



Crowley, S., Porter, R., Taunton, D., & Wilson, P. (2018). Modelling of the WITT wave energy converter. *Renewable Energy*, *115*, 159-174.
<https://doi.org/10.1016/j.renene.2017.08.004>

Peer reviewed version

Link to published version (if available):
[10.1016/j.renene.2017.08.004](https://doi.org/10.1016/j.renene.2017.08.004)

[Link to publication record in Explore Bristol Research](#)
PDF-document

This is the accepted author manuscript (AAM). The final published version (version of record) is available online via Elsevier at <https://doi.org/10.1016/j.renene.2017.08.004> . Please refer to any applicable terms of use of the publisher.

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/pure/about/ebr-terms>

Modelling of the WITT wave energy converter

S. Crowley^a, R. Porter^{a,*}, D.J. Taunton^b, P.A. Wilson^b

^a*School of Mathematics, University of Bristol, University Walk, Bristol, BS8 1TW, UK.*

^b*Faculty of Engineering and the Environment, University of Southampton, Lanchester Building, Southampton, SO17 1BJ, UK.*

Abstract

The paper describes the theoretical modelling and experimental validation of a novel design of ocean wave energy converter which is comprised of a floating, moored, spherical hull containing a mechanical pendulum arrangement from which power is taken when excited by incident waves. Experimental results are shown to compare favourably with those predicted by the theory. An explicit expression is derived for the capture width of the proposed device in terms of physical and hydrodynamic parameters. This exposes the multiple resonant characteristics of the device which enable it to operate effectively over a broad range of wave periods. The subsequent efficient computations allows a numerical optimisation of the design to be performed over a large space of device parameters and model sea spectrum. The work is focussed towards producing reliable estimates for the power capacity of different sized devices deployed at the EMEC site in Scotland. Predictions compare favourably with existing wave energy converter concepts.

Keywords: wave energy converter, floating sphere, internal pendulum, coupled resonances

1. Introduction

1 The WITT (Whatever Input to Torque Transfer; see <http://www.witt-energy.com/>; Fig. 1) is
2 a proprietary mechanical device for converting kinetic energy into electrical energy. It is comprised
3 of a heavy compound pendulum connected through a gearbox so that its rotary motion about
4 either of two perpendicular horizontal axes is transferred to a single unidirectional output through
5 a primary axis from which the energy of motion can subsequently be harvested. The WITT is
6 currently being considered for use in a range of small to large scale applications.

7 The authors on this paper have been involved as part of a wider project to investigate the
8 feasibility of using a WITT housed within a sealed hull to harness the motions induced by ocean
9 waves and convert them into electrical energy. The present paper details a theoretical model which
10 has been developed to describe the operation of a WITT Wave Energy Converter (WITT WEC)
11 and experiments performed to validate its predictions.

12 The concept of using a mechanical device with heavy counterweights operating inside a sealed
13 hull to absorb wave energy is not new; for example the SEAREV [see 5] and the [18] Penguin.
14 The principle underpinning the successful capture of ocean wave energy lies in amplifying and

*Corresponding authors

Email address: richard.porter@bristol.ac.uk (R. Porter)

15 converting the energy in low frequency, low amplitude waves. The Wello Penguin device appears
16 to do this using the instability of a hull to pitching and rolling motions of a large weight mounted
17 on a vertical axis. In contrast, SEAREV uses the more conventional approach in WEC design of
18 exploiting resonance of the hull and an internal pendulum rotating about a horizontal axis. It is
19 this latter approach which also underpins the current design analysis of the WITT WEC and one
20 which allows us to use existing methodology based on linearised (or small amplitude) theory to
21 predict power capture and device motions across an irregular sea state. It is the primary purpose
22 of this paper to demonstrate how this is done and to provide initial estimates as to the potential
23 power output from a WITT WEC.

24 There are several ways in which the WITT WEC design varies from the SEAREV design. Both
25 are designed to operate in the surface of the ocean, but the ability of the pendulum in the WITT
26 to operate about both horizontal axes allows it to extract power from from all wave headings.
27 Moreover, the design of the WITT WEC integrates the sealed hull with a heavy chain catenary
28 mooring system which is crucial in providing device resonances. In contrast, the SEAREV mooring
29 is not an active design component. Thus, we will show later the WITT WEC can exhibit resonance
30 at three distinct periods and this consequently gives the WITT WEC a broadbanded response
31 to incident waves. In contrast the SEAREV [4, Fig. 4], [5, Fig. 12] possesses just two device
32 resonances. There are other differences, perhaps the key one being that the WITT is able to rotate
33 fully through 360 degrees about either axis which means its motion is not mechanically limited, a
34 common problem in converter design.

35 The theoretical development of the WITT WEC design uses a number of assumptions and
36 approximations which are outlined through the paper. Many of these are based on the use of
37 first order, or small amplitude, theory. These assumptions are made in the hydrodynamic theory
38 describing the manner in which waves interact with the sphere, requiring wave steepness and device
39 motions to be sufficiently small. They are also made in the theoretical model of a catenary mooring
40 system whose first order approximation results in a Hooke's law relation. Moreover, pendulum
41 motions are also assumed to be small to allow linearisation of the underlying mechanical equations.
42 Finally, we assume a simple linear power take-off (PTO) system. In addition, various simplifying
43 model assumptions are made throughout justified as having captured the most important effects.
44 These include, for example, constraining the pendulum to move about only one of its two axes on
45 the assumption that a deployed device would be aligned to operate in a marine environment with
46 a well-understood directional sea state.

47 The effect of these assumptions are tested by comparing device RAOs to a series of experiments,
48 described later in the paper, which again focus on the main operational elements of the model rather
49 than a fully developed scale model of the WITT WEC.

50 The theoretical work is most closely related to recent work by two of the current authors on a
51 theoretical WEC design based on a similar principle. Thus [6] considered a long submerged cylinder
52 containing a heavy pendulum allowed to rotate around a single axis and which was tethered to the
53 sea bed and whose cylindrical hull operated as an inverted pendulum using an assumed buoyancy
54 acting to provide a restoring force. That work demonstrated that multiple resonances could be
55 achieved and an optimisation over physically realisable parameters allowed the proposed device to
56 operate close to a theoretical maximum over a broad range of wave periods (roughly 5-11s). The
57 design of [6] is quite different to the WITT WEC the former device principally acting as an attenuator
58 and the latter as a point absorber. However, the concept of using a counterweight is common to
59 both and mathematical ideas developed in [6] can be extended to the moored floating hull design
60 of the WITT WEC. The mathematics here is more complicated with four degrees of freedom here



Figure 1: A photo of a prototype of the WITT pendulum and gearbox mechanism

61 (hull pitch, heave, surge and pendulum pitch) replacing the two (device surge and pendulum pitch)
62 in [6]. Nevertheless, it is shown that explicit expressions for capture width (a standard measure
63 of power absorption capacity) can be attained in terms of physical and hydrodynamic coefficients.
64 This allows us to theoretically identify aspects of the design which are useful in describing the device
65 operation. Moreover, the computational efficiency offered by the theoretical results used allows us
66 to numerically optimise over physical design parameters and over a realistic wave energy spectra.

67 Related work on the use of moored spheres as a wave power absorber include [16] who considered
68 a sphere held submerged below the surface with the PTO incorporated into a mooring system. Also,
69 [7] made a theoretical assessment of the impact on power absorption of placing motion constraints
70 on the operation of WECs including a semi-submerged spherical WEC. That study showed, for
71 example, that a surging sphere whose motion is limited to a wave amplitude cannot extract more
72 than 70% of the power incident on the sphere. This rises to 108% when surge motion is limited to
73 two wave amplitudes.

74 The layout of the paper is as follows. In Section 2, we describe the proposed operation of the
75 WITT WEC and define some of the parameters adopted in the modelling. In Section 3, an outline
76 of the mathematical model is presented focussing on the mechanics and the derivation of expressions
77 useful for calculating the power. The derivation of the equations of motion, the modelling of the
78 mooring system coefficients and the description of a model sea state are relegated to Appendices but
79 will be useful to researchers wanting to follow in detail the modelling and its assumptions. Section
80 4 describes the modelling used in the wave tank tests and compares experimental results with those
81 predicted by the model. In Section 5 we use the model to predict results for an optimised full scale
82 device and finally in the Conclusions summarise the paper and discuss the proposed design in a
83 wider context of WEC design (see [1]) and describe the direction in which further work will be
84 pursued.

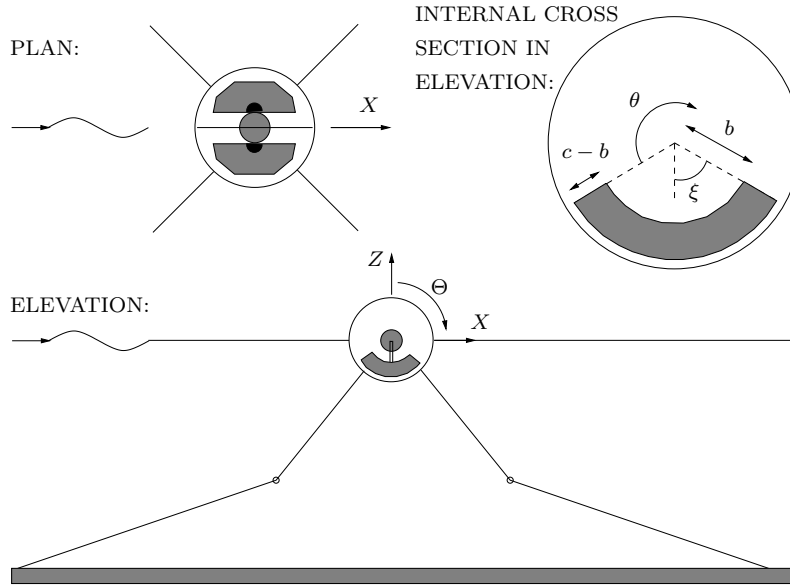


Figure 2: Sketch in plan, elevation and internal cross section (in elevation) of the system, showing directions of modes of motion and definition of the pendulum geometry.

85 **2. Device description**

86 This paper addresses the modelling of a specific embodiment of the proposed WITT WEC in
 87 which a WITT device is placed within a semi-immersed sealed spherical hull, which is able to move
 88 in heave, surge, and pitch but is restrained by a four-point catenary mooring system in which splayed
 89 heavy chains connect the hull of the WEC to the sea floor. This mooring system has the obvious
 90 practical role of preventing the WITT WEC from drifting away from its installation site, but it
 91 also supplies spring restoring forces to the device when it moves in response to waves. A realistic
 92 mooring would include clump weights along an extended section of the mooring line resting on the
 93 sea bed which would provide a stiffening of the restoring force for larger device motions anticipated
 94 under heavier seas. In our model, we use point masses placed on inextensible light lines to represent
 95 the effect of a heavy catenary chain (see Fig. 2).

96 Internal to the sphere the WITT pendulum which is designed to rotate about both horizontal axis
 97 by any amount (see Fig. 1). We model it as a compound pendulum which is vertically axisymmetric
 98 formed by an annular sector in cross section. The gearbox within the WITT device selects the input
 99 possessing the greatest angular velocity from the two axes to drive the output rather than combining
 100 them additively. In practice, this means that a WITT with its primary axis aligned with a principal
 101 direction of incoming waves will operate predominately in a single degree of freedom (which we refer
 102 to as pendulum pitch) and will only extract energy from pendulum roll motion for wave headings
 103 beyond 45° or in the possible event of the onset of parametric instabilities. Therefore, in this
 104 model, the pendulum is allowed to rotate in pitch about just one central horizontal axis of the
 105 sphere aligned with the crests of the predominant incoming waves (see Fig. 2). Power is assumed
 106 to be extracted from a linear damper which acts in proportion to the relative angular velocity of
 107 the pendulum with respect to that of the sphere. In our model, two point masses are positioned

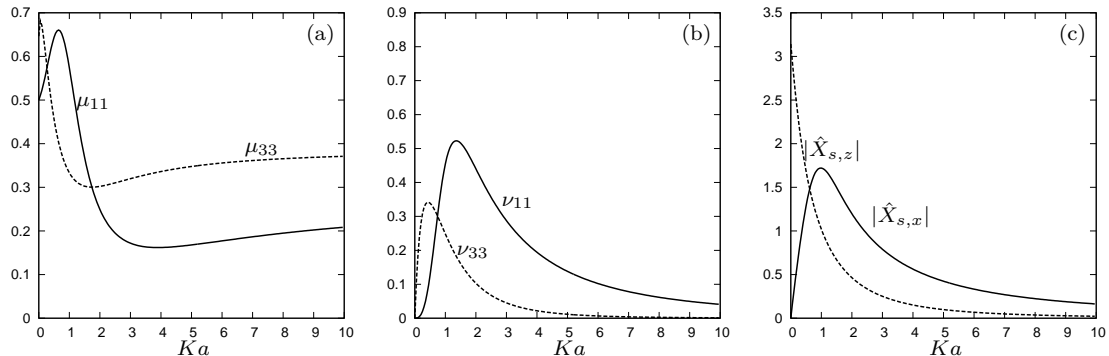


Figure 3: In (a), (b) the non-dimensional added mass and radiation damping coefficients against a dimensionless frequency parameter, Ka , for a surging (solid lines) and heaving (dashed lines) sphere. In (c) the magnitude of the non-dimensional surge (solid) and heave (dashed) wave exciting force.

