Boylan, M., & Coles, A. T. (2017). Is another mathematics education possible? an introduction to a Special Issue on Mathematics Education and the LIVING WORLD . *Philosophy of Mathematics Education Journal*, *32*.

Peer reviewed version

IS ANOTHER MATHEMATICS EDUCATION POSSIBLE? AN INTRODUCTION TO A SPECIAL ISSUE ON MATHEMATICS EDUCATION AND THE LIVING WORLD

Mark Boylan[1] and Alf Coles[2]

Sheffield Hallam University[1], University of Bristol [2]

m.s.boylan@shu.ac.uk[1], alf.coles@bris.ac.uk[2]

Special issue editors: Alf Coles, Mark Boylan
(in cooperation with the founding editor Paul Ernest)

Editorial panel: Yasmine Abtahi; Janet Ainley; Richard Barwell; Annie Savard

Introduction

The slogan 'Another world is possible' was taken up at the start of this century by the World Social Forum and became a rallying cry for social, economic and climate justice. The authors and editorial panel's collective sense that mathematics education needs to come into dialogue with current global and societal challenges motivates this special issue, and hence the title for this introduction to the special issue. Further, we are concerned that this conversation is not yet taking place with sufficient urgency. We believe that the collection of papers in this special issue represents a diverse and powerful contribution to this conversation and to the question 'is another mathematics education possible?'. In this introduction, we provide detail on the background to how this special issue manifested in the form it did and offer a brief overview of how issues of the environment have previously been taken up in mathematics education. We have synthesised five themes across the articles and offer comments and questions related to each one. The themes are not all addressed by all articles, but capture, for us, key elements that link and connect across the special issue, these are: ethics and theories of ethics; extending and relating to critical mathematics education; implications of climate change and the Anthropocene; re-thinking the nature of mathematics, curriculum and pedagogy; theory into practice. We then turn from the contributions to offer reflections on possible directions for future research and practice - a mapping for thinking about where we could go, collectively, in mathematics education - before concluding.

Background

The group of authors represented in these articles grew out of a symposium at the British Educational Research Association (BERA)'s annual conference in 2016, hosted by BERA's Mathematics Education Special Interest Group. At that symposium, papers were presented by several contributors to this special issue and, in discussion after the event, we decided to continue and expand our collaboration and a proposal for a special issue of this journal was agreed by the editor, Paul Ernest. The call for articles stated:

We invite submissions of proposals for articles to form a special issue of the 'Philosophy of Mathematics Education Journal' on the theme of ethics, uncertainty and complexity in relation to the link between mathematics education and the global environmental crisis (e.g. climate change, food security, water, future cities, etc.).

As the range and scope of challenges to ecological survival extend, at what point must this impinge on mathematics taught in schools, or the way we prepare mathematics teachers for their roles, or the research done in mathematics education? Given the accountability pressures on school and universities, where are the spaces for considering the ethical implications of the choices we make within mathematics? the spaces for working with uncertainty? and with complex situations where we cannot predict the results of our actions? Specific questions addressed in the Special Issue might be (but are not limited to):

- *What is the role of mathematics and mathematics education in the global environmental crisis? What role might there be in responding to this crisis?*
- *How can inter-disciplinary work promote awareness of the role of mathematics in complex, ethical decision-making?*
- *How do global crises challenge our values and ethics as mathematics educators?*
- *How can mathematics education address the risk and uncertainty in the current environmental crisis?*

The nine articles that follow address these questions in a range of ways. Six of them involve authors present at the 2016 BERA conference and three arose from this wider call. Some of the articles are clearly philosophical in nature, others point to the interface of philosophy and practice.

Mathematics education and the living world[1]

There has, in general, been a lack of engagement with issues of sustainability within mathematics education (Renert 2011) with several leading mathematics education journals having, to date, published no articles with an ecological or environmental focus. We find this a surprising omission; the lack of engagement was one of the motivations behind the special issue. However, looking more broadly, there has been considerable thinking about the place and role of mathematics in the world, which could be seen as relating to global ecological concerns. We have observed four strands of work that could be seen as related to the living world, which we briefly sketch out here.

[1] Following Monbiot (2017) we are exploring the use of phrases such as 'the living world' in a search for language that is more evocative than writing of 'the environment' and less human-centric than 'sustainability'.

In the 1980s and 1990s, Skovsmose introduced the idea of a critical mathematics education (e.g., Skovsmose 1994) which entailed a classification of types of knowing into: mathematical knowing (formal symbol system rules and techniques); technological knowing (applying mathematics to solve real world problems, including mathematical modelling); and, reflective knowing (including awareness of the purposes of modelling and consideration of the 'formatting power' of mathematics). Given the accelerating ecological and social crisis, the projects of critical mathematics education become more urgent and important, including recognising mathematics and mathematics education as cultural, historical, social and political activities, interrogating the relationship between mathematics education and prevailing norms, ideologies, discourses and structures of society, contributing to learner empowerment and citizenship and, fostering social justice and social change (Ernest, Sriraman and Ernest 2015). However, it is notable that in two recent edited collections of critical mathematics education papers (Ernest, Sriraman and Ernest 2015; Alrø, Ravn, and Valero 2010) only one of 34 chapters specifically related to the ecological crisis (D'Ambrosio, 2010). Thus, as well as being absent from 'mainstream' mathematics education journals as noted above, ecological issues have, as yet, not been addressed to any great extent in critical mathematics education.

