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An Integrated Modelling Approach for the Bicriterion Vehicle Routing and Scheduling Problem with Environmental Considerations

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ABSTRACT

The consideration of pollution in routing decisions gives rise to a new routing framework where measures of the environmental implications are traded off with business performance measures. To address this type of routing decisions, we formulate and solve a bi-objective time, load and path-dependent vehicle routing problem with time windows (BTL-VRPTW). The proposed formulation incorporates a travel time model representing realistically time varying traffic conditions. A key feature of the problem under consideration is the need to address simultaneously routing and path finding decisions. To cope with the computational burden arising from this property of the problem we propose a network reduction approach. Computational tests on the effect of the network reduction approach on determining non-dominated solutions are reported. A generic solution framework is proposed to address the BTL-VRPTW. The proposed framework combines any technique that creates capacity-feasible routes with a routing and scheduling method that aims to convert the identified routes to problem solutions. We show that transforming a set of routes to BTL-VRPTW solutions is equivalent to solving a bi-objective time dependent shortest path problem on a specially structured graph. We propose a backward label setting technique to solve the emerging problem that takes advantage of the special structure of the graph. The proposed generic solution framework is implemented by integrating the routing and scheduling method into an Ant Colony System algorithm. The accuracy of the proposed algorithm was assessed on the basis of its capability to determine minimum travel time and fuel consumption solutions. Although the computational results are encouraging, there is ample room for future research in algorithmic advances on addressing the proposed problem.

Keywords: Routing in Congested Networks, Distribution Planning, Network Reduction, Environment, Bi-criterion Vehicle Routing

1. INTRODUCTION

Road freight distribution activities are responsible for a significant share of energy consumption and production of Greenhouse Gas (GHG) emissions. A major pro-active measure dealing with GHG emissions relates to planning environment-friendly distribution routes by incorporating emissions (or fuel consumption) as an explicit routing criterion. Substantial research work has been focused on vehicle routing models which treat truck emissions as a routing criterion or as a component of total routing cost (Demir et. al., 2012). However, limited work has been devoted to exploring the trade-off between emissions criterion and conventional business criteria (e.g., travel time). In particular, no relevant research work addresses this issue under time varying traffic conditions.

Most of the relevant studies make use of the Comprehensive Modal Emissions Model (CMEM) formulae (Barth et al., 2004) and express the emissions routing criterion as a function of the following vehicle and road network characteristics: i) travel speed, ii) the payload of the vehicle, iii) the physical characteristics of the road path traversed by the vehicle (like the rolling resistance and the road horizontal grade), and iv) certain vehicle's physical and mechanical characteristics like horsepower, type of engine, and vehicle frontal area. There are three basic elements of routing decisions under time varying traffic conditions that have a direct bearing on emissions: i) the sequence that customers are visited which affects the payload between consecutive visits, ii) the selection among various alternative road paths between customers which posses different road characteristics e.g., road gradient, travel speed profile, and iii) the determination of the departure time for each route which results to different travel speeds due to time varying traffic conditions (e.g., traffic congestion). Hence, the incorporation of fuel consumption as an explicit criterion into routing decisions under time varying traffic conditions gives rise to a new class of vehicle routing models which involve multiple time, load, and path dependent criteria. The objective of this paper is to fill this gap by developing and solving a bi-objective time and load dependent vehicle routing problem with time windows (BTL-VRPTW). The proposed model considers simultaneously fuel consumption (as a proxy for emissions) and travel time objectives.

The BTL-VRPTW involves planning routes for a homogeneous fleet of trucks of known capacity for servicing a set of customers with known demand and strict service time windows. Each route starts at a given origin and terminates at given destination (not necessarily different from the origin). In the remainder of this paper, the customer locations, the origin and the destination nodes of the problem are referred to as *stops*. Travel between stops takes place on the underlying road network. Following the common practice, the effect of time varying traffic conditions is incorporated in the relevant routing model by considering time-dependent travel speeds between each pair of customers (Sbihi and Eglese, 2007). In this work, the average travel speeds for the links of the underlying road network are assumed piece-wise linear functions of time (Horn, 2000). It is worth noting that this is the first study on routing decisions that uses the Horn's travel time model. The CMEM (Barth et al., 2004) is used to model fuel consumption. It is assumed that waiting is allowed only at the origin or the customer location. Each solution of the proposed vehicle routing and scheduling model is fully defined by: i) a set of routes (sequence of stops) covering demand, ii) the road path used to travel between consecutive stops, iii) the departure time for each route, and iv) the departure time from each stop (apart from the destination). The objective of the proposed model is to determine the non-dominated solutions that aim to minimize the total travel time and the associated fuel consumption. To the best of our knowledge, no work has been found in the literature that deals with the proposed BTL-VRPTW.

The remainder of this paper consists of six sections. Section two discusses previous related work and highlights the contribution of this paper. Section three provides the description of the proposed model. Section four describes a generic solution framework and places emphasis on its major features. Section five illustrates the implementation of the solution framework by combining the ACS (Ant Colony System) technique with a routing and scheduling routine. Section six presents a network reduction approach for reducing the computational requirements of the solution approach and reports on its effect on the quality of solutions. Finally, section seven summarizes the conclusions of this work and provides directions for future research.

2. PREVIOUS RELATED WORK

In recent years, the literature on integrating environmental considerations into transportation planning decisions has grown rapidly (Lu et al., 2016; Li et al, 2016; Demir et. al., 2014). The main thrust of the research has been placed on the green vehicle routing problem. Three survey papers have been published, summarizing research efforts related to the broader area of Green Freight Transportation (Demir et. al., 2014), the Green Vehicle Routing Problem (G-VRP) (Lin et al., 2014), and G-VRP solution approaches (Park and Chae, 2014). The incorporation of environmental considerations into vehicle routing problems has led to the development of three categories of routing models, depending on how emissions or energy consumption are modeled: i) time-independent emissions (or energy) minimising vehicle routing models (Peng and Wang, 2009; Fagerholt et al., 2010; Urquhart et al., 2010; Suzuki 2011; Rao and Jin, 2012), ii) time-dependent emissions minimising vehicle routing models (Palmer, 2007; Figliozzi, 2010; Jabali et al., 2012), and iii) load-dependent vehicle routing models (Bektas and Laporte, 2011; Franceschetti et al., 2013; Demir et al., 2014; Ehmke et al, 2016). In this last category, called Pollution Routing models, there are time-dependent (Franceschetti et al., 2013; Ehmke et al, 2016) and time-independent (Bektas and Laporte, 2011; Demir et al., 2014) formulations depending on whether time varying traffic conditions are taken into account on estimating emissions or not. The models in category (i) ignore the effect of traffic congestion and the vehicle load on the expected emissions and thus they are applicable to cases where both features are not binding. Models in category (ii) account for traffic congestion but ignore the effect of the payload on the expected emissions. In what follows, we discuss research results that are most relevant to the Pollution Routing models (iii), we identify the literature gaps and elaborate on the novelties introduced by this paper.

The Pollution Routing Problem (PRP) was introduced and studied by Bektas and Laporte (2011). In this work, four variants of the vehicle routing problem with time windows were examined using different objective functions namely, distance, load-distance (product of distance traversed with vehicle's load), energy consumption, and total cost (including fuel cost, driver's cost and emissions' cost). The distinctive features of this modelling approach are that the travel speed between consecutive stops is treated as a decision variable while the emissions objective function is time-independent and expressed by a simplified version of the CMEM formula for the Heavy Duty Vehicles (Barth et al., 2004). The emissions objective function is applicable when non-congested traffic conditions are assumed throughout the network under study.

Franceschetti et al. (2013) enhanced the work of Bektas and Laporte (2011) on the PRP by considering the effect of traffic congestion on emissions. A single-objective, time and load dependent vehicle routing model with time windows is proposed aiming to route a homogeneous fleet of vehicles by minimizing the total emissions' and drivers' costs. The model incorporates the effect of traffic congestion on emissions by considering time-dependent average travel speed between each pair of customers. The definition of the travel speed function is facilitated by splitting the time horizon of the problem into two periods. In the first period, any arc of the network is assumed congested with a known fixed average travel speed. Following the congestion period, there exists a period of free flow conditions where the travel speed is treated as a decision variable ranging between an upper and lower speed limit. Franceschetti et al. (2013) also investigate analytically the fixed route version of the problem where the objective is to determine the departure times and travel speeds for traversing a given route at the minimum total cost.

Demir et al. (2012) extend the work in Bektas and Laporte (2011) by modelling the PRP as a bi-objective (time-independent) vehicle routing problem with time windows. The objectives considered by this model are fuel consumption and driving time. As in Bektas and Laporte (2011) and Franceschetti et al. (2013), travel speed between each pair of stops under free flow conditions is treated as a decision variable. Four a-posteriori bi-objective programming methods are used to solve the problem namely, the weighted method, the weighted method with normalization, the ε-constraint method, and a hybrid method. An Adaptive Large Neighbourhood Search (ALNS) Algorithm is used to solve the resulting formulations. To the best of our knowledge, this is the only paper (Demir et al., 2014) focused on the Pollution Routing problem with multiple objectives. However, the relevant model is time-independent and hence its applicability is narrowed to problems where time varying traffic conditions (e.g., traffic congestion) are not relevant.

Recently, Xiao and Konak (2016) provided a single objective time-dependent vehicle routing and scheduling model for minimising emissions which allows for heterogeneous fleet of trucks and waiting of a truck while en-route. They propose a hybrid solution method combining the use of an MIP solver with Iterated Neighbourhood Search to deal with the emerging problem.

A major limitation of the studies reported so far is that they consider an a-priori determined single road path for travelling between each pair of customers. This approach implies that the relevant routing decision is only concerned with sequencing of servicing customers ignoring the intermediate road path finding problem between customers. However, the emissions-optimal path between any pair of stops may differ for different payload. Ehmke et al (2016) identified this issue and proposed an emissions-minimizing time and load dependent vehicle routing and scheduling model that accounts explicitly for the path finding problem between stops. The formulation of Ehmke et al. (2016) aims to determine a set of routes and the road paths between consecutive stops that minimise emissions (defined through CMEM) assuming a fixed departure time for all routes. They claim that the emissions-optimum road paths between any pair of stops cannot be pre-computed since the load of the vehicle travelling between two stops is not known until the time that their position on a route is specified. However, a key finding of their work is that if the emissions-optimum path between a pair of stops remains the same whether the vehicle is empty or fully loaded, then the same path is optimal for any intermediate value of the payload. Ehmke et al. (2016) take advantage of this finding and pre-compute optimal paths between stops wherever this is possible.

Table 1 presents a summary of the features of the papers presented above. We have marked in bold the features of the reported papers that comply with the features of our work. While Demir et al. (2012) model explicitly the trade-off between these two competing objectives, they do not consider the time-dependent nature of travel speeds (e.g., due to congestion) in determining fuel consumption and driving time. This modelling approach may not represent realistic solutions as the prevailing traffic conditions in the different arcs of the network throughout the day may not allow the attainment of the optimal (free-flow) speeds. Therefore, the specification of the trade-off between travel time and emissions (or fuel consumption) in time and load dependent vehicle routing and scheduling problems is still an open issue. On the other hand, despite the fact that the models proposed by Franceschetti et al. (2013), Xiao and Konak (2016) and Ehmke et al. (2016) incorporate the effect of traffic congestion into the PRP, they do not examine the trade-off between fuel consumption and travel time. In particular, the work in Franceschetti et al. (2013) is applicable only when the routing of delivery vehicles must be completed within the first half of the day. This means that distribution should start during the congestion period represented by the morning peak, and should be concluded no later than the end of the period of free flow conditions that follow the morning peak. In a nutshell, the research reported in Franceschetti et al. (2013), Demir et al. (2012), Xiao and Konak (2016), and Ehmke et al. (2016) is very relevant to the work presented in this paper. However, none of these studies deals explicitly with the bi-objective Vehicle

Routing and Scheduling Problem taking into account the effect of time varying traffic conditions on both travel time and fuel consumption (or emissions).

Features	Bektas & Laporte (2011)	Franceschetti et al. (2013)	Xiao & Konak (2016)	Ehmke et al. (2016)	Demir et al. (2014)	Proposed Work
# Criteria	Single	Single	Single	Single	**Bi-criteria**	**Bi-criteria**
Criteria	FC & TT Cost	FC & TT Cost	Emissions Cost	Emissions Cost	**FC vs. TT**	**FC vs. TT**
Time Varying	No	**Partially**	**Yes**	**Yes**	No	**Yes**
Travel speed (Free Flow vs. Constrained)	Free Flow	Both (Step function)	Both (Step function)	**Constrained (Step function)**	Free Flow	**Constrained (Piecewise Linear)**
Multiple paths between stops	No	No	No	**Yes**	No	**Yes**
Waiting time at Depot	Non Applicable	**Allowed**	**Allowed**	Not Allowed	N/A	**Allowed**
Waiting time at Customers	Non Applicable	**Allowed**	**Allowed**	Not Allowed	N/A	**Allowed**

Table 1. Overview of the major features of relevant previous related work. (where FC: Fuel Consumption, TT: Travel Time)

The model presented in this paper deals explicitly with the trade-off between travel time and fuel consumption (emissions' proxy) taking into account time varying traffic conditions, the effect of dynamic payload of the vehicle, and the complexities of the intermediate path finding problems between consecutive stops. Both objective functions are defined on the basis of the travel time model of Horn (2000), not previously used in the relevant literature. The added value of the proposed travel time model is that in addition to time-dependent travel speeds, it also accounts for the average acceleration/ deceleration rate for travelling a road segment, which is a basic parameter in estimating fuel consumption (emission) according to the CMEM. Furthermore, the proposed routing model aims to: i) determine the most appropriate departure time for each route rather than using a fixed departure time, and ii) determine the number of vehicles needed as opposed to the models cited above which assume a fixed number of vehicles used. The proposed modelling approach leads to a bi-objective time, load and path dependent vehicle routing and scheduling problem which has not been up-to-date reported in the literature of Vehicle Routing Problems.

3. MODEL DEFINITION AND CONTEXT

3.1 Model Assumptions

The proposed vehicle routing and scheduling problem involves a set C of n customers with known demand d_i for $i \in C$, and a homogeneous fleet of vehicles Ω of known capacity Q. All routes should start from a given origin i_0 and terminate at a given destination i_{n+1}. It is reiterated that the customer locations, the origin and the destination nodes of the problem are referred to as *stops*. The demand of each customer is served by exactly one vehicle visiting the corresponding delivery location once. The number of vehicles that will service demand must not exceed the size of the fleet of vehicles (i.e., the number of available vehicles). The departure from the origin and the arrival at the destination may occur within specific time windows denoted by $[a_0, b_0]$ and $[a_{n+1}, b_{n+1}]$ respectively. In addition, each customer i is associated with service duration st_i and a hard service time window $[a_i, b_i]$, where a_i denotes the earliest service start time at customer i, and b_i denotes the corresponding latest service start time. If a vehicle arrives at customer i before a_i, then the service of the customer will be postponed to time a_i. The vehicle is not allowed to arrive at customer i later than b_i.

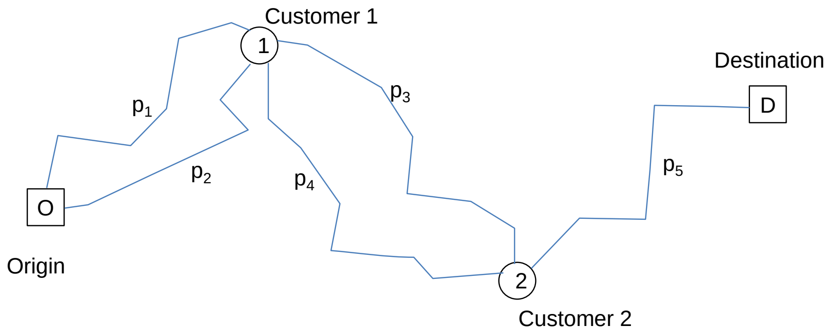
Travel between any pair of stops can be performed through the underlying road network denoted by $G(N, A)$ where N is the set of nodes and A is the set of road links. Any stop i of the problem is associated to a node $l(i) \in N$ on the road network. Hence a *road path* connecting a pair of stops (i, j) is denoted by $p_{ij} = [(l(i), l_1), (l_1, l_2), .., (l_{\eta-1}, l_\eta), .., (l_\nu, l(j))]$, where $l_\eta \in N, \eta = 1, .., \nu$. Since the road links attributes are time-dependent, travel between consecutive stops is fully defined by a road path enhanced with the schedule of traversing it, called *scheduled (road) path*. It is worth noting that the schedule of traversing a road path p_{ij} is fully defined by the departure time $\tau_{l(i)}$ from the origin $l(i)$ (no waiting time is allowed at the intermediate nodes of the road path). Hence, a scheduled road path is denoted by $sp_{ij} = (\tau_{l(i)}, [(l(i), l_1), (l_1, l_2), .., (l_{\eta-1}, l_\eta), .., (l_\nu, l(j))])$.

The objective of the BTL-VRPTW is to determine the service routes and identify the scheduled road paths for travelling between consecutive stops that minimise the total travel time and fuel consumption. It is worth noting that the departure time of each route is considered as a decision variable as well.