108 at the centre and bottom of the sphere to represent the WITT gearbox and power take-off (PTO)
 109 machinery and ballast respectively. The centre of gravity of the hull thus lies a distance below the
 110 centre of the sphere. Resolving the vertical forces on the sphere and mooring lines determines the
 111 mass of ballast required for the device to be semi-submerged when in equilibrium.

112 3. Mathematical modelling

113 The mathematical model of the device described in Section 2 can be broken down into three
 114 components: (i) the hydrodynamic response of a sphere in waves (described later); (ii) the mathe-
 115 matical model of the mooring system (described in detail in Appendix A); and (iii) the dynamics
 116 of the fully coupled mechanical system of sphere - mooring - pendulum (described in detail in
 117 Appendix B).

118 Underpinning each element is a small amplitude assumption, a routine first step in the analysis
 119 of WECs as it allows the equations describing the device motion to be linearised and thus solutions
 120 can be sought by factorising a time-harmonic variation with radian frequency ω from the dynamic
 121 variables. The small amplitude assumption manifests itself in different ways when applied to each
 122 of the different elements of the design. Principally, the incident waves which excite the motion are
 123 assumed to be of small steepness. We also require device motions to be small enough to justify the
 124 use of linearised hydrodynamic theory and linearised elastic behaviour in the mooring model. The
 125 response of the internal pendulum must also be of sufficiently small amplitude. These assumptions
 126 will all be tested at device resonance which is an integral part of WEC design.

127 It is shown in Appendix B that the motion can be described by two uncoupled sets of equations.
 128 The vertical motion is described by the third equation in (B.15) where $\text{Re}\{V e^{-i\omega t}\}$ is the vertical
 129 velocity and does not contribute to power production under the small amplitude assumption. The
 130 surge and pitch motions of the hull, encoded in the time-independent quantities U and Ω , are
 131 coupled to the rotation of the internal pendulum in the equation of motion

$$-i\omega M U = X_w - \frac{i}{\omega} (C + K) U - \gamma G U \quad (1)$$

132 (see Appendix B). The complex velocity vector U is given by $U = (U, \Omega, \Omega_r)^T$ where Ω_r encodes
 133 the rate of rotation of the pendulum relative to the hull. The inertia matrix M , mass restoring

134 force matrix C and mooring force matrix K are defined by (B.19) and (B.20) whilst G is also
 135 given by (B.20) and is pre-multiplied in (1) by the PTO parameter, γ , which we are free to tune.
 136 All matrices are real and symmetric and determined by geometric parameters of the problem.
 137 The vector $\mathbf{X}_w = (X_{w,x}, 0, 0)^T$ where $X_{w,x}$ is the time-independent surge component of the wave
 138 exciting force on the hull. The heave exciting force $X_{w,z}$ is also needed to determine V in (B.15).
 139 Both can be decomposed using linearity in the usual way into forces on the static hull and radiation
 140 forces due to the motion of the hull. Thus we write

$$X_{w,x} = X_{s,x} \cos \beta + (i\omega A_{11} - B_{11})U - D_{11}U \quad (2)$$

141 where β is the incident wave direction and

$$X_{w,z} = X_{s,z} + (i\omega A_{33} - B_{33})V - D_{33}V. \quad (3)$$

142 The forces $X_{s,x}$ and $X_{s,z}$ and the added mass and radiation damping coefficients A_{ii} and B_{ii} are
 143 calculated here following [11] which assumes water of infinite depth. Note there are no wave forces
 144 in the pitch mode of motion owing to the hull being spherical. Forces accounting for hydrodynamic
 145 drag resulting from the differential flow between the fluid and the sphere are modelled in (2) and
 146 (3) by terms linearly proportional to device velocities and with constants of proportionality D_{11}
 147 and D_{33} . A brief description of the derivation of this drag model and estimates to the values of D_{ii}
 148 will be discussed later in §3.4.

149 Tabulated values of the added mass and radiation damping coefficients are provided in [11], but
 150 the wave exciting forces are not calculated explicitly. In Fig. 3 the non-dimensional equivalents of
 151 these quantities are plotted against the non-dimensional wave frequency $Ka \equiv \omega^2 a/g$, where

$$\mu_{ii} = \frac{A_{ii}}{M_w}, \quad \nu_{ii} = \frac{B_{ii}}{M_w \omega}, \quad \hat{X}_{s,\alpha} = \frac{X_{s,\alpha}}{\rho a^2 \omega}, \quad (4)$$

152 with $\alpha = x, z$ for surge and heave respectively and where $M_w = \frac{2}{3}\pi\rho a^3$ is the mass of water, density
 153 ρ , displaced by the sphere which has radius a .

154 3.1. Device motion

155 Using equation (2), the equation of motion (1) can be inverted to give the response as

$$\mathbf{U} = \mathbf{E}\mathbf{X}_s, \quad (5)$$

156 where $\mathbf{X}_s = (X_{s,x} \cos \beta, 0, 0)^T$,

$$\mathbf{E} = (\mathbf{Z} + \gamma\mathbf{G})^{-1}, \quad (6)$$

157 and

$$\mathbf{Z} \equiv \mathbf{B} - i\omega(\mathbf{M} + \mathbf{A} - (\mathbf{C} + \mathbf{K})/\omega^2). \quad (7)$$

158 after defining

$$\mathbf{A} = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} B_{11} + D_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

159 For the heave response, the third equation in (B.15) is similarly used with (3) to give

$$V = \frac{X_{s,z}}{(B_{33} + D_{33} - i\omega(M + A_{33} - (K_{33} + \rho g S)/\omega^2))} \quad (9)$$

where M and S are defined in Appendix B and K_{33} is defined in (A.14). Subsequently, the sphere and pendulum motions are given by

$$\begin{aligned} X(t) &= \text{Re}\{(gA/\omega^2)Ue^{-i\omega t}\}, & \Theta(t) &= \text{Re}\{(gA/\omega^2)(\Omega/l)e^{-i\omega t}\}, \\ Z(t) &= \text{Re}\{(gA/\omega^2)Ve^{-i\omega t}\}, & \theta(t) &= \text{Re}\{(gA/\omega^2)((\Omega + \Omega_r)/l)e^{-i\omega t}\}. \end{aligned} \quad (10)$$

160 following (B.14) where A is the wave amplitude, g is gravitational acceleration and l is the natural
161 length of the pendulum, (B.3). Device RAOs (Response Amplitude Operators) are defined as the
162 maximum excursion per unit wave amplitude.

163 3.2. Power calculation

164 The mean power (time averaged over a period, $T = 2\pi/\omega$) per unit crest length of an incident
165 wave of amplitude A is given as

$$W_{inc} = \frac{1}{2}\rho g|A|^2 c_g \quad (11)$$

166 where c_g is the group velocity given by $\frac{1}{2}(g/\omega)$ in deep water.

167 The mean power absorbed by the device is equivalent to the mean rate of working of the wave
168 forces (see Appendix B) against the device motion, that is

$$W = \frac{1}{T} \int_0^T (F_{w,x}(t)\dot{X}(t) + F_{w,z}(t)\dot{Z}(t))dt$$

and can be expressed, after use of the decomposition in (B.14), as

$$\begin{aligned} W &= \frac{1}{2}(g^2|A|^2/\omega^2)\text{Re}\{X_{w,x}^*U + X_{w,z}^*V\} = \frac{1}{2}(g^2|A|^2/\omega^2)\text{Re}\{\mathbf{X}_w^*\mathbf{U}\} \\ &= \frac{1}{2}(g^2|A|^2/\omega^2)\text{Re}\{\gamma\mathbf{U}^*\mathbf{G}\mathbf{U}\} \\ &= \frac{1}{2}(g^2|A|^2/\omega^2)\text{Re}\{\gamma|\Omega_r|^2\} \end{aligned} \quad (12)$$

169 where $*$ denotes the complex conjugate transpose. In the above the third equation in (B.15) is used
170 with (1) and the fact that the elements of M , K and C are all real. We recognise the last equation
171 in (12) as the mean power developed by the rotation of the pendulum relative to the sphere, that is

$$W = \frac{1}{T} \int_0^T \gamma l^2 (\dot{\theta} - \dot{\Theta})^2 dt.$$

172 As expected we have demonstrated the equivalence of the power generated by the waves acting on
173 the hull to the power generated by the PTO machinery.

174 Using (11) in (12) we can define the capture width as

$$l(T, \beta) = \frac{W}{W_{inc}} = \frac{g}{\rho\omega^2 c_g} \text{Re}\{\gamma|\Omega_r|^2\} \quad (13)$$

175 being the equivalent length of incident wave from which all energy is absorbed. Assuming a fixed
176 power take-off parameter, γ , the capture width is a function of wave period, T , and wave heading,
177 β . Although (13) can be computed in the form presented further useful progress can be made.

178 We denote the i, j th element of E defined in (6) by E_{ij}/Δ where $\Delta = \det(E)$ and the i, j th
179 element of Z defined in (7) by Z_{ij} . Since $(Z + \gamma G)E = I$, the 3×3 identity matrix, it follows that

$$\Delta = E_{33}(\gamma + Y). \quad (14)$$

180 where

$$Y = Z_{33} + (Z_{13}E_{13} + Z_{23}E_{23})/E_{33} \quad (15)$$

which, crucially for what follows, is independent of γ since

$$E_{13} = Z_{12}Z_{23} - Z_{13}Z_{22}, \quad E_{23} = Z_{13}Z_{12} - Z_{11}Z_{23}, \quad E_{33} = Z_{11}Z_{22} - Z_{12}^2. \quad (16)$$

181 From (5), $\Omega_r = (E_{13}/\Delta)X_{s,x} \cos \beta$ and so (13) becomes,

$$l = \frac{g}{\rho\omega^2 c_g} \operatorname{Re} \left\{ \frac{\gamma |X_{s,x}|^2 |E_{13}|^2 \cos^2 \beta}{|E_{33}|^2 |\gamma + Y|^2} \right\} \quad (17)$$

182 after using (14). The Haskind relation (e.g. see [8]) provides the following

$$B_{11} = |X_{s,x}|^2 / (8\rho c_g) \quad (18)$$

183 which allows (17) to be written as

$$l = \frac{8B_{11}}{K} \operatorname{Re} \left\{ \gamma \frac{|E_{13}|^2 \cos^2 \beta}{|E_{33}|^2 |\gamma + Y|^2} \right\}. \quad (19)$$

184 where $K = \omega^2/g = 2\pi/\Lambda$ and Λ is the incident wavelength. If we assume γ to be real we can use
185 the general identity,

$$\frac{2\gamma}{|\gamma + Y|^2} = \frac{1}{(|Y| + \operatorname{Re}\{Y\})} \left(1 - \frac{(\gamma - |Y|)^2}{|\gamma + Y|^2} \right), \quad (20)$$

186 as in [9] allowing us to rewrite (17) as,

$$l = \frac{2\Lambda B_{11} |E_{13}|^2 \cos^2 \beta}{\pi |E_{33}|^2 (|Y| + \operatorname{Re}\{Y\})} \left(1 - \frac{(\gamma - |Y|)^2}{|\gamma + Y|^2} \right). \quad (21)$$