Drawing on some similar roots, a second strand of thinking around sustainability is linked to the much broader tradition of mathematical modelling, within mathematics education research. Kaiser and Siriman (2006) identify 'socio-critical modelling' and an 'emancipatory perspective' (p.304) as one of six approaches to modelling, in their global survey. The emancipatory perspective on modelling is linked to ethno-mathematics (D'Ambrosio, 1985) and, through D'Ambrosio, to the work of Freire (1970) who proposed education as a mechanism by which oppressed people could come to view critically their lived reality and, as a consequence, work towards a transformation of society.

More recently, Renert (2011) has suggested the need for a 'sustainable mathematics education' and proposed a set of three stages or levels of approach: accommodation (sustainability issues are used as a context for teaching mathematical skills, e.g., statistical skills are taught using data relevant to sustainability but the meaning or implications of the data are not discussed); reformation (in which some critical thinking and discussion of values are included as a valid aspect of learning mathematics); transformation (teaching and learning mathematics becomes subordinate to a process of students becoming engaged and critical citizens, able to critique the status quo and participate in social action). A transformational approach has clear connections to Skovsmose's reflective knowing and, indeed, in a response to Renert, Gellert (2011) suggested the importance for this proposed new movement to make explicit connections with the work of critical mathematics education (Skovsmose 1994).

The three strands of thinking above share an emancipatory agenda and common links to the work of Freire (1970). The final strand we identified is linked to the work of Barwell, Hauge and colleagues (e.g. Barwell 2013; Hauge, et al. 2015) who have engaged specifically in the issue of climate change and mathematics education. These authors engage in study of climate

change both within the perspective of critical mathematics education and from a discursive psychology stance. A discursive approach echoes the work of Barbosa (2006), within the emancipatory perspective of mathematical modelling. In Barbosa's work discussions amongst students, that went beyond the pure mathematics of the model being developed, were central to the process of supporting students to become critically engaged citizens. However, a discursive approach represents a departure in the sense of moving away from a focus on desired or ideal methods of classroom organisation, something echoed in several papers in this special issue as in keeping with the current times of partiality and paradox (see Coles, this issue; Mikulan and Sinclair, this issue).

The sparsity of existing literature suggests there is much work to be done in this field. In preparing this introduction, we have not been exhaustive in our search and a comprehensive survey of what has been done within mathematics education, linked to the living world, would be a welcome addition. The literature discussed in the papers in the special issue offers a good starting point for this. With these thoughts in mind, we now move to an elaboration of the five themes (numbered at the ends of their titles) we identified across the special issue articles.

Ethics and theories of ethics (1)

It is perhaps not surprising that questions of ethics were an important aspect of many contributions, given the invitation to contribute to a journal focused on the *philosophy* of mathematics education and that the environmental crisis presents us with far reaching questions about what actions ought to be taken in response to it. The articles in the SI add to a growing concern for issues of ethics in mathematics education. Recent contributions have drawn on ethical thought of, variously, Levinas, Bakhtin and post-structuralist ethical thinking (see, for example: Atweh 2014; Atweh & Brady 2009; Boylan 2016; Ernest 2012; Roth 2013; Walshaw 2013). It is notable that, taken together, the papers in this special issue further extend the range of ethical sources for mathematics educators to consider.

Savard's article offers an example of a project taking place in the context of a school that was part of a movement - Green Institution Brundtland - that aimed to promote ecocitizenship including values of cooperation, equity, solidarity and respect. Thus, it offers an example of a, potentially, ethically informed intervention, though she reports a gap between teachers' sensitivity to sustainability issues and the espoused values that the school had committed to.

Steffensen offers a review of literature on critical mathematics education and post-normal science (PNS). This follows recent engagement by mathematics educators and PNS (Hauge and Barwell 2017). Many of the themes addressed by proponents of PNS resonate with those highlighted in critical mathematics education, including the importance of values and ethical concerns. However, Steffensen highlights the different nuances given to the importance of plurality of perspectives and the importance of democratization of debates about climate change.

As a counterpoint to this, Abtahi, Gøtze, Steffensen, Hiis Hauge and Barwell offer a valuable summary of some of the current 'mainstream' discussion of the ethics of climate change. They discuss how the complexity of the distributed relationships between what actions produce climate change and the effects of those actions necessarily leads to an uncertainty that challenges ethical thinking. Possibilities to address this include the development of a collective vision of a desirable future that can motivate action towards it, an ethics of the commons and a Confucian relational ethics. Informed by these perspectives there are echoes of previous discussions of the importance of responsibility (Atweh 2014; Atweh & Brady 2009) and the ethical dilemmas that may arise (Boylan 2016).