3.2 Network Reduction

In single-objective time-dependent vehicle routing problems (with the exception of Ehmke et al., (2016)), any optimal solution comprise optimal scheduled road paths between consecutive stops. Hence the relevant path finding problems defined between each pair of stops can be addressed at a pre-processing stage by pre-computing optimal scheduled shortest paths between all pairs of stops and for each possible departure time. Similarly, in relevant multi-objective time-independent vehicle routing problems, any non-dominated solution comprise non-dominated paths between consecutive stops. Hence, the intermediate multi-objective shortest path problems between stops can be solved in advance, at a preprocessing stage (Pradhananga et al., 2010). However, we show below that in the proposed BTL-VRPTW, the scheduled road paths between consecutive stops included in a non-dominated solution cannot be determined in advance. It is worth noting that the reason for this is not just the load dependency of emissions as indicated by Ehmke et al. (2016) for the emissions minimizing time and load dependent vehicle routing problem. There is an inherent issue for the bi-objective time-dependent vehicle routing problems in general. In particular, non-dominated solutions do not necessarily comprise non-dominated scheduled road paths connecting consecutive stops. As it is shown in the Example 1 below, a non-dominated solution may also involve dominated scheduled road paths between consecutive stops. This observation leads to a key feature of the problem under consideration which implies the need to address simultaneously routing and path finding decisions.

Example 1. Figure 1 presents a route starting from the origin O, passing through customers 1 and 2 and terminating at the destination D. For simplicity, we assume that two alternative road paths are eligible for traveling from the origin to customer 1 (i.e., p_1 and p_2) and from customer 1 to customer 2 (i.e., p_3 and p_4). A single path is considered for traveling from customer 2 to the destination. The table in Figure 1 presents the travel time and fuel consumption values on paths p_1-p_5 as functions of departure time from the upstream node. Note that travel time and fuel consumption on p_1, p_2 and p_5 are time-independent. Table 2 provides the alternative solutions starting at time 0 that can be formed by combining the alternative intermediate paths between stops. Without loss of generality we assume 0 waiting time and service duration at the customers. It is worth noting that comparing p_2 with p_1 for departure time 0 from the origin O, it can be verified that p_2 outperforms p_1 on the basis of travel time and fuel consumption. Hence p_1 is a dominated path for departure time 0. However, non-dominated solution 1 (Table 2) includes p_1 at departure time 0. This example implies that dominated intermediate paths cannot be excluded from consideration in determining non-dominated solutions, and thus addressing the intermediate bi-objective shortest path problem individually on a per pair of stops basis is not valid for determining the non-dominated solutions of the proposed routing and scheduling problem.



Path	Departure Time	Travel Time (min)	Fuel Consumption (gr)
p_1	0-60	20	300
p_2	0-60	15	200
p_3	0-17 18-60	25 18	600 400
p_4	0-20 21-60	32 24	800 500
p_5	0-60	10	200

Figure 1. Alternative road paths considered for route $\{0, 1, 2, D\}$, and their travel time and fuel consumption values for different departure times.

Solutions	Route	Travel Time (min)	Fuel Consumption (gr)
Solution 1	$0 \xrightarrow{p_1} 1 \xrightarrow{p_3} 2 \xrightarrow{p_5} D$	48	900
Solution 2	$0 \xrightarrow{p_1} 1 \xrightarrow{p_4} 2 \xrightarrow{p_5} D$	62	1300
Solution 3	$0 \xrightarrow{p_2} 1 \xrightarrow{p_3} 2 \xrightarrow{p_5} D$	50	1000
Solution 4	$0 \xrightarrow{p_2} 1 \xrightarrow{p_4} 2 \xrightarrow{p_5} D$	57	1200

Table 2. Alternative solutions traversing route $\{0, 1, 2, D\}$ departing at time 0, and their total travel time and fuel consumption values.

To cope with the computational burden arising from this property of the problem we propose a network reduction approach. The main idea of the proposed network reduction approach is to define a sub-network for each pair of stops (i, j) formed by the road links of the K a priori determined shortest distance road paths, called *eligible road paths* and denoted by $\wp_{ij}$. The path finding problems between consecutive stops are then solved on these sub-networks reducing the relevant computational requirements. The effect of this approach on the quality of the BTL-VRPTW solutions is investigated later in this paper.

3.3 Model Characteristics

The description of any feasible solution σ of the proposed problem is facilitated by the notions of *route-paths* and *route-trajectories*.

Definition 1. A *route-path* P_r of route r, is a road path that traverses the entire route r, i.e., it starts from the origin i_0 and passes through each customer of the route (exactly once), terminating at the destination i_{n+1}.

Under the proposed network reduction, a route-path P_r is fully specified by a combination of the eligible road paths $p_{ij}^k \in \wp_{ij}$ used between each pair of consecutive stops (i, j) in route r. Hence, a route may be traversed by various alternative route-paths depending on the combination of the eligible road paths selected for each pair of consecutive stops of the route. Figure 2 presents a route of three customers $r = \{O, 1, 2, 3, D\}$ (where O and D denote the origin and destination nodes respectively). Each pair of consecutive stops involves two (a priori determined) eligible road paths with the exception of the last part of the route between customer 3 and D where there is a single eligible road path. The eligible road paths indicated with bold lines constitute a route-path associated to route r denoted as follows: P_r =$\{(O, 1, p_2)$, $(1,2, p_4)$, $(2, 3, p_5)$, $(3, D, p_7)\}$.

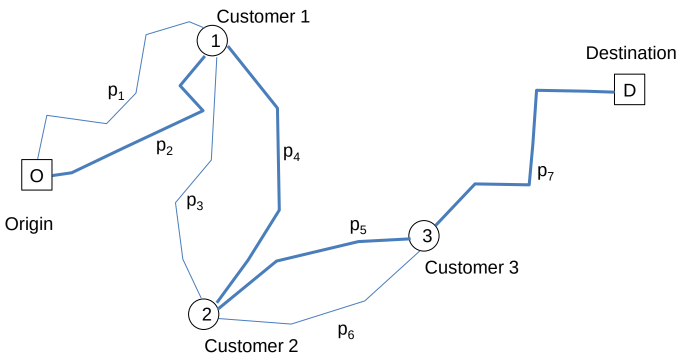


Figure 2. A route-path (in bold) associated to route $\{ O, 1, 2, 3, D \}$.

Definition 2. A *route-schedule* $S_{r\tau}$ associated to route r is defined as the set of departure times from the customers of r assuming that the departure time from the origin i_0 takes place at time τ.

Definition 3. A *route-trajectory* $R(P_r, S_{r\tau})$ of route r is defined by route-path P_r enhanced with the route-schedule $S_{r\tau}$ of traversing it, i.e., the departure time from each customer of the route-path. In practice, a route trajectory is a scheduled route-path.

It is worth noting that $S_{r\tau}$ is assumed to be feasible with respect to the temporal constraints of the stops in r. Example 2 below clarifies the definition of route-trajectories.

Example 2. Assume routes r_1=$\{O, 1, 2, D\}$ and r_2=$\{O, 3, 4, D\}$. Figure 3 presents the routes r_1 and r_2 and depicts the eligible road paths between each pair of consecutive stops. The bold-shaded paths indicate one route-path for each route: $P_{r_1} = \{ (O, 1, p_1), (1, 2, p_4), (2, D, p_5) \}$ for route r_1 and P_{r_2} =$\{(O, 3, P_6)$,$(3, 4, P_9)$, $(4, D, P_{11})\}$ for r_2. Table 3 presents the time dependent travel time and fuel consumption values of the road paths p_1, p_4, p_5, p_6, p_9, and p_{11} that form route-paths P_{r_1} and P_{r_2}. It is evident that different combinations of eligible road paths between consecutive stops of a route lead to different route-paths. Moreover, varying the departure time for any route-path (retaining time feasibility), results to different route trajectories. For instance, assume route-schedule $S_{r_1 5} = (23, 45)$, which implies that the departure time from the origin takes place at time 5 while the departure time from customers 1 and 2 of route r_1 is at times 23 and 45 respectively. It is easy to verify that $S_{r_1 5}$ is feasible for traversing route-path P_{r_1}, i.e., the departure time from customer 1 or 2 (23 and 45) takes place no earlier than the corresponding arrival times (23 and 45 respectively). Hence, enhancing route-path P_{r_1} with route-schedule $S_{r_1 5}$ results to route-trajectory $(P_{r_1}, S_{r_1 5}) = \{((O, 1, p_1), 5), ((1, 2, p_4), 23), ((2, D, p_5), 45)\}$. Moreover, route-schedule $S_{r_2 3} = (22, 44)$ is feasible for route-path P_{r_2}. Hence, enhancing route-path P_{r_2} with route-schedule $S_{r_2 3}$ results to route-trajectory $R(P_{r_2}, S_{r_2 3}) = \{ ((O, 3, p_6), 0), ((3, 4, p_9), 22),$

$\left((4, D, p_{11}), 44\right)\}$ for P_{r_2} . Route-trajectories $R\left(P_{r_1}, S_{r_1 5}\right)$ and $R\left(P_{r_2}, S_{r_2 3}\right)$ constitute a solution of the specific instance of the BTL-VRPTW.

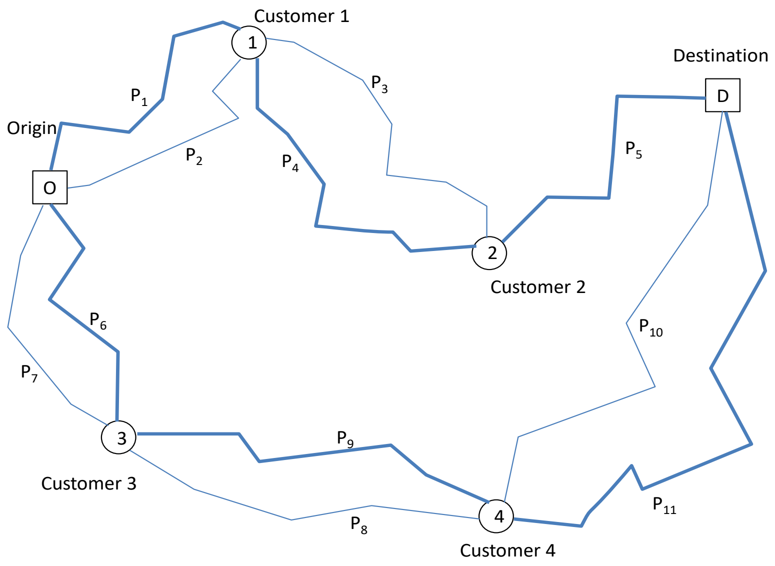


Figure 3. Representation of routes r_1 and r_2 with their constituent eligible road-paths.

Path	Departure Time	Travel time (min)
P1	1-20	18
	31-60	12
P4	1-25	22
	26-60	18
P5	1-35	19
	36-60	10
P6	1-30	18
	31-60	15
P9	1-30	12
	31-60	8
P11	1-30	23
	31-60	15

Table 3. Travel time for the road-paths participating in route-paths P_{r_1} . and P_{r_2} .

Expressions (1) provides a formal representation of route-paths, route-schedules and route-trajectories associated to route $r = \{i_0, i_1, .., i_\gamma, i_{n+1}\}$ where $i_1, .., i_\gamma$ are the customers included in the route.

$$P_r = \{(i_0, i_1, k_0), (i_1, i_2, k_1), \dots, (i_\gamma, i_{n+1}, k_\gamma)\}$$
$$S_{r\tau} = (\tau_{i_1}, \tau_{i_2}, \dots, \tau_{i_\gamma}) \tag{1}$$
$$R(P_r, S_{r\tau}) = \left\{\left((i_0, i_1, k_0), \tau\right), \left((i_1, i_2, k_1), \tau_{i_1}\right), \dots, \left((i_\gamma, i_{n+1}, k_\gamma), \tau_{i_\gamma}\right)\right\}$$

Note that any eligible path between consecutive stops can be represented by either its rank (e.g., k_3) in the relevant list of K eligible paths or its full notation (e.g., p_3).

It is evident that any solution of the proposed problem can be represented by a set of route-trajectories denoted by $\sigma \coloneqq \left\{R\left(P_{r_1}, S_{r_1\tau_1}\right), R\left(P_{r_2}, S_{r_2\tau_2}\right), .., R\left(P_{r_m}, S_{r_m\tau_m}\right)\right\}$, where τ_j denotes the departure time from the origin for route r_j. Below we define non-dominated route-trajectories. A key characteristic of a non-dominated solution of the proposed problem is that it consists of non-dominated route-trajectories.

Definition 4. Assume route r and route-path P_r of r. A route-trajectory $R(P_r, S_{r\tau})$ defined on route-path P_r is *non-dominated* if and only if there is no other route-trajectory $R'(P_r', S'_{r\tau'})$ of route r, that dominates it, i.e., $t(R'(P_r', S'_{r\tau'})) \leq t(R(P_r, S_{r\tau}))$, $f(R'(P_r', S'_{r\tau'})) \leq f(R(P_r, S_{r\tau}))$ and $\left(t\left(R(P_r, S_{r\tau})\right), f\left(R(P_r, S_{r\tau})\right)\right) \neq \left(t\left(R'(P_r', S'_{r\tau'})\right), f\left(R'(P_r', S'_{r\tau'})\right)\right)$, where $(t(\ ), f(\ ))$ denote the total travel time and fuel consumption of a route-trajectory.

Property 1: If solution $\sigma \coloneqq \left\{R_1(P_{r_1}, S_{r_1\tau_1}), R_2(P_{r_2}, S_{r_2\tau_2}), .., R_j\left(P_{r_j}, S_{r_j\tau_j}\right), .., R_m(P_{r_m}, S_{r_m\tau_m})\right\}$ of the BTL-VRPTW is non-dominated, then any of its constituent route-trajectories $R_j(P_{r_j}, S_{r_j\tau_j})$ for $j = 1, .., m$ of route r_j is non-dominated (among route-trajectories of the same route).

Proof. Suppose there exists another route-trajectory $R_j'\left(P_{r_j}', S'_{r_j\tau_j}\right)$ that dominates $R_j\left(P_{r_j}, S_{r_j\tau_j}\right)$, i.e.,

$f\left(R_j'\left(P_{r_j}', S'_{r_j\tau_j}\right)\right) \leq f\left(R_j\left(P_{r_j}, S_{r_j\tau_j}\right)\right)$,

$t\left(R_j'\left(P_{r_j}', S'_{r_j\tau_j}\right)\right) \leq t\left(R_j\left(P_{r_j}, S_{r_j\tau_j}\right)\right)$ and

$\left(t\left(R_j\left(P_{r_j}, S_{r_j\tau_j}\right)\right), f\left(R_j\left(P_{r_j}, S_{r_j\tau_j}\right)\right)\right) \neq \left(t\left(R_j'\left(P_{r_j}', S'_{r_j\tau_j}\right)\right), f\left(R_j'\left(P_{r_j}', S'_{r_j\tau_j}\right)\right)\right)$.

Assume solution $\sigma' \coloneqq \{R_1(P_{r_1}, S_{r_1\tau_1}), R_2(P_{r_2}, S_{r_2\tau_2}), .., R_j'\left(P_{r_j}', S'_{r_j\tau_j}\right), .., R_m(P_{r_m}, S_{r_m\tau_m})\}$. The fuel consumption $f(\ )$ and travel time $t(\ )$ of both solutions σ and σ' are computed as follows:

$$f(\sigma) = f\left(R_1(P_{r_1}, S_{r_1\tau_1})\right) + f\left(R_2(P_{r_2}, S_{r_2\tau_2})\right) + \cdots + f\left(R_j\left(P_{r_j}, S_{r_j\tau_j}\right)\right) + \cdots + f\left(R_m(P_{r_m}, S_{r_m\tau_m})\right)$$

$$f(\sigma') = f\left(R_1(P_{r_1}, S_{r_1\tau_1})\right) + f\left(R_2(P_{r_2}, S_{r_2\tau_2})\right) + \cdots + f\left(P_{r_j}', S'_{r_j\tau_j}\right) + \cdots + f\left(R_m(P_{r_m}, S_{r_m\tau_m})\right)$$

$$t(\sigma) = t\left(R_1(P_{r_1}, S_{r_1\tau_1})\right) + t\left(R_2(P_{r_2}, S_{r_2\tau_2})\right) + \cdots + t\left(R_j\left(P_{r_j}, S_{r_j\tau_j}\right)\right) + \cdots + t\left(R_m(P_{r_m}, S_{r_m\tau_m})\right)$$

$$t(\sigma') = t\left(R_1(P_{r_1}, S_{r_1\tau_1})\right) + t\left(R_2(P_{r_2}, S_{r_2\tau_2})\right) + \cdots + t\left(P_{r_j}', S'_{r_j\tau_j}\right) + \cdots + t\left(R_m(P_{r_m}, S_{r_m\tau_m})\right)$$

It is evident that $f(\sigma') \leq f(\sigma)$, $t(\sigma') \leq t(\sigma)$ and $\left(t(\sigma), f(\sigma)\right) \neq (t(\sigma'), f(\sigma))$, and thus, σ' dominates σ which contradicts the hypothesis that solution σ is non-dominated. ■

Property 1 implies that any route-trajectory included in a non-dominated solution, it is non-dominated (in the context of Definition 4) as well. Moreover, property 1 implies that if a route-trajectory is dominated, it will never be part of a non-dominated solution. Note however that a solution that comprise non-dominated route-trajectories, is not necessarily non-dominated. For instance, assume two non-dominated route-trajectories of a route r_1, $R(P_{r_1}, S_{r_1\tau_1})$, $R'\left(P_{r_1}', S'_{r_1\tau_1'}\right)$, with objective function values (100, 30), (90, 40) respectively and two non-dominated route-trajectories of route r_2, $R(P_{r_2}, S_{r_2\tau_2})$, $R'\left(P_{r_2}', S'_{r_2\tau_2'}\right)$, with objective function values (80, 70), and (70, 60) respectively. It is further assumed that combining the route-trajectories of these two routes one gets four alternative feasible solutions: σ_1 that comprise route trajectories ($R(P_{r_1}, S_{r_1\tau_1}), R(P_{r_2}, S_{r_2\tau_2})$), σ_2 with ($R(P_{r_1}, S_{r_1\tau_1}), R'\left(P_{r_2}', S'_{r_2\tau_2'}\right)$), σ_3 with ($R'\left(P_{r_1}', S'_{r_1\tau_1'}\right), R(P_{r_2}, S_{r_2\tau_2})$), and σ_4 with ($R'\left(P_{r_1}', S'_{r_1\tau_1'}\right), R'\left(P_{r_2}', S'_{r_2\tau_2'}\right)$). The corresponding objective function values for the four alternative solutions are then: (180, 100), (170, 90), (170, 110), and (160, 100) respectively. It is evident, that although solution σ_1 consists of non-dominated route-trajectories, eventually it is dominated by solution σ_2.

