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Endogenous Intermediation in Over-the-Counter Markets[*]

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March 18, 2016

Abstract

We provide a theory of trading through intermediaries in OTC markets. The role of intermediaries is to sustain trade, when trade is beneficial. In our model, traders are connected through a network. Agents observe their neighbors' actions, and can trade with their counterparty in a given period through a path of intermediaries in the network. However, agents can renege on their obligations. We show that trading through a network is essential to support trade, when agents infrequently meet the same counteparty in the market. However, intermediaries must receive fees to have the incentive to implement trades. Concentrated intermediation, as represented by a star network, is both a constrained efficient and a stable structure, when agents incur linking costs. Moreover, the center agent in a star can receive higher fees as well.

Keywords: over-the-counter trading; strategic default; dynamic network formation.

JEL: D85; G14; G21.

1 Introduction

Many financial transactions take place in over-the-counter (OTC) markets where counterparties can choose whom they trade with. Often, markets participants develop long-lived trading relationships. For instance, Afonso, Kovner and Schoar (2014) find evidence that participants in the Fed Funds market frequently choose to interact with the same counterparty over time. Moreover, in various markets a relatively small group of dealers intermediate persistently the majority of trades. This concentrated intermediation structure has been documented in markets for CDS contracts (Duffie, Scheicher and Vuillemey, 2015), muni bonds (Li and Schürhoff, 2014), or securitized products (Hollifield, Neklyudov and Spatt, 2014). These regularities lead to questions about the role of intermediation and its connection to relationship trading in OTC markets.

This paper addresses these questions by proposing a theory of endogenous intermediation in OTC markets. In particular, we study the impact of trading through a network of intermediaries on the efficiency of trade, in an environment with limited commitment and limited information about agents' past actions. Intermediaries in our model can alleviate these frictions and sustain (unsecured) trade. However, intermediaries affect the division of the surplus. We show that intermediaries must be compensated to ensure they have the incentive to implement trades. The share of surplus that accrues to intermediaries is endogenously determined by incentive compatibility, and depends on the network structure. Our main results state that star networks, in which one agent intermediates all transactions, are both constrained efficient and stable structures in large economies, even as traders incur small linking costs.

To study relationship trading, we consider a dynamic setting in which agents trade bilaterally. At each date half of the agents have liquidity surpluses and half have investment opportunities. An agent with a liquidity surplus is randomly paired with an agent with an investment opportunity at each period. The liquidity agent is endowed with one unit of cash that may be lent to the paired investment agent to finance his investment opportunity, whose return depends on the amount of the borrowing.

In this environment, we consider two frictions. First, we assume that there is limited commitment, and that agents can renege on due payments at the end of the period. This

friction captures the fact that agents in financial markets can strategically default and benefit from it at the expense of their counterparties. For instance, in the Fed funds market banks can delay the delivery of overnight loaned funds until the afternoon hours, while in the repo markets agents strategically postpone both the delivery of the collateral (failure to deliver) and repaying the loan (failure to receive).[1] More generally, agents can use the funds borrowed to engage in excessive risk taking activities that would preclude them from repaying their debts.

Second, we consider that agents have limited access to information about other agents' past behavior. This assumption is motivated by the fact that, while OTC markets are opaque and information about the terms of trade is not public, financial institutions may nevertheless have access to soft information about their long-term trading partners.[2] In particular, we consider that traders are connected through an informational *network* that allows each agent to observe the repayments that his neighbors make.

In the presence of limited commitment, agents have to rely on self-enforcing contracts to implement trades. In particular, repayments may be enforced if agents can be threatened with exclusion from the market in case they default on their obligations. The information observed through the network allows agents to implement such threats. For this, however, transactions must take place through intermediaries in the network. Unless contracts are self-enforcing, trade breaks down.[3]

We obtain three sets of results. The first set highlights the role of intermediaries in sustaining trade. We start by showing that trade is not sustainable in large economies in which no agent is linked to any other agent. At the same time, we show that a star network can sustain trade, no matter how large the number of market participants is. However, the center agent in the star must be compensated to ensure he has the incentive to intermediate trades. In particular, since the center agent transfers funds between liquidity

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[1] Bartolini, Hilton and McAndrews (2010) document settlement delays in the money market, while Gorton and Muir (2015) present evidence of fails in repo markets.

[2] For instance, Du et al. (2015) show that participants in the CDS market choose their counterparties based on their risk profile.

[3] A credit bureau that collects and makes credit records public can make intermediaries redundant. However, there are significant difficulties associated with creating such institution. Typically, financial market participants are reluctant to disclose to regulators not only information about themselves, but also information about their counterparties. Indeed, financial institutions see putting a counterparty into default as a very serious step.

and investment agents, he must receive appropriate fees to overcome the temptation to retain the funds for himself. The fees in our model are endogenously determined by incentive compatibility. The incentive compatibility constraint for agents who use the intermediation service sets an upper bound for the fees the center agent receives, while the incentive compatibility constraint for the center agent himself sets a lower bound. In addition, by comparing different network structure we highlight the relative advantage that network positions offer some agents over others. We find that the center agent in a star network can receive a higher fee than any intermediary in other classes of networks we study.

The second set of results focuses on welfare improvements that trading through a network can bring in the presence of linking costs. When taking linking costs into account, maximizing expected welfare involves a trade-off. On the one hand, a higher level of investment increases welfare. On the other hand, a network that implements a high level of investment may involve a higher linking cost. We show that the star network is a constrained efficient network when it can sustain a level of investment sufficiently close to the first-best, provided that the linking costs are not too high and that the market size is large.

The third set of results concerns network formation and stability, when agents incur linking costs. In particular, we investigate whether agents have an incentive to participate in a network and identify structures that are stable when traders are allowed to change their links. We propose a dynamic network formation game, and introduce an appropriate stability concept. We show that a star network is stable.

Although stylized, our results are consistent with the observed features of OTC markets we have described above. In our set-up, a star network is both stable and constrained efficient. This is consistent with observations about the pattern of trades in OTC markets. For instance, Li and Schürhoff (2014) show that nearly 80% of the trades are intermediated by only one dealer, with the remainder involving longer intermediation chains. For our results, we concentrate on characterizing equilibria in which all trades take place without collateral. Since we do not aim to make quantitative statements, this simplifies the analysis without losing insights. However, while in some markets, such as the Fed Funds market, trade is unsecured, this is not always the case. In Section 5 we discuss an extension

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of our model in which transactions are collateralized. Moreover, our insights can be readily transferred into a more realistic but less tractable model that allows for partial collateralization in transactions.

Related Literature

This paper relates to several strands of literature. The more relevant studies are those on intermediation in OTC markets, trading in networks and contract enforcement.

A series of papers, starting with Duffie, Garleanu and Pedersen (2005), has studied trading in OTC markets. While initially these studies have been concerned with explaining asset prices through trading frictions, several recent additions to the literature are interested in the role of intermediaries in OTC markets. Hugonnier, Lester and Weill (2014), Neklyudov (2014) and Chang and Zhang (2015) propose models in which intermediaries facilitate trade between counterparties that otherwise would need to wait a long time to trade. In our model, agents also trade through intermediaries to overcome frictions that arise from search. However, our focus is on informational frictions, as is in Glode and Opp (2015) and Fainmesser (2014). While in the first paper the role of intermediaries is to reduce adverse selection and restore efficient trading, in the second one intermediaries can informally enforce the repayment of loans by borrowers, as in our model. In these studies, however, intermediaries are exogenously determined. In contrast, in our model, certain agents endogenously assume the role of intermediaries to facilitate repeated interactions between traders in the market. Di Maggio and Tahbaz-Salehi (2015) show that intermediaries can alleviate moral hazard problems in the economy if trade is collateralized. However, the intermediation capacity is bounded when there are collateral shortages. We show that intermediaries can alleviate inefficiencies in OTC markets even if such a case were to arise. In addition, we allow agents to choose how to form links and analyze which networks are stable.

There is a growing literature that studies trading in financial networks (e.g. Colla and Mele, 2010, Ozsoylev and Walden, 2011, Babus and Kondor, 2013, Zawadowski, 2013, Gofman (2014), Malamud and Rostek, 2014). These papers typically model trades that take place either sequentially or in a spot market. Either way, trading relationships are not considered. In contrast, the role of repeated interactions is at the core of our analysis.

The literature on contract enforcement is substantial. The general aim of this literature is to show that repeated interactions alleviate problems that arise when there is limited enforcement of contracts. Allen and Gale (1999) propose a model where two parties that interact repeatedly can implement the first-best contract, even though contracts are incomplete. Other papers depart from the assumption that the same two parties interact with each other, and consider a large population of agents that are matched at random to interact every period. In this case, whether contracts can be enforced or not depends crucially on how much information is available to each agent. Greif (1993) and Tirole (1996) propose an enforcement mechanism based on community reputation, while Klein and Leffler (1981) rely on costless communication between consumers to enforce that firms supply products of high quality to the market. In this paper we also study whether it is possible to enforce first-best contracts through repeated interactions when agents are randomly matched to trade. However, in our model agents have access to information via a network of bilateral relationships. We provide conditions under which agents rely on the network to trade the efficient contracts. In addition, we allow agents to choose how to form these relationships and analyze which networks structures are stable.

The paper is organized as follows. The next section introduces the model set-up. In Section 3 we describe in detail the trading protocol and analyze when unsecured trade is implementable, as well as the efficiency of trading through networks. We propose concepts for network formation and show which networks are stable in Section 4. Section 6 concludes.

2   The Environment

Time is discrete and has an infinite horizon. A set of agents, $N = \{1, ..., 2n\}$, participate in the market at each date t. All agents are risk-neutral, infinitely lived, and discount the future with the discount factor $\beta = 1/(1 + \phi)$, where ϕ is the discount rate. At the beginning of each period, uniformly at random, half of the agents are assigned a liquidity surplus, and the other half are assigned an investment opportunity. Let $\mathcal{L}^t$ be the set of agents with liquidity surpluses in period t (henceforth, *liquidity agents*), and $\mathcal{I}^t$ be the set of agents with investment opportunities in period t (henceforth, *investment agents*).