Karrow, Khan and Fleener agree that addressing the mathematical relationship with climate change is a central ethical imperative for mathematics education. From an epistemological perspective, rooted in complexity theory, they argue that dominant ways of knowing in school should be challenged to take into account interconnectedness, relationship and uncertainty. Mathematics education has a role in critiquing what *is,* and so orientating "consciousness to what ought to be" (p.39), alongside the potential to support the development of an ethic of care, founded on characteristics or virtues of care, patience, nurturing, self-sacrifice.

Wolfmeyer and Lupinacci also critique dominant logics of mathematics education and argue for an ecocritical alternative that interrelates issues of social and environmental justice. The 'ecojustice' framework is informed by ecofeminist ethics that seeks to identify, make transparent and counter 'logics of domination'. Given the power of critiques of the gendered nature of school mathematics that entail a disembodied rationalism, it is perhaps surprising that ecofeminist thought has not been more prominent in mathematics education that has addressed sociopolitical issues. Ecofeminist thinking also influences Boylan's argument for a mathematics education for ecological selves. However, important too are new materialist conceptions as the basis for a relational ethics that can inform mathematics education.

Whilst Boylan seeks to bring together different ethical perspectives, Mikulan and Sinclair, drawing on Whitehead and Deleuze focus attention on the ethical implications of rethinking the nature of mathematics itself (discussed more below). The implication of this is that Boylan's concern for ecological selves is misplaced as it remains overly anthropocentric. They argue that the Anthropocene and the possibility of human extinction requires thinking about education including mathematics education in very different ways - and in particular not seeking to reform it from an ethical imperative of sustaining human life.

Gutiérrez takes a different approach. She draws on indigenous epistemology, ontology and values to consider how mathematics education can support a sense of, in summary and oversimplified - interdependence (‘In Lak'ech’ in Mayan), indeterminacy (‘Nepantla’ in Nahautl) and reciprocity. There are resonances here with some recent influences of European relational ethical thought but from an indigenous/post-colonial perspective.

Extending and relating to critical mathematics education (2)

A number of the papers, in different ways, argue that critical mathematics education (CME) should be extended, not only by giving greater attention to ecological issues, but also in how the purpose and scope of critical mathematics education is theorised. Wolfmeyer and Lupinnacci start from an EcoJustice educational framework (Matusewicz et al. 2015) to develop an ecocritical framework informed by ecofeminism. They use this to critique initiatives in STEM education in which environmental concerns are used as a vehicle for student motivation as much as engagement in substantive ecopolitical thinking. They also consider the relationship between their more generic curriculum studies framework and existing relevant examples of mathematics education scholarship, including those that explicitly reference critical mathematics education. They argue for the importance of linking social and environmental issues together and develop an example that addresses incarceration of humans and non-humans in order to surface logics of domination.

Karrow, Khan and Fleener, as stated above address issues from an ethical perspective and do not directly address critical mathematics education. However, their critique of mathematics education in the context of capitalism and specifically neo-liberalism, returns to important foundational arguments in critical mathematics education about the role of mathematics and mathematics education in a market-based economy.

As already noted, Steffensen's review suggests that bringing critical mathematics education and post-normal science into dialogue can potentially enrich both traditions. Reading across these two traditions, she identifies the following key concepts: wicked problems, uncertainty, complexity, controversy, risks, multi/inter/transdisciplinarity, critical citizen, extended peer community, mathematical literacy, reflective knowing, critical agency, critique and dialogue, power, formatting power, responsibility, ethics, value, democratization and global society. Given the review approach, to readers with knowledge of critical mathematics education, many of these concepts will be familiar, however, others appear more novel in the context of mathematics education, addressing issues in new or different ways.

Gutiérrez's contribution is also novel, though framed more explicitly in relation to ethnomathematics than critical mathematics education. She examines the connections between queer and related theories that disrupt normalising discourses, the philosophy/nature of mathematics and debates about ethnomathematics, as well as issues of the relationship between mathematics education and the living world. As noted above, central to her argument and project in the paper is to introduce indigenous perspectives to discussion about the environment and mathematics as well as to mathematics education more generally. She argues for an epistemological pluralism, and one that includes different ways of knowing, through mathematics and as mathematics of other than human beings and other humans. Relational knowing, is a theme found also in Boylan's development of an ecological pedagogy that, he argues, is a necessary extension of forms of knowledge recognised in critical mathematics education.

Implications of climate change and the Anthropocene (3)

There are clearly many ecological or global or societal challenges facing the planet and humanity at the present time. It also seems clear that climate change is the one issue that exacerbates all the others (Pancost 2017) in the sense of adding extra layers of uncertainty and potentially contributing to harm and danger. So, to take just two examples, there are already water shortages in some parts of the world, climate change adds to the uncertainty and heightens risks; we are already witnessing forced human migration on a scale rarely seen before, climate change adds to the uncertainty and heightens risks. It is perhaps unsurprising, therefore, that climate change, specifically, features in a number of contributions.

Abtahi et al. start from UNESCO's draft Declaration on Ethical Principles in Relation to Climate Change. This body views the implications of climate change as the following:

- Safeguarding the interests of present and future generations
- Polluters should pay the price of the damage they cause
- Recognition of the interdependence of life on earth
- The duty to share scientific knowledge. (UNESCO n. d.)

