

Cassini observations of aperiodic waves on Saturn's magnetodisc

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Key Points:

- Saturn's current sheet is distorted by many aperiodic waves in all local time sectors
- The properties of these aperiodic waves can be found by deforming a current sheet model with a Gaussian wave pulse
- Waves predominantly propagate radially outwards on a thickening current sheet with increasing amplitude with radial distance

Abstract

The location and motion of Saturn’s equatorial current sheet is the result of an interplay between a quasi-static deformation that varies in radial distance and local time, impulsive perturbations that produce large-scale displacements, quasi-periodic perturbations near the planetary rotation period, and wave-like structures on shorter timescales. This study focuses on the latter, aperiodic wave pulses with periods from 1-30 minutes, that are unrelated to the quasi-periodic ‘flapping’ with a period near that of Saturn’s rotation. Cassini magnetometer data were surveyed for these aperiodic structures and then fitted to a simple model in order to estimate the properties of the waves. The model consists of a modified Harris current sheet model deformed by a Gaussian pulse wave function. This then allows for the extraction of wave parameters and current sheet properties. In particular we show an increase in current sheet scale height with radial distance from Saturn, an increase in the wave amplitude with radial distance, and the resolution of propagation directions using the wave vector fitted by the model. The dominant propagation direction is found to be radially outwards from Saturn.

1 Introduction

Saturn’s magnetosphere is, to an important extent, rotationally driven [*Southwood et al.*, 2001] and contains internal plasma sourced from the moon Enceladus and to a lesser extent the rings, other satellites and the planet itself [e.g. *Pontius et al.* [2006], *Tokar et al.* [2005], *Jurac et al.* [2002] & *Felici et al.* [2016]]. The magnetosphere outside of $20R_S$ (Saturn radii) is dominated by the magnetodisc current sheet, a washer shaped sheet of particles caused, in part, by the centrifugal stresses of the fast-rotating magnetosphere [*Arridge et al.*, 2007]. Although other particle stresses are imparted, the disc-like current sheet forms at the point that centrifugal stresses dominate over magnetic tension and pressure gradients, which occurs at $\sim 15R_S$, and extends to the magnetopause. This azimuthal current sheet causes the magnetic field of Saturn to appear radially distended, similar to that of Jupiter’s middle and outer magnetosphere.

Additionally, the azimuthal magnetic field structure is affected by rotation and solar wind compression. At large radial distances, magnetic field lines are ‘swept back’ due to plasma sub-corotation, causing an increasing azimuthal field component with radial distance. However, on the dusk sector of the magnetosphere, the confinement from the solar wind causes a ‘swept forward’ field that is pushed forward in the direction of co-rotation.

42 Dynamic pressure of the solar wind acting on the dayside magnetosphere also plays a role
43 on the structure of the azimuthal field. Under higher solar wind dynamic pressures, the
44 dayside will be more compressed, moving the region of corotation breakdown closer to the
45 magnetopause and leading to a more corotating dayside with less swept-back field lines,
46 or swept-forward field lines on the dusk flank due to the effect of the Chapman-Ferarro
47 currents. Under transient solar wind compressions the dayside magnetosphere might un-
48 dergo a short period of super-corotation which may produce swept-forward field lines in
49 the noon sector [e.g. *Southwood et al.* [2001]]. Saturn's equatorial current sheet has been
50 found in all local time sectors, however the dayside magnetosphere sometimes exhibits
51 no ordered current sheet structure when the magnetosphere is compressed [*Arridge et al.*,
52 2008b].

53 Many processes cause the current sheet to be displaced from the rotational equator.
54 Seasonal differences combined with the influence of solar wind pressure [*Arridge et al.*,
55 2008a] will cause the current sheet to be pushed above the rotational equator in North-
56 ern winter time and pushed below the equator in Northern summer time, hence forming
57 a 'bowl shape'. Hence, if Cassini is in the same position for a Saturnian year, the mag-
58 netometer will read one wavelength of the current sheet bowl movement. Therefore, Sat-
59 urn's current sheet experiences a periodic wave with a time period of 30 years. Seasonal
60 changes also affect the current sheet thickness, *Sergis et al.* [2011] has shown that the cur-
61 rent sheet thickness will be highly variable during the seasons and even between subse-
62 quent orbits, and will exhibit north-south asymmetries in the thickness of plasma. Temper-
63 ature, density and pressure, however, remain unchanged.

64 Another periodicity apparent in the magnetometer data is the ~ 10.7 hour rotation
65 rate flapping motion [*Arridge et al.* [2011] & *Provan et al.* [2012]]. The origin of this flap-
66 ping motions is thought to be caused by rotating magnetic perturbations in each hemi-
67 sphere [e.g. *Arridge et al.* [2008a], *Andrews et al.* [2010], *Jia and Kivelson* [2012]]. This
68 flapping motion is also thought to superpose with thickness variations of the current sheet
69 as it propagates [*Thomsen et al.*, 2016].

70 The focus of this study is the much shorter timescale, aperiodic waves that occur in
71 addition to the waves discussed previously. These waves occur over time periods of 1-30
72 minutes and appear in all areas where a current sheet is present but do not repeat period-
73 ically or show a sinusoidal period. The time period refers to the length of time that the

74 wave is detected. *Arridge et al.* [2007] previously used these waves to calculate the stress
 75 balance in Saturn’s magnetosphere but their origins or properties are yet to be explored.
 76 These features also occur frequently in Earth’s magnetotail current sheet [*Sergeev et al.*,
 77 2004] and have been seen on Jupiter’s magnetodisc [*Russell et al.*, 1999]. They were also
 78 present in Pioneer 11 magnetometer data at Saturn [*Smith et al.*, 1980]. Cassini’s varied
 79 orbital trajectory and spatial coverage give an unique opportunity for studying these waves
 80 on a large scale.

81 Small-amplitude periodicities and fluctuations are also present within Saturn’s mag-
 82 netosphere, an example of such are the quasi-periodic waves discussed within *Mitchell et*
 83 *al.* [2016], *Palmaerts et al.* [2016] & *Yates et al.* [2016] where the authors present quasi-
 84 hourly pulsations in UV auroral observations along with particle and magnetic field data.
 85 These waves appear similar to the waves presented within this study, however only waves
 86 that are singular and non-repeating are included within the study. Similar structures are
 87 also found in the jovian magnetosphere [*Khurana and Kivelson*, 1989]. *von Papen et al.*
 88 [2014] & *von Papen & Saur* [2016] discuss fluctuations and magnetic turbulence within
 89 the middle magnetosphere. These studies measure fluctuations in the magnetometer data
 90 in 10 minute bins, where the spacial range of *von Papen & Saur* [2016] overlaps with the
 91 spacial range of this study, it is possible that the aperiodic waves we discuss in this paper
 92 fall within the fluctuations of the current sheet shown in *von Papen & Saur* [2016] at the
 93 inner bound of this study.

94 **2 Magnetometer Data**

95 We use Cassini’s onboard fluxgate magnetometer [*Dougherty et al.*, 2004] to detect
 96 the signatures of aperiodic waves using data with a time resolution of 1 Hz. Magnetome-
 97 ter data and the following model are presented in spherical KRTP coordinates (Kronian
 98 radial, theta, phi) where the first component, \hat{r} , is from the centre of Saturn radially out-
 99 wards. $\hat{\theta}$ is positive Southward at the equator, and $\hat{\phi}$ is positive in the direction of co-
 100 rotation. Additionally, a Cartesian local coordinate system ($\mathbf{x}, \mathbf{y}, \mathbf{z}$) is used when forming
 101 the model and fitting to it. This system gives $\hat{\mathbf{x}}$ in the radial direction, $\hat{\mathbf{y}}$ in the direction
 102 of co-rotation, this is assumed as Cartesian in the local system, and $\hat{\mathbf{z}}$ is equivalent to $-\hat{\theta}$
 103 at the equator in the spherical polar system. All results are converted back to spherical
 104 polar coordinates. Furthermore, the KSM (Kronocentric Solar Magnetospheric) ($\hat{\mathbf{X}}, \hat{\mathbf{Y}}, \hat{\mathbf{Z}}$)
 105 system is useful in interpretation. Where $\hat{\mathbf{X}}$ is the vector pointing to the Sun, $\hat{\mathbf{Y}}$ is perpen-

106 dicular to the rotation axis towards dusk and $\hat{\mathbf{Z}}$ is northwards so that the rotation axis is in
107 the X - Z plane.

108 Aperiodic waves have a distinct signature in magnetic field data. As an aperiodic
109 wave passes Cassini we find that the B_r and B_ϕ components have an anti-phase relation-
110 ship due to the largely swept backwards nature of the field lines. During the passage of an
111 aperiodic wave Cassini will be embedded in either lobe of the magnetosphere and as the
112 wave passes, the spacecraft will sample the current sheet, the opposing lobe (if the wave
113 has sufficient amplitude), the current sheet again, and finally the original lobe.

114 As Saturn's magnetic field at distances of larger than $20R_S$ is mainly radial with
115 a small contribution from the azimuthal component, the magnetometer data will see the
116 passing of a wave in the radial component. If Cassini is originally in the Southern lobe
117 B_r will be negative and will increase through zero as Cassini traverses the centre of the
118 current sheet to positive B_r in the Northern lobe. The radial component will then decrease
119 from a maximum back to the starting value, again crossing zero as Cassini crosses the
120 current sheet centre. The azimuthal component follows a similar profile, but with an oppo-
121 site polarity when the field is swept-back. So starting in the Southern lobe, B_ϕ would be
122 positive and would decrease through zero to a negative value when a maximum positive
123 value is reached in B_r . This signature is shown in figure 1.

124 The third magnetic field component, B_θ varies depending on various other pro-
125 cesses. As $\hat{\theta}$ is positive Southward at the equator, this means that with no external pro-
126 cesses occurring B_θ will be a constant positive. However, during the passage of an aperi-
127 odic wave, the θ component may show deviations that differ depending on the parameters
128 of the wave that is distorting the current sheet. Figure 1 shows magnetic field signatures
129 of a solely radially propagating wave and a solely azimuthally propagating wave. Many
130 examples show a mixture of radial and azimuthal propagation along with various other
131 processes, such as dipolarization and guide fields, that affect the θ component.

135 These aperiodic waves are detected by their distinct characteristics. We select field
136 perturbations that have a smaller time period than the global flapping waves, are non-
137 repeating, and show a deflection in the radial magnetic field of over 1nT. The event must
138 also occur inside of the magnetopause position found by an examination of the magnetic
139 field data. In total, 1461 events fit these criteria from all revolutions of Cassini that occur
140 between $\pm 10R_S$ of the rotational equator from January 2005 to December 2012.

3 Current Sheet Model

To determine the propagation and properties of a wave on a current sheet from single spacecraft measurements we use a simple local current sheet model. We require a model that includes the variation of radial magnetic field from positive above the equator to negative below the equator. Hence, a Harris current sheet model [Harris, 1962] is used for the radial component of the magnetic field, given in equation 1. Equation 2 is used for the azimuthal field component, where if the field is swept backwards we will see a negative B_{y0} , and where the field is swept forward we have a positive B_{y0} . The third and final magnetic field component, the component normal to the current sheet, is modelled as a constant. Deformation of the current sheet will generate a non-constant B_z (Equation 3). This model is true solely at close proximity to the current sheet and outside of the dipolar region of Saturn's magnetosphere. The model also relies on the thin sheet approximation [Vasyliunas, 1983]. Because of these reasons the model is presented in Cartesian local coordinates (x, y, z) .

$$B_x = B_{x0} \tanh\left(\frac{z - z_0}{H_x}\right) \quad (1)$$

$$B_y = B_{y0} \tanh\left(\frac{z - z_0}{H_y}\right) \quad (2)$$

$$B_z = B_{z0} \quad (3)$$

B_{x0} , B_{y0} and B_{z0} are lobe values (or asymptotic value of the hyperbolic tangent function). The sign of B_{y0} is related to the sweep-back and sweep-forward of the magnetic field. A negative value of B_{y0} means that below the current sheet B_y will be positive and above the current sheet B_y will be negative and the field will be swept backwards if we assume B_x is positive above the sheet and negative below. The converse applies if B_{y0} is positive. The current sheet scale height, or how quickly the hyperbolic tangent function reaches its asymptotic value, is given by H_x and H_y . The displacement of the current sheet from the rotational equator, for example by the global flapping and bowl shape, is included through the z_0 parameter. These parameters all describe the local properties of the current sheet, all distances are fitted in units of Saturn radii and all magnetic field quantities in units of nT.

This local model of the magnetic field must then be deformed by a wave. We use a Gaussian pulse wave function as it describes a singular non-repeating wave which can

170 be easily modified to fit magnetometer data. Equation 4 describes the displacement of the
 171 current sheet as $z(x, y, t)$ from $z=0$.

$$z = A \exp [-(\mathbf{k} \cdot \mathbf{r} + \mathbf{k} \cdot \mathbf{u}t - \omega t - \Phi_0)^2] \quad (4)$$

172 Where A is the amplitude of the wave, \mathbf{k} is the wave vector, $\mathbf{k} \cdot \mathbf{u}t$ is the Doppler
 173 shift due to the movement of plasma, ω is the angular frequency and Φ_0 is the phase of
 174 the wave.

175 We now have the static current sheet model and a wave to deform it. The deforma-
 176 tion of the current sheet is carried out using the general deformation method found in
 177 *Tsyganenko* [1998]. This method is ideal as it uses Euler potentials to find the deformed
 178 coordinate system whilst keeping the field divergence free, but requires no knowledge of
 179 what the Euler potentials are.

