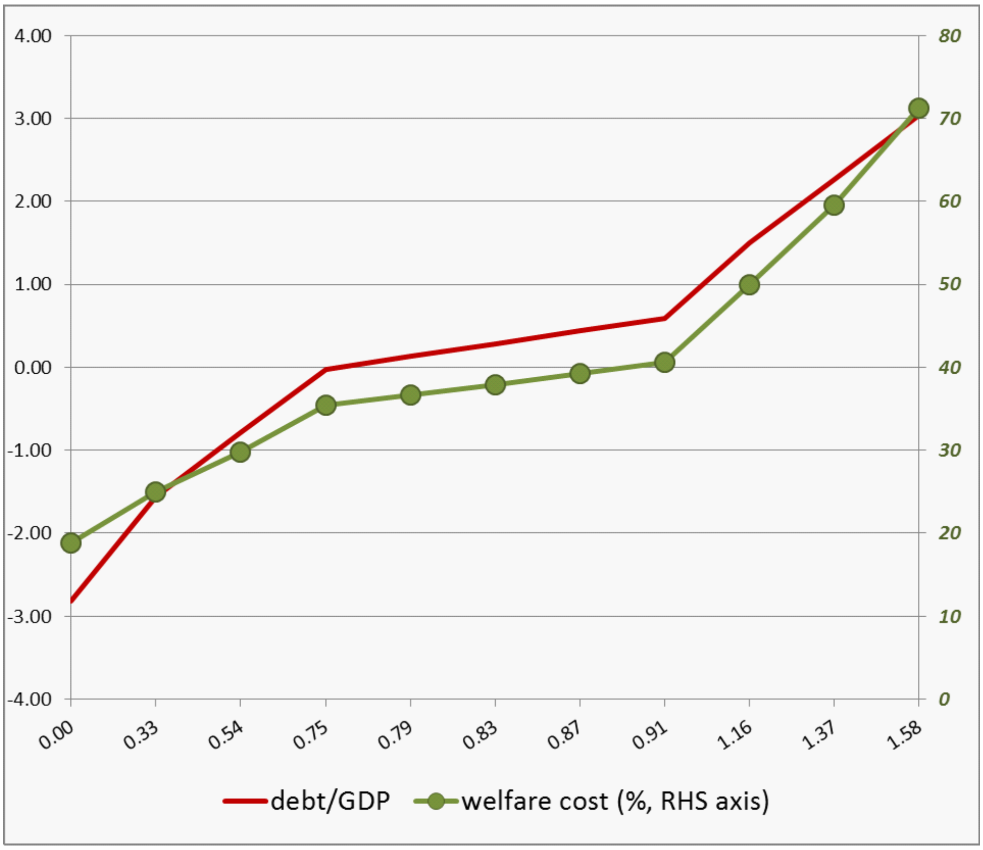

Figure 1: Optimal values of debt-to-GDP ratios and the welfare costs of varying the myopia of policy makers. The horizontal axis gives the annualized increase in discounting the future by the Ramsey planner, relative to individual households, as a percentage.

Notes to Dynamics Figures 2–5: Solid red line - benchmark model with distortionary taxation; dashed green line - lump sum taxation; dotted blue line - social planner's allocation.