Schooling practices are implicated clearly in the first, third and fourth of these. In arguing the question of whether mathematics education has a responsibility to engage in issues around climate change, Abtahi et al., make the point that to separate mathematics teaching and learning from politics and controversial topics is, in itself, a political stance. We cannot, as teachers, avoid taking a political position in relation to climate change – the question becomes what stance is ethical to take, linking back to the theme elucidated above.

One possible stance is stated clearly in Karrow et al., who argue for the ethical *imperative* that mathematics education concern itself with coming into relation with the planetary ecosystem and, in particular, climate change. They propose that this stance is within an ethics of care. These authors bring the tools of complexity theory to help think through how mathematics education might then respond to climate change.

In the two articles that specifically focus on climate change, there are novel images of pedagogy, a theme discussed below. Focusing on climate change also has implications for the context of schooling and even for the organisation of schooling, given that the topic cuts across many traditional subject boundaries. The challenge of such interdisciplinary work is a theme picked up in the fifth strand, below, Theory into Practice.

The full range of societal and global challenges facing the living world is linked to the recognition that we have entered a new era of geological time, the Anthropocene, marked in part by the visible effects of human activity on rock strata. The Anthropocene, as a concept, is central to two further articles, Mikulan and Sinclair, and Coles. Both these articles draw on the writer Colebrook (2016) and her depiction of the Anthropocene as a concept that finally allows us to move away from anthropocentric focused thought that was a shadow, even in the

post-humanist turn of the social sciences. Mikulan and Sinclair argue for a mathematics education that allows us to think about a future without humans. Coles takes the Anthropocene to mark the awareness that there is no longer a ‘pristine’ state to which the living world can now return. We can no longer think about ‘over-coming’ crises but rather, how we live with them as our current and permanent condition. Drawing an analogy to mathematics education, Coles argues that a parallel realisation is that it is no longer useful to invoke ideal ‘ends’ for the classroom. Both articles call for an embrace of paradox and uncertainty and for the energizing, rather than demoralizing, effect this can have.

Re-thinking mathematics, the curriculum and pedagogy (4)

All the articles share and contribute to a critique of how mathematics is conceived in school mathematics and so the mathematics curriculum and pedagogy. A number of them (Boylan; Karrow, Khan and Fleener; and Coles) suggest that it is important that complexity is taken up in the mathematics classroom. Boylan suggests that mathematics education should be reconfigured around a relational pedagogy that provides opportunities for embodiment, expression of emotionality, enchantment, ensemble experiences and expansiveness, particularly of timescapes.

Gutiérrez similarly argues for fundamentally rethinking the nature of mathematics and so the curriculum. The new proposed mathematics is represented by the name 'Mathematx' - pronounced as 'Mathematesh'. The argument is based on the need to rethink mathematics to support new relationships between humans, mathematics and the planet. The 'x' in 'mathematx' - has two sources. The first is the use of 'x' in critical/emancipatory movements linked to queering gender and identity, and the ‘x’ as a political disruption (following Malcolm X). The second is that 'x' is found in the Nahautl language. An important source both for the argument and the composition of mathematx are principles and beliefs drawn from indigenous cultures - 'In Lak'ech', Nepantla and reciprocity. Thus, her paper offers a distinctive and welcome contribution to a range of important discussions in mathematics education, including but not limited to: critiques of school mathematics, connections between critical mathematics education and queer and related theories that disrupt normalising discourses, the philosophy/nature of mathematics, debates about ethnomathematics, as well as issues of the relationship between mathematics education and the environment. As noted earlier Mikulan and Sinclair offer a provocative decentering of humans from the purpose of mathematics education: mathematics can “help us imagine the world without us". Ways of doing this are by "privileging chance encounters, singularities and ambiguities over curricular-ly developmentally and pedagogically fixed points” (p.11).

Similarly, Coles suggests that starting from desired end points in education is not helpful. Rather, by focusing on changing educational (and life) habits, new and different relationships may be explored and themselves become habitual; both these accounts start less from a design, but rather actions informed by current societal and ecological conditions. Notably, in both Mikulan and Sinclair, and Coles' suggestions as well as Gutiérrez's proposal for a new mathematics, embracing uncertainty and disruptive action are found.

Theory into practice (5)

Several articles begin by setting out philosophical stances or issues and then relate the implications directly to the classroom. A common theme across the articles that do this, is just the difficulties and tensions that arise for any teacher wanting, for example, to broach ecological issues in a mathematics lesson (see Abtahi et al.; Coles; Savard; Wolfmeyer and Lupinacci).

There are some common findings in relation to how ecological issues might be approached in classrooms. Abtahi et al. report on teachers concerned that students do not respond well to gloom and doom. Similarly, Coles reports on a teacher who realised that the line of argument: "It is your generation who need to sort out these problems" was both alienating and disengaging. And, thirdly, Boylan recognises the danger in presenting catastrophe scenarios or narratives of 'disenchanting' the world in a self-defeating manner. Abtahi et al. discuss a survey of Canadian and Norwegian mathematics teachers about how those teachers address (or not) issues of climate change. Given that the respondents were motivated enough to participate in the survey then they, presumably, represent those that are most likely to be engaged in practice on this issue. However, the findings are that teachers report being unsure how to act and, for many who would like to address climate change, they experience a variety of constraints.