For the convenience of the reader, Table 4 presents the notation used in this paper.

Notation	Definition
Network Parameters	
$\boldsymbol{N}$	Set of nodes of the road network
$\boldsymbol{A}$	Set of arcs of the road network
$\boldsymbol{N_s}$	Set of stops of the problem (origin node, destination node, or customer location)
$\boldsymbol{A_s}$	Set of arcs connecting stops
$\boldsymbol{K}$	Number of eligible road paths between stops.
$\boldsymbol{i_0}$	Origin node
$\boldsymbol{i_{n+1}}$	Destination node
$\boldsymbol{p_{ij}^k}$	The kth alternative road path connecting stop i with stop j
$\boldsymbol{t_{ijk}^\tau}$	Travel time from stop i to stop j through path p_{ij}^k for departure time τ
$\boldsymbol{P(r)}$	Route-path associated to route r
$\wp_{ij}$	The set of K road paths connecting stops i and j
$\boldsymbol{t_p(\tau)}$	Travel time on path $p \in P_{i_q i_{q+1}}$ for departure time τ
$\boldsymbol{f_p(\tau)}$	fuel consumption on path $p \in P_{i_q i_{q+1}}$ for departure time τ
Vehicle Parameters	
$\boldsymbol{\mu_c}$	Curb weight of a vehicle.
$\boldsymbol{\Omega}$	Set of available vehicles
$\boldsymbol{Q}$	The capacity of any vehicle
$\boldsymbol{\kappa}$	Engine friction factor
$\boldsymbol{\Psi}$	Engine Speed
V	Engine Disposition
Phi($\boldsymbol{\phi}$)	Equivalence ratio
Ita ($\boldsymbol{\eta}$)	Engine Efficiency
$\boldsymbol{C_d}$	Aerodynamic drag co-efficient
$\boldsymbol{E}$	Frontal face area
Customers Parameters	
$\boldsymbol{d_i}$	Demand of customer i
$\boldsymbol{st_i}$	Service duration at customer i
$\mathbf{N_c}$	Set of stops representing the customer locations ($\mathrm{N_c} = \mathrm{N} \backslash \{\mathrm{i_0}, \mathrm{i_{n+1}}\}$)
$\boldsymbol{U}$	Set of unserviced customers
Route Parameters	
$\boldsymbol{r_j}$	Route j
$\boldsymbol{i_{vj}}$	The v^{th} stop of route j
$\boldsymbol{m_j}$	The number of customers in route j.
$\boldsymbol{R_j\left(\tau_{i_{0j}}\right)}$	Route trajectory of route j with departure time $\tau_{i_{0j}}$ from the origin.
Decision Variables	
$\boldsymbol{x_{ijkv}^\tau}$ $(\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}) \in \mathbf{A}$, $\boldsymbol{k} \in \{\mathbf{1},..,\boldsymbol{K}\}$, $\boldsymbol{v} \in \mathbf{\Omega}$	Binary variable that takes which take value 1 if vehicle v departs from node i at time τ heading to node j through path p_{ij}^k, and 0 otherwise
$\boldsymbol{y_i}$, $\boldsymbol{i} \in \boldsymbol{N_c}$	Non-negative variables expressing the departure time from customer i
$\boldsymbol{y_0^v}$, $\boldsymbol{v} \in \mathbf{\Omega}$	Non-negative variables expressing the departure time of vehicle v from the origin i_0.
$\boldsymbol{y_{n+1}^v}$, $\boldsymbol{v} \in \mathbf{\Omega}$	Non-negative variables expressing the arrival time of vehicle v at destination i_{n+1}
$\boldsymbol{w_{ijkv}^\tau}$ $(\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}) \in \mathbf{A}$, $\boldsymbol{k} \in \{\mathbf{1},..,\boldsymbol{K}\}$, $\boldsymbol{v} \in \mathbf{\Omega}$	non-negative variables expressing the load of vehicle v traversing road path p_{ij}^k departing from stop i at time τ
Routing and Scheduling	
$\boldsymbol{B_{i_q}^\tau}$	Bucket of non-dominated labels associated to node i_q and departure time τ
$(\boldsymbol{\lambda_{i_q}^t(\tau)}, \boldsymbol{\lambda_{i_q}^f(\tau)})$	label associated to node i_q at time τ, $\lambda_{i_q}^t(\tau)$ represents the total travel time value while $\lambda_{i_q}^f(\tau)$ the fuel consumption value.
$\boldsymbol{\tau_{i_q}^{arr}}$	Arrival time at node i_q
General Parameters	
θ	Road gradient

g	Gravity Acceleration
$\boldsymbol{\rho}$	Air Density (kg/m3)
$\boldsymbol{T}$	Discretised time horizon of the problem.

Table 4. Notation used in the paper.

3.4 Modeling Travel Time

Various travel time models have been proposed for time-dependent routing and scheduling problems (Malandraki and Daskin, 1992; Fleischmann et al., 2004; Horn, 2000; Ichoua et al., 2003; Haghani and Jung, 2005). In this work, the travel time on any path $p_{ij}^k \in \wp_{ij}$ between stops (i, j) is calculated on the basis of the model introduced by Horn (2000). A distinctive feature of the proposed travel time model is that average travel speed on any link of the road network is modeled as a continuous piecewise linear function of the time of the day. The continuity of this travel speed function signifies the capability of the model to provide a realistic representation of average travel speed. This feature plays also a significant role in the quality of the estimates for fuel consumption (presented later in this section) since travel speed is a basic parameter in the CMEM formula. Given that the relationship between fuel consumption and travel speed is non-linear (as shown below), the realistic representation of travel speeds is imperative in obtaining realistic estimation of fuel consumption. In what follows there is an exposition of how Horn's travel time model is incorporated in the BTL-VRPTW.

The calculation of travel time on $p_{ij}^k = \{(l(i), l_1), (l_1, l_2), , (l_{\eta-1}, l_\eta).., (l_v, l(j))\}$ for a given departure time τ (where $l(i)$ and $l(j)$ are the roadway nodes that host stops i and j respectively), involves summing up the travel times on the relevant constituent road links $(l_{\eta-1}, l_\eta)$. The proposed travel time model is based on the assumption that historical measurements of travel speed are available for any road link $(l_{\eta-1}, l_\eta)$ (e.g., from a traffic management center) at discrete times $T = \{\tau_0, \tau_1, \tau_2, \ldots, \tau_{|T|}\}$ (e.g., every 15 min). An estimate of the expected travel speed at time τ_κ denoted by u_κ^o is then estimated by the average of the corresponding historical speed observations for time τ_κ, resulting to the set $U^o = \{u_0^o, u_1^o, u_2^o, .., u_\kappa^o, .., u_{|T|}^o\}$. Based on Horn (2000), the travel speed function $u_{l_{\eta-1}, l_\eta}(t)$ on road link $(l_{\eta-1}, l_\eta)$ is defined for each time interval $[\tau_\kappa, \tau_{\kappa+1})$ by formula (2):

$$u_{l_{\eta-1}l_\eta}(t) := u_\kappa^o + a_{l_{\eta-1}l_\eta\kappa}(t - \tau_\kappa) \quad t\epsilon[\tau_\kappa, \tau_{\kappa+1}) \tag{2}$$

$$a_{l_{\eta-1}l_\eta\kappa} = \frac{u_{\kappa+1}^o - u_\kappa^o}{\tau_{\kappa+1} - \tau_\kappa} \tag{3}$$

where $a_{l_{\eta-1}l_\eta\kappa}$ is the average travel acceleration/deceleration rate on road link $(l_{\eta-1}, l_\eta)$ estimated by formula (3). Formula (2) is based on the implicit assumption that any vehicle is smoothly accelerating/ decelerating from u_κ^o at τ_κ to $u_{\kappa+1}^o$ at $\tau_{\kappa+1}$, with rate $a_{l_{\eta-1}l_\eta\kappa}$. The graph on the top of the Figure 4 presents the average travel speeds $u_\kappa^o \in U^o$ calculated based on historical speed observations for a given road link. The graph at the bottom of Figure 4 presents the graphical representation of formula (2) emerging from the average travel speed values u_κ^o. It is evident that the graphical representation of formula (2) is formed by the linear segments that connect the consecutive average travel speed observation points u_κ^o.

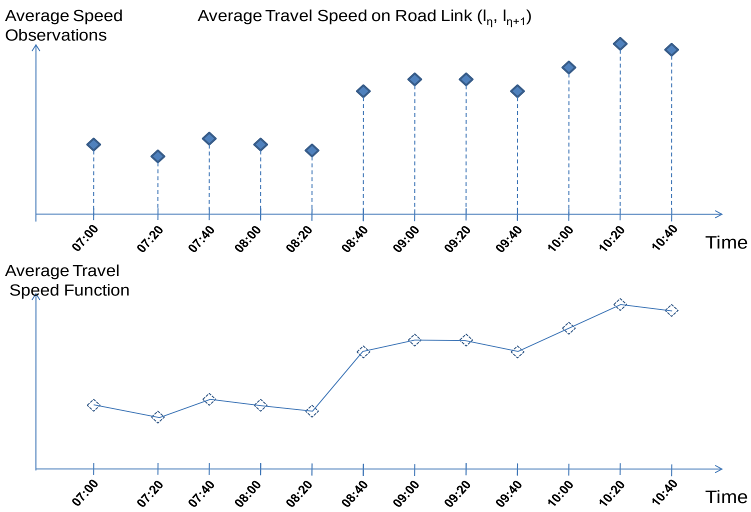


Figure 4. Graphical representation of the travel speed function of a road link.

The calculation of the travel time $t_{l_{\eta-1},l_\eta}(\tau)$ on road link $(l_{\eta-1}, l_\eta)$ for a given departure time τ from stop $l_{\eta-1}$ is performed by iteratively integrating formula (2) throughout any time interval elapsed until the node l_η is reached (Horn, 2000). An efficient calculation procedure for the link and the entire path travel time is proposed in Androutsopoulos and Zografos (2012).

Although, the proposed travel time model is computationally intensive as compared to other relevant models, it has various desirable features that fit with the travel time requirements of the proposed BTL-VRPTW. It satisfies the FIFO property (Horn, 2000) while it provides a smooth and realistic representation of travel speed changes due to traffic congestion or any other reason. Moreover, it takes into account the average acceleration/deceleration rate a_κ throughout any road path, which constitutes a significant contributor to the fuel consumption of a vehicle especially in the presence of congestion. The use of this travel time model facilitates the realistic modeling of the proposed routing problem. Thus, it is expected that the use of this travel time model tends to increase the likelihood of producing route schedules that will retain feasibility when they will be executed, i.e., customers' time window constraints will not be violated by the actual routes.

3.5 Modeling Fuel Consumption

The total fuel consumption (treated as a proxy for emissions) of a BTL-VRPTW solution is equal to the sum of the fuel consumption in each constituent route-trajectory. Given that a route trajectory consists of a sequence of scheduled road paths between consecutive stops of a route, its fuel consumption is equal to the sum of the relevant consumption over each of these scheduled road paths when traversed at the corresponding departure time. In this paper we incorporate the travel speed formulation of Horn (2000) in the CMEM formula and derive a closed formula for calculating the fuel consumption of a vehicle through a road link for a given departure time and payload. The fuel consumption over a scheduled road path sp_{ij} for departure time τ_d is given by formula (4):

$$\boldsymbol{F}_{ij}(\tau_d) \coloneqq \boldsymbol{\Gamma}_{ij}(\tau_d) + M \cdot \boldsymbol{Z}_{ij}(\tau_d) \tag{4}$$

where $\boldsymbol{\Gamma}_{ij}(\tau_d)$ and $\boldsymbol{Z}_{ij}(\tau_d)$ are time and path dependent parameters that can be pre-computed, and M is the total mass of the vehicle. Appendix I provides an exposition of how formula (4) emerges and how parameters $\boldsymbol{\Gamma}_{ij}(\tau_d)$ and $\boldsymbol{Z}_{ij}(\tau_d)$ can be computed.

3.6 Mathematical Formulation

Assume graph $G_s(N_s, A_s)$ where N_s is the set of stops of the problem, and A_s is the set of arcs. The set of customers is denoted by $N_c \subset N_s$. Note that under the proposed network reduction technique, each arc $(i, j, k) \in A_s$, for $i, j \epsilon N_s$, $k = 1, .., K$, represents a road path $p_{ij}^k \in \wp_{ij}$ among the K eligible road paths connecting stops i and j. Moreover, each arc (i, j, k) is associated with a travel time function $t_{ijk}(\tau)$ and fuel consumption parameters $\boldsymbol{\Gamma}_{ijk}(\tau)$ and $\boldsymbol{Z}_{ijk}(\tau)$ where $\tau \in T$ denotes the departure time from i (T denotes the discretised time horizon of the problem). The mathematical formulation of the proposed bi-objective time and load dependent vehicle routing and scheduling problem is based on three groups of variables:

- x_{ijkv}^{τ} $(i, j, k) \in$ A , $k \in K_{ij}$, $v \in \Omega$, binary variables which take value 1 if vehicle v departs from node i at time τ heading to node j through path p_{ij}^k, and 0 otherwise (K_{ij} is the set of indices of the alternative road paths connecting stop i to stop j)
- y_i $i \in N_c$ non-negative variables expressing the departure time from customer i
- y_0^v, $v \in \Omega$, non-negative variable expressing the departure time of vehicle v from the origin i_0
- y_{n+1}^v, $v \in \Omega$, non-negative variable expressing the arrival time of vehicle v at the destination i_{n+1}
- w_{ijkv}^{τ} $(i, j, k) \in$ A, $k \in K_{ij}$, $v \in \Omega$, non-negative variables expressing the load of vehicle v traversing road path p_{ij}^k departing from stop i at time τ.

A slight modification of the graph $G_s(N_s, A_s)$ is required in order to allow for determining the number of vehicles required through the proposed mathematical model. In particular, a fictitious arc $(i_0, i_{n+1}, 1)$ is added on the graph connecting the origin i_0 directly to the destination i_{n+1}. The travel time and the fuel consumption on this arc are set to 0. Any vehicle that is assigned the route from origin i_0 directly to the destination i_{n+1} is practically not used. The curb weight of the vehicle is denoted by μ_c . The mathematical formulation of the proposed problem is given by (5)-(21). Objective function (5) expresses the total duration of the routes of the solution. Objective function (6) expresses the corresponding total fuel consumption.

$$Min \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$F_1 = \sum_{v \in \Omega} (y_{i_{n+1}}^v - y_{i_0}^v) \tag{5}$$

$$F_2 = \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{i \in N_s} \sum_{j \in N_s} \sum_{v \in \Omega} \boldsymbol{\Gamma}_{ijk}(\tau) x_{ijkv}^{\tau} + \boldsymbol{Z}_{ijk}(\tau)(w_{ijkv}^{\tau} + \mu_c) \tag{6}$$

Routing Constraints

$$\sum_{\tau \in T} \sum_{v \in \Omega} \sum_{\mathrm{i} \in \mathrm{N} \setminus \{\mathrm{i}_{\mathrm{n}+1}\}} \sum_{k \in \{1,..,K\}} x_{ijkv}^{\tau} = 1, \quad j \in N_c \tag{7}$$

$$\sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{i \in N_s \setminus \{i_{n+1}\}} x_{ijkv}^{\tau} - \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{i \in N_s \setminus \{i_0\}} x_{jikv}^{\tau} = 0, \qquad j \in N_c, \qquad v \in \Omega \tag{8}$$

$$\sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{j \in N_s \setminus \{i_0\}} x_{i_0 jkv}^{\tau} = 1 \qquad v \in \Omega \tag{9}$$

$$\sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{j \in N_s \setminus \{i_{n+1}\}} x^{\tau}_{j i_{n+1} k v} = 1 \qquad v \in \Omega \tag{10}$$

Constraint (7) imposes that exactly one vehicle visits each customer at a single point in time. Constraint (8) implies that any vehicle v arriving at a customer, it has to leave the customer as well. Constraints (9) and (10) impose that each vehicle (v) should exit the origin and enter the destination exactly once. Note however that given the addition of the fictitious arc (i_0, i_{n+1}), imposing each vehicle to leave the origin does not necessarily mean that all vehicles must be used to service demand. Any vehicle not used for servicing demand will be assigned to use the arc $(i_0, i_{n+1}, 1)$.