180 Firstly, we need to evaluate the magnetic field in a co-ordinate system that has been
 181 deformed by the wave function. This is done by finding the normal to the wave, using the
 182 differentials of the original wave function in x and y . This normal can then be used to
 183 find the new radial (X) axis and azimuthal (Y) axis using cross products to retain a right-
 184 handed system. Now we can find the positions in the new coordinate system, which are
 185 then put into the Harris current sheet equation with initial estimates of scale heights and
 186 lobe values to find the magnetic field components in the new coordinate system (\mathbf{B}^*).

$$\mathbf{B}' = \hat{\mathbf{T}}\mathbf{B}^* \quad (5)$$

187 The general deformation method is then used to deform this magnetic field using
 188 Equations 6 in *Tsyganenko* [1998]. The matrix these equations form, $\hat{\mathbf{T}}$, can then be used
 189 to find the final deformed magnetic field components using equation 5. Since the coordi-
 190 nate system is a local current sheet system based on the current sheet normal vector, these
 191 derivatives are found using centred finite differences. B^* denotes the magnetic field in the
 192 new coordinate system and B' shows the new deformed magnetic field.

193 With reasonable initial guesses, this local model of the current sheet deformed by a
 194 Gaussian wave function can model accurately the changes of a magnetic field during the
 195 passage of an aperiodic wave shown in figures 2 and 3.

4 Fitting Magnetometer Data to Model

The local current sheet model has 11 free parameters; six parameters describing the undisturbed planar current sheet (B_{x0} , B_{y0} , B_{z0} , H_x , H_y , z_0) from equations (1,3), and five parameters describing the properties of the wave (k_x , k_y , ω , Φ_0 , A) from equation (4)

Due to the large number of variables that require fitting and the possible interdependence of the variables (for example, a negative scale height arising in y from the fact that the lobe magnetic field B_{y0} should be negative instead), an iterative process is used which fits half the variables using a Levenberg-Marquardt nonlinear least squared fitting algorithm with the other variables held constant, and then vice versa. The variables are treated in two distinct groups (B_{y0} , H_y , k_x , k_y and A) and (B_{x0} , B_{z0} , H_x , ω , and z_0). Changing the order of the group that fits first and second has no effect on the final fit, however changing which variables are in each group does change the result.

The groups given previously were used as they return the most fits with the lowest mean squared error (MSE) of the fit, identified by trial and improvement. The process of fitting half and holding constant the remaining variables repeats until the MSE of the fitting is below a threshold of $0.1 nT^2$ as during testing, fits with below this values were more likely to be considered good. This process is then iterated over to find the phase, Φ_0 , between $0 - 2\pi$ that gives the lowest MSE, and hence the most accurate fitting. This entire algorithm is then run four times with a different combination of positive and negative wave numbers as initial guesses to reduce any bias in wave number selection. The remaining values for initial guesses were taken from preliminary experiments that tested the sensitivity and ability of the algorithm to fit the different parameters. The run with the smallest MSE will be the final selected fitting giving the values of the 11 parameters.

In total, 793 out of 1461 wave events can be fitted using this method. This number of events being fitted is expected as many events exhibit multiple crossings of the current sheet leading to the conclusion that other processes are occurring at the same time or multiple waves are superposed during the event time window, and hence cannot be fitted by just one wave function. Some events, although they fit the criteria of selection, on further inspection do not follow the general signature of a wave but that of an O- or X-line traversal. These are easily filtered out as the local model will not fit the data well as the B_θ component of magnetic field will change sign rapidly and then remain constant at the new value throughout the time frame. Additionally, each fitting is associated with a MSE result

228 of how well the model fits the magnetometer data as well as each event being manually
 229 inspected to confirm a correct fitting has occurred.

230 All fitting is done in Cartesian local coordinates, so magnetometer data is converted
 231 to Cartesian before the fitting procedure, but to be in line with convention both magne-
 232 tometer data and model are converted to and presented in spherical KRTP coordinates.

233 Figures 2 and 3 show two examples of aperiodic waves detected in the magnetome-
 234 ter data (black) fitted with the local model (red). Figure 2 shows a wave that causes Cassini
 235 to encounter the current sheet, with little in the way of variation in B_θ , this example has a
 236 MSE value of $0.06 nT$. Figure 3 shows a wave that displaces the current sheet towards
 237 Cassini, but Cassini does not encounter it. This example shows a swept forward field in
 238 B_ϕ and B_r , along with variations in B_θ . This example has an MSE value of $0.05 nT^2$.

247 Uncertainties in the values of the fitted parameters are extracted from the covari-
 248 ance matrix output of the non-linear least squares fitting, where the standard deviation of
 249 the fitted parameters are the square roots of the diagonal elements of the matrix. The per-
 250 centage uncertainties for a successful fit lie between 1-5% for current sheet properties and
 251 1-10% for wave parameters. The uncertainties are related to the multi-parameter χ^2 space
 252 which is related to the previously mentioned MSE, both of which are measurements from
 253 goodness of fit.

256 For example, we can plot the 2-dimensional χ^2 space for only the two wave num-
 257 bers to ascertain if the fitting for the wave parameters is successful. A successful fit will
 258 show the final values in a unique local minimum (dark blue area) which is not associated
 259 with noise. This parameter space is formed by calculating the χ^2 value for each k_x and
 260 k_y values while all the other parameters remain constant. Figure 4 shows an example of
 261 this space, we see that the model has placed the values (red dot) in a low area of χ^2 how-
 262 ever, there are other possibilities, these other possibilities may later reduce when including
 263 the χ^2 space of the other variables. The model gives a χ^2 value of 2.33, which is smaller
 264 than the majority of the 'blue valley' which give χ^2 values of around 10 to 50.

265 5 Results

266 Firstly, we explore results on the properties and structure of the current sheet that
 267 appear in the equations for the modified Harris current sheet (1, 2, 3). Figures 5, 6, and
 268 7 show the azimuthal lobe field component, B_{y0} , mean scale height, H , and offset from

269 the equatorial plane, z_0 , respectively, projected into the X-Y plane of the KSM coordinate
 270 system. This overview plots all values of a specified parameter with respect to the position
 271 of Cassini during the event. Each box shows the mean of the parameter for the number
 272 of events that occur within it. The mean is weighted by the inverse of the MSE for each
 273 event, allowing for better fits to more strongly influence the mean.

274 Below the overview plot, the sector plots (b-e) show the relationship of the parame-
 275 ter of interest with radial distance in different local time sectors. ‘Night’ refers to 21:00 to
 276 03:00 SLT (Saturn Local Time), ‘Morning’ refers to 03:00 to 09:00 SLT, ‘Noon’ refers to
 277 09:00 to 15:00 SLT and ‘Evening’ refers to 15:00 to 21:00 SLT. Additionally, each point
 278 on the plots has a color associated with the revolution number of Cassini, which allows
 279 for temporal differences to also be viewed. Early Cassini orbits in 2005 are colored blue,
 280 whereas late orbits in 2012 are colored yellow.

281 If a correlation coefficient of more than 0.25 is found for any of the sectors, a linear
 282 fit shows the increase or decrease of parameter of interest with radial distance. The uncer-
 283 tainties in the linear fitting parameters shown as dashed lines either side of the fit. This
 284 linear fit is also weighted by the inverse of the MSE values for each event, so events with
 285 better fits will more strongly determine the linear fit.

286 We find that the lobe values of the magnetic field components with radial distance
 287 have an expected decrease. Negative values of B_{y0} are most common in the night, morn-
 288 ing and noon sectors, relating to the sweptback features of the field. Positive values of B_{y0}
 289 are only found near the post midday flank of the magnetosphere, the area most commonly
 290 found to have swept forward field lines, all shown in figure 5.

297 The scale height of the overall current sheet is found by calculating the geometric
 298 mean of H_x and H_y , the two scale heights fitted are usually within uncertainties of each
 299 other for good fits. Figure 6 shows that the scale height increases from $2R_S$ to $6R_S$ as it
 300 approaches the magnetopause in the morning sector, and increases from $1R_S$ to $5R_S$ in
 301 the night sector. The evening and noon sectors show little or no correlation with radial
 302 distance.

305 The value of z_0 varies radially, azimuthally and with time due to the seasonal bowl
 306 shape of the current sheet discussed in section 3. Figure 7 shows increases in the morning
 307 and night sectors with increasing radial distance.

309 We now move onto the wave parameters, amplitude, k_x , k_y , ω and Φ_0 . Most pa-
 310 rameters show little spatial variation within Saturn's magnetosphere, but most have a large
 311 range of values. For example, angular frequency ω has a median of 0.007 s^{-1} with an in-
 312 terquartile range of 0.007 s^{-1} .

313 One parameter that does exhibit considerable variation in radial distance and local
 314 time is the wave amplitude. Figure 8 shows the spatial distribution of amplitude around
 315 Saturn. The wave amplitude becomes increasingly negative with radial distance in the
 316 morning and night sectors, from $-1R_S$ to $-2R_S$. A negative amplitude is associated
 317 with the current sheet moving to a negative position in z as an aperiodic wave passes.

320 Wave numbers k_x and k_y can be used to find the direction that a wave is propagat-
 321 ing along the current sheet. Figure 9 gives an overview of the direction of propagation
 322 of all fitted waves. Again each section is split into SLT sectors as described previously,
 323 and then divided into three radial distance groups: inner ($< 20R_S$), middle ($20 - 40R_S$)
 324 and outer ($> 40R_S$). The red curve in each subplot shows an estimate of the probability
 325 distribution function for the wave propagation direction, produced by kernel smoothing.
 326 These are produced from a superposition of normal distributions with the mean of the an-
 327 gles of propagation and its standard deviation of the uncertainty on the angle. The angle
 328 of propagation is calculated using k_x and k_y , with respect to the Saturn-Sun line (0°), and
 329 its uncertainty is propagated through from the uncertainties on k_x and k_y . The bottom left
 330 plot in figure 9 shows the key and describes the direction of which waves in the specified
 331 area are propagating. The probability distribution for the wave that has equal probability
 332 of propagating in any direction is shown as the black circle.

333 All sectors and radial distances, apart from the noon sector, have statistically skewed
 334 probability distributions, skewed towards outwards radial propagation and slight azimuthal
 335 propagation in the direction of corotation. This means that in the evening sector the distri-
 336 bution is skewed dusk-wards, in the night sector the distribution is skewed anti-sunwards
 337 and in the morning sector the distribution is skewed dawn-wards. This measure of statis-
 338 tical skewness is tested against the null hypothesis of an isotropic probability distribution
 339 using the cumulative distribution function. This function uses the χ^2 value of the null hy-
 340 pothesis and the statistical distribution of the direction of propagation to find the probabili-
 341 ty that it could agree with the null hypothesis. The evening, night, and morning sectors all
 342 have skewed distributions that are statistically significant at the 1% level. Concluding that

343 these sectors are all significantly skewed. The noon sector is not statistically significant
 344 at the 1% or 10% levels and so we conclude that there is no preferred wave propagation
 345 direction in the noon sector.

350 **6 Discussion**

351 The scale height of the current sheet was found to increase in the morning and noon
 352 sectors from $2R_S$ to $6R_S$ and $5R_S$ respectively. It is important to note that this scale height
 353 is for the magnetic field, and does not include analysis from plasma data, hence it is not
 354 directly comparable to thickness measurements made using plasma data. However the two
 355 are correlated and similar trends should be present in both magnetic and plasma data. Further
 356 discussion on this topic can be found in *Sergis et al.* [2009] and *Sergis et al.* [2011].

357 At Saturn, with the use of highly inclined revolutions of Cassini, *Kellett et al.* [2009]
 358 and *Kidder et al.* [2009] previously found that the vertical distribution of plasma at the
 359 current sheet extends from $1.5 - 2.5R_S$ in half-thickness. *Giampieri et al.* [2004] use a ring
 360 current model from *Connerney et al.* [1983] to conclude from Voyager and Pioneer flybys
 361 that the current sheet thickness increases with radial distance. *Carbary et al.* [2012] show
 362 a scale height of $1.5R_S$ using magnetometer data. The scale height result is similar to that
 363 found in the Jovian magnetosphere where *Khurana and Kivelson* [1989] have shown that
 364 the half-width of the plasma sheet grows from $4R_J$ at $20R_J$ to $7.5R_J$ at $100R_J$.

365 Use of a modified Harris current sheet to model the current sheet allows fitting of
 366 the z_0 value. This variable represents the distance that the current sheet is displaced along
 367 the z-axis from the equator due to other processes before the aperiodic wave passes. We
 368 assume that this value of z_0 is constant during over the period of the wave, such that the
 369 period is much shorter than the previously discussed known distortions such as the bowl
 370 and flapping waves. We find that the revolutions of Cassini that occur before 2009 (Saturn
 371 equinox) give a value of positive z_0 . These revolutions occur mainly within the morning
 372 and night sectors, and during this time we would expect the current sheet to be pushed
 373 upwards into a bowl shape [*Arridge et al.*, 2008a]. Conversely, the evening and noon sec-
 374 tor revolutions mainly occur after equinox, and so the opposite is true, z_0 is found to be
 375 negative and the current sheet is pushed below the rotational equator. Additionally, we
 376 find that there is an increase in the absolute values of z-axis offset z_0 in each local time
 377 sector with radial distance, showing that the current sheet is taking the shape of a bowl.