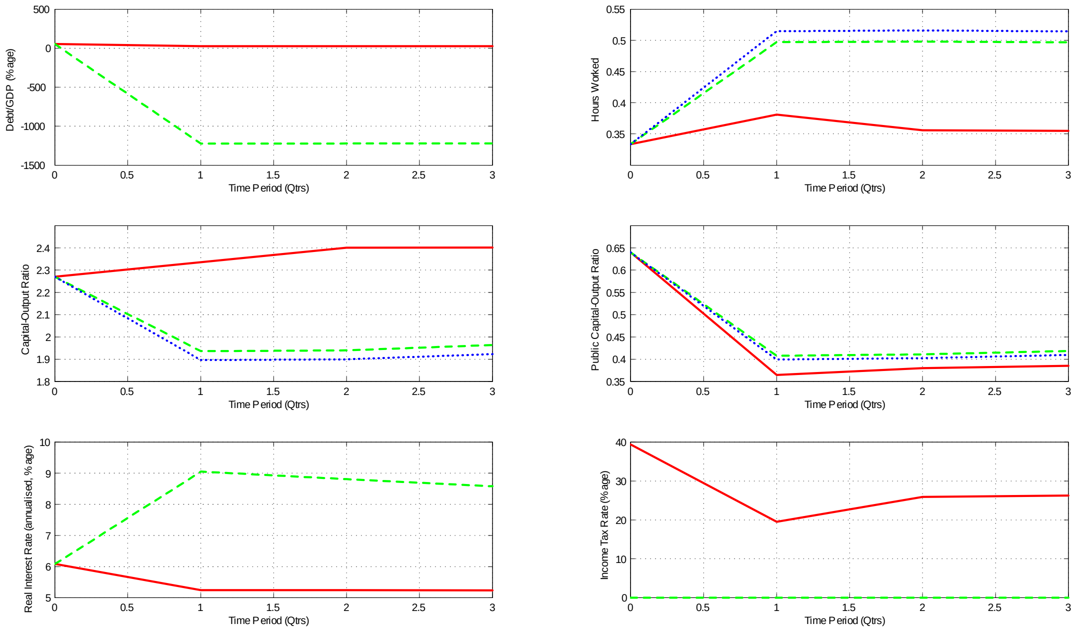

Figure 2: Ramsey Dynamics in the First Year I

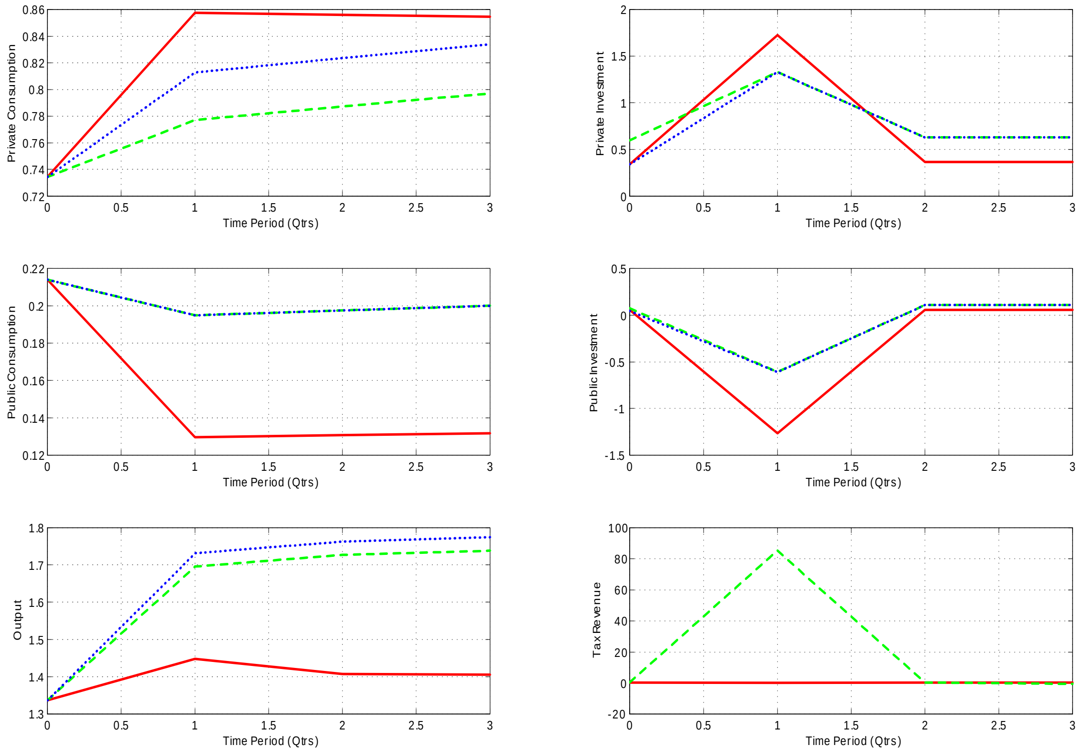

Figure 3: Ramsey Dynamics in the First Year II