Scheduling Constraints

$$y_i + \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{v \in \Omega} x^{\tau}_{ijkv} \left(t^{\tau}_{ijk} + st_j\right) - \left(1 - \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{v \in \Omega} x^{\tau}_{ijkv}\right) \mathcal{M} \leq y_j \quad ,(i,j,k) \in A_s \tag{11}$$

$$y^v_0 + \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} x^{\tau}_{i_0 jkv} \left(t^{\tau}_{i_0 jk} + st_j\right) - \left(1 - \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} x^{\tau}_{i_0 jkv}\right) \mathcal{M} \leq y_j \quad ,j \in N_c\,, v \in \Omega \tag{12}$$

$$y_j + \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} x^{\tau}_{j i_{n+1} kv} \left(t^{\tau}_{j i_{n+1} jk} + st_j\right) - \left(1 - \sum_{\tau \in T} \sum_{k \in \{1,..,K\}} x^{\tau}_{i_0 jkv}\right) \mathcal{M} \leq y^v_{n+1} \;, j \in N_c, v \in \Omega \tag{13}$$

$$a_i + st_i \leq y_i \quad \leq b_i + st_i \quad \forall\, i \in N_c, \tag{14}$$

$$a_0 \leq y^v_0 \leq b_0 \quad v \in \Omega, \tag{15}$$

$$a_{n+1} \leq y^v_{n+1} \leq b_{n+1} \quad v \in \Omega, \tag{16}$$

$$\sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{v \in \Omega} \sum_{j \in N_s \setminus \{i_o\}} (x^{\tau}_{ijkv} * \tau) = y_i \;, \qquad i \in N_c \tag{17}$$

$$\sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{j \in N_s \setminus \{i_o\}} (x^{\tau}_{i_o jkv} * \tau) = y^v_o \;, v \in \Omega \tag{18}$$

$$\sum_{\tau \in T} \sum_{k \in \{1,..,K\}} \sum_{j \in N_s \setminus \{i_{n+1}\}} x^{\tau}_{j i_{n+1} kv} * (\tau + t^{\tau}_{j i_{n+1} k}) = y^v_{n+1} \;, v \in \Omega \tag{19}$$

The left part of the inequality in constraint (11) expresses the arrival time of a vehicle at node j when it leaves node i heading to node j. Thus, constraint (11) states that the departure time from node j either coincides or it takes place later than the corresponding service finish time at that node. In constraint (11), $\mathcal{M}$ represents a large number. With no lack of generality $\mathcal{M}$ could be set equal to $|\Omega| b_{n+1}$. Constraint (12) implies that if j is the first customer visited in a route, then the departure time from j either coincides or it takes place later than the corresponding service finish time at that customer. Constraint (13) defines the arrival time at the destination. Constraint (14) expresses the service time windows constraints on customers while constraints (15) and (16) define the time window constraints on the origin and destination nodes respectively. Finally, constraints (17)-(19) ensure the compatibility between x and y variables.

Capacity Constraints

$$\sum_{\tau\in T}\sum_{k\in\{1,..,K\}}\sum_{i\in N_s\setminus\{i_{n+1}\}}\sum_{v\in\Omega} w_{ijkv}^{\tau} - \sum_{\tau\in T}\sum_{k\in\{1,..,K\}}\sum_{p\in N_s\setminus\{i_0\}}\sum_{v\in\Omega} w_{jpkv}^{\tau} \geq d_j \quad ,j\in C \tag{20}$$

$$x_{ijkv}^{\tau} Q \quad \geq w_{ijkv}^{\tau} \qquad i,j\in N_c,\ k\in\{1,..,K\},\ t\in T \tag{21}$$

Constraint (20) implies that the difference in the load of a vehicle before and after visiting a customer j is due to servicing the demand of the customer. Constraint (21) facilitates the definition of the load variables w_{ijkv}^{τ}, while it imposes that the maximum load carried by a vehicle should not exceed its capacity. In more detail, constraint (21) imposes w_{ijkv}^{τ} (the quantity on vehicle v transferred from node i to node j through path k departing at time τ) to be equal to 0 if arc (i,j,k) is not used by vehicle v at time τ. If, on the other hand, arc (i,j,k) is used by vehicle v at time τ, then the vehicle load transferred on that arc should not exceed the capacity of the vehicle.

4 DEVELOPING A GENERIC SOLUTION APPROACH

A significant feature of the BTL-VRPTW is that it encompasses three sub-problems: i) service routes determination, ii) specification of the route-path (i.e., the combination of road paths for travelling between consecutive stops) for each route, and iii) scheduling the traversal of the route-paths resulting to route-trajectories. The solution approach proposed for addressing the BTL-VRPTW involves the co-operation of two solution techniques: i) a heuristic method that deals with subproblem (i) creating capacity-feasible standard routes where each customer is visited exactly once, and ii) a routing and scheduling technique that addresses sub-problems (ii) and (iii) by transforming routes to feasible solutions of the BTL-VRPTW. This solution framework may incorporate any of the existing heuristic methods developed for the standard VRP in order to build routes. However, converting standard routes to feasible BTL-VRPTW solutions is not straightforward. In what follows, we show that transforming a set of capacity-feasible routes to BTL-VRPTW solutions implies a bi-objective time-dependent shortest path problem with time windows on a network with special structure. Moreover, we propose a backward label setting routine for solving the emerging shortest path problem. The implementation of this generic solution framework for the BTL-VRPTW is illustrated later in this paper.

4.1 Emerging Shortest Path Problem

We claim that building non-dominated solutions of the BTL-VRPTW associated to a given set of capacity-feasible routes pertains to a bi-objective time-dependent shortest path problem with time windows. Example 3 illustrates this issue.

Example 3. Assume two routes, $r_1 = \{O, 1, 2, D\}$ and $r_2 = \{O, 5, 3, 4, D\}$. Figure 5 presents the stops and the eligible road paths $(p_1 - p_{13})$ between each pair of consecutive stops of r_1 and r_2. Routes r_1 and r_2 can be represented by the graph at the bottom of Figure 5 that is produced through the following rules: i) each stop of a route is represented by a node (the origin and destination are the same for both routes and thus they are denoted by O and D for route r_1 and O' and D' for route r_2 in order to avoid confusion), ii) each eligible road path between consecutive stops is represented by an arc, iii) the destination node of route r_1 is connected to the origin node of route r_2 with a fictitious arc, and iv) each arc is associated to the travel time and fuel consumption functions of the corresponding road paths. The nodes of the graph that represent customers are also associated with the corresponding customers' service time windows, while the nodes that represent the origin or destination of a route are associated to the corresponding facilities time windows (opening/closing times). It is worth noting that the fictitious arc from D to O' does not represent any actual movement between these two nodes and therefore the relevant travel time and fuel consumption values are set to zero. The emerging graph is named *routes-graph*. By definition, any solution of the BTL-VRPTW problem associated to routes r_1 and r_2 comprise two route-trajectories associated to routes r_1 and r_2 respectively. Moreover, any route trajectory of route r_1 corresponds to a scheduled road path from O to D on the routes-graph of Figure 5. Similarly, any route trajectory of route r_2 corresponds to a scheduled road path from O' to D'. Thus, any solution of the BTL-VRPTW problem corresponds to a scheduled road path on the routes-graph, from node O to node D'. It is evident that the fictitious link (D, O') is only used to facilitate the representation of a BTL-VRPTW solution by a single scheduled road path defined on the relevant routes-graph. Therefore, determining non-dominated BTL-VRPTW solutions associated to these two routes is equivalent to solving the bi-objective time dependent shortest path problem with time windows on the routes-graph presented in Figure 5.

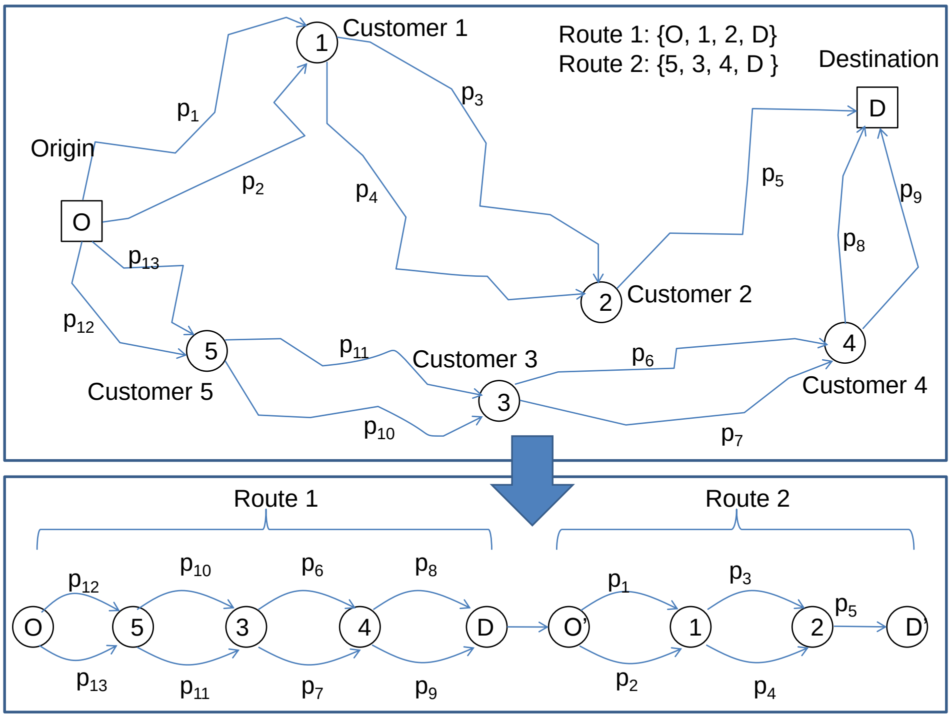


Figure 5. Transformation of routes r_1 and r_2 to a mathematical graph (routes-graph).

We now generalize the findings of Example 3. Assume a set of routes $r_1, r_2, \ldots, r_{q-1}, r_q$ each one represented by a sequence of stops as indicated in (22). To distinguish between stops of different routes, the ξ^{th} stop of route r_j is denoted by $i_{\xi j}$. Note that route r_j includes m_j customers while i_{0j} and $i_{(m_j+1)j}$ denote the origin and destination nodes.

$$r_j = \{i_{0j}, i_{1j}, i_{2j}, \ldots, i_{\xi j}, i_{(\xi+1)j}, \ldots, i_{m_j j}, i_{(m_j+1)j}\},\ j = 1, \ldots, q \tag{22}$$

The relevant routes-graph (Figure 6) is formed by putting the routes in an arbitrary order and placing the stops of the routes in a single series keeping the order that they are visited. The routes-graph emerges directly by representing the route stops by a series of nodes and the eligible road paths between consecutive stops by arcs. Figure 6 presents the routes-graph that emerges from routes $r_1, r_2, \ldots, r_{q-1}, r_q$ when placed in the reverse order of their numbering (i.e., the q^{th} route is considered 1st, the $(q-1)^{th}$ is 2nd etc.). We claim that any solution of the BTL-VRPTW is represented through a scheduled road path on routes-graph of Figure 6 from node i_{0q} to node $i_{(m_1+1)1}$ and vice versa. By definition, any solution σ of the BTL-VRPTW that corresponds to these routes may be represented by (23) as a list of route-trajectories. Expression (24) provides the representation of route-trajectory $R_j\left(P_{r_j}, S_{r_j\tau_j}\right)$ defined over route r_j following the notation of (1). It is worth noting that, τ_j is the departure time of the route-trajectory associated to route r_j.

$$\sigma = \{R_1(P_{r_1}, S_{r_1\tau_1}), \ldots, R_j\left(P_{r_j}, S_{r_j\tau_j}\right), \ldots, R_{q-1}\left(P_{r_{q-1}}, S_{r_{q-1}\tau_{q-1}}\right), R_q\left(P_{r_q}, S_{r_q\tau_q}\right)\} \tag{23}$$

Where,

$$\begin{aligned} &P_{r_j} = \left\{\left(i_{0j}, i_{1j}, p^{k_0}_{i_{0j}i_{1j}}\right), \left(i_{1j}, i_{2j}, p^{k_1}_{i_{1j}i_{2j}}\right), \ldots, \left(i_{m_j j}, i_{(m_j+1)j}, p^{k_{m_j}}_{i_{m_j j}i_{(n+1)j}}\right)\right\} \\ &S_{r_j\tau_j} = \left(\tau_{i_{1j}}, \tau_{i_{2j}} \ldots, \tau_{i_{m_j j}}\right) \\ &R_j\left(P_{r_j}, S_{r_j\tau_j}\right) = \left\{\left(\left(i_{0j}, i_{1j}, p^{k_0}_{i_{0j}i_{1j}}\right), \tau_j\right), \left(\left(i_{1j}, i_{2j}, p^{k_1}_{i_{1j}i_{2j}}\right), \tau_{i_{1j}}\right), \ldots, \left(\left(i_{m_j j}, i_{(m_j+1)j}, p^{k_{m_j}}_{i_{m_j j}i_{(n+1)j}}\right), \tau_{i_{m_j j}}\right)\right\} \\ &j = 1, \ldots, q \end{aligned} \tag{24}$$

It is evident that each segment of a route-trajectory $\left(\left(i_{\xi j}, i_{\xi+1 j}, p^{k_v}_{i_{\xi j}i_{v+1 j}}\right), \tau_{i_{\xi j}}\right)$ corresponds to arc $(i_{\xi j}, i_{\xi+1 j}, k_v)$ of the routes-graph enhanced with departure time $\tau_{i_{\xi j}}$. Given that a scheduled road path in the routes-graph is a chain of arcs enhanced with the departure time from the tail node of each arc, the one to one correspondence between BTL-VRPTW solutions σ and scheduled road paths on the routes-graph of Figure 6 is straight forward. The sequence of routes we use in order build the routes-graph does not affect this correspondence between BTL-VRPTW solutions and the scheduled road paths the corresponding routes-graph.

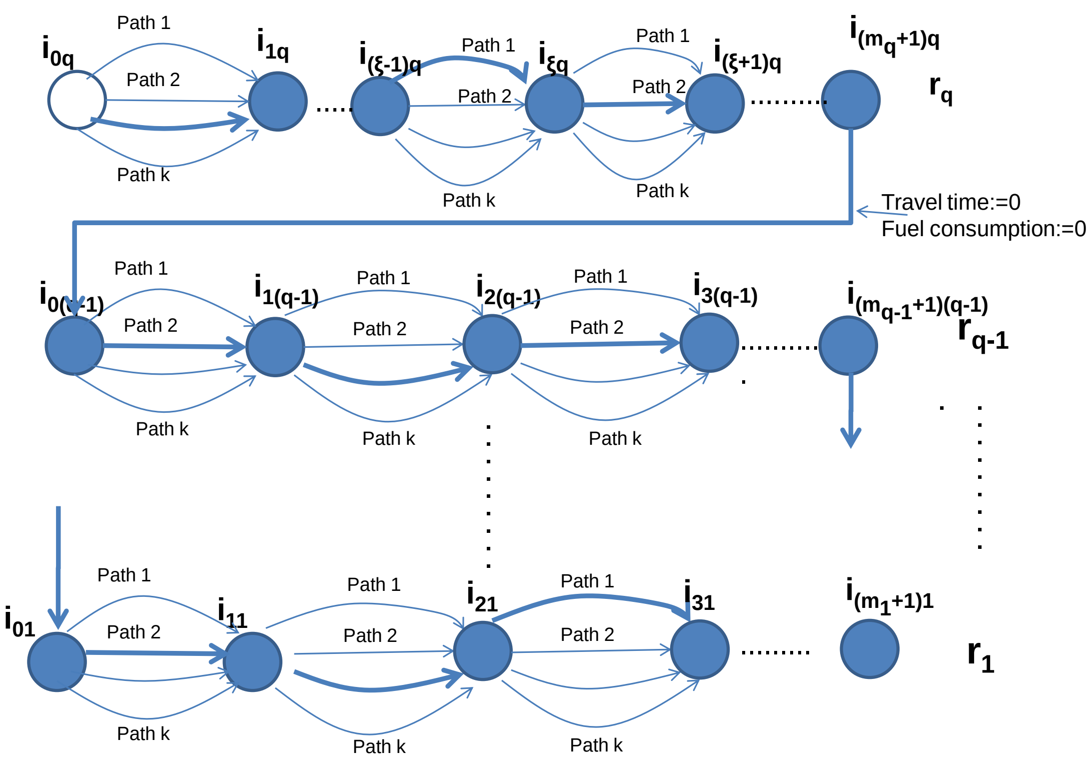


Figure 6. Routes-graph associated to routes $\{r_1, r_2, \ldots, r_q\}$.

It is worth noting that the arrival time at each node is constrained from the service time window of the corresponding stop. Hence, the determination of the non-dominated solutions associated to the given set of routes involves a bi-objective time-dependent shortest path problem with time windows on the routes-graph depicted in Figure 6.

4.2 Routing and Scheduling Method

We provide a routing and scheduling method that generates non-dominated BTL-VRPTW solutions associated to a given set of capacity-feasible routes. As already stated, generating feasible solutions for the BTL-VRPTW associated to a set of capacity-feasible routes can be accomplished by solving a bi-objective time-dependent shortest path problem with time windows on the corresponding routes-graph. The generic version of the emerging time-dependent shortest path problem has been studied by Hamacher et al. (2006). Below we customize the findings of that study in the context of the emerging bi-objective time-dependent shortest path problem. A key finding in (Hamacher et al., 2006) is that if a scheduled road path (i.e., a path enhanced with the arrival and departure times at each constituent node) is non-dominated then any of its scheduled sub-paths from an intermediate node to the destination node, is non-dominated as well. Moreover, in the same work, it was proven that this result is not necessarily true for the scheduled sub-paths starting from the origin and reaching an intermediate node of the scheduled path. Based on these properties, Hamacher at al. (2006) provide a backward dynamic programming algorithm that solves the multi-objective time-dependent shortest path problem.