187 From (15), considerable algebra leads to the relation

$$\operatorname{Re}\{Y\} = \frac{(B_{11} + D_{11})|E_{13}|^2}{|E_{33}|^2} \quad (22)$$

188 which, when used in (21), gives our final expression for the capture width as

$$l(T, \beta) = \frac{\Lambda}{\pi} \frac{B_{11}}{(B_{11} + D_{11})} \frac{2\operatorname{Re}\{Y\}}{(|Y| + \operatorname{Re}\{Y\})} \left(1 - \frac{(\gamma - |Y|)^2}{|\gamma + Y|^2} \right) \cos^2 \beta. \quad (23)$$

189 Thus, the original expressions for the mean absorbed power given in (12) has been reduced to
190 (23) with $W = lW_{inc}$ and the dependence on the PTO parameter, γ , has been made explicit in
191 the final bracket of equation (23). Consequently the power is maximised, when $\gamma = |Y|$ and the
192 hydrodynamic damping $D_{11} = 0$, to the value which we will call the optimal capture width and
193 label

$$l_{opt} = \frac{\Lambda}{\pi} \frac{2\operatorname{Re}\{Y\}}{|Y| + \operatorname{Re}\{Y\}} \cos^2 \beta. \quad (24)$$

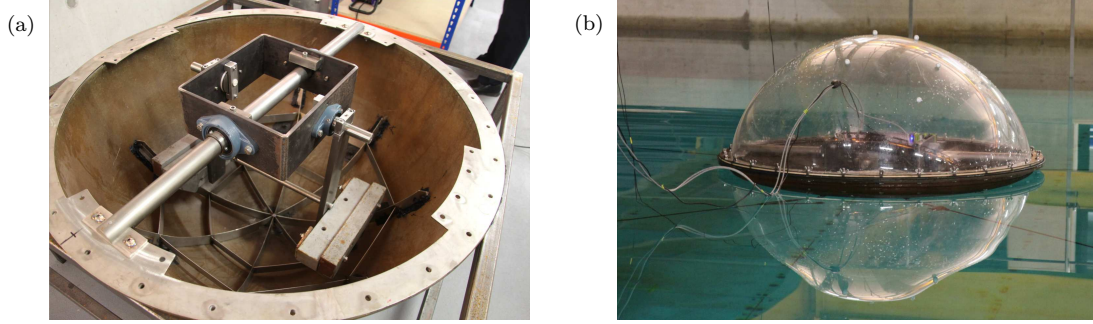


Figure 4: Images of the experimental set up: In (a) the internal arrangement of the sphere used in the experimental tests and (b) the sphere, semi-submerged in calm water, during tank testing.

194 If, additionally, $\text{Im}\{Y\} = 0$, then

$$l_{opt} = l_{max} \equiv \frac{\Lambda}{\pi} \cos^2 \beta \quad (25)$$

195 which is the maximum theoretical capture width that a vertically axisymmetric wave power device
 196 operating in surge/pitch can achieve, a well-known result – see [8] or [13].

197 In other wave energy problems with simpler mechanical components it is easy to identify the
 198 condition under which $l_{opt} = l_{max}$ with a resonant condition being met (e.g. [9]). Often this is
 199 a balance between inertia – including hydrodynamic inertia – and spring forces. Because of the
 200 complexity of Y in (15) it seems unlikely that a similar connection can be made here. However, by
 201 analogy with these simpler systems we will refer the condition for device resonance as $\text{Im}\{Y\} = 0$
 202 at which $l_{opt} = l_{max}$. We recall the tuning condition for optimal power is $\gamma = |Y|$ when $l = l_{opt}$.
 203 Thus if both tuning and resonance conditions are satisfied at the same frequency, $l = l_{max}$.

204 We remark that for axisymmetric devices taking power in heave only l_{max} is half that reported
 205 above whilst a device capable of taking power in both surge/pitch and heave motions the value of
 206 l_{max} reported above is increased by a factor of 1.5 ([8] or [13]).

207 3.3. Non-dimensionalisation

208 In order to solve (5) we define dimensionless variables using $\hat{Z} = Z/(M_w\omega)$, $\hat{X}_s = a\mathbf{X}_s/(M_w\omega)$,
 209 $\hat{U} = a\mathbf{U}$ and $\gamma/(M_w\omega) = \hat{\gamma}/\sqrt{Ka}$ so that a fixed $\hat{\gamma}$ implies a fixed physical PTO damping constant,
 210 γl^2 .

211 Consequently the RAOs in surge and heave are defined from (10) as dimensionless quantities
 212 $\mathcal{X} = |\hat{U}|/Ka$ and $\mathcal{Z} = |\hat{V}|/Ka$ and the hull pitch and pendulum RAOs are $\vartheta = |\hat{\Omega}|/(Kal)$ and
 213 $\vartheta_p = |\hat{\Omega} + \hat{\Omega}_r|/(Kal)$ and are not dimensionless, but measured per metre of wave amplitude.

214 Results presented later will be expressed in terms of dimensionless capture width ratios – or
 215 capture factors – where capture widths are divided the by the device diameter, $2a$. These are
 216 defined as $\hat{l} = l/2a$, $\hat{l}_{opt} = l_{opt}/2a$ and $\hat{l}_{max} = \Lambda \cos^2 \beta / (2\pi a)$ These are all frequency and incident
 217 angle dependent and should not be confused with the *mean* capture factor \bar{l} (defined in §5) which
 218 encodes the similar information but is averaged over all incident frequencies and angles.

219 3.4. Hydrodynamic drag

220 We assume that the total hydrodynamic drag is dominated by turbulent drag and adopt, as a
 221 starting point, a quadratic law to capture its effect. For surge motions, this drag force is approxi-

222 mated by $\frac{1}{2}\rho C_D \mathbb{A} \dot{X}(t) |\dot{X}(t)|$ where $C_D \approx \frac{1}{2}$ is the drag coefficient for a sphere and $\mathbb{A} = \frac{1}{2}\pi a^2$ is its
 223 frontal area. This approximation neglects the effect of background flow velocity. Using the Lorentz
 224 principle of equivalent work over a cycle, the linearised version of this drag is $4\rho C_D \mathbb{A} A \omega U / (3\pi)$
 225 and this, with reference to (2), gives $D_{11} = 4\rho C_D \mathbb{A} A \omega / (3\pi)$ having assumed a characteristic device
 226 velocity based on the background wave field. The dimensionless drag coefficient is therefore

$$\hat{D}_{11} = D_{11} / (M_w \omega) \approx 0.16A/a. \quad (26)$$

227 Dedicated experimental studies, beyond the scope of the current project, can be used to parametrise
 228 \hat{D}_{11} accurately and (26) should be regarded only as a simple first attempt at capturing the correct
 229 order of magnitude of the drag effects.

230 For example, with 2m wave amplitude and a 7.5m diameter sphere, $\hat{D}_{11} \approx 0.04$ and it can
 231 be seen from Fig. 3 that this is small percentage of $\nu_{11} = B_{11} / (M_w \omega)$ to which \hat{D}_{11} is added in
 232 calculations (equations (8), (23)).

233 Although heave motion is affected by drag, it does not contribute to power absorption and hence
 234 we do not consider the influence of D_{33} here.

235 Because of the uncertainty with setting an accurate representation of drag, all calculations in
 236 the main results section are made with $D_{11} = 0$. That is, we do not want to misrepresent our results
 237 or analysis. However, some brief comments on the effect of including drag into the calculations are
 238 made in the Conclusions.

239 4. Preliminary model validation with experimental results

240 Scale tank test experiments were conducted in the UK's Plymouth University ocean wave basin,
 241 in order to validate the RAOs of the device predicted by the mathematical model.

242 The following experimental set-up was chosen. A 1.2m diameter spherical hull was constructed
 243 from bolting a lower hemisphere made of steel to an upper hemisphere of perspex allowing observa-
 244 tions of the pendulum motion to be made. The hull contained a simplified pendulum model of the
 245 WITT as no suitable WITT unit was available for testing. Two pendulums were suspended and free
 246 to move independently about a single common axis perpendicular to the incident wave direction,
 247 see Fig. 4(a). Observations made during the tests confirmed motions of the two pendulums were
 248 synchronised. No rotation was allowed about the the transverse axis, as per assumptions in the
 249 analysis. No power take-off device was attached to the pendulums.

250 The hull was moored to the bottom of the wave tank (which was filled to a depth of 3m) using
 251 pre-tensioned elastic bungee cord. Sufficient lead ballast was placed at the bottom of the hull to
 252 ensure near semi-immersion of the hull, see Fig. 4(b). The sphere was raised about 50mm above the
 253 level of semi-immersion to help mitigate against the effects from the protruding lip of the sphere
 254 formed where the upper and lower sections were bolted together. Four elastic cords were splayed
 255 symmetrically left and right and fore and aft of the hull and pre-tensioned according to the model
 256 outlined in Appendix A. Small variations (less than 5%) in the pre-tensioning in each cord was
 257 required to configure an aligned and level static configuration which suppressed unwanted yaw and
 258 roll effects in motion. These adjustments were needed to account for small misalignments in the
 259 positioning of eye-bolts on the sphere for the mooring line attachments. Static loadings were applied
 260 to the bungee cord to determine its elastic modulus and confirm that the behaviour of the cord in
 261 motion was Hookean.

262 Some of the modelling outlined in the main body of the paper has been altered to reflect the
 263 experimental model. This involved the use of the elastic mooring model, described at the end of

264 Appendix A and the setting of the PTO parameter γ to zero in addition to determining appropriate
 265 moments of inertia and centres of gravity specific to the experimental model.

266 Tests were run across a range of single wave frequencies, with particular focus around the
 267 resonant frequency of the internal pendulum. These tests were repeated at three different wave
 268 amplitudes: 50mm, 100mm and 200mm. All tests were performed with normally-incident waves.

269 The experimentally measured device RAOs are plotted in Fig. 5 using symbols, along with
 270 the output from the mathematical model (with damping $D_{11} = 0$) using solid lines. The root
 271 mean square (RMS) surge and heave values have been non-dimensionalised by the RMS wave
 272 amplitude and the pitch and pendulum RMS values, measured in radians, have been normalised
 273 by the non-dimensional RMS wave steepness, $\omega^2 A/g$ in order to obtain the RAOs.

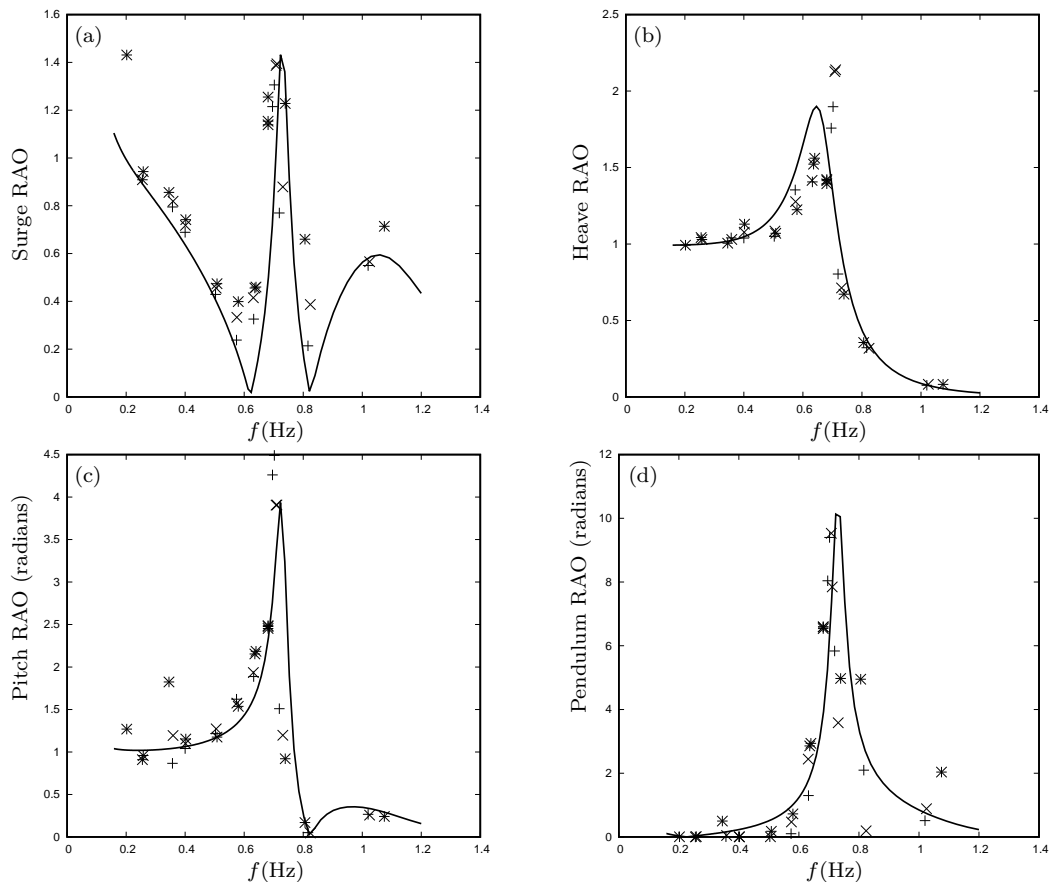


Figure 5: Comparison of the theoretical (line) and experimental (points) results for (a) surge, (b) heave, (c) pitch and (d) relative pendulum RAOs against wave frequency in Hertz. The different symbols represent results from different incident wave amplitudes used in the experiments: $A = 50\text{mm}$ (+); $A = 100\text{mm}$ (\times) $A = 200\text{mm}$ (*).