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A liquidity agent is endowed with one unit of cash, which can be stored at no cost until the end of the period. An investment agent has an opportunity to invest in an asset that matures at the end of the period. The investment in the asset is scalable: if an amount $q \in [0, 1]$ is invested, the asset yields a return $R(q)$. We assume that R is strictly increasing, strictly concave, twice differentiable with $R'(1) \geq 1$ and $R(0) = 0$.

To exploit the investment opportunity, an investment agent $i \in \mathcal{I}^t$ needs to borrow funds from some liquidity agent $\ell \in \mathcal{L}^t$ at the beginning of each period, t. Typically, in OTC markets parties trade customized contracts. To capture this feature, we assume that once agents have been assigned a type (liquidity or investment), liquidity and investment agents are matched uniformly at random, and each investment agent can borrow only from the liquidity agent he is matched with. The debt must be repaid at the end of the period.

Formally, a matching $\mathbf{m}^t$ is a subset of $\mathcal{L}^t \times \mathcal{I}^t$ such that for each liquidity agent $\ell \in \mathcal{L}^t$, there is a unique investment agent $i \in \mathcal{I}^t$ for which the pair $m^t = (\ell, i) \in \mathbf{m}^t$. At each date t, a matching $\mathbf{m}^t$ is randomly drawn from the set of all possible matches at date t. The probability that a pair of agents $(k, k') \in \mathcal{N} \times \mathcal{N}$ is matched at date t is then[4]

$$\Pr[(k, k') \in \mathbf{m}^t] = \frac{1}{2(2n - 1)}.$$

For the remainder of the paper, we refer to a pair of agents before any uncertainty is realized as (k, k'), and to a matched pair of liquidity and investment agents as (ℓ, i).

In this environment, we consider two frictions. First, we assume that there is limited commitment, and that agents can renege on obligations at the end of the period.[5] Second, we consider that agents have limited access to information about other agents' past behavior. In particular, we consider that agents are connected through an informational *network* that allows each agent to observe the unilateral actions that his neighbors take. A network, g^t, is a graph $(\mathcal{N}, \mathcal{E}^t)$, where $\mathcal{N}$ is the set of nodes, and $\mathcal{E}^t \subset \mathcal{N} \times \mathcal{N}$ is the set of links that exist between agents at date t. The set of agents who have a link with agent k in the network g^t, or, the set of agent k's neighbors, is denoted by $\mathcal{N}_k^t$. The information

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[4]This is because the probability that k is a liquidity agent is $\frac{1}{2}$. Then, conditional on being a liquidity agent, the probability that he is matched with k' as an investment agent is $1/(2n - 1)$.

[5]In our environment agents do not have collateral to secure trades. We discuss collateralized trades in Section 5.

that agents observe is described in detail in Section 3.1.

Trade may break down in the presence of limited commitment. To counteract this problem agents can use the information they access through the network and trade (without collateral) by relying on self-enforcing contracts. In particular, agents have the option to trade through the informational network. Given a network g^t and a realization of the matching $\mathbf{m}^t$, the pairs that are matched at date t may or may not be connected by a link. If a matched pair (ℓ, i) has a link in the network g^t, they can trade directly through their link. If a matched pair (ℓ, i) does not have a link in the network g^t, they can trade through a *path of intermediaries*. A path of intermediaries between a pair $(k, k') \in \mathcal{N} \times \mathcal{N}$ in a network g^t is a sequence of agents $(j_1, j_2, ..., j_v)$ such that the links $(k, j_1), (j_1, j_2), ..., (j_v, k') \in \mathcal{E}^t$. We use $\mathbb{P}^t(k, k')$ to denote the set of paths from k to k' in the network g^t, and $\mathcal{P}^t(k, k')$ to denote a generic path. Similarly, once the matching $\mathbf{m}^t$ is realized, we use $\mathbb{P}^t(m^t)$ to denote the set of paths that can be used to intermediate trade between a matched pair $m^t = (\ell, i)$, and $\mathcal{P}^t(m^t)$ to denote a generic path. The trading protocol is described in detail in Section 3.1. The network has, thus, both a trading and an informational function.

Links in the network are costly. In particular, each agent, k, incurs a *linking* cost for each link he has in the network that has two components: a recurrent component, c_l, that is paid every period, and an idiosyncratic component, c_m, that is paid only in the periods in which the link is used in a transaction. A link can be used in a transaction when it connects a pair of matched agents, or when it connects agents that intermediate trade between a matched pair. Thus, the total cost that an agent pays in any given period t depends not only on his position in the network, but also on the realized matching $\mathbf{m}^t$ and the path of intermediaries used to trade. The motivation behind the structure of the linking costs is related to the two functions that a network has. The idiosyncratic component, c_m, can be interpreted as a *transaction cost*, while the recurrent component, c_l, can be interpreted as a cost to access information, or *informational cost*.

We study when the first-best allocation can be decentralized, and characterize second-best outcomes as well.

3 The (Repeated) Trading Game

In this section we take the network $g = (\mathcal{N}, \mathcal{E})$ as fixed for all periods.[6] We analyze the set of *financial contracts* for which trade takes place, if the level of investment is $q \in [0, 1]$. The *level of investment*, q, is defined as the amount that each investment agent borrows from the liquidity agent with whom he is matched, and invests in the asset.

We begin by specifying the contracts and the trading game, and define strategies and equilibrium. We characterize the level of investment that is implementable in equilibrium. Then, we proceed to analyze the efficiency of financial networks.

3.1 Financial Contracts and Trading Procedure

For each investment level q, the terms of trade between a matched pair is determined by a *financial contract* which has two components. The first component specifies an amount, $d \in [q, R(q)]$, that an investment agent should repay a liquidity agent with whom he is matched in exchange for borrowing q units of funds. The second component specifies a fee $f \in \mathbb{R}_+$ that intermediaries can receive. More precisely, if a pair (k, k') is matched and trade through a path $\mathcal{P}(k, k') = (j_1, j_2, ..., j_v)$, then the investment agent should repay in total $d + \sum_{s=1}^{v} f$, such that an intermediary j_s receives f, for any $s = 1, ..., v$.

The financial contract, (d, f), is independent of the position of the agents in the network. An agent's position in the network is only reflected in the total payoff he expects to receive in a given period. However, a crucial feature of our analysis is that the financial contract depends on the network structure g. By comparing different network structure we highlight the relative advantage that network positions offer some agents over others. We also allow the financial contract to depend on the level of investment, q.

In the presence of limited commitment, the incentive of intermediaries to transfer the repayments to the next agent depends on the future benefits they expect to receive from trade. In particular, an agent with a liquidity surplus who is an intermediary may find it optimal to keep the repayments for himself, without the expectation of receiving fees in the future. The fees in the financial contracts have then to be adequately designed to deter the incentives of the intermediaries to renege on their obligations, with respect to

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[6]In Section 4 we relax this assumption and analyze issues related to network formation and stability.

the information obtained from the network.

The trading procedure at date t, is given as follows. First each agent is assigned a type (liquidity or investment), and the matching $\mathbf{m}^t$ realizes. These realizations are common knowledge among all agents.

Then, for each matched pair $m^t = (\ell, i) \in \mathbf{m}^t$, the investment agent i proposes a path $\mathcal{P}(m^t) = (j_1, j_2, ..., j_v)$ through which to trade with ℓ. We allow the investment agent i to propose the *empty path*, that is, to propose to trade directly with ℓ even if they don't have a link, and circumvent the intermediaries given by the network g. We assume that this proposal is common knowledge to all agents. Each agent on the path then sequentially responds with an *yes* or *no*, starting from j_1 and ending with ℓ. If all agents on the path respond with *yes*, then trade takes place and the liquidity agent, ℓ, transfers q units of cash to the investment agent, i, through the path. Otherwise, there is no trade between the matched pair m^t along the proposed path.

If trade takes place, each agent on the path has a debt obligations to the next one according to the financial contract, (d, f), as follows. The agent i is obligated to repay $[d + v \cdot f]$ to j_1. Further, each intermediary $j_{v'}$ is supposed to receive $[d + (v - s' + 1)]$ from $j_{s'-1}$ and is obligated to repay $[d + (v - s') \cdot f]$ to $j_{s'+1}$, with $j_0 = i$ and $j_{v+1} = \ell$.

After the investment realizes its payoff, each agent on the path decides whether he repays his debt obligation. The decision depends on both the agent's willingness to repay and the resources that are available to him. In particular, an intermediary may not have sufficient resources to honor his obligations if the agents before him on the path do not honor theirs. In what follows we assume that an agent either repays in full or repays nothing. This assumption will simplify our notation without losing any insights.

If there is no trade, the liquidity agent retains the unit of cash and the opportunity to invest is foregone. Intermediaries receive no fees on the path $\mathcal{P}(m^t)$.

Next, we describe the information structure in detail. As we discussed earlier, an agent j can observe each of his neighbors' unilateral actions, as well as information that is common knowledge, which includes the realized types of the agents, the matching, and the proposed paths by each investment agent. For each agent k, his unilateral actions in the network g at date t, denoted by a_k^t, include the following elements: (i) his responses on the proposed trading paths that he is involved; (ii) whether he repays in full to each of his

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neighbors, if he is either an intermediary and/or an agent with an investment opportunity. If an agent repays directly to agents other than his neighbors, his action is not observed by his neighbors. Let $\mathbf{a}_k^t = \left(a_k^0, ..., a_k^t\right)$ be the unilateral actions taken by agent k up to date t, and let $\mathbf{a}_0^t = \left(a_0^0, ..., a_0^t\right)$ be the commonly known information up to date t. Then, the history that an agent k observes at date t is given by $h_k^t = \left\{\mathbf{a}_j^t : (j,k) \in \mathcal{E}\right\} \cup \{\mathbf{a}_0^t\}$.

Because an agent may be involved in multiple trading paths, we need to specify a timing for their responses and repayments. For each proposed trading path $\mathcal{P} = (j_1, j_2, ..., j_v)$ between a matched pair $m = (i, \ell)$, agents in position j_1 respond simultaneously first, then agents in position j_2, etc. Similarly, for repayment decisions, investment agents decide first simultaneously, and then agents in position j_1, depending on the resources repaid by investment agents, and then agents in position j_2, etc.