Transferring these results for the proposed bi-objective time-dependent shortest path problem defined on a routes-graph, we argue that if a scheduled road path in the routes-graph (Figure 6) from node $i_{\xi j}$ to the destination node of the problem $i_{(m_1+1)1}$ is non-dominated for departure time $\tau_{i_{\xi j}}$ then any of its scheduled subpaths from any constituent node $i_{\xi' j'}$ to destination node $i_{m_1 1}$ with departure time $\tau_{i_{\xi' j'}}$ is non-dominated as well (for the specific departure time $\tau_{i_{\xi' j'}}$). On the other hand it cannot be guaranteed that any scheduled sub-path from node $i_{\xi j}$ to any intermediate node $i_{\xi' j'}$ departing at time $\tau_{i_{\xi j}}$ is non-dominated for time $\tau_{i_{\xi j}}$. This finding implies that if the non-dominated scheduled road paths from node $i_{\xi j}$ to destination $i_{(m_1+1)1}$ are known for any possible departure time from $i_{\xi j}$, then one could compute the non-dominated scheduled road paths from node $i_{(\xi-1)j}$ to destination by simply extending these scheduled road paths backwards (Hamacher et al., 2006). Moreover, given the structure of the underlying routes-graph which is formed by a chain of nodes (while arcs exist only between consecutive nodes), the emerging bi-objective time-dependent shortest path problem with time windows defined on the routes-graph can be decomposed to a series of nested bi-objective time-dependent shortest path problems between pairs of consecutive nodes starting from the last pair and moving backwards to the origin node of the routes-graph.

A label setting routine is iteratively applied to extend any non-dominated scheduled road path starting from node $i_{\xi j}$ in the backward direction. To facilitate the comprehensive presentation of the algorithm we simplify the notation of the nodes of routes-graph in Figure 6 by renumbering them from 1 to Λ in the sequence that they appear on the routes-graph. Thus node 1 corresponds to node i_{0q} and node Λ to $i_{m_1 1}$. Based on this renumbering of the nodes, the i^{th} iteration of the algorithm calculates the non-dominated scheduled road paths for each possible departure time from node $(\Lambda - i + 1)$ to node Λ, passing through nodes $\Lambda - i + 1$, $\Lambda - i + 2, .., \Lambda - 1$. The algorithm starts with node Λ and moves backwards in the graph until it reaches node 1.

The proposed algorithm works with labels. Each label $\underline{\boldsymbol{\lambda}}_{li}(\tau)$ $=(\lambda_{li}^{t}(\tau), \lambda_{li}^{f}(\tau))$ is associated to node i of the routes-graph and departure time τ and corresponds to scheduled road path from node i to node Λ (recall that node Λ corresponds to destination node $i_{(m_1+1)1}$) departing at time τ. $\lambda_{li}^{t}(\tau)$ represents the travel time and $\lambda_{li}^{f}(\tau)$ the fuel consumption of the corresponding scheduled road path. Each label is also associated to a pair of pointers $(p_{li}^{1}(\tau), p_{li}^{2}(\tau))$ where $p_{li}^{1}(\tau)$that points to the arc (i,j,k) that succeeds node i in the corresponding scheduled road path and $p_{li}^{2}(\tau)$ points to the corresponding label at node j. These pointers are used in order to enable backtracking for any scheduled road path. All non-dominated labels associated to node i and departure time τ are kept in a bucket of labels denoted by B_i^{τ}. The list of buckets B_i^{τ}, for all possible departure times τ from node i, is denoted by Ξ_i. It is worth noting that Ξ_i contains buckets B_i^{τ} for values of τ in the set of discrete times between the earliest possible departure time from i, i.e., $a_i + st_i$, and the latest possible departure time from i, i.e., $b_i + st_i$.

The first iteration of the algorithm creates a single label $(\lambda_{1\Lambda}^{t}(\tau), \lambda_{1\Lambda}^{f}(\tau))$ for each possible arrival time τ at node Λ and adds it to the corresponding bucket B_{Λ}^{τ}. Both elements of each new label are set to zero. For convenience, we say that a node is "checked" when the non-dominated labels of this node have been calculated. By the end of the first iteration node Λ has been checked. Hence, each of the remaining iterations of the algorithm take the most recently checked node i and computes the non-dominated labels for the preceding node $i-1$. The computation of the labels for the preceding node $i-1$ involves the use of the labels found for node i, which in practice is equivalent to the backward extension of the non-dominated scheduled road paths determined for node i.

We now provide an exposition of an indicative iteration of the proposed label setting routine. Given the list of buckets of labels Ξ_i (computed from the previous iteration) for node i, the algorithm determines the non-dominated labels for the preceding node $i-1$ for each possible departure time between $a_{i-1} + st_{i-1}$ and $b_{i-1} + st_{i-1}$. After the execution of this iteration, node $i-1$ becomes checked. In what follows we present the computations performed in a single iteration of the label setting routine in which the non-dominated labels associated to node $i-1$ are determined. For each possible departure time τ, starting from the last possible departure time $b_{i-1} + st_{i-1}$ and moving backwards in time upto time $a_{i-1} + st_{i-1}$, the following steps are performed:

(i) If $B_{i-1}^{\tau+1} \neq \emptyset$, (i.e., there exists at least one label for time $\tau+1$ at node $i-1$), then for each existing label $(\lambda_{li-1}^{t}(\tau+1), \lambda_{li-1}^{f}(\tau+1)) \in B_{i-1}^{\tau+1}$, a new label $(\lambda_{li-1}^{t}(\tau), \lambda_{li-1}^{f}(\tau))$ is created for node $i-1$, where $\lambda_{li-1}^{t}(\tau) \coloneqq \lambda_{li-1}^{t}(\tau+1) + 1$, and $\lambda_{li-1}^{f}(\tau) \coloneqq \lambda_{li-1}^{f}(\tau+1)$. Each new label created in this step corresponds to a scheduled road path that emerges if one time unit of waiting time at node $i-1$ is added on the existing scheduled road path associated to $(\lambda_{li-1}^{t}(\tau+1), \lambda_{li-1}^{f}(\tau+1))$. Each new label is inserted in bucket B_{i-1}^{τ}. Finally, the relevant pointers are set as follows $p_{li-1}^{1}(\tau) \coloneqq p_{li-1}^{1}(\tau+1)$, and $p_{li-1}^{2}(\tau) \coloneqq p_{li-1}^{2}(\tau+1)$.

(ii) For each arc $(i-1, i, \delta)$ (representing the δ^{th} eligible road path between stops $(i-1)$ with i, where $\delta = 1,..,K$), the following steps are performed:
   a. Estimation of the arrival time $(\tau_i^{a}(\delta))$ at node i through $(i-1, i, \delta)$ and departure time τ from node $i-1$. If $\tau_i^{a}(\delta) \leq b_i$ then move to (b).
   b. A set of new labels are calculated for node $i-1$ as follows:
      - Determine the ready (for departure) time $\tau' = \max\{\tau_i^{a}(\delta), a_i\} + st_i$ at node i if the truck arrives at that node at time $\tau_i^{a}(\delta)$ found in (a).
      - For each label (e.g., the h^{th}label) $(\lambda_{hi}^{t}(\tau'), \lambda_{hi}^{f}(\tau'))$ in bucket $B_i^{\tau'}$ of node i, a new label $(\lambda_{l(i-1)}^{t}(\tau), \lambda_{l(i-1)}^{f}(\tau))$ is calculated for node $i-1$ and departure time τ as follows:

$$\lambda_{l(i-1)}^{t}(\tau) \coloneqq \begin{cases} \lambda_{hi}^{t}(\tau') + t_{(i-1)i\delta}(\tau) + st_i, & if\ \tau_i^{a}(\delta) \in [a_i, b_i] \\ \lambda_{hi}^{t}(\tau') + t_{(i-1)i\delta}(\tau) + st_i + (a_i - \tau_i^{a}(\delta)), & if\ \tau_i^{a}(\delta) < a_i \end{cases} \tag{25}$$

$$\lambda_{l(i-1)}^{f}(\tau) \coloneqq \lambda_{hi}^{f}(\tau') + f_{(i-1)i\delta}(\tau) \quad (26)$$

where $t_{(i-1)i\delta}(\tau)$ and $f_{(i-1)i\delta}(\tau)$ are the travel time and fuel consumption values for traversing arc $(i-1, i, \delta)$ at time τ. Each label constructed for customer $(i-1)$ at departure time τ is inserted in list $B_{(i-1)}^{\tau}$ .Moreover, $p_{li-1}^{1}(\tau)$ points to $(i-1, i, \delta)$ and $p_{li-1}^{2}(\tau)$ points to $(\lambda_{hi}^{t}(\tau'), \lambda_{hi}^{f}(\tau'))$.

It should be clarified that any bucket B_i^{τ} used in this routine retains only the non-dominated labels. Thus, whenever a new label is found and added to a bucket, it is compared with the existing ones in order to discard any dominated labels. At the end of the step described above a separate list of non-dominated labels is specified for node $(i-1)$ and each possible departure time τ. The algorithm is terminated when the non-dominated labels for node $i = 1$ (which represents the first node of the routes-graph) for each possible departure time are computed. The labels identified correspond to the non-dominated scheduled road paths of the bi-objective time-dependent shortest path problem defined on the routes-graph. The transformation of each of these paths to a solution to the proposed BTL-VRPTW is straightforward. Putting all these labels in buckets B_i^{τ} in a single list and removing any dominated labels leads to the set of non-dominated solutions (irrespective to departure time) of the problem associated to the given set of routes. It is worth noting that if any step of the above routine returns empty buckets, it can be deduced that no feasible solution exists for the specific set of routes.

5 IMPLEMENTATION OF THE SOLUTION FRAMEWORK

The proposed generic solution framework has been implemented by combining the Ant Colony System (ACS) technique (Dorigo and Stutzle, 2004) for building capacity feasible routes with the routing and scheduling method presented in the previous section. However, instead of applying these two techniques in two different stages, we incorporate the routing and scheduling method into the ACS.

The ACS technique has been previously used to solve vehicle routing problems (Donati et al., 2008; Gambardella et al., 1999). In single objective vehicle routing problems, an ACS algorithm iteration uses a set of agents to build a number of solutions. Each agent determines a single solution by applying a semi-randomized route construction routine. Routes are built sequentially by iteratively selecting and inserting a new un-serviced customer at the end of the route under construction. A customer is selected either probabilistically or on the basis of an insertion metric. The insertion process is iterated until all customers have been included in a route. The insertion metric and the probability distribution used for selecting a customer, incorporate a dynamic arc attribute called pheromone. The pheromone value of an arc of the complete graph of the vehicle routing problem expresses the desirability of using the specific arc within a solution. When all agents terminate, the identified solutions are compared with the currently best solution on the basis of the objective function of the problem. Whenever a new solution outperforms the currently best solution, its objective function value is used to update (increase) the pheromone value of the arcs included in that solution. In order to avoid being trapped in a local minimum, the ACS technique incorporates a pheromone evaporation procedure within the route construction routine, where the pheromone of an arc is decreased whenever that arc is selected by an agent as part of a solution. The evaporation procedure aims to restrain the agents from using the same arcs in the problem solutions (and thus avoiding being trapped in a local optimum).

In this work, each agent of the ACS technique builds routes sequentially by iteratively adding a new stop at the start of the route under construction. Whenever a candidate stop is considered for insertion in the route under construction, the routing and scheduling method is called to calculate the emerging non-dominated partial solutions traversing the current partial route under construction and any other route already formed. The value of the insertion metric for the candidate customer is calculated based on the travel time and fuel consumption performance of the emerging partial solutions and the relevant pheromone values. Hence, upon termination of the operations of an agent, apart from the routes, a set of non-dominated BTL-VRPTW solutions (corresponding to the formed routes) are also readily available. We now present the details of the emerging hybrid algorithm and an assessment of the quality of solutions determined for the BTL-VRPTW.

5.1 Solution Algorithm Outline

Given graph $G_s(N_s, A_s)$ the objective of the proposed algorithm is to determine a set of non-dominated solutions of the relevant BTL-VRPTW. It is worth noting that in addition to travel time and fuel consumption functions, each arc (i, j, k) (representing the kth eligible road path connecting stops i and j) is associated to a pheromone value denoted by φ_{ijk}^{τ} for each possible departure time τ. The pheromone value is dynamic (it changes throughout the algorithm execution) and expresses a measure of desirability of using that arc for the specific departure time.

The emerging hybrid algorithm inherits the core structure from the ACS technique and performs ϑ iterations by utilizing ζ agents (both ϑ and ζ are specified in advance). Each of the ζ agents executes a sequential route construction routine which apart from building routes, determines the non-dominated partial solutions that traverse the route under construction and any other routes already formed by the agent. Upon termination of each agent routine, the solutions identified are inserted in a list of non-dominated solutions Υ. The list is updated accordingly by removing any dominated solutions. The pheromone trail of the new members in Υ is then updated (increased). Figure 7 provides the major steps of the emerging algorithm in the form of a high-level logical diagram.

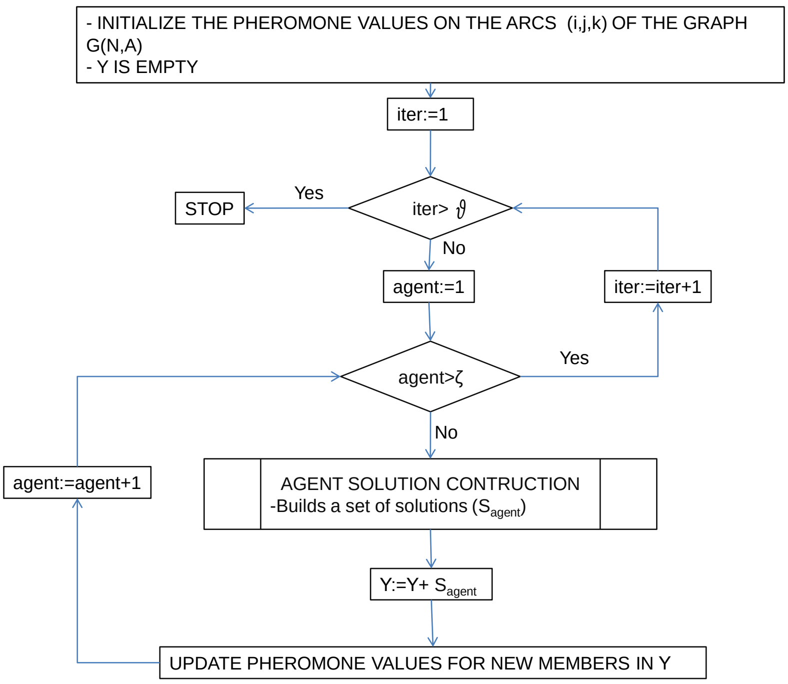


Figure 7. Logical diagram of the hybrid algorithm.

In a nutshell, the proposed hybrid algorithm differs from the standard ACS technique for vehicle routing problems in the following aspects:

(i) The proposed hybrid algorithm involves a route construction heuristic which in addition to iteratively inserting a new customer in the route under construction, computes the non-dominated partial solutions of the problem associated to the currently formed routes.
(ii) For reasons of computational efficiency explained later in this section, the route construction heuristic routine iteratively selects an unserviced customer and places it at the first position of the partial route under construction rather than the last position (i.e., between the currently last customer and the destination) as in standard implementations (Donati et al., 2008; Gambardella et al., 1999).

In the remainder of this section we elaborate on the major components of the proposed hybrid heuristic algorithm including the route construction heuristic used by agents, placing special emphasis on how the routing and scheduling method is incorporated in it.

5.2 Construction Heuristic Routine

The proposed construction heuristic builds routes sequentially while in parallel it transforms the emerging routes to partial BTL-VRPTW solutions. Hence, the output of the heuristic includes a set of capacity feasible routes and a set of BTL-VRPTW solutions associated to these routes. Each iteration of the heuristic involves the execution of the following operations: i) temporary insertion of each candidate customer in the first position of the route under construction and determination the non-dominated partial BTL-VRPTW solutions that traverse the emerging route under construction and any other route formed so far, ii) calculation of the insertion metric for each candidate customer based on the travel time and fuel consumption of the partial solutions determined in (i), and iii) selection and insertion of one of the candidate customers in the first position of the route under construction. The intuition behind this construction technique is to secure that the capacity-feasible routes which are built, may also provide temporal-feasible BTL-VRPTW solutions. In this way the agent avoids the situation of building capacity feasible routes for which no BTL-VRPTW solutions exist.