274 Generally, the results show good agreement. The frequency of the resonant peak is offset very
 275 slightly between experiments and theory and this is probably due to small errors made in esti-
 276 mating measurements of the hull/ballast/pendulum/mooring configuration. The experiments also
 277 demonstrate that the response of the buoy and the pendulum is generally linear with increasing

$h = 3\text{m}$	$T_0 = 70\text{N}$	$m_s = 75\text{kg}$
$a = 0.6\text{m}$	$L_0 = 8.4\text{m}$	$I = 211\text{kgm}^2$
	$\lambda = 150\text{N/m}$	
$\alpha = 48.2^\circ$		$m_b = 309\text{kg}$
$\chi = 71.3^\circ$	$l = 0.335\text{m}$	$L = 0.44\text{m}$
$\eta = 67.5^\circ$	$m = 77\text{kg}$	
$\zeta = 34.4^\circ$	$mk^2 = 9.5\text{kgm}^2$	$M = 452\text{kg}$

Table 1: Table of measured and calculated experimental parameters used to generate data and curves in Fig. 5.

278 wave height and it is only at the highest wave amplitude of 200mm (so that the wave height is one
279 third of the hull diameter) is there any notable difference. Here the resonant peaks in RAOs drop
280 significantly lower than for the two other wave heights. The inclusion of a linear damping term
281 using (26) in the theoretical model reduces the peak response in the surge and pendulum motions
282 for the largest 20mm wave amplitudes by roughly 8% whilst the response elsewhere changes very
283 little. Thus, it seems that hydrodynamic drag is not an important factor in these experiments.
284 Other non-linear effects may be influential. For example, for the largest 200m wave amplitudes
285 linear theory predicts a pendulum amplitude of 43° at peak resonance, around 0.73Hz. The time
286 series from the experimental results for these largest waves revealed that the pendulum motion be-
287 came asymmetric around the peak resonant frequency (the most extreme records showing motions
288 between -20° and 50° for the relative motion of the pendulum against the hull). The displayed
289 experimental results have been adjusted to account for this offset in the mean position of the mo-
290 tion. The non-linear effects described above may have been caused by a lip around the equator of
291 the hull (Fig. 4) which was observed to slam hard against the water surface especially at resonance
292 in larger amplitude waves.

293 Similar results, not shown here, have been obtained for a different splay and pre-tensioning of
294 mooring lines with similar agreement observed.

295 The generally good correlation between theory and experiments provide us with confidence that
296 the theoretical model can make reasonable predictions about a full scale WITT WEC apart from
297 at the largest amplitudes. In particular, experiments have indicated that predictions using linear
298 theory of pendulum pitch motions beyond roughly 30° show a loss of accuracy and this will act as
299 an important guide later.

300 5. Optimisation of full scale devices

301 In order to demonstrate the efficacy of the proposed design, we set out to optimise the device
302 performance over a wave energy spectrum for a given device test site. We have considered the Billia
303 Croo EMEC test site, on the western edge of the UK's Orkney mainland. For context, the EMEC
304 site has an annual average wave power of 21kW/m and an average water of depth 50m, justifying
305 the earlier deep water modelling assumptions in the calculation of the hydrodynamic coefficients.

306 A scatter diagram of probabilities of expected sea states is replicated in Fig. C.12 but can also
307 be found in [14], allowing us to define a joint probability function $P(H_s, T_p)$ of the likelihood of
308 occurrence of a pair of parameter values describing a particular sea state, where H_s is the significant
309 wave height and T_p the peak wave period.

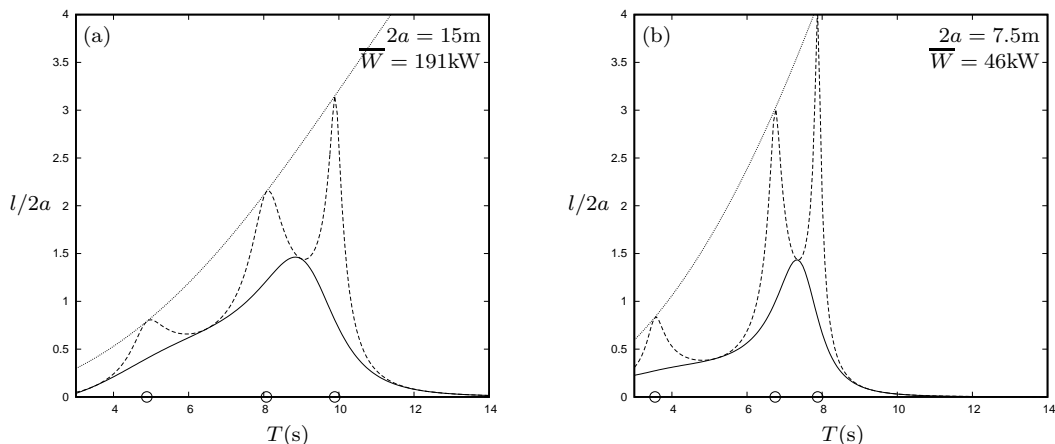


Figure 6: Maximum (dotted), optimum (dashed) and achievable (solid) capture width ratio for a device of diameter (a) $2a = 15\text{m}$ and (b) $2a = 7.5\text{m}$, under head seas. The circles indicate periods at which $\text{Im}\{Y\}$ vanishes.

310 We employ the two parameter spectrum developed by [3] and using the probability function
 311 $P(H_s, T_p)$ along with a function $G(\theta)$ to describe the angular spread of the energy density of the
 312 incident wave field, define a modified spectrum, $\tilde{S}(T, \theta)$. Expressions for these functions can be
 313 found in Appendix C, equations (C.1)–(C.3).

314 The total mean power absorbed by a device of width $2a$ is then

$$\overline{W} = \rho g \int_{-\pi}^{\pi} \int_0^{\infty} c_g(T) \tilde{S}(T, \beta) l(T, \beta) T^{-2} dT d\beta, \quad (27)$$

315 where $l(T, \beta)$ is given in (23) and $c_g = Tg/4\pi$ in deep water. The explicit $\cos^2 \beta$ variation in l
 316 in (23) combined with the model spread in (C.3) allows the β dependence (27) to be integrated
 317 analytically to $25/26$. Thus, only integration over period is required and multiplication by $25/26$
 318 accounts for spreading.

319 We can also define a dimensionless *mean* capture factor,

$$\bar{l} = \frac{\overline{W}}{\overline{W}_{inc} 2a}, \quad (28)$$

320 which describes the mean proportion of incident wave power absorbed per unit width of the device,
 321 where \overline{W}_{inc} has been defined in (C.5).

322 With many free parameters in this problem, we employ a numerical optimiser from the NAG
 323 library (E04JYF) to determine the design parameter values which maximise the mean capture
 324 factor, \bar{l} , over a given wave energy spectrum. In order to reduce the numerical effort required, a
 325 small number of parameters are fixed: for example, the density of the pendulums are set to that
 326 of concrete and the spherical structure is assumed to be equivalent to a shell of thickness 0.001%
 327 of the sphere diameter (e.g. 10mm for a 10m sphere) and to be made of steel. Upper and lower
 328 bounds are also imposed on the optimisers free parameters to ensure that optimised configurations
 329 are physically sensible. These include bounds on the inclination of the mooring lines, the slack in
 330 the static mooring line configuration and the position of masses along the mooring line. Some of

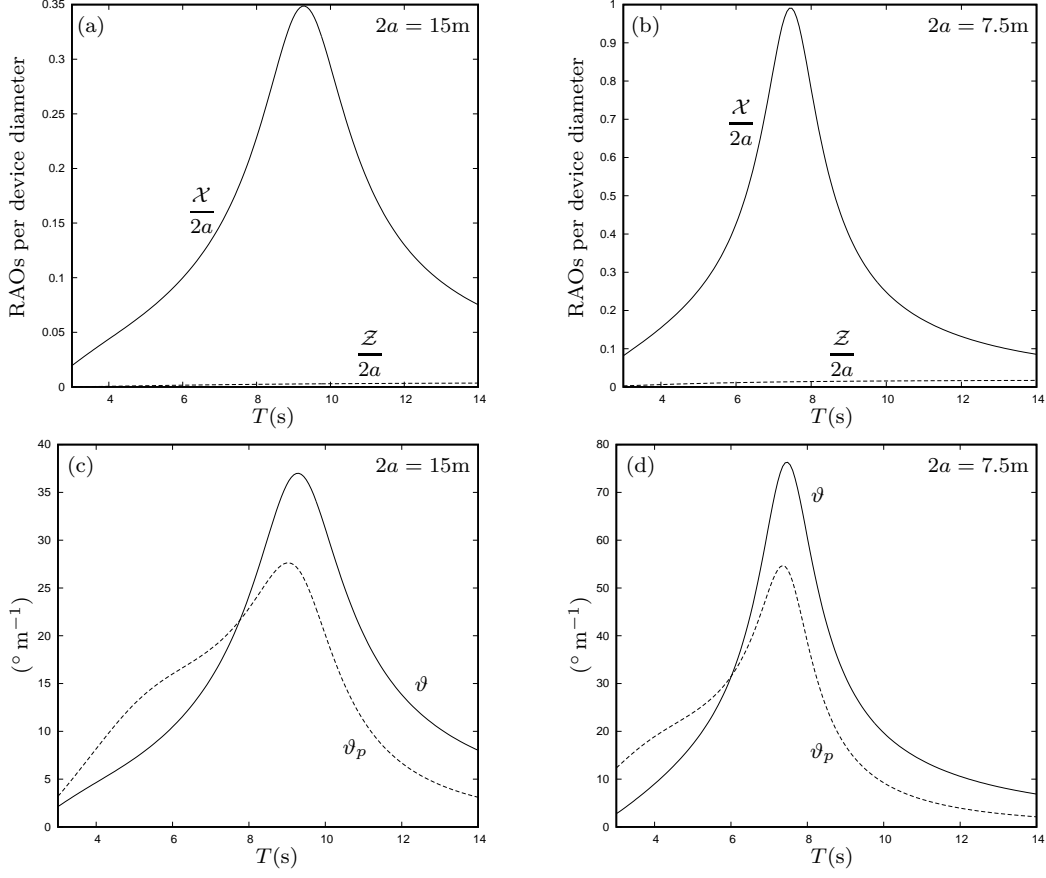


Figure 7: The RAOs in surge, \mathcal{X} (solid lines), and heave, \mathcal{Z} (dashed lines), per device diameter for (a) 15m and (b) 7.5m diameter WITT WECs; the angular pitch, ϑ (solid lines), and pendulum, ϑ_p (dashed lines), RAOs, for (c) 15m and (d) 7.5m diameter WITT WECs

331 these have been guided by advice from marine engineers employed as part of the current project.
 332 This includes weights of mooring lines.