Gutiérrez's re-conceptualising of mathematics, as mathematx, includes an explicit desire to intervene in reality, to usher in a living mathematics that reflects multiple knowledges and with a shift in teaching from telling to 'living alongside'. The idea of living alongside finds an echo in Boylan's suggestions of what it would mean for teaching and learning mathematics to contribute to a re-enchantment with the world, including the human within it:

- as a site to practice or develop dispositions and capacity for wonder and enchantment, with mathematics itself as the ethical Other
- when the experience of mathematics supports a conception of being human which decentres the individualist sense of self
- when mathematics is used in order to explore, engage with and deepen understanding of the other than human world (p.9).

Other authors list the needs for the future differently, reflecting the different intellectual traditions they draw from. So, Karrow et al., drawing on complexity theory, propose the need for:

> fostering an emergentist epistemology, true to the elements of randomness, unpredictability, iteration and recursiveness; and, accordingly, the commonplaces of mathematics education, including teaching, learning, and curriculum that reflect this way of coming to know. (p.22)

Any one of these lists provide important avenues for future research. And, at the same time, Savard reports on the difficulties of engaging in interdisciplinary projects in school, for example, the level of subject expertise needed to draw out subject issues from broader topics. This is an important and salutary counter-point to the hopes and aspiration of some of the articles in this collection.

Mikulan and Sinclair suggest that the very concepts of mathematics come to be seen in a new manner, if we give up on the quest for over-arching visions or grand understanding. They link to the practice of 'problem-posing' in the classroom and offer examples of what this might look like in practice. A concern raised by Coles about the introduction of environmental issues into the mathematics classroom is the potential to place students in a "double bind" in which they are encouraged to act, but given no realistic avenue in which to act, and where there is little opportunity for them to communicate about this bind. Imagining different futures, rather than dwelling on the problems of the present, is offered as one way out of these binds.

Alongside Savard, who elaborates in some detail a classroom project, another article that does this is Wolfmeyer and Lupinacci. They do not offer the detail of the teaching but more the complexity of their journey towards finding a starting point for work in a classroom that would fit their ecocritical perspective. They offer 'A story of incarceration' as an example of an activity suitable for an ecocritical curriculum. The project includes a study of:

> differential incarceration rates (worldwide) by race and class, the engineering of prisons, the panopticon, the concept of open prisons, and the incarceration of animals in three ways: agribusiness, animal experimentation, and animal entertainment

There are clear links, potentially, between this project and the suggestions of Gutiérrez, Boylan, Mikulan and Sinclair, and Karrow et al., in that the topic could allow a deepening of understanding of not just other humans but the other-than-human world. The link between human and animal incarceration seems particularly pertinent (see also Boylan (2016) for discussion of beginning teachers' responses to an activity focused on industrial poultry farming).

A final theme to draw out, from the articles that make strong links to practice, is that of time. Gutiérrez, Boylan and Karrow et al. all point to the need to slow down education. If students are to engage with care, and if teachers are to work with uncertainty as a productive

disposition, and deeper connection made through mathematics with other than human beings, then these are not things that can be packaged into small chunks of learning with explicit learning objectives. Tasks that extend over time are needed and, implicit in this, is a degree of choice for students and a necessary contingency on the part of the teacher. Conversely, Coles reports how a group of educators have experimented with creating short tasks that can create new habits around the kinds of content that can be addressed in the classroom and can be used within current dominant temporal cultures within mathematics classrooms.

Mapping a future for mathematics education in the Anthropocene?

We began this introduction noting that research and scholarship linking mathematics education and the living world was relatively sparse and the dialogue, we think is an urgent one, is only just beginning. We stated that we believed the contributions to the special issue take the conversation forward, and hope you agree. We turn now to the future and possibilities for both research and practice in this area. These are not trivial tasks and, later in this section, we offer the notion of a 'map' to help think about how we move from where we are, to where we could go. So far in this introductory review we have considered previous contributions focused on environmental issues and mathematics education and discussed themes found in papers in the special issues. We now turn to considering the special issue as a whole.

The authors draw on multiple strands of thinking to link mathematics education to global issues facing the planet and these strands provide fertile ground for further development. The strands include: critical mathematics education, post-normal science, deep ecology, ecofeminism, post-colonial thinking, new materialism, the philosophy of Deleuze and later thinkers such as Colebrook, systems thinking and complexity science. These articles contain many suggestions for the future as we have tried to draw out in our synthesis above. We are struck by all the authors in this collection being outward facing, in terms of where they are drawing inspiration. What we mean is that the references in the articles are generally not within mathematics education. This stance is surely an important one in terms of facing the kinds of crisis being discussed here; whatever perspectives are taken on the causes, solutions or alternatively learning to live with the current situation, responses cannot be contained within disciplinary boundaries. We believe that addressing ecological issues can similarly generate engagement with important currents of philosophical and social theoretical thought that have hitherto not been part of conversations in mathematics education.