Next we introduce strategies and the equilibrium concept. First we define strategies. For each agent k, his strategy in period t, denoted by s_k^t, has three components:

- $s_k^{t,1}$ maps the history h_k^{t-1} he observes, the realization of agents' type, and the matching $\mathbf{m}^t$ to a proposed path, if he is an investment agent;

- $s_k^{t,2}$ maps the history h_k^{t-1} he observes, the commonly known information $\mathbf{a}_0^t$, and the responses of his neighbors before him on the paths that involve him to his responses, if he is a liquidity agent and/or an intermediary;

- $s_k^{t,3}$ maps the history h_k^{t-1} he observes, the commonly known information $\mathbf{a}_0^t$, and the repayments of his neighbors before him on the paths that involve him to his repayment decisions on all trading paths he is involved, if he is an investment agent and/or an intermediary. Note that his repayment decision is constrained by repayment decisions of agents before him on the trading paths.

We use Perfect Bayesian Equilibrium (PBE) as the solution concept. We restrict attention to equilibria that satisfy the following properties.

(A1) **No default**. Every agent consents to trade according to the contract (d, f) and there is no default in equilibrium plays.

(A2) **Shortest path.** The shortest paths in the network g are always proposed in equilibrium. When there are multiple shortest path between a matched pair, they are proposed

with equal probabilities in equilibrium.

(A3) **Stationary equilibrium allocation**. The level of investment, q, is constant across realized matches and across periods.

Definition 1 *A PBE equilibrium satisfying (A1)-(A3) is called a **simple equilibrium**.*

Condition (A1) is a symmetry requirement, as it rules out the possibility that only a subset of agents trade. Similar considerations motivate condition (A3). Condition (A2) requires that the equilibrium trading paths are the shortest ones. This assumption simplifies our analysis, since in general networks multiple paths may be used to trade, but only the shortest one minimizes the expected transaction cost, c_m.

3.2   Implementation and Constrained Efficiency

In this section we characterize contracts that can be implemented in equilibrium and analyze their welfare properties. We conclude the sections by with some observations about the compensation that intermediaries receive.

3.2.1   Contract implementation

We start by exploring the role of networks in supporting trade in equilibrium. We first describe how the gains from trade depend on the level of investment q. We then characterize the investment level, q, that is implementable in a given network g. Focusing on the level of investment, q, provides a rich metric to differentiate across those network structures in which trade can be sustained.

Definition 2 *A level of investment, q, is **implementable** in a network g if it is supported in a simple equilibrium for some associated financial contract (d, f).*

Abstracting from linking and transaction costs, trade is always beneficial. In particular, when all matched pairs trade and the level of investment is q, then the average surplus generated at each date is

$$\Delta(q) = R(q) - q. \tag{1}$$

Since the return $R(\cdot)$ is strictly concave and increasing, the condition $R'(1) \geq 1$ ensures that $\Delta(\cdot)$ is increasing in $q \in [0, 1]$ with $\Delta(q) > 0$ for all $q \in (0, 1]$. The gains from

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trade are maximized when $q = 1$. This implies that $q = 1$ represents the *first-best* level of investment.

Although trade generates a positive surplus, it is not necessarily the case that it can be supported in equilibrium. Even in the least restrictive case of complete information, when all histories are publicly observable, trade can be supported in equilibrium for an investment level q if and only if[7]

$$\phi q \leq \frac{1}{2}\Delta(q). \tag{2}$$

The intuition is simple. Agents weigh the long-term benefit from participating in the market against the one time gain of retaining all the return of the asset and paying 0. In particular, when an investment agent decides whether to repay at the end of the period, he takes into consideration he will be excluded from the market at all future dates as an investment agent, if he defaults on his obligations.

When there is incomplete information, condition (2) is no longer sufficient. In this case, the frequency with which an agent trades with a counterparty affects his incentives to default on his obligations. As we show below, networks may implement an investment level q for which there are positive gains from trade, particularly when the number of market participants grows large.

To understand the role of networks in supporting trade, we first explore the *empty network* benchmark. In an empty network, no agent is linked to any other agent. In this case, once the agents' type has been assigned and the matching has been realized, an investment agent can only propose to trade directly (i.e. the empty path) with the liquidity agent he has been matched with. The liquidity agent can then respond either *yes* or *no*. No agent intermediates trades. Aside from the information that is common knowledge, each agent observes only the action of his counterparty at a given date t. Note that this trading procedure is a special case of the trading procedure described in Section 3.1. The following lemma characterizes the level of investment that is implementable in the empty network.

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[7]We do not provide a proof for this statement, as the result is standard.

Lemma 1 *Let agents trade in an empty network.*

(i) A level of investment, q, is implementable if

$$\phi q \le \frac{1}{2(2n-1)} \Delta(q).$$ (3)

(ii) For any level of investment $q > 0$, there exists $\bar{n}$ such that q is not implementable for all $n \ge \bar{n}$.

The lemma shows that the level of investment that is implementable when no information (other than agents' own past trades) is observable depends on how large the economy is. This is because the market size affects how likely it is that two counterparties who trade at date t, meet again in a given future period. When n grows large, the probability of meeting the same agent in future periods is small. Thus, if an agent defaults on his current obligation but repays in future trades with other counterparties, the threat that he will not trade when he meets his date-t counterparty again is not binding as n grows large. Hence, he cannot overcome his temptation to default. As a result, when the market size increases, no level of investment is implementable in an empty network, even though there may be positive gains from trade.[8]

In a stark contrast with the empty network is the level of investment implementable in a star network, which we characterize next. A network is a star if there exists an agent k_C such that

$$\mathcal{E} = \{(k_C, j) : j \in \mathcal{N}, j \ne k\}.$$

We refer to agent k_C in a star network the *center agent*. All other agents in the star network are *periphery agents*. A star network with $2n$ agents is denoted g_n^*. Figure 1(a) illustrates a star network.

When analyzing implementation in networks, such as the star or more general structures, linking costs also affect agents' incentives to make repayments. In particular, the

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[8]Lemma 1 also implies that the trading procedure we consider, in which each matched pair trades through the network, is without loss of generality. Recall that, in a network, an agent's repayment to agents other than his neighbors is not observed by his neighbors. In particular, we cannot implement unsecured trades in which an investment agent repays directly to the liquidty agent he is matched with and who is not his neighbor, when n is sufficiently large.
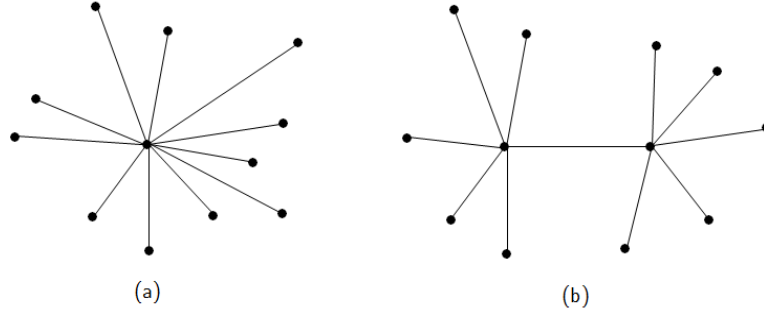
Figure 1: This figure illustrates two types of networks with the same number of agents. Panel (a) shows a star network. Panel (b) illustrates an inter-linked star network.

transaction cost, c_m, is consequential, since an agent incurs it for each of his links that is used to trade in a given period. In contrast, the agent incurs the informational cost, c_l, for each of his links every period, and hence it does not affect his repayment decision. While only transaction costs play a role in implementation, both costs influence significantly welfare and the stability of networks, as discussed later.

To characterize equilibria in networks for the remainder of the paper, we restrict our attention to financial contracts with the property that $d \geq q + c_m$. We use this restriction for simplicity, as it ensures that the liquidity agent is willing to lend to the investment agent through the network, provided that he believes that his counterparties will repay their debts. No insights are lost if we relax the assumption.

The next proposition characterizes the levels of investment that can be implemented under a star network.

Proposition 1 *Let agents trade in a star network g_n^*. Then, a level of investment, q, is implementable if*

$$\phi(q + c_m) + 2c_m \leq \frac{1}{1/2 + \phi}\left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m\right]. \tag{4}$$

Proposition 1 provides a sufficient condition for a star network to implement a given level of investment q, that is independent of the number of market participants. Thus, even as n grows large, agents can still trade as long as the level of investment q satisfies (4).

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We obtain condition (4) by ensuring that both center and periphery agents have the incentive to repay their obligations. Consider first the incentives of a periphery agent. On the one hand, the largest amount that a periphery agent can retain if he reneges on his obligations is $(d + f)$. On the other hand, the expected discounted future benefit of trading in the star network relative to no trade is at least $\frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) - \frac{1}{2}f - c_m\right]$. Indeed, the first term, $\frac{1}{2}\Delta(q)$, reflects the gains from trading weighted by the probability that the agent is assigned the investment role. The second term, $\frac{1}{2}f$, reflects the expected fee that an agent must pay to the center agent, when he is an investment agent matched with another periphery agent. The third term reflects the transaction cost. Thus, if

$$-(d + f) + \frac{\beta}{1 - \beta}\left[\frac{1}{2}\Delta(q) - \frac{1}{2}f - c_m\right] \geq 0,$$

or

$$f \leq \frac{1}{1/2 + \phi}\left[-\phi d + \frac{1}{2}\Delta(q) - c_m\right], \tag{5}$$

then a periphery agent has incentive to make repayments.

Next, consider the incentive of the center agent. On the one hand, the largest amount that the center agent can retain if he reneges on his obligations is nd. On the other hand, the expected discounted future benefit from trading and intermediating relative to not trading in the star network is $\frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) + (n - 1)f - (2n - 1)c_m\right]$. As before, the first term, $\frac{1}{2}\Delta(q)$, reflects the relative gains from trading weighted by the probability that the agent is an investment agent. In addition, every period he receives an amount $(n - 1)f$ in fees, while his total transaction cost is $(2n-1)c_m$. Thus, the center agent has an incentive to make repayments if

$$-nd + \frac{\beta}{1 - \beta}\left[\frac{1}{2}\Delta(q) + (n - 1)f - (2n - 1)c_m\right] \geq 0,$$

which holds when

$$f \geq \phi d + 2c_m, \tag{6}$$

since $-\phi d + \frac{1}{2}\Delta(q) - c_m \geq 0$ from (5). Setting $d = q + c_m$, condition (4) ensures that there exists a fee f that satisfies the two inequalities (5) and (6).