333 In Fig. 6 curves of the theoretical maximum, the optimal and the actual capture width ratio
 334 (i.e. for fixed PTO parameter) plotted for two numerically optimised WITT WEC devices of 15m
 335 and 7.5m diameter.

336 In both plots, the optimal capture width ratio $l_{opt}/2a$ possess three peaks which extend to
 337 the theoretical maximum. As noted in Section 3.2, these peaks are associated with the resonant
 338 condition $\text{Im}\{Y\} = 0$, which are indicated in Fig. 6 by the circles on the period axis.

339 The numerical optimisation has distributed these resonances across the range of periods and
 340 selected the particular PTO parameter, γ , such that the realisations given by the solid curves
 341 maximises the mean power, \overline{W} , (i.e. averaged over the wave energy spectrum). Those values are
 342 indicated within the figures. The multiple resonances can be seen to broaden the capture width
 343 ratio over a range of wave periods so that, for example, for the 15m device, the capture width ratio,

Diam.	Unconstrained		Case (i)		Case (ii)	
	\overline{W}	\overline{l}	\overline{W}	\overline{l}	\overline{W}	\overline{l}
6	27	0.22	22	0.17	14	0.12
9	70	0.37	63	0.34	57	0.31
12	127	0.51	122	0.49	115	0.46
15	191	0.61	190	0.61	183	0.59
18	258	0.69	258	0.69	254	0.68
21	325	0.74	325	0.74	325	0.74
24	387	0.77	387	0.77	387	0.77

Table 2: Table of mean capture factor \overline{l} , and mean power \overline{W} (kW) for different diameters (m) with and without motion constraints.

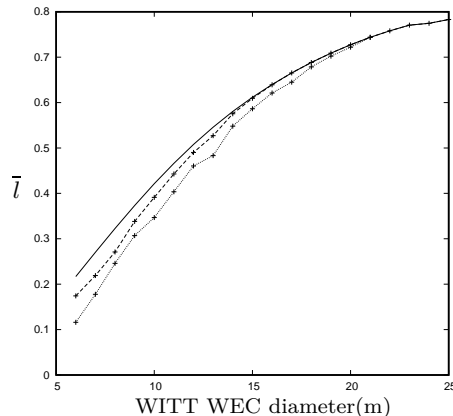


Figure 8: Mean capture factor against device size: Unconstrained (solid); Case (i) (dashed); Case (ii) (dotted).

344 $l/2a$, is greater than unity across 8–10s waves and is above a half for a 5-second range of wave
345 periods. These ranges are smaller for the half-size 7.5m device, as might be expected. The other
346 effect of the optimisation is that it selects a configuration and PTO characteristics which smooth
347 out the power capture over the range of wave periods. In other words the device de-tuned away
348 from the peak resonances, but is near resonant over a broad range of periods instead. This, in turn,
349 also helps smooth out the RAOs and the loads on the mooring.

350 Selecting other non-optimal fixed values of PTO parameter, γ , shifts the solid curve around
351 under the optimal curve resulting in lower overall mean power, \overline{W} . This procedure has a useful
352 purpose, allowing us to tune the device to the each of the three resonant peaks in turn as a means
353 of understanding the characteristic device behaviour associated with these resonances. Thus, the
354 lowest resonant period is found to be associated with a resonant motion of the pendulum only.
355 This tallies with (B.5) which provides the natural resonant period of the pendulums in isolation for
356 the two optimised 15m and 7.5m spheres as 5.29s and 3.79s, close to the values indicated on the
357 horizontal axis. The other two peaks at higher periods excite large motions in all three components
358 of pendulum, WEC pitch and surge motions. This is reminiscent of the effects observed in ‘tuned
359 mass dampers’ (see, for example, [12]) where the coupling of different independent motions leads to
360 bifurcation of resonances. Under linear theory heave is decoupled from pitch, surge and pendulum
361 rotation and consequently no resonant heave motions are observed in any results.

362 The RAOs corresponding to the optimised solid curves in Fig. 6 are given in Fig. 7. They show,
363 respectively, the surge and heave RAOs per device diameter and the angular RAOs of the internal
364 pendulum and pitch of the device, measured in degrees per metre wave amplitude (see §3.3). The
365 RAOs for the 7.5m sphere are more than double that for the 15m sphere. This is not surprising
366 (indeed, it is well known) that smaller devices optimised to maximise power output have larger
367 responses than larger devices; see [7] for example. We notice that the heave motion is very small
368 in both cases; this is in part due to the optimal mooring line configuration which nearly always
369 favours the steepest allowable mooring angle (30° to the vertical in our design space) at the point
370 of attachment to the sphere. The surge RAO for the 15m optimal design is not exceptionally large.
371 However, the pitch and pendulum RAOs suggest that linear theory could only be expected to hold

372 in low to moderate wave heights. The 7.5m optimal design has RAOs which are well outside the
 373 range of validity of linear theory apart from in low seas.

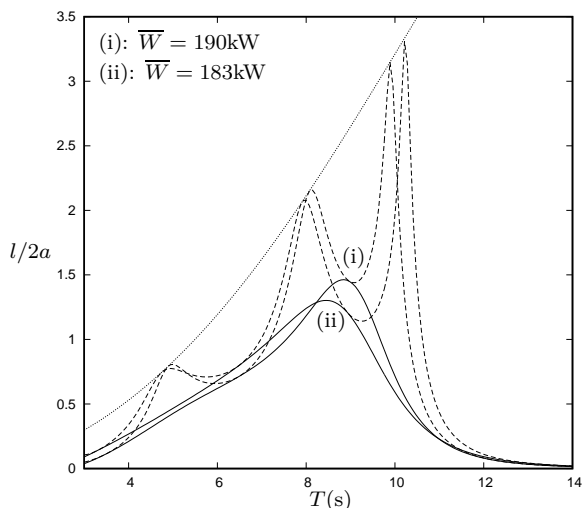


Figure 9: The maximum (dotted), optimal (dashed) and actual (solid) capture width ratios for a 15m diameter WITT WEC optimised under Case (i) and (ii) motion constraints.

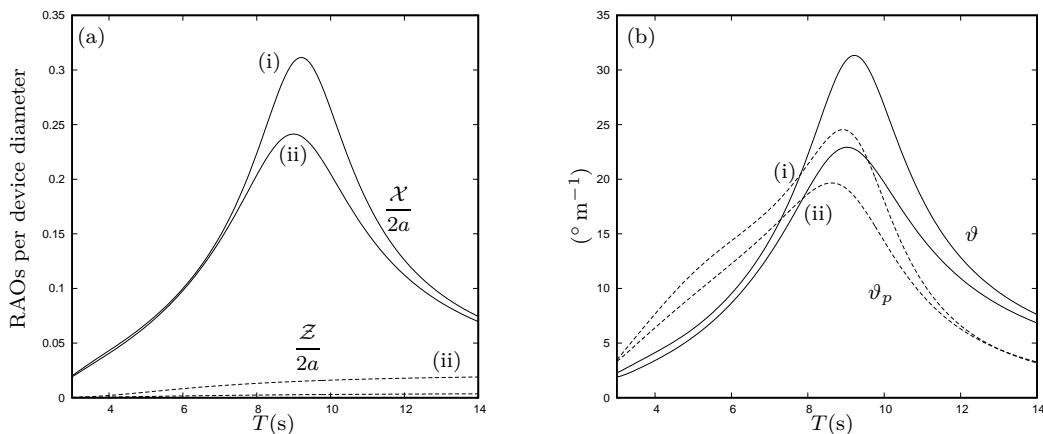


Figure 10: RAOs for 15m diameter WITT WEC optimised under motion constraints Case (i) and (ii): (a) surge, \mathcal{X} (solid lines), and heave, \mathcal{Z} (dashed lines) measured per device diameter and (b) the pitch, ϑ (solid lines), and pendulum rotation, ϑ_p (dashed lines) measured in degrees per metre wave amplitude.

374 5.1. Optimisation with motion constraints

375 Theoretical work to include the effect of motion constraints on power output have been con-
 376 sidered in, for example, the work of [7] and [15]. In Fig. 1 of [7] it was shown that restricting the
 377 vertical motion of a heaving semi-immersed spherical WEC to different proportions (results for 0.5,

1 and 2 are shown) of the wave amplitude leads to a reduction in the capture width ratio from its maximum $\Lambda/2\pi$ for unconstrained motions as Λ increased.

We have not attempted to follow [7] and apply theoretical motion constraints here. Since a numerical optimisation is already being used with constraints on input design parameters, we have considered restricting the output RAOs as part of the optimisation. This is done by including a smooth penalty function into the optimiser’s objective function which is set to penalise motions above threshold which can be set arbitrarily.

As an illustration, each WEC size has been optimised in the same manner as before but subject to two different sets of constraints, one more severe than the other. In Case (i) the penalty threshold of heave and surge RAOs is set at the half the device diameter, and the pendulum and hull pitch RAOs set at 30° per metre wave amplitude (i.e. per 2m wave height). Case (ii) restricts heave and surge RAOs to 25% of device diameter and angular rotations to 22.5° per metre wave amplitude. These are somewhat arbitrary, although they are influenced by the range of validity indicated by the experimental results and the distribution of significant wave heights at the EMEC test site, Fig. C.12. Moreover, they provide a good indication of the effects that motion constraints introduce.

Numerical results are presented in Table 2 and Fig. 8 for the mean power and the capture width ratios for both constrained cases alongside the unconstrained device motion. The optimiser finds it less easy to converge to an optimal solution when a penalty function is introduced and the data for Cases (i) and (ii) is not particularly smooth as a result.

The results demonstrate that motion constraints increase the percentage loss in power as the device gets smaller. Thus, Case (ii) constraints applied to a 6m device leads to roughly a 50% loss in power. However, for larger devices the loss incurred by imposing motion constraints is actually quite small, and for devices larger than 18m optimised motions fall within the bounds of both Case (i) and (ii) constraints.

For devices smaller than 6m the motion constrained devices generate very little power, suggesting that linear analysis is not a useful tool for analysing and optimising smaller devices. This adds to the fact that we have neglected drag forces and these play an increasingly prominent role in smaller devices.

Hull diam. (m)	S. area (m ²)	Mass (t)	Case (i)			Case (ii)		
			\bar{l}	\bar{l}_s	\bar{l}_m	\bar{l}	\bar{l}_s	\bar{l}_m
6	56.54	57.95	0.17	3.53	3.33	0.12	2.16	2.11
9	127.2	195.6	0.34	4.33	2.82	0.31	3.92	2.55
12	226.2	463.7	0.49	4.72	2.30	0.46	4.45	2.17
15	353.4	905.7	0.61	4.70	1.83	0.59	4.53	1.77
18	508.9	1595	0.69	4.44	1.41	0.69	4.44	1.41

Table 3: Alternative measures of device performance based on [2] for devices optimised under Case (i) and (ii) motion constraints: annual absorbed energy per submerged surface area \bar{l}_s (Mwh/m²) and per displaced mass \bar{l}_m (Mwh/t).

Results are only now plotted in Fig. 9 and Fig. 10 for the 15m device under Case (i) and (ii) motion constraints. Although there is only a small reduction in overall mean power, Fig. 9 illustrates the fairly significant shift in the capture width ratios between Cases (i) and (ii) which are broader and lower in the latter. The result of this on the RAOs is evident in Fig. 10. The Case (i) curve

410 in Fig. 10(b) has the 30° m^{-1} threshold imposed and Case (ii) curves are limited by the 0.25 surge
411 RAO limit and the $22.5^\circ \text{ m}^{-1}$ pitch and and pendulum limit.

412 In summary, for the 15m device, the optimisation under Case (ii) constraints has resulted in
413 a reduction of under 5% in mean power which has required a reduction in maximum RAOs of
414 30 – 40%.

415 The capture factor is arguably not the most appropriate measure for the comparison of the
416 relative performance of WECs of different sizes, especially for a spherical device. [2] suggests some
417 alternative measures, including the annual absorbed energy per characteristic submerged surface
418 area, \bar{l}_s , and per characteristic mass, \bar{l}_m . These measures are shown for the 6m–18m devices subject
419 to optimised Case (i) and (ii) motion constraints in Table 3. Here, we have defined the characteristic
420 mass as the hull displacement. Unlike the capture factor which is seen to increase as the device size
421 increase, both of these alternative measures take their maximum at intermediate diameters.