However, on a less optimistic note, it is sobering that the nine papers in this special issue represent a significant increase in the number of texts addressing similar issues. It is also sobering that whilst a number of papers do report examples from practice, these are limited and fragmentary. In part, this may be because this is a special issue of the journal of the *philosophy* of mathematics education. Borko (2004) proposes a three-fold scale for forms of research on teacher professional development: studying programmes on a single site; studying a single programme enacted at more than one site; or comparing different

programmes at different sites. Applying this to research on environmental issues and mathematics education, it is clear that as yet there is still some way to go to develop a body of research or, for that matter, programmes focused on single sites.

We also note that it is more than 30 years since Skovsmose's (1985) seminal work discussing the relationship between critical education and mathematics education. Since then critical mathematics education has developed into a recognised tradition within mathematics education: a research programme; multiple texts; conferences such as Mathematics Education and Society where it is represented; as well as, influencing and being influenced by related movements/networks such as mathematics for social justice in the US and ethnomathematics. However, certainly in the UK, it has limited influence on school mathematics or mathematics teacher education.

Similarly, environmental education has a long history and apparently greater success than critical mathematics education, as evidenced by its embrace from transnational organisations such as UNESCO (2006). But still, practitioners consider they are "primarily a cottage industry out in the hinterlands of the formal and informal educational systems—struggl[ing] to gain purchase despite decades of effort on many fronts" (Henderson and Zarger 2017 p.285). However, we think it is significant that in general the contributions to this special issue are informed by critical perspectives, or perhaps in the case of Mikulan and Sinclair and Coles, post-critical perspectives.

Within environmental education there are existing frameworks that support taking an overview of the papers in the special issue, and other recent contributions. Various typologies have been proposed (see Chubbuck 2010; Gorski 2009; Kumashiro 2000) and have been synthesised by considering three orientations (Boylan 2017):

- conservative: focused on improving learning opportunities and outcomes for those who experience injustice, whilst maintaining education as it is (education for and about the other).
- socially liberal: recognises that education needs transforming, that teaching should be culturally sensitive and multi-culturally competent; privilege and othering should be challenged (Kumashiro 2000).
- critical: recognises that social structures beyond school need transforming, socio-political contexts need to be recognised within education, thus, seeking to allow change for students and society, and interrupt processes of othering (Boylan 2017 p.370).

This typology can be extended to consider issues of ecological justice. More conservative orientations will include environmental issues in the curriculum but with an underlying message or paradigm of individual responsibility and amelioration. Socially liberal responses might lead to a re-imagining of the curriculum as a whole and a recognition that there are issues of social justice bound up with environmental issues. Critical responses envisage radically different eco-related pedagogies and a recognition that ecological justice requires

profound social transformation. We might add a fourth category of post-critical response, that begins to question the focus on particular 'ends' in the first place, and attempts to engage in disruptive and creative actions within an unendingly broken and compromised world.

Table 1: Mapping possible future developments of practice and research for mathematics education and the living world.

	Development of curriculum, pedagogy and practice	**Teacher education and professional development**	**Creation and fostering of networks**	**Research including theoretical work and empirical study**
Starting points	Small-scale, classroom-based projects take place, using resources that open up mathematics to global issues, in the context of a subject-oriented curriculum.	Small-scale programmes are developed, on isolated sites, linking learning to teach mathematics with a questioning of the role of mathematics in the world.	A mathematics education conference hosts a symposium on mathematics education and the living world. Mathematics educators connect with others engaging in environmental education. There is a global sharing, online, of resources and experiences.	A research agenda is established to explore ways in which mathematics classrooms can become sites for exploring uncertainty, risk and global, ecological issues.
Beginning to influence mathematics education	Some schools experiment with a curriculum in which mathematical development is balanced with interdisciplinary work. A range of models are tried and tested globally and the results are shared.	Teacher training courses internationally begin to include how the study of mathematics can relate to wider ecological issues and the living world.	Scholars from across the globe document, trial and share experiences of linking mathematics education to the living world, both within their own communities and internationally.	The first large scale research project is funded, to study the ways in which mathematics education can broaden to encompass ecological awareness.
Mathematics education changes to address the living world	One country changes its national curriculum to put ecological awareness and stewardship of the planet at the core of all teaching and learning.	Teacher education and professional development, that supports inter-disciplinary ways of working in schools, becomes a norm, on multiple sites.	Research in this field, internationally, matures providing a range of forums for the on-going linking of teaching and learning mathematics to inter- and trans-disciplinary ways of working and the living world.	Multiple schools globally become sites where new research takes place on trans-disciplinary questions and issues. Students and teachers and researchers collect evidence in their own communities about change, documenting oral histories and sharing findings.

It is in a critical spirit that we offer a “map” (Table 1, above) for how we might move from the current context of schooling in the affluent countries of the world, towards some of the images of mathematics education in this special issue.