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The condition for implementation of an investment level, q, in a star network is comparable to the complete information case. Indeed, if we take $f = 0$ and $c_m = 0$ in inequality (5) we obtain condition (2) as when there is complete information. However, condition (6) reflects a crucial distinction that arises because of the asymmetry in the information that center and periphery agents can access in a star network. While the center agent has information about all other agents in the economy, a periphery agent has information only about the center. Thus, the center agent has the incentive to repay only when he expects to receive a positive fee. In fact, condition (6) is a lower bound and condition (5) is an upper bound for the fee that the center agent must receive, in the limit as the number of market participants grows large.

3.2.2 Welfare and efficiency

Next we turn to the welfare properties of networks, taking the linking costs into account. Our aim is to characterize constrained efficient networks. We begin with our welfare criterion. Given a network g, and an investment level q, the expected aggregate welfare when trades take place is given by

$$W(g,q) = \sum_{t=0}^{\infty} \beta^t n \left\{ R(q) - q + 1 - 4\eta_g c_l - 2(v_g + 1)c_m \right\} \tag{7}$$

where η_g represents the *average number of links*, and v_g represents the *average number of intermediaries* between pairs of agents in g, respectively.

As it is evident from (7), the direct effect of a network structure, g, on welfare can be summarized by only two variables, η_g and v_g. Given the network g, the total informational cost per period is $|\mathcal{E}| \cdot (2c_l) = (2n) \cdot \eta_g \cdot (2c_l)$. The total transaction cost depends on the realization of the matching. However, in expectation, in any given period, it only depends on the average number of intermediaries, and hence the total expected transaction cost is $n \cdot (v_g + 1) \cdot (2c_m)$. Next, we define constrained efficiency.

Definition 3 *A network g and an investment level q is a constrained efficient arrangement if it maximizes $W(g,q)$ over the space of connected networks and investment levels such that q is implementable in g.*

Maximizing expected social welfare involves a trade-off. On the one hand, the welfare function (7) is increasing in the level of investment, q. On the other hand, there may be high linking costs associated with a network that implements a higher q. For instance, while it is possible to implement the first-best level of investment in the complete network, the linking costs become infinitely large as the number of market participants grows.

A good candidate for a constrained efficient arrangement is a network that can implement high levels of investment at low linking costs, such as the star network. Indeed, let q^* be the largest investment level that can be implemented asymptotically in the star network, that is,

$$q^* = \arg\max \Delta(q)$$

$$\text{s.t. } \phi(q + c_m) + 2c_m \leq \frac{1}{1/2 + \phi}\left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m\right].$$

We have the following result.

Proposition 2 *Suppose the first best level of investment is implementable in a star network g_n^*, or $q^* = 1$. Then (g_n^*, q^*) is the unique constrained efficient arrangement for sufficiently large n.*

The intuition is as follows. From Proposition 1, we know that a star network can implement the first-best investment level when (4) holds for $q = 1$. Thus, we only need to show that the star network minimizes the linking costs relative to all other connected networks. The key trade-off then is between the transaction and informational costs, and we prove that it never pays off to decrease the transaction costs while increasing informational costs for large n's, independently of c_m and c_l. The class of networks that have the lowest informational costs is the class of *minimally connected networks*. In a minimally connected network there exists a unique path between any pair of agents. A star networks is clearly a member of this class. Lastly, we show that transaction costs are minimized in the star among all minimally connected networks for sufficiently large n.

When the first-best is not implementable in a star network ($q^* < 1$), the trade-off between the level of investment and linking costs that the welfare function (7) embeds is even more pronounced. In particular, the gains from trade in a connected network must be sufficiently high to compensate for the linking costs that agents incur each period. We

analyze the resolution of this trade-off asymptotically. For this purpose, we first introduce the following definition.

Definition 4 *Let $\{g_n\}_n$ be a sequence of networks. Then, a level of investment, q, is **asymptotically implementable** in $\{g_n\}_n$ if there exists $\bar{n}$ such that q is implementable in g_n for all $n \geq \bar{n}$. The sequence $\{g_n\}_n$ and the investment level q is an **asymptotically constrained efficient** arrangement if for any sequence of connected networks $\{g'_n\}_n$ and any q' asymptotically implementable under $\{g'_n\}_n$, we have that $W(g_n, q) \geq W(g'_n, q')$ for all large n.*

Under asymptotic implementability, a level of investment is implementable only if it is implementable in a sequence of networks in sufficiently large economies. The next result shows that a star network can be asymptotically constrained efficient even when the first best level of investment is not implementable.

Proposition 3 *There exists a threshold $\widehat{q} < 1$ such the star network, g_n^*, and the level of investment, q^*, is an asymptotically constrained efficient arrangement whenever $q^* \geq \widehat{q}$.*

Proposition 3 extends the result in Proposition 2 to the case when the star network cannot implement the first best level of investment.

Before we lay out the intuition for this result, we need to introduce a class of networks as follows. In Lemma A.1 in the Appendix we show that there exist $\bar{\eta} > 1$ and $\bar{v} > 1$ such that for any sequence of networks $\{g_n\}$ in which $\eta_{g_n} \leq \bar{\eta}$ and $v_{g_n} \leq \bar{v}$ for all n, if a level of investment, q, is asymptotically implementable in $\{g_n\}$, then it is asymptotically implementable in star networks as well. We refer to a network g with $\eta_g \leq \bar{\eta}$ and $v_g \leq \bar{v}$, as a *small network*. Given this result, the threshold $\widehat{q}$ is determined as the minimum q such that

$$W(1, 1, q) \geq \max\{W(1, \bar{v}, 1), W(\bar{\eta}, 1, 1)\}, \tag{8}$$

where $W\left(\eta_g, v_g, q\right)$ is the welfare in a network g with average number of links, η_g, and with average number of intermediaries, v_g, when the investment level is q.

When $q^* < 1$, the trade-off between the level of investment that is implementable and linking costs for networks outside the class of small networks is resolved in favor of the

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star network, as long as $q^* \geq \widehat{q}$. Indeed, condition (8) ensures that the potential increase in the implementable investment level is offset by the increase in linking costs for any network that is not small, relative to the star. Thus, we just need to prove that the star network is the constrained efficient one among the small networks. For this, we use Lemma A.1 which shows that an investment level higher than q^* is not implementable in networks in which linking costs are bounded by $\bar{\eta}$ and $\bar{v}$.[9] Hence, the star network can (asymptotically) implement the highest investment level among all small networks. Then, using similar arguments as for Proposition 2, we show that the star network is also the most efficient one in terms of linking costs among all small networks for sufficiently large n.

Note that according to Definition 3, a constrained efficient network maximizes welfare relative to all other connected networks. Thus, the results described in Proposition 2 and 3 do not require any condition for the value of the linking costs, c_m and c_l. Of course, the implementability requirement places an upper bound on the transaction cost, c_m. However, as either informational or transaction costs rise, then the empty network may yield higher welfare even when no trade takes place.

3.2.3 Intermediary fees

Another interesting implication that arises in our setup is related to the fees that the intermediaries receive. In particular, the following result illustrates how the network structure favors some intermediaries with respect to the fees they receive.

Corollary 1 *Let $f_g^{\max}(q)$ be the maximum fee an intermediary can receive in a network g, for a given implementable investment level q. Then, for any sequence $\{g_n\}_n$ of minimally connected networks or small networks,*

$$f_{g_n^*}^{\max}(q) \geq f_{g_n}^{\max}(q)$$

for all asymptotically implementable investment levels q in $\{g_n\}_n$ and for all n sufficiently large.

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[9] Since in a "small" network the average number of links and the average number of intermediaries are bounded by $\bar{\eta}$ and $\bar{v}$, respectively, then aggregate informational cost is at most $4n\bar{\eta}c_l$, while the aggregate transaction cost is at most $2n(\bar{v}+1)c_m$.

Corollary 1 shows that the center agent in a star network can receive a higher fee than any intermediary in a minimally connected or small network. While the result holds when the same level of investment is asymptotically implemented in the star network as in a minimally connected or small network, an additional mechanism strengthens this finding. By Lemma A.1, the level of investment that is asymptotically implementable in a star network is at least as high as in a small network (we show a similar result for minimally connected networks in Babus and Hu, 2015). This implies that the surplus generated by trading, as defined in (1), is at least as large in a star network. Since the fee represents a division of the surplus between the center the periphery agents, a larger surplus makes it feasible for center agent to receive higher fees.

Corollary 1 focuses on asymptotically implementable investment levels. For exactly implementable investment levels, the maximum fee each of the two intermediaries in an inter-linked star, represented in Figure 1(b), can receive is strictly smaller than the maximum fee the center agent in a star can receive. This is because a periphery agent in an inter-linked star needs to pay fees to two intermediaries in any period with probability half, when he has an investment opportunity. Thus, ensuring he has incentives to repay places a tighter constraint on the fees that each intermediary receives. We can generalize this argument and show that the analogous result holds for any interlinked stars with a finite number of centers.[10]

4   Network Stability

We have demonstrated that the star network is the constrained efficient arrangement. In this section we investigate whether agents have incentives to participate in such a network when traders are allowed to change their links. For this purpose, we first introduce the network formation game, and then propose a stability concept.

We consider the following network-formation game. At date 0, fix a network g. At the beginning of each even period $t = 0, 2, ...$, one agent k, selected at random, is allowed to sever one or more of his links. At the beginning of each odd period $t = 1, 3, ...$, one pair of agents (k, k'), selected at random, are given the opportunity to form a link, if they do

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[10]For a formal argument in a related model, see Babus and Hu (2015).

not have one. If both agents agree, the link is formed. At each period t, agents' linking decisions result in a new network g^t.

After agents make their linking decisions, their types (liquidity or investment) are assigned, and the matching realizes. In the new network g^t, an agent only observes each of his current neighbors' unilateral actions, as well as information that is common knowledge. Then, the agents trade according to the trading procedure described in Section 3.1. Consistent with the previous section, we allow the financial contract and the level of investment to depend on the network structure. In particular, we consider a function $\mathcal{C}(g^t)$ that assigns to a network g^t a contract, (d_{g^t}, f_{g^t}), and an investment level q_{g^t}, that specify the terms of trade. The function $\mathcal{C}(\cdot)$ allows agents to evaluate their continuation payoff for each linking decision they can take at date t, given the distribution of networks that may arise at each future date τ, and given the actions that other agents are expected to take in the trading game in each possible network g_τ.