378 Wave numbers found from the fitting of a Gaussian pulse wave function to magne-
 379 tometer data are used to find the propagation direction. We find that in each local time
 380 sector and radial distance bin, excluding the noon sector, the probability distributions
 381 show that waves propagate radially outwards and azimuthally in the direction of corota-
 382 tion. In the noon sector we find that, with lower numbers of events occurring within each
 383 bin, the distributions have more than one primary direction. Inwards of $20R_S$ we see a bi-
 384 directional distribution with a majority travelling inwards. This distribution may be due to
 385 magnetopause compression and expansion causing waves to travel from the magnetopause
 386 inwards, giving a link between solar wind conditions and aperiodic waves near the nose of
 387 the magnetosphere [e.g. *Arridge et al.* [2006], *Clarke et al.* [2006] & *Kanani et al.* [2010]].
 388 Other sources of aperiodic waves may originate from reconnection within the tail region,
 389 where waves are induced by an explosive reconnection event and travel away from the re-
 390 connection site [*Arridge et al.*, 2016]. This may account for tail-ward waves, but would
 391 also induce planet-ward travelling waves as well, perhaps due to dipolarizations travelling
 392 planet-ward.

393 For the waves to be travelling radially away from the planet, a source of waves must
 394 be found in the inner dipolar magnetosphere or in the planet itself. Enhancement of the
 395 ring current may induce a large enough perturbation of magnetic field to produce an aperi-
 396 odic wave [*Bunce et al.*, 2007; *Kellett et al.*, 2011]. We also suggest that centrifugal in-
 397 terchange processes may also provide a sufficient disturbance to the system in the inner
 398 magnetosphere to produce aperiodic waves travelling radially outwards, these processes
 399 have been studied both observationally by [e.g. *Burch et al.*, 2005; *Mauk et al.*, 2005] and
 400 through numerical simulation [e.g. *Kidder et al.*, 2009].

401 The next radially varying parameter is the amplitude of the waves. We find that in
 402 the morning and night sectors the amplitude increases from $1R_S$ to $2R_S$ over radial dis-
 403 tances from $15R_S$ to $50R_S$. To test that this behavior is not a viewing bias introduced
 404 by the vertical separation of Cassini from the current sheet in the outer magnetosphere,
 405 we checked the coverage of in situ measurements and z_0 fitting. Cassini covers a range
 406 from $-10R_S$ to $10R_S$ in z , which is greater than the largest amplitudes found and so will
 407 be close enough to the current sheet at large radial distances to be able to see the smaller
 408 amplitude waves, but does not. This was tested by considering the position of Cassini sub-
 409 tracted from the position of current sheet in z against amplitude, of which there is no cor-
 410 relation. We may conclude that the lack of smaller amplitude waves at larger radial dis-

411 tances must be caused by the amplitude increasing due to a decrease in density and is not
412 a viewing bias caused by the distance of the current sheet from Cassini.

413 Having ruled out an observer bias, we now seek a physical interpretation for the in-
414 creasing amplitude. To conceptually understand the origin of the increasing amplitude,
415 we use a physical analogy of a wave on a string or a wave in water and draw a compari-
416 son with water shoaling. The energy of the wave is related to the linear mass density, the
417 frequency of the wave and the amplitude of the wave. Energy of the wave must remain
418 constant, and so if the linear mass density is decreasing [Arridge *et al.*, 2011] either the
419 amplitude or frequency of the wave must change. We measured no radial dependance on
420 frequency, but a increasing trend with radial distance in amplitude, hence we suggest that
421 the amplitude increase is due to the decrease in plasma density.

422 An underlying assumption of our model is that the wave can be described by a Gaus-
423 sian pulse. However, the wave could be sinusoidally shaped, perhaps with a time-dependent
424 amplitude. If Cassini is below the current sheet it will only view wave motion in the nega-
425 tive z direction as motions in the positive z direction will be invisible, or at least damped,
426 by the asymptotic behavior of the hyperbolic tangent function in the modified Harris cur-
427 rent sheet. This wave shape can be simulated by a Gaussian differential wave function (the
428 differential of the Gaussian pulse wave function described previously) using the same pro-
429 cess applied in section 4. We find that the Gaussian differential wave function fits some
430 of the magnetometer data, however it gives a higher MSE and higher uncertainties on pa-
431 rameters that are found. On events where both waves fit the magnetometer data, the pa-
432 rameters are comparable within uncertainties. Thus we conclude that the Gaussian wave
433 function is appropriate to characterise the magnetometer data. The Gaussian differential
434 wave function also fails to fit the majority of events in the evening and noon sector.

435 As shown in figures 6 and 8, current sheet scale height and wave amplitude param-
436 eters in the evening and noon sectors show almost no variation with radial distance. The
437 color scale shows what year and what number revolution Cassini detected the event on.
438 In the morning and night sectors the majority of events are blue colored meaning that
439 they all occur on subsequent revolutions in 2005. However, in the evening we see a larger
440 range of revolutions over a larger time period (2007-2012) meaning that we are seeing
441 temporal variations in addition to spatial differences. At the extreme we have the noon
442 sector data which is only from the initial revolutions in 2005 and the final revolutions

443 in 2012. We can see that the two time separated subsets of waves produce two different
444 trends with radial distance.

445 Additionally, there are numerous lines of evidence to suggest that the dusk sector,
446 and occasionally the noon sector, may have a thicker current sheet as summarised in fig-
447 ure 10 of *Arridge et al.* [2015]. This is consistent with jovian observations by *Krupp et al.*
448 [1999]. The thin current sheet approximation, therefore, may not be valid for a number of
449 events, or may provide an additional source of error for events on the dusk flanks of the
450 magnetosphere and leads to larger uncertainties. A future analysis of the evening and noon
451 sectors should take into account the temporal differences, assess whether the thin current
452 sheet approximation is valid, and plot singular spacecraft orbits to decipher a correlation
453 between a specified parameter and radial distance, if there is one present.

454 7 Summary

455 Cassini magnetometer data was surveyed for short duration aperiodic waves in Sat-
456 urn’s magnetosphere and a catalogue of 1461 events were found in data from January
457 2005 to December 2012. Assuming wave-like perturbations that could be adequately mod-
458 eled by a Gaussian wave pulse, a local current sheet model was constructed and fitted to
459 the data to estimate current sheet, magnetic field, and wave properties.

460 The local current sheet model consisted of a Harris current sheet for the radial and
461 azimuthal components of the magnetic field and a constant normal component. This model
462 was then deformed by a Gaussian wave pulse using the general deformation technique
463 [*Tsyganenko, 1998*] which imposes $\nabla \cdot \mathbf{B} = 0$. This model was fitted to these events us-
464 ing an automated technique which resulted in 742 wave events having a good fit to the
465 data.

466 Use of the Harris current sheet allows for current sheet and magnetic field param-
467 eters to be fitted to magnetometer data, we find that a) lobe magnetic field values showed a
468 decrease with radial distance, as expected; b) the current sheet scale height increased with
469 radial distance in the morning and night sectors, consistent with previous studies; c) the
470 stationary offset of the current sheet from the equatorial plane was fully consistent with
471 seasonal changes in the position of the bowl-shaped current sheet [*Arridge et al., 2008a*].

472 Wave properties from the Gaussian wave function showed that waves were most
473 often radially propagating outwards from the planet and toward the direction of corota-

474 tion, arguing that a source of the waves must lie in the inner magnetosphere. A statisti-
475 cally significant single propagation direction in the noon sector was not found. This result
476 was interpreted as evidence that multiple sources of waves are present in the noon sec-
477 tor. Sources such as compression and expansion of the magnetopause may cause waves
478 to travel inwards away from the magnetopause, in addition to the aforementioned inner
479 magnetospheric source. Although we speculated that tail reconnection may drive both in-
480 ward and outward propagating waves (by analogy with results from the terrestrial magne-
481 tosphere), we argued that perturbations associated with the centrifugal interchange instabil-
482 ity may provide an inner magnetospheric source for waves propagating radially outward.

483 The wave amplitude was found to increase with radial distance. An observer bias
484 was ruled out and so this finding was interpreted as being due to the conservation of wave
485 energy in an environment where the plasma density and magnetic field strength were de-
486 creasing. We find a range of angular frequencies, ω , from 0.0005 to 0.0750 s⁻¹ which does
487 not show any variation with local time or radial distance.

488 Further work on these events includes a) case studies in order to understand unusual
489 events, for example where multiple current sheet encounters are found which may be bet-
490 ter fitted by a sinusoidal wave function as well as singular events which may fit a different
491 wave profile; b) incorporating plasma data into case studies or fits; c) numerical studies on
492 the wave modes in order to better understand the origin and propagation of these waves.

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132 **Figure 1.** Figure showing the signature of magnetic field components (r, θ, ϕ), and the geometry of the
 133 wave as it propagates, as a purely radially propagating wave (a) passes and a purely azimuthally propagating
 134 wave (b) passes Cassini.

239 **Figure 2.** Figure showing the magnetometer data in black, and the fitted model in red for spherical mag-
 240 netic field components, B_r , B_θ and B_ϕ along with the residuals. To the right, three hodograms display the
 241 relationship between the three components. This example occurred on 18th Feb 2005, at $18.7R_S$ and 5.25
 242 SLT

243 **Figure 3.** Figure showing the magnetometer data in black, and the fitted model in red for spherical mag-
 244 netic field components, B_r , B_θ and B_ϕ along with the residuals. To the right, three hodograms display the
 245 relationship between the three components. This example occurred on 10th Sept 2011, at $28.7R_S$ and 16.7
 246 SLT

254 **Figure 4.** Figure showing the χ^2 space of the wave numbers fitted by the local model, the red dot is the best
 255 fit according to the model.

291 **Figure 5.** Figure showing spacial distribution of B_{y0} of the magnetic field during the passing of aperiodic
 292 waves. a) shows an overall view of the magnetosphere with the orbits of Rhea ($9R_S$) and Titan ($20R_S$), and a
 293 minimum and maximum magnetopause position using the magnetopause model of *Arridge et al.* [2006]. Each
 294 colored box shows the mean of the B_{y0} projected onto the equatorial (X-Y) plane in that $1R_S$ bin. Below in
 295 b), c), d) and e) show the variability of B_{y0} with radial distance in each local time segment, color-coded by
 296 Cassini Rev number as indicated by the bar on the right hand side.

303 **Figure 6.** Figure showing spacial distribution of scale height of the magnetic field during the passing of
 304 aperiodic waves. Format is the same as figure 5

308 **Figure 7.** Figure showing spacial distribution of z_0 . Format is the same as figure 5

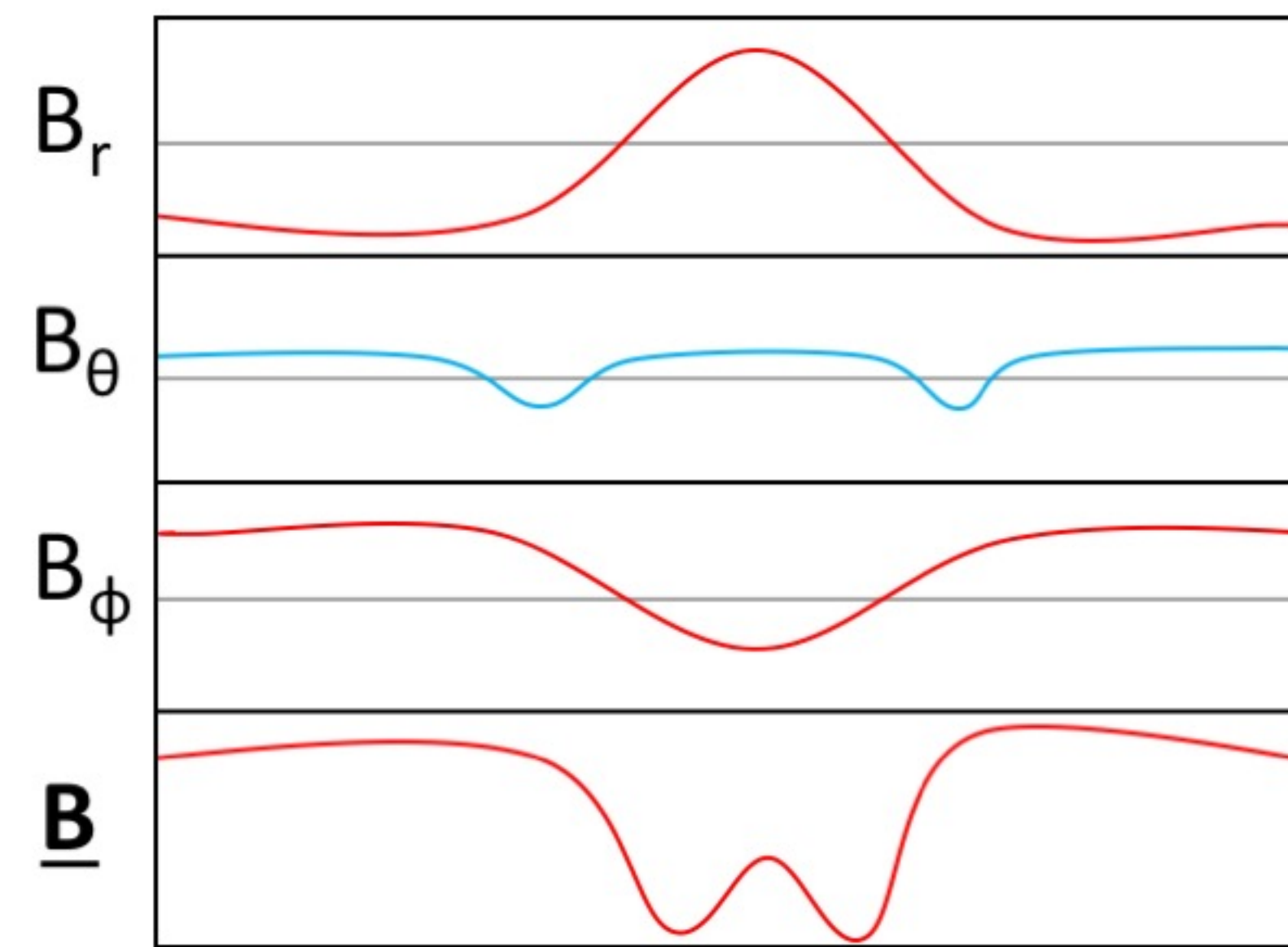
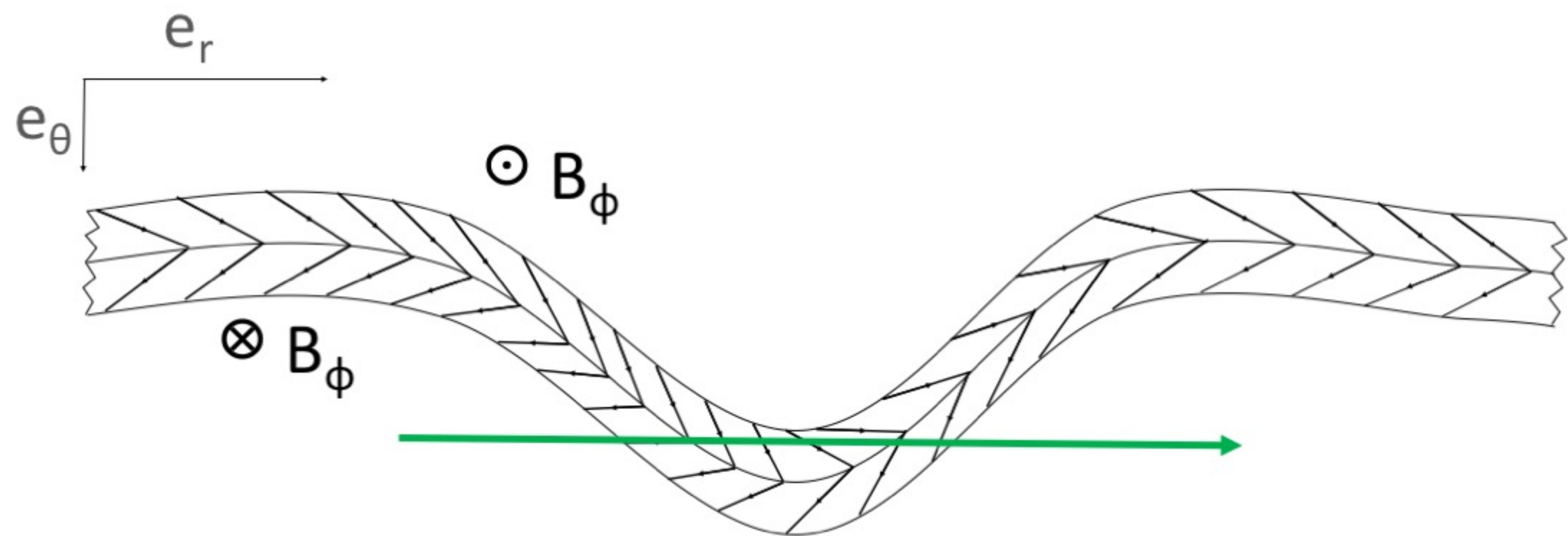
318 **Figure 8.** Figure showing spacial distribution of amplitude of aperiodic waves. Format is the same as figure