422 Based on Case (i) motion constraints the results in Table 3 might suggest a roughly 9m sphere
423 to be most favourable. Under more plausible Case (ii) motion constraints, the measures perhaps
424 favour a slightly larger device, roughly 12m in size.

425 6. Conclusions

426 In this paper we have outlined a mathematical model of a novel design of wave energy converter
427 (WEC) in which a sealed hull containing a heavy pendulum representative of the WITT energy
428 harvesting device operates on the surface of the ocean. The modelling has been carried out using
429 small amplitude theory and care has been taken in the results to assess the WEC performance
430 under the conditions assumed in the model. Furthermore, experimental results have confirmed the
431 key elements of the WEC operation predicted by the theory: the coupling of device motions with
432 an internal pendulum under a four-point mooring system, as assumed in the model.

433 The derivation of model mooring systems and the equations of motion along with key results of
434 the hydrodynamic modelling are all provided in the paper. The main part of the paper focusses on a
435 derivation of an analytic expression for the capture width which have allowed us to both understand
436 the operation of the WITT WEC and perform rapid computations in numerical optimisation over
437 many device design parameters under a model wave climate. In particular, the ability to be able to
438 calculate the maximum achievable power and its corresponding power take off tuning condition has
439 also allowed us to identify how the system operates, by spreading three resonances across a range
440 of periods allowing broad banded power capture characteristics to be obtained with a smooth RAO
441 response.

442 Numerical results have focussed on numerically optimised design of WITT WECs with diameters
443 between 6m and 24m operating in the EMEC test site in Orkney, UK. The optimisation is performed
444 over many free parameters of the design including the pendulum shape and mooring configuration
445 and allows thresholds on the motions of elements of the design to be set to ensure the underlying
446 small amplitude theory is not compromised. The imposition of motion constraints to limit the
447 RAOs to operating conditions in all but very heavy seas to ensures that the model assumptions
448 have not been violated and means that the mean power estimates provided for different device
449 diameters are realistic. In illustrative calculations we have focussed on a 15m device, which has
450 been estimated to produce an annual mean power output of 188kW under motion constraints in
451 the 21kW/m wave climate of EMEC, equivalent to a mean capture factor of 0.59.

452 According to Fig. 16 of [1] such a predicted device performance competes favourably amongst
453 existing WEC designs especially considering it does not technically belong in the the class of oscil-

454 lating wave surge converter which emerge from the study of [1] as on average the best performing
455 WEC.

456 However, we have not included effects of hydrodynamic drag (which [1] does by using a quadratic
457 drag law and solving in the time domain) or mechanical losses into our device operation and this
458 will have some impact on its performance. Computations which include a linearised hydrodynamic
459 drag term with the definition (26) in the numerical optimisation without motion constraints show,
460 for example, show the following reduction in mean power: from $\bar{W} = 27\text{kW}$ to $\bar{W} = 17\text{kW}$ for a 6m
461 sphere (with $\hat{D}_{11} = 0.075$); from $\bar{W} = 191\text{kW}$ to $\bar{W} = 167\text{kW}$ (or $\bar{l} = 0.53$) for a 15m sphere (with
462 $\hat{D}_{11} = 0.030$); from $\bar{W} = 383\text{kW}$ to $\bar{W} = 363\text{kW}$ (or $\bar{l} = 0.72$) for a 24m sphere (with $\hat{D}_{11} = 0.018$).
463 Additionally the power take off (PTO) model assumed here is idealised; it includes no mechanical
464 or electrical losses and may not be representative of a practical implementation. Modifications
465 to account for these factors including the influence of random seas, will require further modelling
466 beyond the current first order model most likely performed in numerical time domain simulations
467 and experimental tests.

468 In the Introduction we highlighted the SEAREV as being a comparable device, employing a
469 counterweight in a sealed hull. In [5], the SEAREV G1 design with roughly the same displacement
470 as a 15m WITT WEC is reported to produce a mean power of 70kW in a wave resource of 25kW.
471 Its width is 14m and the figures above equate to a mean capture factor of $\bar{l} \approx 0.2$.

472 Although not reported in any detail here, other hull geometries have been considered to inves-
473 tigate whether any significant improvement in the mean absorbed power could be made including:
474 (i) an upright cylindrical hull floating in the surface and moored using the same four-point mooring
475 system as the spherical hull; and (ii) a submerged spherical hull moored to the sea bed via taut lines
476 held under tension by the assumed buoyancy of the hull. Both designs have been modelled using
477 the same underlying principles and assumptions as for the original spherical hull. The performance
478 of each design has been optimised in a similar manner to that described here. Results suggest that
479 there is little difference in the predicted power output from the mathematical model at the scale of
480 10m–25m diameter device where the small amplitude assumptions can be reasonably applied.

481 Appendix A. Mooring model

482 In this Appendix, two different mooring models are outlined. In the first, a simple mathematical
483 model of a four-point heavy catenary mooring system is described, in which a point mass is placed
484 some distance along each of the light, inextensible mooring lines connecting the hull to the sea bed.
485 In the second, taut elastic mooring lines replace the catenary lines: this has been introduced to
486 mimic the experimental set-up. Both models are illustrated in Fig. A.11.

487 In both cases, it is assumed that the mooring limbs are arranged symmetrically about the sphere,
488 their horizontal projection making an angle ζ to the primary incident wave direction. The mooring
489 points on the spherical hull are described by the two angles α and η as shown in Fig. A.11. The
490 vertical distance from the point of attachment to the seabed is given by $h_1 = h - a \cos \alpha$, (h is
491 the water depth and a is the radius of the hull) and the horizontal distance along the mooring line
492 between these two points $h_2 = h_1 \tan \chi$ so that χ is the angle that the imaginary line from the
493 points of attachment on the hull to the bed make with the vertical. We shall assume that all angles
494 α , η , ζ and χ are given, in addition to h and a .

495 The hull can move along the horizontal x axis by $X(t)$ (surge) and along the vertical z axis
496 by $Z(t)$ (heave) and it can rotate by an angle $\Theta(t)$ about the y -axis (pitch, measured clockwise).
497 As it moves in each of these three modes of motion, the tension in each of the four mooring lines

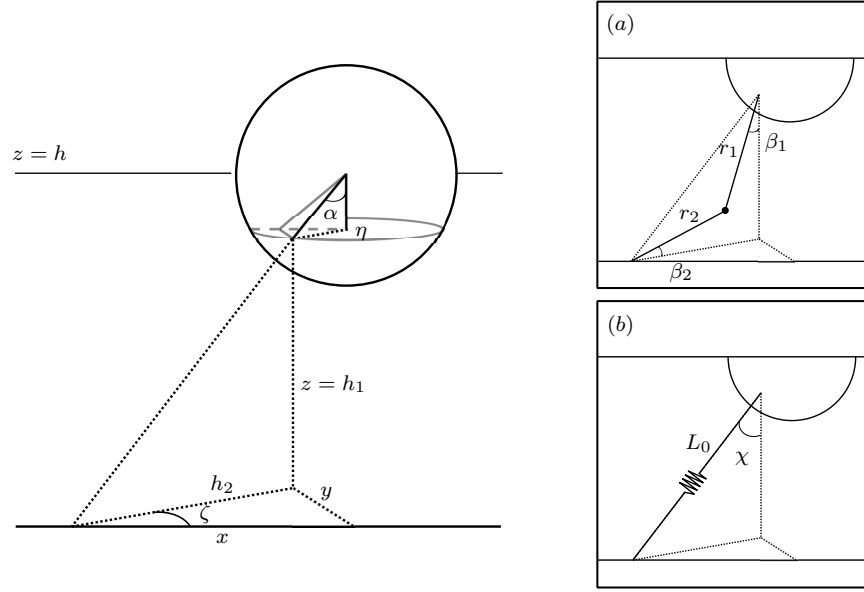


Figure A.11: A basic definition sketch of a single mooring limb of the four point mooring system in (a) the catenary type mooring system, in (b) the taut, extensible mooring system.

498 attached to the hull will also change and this will provide reactive force/moments on the hull. It is
 499 the relation between the motion and these force/moments we set out to derive here.

500 *Appendix A.1. Catenary mooring system*

501 Each mooring limb comprises two straight, massless, line segments of fixed lengths r_1 and r_2
 502 with a single point mass m_l at their intersection, see Fig. A.11(a). The ratio $\hat{r} = r_1/r_2$ is assumed
 503 to be a given mooring parameter, as is m_l .

504 Consider just one mooring limb connected to the hull. The tension in the upper segment is
 505 denoted T_1 in Fig. A.11 and so the components of the tension in the x and z directions are

$$T_{1x} = T_1 \sin \beta_1 \cos \zeta, \quad T_{1z} = T_1 \cos \beta_1, \quad (\text{A.1})$$

506 along with the pitch moment about the centre of the sphere,

$$M_{1\theta} = T_1 a (\cos \eta \sin \alpha \cos \beta_1 - \cos \zeta \cos \alpha \sin \beta_1), \quad (\text{A.2})$$

507 which, we note, may be written

$$M_{1\theta} = a (T_{1z} \cos \eta \sin \alpha - T_{1x} \cos \alpha). \quad (\text{A.3})$$

508 Additionally there is a component of tension in the y direction and yaw and roll moments non of
 509 which contribute to the dynamics.

510 When the hull is in motion, Cartesian components of distance between the mooring points on
 511 the hull and the bed can be calculated as

$$l_x = X + h_2 \cos \zeta + a \sin \alpha \cos \eta (1 - \cos \Theta) - a \cos \alpha \sin \Theta$$

512 where $h_2 \equiv (h - a \cos \alpha) \tan \chi$ in terms of prescribed parameters,

$$l_y = h_2 \sin \zeta, \quad \text{and} \quad l_z = Z + h - a \cos \alpha \cos \Theta + a \sin \alpha \cos \eta \sin \Theta.$$

513 Further elementary geometry applied to the two mooring line segments provides the two relations

$$r_2(\hat{r} \sin \beta_1 + \cos \beta_2) = \sqrt{l_x^2 + l_y^2} \quad (\text{A.4})$$

514 and

$$r_2(\hat{r} \cos \beta_1 + \sin \beta_2) = l_z \quad (\text{A.5})$$

515 where β_1 is the angle the upper mooring line makes to the vertical at the hull and β_2 is the angle the
516 lower mooring line makes with the horizontal at the bed. These two relations implicitly determine
517 β_1 and β_2 in terms of (X, Θ, Z) .

518 The tensions T_1 and T_2 in upper and lower lines follow from a quasi-static force balance (i.e. we
519 assume no inertial effects from the moving lines in this model) to give

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \frac{m_l g}{\cos(\beta_1 + \beta_2)} \begin{pmatrix} \cos \beta_2 \\ \sin \beta_1 \end{pmatrix}. \quad (\text{A.6})$$

520 Thus l_x, l_y, l_z and β_1, β_2, ζ are all functions of (X, Θ, Z) and hence so are T_1, T_2 . To determine the
521 static configuration, including static tensions, we substitute $(X, \Theta, Z) = (0, 0, 0)$, we fix the angle
522 $\beta_1 = \lambda \chi$ where $\lambda < 1$ becomes the final mooring parameter.

523 Thus the static angle β_2 can be deduced from

$$\cos(\beta_2 + \chi) = \hat{r} \sin(\chi(1 - \lambda)). \quad (\text{A.7})$$

524 which combine (A.4) and (A.5), and then

$$r_2 = (h - a \cos \alpha) / (\hat{r} \cos \lambda \chi + \sin \beta_2) \quad (\text{A.8})$$

525 from which $r_1 = r_2 \hat{r}$.

526 Meanwhile, a small amplitude assumption allows us to approximate the dynamic elements of
527 the tension components T_{1x} as

$$\left(X(t) \frac{\partial T_{1x}}{\partial X} + \Theta(t) \frac{\partial T_{1x}}{\partial \Theta} + Z(t) \frac{\partial T_{1x}}{\partial Z} \right)_{(0,0,0)} \quad (\text{A.9})$$

528 (similarly for T_{1z} and hence $M_{1\theta}$ from (A.2)).