In more detail, the construction heuristic routine starts by initializing a new route. This is achieved by inserting the destination stop into the empty route. In a generic iteration of the construction heuristic it is assumed that routes $r_1, r_2, \ldots, r_{q-1}$ have been constructed and partial route $r_q \coloneqq \{i_{1q}, i_{2q}, .., i_{vq},\ i_{(m_q+1)q}\}$ is under construction. If there are still customers left unserviced, then each of them (denoted by i_u) is temporarily placed in the first position of the route under construction, i.e., right before stop i_{1q}. Selecting one of them for insertion is performed in two stages: i) for each candidate customer i_u we compute the non-dominated partial solutions that correspond to the currently formed routes and the route under construction enhanced with i_u in the first position of the route, and ii) selection of the candidate customer based on a semi-randomized procedure. A random number μ is drawn within range [0,1]. If $\mu < \mu_0$ (where μ_0 is a fixed threshold value within range (0,1)) then an insertion cost metric is calculated for each candidate customer i_u and the customer with the maximum value is selected for insertion. Otherwise the selection of the next customer is performed probabilistically. Given the non-dominated partial solutions calculated for candidate stop i_u in the route under construction and for each possible departure time τ, the corresponding insertion metric value for stop i_u is given by (27):

$$m(i_u) \coloneqq \max_{\tau,h}\left\{\frac{\varphi^{\tau}_{i_u i_{1q}\kappa_h}}{[\lambda^t_{hi_u}(\tau)]^{\delta_1}\cdot[\lambda^f_{hi_u}(\tau)]^{\delta_2}}\right\} \tag{27}$$

where κ_h corresponds to the path $p^{\kappa_h}_{i_u i_{1q}}$ between i_u and i_{1q} in the h^{th} non-dominated partial solution with departure time τ, $\lambda^t_{hi_u}(\tau)$ and $\lambda^f_{hi_u}(\tau)$ denote the travel time and fuel consumption values of the h^{th} non-dominated partial solution for departure time τ. $\varphi^{\tau}_{i_u i_{1q}\kappa_h}$ denotes the pheromone of the arc (i_u, i_{1q}, κ_h) at departure time (τ). The parameters δ_1 and δ_2 are used as scaling factors in order to bring both travel time and fuel consumption values to a common scale. Customer i^* with the maximum metric value (28) is selected to be inserted in the existing partial route.

$i^* :=\text{argmax}\{m(i_u): i_u \in U\}$ (28)

where U is the set of un-serviced customers that can be feasibly inserted in the route under construction. Thus, the metric for the selection of an un-serviced customer in the current route favors customers for which the product of total travel time and fuel consumption of the emerging partial solutions from node i_u is low and the corresponding pheromone value is high. If however the selection of the next customer is performed probabilistically, the probability distribution defined by (29) is used.

$$p(i_u) := \frac{m(i_u)}{\sum_i m(i_u)} \quad (29)$$

Upon selecting a new customer, the route is updated accordingly. It is worth noting that the non-dominated partial solutions corresponding to the emerging set of routes have been already calculated (for the sake of the computation of the insertion metric during the selection process) and thus they are readily available.

Finally, a typical iteration of the proposed construction heuristic concludes with updating (evaporating) the pheromone trail on arcs (i^*, i_{1q}, δ) and departure time τ that define a segment of at least one non-dominated partial solution from node i^* according to formula (30).

$$\varphi^\tau(i^*, i_{1q}, \delta) := (1-\rho)\cdot \varphi^\tau(i^*, i_{1q}, \delta) + \rho\varphi_0 \quad (30)$$

where $\rho \in (0,1)$ is the evaporation co-efficient and φ_0 is the initial pheromone value.

The construction of a route is terminated if at least one of the following conditions holds: i) the list of un-serviced customers U has become empty, or ii) none of the remaining un-routed customers can be feasibly inserted in the 1st position of the route. When the construction of a route is terminated, the following actions are performed: i) insertion of the origin node i_{0q} at the front part of the route r_q and ii) calculation of the non-dominated partial solutions for any possible departure time from the origin i_{0q} with the use of the routing and scheduling method. If the list of un-serviced customers is not empty, then a new route is initialized. The overall insertion routine terminates when all customers have been included in a route. The termination of the routine results to a number of lists of solutions of the BTL-VRPTW, each one corresponding to a different possible departure time τ. The solutions within each list are non-dominated for the corresponding departure time τ.

5.3 Computing Partial Solutions

A basic task performed by the construction heuristic is to determine the non-dominated partial solutions associated to the currently formed routes. As already discussed, the computation of the non-dominated solutions of BTL-VRPTW associated to a given set of routes can be accomplished by solving a bi-objective time-dependent shortest path problem with time windows defined on the corresponding routes-graph through the proposed label setting routine. We show how the proposed label setting routine can be efficiently integrated in the construction heuristic.

The example that follows replicates a typical (temporary) insertion of an un-serviced customer in the route under construction and clarifies how the partial solutions associated to the emerging routes can be efficiently determined. Assume two routes, namely route 1 and 2 presented at the top of Figure 8. Route 1 includes stops $\{O, 1, 2, D\}$ and it is considered complete while route 2 includes stops $\{3, 4, D\}$ and it is still under construction. Note that O denotes the origin and D denotes the destination stop. The eligible road paths between the consecutive stops within these two routes are denoted by p_1-p_9. The lower part of Figure 8 presents the routes-graph that emerges from routes 1 and 2. It is worth noting that the destination node of route 2 is denoted by D' in the routes-graph in order to avoid confusion with the node representing the destination of route 1. However, both nodes represent the same stop. Solving the bi-objective time-dependent shortest path problem with time windows defined on the routes-graph from node 3 to node *D* for any possible departure time from node 3 leads to a set of non-dominated scheduled road paths. As already discussed in this paper, the emerging scheduled road paths constitute the partial solutions of the BTL-VRPTW that are associated to routes 1 and 2.

We now illustrate the path finding computations required in order to insert stop (customer) 5 at the first position of route 2. If customer 5 is temporarily placed in the front of route 2, then determining the partial solutions corresponding to routes 1 and 2, involves a new bi-objective time-dependent shortest path problem with time windows defined on the network illustrated in Figure 9. The new routes-graph in Figure 9 is derived from the routes-graph of Figure 8 by adding node 5 in front of node 3. Hence, the partial solutions associated to updated route 2 and route 1 involves solving the bi-objective time-dependent shortest path problem on the graph of Figure 9 from node 5 to node *D* for any possible departure time.

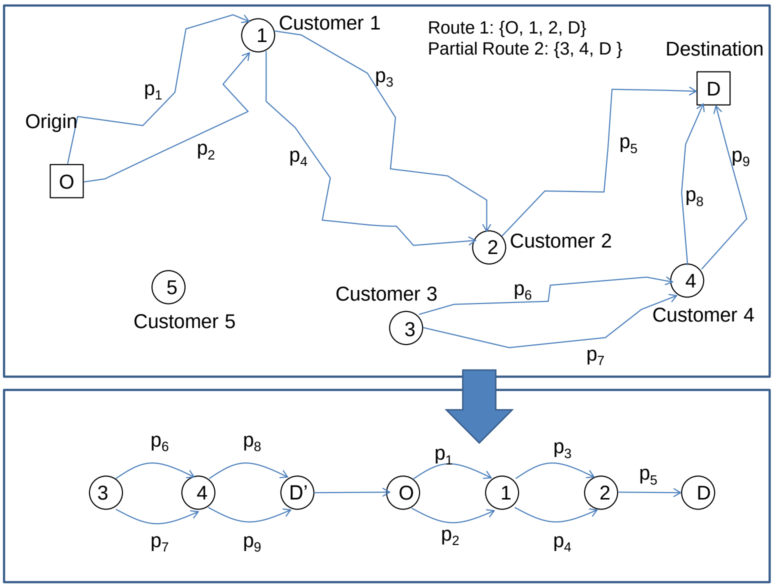


Figure 8. Transformation of route 1 and partial route 2 to a routes-graph.

The emerging shortest path problem can be solved by applying the label setting routine presented in the previous section. However, the observations that follow lead to a substantial simplification of this method:

- The non-dominated partial solutions (i.e., scheduled road paths on the relevant routes-graph) already identified for node 3 are still valid when the insertion of node 5 is considered in the first position of route 2. The additional load on the truck (weight of customer 3 order) when traversing a path from node 5 to node 3 is unloaded upon arrival at node 3 and thus any fuel consumption computations performed after node 3 remain valid.
- Given that the non-dominated partial solutions from node 3 to node *D* for any possible departure time are available, the partial solutions (scheduled road paths) from node 5 to node *D* for any possible departure time may be computed by simply extending backwards the partial solutions (scheduled road paths) from node 3. This can be achieved by performing a single iteration of the backward label setting routine presented in the previous section.

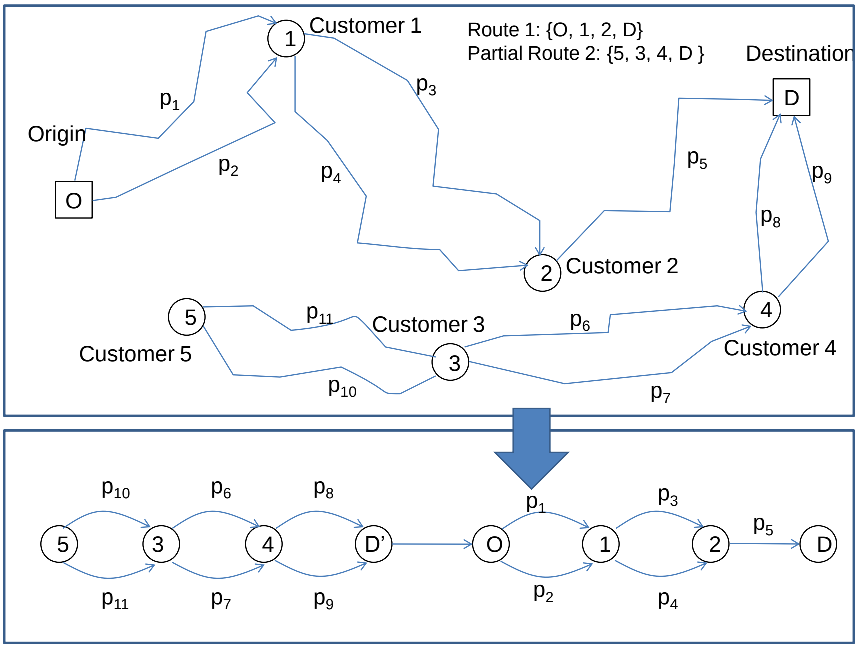


Figure 9. Transformation of route 1 and updated partial route 2 to a routes-graph

Based on the above observations, we claim that building partial BTL-VRPTW solutions after temporarily inserting a candidate customer in a route under construction can be efficiently performed by a substantially simplified version of the label setting routine presented in the previous section of this paper. In particular, the determination of the non-dominated partial solutions starting from (i_u) can be accomplished by simply extending backwards the corresponding partial solutions that start from the currently first node (i_{1q}) of the route under construction. The example that follows illustrates how the proposed label setting routine is customized in order to solve the bi-objective time dependent shortest path problems arising within the proposed construction heuristic.

Figure 10 presents three un-serviced customers, an initial route containing only the destination node $\{D\}$, and the alternative road paths that connect each of the unserviced customers with node D. According to the proposed construction heuristic, one of the three customers will be selected and added to the route. This procedure however takes into account the performance of the non-dominated partial solutions that emerge after trying the insertion of each of the unserviced customers in the first position of the current route. Hence, three bi-objective time-dependent shortest path problems have to be solved on the routes-graphs illustrated in Figure 10.

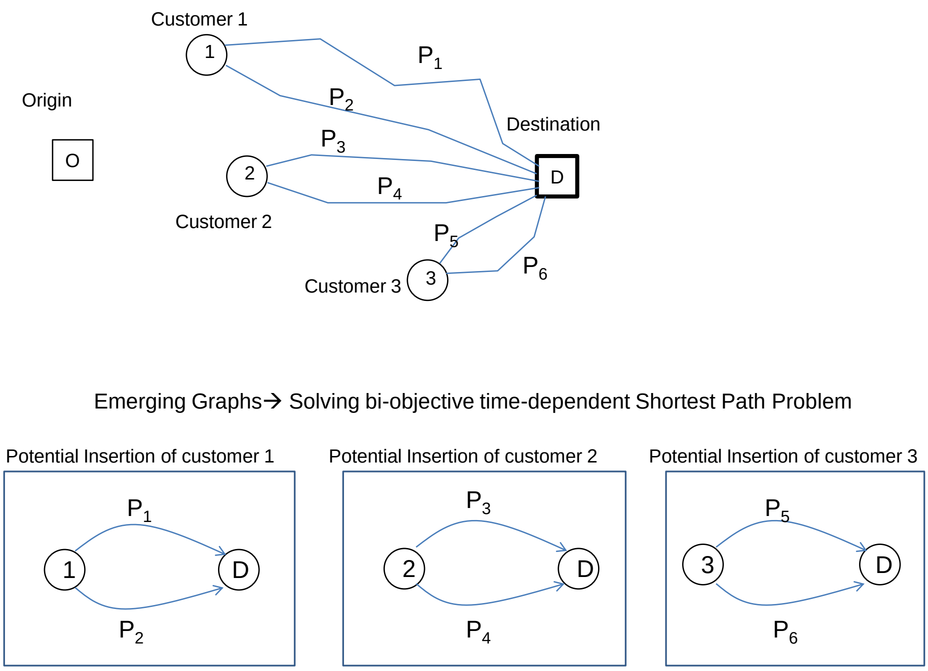


Figure 10. Route-graphs emerging for the insertion of the first customer in an empty route.

Assuming that the relevant shortest path computations on these routes-graphs have been performed and customer 2 has been selected, the route becomes $\{2, D\}$. The next iteration would be to select one of the two remaining customers 1 or 3 and place it in front of customer 2 (assuming that their insertion satisfies the capacity constraint of the truck). This step involves solving the bi-objective time-dependent shortest path problems on the routes-graphs illustrated in Figure 11. However, based on the previous discussion, the solution process may be simplified by utilizing the non-dominated partial solutions found in the previous step for route $\{2, D\}$. Hence, one does not have to solve the emerging shortest path problem from scratch, each time a candidate customer is temporarily inserted in a route under construction. It is sufficient to use the non-dominated labels of the first stop of the route currently under construction and perform a single iteration of the label setting routine in order to identify the non-dominated labels for the candidate customer for insertion.

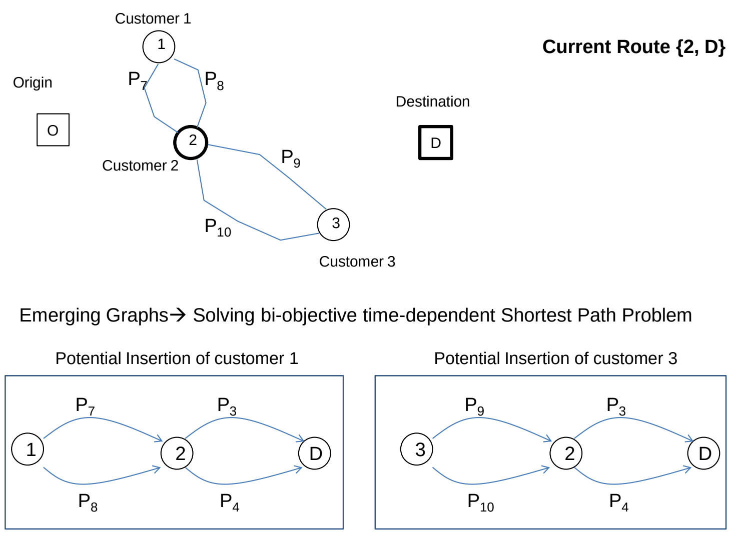


Figure 11. Alternative routes-graphs emerging for the insertion of the second customer in the route under construction.

Figure 12 provides the pseudo-code for the label setting routine that determines the non-dominated labels of node i_q for any possible departure time $\tau \in [a_{i_q} + st_{i_q}, b_{i_q} + st_{i_q}]$ given the relevant non-dominated labels at node i_{q+1}.

```
LABEL SETTING ROUTINE (i_q, i_{q+1})

Set τ:= b_{i_{q+1}}
WHILE (τ ≥ a_{i_q} + st_{i_q}) DO
  BEGIN
    GET LIST OF PATHS ℘_{i_q i_{q+1}}
    FOR (p ∈ ℘_{i_q i_{q+1}}) DO
      BEGIN
        Calculate arrival time : τ^{arr}_{i_{q+1}} := τ + t_p(τ)
        IF (τ^{arr}_{i_{q+1}} ≤ b_{i_{q+1}}) THEN
          BEGIN
            IF (τ^{arr}_{i_{q+1}} < a_{i_{q+1}}) THEN
              BEGIN
                Set τ' := a_{i_{q+1}} + st_{i_{q+1}}
                GET BUCKET B^{τ'}_{i_{q+1}}
                FOR (Label λ ∈ B^{τ'}_{i_{q+1}}) DO
                  BEGIN
                    λ^t_{i_q}(τ) := λ^t_{i_{q+1}}(τ') + t_p(τ) + st_{i_{q+1}} + (a_{i_{q+1}} − τ^{arr}_{i_{q+1}})
                    λ^f_{i_q}(τ) := λ^f_{i_{q+1}}(τ') + f_p(τ)
                    B^τ_{i_q} := B^τ_{i_q} ⊕ (λ^t_{i_q}(τ), λ^f_{i_q}(τ))
                  END
              END
                ELSE
                  BEGIN
                    Set τ' := τ^{arr}_{i_{q+1}} + st_{i_{q+1}}
                    GET BUCKET B^{τ'}_{i_{q+1}}
                    FOR (Label λ ∈ B^{τ'}_{i_{q+1}}) DO
                      BEGIN
                        λ^t_{i_q}(τ) := λ^t_{i_{q+1}}(τ') + t_p(τ) + st_{i_{q+1}}
                        λ^f_{i_q}(τ) := λ^f_{i_{q+1}}(τ') + f_p(τ)
                        B_{i_q}(τ) := B_{i_q}(τ)⊕ (λ^t_{i_q}(τ), λ^f_{i_q}(τ))
                      END
                  END
              END
      END
  END
```

Figure 12. Pseudocode of the routine that determines the non-dominated labels of node i_q given the relevant non-dominated labels at node i_{q+1}.