319 5

346 **Figure 9.** Figure showing spacial distribution of propagation direction of aperiodic waves split by SLT and
347 radial distance. Propagation direction angle is from the Saturn-Sun line (0°). The bottom left plot shows the
348 directions at which the plots are skewed, along with an example of an isotropically propagating distribution
349 shown in black. Arrows are shown to guide the eye.

Figure 1.

a) Radially propagating wave



b) Azimuthally propagating wave

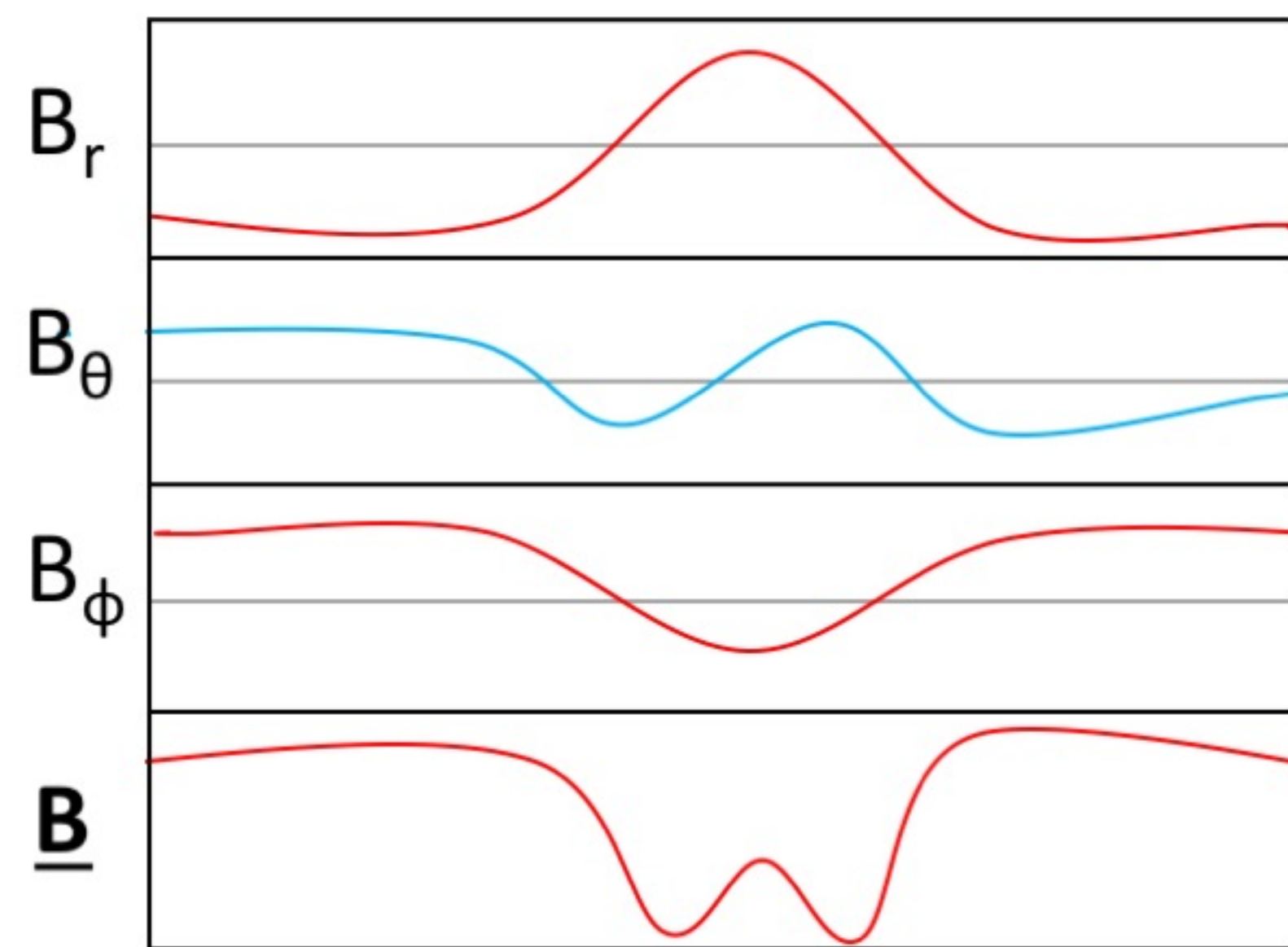
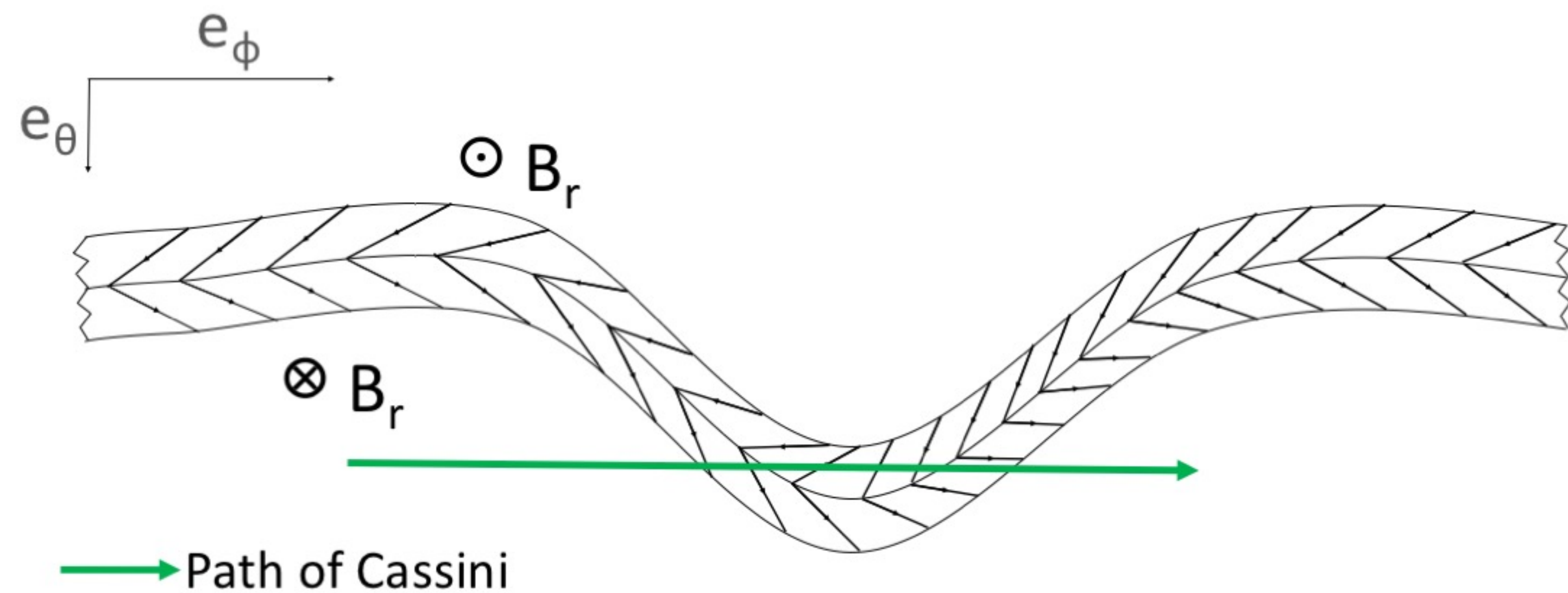


Figure 2.