In a sense, the map is a thought experiment and we offer it in the spirit of continuing dialogue rather than in any normative gesture. We consider four areas: development of curriculum, pedagogy and practice; teacher education and professional development; creation and fostering of networks; research including theoretical work and empirical study. The content of the table provides some possible 'landscape features' that might be found and by no means are an exhaustive set of possible activity. The rows aim to represent a deepening engagement as they go down and there are, no doubt, many intermediate steps and alternative routes and rows before and beyond; we believe we are likely on, or before, the first row in each area. We imagine that readers might consider the map either as they are reading an article, or afterwards, or as they are conducting their related research questions. We invite readers to consider their own work and where they might situate themselves on the map? What challenges are most pertinent to connecting your work to the map? For example, we (authors) are involved in the professional development of teachers and the map provokes us to consider how our own work currently asks or expects teachers to question the role of mathematics in the world, and whether there are ways we can expand what we are doing, having read the articles in this issue. As a reader who is interested, or invested, or an expert on an issue, what seems to be missing from the map and why is that important? We would love to hear from you.

Conclusion

As noted already, there have to date been sparse connections between global ecological issues and mathematics education; a key purpose in putting together this special issue has been an attempt to open out and broaden the strands of conversation that have existed. We are grateful to Paul Ernest and the Philosophy of Mathematics Education Journal for supporting this special issue, and so supporting the authors' contributions to enter the mathematics education field soon after writing.

The papers in this special issue suggest the possibility of enriching or extending the sociopolitical turn in mathematics education (Gutiérrez 2010/13) with an ecological turn to contribute to the development of critical ecopedagogy (Kahn 2017). These contributions point to the importance of new forms of mathematical literacy to support global citizenship, for example, by disrupting the myths of hegemonic economics and to allow people to engage in more informed ways with personal, community, societal and global challenges. Extending the content of critical mathematics education to embrace ecological concerns is a necessary task for mathematics educators. However, it is not sufficient. The ecological crisis also gives impetus for the reconstitution of mathematics in school, in ways that help support different relationships than currently predominate, both to mathematics itself and to others in the living world.

References

Abtahi, Y., Gotze, P., Steffensen, L., Hauge, K. H., and Barwell, R. (this issue) 'Teaching climate change in mathematics classrooms: An ethical responsibility', *Philosophy of Mathematics Education Journal,* No.33.

Alrø, H. Ravn, O. and Valero, P. (2010) 'Inter-viewing critical mathematics education', in, H. Alrø, O. Ravn, and P. Valero (eds.), *Critical Mathematics Education: Past, Present and Future*, Sense Publishers: Rotterdam, 1–9

Atweh, B. (2014) 'Mathematics education and democratic participation between the critical and the ethical: a socially response-able approach', in O. Skovsmose and B. Greer, Eds., *Opening the cage: critique and politics of mathematics educatio*n, Rotterdam: Sense Publishers, 325–342.

Atweh, B. and Brady, K. (2009) 'Socially response-able mathematics education: implications of an ethical approach', *Eurasia Journal of Mathematics Science and Technology Education,* Vol. 5, No.3, 267–276.

Barbosa, J.C. (2006) 'Mathematical Modelling in classroom: a critical and discursive perspective', *Zentralblatt für Didaktik der Mathematik,* Vol.38, No.3, 293-301.

Barwell, R. (2013) 'The mathematical formatting of climate change: critical mathematics education and post-normal science', *Research in Mathematics Education,* Vol.15, No.1, 1-16.

Borko, H. (2004) 'Professional development and teacher learning: Mapping the terrain', *Educational Researcher*, Vol.33, No.8, 3–15.

Boylan, M. (2016) 'Ethical dimensions of mathematics education', *Educational Studies in Mathematics*, Vol.92, No.3, 95-409.

Boylan, M. (2017) 'Disrupting Oppressive Views and Practices through Critical Teacher Education: Turning to Post-Structuralist Ethics'. In D. J. Clandinin, and J Husu, (Eds.), *The SAGE Handbook of Research on Teacher Education,* Thousand Oaks, Sage.

Boylan, M. (this issue) 'Towards a mathematics education for ecological selves', *Philosophy of Mathematics Education Journal*, No.33.

Chubbuck, S. (2010) 'Individual and structural orientations in socially just teaching: Conceptualization, implementation, and collaborative effort', Journal of Teacher Education, Vol.61, No.3, 197–210.

Colebrook, C. (2016) 'A Grandiose Time of Coexistence: Stratigraphy of the Anthropocene'. *Deleuze Studies*, Vol.10, No.4, 440–454. https://doi.org/10.3366/dls.2016.0238

Coles A. (this issue) 'Habits and binds of mathematics education in the Anthropocene', *Philosophy of Mathematics Education Journal*, No.33.

D'Ambrosio, U. (1985) 'Ethnomathematics and its place in the history and pedagogy of mathematics', *For the Learning of Mathematics*, Vol.5, No.1, 44-48.

D'Ambrosio, U. (2010) 'Mathematics education and survival with dignity'. In H. Alrø, O. Ravn & P. Valero. (eds.) (2010). *Critical mathematics education: Past, present and future* (pp. 51-63). Rotterdam: Sense Publishers.