We say that the function $C(\cdot)$ is tight if q_{g^t} is the highest level of investment that is implementable in g^t, provided the set of implementable investment levels is non-empty. Given a tight function $\mathcal{C}(\cdot)$, a trading strategy profile is *tight* w.r.t. $\mathcal{C}(\cdot)$ if agents in any connected component of the network g^t accept to trade among themselves, in each period t when q_{g^t} is implementable in g^t, and after any possible partial history of the network-formation game (both on and off equilibrium paths).

Definition 5 *A network g is **stable** under $[q, (d, f)]$ if there exist a tight function, with $\mathcal{C}(g) = [q, (d, f)]$, and a Nash equilibrium in the network-formation game such that no agent severs a link and no pair of agents forms new links, and agents use a tight trading strategy profile.*

The notion of stability that we propose here is consistent with the welfare analysis we have developed in Section 3.2. In particular, it allows us to check whether constrained efficient networks are also stable. Moreover, focusing on a function that selects the highest implementable level of investment for a given network, we are able to conceptualize the value of a link in a dynamic setting. Indeed, as agents change links, they are still able to extract the maximum surplus in the new network. This implies that the relative benefit an agent obtains by maintaining a link represents a lower bound for the value of the respective

link.[11]

This notion of stability allows us to narrow down the set of stable networks in a meaningful way. For instance, suppose we relax the requirement that agents use a tight trading strategy. Instead, consider that agents refuse to trade with other agents that have changed their links. Facing a severe punishment, agents may be deterred from severing or forming new links. We conjecture that, in this case, most networks that can implement positive levels of investment are stable. The requirement to use a tight strategy rules out this type of unreasonable punishments.

We proceed to show that the star networks are stable. Let q_n^* be the level of investment such that

$$q_n^* = \arg \max \Delta(q)$$

$$\text{s.t.} \quad \min \left\{ \frac{1}{n-1} \left[n\phi(q + c_m) - \frac{1}{2}\Delta(q) + 2nc_m \right], 0 \right\} \leq \frac{1}{\frac{n-1}{2n-1} + \phi} \left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m \right]. \tag{9}$$

In the proof of Proposition 1 we provide in the appendix, we show that (9) is the necessary and sufficient condition for a level of investment q to be implementable in star network with $2n$ agents. Thus, q_n^* is the highest investment level that can be implemented in a star network, for each n. Further, let $f_{g_n^*}^{\max}$ and $f_{g_n^*}^{\min}$ to denote right-hand side and the left-hand side of condition (9) evaluated at q_n^*, respectively. Hence, $f_{g_n^*}^{\max}$ and $f_{g_n^*}^{\min}$ are the upper and lower bounds for the fee that the centre agent in a star with $2n$ agents receives. Note that $f_{g_n^*}^{\max} > f_{g_n^*}^{\min}$ only if $q_n^* = 1$. We have the following result.

Proposition 4 *Suppose that* $0 < c_l \leq \frac{1}{2}\phi(q_n^* + c_m)$. *Then,* g_n^* *is stable under* $[q, (d, f)] = [q_n^*, (q_n^* + c_m, f_n^*)]$ *for any* $f_n^* \in [f_n^{\min}, f_n^{\max}]$, *provided that* n *is sufficiently large.*

Proposition 4 shows that a star network is stable, if the informational cost, c_l, is small and the economy size, $2n$, is large. It is indeed expected that if c_l is high, then agents are better off by not trading, and, hence, the resulting equilibrium in the network formation

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[11]Our notion of stability is closely related to the pairwise stability concept introduced by Jackson and Wolinski (1996). While pairwise stability is a static concept, ours is a dynamic notion based on noncooperative game reasoning. In particular, the random selection of an agent to sever his links ensures that a stable network according to our definition is also individually rational given the linking choices of others. Similarly, the random selection of a pair of agents to form new links ensures that our stability is robust to a size-two group defection.

game will leave the network an empty one. Similarly, if n is small, agents can enforce contracts and trade in the empty network, saving on linking costs.

The intuition for the stability result is as follows. To see whether a star network is stable, we need to study agents' incentives to sever or form links. In particular, there are two main cases we need to consider. First, we need to show that the center agent has no incentive to delete any link. Second, we need to show that no periphery agent has an incentive to form a new link with another periphery agent.

We start with the center agent, k_C. We illustrate how he evaluates his continuation payoff when he makes a linking decision, given the notion of stability we proposed in Definition 5. Suppose that at the beginning of an even date t, the center agent, k_C, is given the opportunity to sever one or more of his links. If he severs links with a set K_C of his neighbors, the new network is $g_-^t = g_n^* - \{(k_C, k') : k' \in K_C\}$. As usual in equilibrium analysis, he considers that all other agents respect their equilibrium linking strategy at future dates. We construct equilibrium linking strategies with the property that agents do not sever or form new links after any partial history. This implies that the center agent expects that $g^\tau = g_-^t$ for any $\tau > t$. Further, he understands that the function $\mathcal{C}(\cdot)$ selects the highest level of investment, $q_{g_-^t}$, that is implementable in g_-^t, that $q_{g_-^t}$ is implemented and trade takes place forever after. Otherwise, if he maintains all his links, the new network is $g^t = g_n^*$, and he reasons in the same way to evaluate his continuation payoff. For the star network to be stable he must find it beneficial to maintain all his links. We show that this is the case by proving that the marginal value of a link for the center is bounded away from zero. Indeed, we find that the highest level of investment in the new network g_-^t can only be lower than q_n^*. In consequence, there exists a function $\mathcal{C}(\cdot)$ that allocates a fee, f, to agent k_C in the new network g_-^t, which ensures a positive lower bound for the marginal value of a link. This implies that the centre has no incentive to sever any link provided that c_l is small.

Next, we discuss the case of periphery agents. When a periphery agent, k, considers whether to form a link with another periphery agent, k', he evaluates his continuation payoff following a similar reasoning process as described above. If, at an odd period t, the agents consent to form a link, the new network is $g_+^t = g_n^* + \{(k, k')\}$. Otherwise, the new network is $g^t = g_n^*$. In the network g_+^t, both agents k and k' are able to trade

directly through their link, without paying a fee to the center agent in those periods when they are matched to trade. However, as n grows large, the probability of avoiding the fee diminishes, which makes the link too expensive to maintain. Thus, the new network g_+^t would be more attractive than g_n^* only if the fee paid to agent k_C is lower. We show that there exists a function $\mathcal{C}(\cdot)$ that allocates a fee, f, to agent k_C in the new network g_+^t which is higher than, or arbitrarily close to f_n^*, as n goes infinity. This ensures that two periphery agents do not have an incentive to form a new link.

We conclude this section with a remark about our notion of stability. A stronger notion of stability that departs from Definition 5 may also seem natural. Under Definition 5, a tight function can assign any fee, f, to intermediaries in a network g such that the level of investment q is implementable in g. In contrast, we can consider that a network is stable only if agents maintain their links for all fees (subject to implementability) that can be assigned in the current network, as well as in networks that arise on the continuation paths that follow deviations. Under this stronger notion, the star network may not be stable when the inequality (4) is slack. Indeed, there may exist a function $C(\cdot)$ which assigns fees to intermediaries in the network resulting from a deviation, in a way that favors the agents who just deviated. For instance, suppose that the center agent deletes one or more of his links. Consider a function $C(\cdot)$ that assigns to the center agent a higher fee than in the original star network. At the same time, the function $C(\cdot)$ can assign a lower fee to the center agent when two periphery agents form a link. For the star network to be stable under this stronger notion, the center agent must not find it beneficial to delete links, which implies that the fee he receives in the original network has to be sufficiently high. Similarly, the periphery agents must not find it beneficial to form a link between themselves, which implies that the fee they pay to the center agent in the original network has to be sufficiently low. Since a fee that meets both requirement may not exist, the star network may not be stable. A similar line of reasoning can be used to argue that an interlinked star is not stable either. In fact, this stronger notion of stability is a very demanding property, and we expect that many networks do not satisfy it. Nevertheless, the proof of Proposition 4 suggests that the star is stable under this stronger notion when condition (4) holds with equality, as this narrows the set of fees a function $C(\cdot)$ can assign.

5   Discussions

5.1   Other Trading Frictions

In our framework each agent has either cash or an opportunity to invest every period.
Moreover, all agents are matched and it is feasible for all pairs to realize the gains from
trade at each date. One can extend our framework and allow for additional frictions in the
matching and trading process. For instance, we can assume that each investment agent
has the opportunity to invest only with probability p at each date. Our framework is the
special case where $p = 1$. The case $p = 1/2n$ is equivalent to assuming that only a pair of
agents trade at any given period.

We conjecture that most of our results go through if this type of trading frictions are
not too severe. For example, we conjecture that the star is still a constrained efficient
arrangement if p is sufficiently close to 1, even as n grows large.

However, a sufficiently high trading frequency is crucial for our results. Indeed, if the
probability, p, of arrival of the investment opportunity is too small, then no investment
level $q > 0$ is implementable under any network. The logic is as follows. Suppose that,
by contradiction, we can implement a contract (d, f) in a network g, when the investment
level is q. Then, consider the incentives to repay of an investment agent who is selected
in a given period to receive the investment opportunity. On the one hand, the agent
can retain at least d if he reneges on his obligations. On the other hand, the expected
discounted future benefit of trading relative to not trading is at most $\frac{p}{2\phi}\Delta(q)$. Then, an
investment agent has the incentive to make repayments only if

$$-d + \frac{p}{2\phi}\Delta(q) \geq 0.$$

For any $q > 0$ and $d \geq q$, this condition is violated if p is sufficiently low.

5.2   Collateralized Trades

In our model, trade can only take place through unsecured transactions. We can extend
the model and incorporate collateralized trades. One way to allow for collateralized trades
is to endow an investment agent with a riskless asset that cannot be used to finance the

investment project but may be used as collateral. As before, the role of the informational network is to facilitate unsecured trade, if there are positive gains from trading without collateral relative to trading against collateral. One potential reason why unsecured trade is welfare improving is that collateral could be inefficiently liquidated by the liquidity agent, when trade is secured.