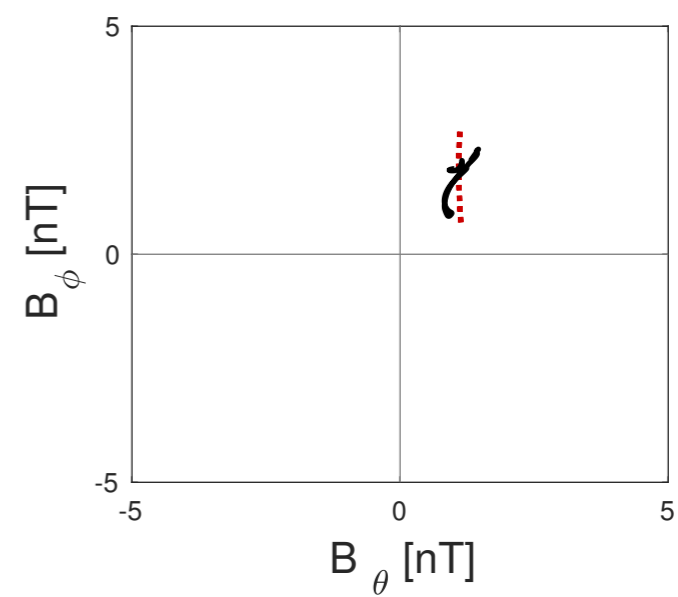
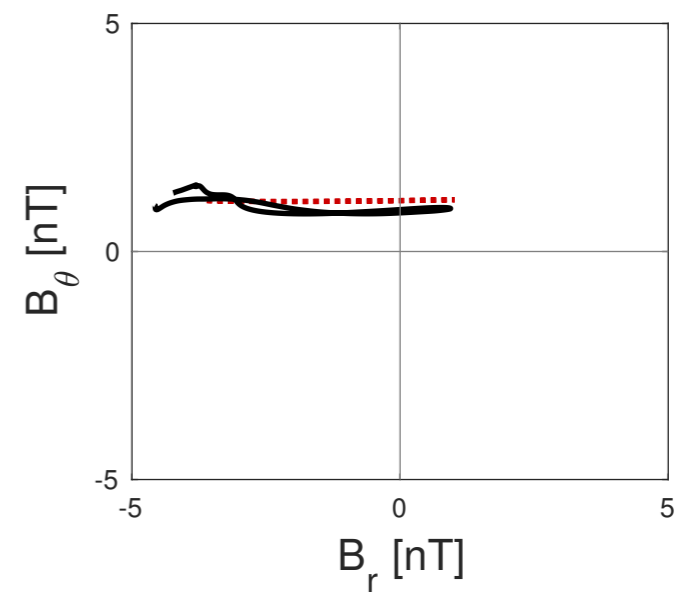
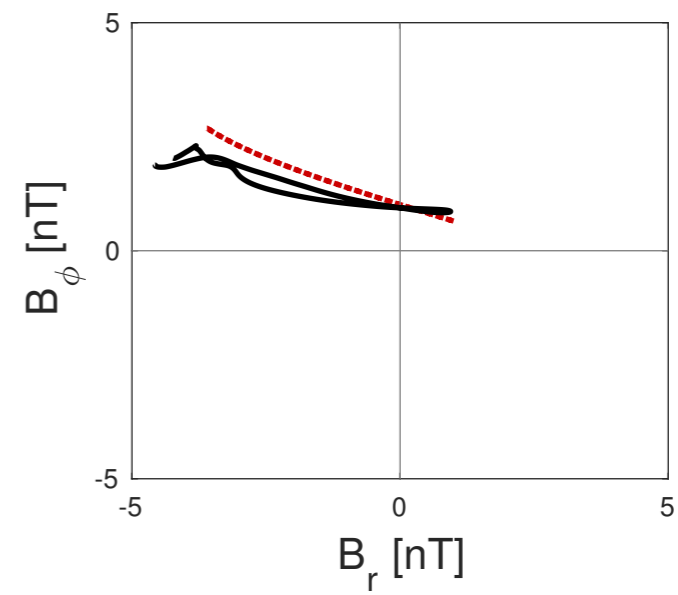
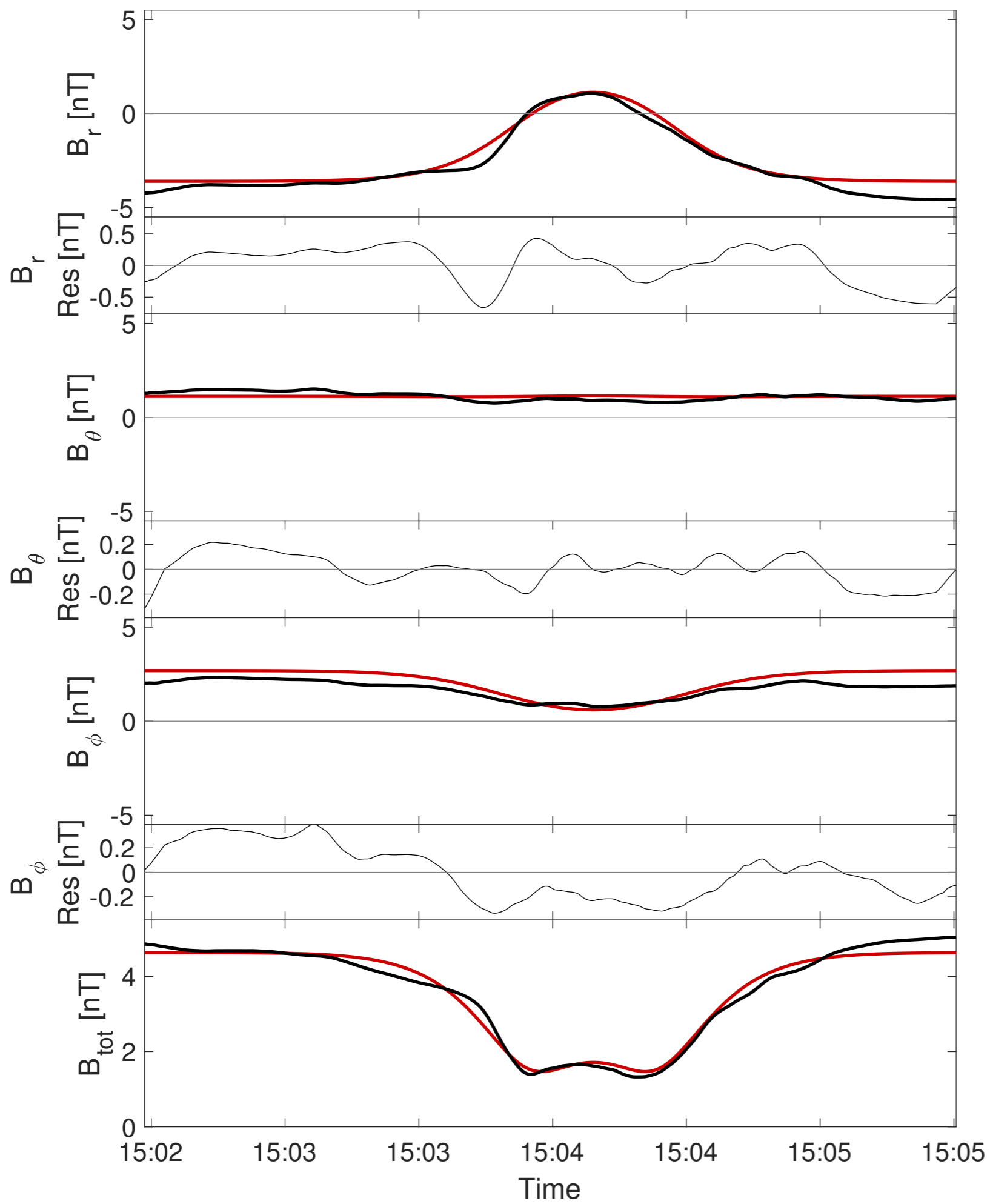


Figure 3.

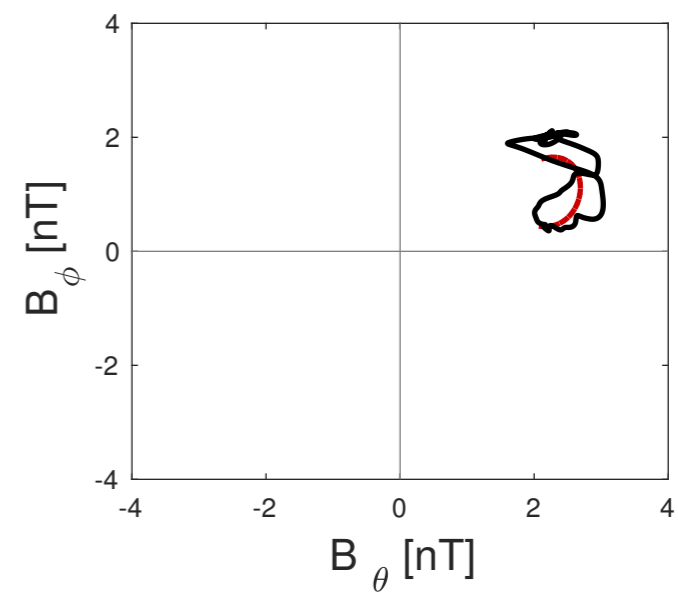
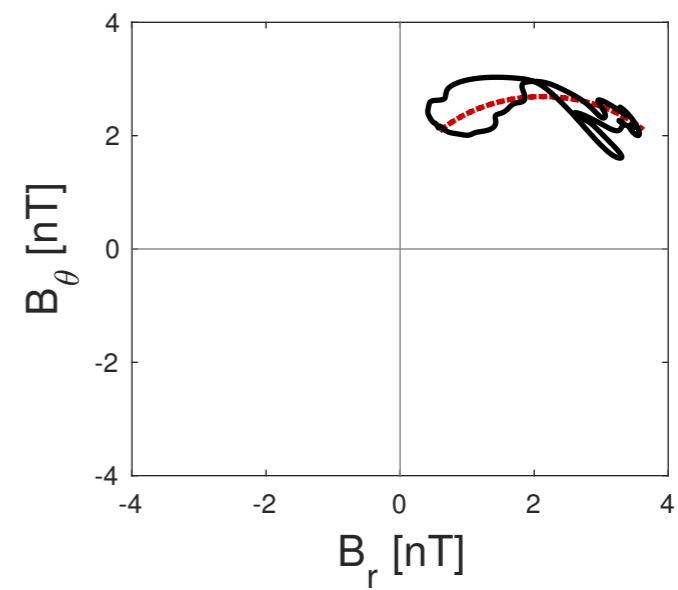
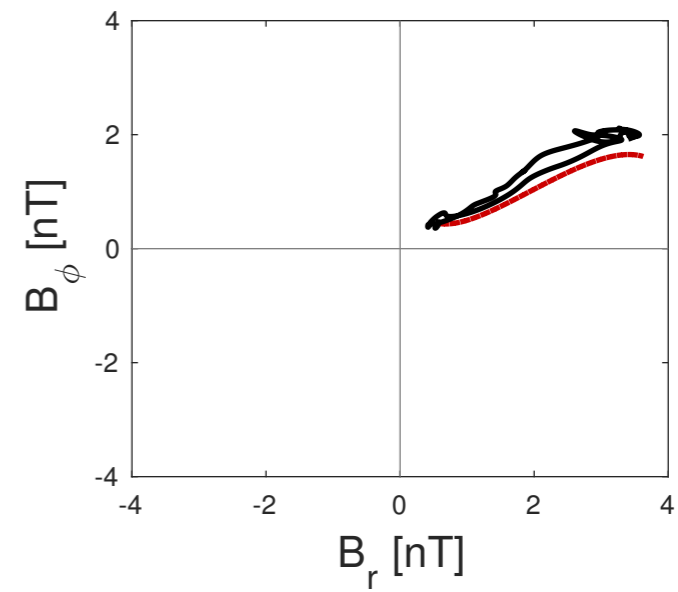
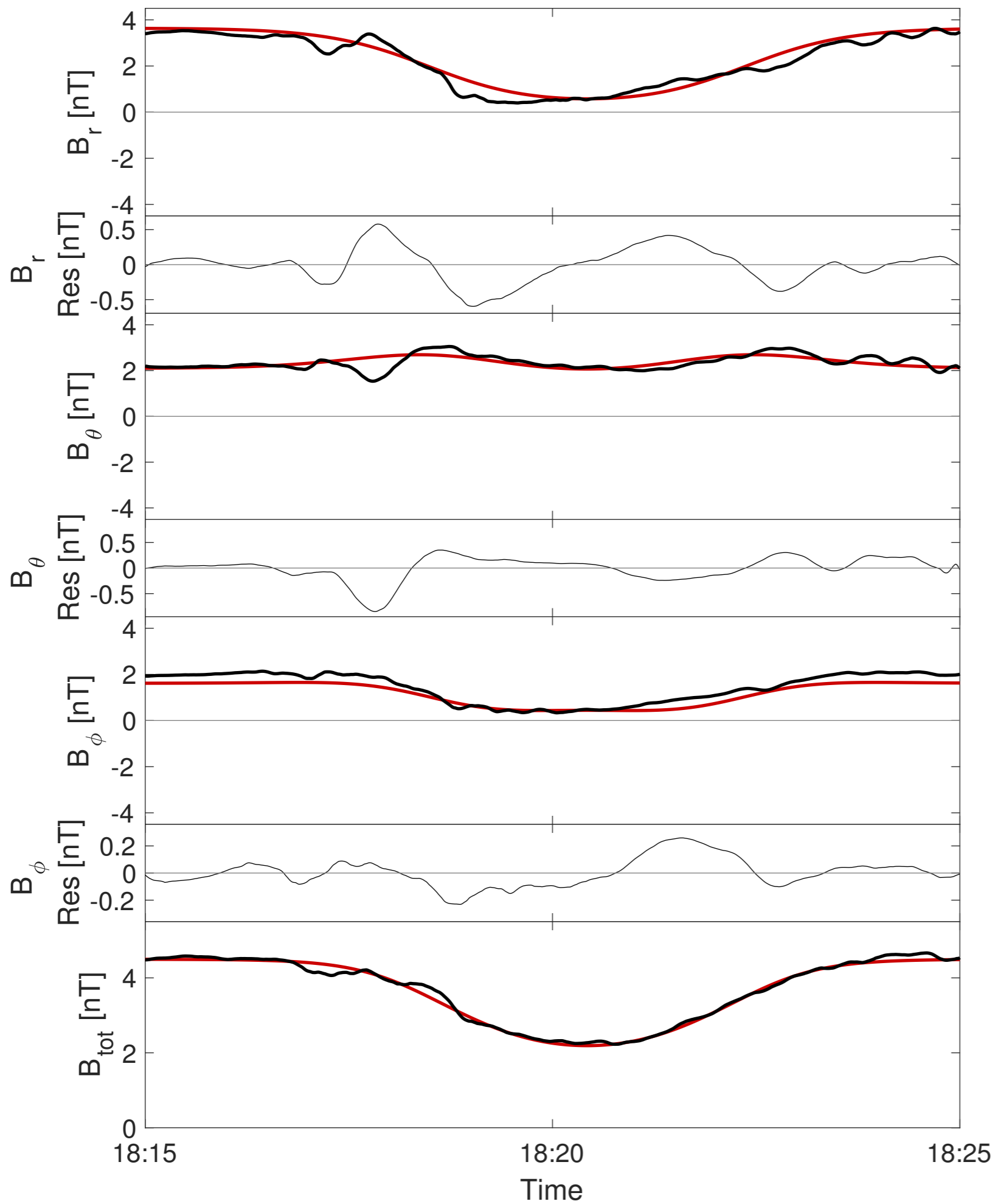


Figure 4.