529 Due to symmetry of the mooring configuration, the net effect of the four lines either reinforces
530 additively or cancels out. For example, heave motions create dynamic tensions in each limb of the
531 mooring line in all three components, but the net surge force and pitch moment induced by this
532 heave motion is zero.

533 Thus, the dynamic components of the forces/moments experienced by the hull in directions of
534 surge, pitch and heave provided by the mooring system due to small displacements of the hull are
535 summarised by the matrix representation

$$\mathbf{X}_m = - \begin{pmatrix} K_{11} & K_{12} & 0 \\ K_{21} & K_{22} & 0 \\ 0 & 0 & K_{33} \end{pmatrix} \begin{pmatrix} X \\ \Theta \\ Z \end{pmatrix}. \quad (\text{A.10})$$

536 where

$$K_{11} = 4 \frac{\partial T_{1x}}{\partial X}, \quad K_{12} = 4 \frac{\partial T_{1x}}{\partial \Theta}, \quad K_{21} = 4 \frac{\partial M_{1\theta}}{\partial X}, \quad K_{22} = 4 \frac{\partial M_{1\theta}}{\partial \Theta}, \quad K_{33} = 4 \frac{\partial T_{1z}}{\partial Z}.$$

Taking partial derivatives of (A.4), (A.5) with respect to X , Z and Θ and evaluating at $(X, \Theta, Z) = (0, 0, 0)$ we find, after considerable algebra

$$\begin{aligned} \frac{\partial \beta_1}{\partial X} &= \frac{\cos \zeta \cos \beta_2}{r_1 \cos(\beta_1 + \beta_2)}, & \frac{\partial \beta_1}{\partial Z} &= \frac{\sin \beta_2}{r_1 \cos(\beta_1 + \beta_2)}, \\ \frac{\partial \beta_2}{\partial X} &= \frac{\cos \zeta \sin \beta_1}{r_2 \cos(\beta_1 + \beta_2)}, & \frac{\partial \beta_2}{\partial Z} &= \frac{\cos \beta_1}{r_2 \cos(\beta_1 + \beta_2)}. \end{aligned} \quad (\text{A.11})$$

537 Additionally, we can deduce that

$$\frac{\partial \zeta}{\partial X} = -\frac{\sin \zeta}{h_2}, \quad \text{and} \quad \frac{\partial \zeta}{\partial Z} = 0 \quad (\text{A.12})$$

538 whilst $\partial_\Theta = -a \cos \alpha \partial_X + a \cos \eta \sin \alpha \partial_Z$.

539 With (A.1) the relations above are enough to determine the elements of the matrix (A.10)

$$K_{11} = \frac{4m_l g \cos^2 \zeta}{r_2 \cos^3(\beta_1 + \beta_2)} \left(\frac{\cos^3 \beta_2}{\hat{r}} + \sin^3 \beta_1 \right) + \frac{4m_l g \sin^2 \zeta \sin \beta_1 \cos \beta_2}{h_2 \cos(\beta_1 + \beta_2)}, \quad (\text{A.13})$$

540

$$K_{33} = \frac{4m_l g}{r_2 \cos^3(\beta_1 + \beta_2)} \left(\frac{\cos \beta_2 \sin^2 \beta_2}{\hat{r}} + \cos^2 \beta_1 \sin \beta_1 \right), \quad (\text{A.14})$$

541 with

$$K_{12} = K_{21} = a(\cos \eta \sin \alpha K_{13} - \cos \alpha K_{11}), \quad (\text{A.15})$$

542

$$K_{22} = a^2(\cos^2 \alpha K_{11} - \cos \eta \sin 2\alpha K_{13} + \cos^2 \eta \sin^2 \alpha K_{33}) \quad (\text{A.16})$$

543 in terms of an intermediary variable

$$K_{13} = \frac{4m_l g \cos \zeta}{r_2 \cos^3(\beta_1 + \beta_2)} \left(\frac{\cos^2 \beta_2 \sin \beta_2}{\hat{r}} + \cos \beta_1 \sin^2 \beta_1 \right). \quad (\text{A.17})$$

544 In these definitions, dynamic variables are evaluated at their static values: in particular $\beta_1 = \lambda \chi$ in
 545 terms of given mooring parameters and β_2 and r_2 are given by (A.7) and (A.8). The symmetry of
 546 the matrix in (A.10) is expected. Simplifications made under special cases have been used to check
 547 the validity of the spring constants.

548 *Appendix A.2. Elastic mooring system*

549 Here a model of a taut mooring system with extensible limbs is considered, see Fig. A.11(b).
 550 This model is used only in the comparison with the experiments in Section 4. There are fewer
 551 model parameters than in the previous system. Each mooring limb comprises a single taut elastic
 552 mooring line of elastic stiffness λ and pre-stressed tension T_0 when extended to length L_0 in the
 553 static configuration. The line makes an angle χ with the vertical at the point of attachment as in
 554 the catenary example. Thus $L_0 = (h - a \cos \alpha) / \cos \chi$ whilst the tension T_1 in the line attached to
 555 the hull is modelled by the Hookean relation

$$T_1 = T_0 + \lambda \left(\sqrt{l_x^2 + l_y^2 + l_z^2} - L_0 \right).$$

A simpler application of the ideas given previously leads to

$$K_{11} = 4\lambda \sin^2 \chi \cos^2 \zeta + \frac{4T_0}{L_0}(1 - \sin^2 \chi \cos^2 \zeta), \quad (\text{A.18})$$

$$K_{33} = 4\lambda \cos^2 \chi + \frac{4T_0}{L_0} \sin^2 \chi \quad (\text{A.19})$$

and the intermediate variable

$$K_{13} = 4 \left(\lambda - \frac{T_0}{L_0} \right) \sin \chi \cos \chi \cos \zeta \quad (\text{A.20})$$

556 allows $K_{21} = K_{12}$ and K_{22} to follow from (A.15) and (A.16) as before.

557 Appendix B. Formulation of governing equations

558 In this Appendix, the equations of motion of the pendulum and the sphere are derived from the
 559 Euler-Lagrange equations. A hollow spherical shell of radius a , mass m_s , thickness d_s and density
 560 ρ_s has power take off machinery represented by a point mass, m_{pto} , positioned at the centre of
 561 the sphere and a point mass m_b representing ballast which is assumed to sit at the bottom of the
 562 sphere. The sum of these three fixed masses is denoted $M = m_s + m_{pto} + m_b$, its centre of mass
 563 located at $z = -L$ where $L = am_b/M$, and its moment of inertia about the origin is I where we
 564 calculate

$$I = a^2 \left(m_b + \frac{2m_s}{5} \left(\frac{1 - (a - d_s)^5/a^5}{1 - (a - d_s)^3/a^3} \right) \right). \quad (\text{B.1})$$

565 The sphere contains a compound pendulum which rotates through an angle $\theta(t)$ clockwise about
 566 the horizontal y -axis. The pendulum is assumed to be rotationally symmetric about the vertical
 567 central axis and has an annular central cross-section; it has a outer radius $c < a$, an inner radius
 568 $b < c$ and subtends to an angle of 2ξ , as shown in Fig. 2. The pendulum has a density ρ_p and has
 569 corresponding mass

$$m = \frac{2}{3} \rho_p (c^3 - b^3) (1 - \cos \xi) \pi. \quad (\text{B.2})$$

570 The natural length l of the pendulum, being the distance from the origin to its centre of mass is
 571 calculated to be

$$l = \frac{3}{8} \frac{(c^4 - b^4)}{(c^3 - b^3)} (1 + \cos \xi). \quad (\text{B.3})$$

572 The pendulum also has a moment of inertia about the origin which we denote by mk^2 , where k is
 573 the radius of gyration of the pendulum defined by

$$k^2 = \frac{1}{10} \frac{(c^5 - b^5)}{(c^3 - b^3)} c^2 (4 + \cos \xi (1 + \cos \xi)). \quad (\text{B.4})$$

574 We note that, in the absence of damping, the resonant period of small amplitude pendulum motions
 575 are given by

$$T = \frac{2\pi}{\omega}, \quad \text{with} \quad \omega^2 = \frac{gl}{k^2}. \quad (\text{B.5})$$

576 In motion, the centre of sphere is $(X(t), Z(t))$ and it rotates through an angle $\Theta(t)$ clockwise
 577 about the origin. The sphere is restrained by mooring lines characterised by linear spring constants

578 K_{ij} representing the force in the direction i due to motion in direction j , $i, j = 1, 2, 3$. These are
 579 given in Appendix A. Thus, the potential energy of the sphere/pendulum/mooring system is then
 580 given by,

$$\mathcal{V}(X, Z, \Theta, \theta) = \frac{1}{2} (K_{11}X^2 + K_{22}\Theta^2 + K_{33}Z^2 + 2K_{12}X\Theta) + \frac{1}{2}\rho gSZ^2 - MgL \cos \Theta - mgl \cos \theta, \quad (\text{B.6})$$

581 where ρ is the density of the fluid and $S = \pi a^2$ is the equilibrium water plane area of the sphere.

The kinetic energy for the system is the sum of the kinetic energies for the sphere/ballast and that of the pendulum given by

$$\mathcal{T}(\dot{X}, \dot{Z}, \dot{\Theta}, \dot{\theta}, \Theta, \theta) = \frac{1}{2}I\dot{\Theta}^2 + \frac{1}{2}mk^2\dot{\theta}^2 + \frac{1}{2}M(\dot{X}^2 + \dot{Z}^2 - 2L\dot{\Theta}(\dot{X} \cos \Theta - \dot{Z} \sin \Theta)) \\ + \frac{1}{2}m(\dot{X}^2 + \dot{Z}^2 - 2l\dot{\theta}(\dot{X} \cos \theta - \dot{Z} \sin \theta)). \quad (\text{B.7})$$

582 A linear damping mechanism is connected to the pendulum, which acts in proportion to the rate
 583 of rotation of the pendulum with respect to the rotation of the sphere in pitch in order to extract
 584 power from the system. The linearised damping is included via the Rayleigh dissipation function
 585 [see 10], and we write

$$\mathcal{D}(\dot{\Theta}, \dot{\theta}) = \frac{1}{2}\gamma l^2(\dot{\theta} - \dot{\Theta})^2, \quad (\text{B.8})$$

586 where γ represents a power take-off parameter.

587 The Euler-Lagrange equations are then given by

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{X}} - \frac{\partial \mathcal{L}}{\partial X} + \frac{\partial \mathcal{D}}{\partial \dot{X}} = F_{w,x}, \quad (\text{B.9})$$

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{\Theta}} - \frac{\partial \mathcal{L}}{\partial \Theta} + \frac{\partial \mathcal{D}}{\partial \dot{\Theta}} = 0, \quad (\text{B.10})$$

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{Z}} - \frac{\partial \mathcal{L}}{\partial Z} + \frac{\partial \mathcal{D}}{\partial \dot{Z}} = F_{w,z}, \quad (\text{B.11})$$

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial \mathcal{D}}{\partial \dot{\theta}} = 0, \quad (\text{B.12})$$

where $\mathcal{L} = \mathcal{T} - \mathcal{V}$ and $F_{w,x}(t)$ and $F_{w,z}(t)$ represent the external horizontal and vertical wave forces acting on the sphere. Applying equations (B.9)–(B.12) to (B.6)–(B.8) and linearising on the assumption of small amplitude motions gives

$$(M + m)\ddot{X} - ML\ddot{\Theta} - ml\ddot{\theta} = F_{w,x} - K_{11}X - K_{12}\Theta \\ I\ddot{\Theta} - ML\ddot{X} = -K_{22}\Theta - K_{12}X - MgL\Theta + \gamma l^2(\dot{\theta} - \dot{\Theta}), \\ (M + m)\ddot{Z} = F_{w,z} - (K_{33} + \rho gS)Z \\ mk^2\ddot{\theta} - ml\ddot{X} = -mgl\theta - \gamma l^2(\dot{\theta} - \dot{\Theta}). \quad (\text{B.13})$$