Ernest, P. (2012) 'What is our First Philosophy in Mathematics Education?', *For the Learning of Mathematics,* Vol.32, No.3, 8-14.

Ernest, P., Sriraman, B., & Ernest, N. (eds.). (2016). *Critical mathematics education: Theory, praxis and reality. Charlotte,* NC: Information Age Publishing.

Freire, P. (1970) *Cultural action for freedom*. Cambridge, Massachussetts, USA: Harvard educational review.

Gellert, U. (2011). 'Now it concerns us! A reaction to sustainable mathematics education', *For the Learning of Mathematics 31* (2): 19-20.

Gorski, P.C. (2009) 'What we're teaching teachers: An analysis of multicultural teacher education coursework syllabi', *Teaching and Teacher Education*, Vol.25, No.2, 309–318.

Gutiérrez, R. (2010/2013) 'The sociopolitical turn in mathematics education', *Journal for Research in Mathematics Education*, Vol.44, No.1, 37-68. Appeared online on 2010; in print in 2013.

Gutiérrez, R. (this issue) 'Living Mathematx: Towards a Vision for the Future', *Philosophy of Mathematics Education Journal,* No.33.

Hauge, K. H., & Barwell, R. (2017) 'Post-normal science and mathematics education in uncertain times: educating future citizens for extended peer communities', *Futures, (in press)*. doi:http://www.sciencedirect.com/science/article/pii/S0016328717300484

Hauge, K. H., Sørngård, M. A., Vethe, T. I., Bringeland, T. A., Hagen, A. A., & Sumstad, M. S. (2015). 'Critical reflections on temperature change'. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education (CERME9, 4-8 February 2015)* (pp. 1577-1583). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

Henderson, J. A., and Zarger, R. K. (2017) 'Toward political ecologies of environmental education, *The Journal of Environmental Education,* Vol. 48, No.4, 285-289.

Kaiser, G., and Sriraman, B. (2006) 'A global survey of international perspectives on modelling in mathematics education', *ZDM,* Vol.38, No.3, 302-310**.**

Karrow, D., Khan, S., Fleener, J. (this issue) 'Mathematics Education's Ethical *Relation With* And *Response To* Climate Change', *Philosophy of Mathematics Education Journal,* No.33.

Kahn, R. (2010) 'Producing crisis: Green consumerism as an ecopedagogical issue', in J. Sandlin, and P. McClaren P. eds. *Critical pedagogies of consumption: Living and learning in the shadow of the "shopocalypse"*, New York: Routledge, 47-57.

Kumashiro, K.K. (2000) 'Toward a theory of anti-oppressive education', *Review of Educational Research*, Vol.70, No.1, 25–53.

Monbiot, G. (2017) 'Forget 'the environment: we need new words to convey life's wonders', *The Guardian*, 9th August 2017. Retrieved from: https://www.theguardian.com/commentisfree/2017/aug/09/forget-the-environment-new-words-lifes-wonders-language

Mikulan, P. & Sinclair, N. (this issue) 'Thinking mathematics pedagogy stratigraphically in the Anthropocene', *Philosophy of Mathematics Education Journal*, No.33.

Pancost, R. (2017) *Resilience in a climatically uncertain world*. Presentation to the Post Graduate Certificate of Education course, University of Bristol. Bristol: Cabot Institute.

Renert, M. (2011) 'Mathematics for life: Sustainable mathematics education', *For the Learning of Mathematics*, Vol. 31, No. 1, 20-26.

Roth, W.-M. (2013) 'Toward a post-constructivist ethics in/of teaching and learning', *Pedagogies An International Journal*, Vol.8, No.2, 103–125.

Savard, A. (this issue) 'Implementing inquiry-based learning situation in science and technology: What are elementary school teachers' learning intensions and mathematics?' *Philosophy of Mathematics Education Journal,* No.33.

Skovsmose, O. (1985) 'Mathematics education versus critical education', *Educational Studies in Mathematics*, Vol.16, 337-354.

Skovsmose, O. (1994) *Towards a Philosophy of Critical Mathematics Education*. Dordrecht: Springer.

Steffensen, L. (this issue) 'Critical mathematics education and post-normal science – a literature overview', *Philosophy of Mathematics Education Journal,* No.33.

Walshaw, M. (2013) 'Post-structuralism and ethical practical action: Issues of identity and power', *Journal for Research in Mathematics Education*, Vol.44, No.1, 100-118.

Wolfmeyer, M. and Lupinacci, J. (this issue) 'A mathematics education for the environment: Possibilities for interrupting all forms of domination', *Philosophy of Mathematics Education Journal,* No.33.

UNESCO. (2006) *Framework for the UNDESD International Implementation Scheme*. Retrieved from Paris: http://unesdoc.unesco.org/images/0014/001486/148650e.pdf

UNESCO (n. d.) *Preparation of a Declaration on Ethical Principles in relation to Climate Change.* Retrieved from: www.unesco.org/new/en/social-and-human-sciences/themes/comest/ethical-principles/