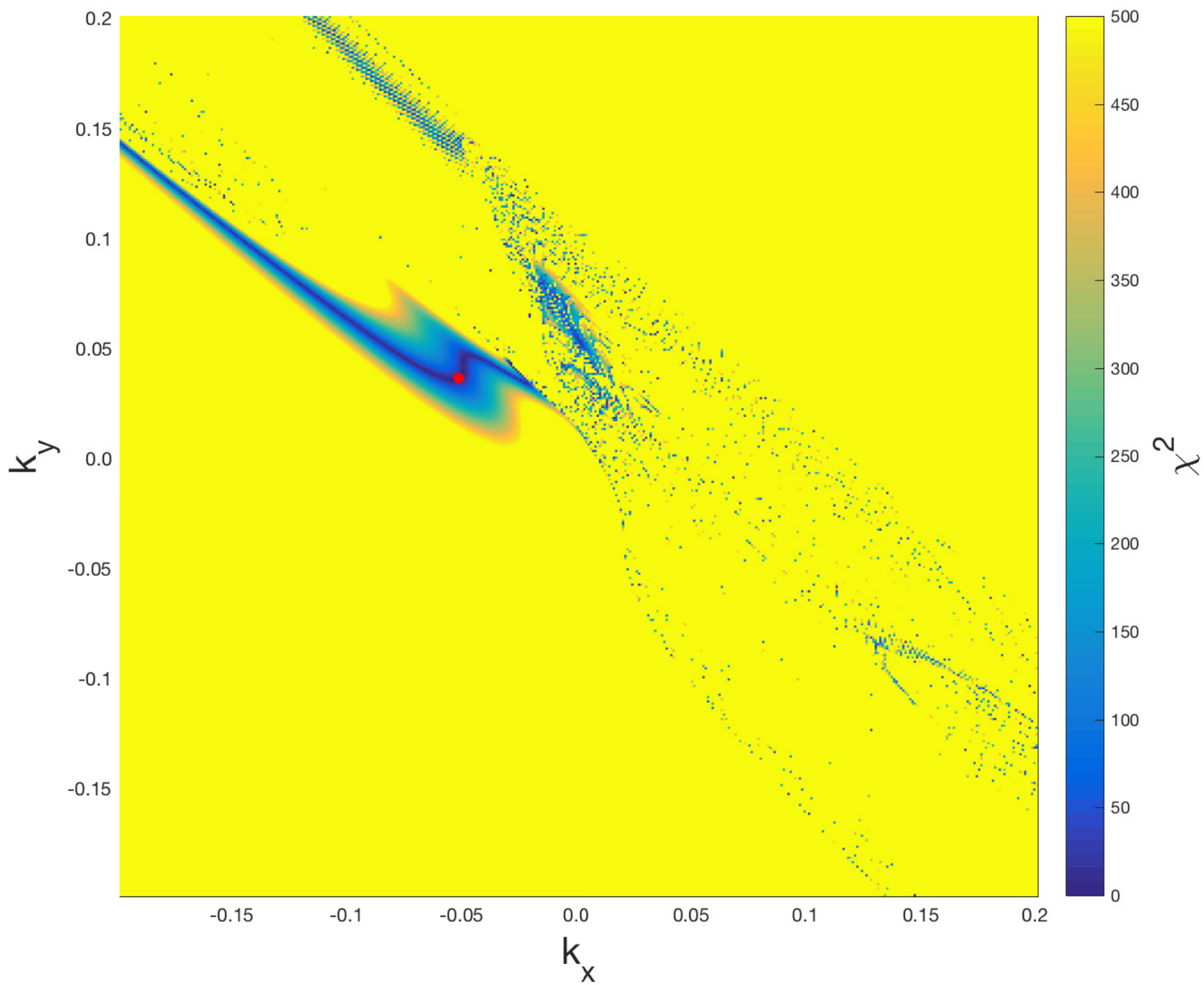


Figure 5.

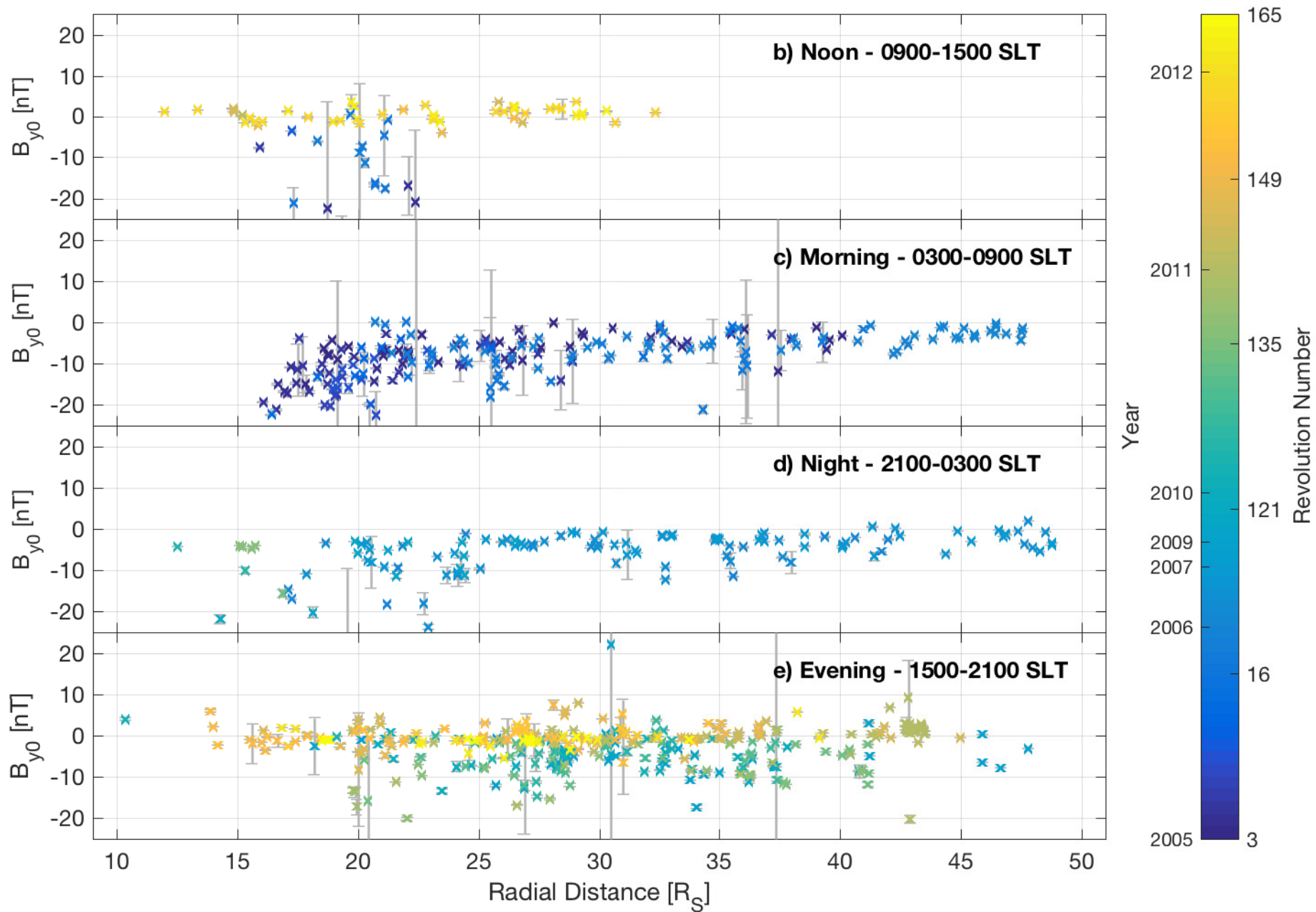
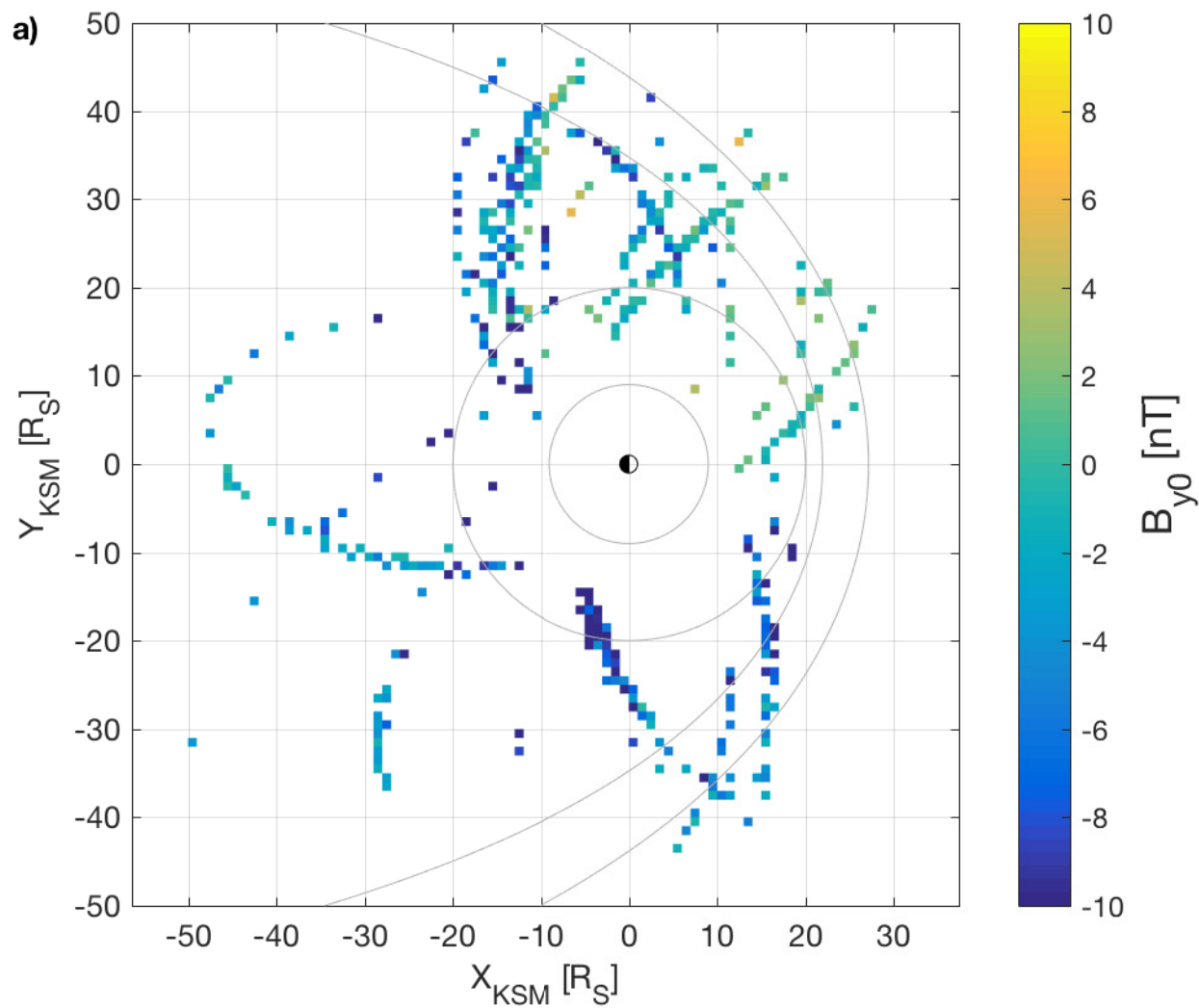


Figure 6.