Babus and Hu (2015) explore this extension with an explicit formulation of gains from unsecured trade relative to secured trade, and show that our results hold. One virtue of this extension is that, depending on the economic fundamentals, the efficient arrangement is either unsecured trade in the star network or secured trade without an informational network (i.e. secured trade in the empty network). In particular, the star network is efficient when the limited commitment issue is not too severe.

6 Conclusion

Our results demonstrate that intermediation can be welfare-improving when OTC trades take place through networks if the market size is large. In our model, networks can provide adequate monitoring to sustain unsecured trade, provided financial contracts are designed to respect traders' incentives. In particular, we show that intermediaries must receive fees to ensure they have the incentive to sustain trade. We characterize an upper and lower bound for the fees intermediaries receive in various networks. We also show that the fee to the intermediary in the star network is the highest relative to how intermediaries in various other networks can be compensated. The way compensation of intermediaries is determined in our model may provide an explanation for the rents intermediaries receive in OTC markets.

Our analysis of the constrained efficient arrangement highlights a trade-off between the cost of maintaining and using a network, and the investment level that is implementable in a network. We provide conditions under which the star is the constrained efficient arrangement among all possible arrangements. We obtain this result when the market size is large, and when the star can implement a level of investment that is close to the first-best level. Finally, we show that the star network is a stable structure as well.

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A   Appendix

Proof of Lemma 1

(i) First we prove sufficiency. Set $d = q$. We construct a strategy profile and show that it is a simple equilibrium, as follows. For each possible match $m = (\ell, i)$, we summarize the observed history of the match at the end of period t (which is observable to the match) with a state $s_{m,t} \in \{G, B\}$. We use m^r to denote the match with the same pair of agents but with their roles reversed, i.e., $m^r = (i, \ell)$. The state is such that $s_{m,t} = s_{m^r,t}$ for all t, and it evolves as follows: $s_{m,0} = s_{m^r,0} = G$; $s_{m,t+1} = G$ if $s_{m,t} = G$ and if either one of the following two conditions holds: (a) neither m or m^r realizes at period $t + 1$, (b) either m or m^r is realized, and the agent assigned to the investment role repays his debt if the unsecured trade is accepted; $s_{m,t+1} = B$ otherwise. Note that for any match $m = (\ell, i)$ at period t, the pair's actions have no effect on states $s_{m',t}$ with m' having agents other than the pair.

For any realized match $m = (\ell, i)$ at period t, the strategy for the pair only depends on $s_{m,t-1}$ as follows: ℓ accepts the proposed trade from i if $s_{m,t-1} = G$ and rejects it otherwise; i repays his debt if $s_{m,t-1} = G$ and does not repay otherwise.

Now we show that this strategy profile is sequentially rational. Consider a realized match $m = (\ell, i)$ at period t. Because state B is self-absorbing, if $s_{m,t-1} = B$, i has no incentive to repay his debt and hence it is optimal for ℓ to reject the trade. Now, suppose that $s_{m,t-1} = G$. By the equilibrium strategy of i, he will repay if his trade is accepted. Moreover, accepting or rejecting the trade has no impact on future states of the match. Thus, the current-period payoff for ℓ to accept the proposed trade is $(d + (1 - q))$ while the current-period payoff to reject the trade is 1; since $d = q$, the two payoffs are the same. Hence, it is optimal for agent ℓ to accept the proposed trade. Finally, assuming that the proposed trade from i was accepted by ℓ, by repaying the debt, i's expected continuation equilibrium payoff is (assuming that the number of other agents $j \neq \ell$ for

which $s_{(i,j),t-1} = B$ is $k \leq 2n - 2$)

$$-d + \frac{\beta}{1-\beta} \left\{ \frac{k}{2n-1}0.5 + \frac{2n-k-1}{2n-1}[0.5(R(q)-d) + 0.5(d+1-q)] \right\}$$

$$= -q + \frac{1}{\phi} \left\{ 0.5[R(q)-q] + 0.5 - \frac{0.5k}{2n-1}[R(q)-q] \right\};$$

in contrast, by not repaying the debt, i's equilibrium strategy implies that his expected continuation payoff is given by

$$\frac{\beta}{1-\beta} \left\{ \frac{k+1}{2n-1}0.5 + \frac{2n-2-k}{2n-1}[0.5(R(q)-d) + 0.5(d+1-q)] \right\}$$

$$= \frac{1}{\phi} \left\{ 0.5[R(q)-q] + 0.5 - \frac{0.5(k+1)}{2n-1}[R(q)-q] \right\}.$$

Recalling that $\Delta(q) = R(q) - q$, by (3), it is optimal for him to repay his debt.

(ii) Now we show that, for any given $q > 0$, it is not implementable for large n's. Here we assume that $\beta > \frac{1}{2}$; the other case can be proved in a similar fashion. Let N be so large that if $K = \log_2(2N-1) - 1$, then

$$\frac{\beta^K}{2\beta - 1} + \frac{\beta^K}{1 - \beta} < \frac{q}{\Delta(q)}. \tag{A.1}$$

Suppose, by contradiction, that q is implementable with $2n \geq 2N$ agents. Now, at period zero, consider an agent with the investment role at the end of period 0 and is supposed to repay his promise, $d \geq q$.

Consider the deviation to default now and, in all future period, behave as a non-defector. The worst scenario for this deviation would be that his current trading partner defects in all future periods, and all who are defected also defect. Thus, at period t, the probability of meeting a defector is at most

$$p_t \equiv \frac{\min\{2^{t-1}, 2n - 1\}}{2n - 1}.$$

Hence, the expected continuation payoff is at least $\sum_{t=1}^{\infty} \beta^t (1 - p_t)\Delta(q)$, and, for the agent

to prefer the equilibrium action than this deviation, it must be the case that

$$-d + \sum_{t=1}^{\infty} \beta^t \Delta(q) \geq \sum_{t=1}^{\infty} \beta^t (1 - p_t) \Delta(q),$$

that is,

$$d \leq \sum_{t=1}^{\infty} \beta^t p_t \Delta(q). \tag{A.2}$$

Now, for any $n \geq N$ (recall that N is defined by (A.1)) and for $k = \log_2(2n - 1) - 1$, we have

$$\begin{aligned}
\sum_{t=1}^{\infty} \beta^t p_t &\leq \frac{(2n - 1)\beta^k - 1}{(2n - 1)(2\beta - 1)} + \frac{\beta^k}{1 - \beta} \\
&\leq \frac{\beta^K}{2\beta - 1} + \frac{\beta^K}{1 - \beta} < \frac{q}{\Delta(q)} \leq \frac{d}{\Delta(q)},
\end{aligned}$$

a contradiction to (A.2). ∎

Proof of Proposition 1

We claim that q is implementable under star if and only if (9) holds. Note that (4) implies (9) for any $n > 0$: first,

$$\frac{1}{\frac{1}{2} + \phi} \left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m \right] \leq \frac{1}{\frac{n-1}{2n-1} + \phi} \left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m \right]$$

since $\frac{1}{2} + \phi \geq \frac{n-1}{2n-1} + \phi$ for any n; second,

$$n\phi(q + c_m) - \frac{1}{2}\Delta(q) + (2n - 1)c_m \leq (n - 1)\left[\phi(q + c_m) + 2c_m \right]$$

since $-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m \geq 0$.

First we prove necessity of (9). Let k_C be the center agent. Suppose that (d, f) with $d \geq q + c_m$ implements q under the star. Consider a periphery agent assigned to the investment role, deciding whether to repay his debt, $f + d$. We consider two choices: (a) repay the debt and follow the equilibrium strategies; (b) do not repay and receive no trade in all following periods. By (A1), the choice (a) has to be better than the choice (b), and

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hence we have

$$-(d+f) + \frac{\beta}{1-\beta} \left\{ \frac{1}{2} \left((R(q) - q + 1) - \frac{2n-2}{2n-1} f \right) - c_m \right\} \geq \frac{\beta}{1-\beta} \frac{1}{2}, \quad \text{(A.3)}$$

that is,

$$f \leq \frac{1}{\frac{n-1}{2n-1} + \phi} \left[-\phi d + \frac{1}{2}\Delta(q) - c_m \right]. \quad \text{(A.4)}$$

Now consider the center agent, k_C, assigned to the investment role and who is at the moment deciding whether to repay his debt, nd. Again, we consider two choices: (a) repay all the debts and follow the equilibrium strategies; (b) do not repay (to any debt) and receive trade in all following periods. By (A1), the choice (a) has to be better than the choice (b), and hence we have

$$-nd + \frac{\beta}{1-\beta} \left\{ \frac{1}{2}[R(q) - q + 1] + (n-1)f - 2nc_m \right\} \geq \frac{\beta}{1-\beta} \frac{1}{2},$$

that is,

$$f \geq \frac{1}{n-1} \left[n\phi d - \frac{1}{2}\Delta(q) + 2nc_m \right]. \quad \text{(A.5)}$$

Combining (A.4) and (A.5) and the fact that $d \geq q + c_m$, we obtain (9).

Now we prove sufficiency. Suppose that (9) holds. First we specify the financial contracts as follows: let $d = q + c_m$ and let $f \geq 0$ satisfy

$$\frac{1}{n-1} \left[n\phi d - \frac{1}{2}\Delta(q) + 2nc_m \right] \leq f \leq \frac{1}{\frac{n-1}{2n-1} + \phi} \left[-\phi d + \frac{1}{2}\Delta(q) - c_m \right].$$

Given the contracts, the liquidity agent is indifferent between no trade and the proposed trades, assuming that the investment agent will repay the trades. We construct equilibrium strategies as follows. Each periphery agent can be one of the two states, G or B. At date 0, all agents are in state G. A periphery agent stays in state G if and only if he repays his debts to k_C when assigned to the investment role in all previous periods; otherwise, he enters state B. An agent who enters state B stays there forever. The state of these agents is only observable to the center agent, k_C. Note that if a periphery agent proposes to trade directly then this choice does not affect his state and a periphery agent's action in liquidity role does not affect his state. Similarly, the center agent, k_C, can also be in

of the two states, G or B. He stays in state G if and only if he repays his debts in all previous periods; otherwise, he enters state B. His state is then observable to all agents.