We shall assume incident waves of a single radian frequency ω and, since our governing equations are linear, a time harmonic dependence can be factorised from all dynamic variables and we write

$$[F_{w,x}, F_{w,z}] = \text{Re}\{(-igA/\omega)[X_{w,x}, X_{w,z}]e^{-i\omega t}\}, \\ [\dot{X}, \dot{Z}, l\dot{\Theta}, l(\dot{\theta} - \dot{\Theta})] = \text{Re}\{(-igA/\omega)[U, Z, \Omega, \Omega_r]e^{-i\omega t}\} \quad (\text{B.14})$$

where A is the incident wave amplitude so that $X_{w,x}$, $X_{w,z}$, U , V , Ω and Ω_r are all complex frequency dependent variables encoding amplitude and phase, respectively, of the surge and heave wave exciting forces, the surge and heave velocities, and scaled angular velocities of the sphere and the pendulum, relative to the sphere. The scaling of angular velocities by l assists with subsequent non-dimensionalisation and the introduction of relative velocity Ω_r as a proxy for the pendulum rotation is important in allowing the governing equations to be expressed in a natural symmetric manner as shown below. Thus, applying the decompositions (B.14) to (B.13) results in

$$\begin{aligned}
-i\omega(M+m)U + i\omega M\hat{L}\Omega + i\omega m(\Omega + \Omega_r) &= X_{w,x} - \frac{i}{\omega}K_{11}U - \frac{i}{\omega l}K_{12}\Omega, \\
-i\omega\hat{I}\Omega + i\omega M\hat{L}U &= -\frac{i}{\omega l}K_{12}U - \frac{i}{\omega}(K_{22}/l^2 + (Mg/l)\hat{L})\Omega + \gamma\Omega_r, \\
-i\omega(M+m)V &= X_{w,z} - \frac{i}{\omega}(K_{33} + \rho gS)V, \\
-i\omega m\hat{k}^2(\Omega + \Omega_r) + i\omega mU &= -\frac{i}{\omega}(mg/l)(\Omega + \Omega_r) - \gamma\Omega_r
\end{aligned} \tag{B.15}$$

588 where $\hat{L} = L/l$, $\hat{I} = I/l^2$ and $\hat{k} = k/l$. It is clear from the third line of (B.15) that the heave
589 motions are independent, or uncoupled, to surge, pitch and pendulum motions.

The result of adding the fourth to the second equation is

$$\begin{aligned}
-i\omega(\hat{I} + m\hat{k}^2)\Omega - im\hat{k}^2\Omega_r + i\omega(M\hat{L} + m)U &= -\frac{i}{\omega l}K_{12}U \\
&\quad - \frac{i}{\omega}(K_{22}/l^2 + (Mg/l)\hat{L} + (mg/l))\Omega - \frac{i}{\omega}(mg/l)\Omega_r.
\end{aligned} \tag{B.16}$$

590 Organising the first line of (B.15), (B.16) and then the last line of (B.15) as a 3×3 matrix equation
591 gives

$$-i\omega\mathbf{M}\mathbf{U} = \mathbf{X}_w - \frac{i}{\omega}(\mathbf{C} + \mathbf{K})\mathbf{U} - \gamma\mathbf{G}\mathbf{U} \tag{B.17}$$

592 where the vectors are

$$\mathbf{U} = (U, \Omega, \Omega_r)^T \quad \text{and} \quad \mathbf{X}_w = (X_{w,x}, 0, 0)^T, \tag{B.18}$$

593 and the matrices are

$$\mathbf{M} = \begin{pmatrix} M+m & -M\hat{L}-m & -m \\ -M\hat{L}-m & \hat{I}+m\hat{k}^2 & m\hat{k}^2 \\ -m & m\hat{k}^2 & m\hat{k}^2 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (Mg/l)\hat{L} + mg/l & mg/l \\ 0 & mg/l & mg/l \end{pmatrix}, \tag{B.19}$$

594 and

$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12}/l & 0 \\ K_{12}/l & K_{22}/l^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.20}$$

595 The system defined by (B.17) is similar to that described in [5] for the SEAREV device.

596 It should be noted that we have not included the dynamic effects of the catenary mooring lines
597 on the equations of motion of the coupled sphere/pendulum system. For heave and pitch motions
598 there will be no net restoring forces from the mooring line mass but there will be extra inertia

599 contributions. In reality, the mooring lines would also exhibit added inertia due to acceleration
600 of the fluid and viscous damping losses due to its motion. These terms have been neglected in
601 the model above for simplicity and on the assumption they will have a small effect on the overall
602 dynamics.

603 Appendix C. Wave climate parametrisation

604 This appendix describes the process used in order to be able to predict the device performance
605 over a given wave energy spectrum for a particular wave energy test site. [14] provide scatter
606 diagrams of sea states for a number of wave energy test sites, including EMEC illustrated in
607 Fig. C.12, which we will use in this study. Data is binned in intervals of 1 s for T_z , the zero-crossing
608 period, and 0.5 m for H_s , and the occurrence of each sea state over a given period is provided.
609 Using this data we define a function $P(H_s, T_p)$ to be the joint probability of the occurrence of a
610 pair of parameter values; it is assumed that $T_p = \sqrt{2}T_z$.

611 We employ the two parameter spectrum developed by [3],

$$S(T) = \frac{5}{16} H_s^2 \frac{T^5}{T_p^4} e^{-\frac{5}{4}(T/T_p)^4}, \quad (\text{C.1})$$

612 where H_s denotes the significant wave height – defined as the mean height of the highest third
613 of waves – and T_p the peak wave period in the spectrum. Then, using the probability function
614 $P(H_s, T_p)$, define a modified spectrum,

$$\tilde{S}(T) = \sum_{H_s} \sum_{T_p} P(H_s, T_p) S(T; H_s, T_p), \quad (\text{C.2})$$

615 where the sums extend over the full range of expected sea states. Thus $\tilde{S}(T)$ takes into account the
616 probability of occurrence of each sea state.

617 We also incorporate a function to describe the spread of the energy density of the incident wave
618 field, such that the incoming waves are no longer assumed to be unidirectional.

619 The directional spread of the incident waves is incorporated using a normalised cosine(2s) func-
620 tion,

$$G(\theta) = \begin{cases} F(s) \cos^{2s}(\theta) & |\theta| < \pi/2, \\ 0 & \text{otherwise,} \end{cases} \quad \text{where} \quad F(s) = \frac{2^{2s-1} \Gamma^2(s+1)}{\pi \Gamma(2s+1)}, \quad (\text{C.3})$$

621 where θ is the angle of incidence of the incoming wave. We have taken the predominant wave
622 direction to be zero. The variable s is known as the spread parameter that can be taken to be
623 constant or frequency dependent. We set $s = 12$, which is commonly used as an estimate for
624 practical purposes, [17]. The function $G(\theta)$ is normalised such that,

$$\int_{-\pi}^{\pi} G(\theta) d\theta = 1. \quad (\text{C.4})$$

625 The mean incident wave power per unit crest length is then given by

$$\overline{W}_{inc} = \rho g \int_{-\pi}^{\pi} \int_0^{\infty} c_g(T) \tilde{S}(T) G(\theta) T^{-2} dT d\theta, \quad (\text{C.5})$$

626 in units of kW/m, where $c_g(T)$ is the group velocity of the waves as a function of period [see 8, for
627 example] which, in deep water is equivalent to $c_g = g/(2\omega)$, and $\tilde{S}(T)$ the modified wave energy
628 density spectrum, given in (C.2).

Tz/Hs	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5
0.25	2.698	3.680	1.976	0.805	0.362	0.254	0.139	0.032	0.002			
0.75	2.312	9.523	5.893	2.098	0.747	0.185	0.086	0.041	0.002			
1.25	0.461	6.595	7.096	2.996	1.150	0.334	0.108	0.039		0.009	0.007	0.002
1.75	0.132	2.070	7.783	2.871	1.047	0.358	0.141	0.021			0.005	
2.25		0.293	5.497	3.904	1.099	0.324	0.187	0.067	0.009			
2.75		0.026	1.154	5.159	1.154	0.238	0.139	0.053	0.017	0.002		0.002
3.25		0.005	0.139	3.334	2.036	0.254	0.108	0.036	0.019	0.007		
3.75			0.014	0.725	2.760	0.422	0.094	0.014	0.009	0.002		
4.25				0.058	1.755	0.559	0.096	0.041	0.019		0.005	0.005
4.75				0.007	0.437	0.876	0.120	0.026	0.014	0.005		
5.25					0.046	0.780	0.146	0.032	0.005			
5.75						0.272	0.259	0.007	0.002	0.005		
6.25				0.002	0.005	0.055	0.231	0.014	0.005			
6.75					0.005	0.005	0.144	0.081	0.005			
7.25							0.067	0.113		0.002		
7.75							0.032	0.111	0.002	0.002		
8.25							0.007	0.032	0.021			
8.75								0.026	0.026			
9.25								0.002	0.019	0.005		
9.75									0.014	0.002		
10.25									0.007	0.002		
10.75										0.002		
11.25										0.005		

Figure C.12: A scatter plot of the probabilities of expected sea states at the EMEC wave site, data from [14].

629 Acknowledgments

630 The authors would like to acknowledge funding from EPSRC grant numbers EP/N508640/1 and
631 EP/N508652/1. The authors are also grateful to WITT Limited and other project partners of the
632 INNOVATE UK consortium for direction and support on this project.

633 References

- 634 [1] Babarit, A., 2015. A database of capture width ratio of wave energy converters. *Renewable*
635 *Energy* 80, 610–628.
- 636 [2] Babarit, A., Hals, J., Muliawan, M., Kurniawan, A., Moan, T., Krokstad, J., 2012. Numerical
637 benchmarking study of a selection of wave energy converters. *Renewable Energy* 41, 44–63.
- 638 [3] Bretschneider, C., 1959. Wave variability and wave spectra for wind-generated gravity waves.
639 Tech. rep., U.S. Army, Corps of Engineers.
- 640 [4] Clément, A., Babarit, A., Gilloteaux, J.-C., Josset, C., Duclos, G., 2005. The SEAREV wave
641 energy converter. In: *Proceedings of the 6th Wave and Tidal Energy Conference, Glasgow,*
642 *UK.*
- 643 [5] Cordonnier, J., Gorintin, F., De Cagny, A., Clément, A., Babarit, A., 2015. SEAREV: Case
644 study of the development of a wave energy converter. *Renewable Energy* 80, 40–52.
- 645 [6] Crowley, S., Porter, R., Evans, D., 2013. A submerged cylinder wave energy converter. *Journal*
646 *of Fluid Mechanics* 716, 566–596.
- 647 [7] Evans, D., 1981. Maximum wave-power absorption under motion constraints. *Applied Ocean*
648 *Research* 3 (4), 200–203.

- 649 [8] Evans, D., 1981. Power from water waves. *Annual Review of Fluid Mechanics* 13 (1), 157–187.
- 650 [9] Evans, D., Porter, R., 2012. Wave energy extraction by coupled resonant absorbers. *Phil.*
651 *Trans. of the Royal Soc. of London A* 370 (1959), 315–344.
- 652 [10] Goldstein, H., 1974. *Classical mechanics*. Addison-Wesley Pub. Co.
- 653 [11] Hulme, A., 1982. The wave forces acting on a floating hemisphere undergoing forced periodic
654 oscillations. *Journal of Fluid Mechanics* 121, 443–463.
- 655 [12] Ibrahim, R. A., 2005. *Liquid sloshing dynamics: theory and applications*. Cambridge University
656 Press.
- 657 [13] Newman, J., 1976. The interaction of stationary vessels with regular waves. In: 11th Symp. on
658 Naval Hydrodynamics.
- 659 [14] Nielsen, K., Pontes, T., 2010. Generic and site-related wave energy data. Report T02-1.1 OES
660 IA Annex II Task.
- 661 [15] Pizer, D., 1993. Maximum wave-power absorption of point absorbers under motion constraints.
662 *Applied Ocean Research* 15 (4), 227–234.
- 663 [16] Srokosz, M., 1979. The submerged sphere as an absorber of wave power. *Journal of Fluid*
664 *Mechanics* 95 (04), 717–741.
- 665 [17] Tucker, M., 1991. *Waves in Ocean Engineering: Measurement, analysis, interpretation*. Ellis
666 Horwood.
- 667 [18] Wello Oy, 2014. The Wello Penguin at EMEC. <http://www.wello.eu/en/penguin.php>