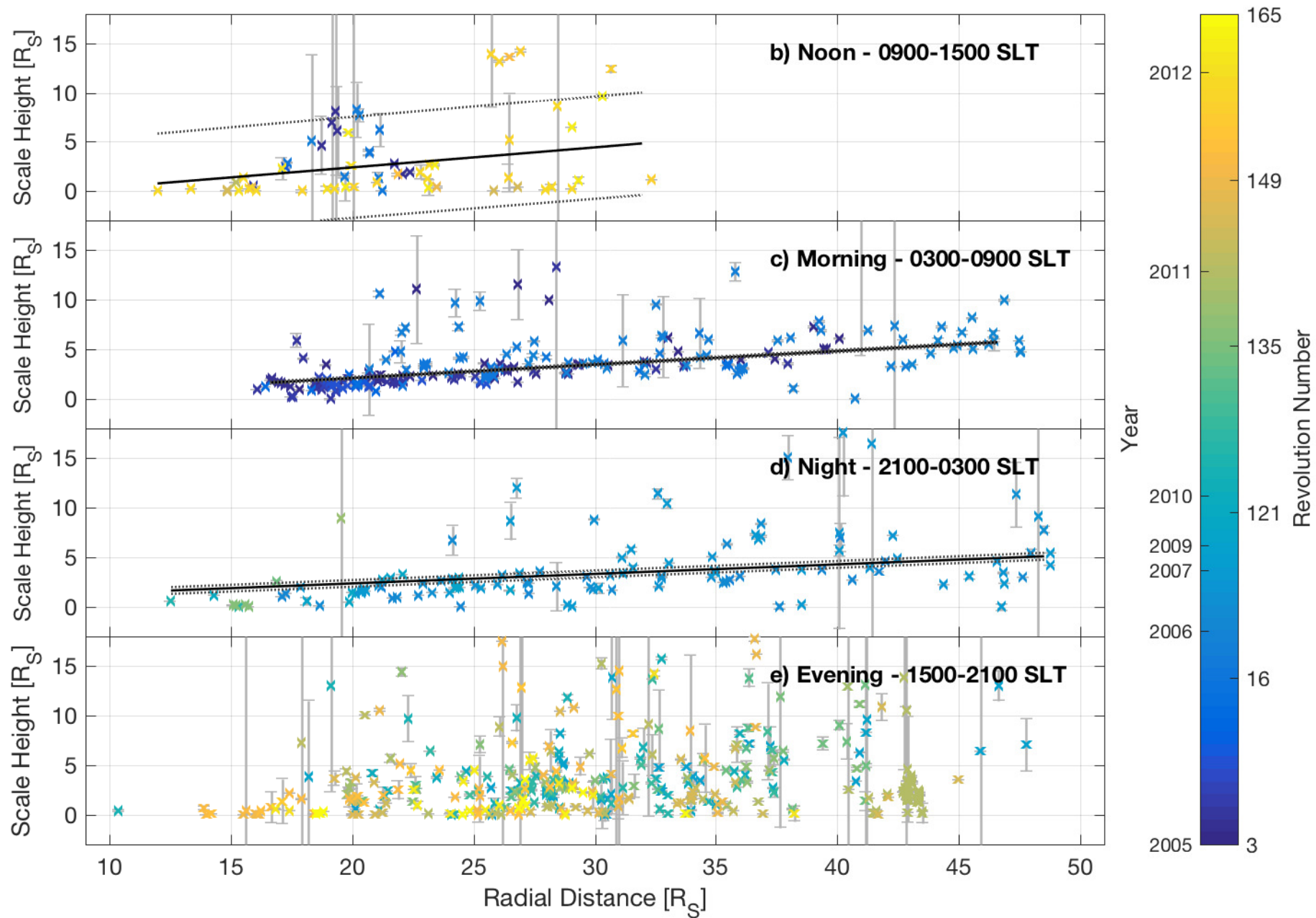
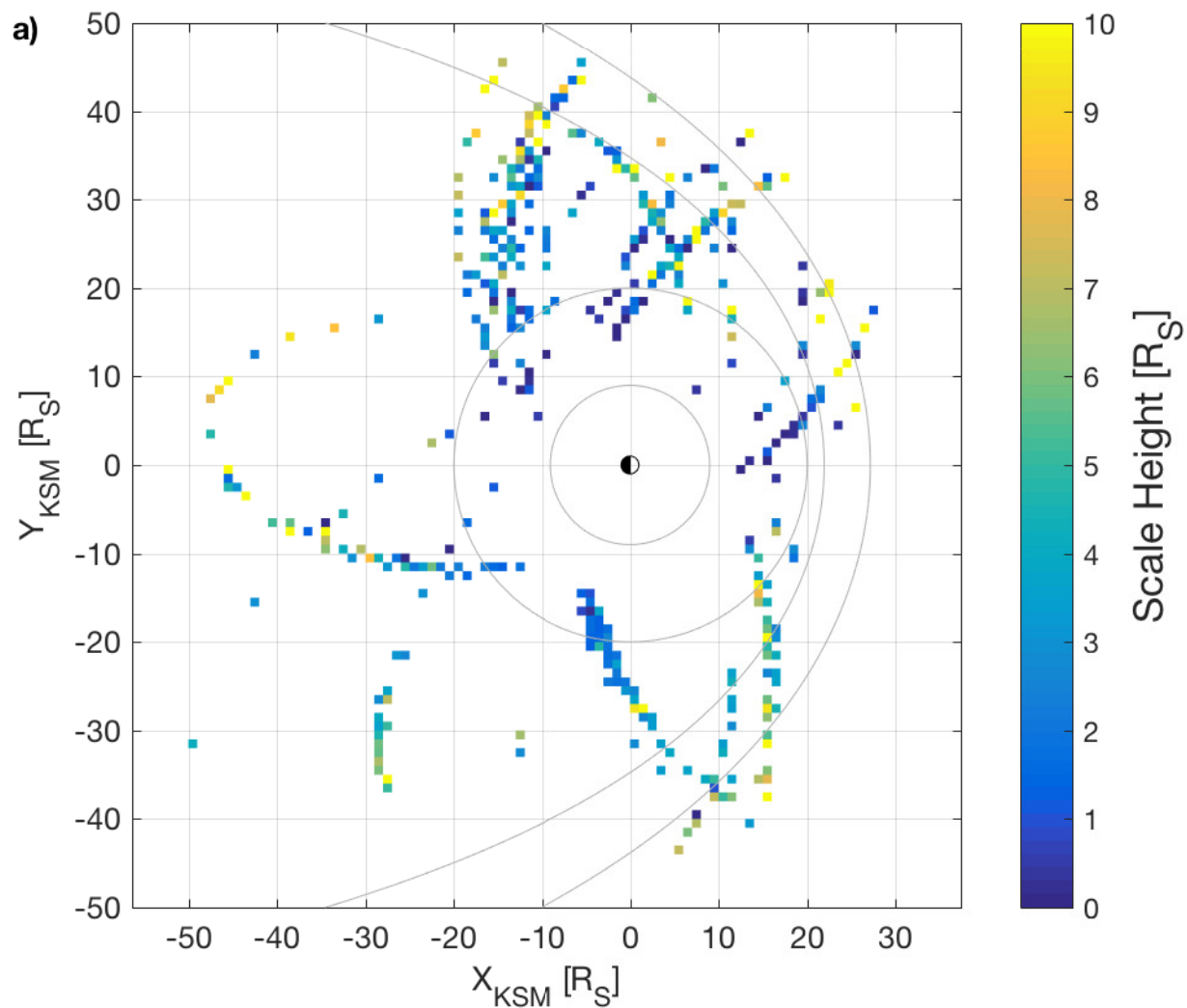


Figure 7.

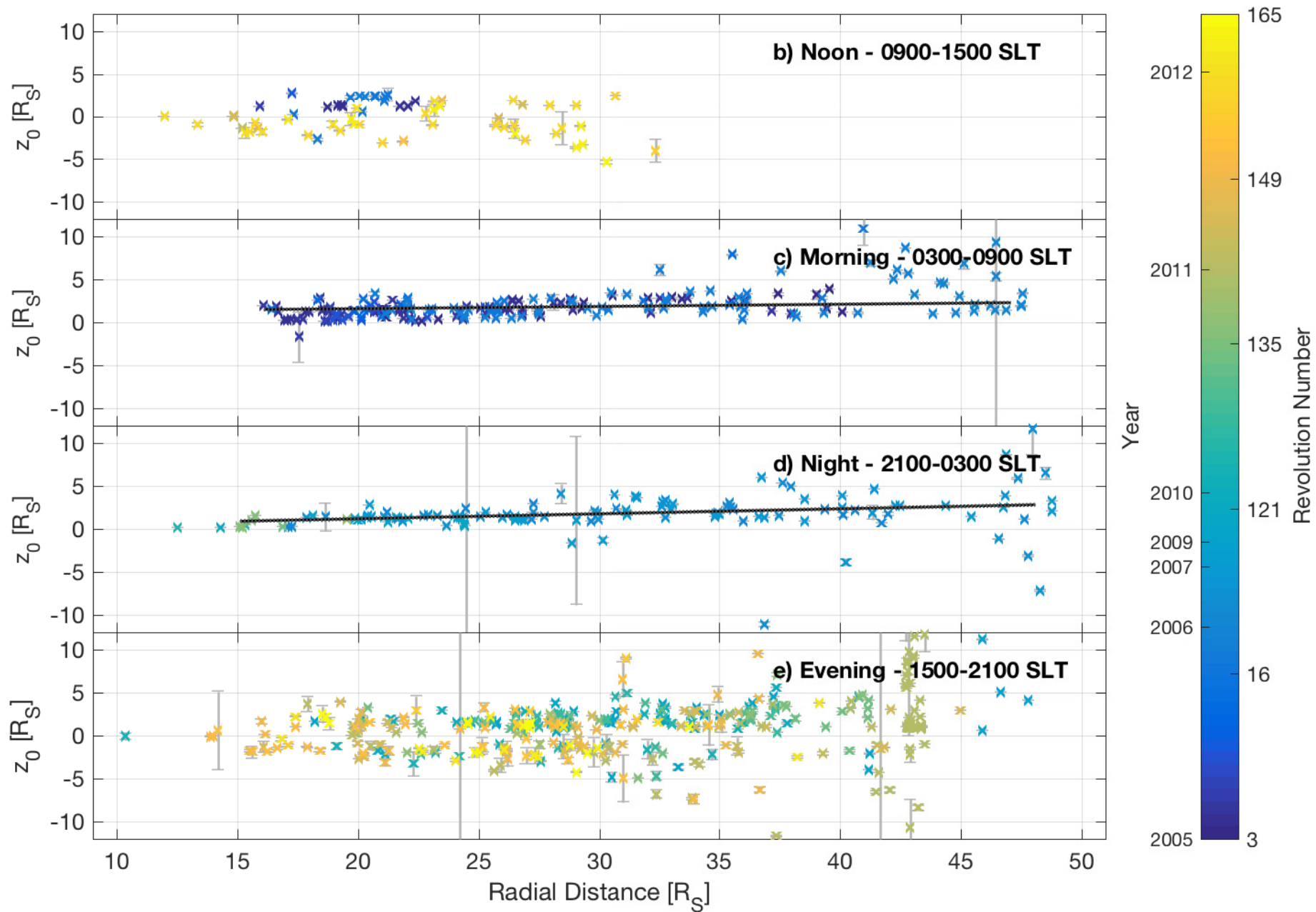
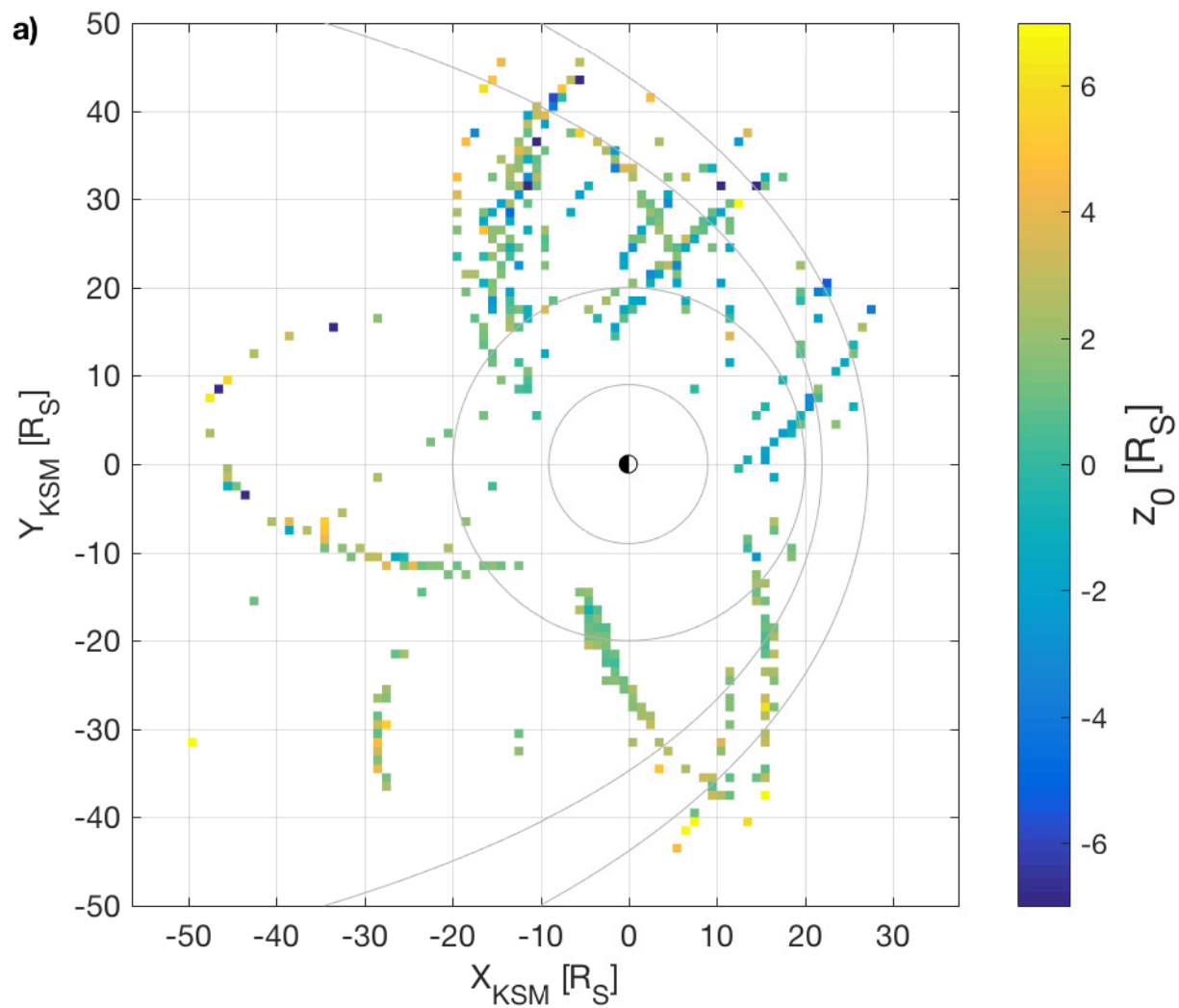


Figure 8.