The strategy of a periphery agent j assigned to the liquidity role in state G is as follows: if k_C is in state G and if j is in state G, then he accepts any trade through k_C; otherwise, he rejects. Moreover, he never accepts if asked to trade directly. A periphery agent j assigned to the liquidity role in state B never accepts trades. The strategy of a periphery agent j in investment role is as follows: if both himself and k_C are in state G, then he propose to trade through k_C and repay his debt; otherwise, he proposes to trade directly and, if his trade is accepted, he does not repay anything. Finally, the strategy of k_C is as follows: if he is in state B, then he never repays anything; otherwise, he accepts trades from a match $m = (\ell, i)$ if and only if both ℓ and i are in state G and rejects it otherwise, and he repays all debts if and only if it is feasible and the number of periphery agents in state G who repays at the current period, denoted by K_1, and the number of loans k_C has, denoted by K_2 (including his own), satisfy

$$-K_2 d + \frac{\beta}{1-\beta} \left\{ \frac{K_1}{2(2n-1)} \left[R(q) - q + 1 + (K_1 - 1)f \right] + \frac{2n - 1 - K_1}{2(2n-1)} \right\} \geq \frac{\beta}{2(1-\beta)}. \tag{A.6}$$

Note that when there are still K_1 periphery agents in state G, the expected fees for each such agent is $(K_1 - 1)f/2(2n - 1)$ and since any such fee is paid to k_C, the expected fee revenue is $K_1(K_1 - 1)f/2(2n - 1)$. Moreover, only with those agents k_C can expect to have trades.

We also need to construct equilibrium beliefs. As agent k_C has complete information, his belief is the actual history. For an periphery agent j, his belief is such that if k_C is in state G, then he believes that all other agents are also in state G. Note that once k_C enters state B, a periphery agent's belief does not matter to his equilibrium strategy any more.

To show that these strategies form a simple equilibrium, first notice that (A1)-(A3) are satisfied. Moreover, the agents' beliefs are consistent with equilibrium strategies. In particular, when a proposed trade is rejected with k_C in state G, it is believed to be a mistake and agents are all in state G and will continue to accept trades and repay from next period on. We use the one-shot-deviation principle to verify sequential rationality.

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By (A.4) and (A.5) and the previous discussion no agent has incentive to deviate along the equilibrium path. On the off-equilibrium path, the history is summarized by the configuration of states. For a periphery agent assigned to the liquidity role, because k_C will not accept any trade from an investment agent in state B, he is indifferent between accepting a unsecured trade with k_C and having no trade so long as k_C is in state G and it is optimal to reject any other trade (note that a periphery investment agent will not repay any debt incurred through direct trading). For a periphery agent in investment role, as their state only depends on whether they repay k_C, their incentive is determined by (A.4). Note that as they believe all other agents are in state G, the continuation payoff is given by the left side of (A.3). Finally, for k_C, (A.6) determines whether he has incentive to remain in state G or not. ∎

Proof of Proposition 2

First note that under the star network, g_n^*, with $2n$ agents, $\eta_{g_n^*} = 1 - 1/2n$ and $\nu_{g_n^*} = 1 - 1/n$. Since $q^* = 1$ and hence the first-best level of investment is implementable, the average welfare is given by

$$W^* = \frac{1}{2} \left\{ [R(1) - 1] + 1 - 4 \left(1 - \frac{1}{2n} \right) c_l - 2 \left(1 - \frac{1}{n} \right) c_m \right\}.$$

Since the star network already implements the first-best level of investment, it remains to show that it minimizes linking costs (both recurrent and idiosyncratic) among all connected networks.

First it is easy to verify that the idiosyncratic costs are minimized under star among all minimally connected networks.

Next, we show that for any connected network g_n with $2n$ agents,

$$\nu_{g_n} + 1 \geq 2 - \frac{2}{2n - 1} \eta_{g_n}.$$

To see this, note that for each agent j, any agent who is directed connected to him has

distance 1 but every other agent has distance at least 2, and hence

$$\nu_{g_n} + 1 \geq \frac{\sum_{j \in \mathcal{N}} \{\deg(j) + 2[2n - 1 - \deg(j)]\}}{2n(2n-1)}$$
$$= \frac{4n(2n-1) - 2(2n)\eta_{g_n}}{2n(2n-1)}$$
$$= 2 - \frac{2}{2n-1}\eta_{g_n},$$

where $\deg(j)$ is the degree of agent j.

Thus, the network costs of g_n per capita, denoted by C_n, satisfies

$$C_n = 4\nu_{g_n}c_l + 2\eta_{g_n}c_m \geq 4\nu_{g_n}c_l + 2\left[2 - \frac{2}{2n-1}\eta_{g_n}\right]c_m.$$

Now, let $C_n^* = 4(1 - \frac{1}{2n})c_l - 2(2 - 1/n)c_m$ be the corresponding cost for the star network, we have

$$C_n - C_n^* \geq S(\eta_{g_n}, n) \equiv 4\left\{\left[\eta_{g_n} - \left(1 - \frac{1}{2n}\right)\right]c_l + \left(\frac{1}{2n} - \frac{1}{2n-1}\eta_{g_n}\right)c_m\right\}.$$

Now, for each n, $S_1(\eta_{g_n}, n) = 4\{c_l - \frac{1}{2n-1}c_m\}$. Then, for all $n > N_2$, $S_1(\eta_{g_n}, n) > 0$ and hence is strictly increasing in η_{g_n}. Since we are only concerned with networks other than the minimally connected one, we may assume that $\eta_{g_n} \geq 1$. Now, for all $n > N_2$,

$$S(\eta_{g_n}, n) \geq S(1, n) \equiv 4\left\{\frac{1}{2n}c_l + \left(\frac{1}{2n} - \frac{1}{2n-1}\right)c_m\right\} > 0.$$

This implies that $C_n - C_n^* > 0$. ∎

Before we prove Proposition 3, we give a lemma.

Lemma A.1 Let $\{g_n\}_n$ be a sequence of networks. There exist $\bar{\eta} > 1$ and $\bar{\upsilon} > 1$ such that, if $\eta_{g_n} \leq \bar{\eta}$ and $\upsilon_{g_n} \leq \bar{\upsilon}$ for all n, then the level of investment q is asymptotically implementable only if (4) holds.

Proof Here we choose $\bar{\eta} = 1.05$ and $\bar{\upsilon} = 1.05$. Let

$$\widetilde{\Lambda}_n = (2n - 1)\frac{2 - \bar{\upsilon}}{6\bar{\eta}} - 1. \tag{A.7}$$

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For $n \geq 100$, $\widetilde{\Lambda}_n \geq 0.3n$. Let g be a given network with $2n$ agents with $\eta_g \leq \overline{\eta}$ and $\nu_g \leq \overline{\nu}$ under which q is implementable with financial contract (d, f), $d \geq q + c_m$. We show that, for $n \geq 100$,

$$\phi d - \frac{1}{0.15n}\Delta(q) + 2c_m \leq f \leq \frac{-\phi d + \frac{1}{2}\Delta(q) - c_m}{\frac{n-1}{2n-1} + \phi}. \tag{A.8}$$

By taking n to infinity in the above inequality and replacing d with $q + c_m$, we obtain (4).

To show the first inequality in (A.8), we find an agent whose incentive is similar to that of the center agent in the star network. We first need a claim about existence of an agent j with large degrees.

Claim 1. Let Λ be the maximum degree in g. Then,

$$\Lambda \geq (2n - 1)\frac{2 - \nu_g}{6\eta_g} - 1. \tag{A.9}$$

Proof. Let δ_j be the degree of agent j. Then,

$$
\begin{aligned}
2n(2n-1)(\nu_g + 1) \quad &\geq \quad 3 \times \left[\sum_{j \in \mathcal{N}} \left(2n - 1 - \delta_j - \sum_{j' \text{ linked to } j} \delta_{j'} \right) \right] \\
&\geq \quad 3 \times \{2n[2n-1] - 2|\mathcal{E}(g_n)| - 2|\mathcal{E}(g_n)|\Lambda\} \\
&\geq \quad 3 \times (2n) \times [2n - 1 - 2\eta_g(1 + \Lambda)].
\end{aligned}
$$

Then, (A.9) follows directly by rearranging terms. □

Since $\eta_g \leq \overline{\eta}$ and $\nu_g \leq \overline{\nu}$, (A.9) also implies that $\Lambda \geq \widetilde{\Lambda}_n$. Hence, we can find an agent j who has degree at least $\widetilde{\Lambda}_n \geq 0.3n$. Now, consider, S, the set of j's neighbors. Since by deleting all the links between agents in S the network is still connected (through j), it follows that the number of those links has to be at most

$$2n\overline{\eta} - (2n - 1) = 2n(\overline{\eta} - 1) + 1 \leq 0.1n + 1.$$

Thus, there are at least $0.15n$ agents in the set S who has no link with any other agent in S. Thus, let K be the maximum number of intermediated trades for agent j and, by (A2), we have $K \geq 0.15n$. Note that the expected number of fees for j is less than K. To

ensure that a simple equilibrium exists, considering j's incentive, it must be the case that

$$-Kd + \frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) + Kf - 2Kc_m\right] \geq 0. \tag{A.10}$$

Since $K \geq 0.15n$, (A.10) implies the first inequality in (A.8).