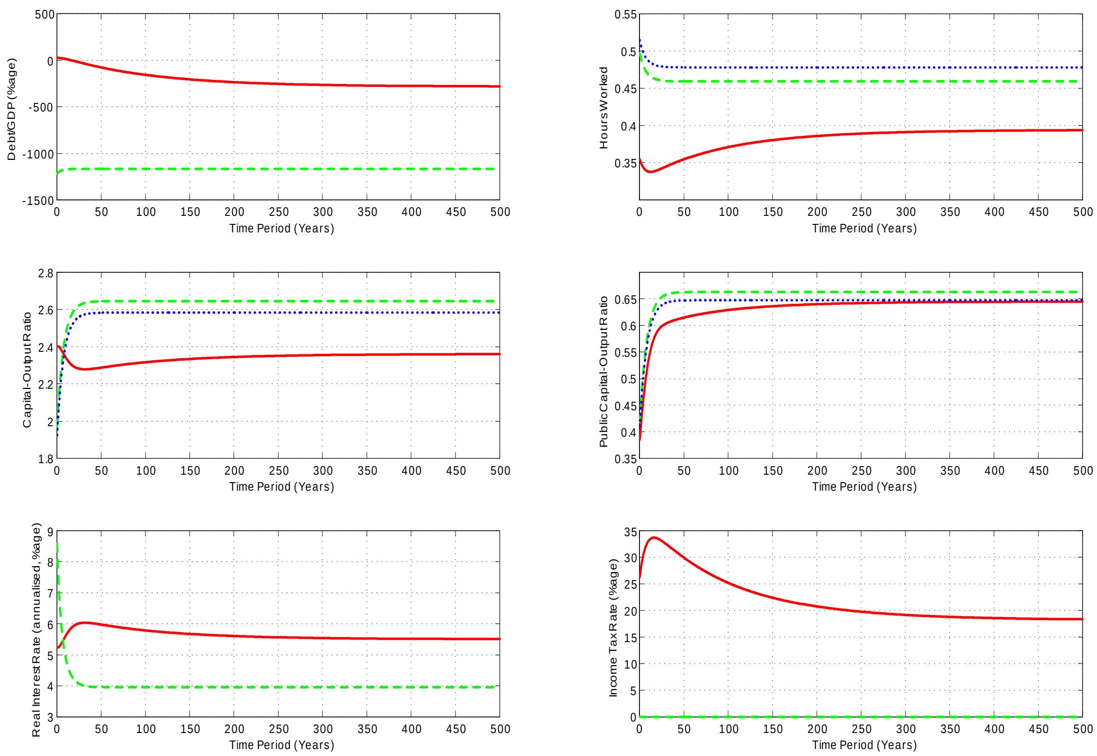

Figure 4: Ramsey Dynamics Beyond the First Year I

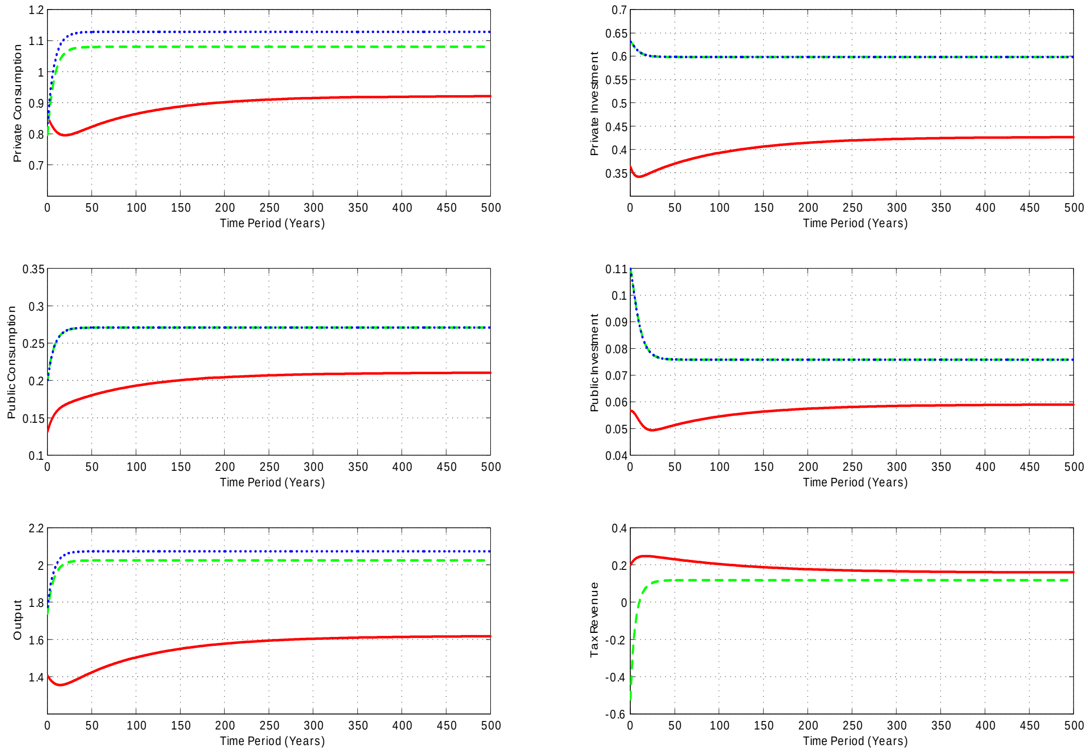

Figure 5: Ramsey Dynamics Beyond the First Year II