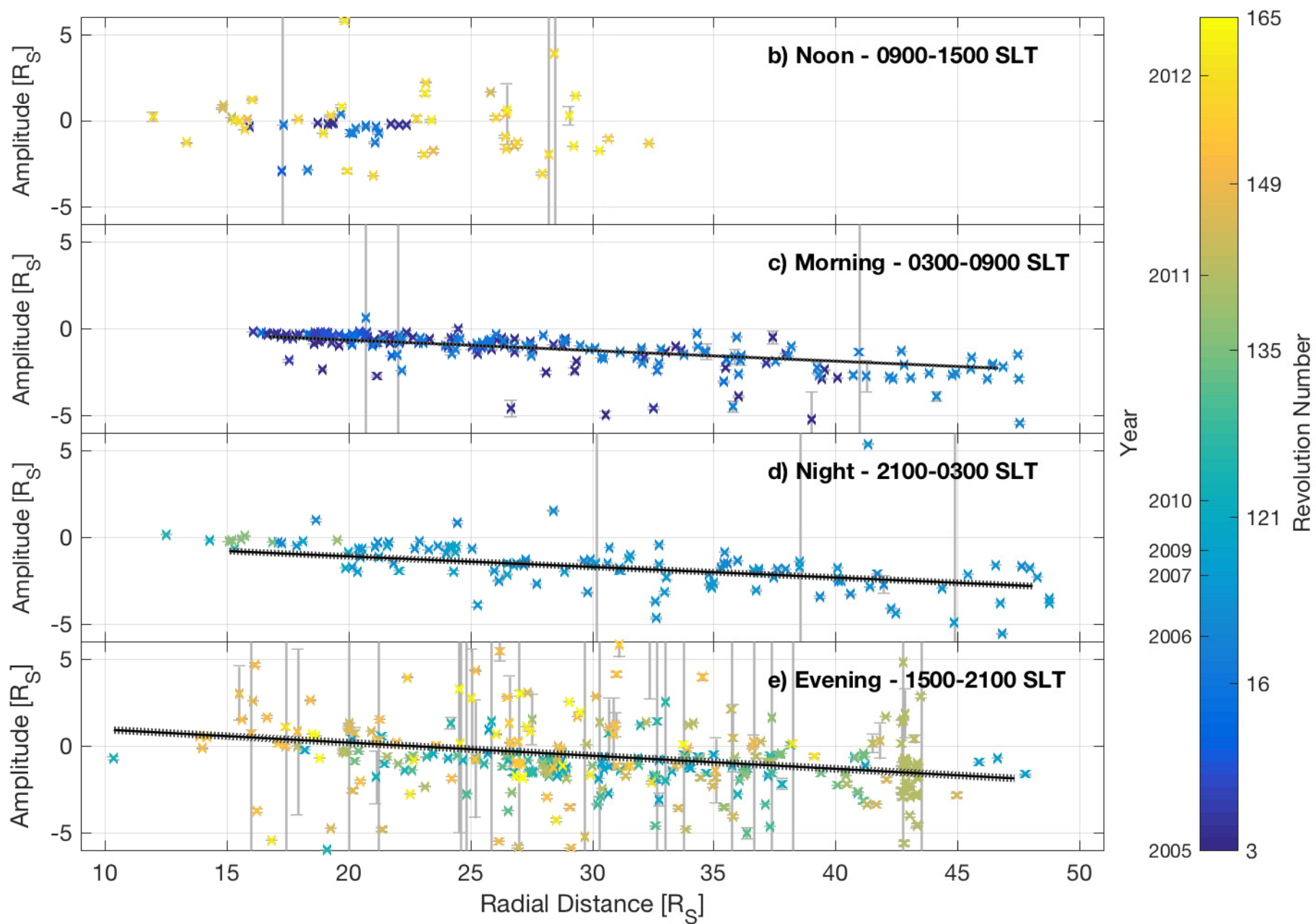
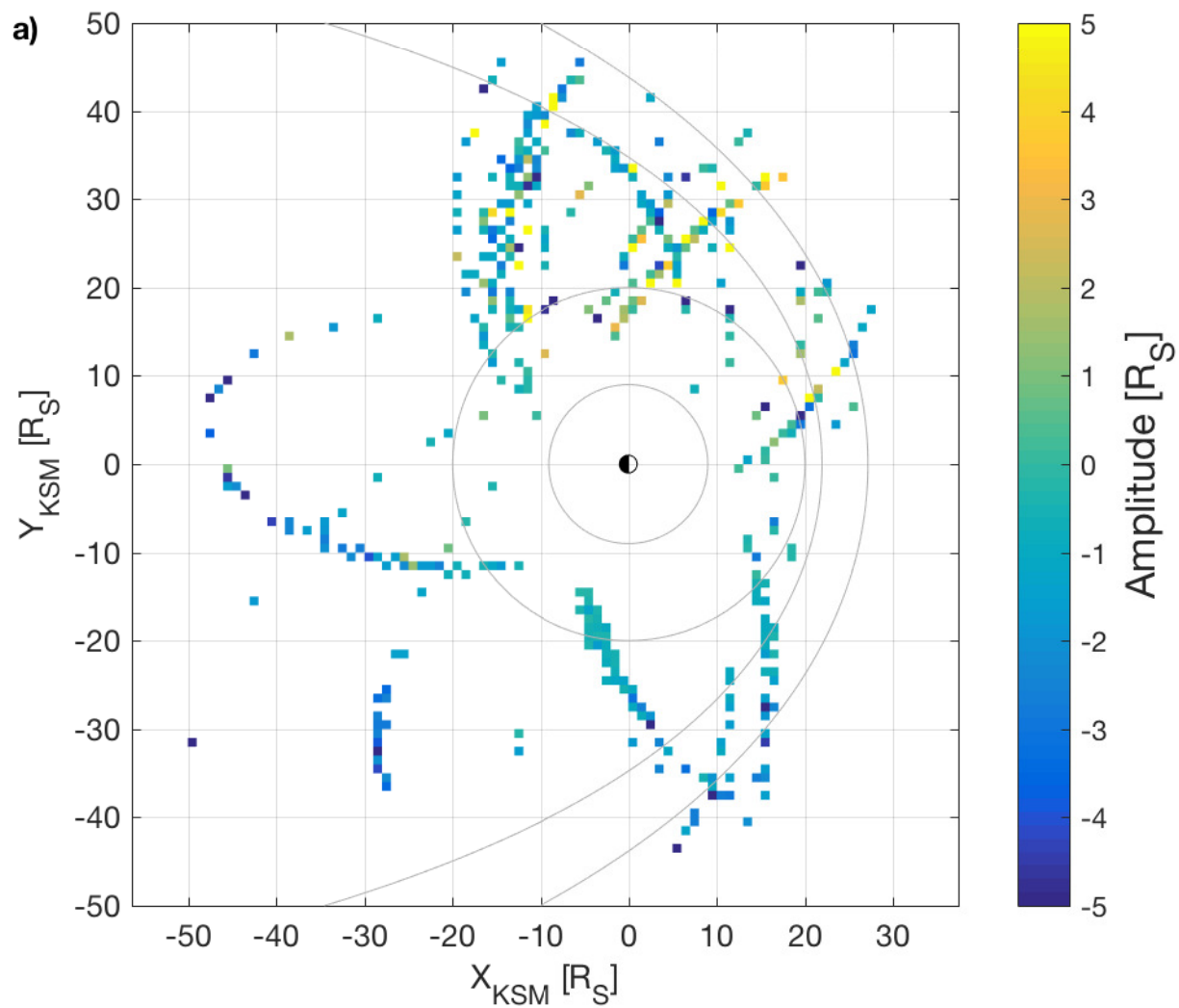
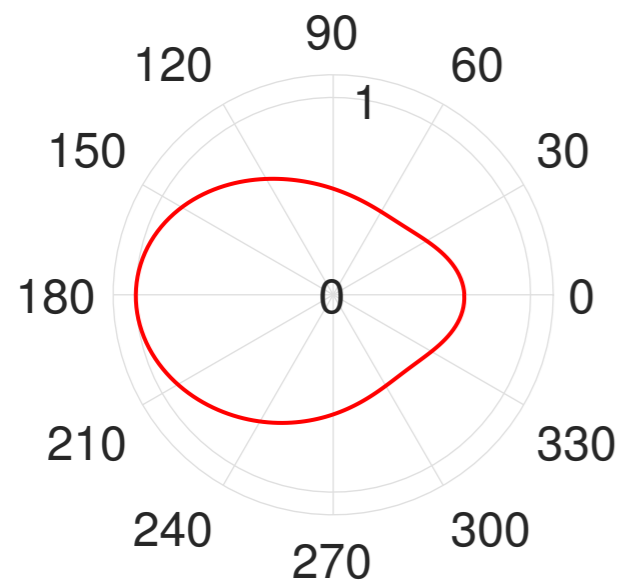
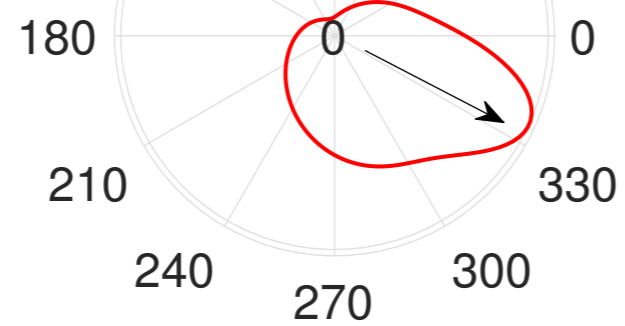
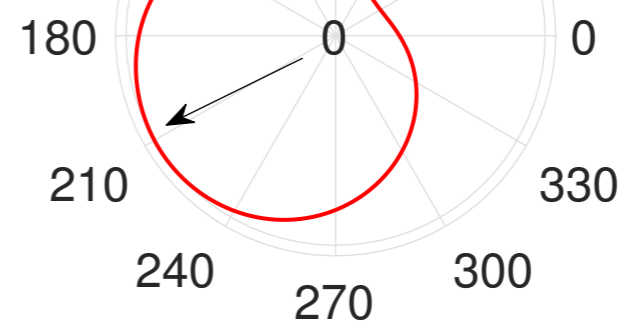
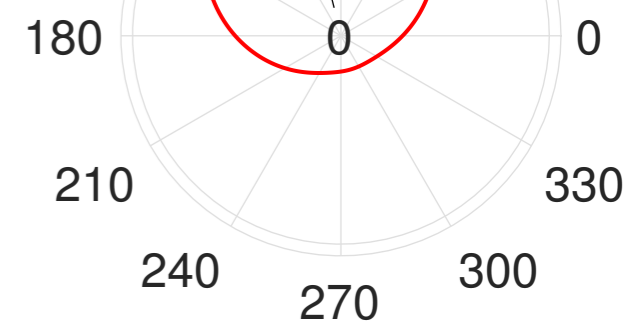
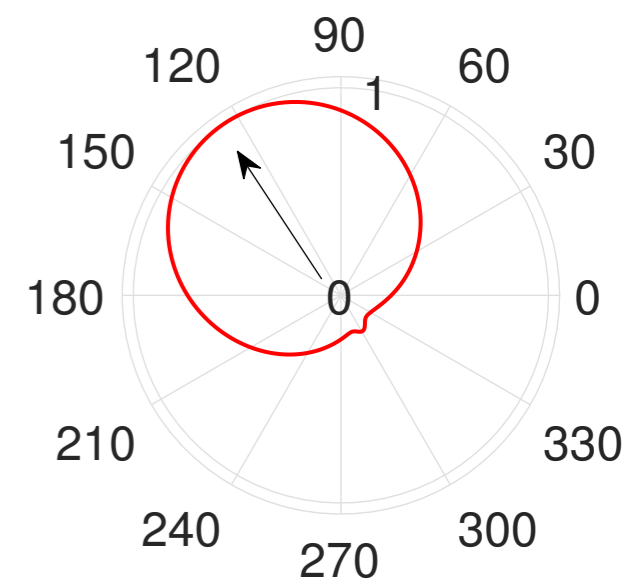
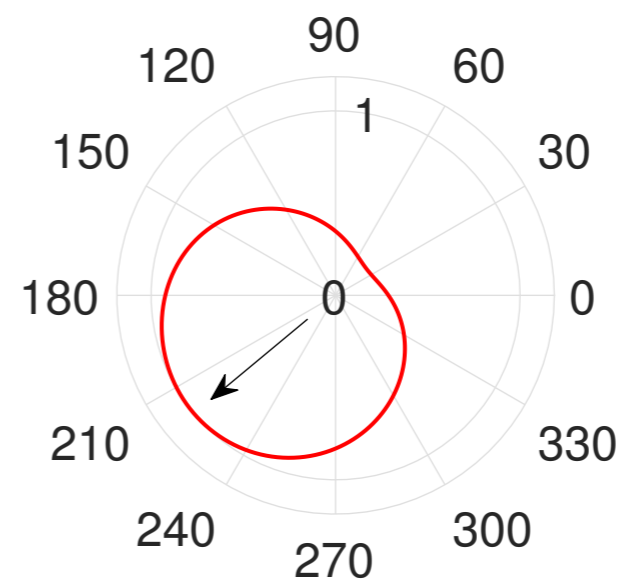
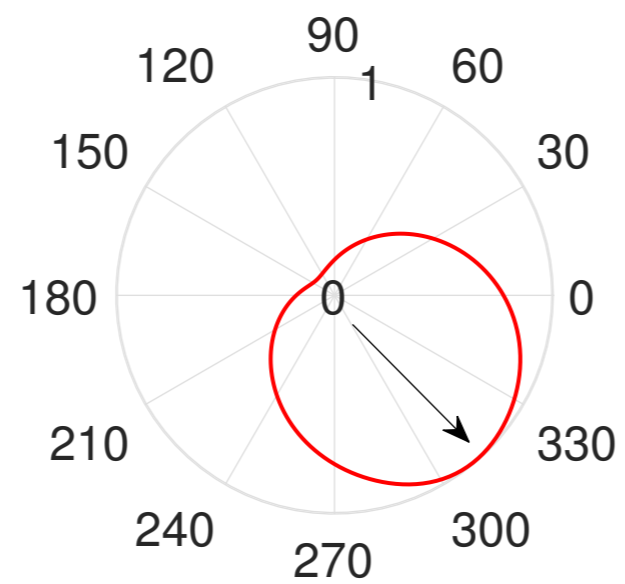
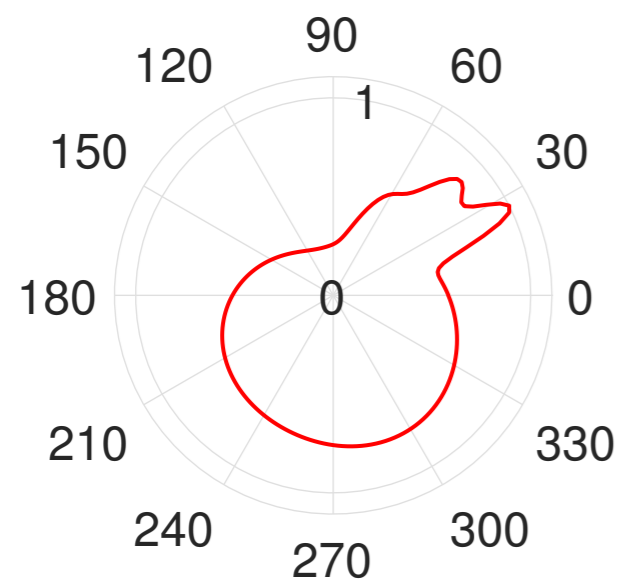


Figure 9.

Noon 09:00 - 15:00 $< 20 R_S$ **Morning 03:00 - 09:00****Night 21:00 - 03:00****Evening 15:00 - 21:00** $20 R_S < R_S < 40 R_S$ **Key** $> 40 R_S$ 