It is worth noting that $B_{i_q}(\tau) \oplus (\lambda^t_{i_q}(\tau), \lambda^f_{i_q}(\tau))$ implies that label $(\lambda^t_{i_q}(\tau), \lambda^f_{i_q}(\tau))$ is added in the bucket of lists $B_{i_q}(\tau)$ if and only if it is not dominated by any label in the bucket. In this case, the bucket is cleared of any existing labels which are dominated by $(\lambda^t_{i_q}(\tau), \lambda^f_{i_q}(\tau))$.

5.4 Pheromone Trail Update

Upon termination of an agent's iteration, a set of non-dominated BTL-VRPTW solutions are determined. The emerging solutions are placed in the list of non-dominated solutions of the problem Υ which is further processed so that any dominated solutions are removed. If a solution is dominated by any of the existing solutions, then it is disregarded from further consideration. If, on the other hand, the new solution σ is not dominated by any of the existing ones, then it is added in the list Υ and the pheromone trail $(\varphi^\tau(i,j,k))$ on its constituent arcs is updated by formula (30) presented previously, where φ_0 is given by (31):

$$\varphi_0 := \begin{cases} \frac{1}{t(\sigma) f(\sigma)\, \gamma}, & (i,j,\tau) \in \sigma \\ 0, & otherwise \end{cases} \tag{31}$$

$t(\sigma)$ is the total travel time and $f(\sigma)$ is the total fuel consumption of partial solution σ, and γ is the number of routes.

5.5 Testing the accuracy and computational performance

The accuracy of the proposed algorithm was assessed by measuring its capability to determine solutions close to the minimum total travel time or the minimum fuel consumption solutions. Given that the proposed problem has not been addressed in the existing literature, no benchmark test problems were found. The authors produced a set of test problems on a grid like network consisting of sixty nodes (arranged in six rows of ten nodes). Each link of the test network was associated with a distance ranging from 1 to 2 km and a randomly generated speed profile, i.e., random speed observations u_k^0 were generated per 10 min intervals for a time horizon of 50 min. For the purpose of generating speed profiles adhering to the usual traffic pattern in an urban environment, the speed values on each link of the network were randomly generated within the following ranges of values: i) 20-30 km/h for the time periods 0-20 min and 30-50 min, and ii) 50-60km/h for the time period between the 20-30 min. Each test problem was created by randomly selecting ten delivery locations, plus the origin and the destination points from the nodes of the test network. Order quantities were randomly set between 1-1.4t. We considered only two eligible paths for each pair of stops. This choice was made because the test problems could not be solved to optimality when three or more paths where considered of each pair of stops. Table 5 provides the values of the parameters that were used for the fuel consumption estimation model (Bektas and Laporte, 2011). Wide time windows were considered (0-50) while the capacity of the vehicles was assumed large enough so that all customers could be feasibly loaded to a single vehicle. The service duration per customer was set equal to 0. The curb weight was assumed equal to 5t.

Parameter	Notation	Value	Parameter	Notation	Value
Engine friction factor	κ	0.2	Coefficient of rolling resistance	Cr	0.01
Engine Speed	Ψ	35	Air Density (kg/m^3)	ρ	1.2041
Engine Disposition	V	4	Frontal face area	E	4
Equivalence ratio	Phi(ϕ)	1	Road gradient	θ	0
Engine Efficiency	Ita (η)	0.4	Gravity Acceleration	g	10
Aerodynamic drag co-efficient	Cd	0.7			

Table 5. Parameters used in the fuel consumption estimation model and the corresponding values used for solving the test problems.

Each test problem was solved to optimality (using ILOG Cplex 12.6) for minimizing: i) travel time and ii) fuel consumption. The relevant solutions determined by the proposed algorithm were achieved for 150 agents and 400 iterations. Table 6 presents the results from solving the test problems. The average percentage deviation of the heuristic solutions closer to the minimum travel time was found 4.6% while the corresponding deviation from the minimum fuel consumption was 9.1%.

Problem ID	Optimizing Travel Time			Optimizing Fuel Consumption		
	Optimum	Heuristic	Deviation(%)	Optimum	Heuristic	Deviation(%)
1	13	13	0.0%	934	1000	7.1%
2	13	14	7.7%	960	1070	11.5%
3	12	13	8.3%	986	1115	13.1%

4	11	11	0.0%	970	1042	7.4%
5	12	12	0.0%	930	1032	11.0%
6	11	11	0.0%	945	1023	8.3%
7	12	12	0.0%	827	897	8.5%
8	13	16	23.1%	942	1033	9.7%
9	14	15	7.1%	1079	1129	9.7%
10	12	12	0.0%	847	967	4.6%
		Average	4.6%		Average	9.1%

Table 6. Travel time and fuel consumption values of the optimal solutions and the heuristic solutions.

Given that the proposed time and load-dependent vehicle routing and scheduling problem has not been addressed previously in the literature, no performance standards have been established regarding the accuracy of the relevant solution methods of the problem. However the results obtained were encouraging.

Additional experiments were performed in order to assess the computational time required by the proposed heuristic algorithm in order to solve various instances of the BTL-VRPTW. The objective of the tests were to explore how the computational time of the proposed heuristic is affected by : i) the number of the customers, and ii) the number K of alternative eligible paths considered between each pair of stops. Problem sets of 20, 30, 40, and 50 customers were produced on a grid like test network of 225 nodes. The time horizon of the problems was 240 min. The speed values on each link of the network were randomly generated within the following ranges of values: i) 20-30 km/h for the time periods 0-100 min and 150-240 min, and ii) 50-60km/h for the time period 100-150 min. Up to four eligible paths were considered between each pair of stops while the capacity of the vehicles was 16t (8t curb weight). The order quantities ranged between 0.8-1.4 tons. A two-hours time window was assumed for each order. The heuristic algorithm employed 100 iterations with 10 agents to solve these test problems. The experiments were performed on a PC with 3 GHz processor and 4GB RAM. Figure 13 presents the average computational time of the heuristic algorithm in solving ten problem instances of 50 customers with vehicle capacity equal to 16t for three different values of parameter K (number of eligible paths between stops). Figure 14 presents the average computational time (over the ten problem instances) of the heuristic algorithm as a function of the number of customers (orders). In addition to using the proposed algorithm, the authors attempted to solve the 20-customers test problems through CPlex. However, Cplex run out of memory before a feasible solution was found.

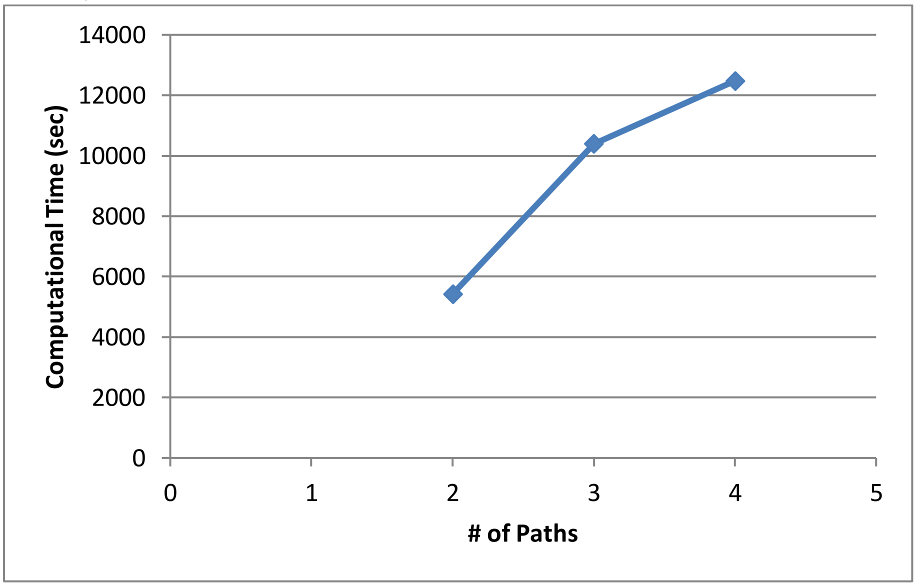


Figure 13. Average computational time of the proposed algorithm for solving ten BTL-VRPTW test problems assuming 2-4 eligible paths between stops.

Figure 14. Average computational time of the proposed algorithm for solving ten BTL-VRPTW test problems with different number of orders.

It is evident that the worst case average computational time reaches 3.5 hours. However, it is worth mentioning that this amount of time cannot be considered prohibitive for the problem under study. The relevant route planning process usually takes place the day before execution implying loose computational time requirements for addressing the associated routing decisions.

6 EFFECTS OF THE NETWORK REDUCTION APPROACH

A major feature of the proposed model for the BTL-VRPTW is that the determination of the scheduled road paths for traveling between consecutive stops is performed on a small part of the road network formed by K pre-specified shortest distance paths. Although this approach reduces the computational requirements for solving the BTL-VRPTW, it leaves open the possibility of excluding part of the non-dominated route-trajectories (especially for small values of K), and thus losing part of the non-dominated solutions of the BTL-VRPTW. On the other hand increasing K is expected to improve the quality of the solutions produced but increase substantially the computational time (Figure 14). We investigate the extent of these potential effects of incorporating the network reduction technique in the BTL-VRPTW, through two series of experiments. The objective of the first series of experiments (Experiments I) is to assess how many non-dominated route-trajectories associated to a given route are lost for various values of K. The second series of experiments (Experiments II) investigates the magnitude of improvement on the quality of solutions (in terms of travel time and fuel consumption) by increasing the size of K. It is worth noting that the current practice in the PRP is to consider a single path between consecutive stops. The results from Experiments II indicate that this practice may lead to substantial economic and environmental losses.

In more detail, Experiments I aim to assess the portion of the entire set of non-dominated route-trajectories associated to a given route that can still be determined after applying the proposed network reduction approach for different values of K. A series of routes were generated as sequences of nodes randomly selected from a 225-nodes grid-like network. The length of the links of the test network range from 2 km to 5 km. The time horizon is five hours. The values of speed parameters u_k^0 were randomly generated for every link within the following ranges: i) 20-30 km/h for the time periods 0-120 min and 180-300 min and ii) 50-60km/h for the time period 120-180 min. The number of customers per route ranged from 4 to 9 and their total demand per route was equal to 15 tons. The origin and destination of each route were also randomly selected. Each experiment involved the following steps:

i) Determination of the non-dominated route-trajectories associated to a given route, assuming that traveling between consecutive stops can be performed through the entire underlying network.
ii) Determination of the non-dominated route-trajectories associated to a given route, assuming that traveling between consecutive stops can be performed through the corresponding K shortest distance paths.
iii) Specification of the number of solutions from step (ii) that coincide with solutions from step (i)

As illustrated previously in this paper, determining the non-dominated route-trajectories associated to a given route involves a bi-objective time-dependent shortest path problem with time windows. For each route generated, the emerging bi-objective time dependent shortest path problem was solved under five different scenarios concerning the number of alternative road paths considered for each pair of consecutive stops. In scenario 1 it was assumed that travelling between consecutive stops can be performed through the entire road network, while in scenarios 2-5, it was assumed that travelling between each pair of consecutive stops of the route could only be performed through the relevant 2-5 shortest distance road paths (respectively). All bi-objective time-dependent shortest path problems under scenario 1 were solved through the implementation of the algorithm presented in (Hamacher et al. 2006). The remaining bi-objective time-dependent shortest path problems under scenarios 2-5 were solved through the label setting routine proposed in this paper (Section 4). Finally, the K-shortest distance paths (for K=2, 3, 4 & 5) between each pair of consecutive stops of the route were determined through Yen's algorithm (Yen, 1971). Depending on the number of stops in the route, six groups of routes were generated. Note that 30 routes were generated for each group of routes.

Table 7 summarizes the results of Experiments I. Column three of Table 7 provides the total number of non-dominated solutions under scenario 1 that were determined for all routes of a given group. Under the sub-column titled "2-paths" we provide the percentage of the total number of non-dominated solutions determined in scenario 1 that were also determined under scenario 2. Similar results are provided in the remaining sub-columns of the Table.

The results of Table 7 indicated that the percentage loss of route trajectories when the proposed network reduction scheme is applied for K equal to 5, ranges from 3% (for 4 customers routes) up to 19% (for 7 customers routes). Hence, only a relatively small part of the non-dominated route-trajectories may be lost due to the proposed network reduction approach.

Group of routes	# of stops per route	Total # of Non-Dominated route-trajectories (scenario 1)	% of non-dominated route-trajectories determined for K-paths networks (scenarios 2-5)			
			2-Paths (scenario 2)	3-paths (scenario 3)	4-paths (scenario 4)	5-paths (scenario 5)
P1	4	1278	79%	91%	94%	97%
P2	5	4567	84%	89%	90%	96%
P3	6	2641	59%	77%	86%	88%
P4	7	3157	54%	71%	75%	81%
P5	8	3855	54%	67%	86%	90%
P6	9	3118	64%	85%	88%	89%

Table 7. Summary of the results from the experiments on the effect of the network reduction approach on the calculation of non-dominated route-trajectories.

The objective of Experiments II was to investigate the effect of the value of parameter K on the quality of solutions for the BTL-VRPTW. Ten test problems were designed and solved for different number of alternative road paths between each pair of stops. Each test problem included 9 customers randomly located on the test network developed for testing the accuracy of the algorithm (Section 5). The order size ranged from 800 to 1200 kg while the service duration of each order was set equal to zero (with no loss of generality). Vehicles capacity was set equal to 15t and the customers time windows were wide enough so that the entire set of orders can be serviced by a single route. Up to four eligible (shortest distance) paths were considered for each pair of stops of the test problems. Four instances were generated for each test problem. Each instance involved different number of eligible paths between each pair of stops, ranging from one to four. Each instance was solved to optimality by ILOG Cplex 12.6 for : i) total travel time, and ii) total fuel consumption. It is worth noting that the average computational time for Cplex to solve instances 1-4 was 2 min, 16 min, 53 min, and 187 min respectively.

Tables 8 and 9 present the results from solving the test problems instances. The first column of Table 8 indicates the problem ID while columns two to five correspond to the travel time of the optimum solutions (i.e., in terms of total travel time) for the problem instances 1-4 respectively. Columns six to eight present the percentage reduction of travel time when comparing the optimal solutions of instances 2, 3 and 4 with the corresponding optimal solution of instance 1. For example, the optimum solution for problem 1 with 4 alternative paths between stops (Instance 4) has travel time 8.33% lower than the travel time of the optimum solution of the same problem considering a single path between each pair of stops (Instance 1). Table 9 presents analogous results that emerged from solving the same problem instances on the basis of minimizing the fuel consumption objective function.

Problem ID	Optimal Total Time (in min) per Instance (# of paths)				% improvement of Total Time (Instance i over Instance j)		
	Inst. 1 (#paths=1)	Inst. 2 (#paths=2)	Inst. 3 (#paths=3)	Inst. 4 (#paths=4)	Inst.2 over Inst.1	Inst.3 over Inst.1	Inst.4 over Inst.1
1	12	12	11	11	0.00%	8.33%	8.33%
2	12	10	10	10	16.67%	16.67%	16.67%
3	14	14	13	13	0.00%	7.14%	7.14%
4	13	12	12	11	7.69%	7.69%	15.38%
5	10	10	10	10	0.00%	0.00%	0.00%
6	14	13	13	12	7.14%	7.14%	14.29%
7	14	13	13	13	7.14%	7.14%	7.14%
8	13	11	11	11	15.38%	15.38%	15.38%
9	17	15	14	14	11.76%	17.65%	17.65%
10	13	12	12	12	7.69%	7.69%	7.69%
				Average	7.35%	9.48%	10.97%

Table 8. Results from solving the instances of the test problems on the basis of minimum the total travel time.

Problem ID	Optimal Fuel Consumption (in gr) per Instance (# of paths)				% improvement of Total Time (Instance i over Instance j)		
	Inst. 1 (#paths=1)	Inst. 2 (#paths=2)	Inst. 3 (#paths=3)	Inst. 4 (#paths=4)	Inst.2 over Inst.1	Inst.3 over Inst.1	Inst.4over Inst.1
1	836.40	793.00	776.30	752.85	5.19%	7.19%	9.99%
2	771.00	758.45	713.53	693.90	1.63%	7.45%	10.00%
3	1020.31	958.98	917.16	899.00	6.01%	10.11%	11.89%
4	806.30	757.67	745.00	738.70	6.03%	7.60%	8.38%
5	662.90	624.20	602.14	589.13	5.84%	9.17%	11.13%
6	942.93	866.62	833.84	830.96	8.09%	11.57%	11.87%
7	947.27	907.47	891.98	796	4.20%	5.84%	15.9%
8	776.55	741.76	723.71	718.58	4.48%	6.80%	7.47%
9	1109.92	1020.39	1007.23	984.04	8.07%	9.25%	11.34%
10	921.62	886.46	872.61	859.39	3.82%	5.32%	6.75%
				Average	5.34%	8.03%	10.47%

Table 9. Results from solving the instances of the test problems on the basis of minimum total fuel consumption.

Based on the results presented in Tables 8 and 9, it can be argued that increasing the number of eligible paths between each pair of stops of the problem may provide substantially improved solutions on the basis of both travel time and fuel consumption. Considering two alternative paths instead of a single one, has led to solutions with 7.35% (on average) lower travel time and 5.34% lower fuel consumption. Moreover, comparing the solutions of problem instances involving four paths with the corresponding solutions with a single path, it was found that both travel time and fuel consumption are improved (on average) by about 10%. These findings imply that oversimplifying the path finding problem between consecutive stops (e.g., considering a single path) may lead to substantial losses in terms of travel time and fuel consumption.

7 CONCLUDING REMARKS AND FUTURE RESEARCH

This paper has been focused on formulating and solving a new bi-objective vehicle routing and scheduling model arising in a contemporary distribution route planning framework where the emissions (expressed by fuel consumption) of shippers'/carriers' transport activities are explicitly traded-off with travel time. The proposed model aims to fill the gap in the relevant research area by considering simultaneously two objective functions (travel time and fuel consumption) under time varying traffic conditions, which realistically represent the urban freight distribution environment. The emerging bi-objective time and load dependent vehicle routing problem with time windows aims to address simultaneously three sub-problems: i) determining service routes, ii) specifying a road path between each pair of consecutive stops, and iii) determining the schedule for traversing each route. A key finding reported in this paper is that the path finding problem between each pair of stops cannot be solved in advance separately from sub-problems (i) and (iii). To address this issue we propose a network reduction approach in which the path finding problem between any pair of stops is solved on a small part of the underlying network formed by the K shortest distance paths between the stops. Computational tests on the effect of the network reduction approach on determining non-dominated route trajectories revealed that considering five shortest distance paths between the consecutive stops of a route is sufficient to determine 80% (in the worst case) of the relevant non-dominated route trajectories. Moreover, we have tested the effect of considering K shortest distance paths (instead of a single one) between any pair of stops of the problem, on the quality of the solutions. Based on the experimental results of the performed tests, there is evidence that using more than one path between the stops of the problem may lead to solutions with substantially improved performance under travel time and fuel consumption.

A generic solution framework is proposed to address the BTL-VRPTW combining any technique that creates capacity-feasible routes with a routing and scheduling method that converts the identified routes to problem solutions. We showed that transforming a set of routes to BTL-VRPTW solutions is equivalent to solving a bi-objective time dependent shortest path problem on a specially structured graph. We proposed a backward label setting technique to solve the emerging problem that takes advantage of the special structure of the graph. The proposed solution framework is implemented by integrating the routing and scheduling method into an ACS technique. The accuracy of the proposed algorithm was assessed on the basis of its capability to determine minimum travel time and fuel consumption solutions. Although the computational results are encouraging, there is ample room for future research in algorithmic advances on the BTL-VRPTW.

The proposed model is applicable to cases where travel speed fully depends on the expected traffic conditions. In cases of free-flow conditions the travel speed is assumed equal to the pre-specified value. Based on the above reasoning, future work could be focused on extending the model covering cases where the vehicle speed (during the non-congested period) may be considered as a decision variable and investigating the effect of various problem features (e.g., load utilization) on the efficient frontier of the problem.

APPENDIX A

Fuel Consumption on a road link

In this paper, the fuel consumption over a road link is calculated through the CMEM formula (A.1). More details regarding formula (A.1) is provided in (Barth et al, 2004).

$$R_f(t) = \beta_0 + \beta_1 \cdot \left(Ma + Mg\sin\theta + \tfrac{1}{2}C_d E\,\rho\,u(t)^2 + Mg\,C_r\cos\theta\right)u(t) \tag{A.1}$$

$$\beta_0 \coloneqq \frac{1}{44}\,\kappa\,\Psi\,V\,\phi \tag{A.2}$$

$$\beta_1 \coloneqq \frac{1}{\eta\,44}\phi 10^{-3} \tag{A.3}$$

ϕ is the equivalence ratio of the stoichiometric air to fuel ratio, K is the engine friction factor, Ψ is the engine speed (in rpm), V the engine displacement (in L), M is the total vehicle mass (including the mass of the vehicle and the load), a is the vehicle acceleration/deceleration rate (in m/sec^2), g is the gravitational constant (9.81 m/sec^2), θ is the gradient of the road, C_d is the aerodynamic drag coefficient, E is the frontal surface area of the vehicle (in m^2), ρ is the air density (in kg/m^3), $u(t)$ is the vehicle speed at time t, and C_r is the coefficient of rolling resistance. Although the engine speed (Ψ) is dynamic and varies throughout the driving cycles, it is assumed constant and is set equal to the value which produces the maximum torque.

In more detail, the fuel consumption on any road link $(l_\eta, l_{\eta+1})$ for departure time τ_d and total vehicle mass M is calculated by processing formula (A.1) as follows: i) substitute average travel speed with expression (2) of Horn's model, and ii) integrate the emerging formula (A.1) from time τ_d up to arrival time τ_a at $l_{\eta+1}$. If a vehicle departs from the upstream node l_η at time $\tau_d \in [\tau_\kappa, \tau_{\kappa+1})$ and its estimated arrival time at downstream node $l_{\eta+1}$ is time $\tau_a \in [\tau_{\kappa+m}, \tau_{\kappa+m+1})$, then the total fuel consumed on $(l_\eta, l_{\eta+1})$, denoted by $F_{l_\eta l_{\eta+1}}(\tau_d)$ is the sum of the fuel consumed over the time intervals $[\tau_d, \tau_{\kappa+1})$, $[\tau_{\kappa+1}, \tau_{\kappa+2})$,…, $[\tau_\xi, \tau_{\xi+1})$, …, $[\tau_{\kappa+m}, \tau_a)$ elapsed between τ_d and τ_a. Since the coefficients of formula (2) differ among different time intervals $[\tau_\xi, \tau_{\xi+1})$, integration of formula (A.1) is performed separately over each time interval elapsed while traveling from l_η to $l_{\eta+1}$. Hence, $F_{l_\eta l_{\eta+1}}(\tau_d)$, is given by formula (A.4).

$$F_{l_\eta l_{\eta+1}}(\tau_d) \coloneqq F^{\kappa}_{l_\eta l_{\eta+1}}(\tau_d, \tau_{\kappa+1}) + \sum_{\xi=\kappa+1}^{\kappa+m-1} F^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) + F^{\kappa+m}_{l_\eta l_{\eta+1}}(\tau_{\kappa+m}, \tau_a) \tag{A.4}$$

where:

$$F^{\kappa}_{l_\eta l_{\eta+1}}(\tau_d, \tau_{k+1}) = \int_{\tau_d}^{\tau_{\kappa+1}} \left(\beta_0 + \beta_1 \cdot \left(Ma + Mg\sin\theta + \frac{1}{2}C_d E\,\rho\,u(t)^2 + Mg\,C_r\cos\theta\right)u(t)\right)\mathrm{dt} \tag{A.5}$$

$$F^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) = \int_{\tau_\xi}^{\tau_{\xi+1}} \left(\beta_0 + \beta_1 \cdot \left(Ma + Mg\sin\theta + \tfrac{1}{2}C_d E\,\rho\,u(t)^2 + Mg\,C_r\cos\theta\right)u(t)\right)\mathrm{dt},\quad \xi \coloneqq \kappa+1, .., \kappa+m-1 \tag{A.6}$$

$$F^{k+m}_{l_\eta l_{\eta+1}}(\tau_{k+m}, \tau_a) = \int_{\tau_{\kappa+m}}^{\tau_a} \left(\beta_0 + \beta_1 \cdot \left(Ma + Mg\sin\theta + \frac{1}{2}C_d E\,\rho\,u(t)^2 + Mg\,C_r\cos\theta\right)u(t)\right)\mathrm{dt} \tag{A.7}$$

Figure A.1 illustrates an actual road link (segment) on which the intermediate time intervals elapsed between τ_d and τ_a are marked. It is worth noting that in this example the size of each time period $[\tau_\xi, \tau_{\xi+1}]$ is twenty minutes. In addition, Figure A.1 shows the segments of the road link traversed within each such time period. Thus, the fuel consumption on road link $(l_\eta, l_{\eta+1})$ departing at time τ_d is the sum of the fuel consumption on each of these road segments. In particular, formula (A.5) calculates the fuel consumption of a vehicle for traveling the first segment of the road link $(l_\eta, l_{\eta+1})$ from node l_η (starting at departure time) until the point where time $\tau_{\kappa+1}$ is reached (and the coefficients of the travel speed function change). The fuel consumption on any of the intermediate segments of road link $(l_\eta, l_{\eta+1})$ corresponding to a time period between τ_ξ and $\tau_{\xi+1}$, is calculated by Formula (A.6). Finally, formula (A.7) calculates the fuel consumption during the final time period of the trip between time $\tau_{\kappa+m}$ and the arrival time τ_a.

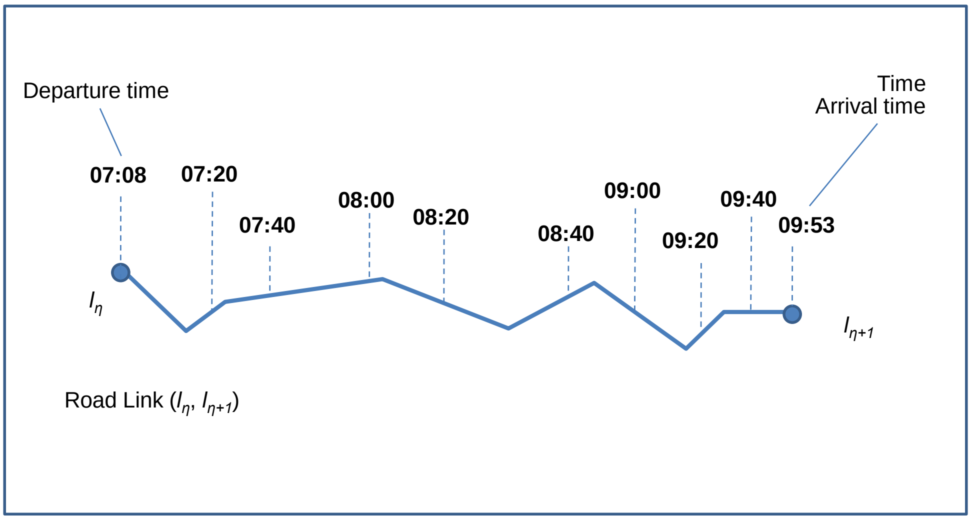


Figure A.1. Decomposition of a road link to segments traversed over different time intervals.

Any of formulae (A.5)-(A7) may be written in the condensed form of (A.8) below, as a linear function of the total mass of the vehicle.

$$F^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) = \gamma^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) + M \cdot \zeta^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) \tag{A.8}$$

where,

$$\gamma^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) =$$
$$\begin{cases} \frac{\phi}{44} \cdot K \cdot \Psi \cdot V \cdot (\tau_{\xi+1} - \tau_\xi) + \frac{\phi}{44\eta 8 a_\xi 10^3} C_d \rho E(u^4_{\xi+1} - u^4_\xi), & \alpha_\xi \neq 0 \\ \frac{\phi}{44} \cdot K \cdot N \cdot V \cdot (\tau_{\xi+1} - \tau_\xi) + \frac{1}{\eta\, 44} \phi 10^{-3} (\frac{1}{2} C_d \rho E u^3_\xi)(\tau_{\xi+1} - \tau_\xi), & \alpha_\xi = 0 \end{cases} \tag{A.9}$$

$$\zeta^{l}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) = \begin{cases} \frac{\phi}{44\eta 10^3} \cdot \left(\frac{a + g sin\theta + g C_r cos\theta}{2 a_\xi}\right) \cdot E(u^2_{\xi+1} - u^2_\xi), & \alpha_\xi \neq 0 \\ \frac{\phi}{44\,\eta 10^3} (g sin\theta + g C_r cos\theta) u_1 (\tau_{\xi+1} - \tau_\xi), & \alpha_\xi = 0 \end{cases} \tag{A.10}$$

where u_ξ and $u_{\xi+1}$ are the travel speeds of the vehicle at times τ_ξ and $\tau_{\xi+1}$ respectively, and a_ξ represents the acceleration/deceleration rate $a_{l_\eta l_{\eta+1} \xi}$ (for simplicity we drop the link indices $l_\eta, l_{\eta+1}$ from travel speeds and acceleration rates). Replacing $F^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1})$ with expression (A.8) in formula (A.4), leads to formula (A.11) for the estimated fuel consumed $F_{l_\eta l_{\eta+1}}(\tau_d)$ over road link $(l_\eta, l_{\eta+1})$.

$$F_{l_\eta l_{\eta+1}}(\tau_d) \coloneqq \left\{\gamma^{\kappa}_{l_\eta l_{\eta+1}}(\tau_d, \tau_{\kappa+1}) + \sum_{\xi=\kappa+1}^{\kappa+m-1} \gamma^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) + \gamma^{\kappa+m}_{l_\eta l_{\eta+1}}(\tau_{\kappa+m}, \tau_a)\right\} + M \cdot \left\{\zeta^{\kappa}_{l_\eta l_{\eta+1}}(\tau_d, \tau_{\kappa+1}) + \sum_{\xi=\kappa+1}^{\kappa+m-1} \zeta^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) + \zeta^{\kappa+m}_{l_\eta l_{\eta+1}}(\tau_{\kappa+m}, \tau_a)\right\} \tag{A.11}$$

Hence, according to formula (A.11), fuel consumption over road link $(l_\eta, l_{\eta+1})$ may be written as a linear function of the total mass M of the vehicle with time dependent coefficients. This is further illustrated if we rewrite (A.11) as follows:

$$F_{l_\eta l_{\eta+1}}(\tau_d) \coloneqq \Gamma_{l_\eta l_{\eta+1}}(\tau_d) + M \cdot Z_{l_\eta l_{\eta+1}}(\tau_d) \tag{A.12}$$

where

$$\Gamma_{l_\eta l_{\eta+1}}(\tau_d) \coloneqq \gamma^{k}_{l_\eta l_{\eta+1}}(\tau_d, \tau_{k+1}) + \sum_{\xi=k+1}^{k+m-1} \gamma^{\xi}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) + \gamma^{k+m}_{l_\eta l_{\eta+1}}(\tau_{k+m}, \tau_a) \tag{A.13}$$

and

$$Z_{l_\eta l_{\eta+1}}(\tau_d) \coloneqq \zeta^{k}_{l_\eta l_{\eta+1}}(\tau_d, \tau_{k+1}) + \sum_{\xi=k+1}^{k+m-1} \zeta^{l}_{l_\eta l_{\eta+1}}(\tau_\xi, \tau_{\xi+1}) + \zeta^{k+m}_{l_\eta l_{\eta+1}}(\tau_{k+m}, \tau_a) \tag{A.14}$$

It is worth noting that time dependent coefficients $\Gamma_{l_\eta l_{\eta+1}}(\tau_d)$ and $Z_{l_\eta l_{\eta+1}}(\tau_d)$ can be pre-computed at a pre-solving time for any possible departure time. This observation facilitates the calculation of the fuel consumption over a road path as indicated below.

Fuel Consumption on a scheduled road-path

For any scheduled road path sp_{ij} between stops i and j rerpesented by (A.15) and departure time τ_d from stop i (or node $l(i)$), the fuel consumption is the sum of the fuel consumption on the constituent road links.

$$sp_{ij} = (\tau_d, [(l(i), l_1), (l_1, l_2),, (l_{\eta-1}, l_\eta).., (l_\nu, l(j))]) \tag{A.15}$$

Based on formula (14) for the fuel consumption on a road link for a given departure time, the fuel consumption over a road path p_{ij} is given by (A.16).

$$\boldsymbol{F}_{ij}(\tau_d) \coloneqq \boldsymbol{\Gamma}_{ij}(\tau_d) + M \cdot \boldsymbol{Z}_{ij}(\tau_d) \tag{A.16}$$

where

$$\boldsymbol{\Gamma}_{ij}(\tau_d) = \Gamma_{l(i)l_1}(\tau_d) + \sum_{\eta=1}^{\nu-1} \Gamma_{l_\eta l_{\eta+1}}\left(\tau_{l_\eta}\right) + \Gamma_{l_\nu l(j)}(\tau_{l_\nu}) \tag{A.17}$$

$$\boldsymbol{Z}_{ij}(\tau_d) = Z_{l(i)l_1}(\tau_d) + \sum_{\eta=1}^{\nu-1} Z_{l_\eta l_{\eta+1}}\left(\tau_{l_\eta}\right) + Z_{l_\nu l(j)}(\tau_{l_\nu}) \tag{A.18}$$

τ_{l_η} represents the departure time from node l_η on road link $(l_\eta, l_{\eta+1})$, which coincides with the arrival time at the same node after traversing the previous road link $(l_{\eta-1}, l_\eta)$. It worth noting that $\boldsymbol{\Gamma}_{ij}(\tau_d)$ and $\boldsymbol{Z}_{ij}(\tau_d)$ are expressed by the sum of the corresponding $\Gamma_{l_\eta l_{\eta+1}}(\tau)$ and $Z_{l_\eta l_{\eta+1}}(\tau)$ parameters calculated for the constituent road links of the path. This observation implies that $\boldsymbol{\Gamma}_{ij}(\tau_d)$ and $\boldsymbol{Z}_{ij}(\tau_d)$ can be pre-computed, and thus calculating the fuel consumption over a road path for a given departure time and total vehicle mass can be efficiently performed (even in real time) by retrieving the appropriate pre-computed values for $\boldsymbol{\Gamma}_{ij}(\tau_d)$ and $\boldsymbol{Z}_{ij}(\tau_d)$ and apply formula (A.16).

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