Now we show the second inequality in (A.8). Since $|\mathcal{E}(g_n)| = 2n\eta_g \leq 2n\bar{\eta} = 2.1n$ and hence the sum of all agents' degrees is less than $4.2n$, and since there exists one agent with degree at least $0.3n$, there exists an agent with degree less than $(4.2 - 0.3)/2 = 1.95$. Hence, there exists some agent with only one link. Since he has only one link, he cannot serve as an intermediary but has to go through an intermediary as an investment agent with probability at least $\frac{n-1}{2n-1}$. Thus, for it to be optimal to repay his largest possible debt, $d + f$, when assigned to the investment role, it must be the case that

$$-(d + f) + \frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) - c_m - \frac{n-1}{2n-1}f\right] \geq 0. \tag{A.11}$$

By rearranging terms, (A.11) implies the second inequality in (A.8). ∎

Proof of Proposition 3

Let $\{g_n\}$ be a sequence of networks and let q^* be defined in the main text. Consider two cases. First, suppose that $\eta_{g_n} \leq \bar{\eta}$ and $\nu_{g_n} \leq \bar{\nu}$ infinitely often. Then, by Lemma A.1 (ii), $q \leq q^*$. Since, by the arguments in Proposition 2, the star minimizes the linking costs among all connected networks for large n's, the candidate arrangement dominates that sequence. Next, suppose that $\eta_{g_n} > \bar{\eta}$ or $\nu_{g_n} > \bar{\nu}$ for all sufficiently large n. Since $q^* \geq \hat{q}$ and hence $W(1, 1, q^*) \geq \max\{W(\bar{\eta}, 1, 1), W(1, \bar{\nu}, 1)\}$, q^* and the star network performs better for large n's. ∎

Proof of Corollary 1

By Lemma A.1, if investment level q is asymptotically implementable in a sequence of small networks, $\{g_n\}$, then (4) holds for q, and, by Proposition 1, it is also implementable

in $\{g_n^*\}$. Moreover, by the proof of Lemma A.1 (the second inequality in (A.8)),

$$f_{g_n}^{\max}(q) \leq \frac{-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m}{\frac{n-1}{2n-1} + \phi},$$

and, by the the proof of Proposition 1 (the second inequality in (9)),

$$f_{g_n^*}^{\max}(q) = \frac{-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m}{\frac{n-1}{2n-1} + \phi}.$$

Thus, $f_{g_n}^{\max}(q) \leq f_{g_n^*}^{\max}(q)$.

For minimally connected networks, Babus and Hu (2015) prove a similar result to Lemma A.1 regarding asymptotic implementability (Proposition 4 (i) there) and a similar result regarding the intermediation fees (Corollary 1 there) in a related setting. The proofs there can be readily modified to fit our current setting. ∎

Proof of Proposition 4

We consider the following strategies and show they constitute a Nash equilibrium and use a tight trading strategy profile. First, agents never sever existing links or form new links in equilibrium. After any deviation, they also never sever existing links or form new links in equilibrium. In the trading game, all connected agents accept unsecured trades from other connected agents as long as the set of implementable investment levels under g^t at period t is non-empty (and conduct secured trades otherwise), both on and off equilibrium paths.

First we show that the center agent has no incentive to delete any link. We begin with a claim about the implementable investment levels in networks where the center has deleted some of his links.

Claim 1. Let g_K be the resulting network by deleting K links from the star. If the set of implementable investment levels is non-empty in g_K, then, the highest level implementable under g_K, denoted by q_K, satisfies $q_K \leq q_n^*$ for n large.

Proof. First we give necessary conditions for implementability. Fix some candidate in-

vestment level and contract, $[q, (d, f)]$. Consider the incentive of a connected periphery:

$$-(d + f) + \frac{\beta}{1 - \beta} \left\{ \frac{2n - K - 1}{2(2n - 1)} \Delta(q) - \frac{2n - K - 2}{2(2n - 1)} f - \frac{2n - K - 1}{2n - 1} c_m \right\} \geq 0.$$

This implies that

$$G(f, K) \equiv - \left\{ \phi + \frac{2n - K - 2}{2(2n - 1)} \right\} f - \phi d + \frac{2n - K - 1}{2(2n - 1)} [\Delta(q) - 2c_m] \geq 0,$$

and that the upper bound for f given q is given by the implicit function $f = g(K)$ such that $G(g(K), K) = 0$. Now, for $K \leq 2n - 2$,

$$G_f = - \left[\phi + \frac{2n - K - 2}{2(2n - 1)} \right] \leq 0$$

and

$$G_K = \frac{1}{2(2n - 1)} f - \frac{1}{2(2n - 1)} [\Delta(q) - 2c_m].$$

Since $g'(K) = -G_f / G_K$, to show that $g'(K) \leq 0$, it suffices to show that $G_K \leq 0$, that is, $g(K) \leq \Delta(q) - 2c_m$, which in turn is equivalent to

$$-\phi d + \frac{2n - K - 1}{2(2n - 1)} [\Delta(q) - 2c_m] \leq \left[\phi + \frac{2n - 2 - K}{2(2n - 1)} \right] [\Delta(q) - 2c_m].$$

Rearranging the terms and taking $d = q + c_m$, it suffices to show that

$$-\phi(q + c_m) + \frac{1}{2(2n - 1)} [\Delta(q) - 2c_m] \leq \phi[\Delta(q) - 2c_m].$$

Note that if $-\phi(q + c_m) + 1/2[\Delta(q) - 2c_m] < 0$, then the proposed trade is not implementable. Let $\underline{q}$ be the lowest q for which $-\phi(q + c_m) + 1/2[\Delta(q) - 2c_m] \geq 0$. Given that $q \geq \underline{q}$, we may replace the right-side with zero and the above inequality holds if

$$2(2n - 1)\phi(q + c_m) - [\Delta(q) - 2c_m] \geq 0,$$

which holds for large n and $q \geq \underline{q}$.

Now, consider the incentive for the center agent. We have

$$-\frac{2n-K}{2}d + \frac{\beta(2n-1-K)}{(1-\beta)(2n-1)}\left\{\frac{\Delta(q)}{2} + \frac{2n-2-K}{2}f - (2n-1-K)c_m\right\} \geq 0.$$

This implies that

$$H(f,K) \equiv (2n-2-K)f + \Delta(q) - \frac{(2n-K)(2n-1)}{(2n-1-K)}\phi d - 2(2n-1-K)c_m \geq 0.$$

The lower bound for f given q is then given by the implicit function $f = h(K)$ such that $H(h(K),K) = 0$, and $h'(K) = -H_K/H_f$. Now,

$$H_f = 2n - 2 - K \geq 0$$

and

$$H_K = -f - \frac{-(2n-1)(2n-1-K)+(2n-K)(2n-1)}{(2n-1-K)^2}\phi d + 2c_m.$$

Note that $h'(K) \geq 0$ if $H_K \leq 0$, which holds if

$$h(K) \geq -\frac{2n-1}{(2n-1-K)^2}\phi d + 2c_m.$$

This holds if

$$-\Delta(q) + \frac{(2n-K)(2n-1)}{2n-1-K}\phi(q+c_m) + 2c_m \geq 0.$$

Again, we have this inequality if $q \geq \underline{q}$ and n large.

Combining these incentives, we have

$$\frac{-\Delta(q) + \frac{(2n-K)(2n-1)}{(2n-1-K)}\phi d + 2(2n-1-K)c_m}{2n-2-K} \leq \frac{-\phi d + \frac{2n-K-1}{2(2n-1)}[\Delta(q) - 2c_m]}{\phi + \frac{2n-K-2}{2(2n-1)}}. \quad \text{(A.12)}$$

By taking derivatives with respective to K, we have verified that the left-hand side is increasing in K while the right-hand side is decreasing in K. Thus, q_K, defined as the maximizer to $\max_q \Delta(q)$ subject to (A.12) with $d = q + c_m$, must satisfy $q_K \leq q_0 = q_n^*$. $\square$

Now, for each K where the set of investment levels is non-empty under g_K, let $\mathcal{C}(g_K) = [q_K, (q_K + c_m, f_K)]$, where f_K corresponds to the left-hand side of (A.12) with $q = q_K$.

When q_K is not implementable, we have agents receive no trade and the center receives no fee.

Then, under the contract $q = q_n^*$, $d = q_n^* + c_m$ and $f = f_n^* \geq f_n^{\min}$, the benefit per period by deleting K links (relative to the star network) is less than

$$\left\{ \frac{2n-1-K}{2(2n-1)} \Delta(q_K) + \frac{(2n-1-K)(2n-2-K)}{2(2n-1)} f_K - \frac{(2n-1-K)^2}{2n-1} c_m \right\}$$
$$- \left\{ \frac{1}{2} \Delta(q_n^*) + (n-1) f_n^{\min} - (2n-1) c_m \right\} + K c_l$$
$$= \frac{2n-K}{2} \phi(q_K + c_m) - n\phi(q_n^* + c_m) + K c_l.$$

Now, by Claim 1, $q_K \leq q_n^*$; hence,

$$\frac{2n-K}{2} \phi(q_K + c_m) - n\phi(q_n^* + c_m) + K c_l \leq K \left[-\frac{\phi}{2}(q_n^* + c_m) + c_l \right] \leq 0,$$

provided that $c_l \leq \frac{1}{2}\phi(q_n^* + c_m)$. This shows that the center agent does not want to sever any link.

Next, we show that when a pair of two leaf agents are chosen in the linking stage, they have no incentive to form a link. Let g' denote the network by having exactly two periphery agents forming a new link between them.

Claim 2. Let g' be the network by having exactly two periphery agents forming a new link between them, and let q' be the highest level of investment implementable under g'.
(a) Suppose that (4) holds for $q = 1$ with strict inequality. Then, for n large, $q' = 1$ and there is a corresponding fee $f' \geq f_n^*$.
(b) Suppose that (4) does not hold for $q = 1$ with strict inequality. Then, for any fee f' corresponding to q', both $|f' - f_n^*|$ and $|q' - q_n^*|$ converge to zero as n goes to infinity.
Proof. Fix a candidate contract, $[q, (d, f)]$ with $d = q + c_m$. Consider the center agent. His incentive requires

$$-nd + \frac{\beta}{1-\beta} \left\{ \frac{1}{2}\Delta(q) + (2n-3)\frac{2n-2}{2(2n-1)} f + 2\frac{2n-3}{2(2n-1)} f - c_m - \frac{n(2n-3)}{2n-1} c_m \right\} \geq 0.$$

This implies that
$$f \geq \frac{\phi nd - \frac{1}{2}\Delta(q) + c_m + \frac{n(2n-3)}{2n-1}c_m}{\frac{n(2n-3)}{2n-1}}.$$

Consider the two periphery agents who are linked. Their incentives require

$$-(d+f) + \frac{\beta}{1-\beta}\left\{\frac{1}{2}\Delta(q) - \frac{2n-3}{2(2n-1)}f - c_m\right\} \geq 0,$$

and hence

$$f \leq \frac{-\phi(q+c_m) + \frac{1}{2}\Delta(q') - c_m}{\phi + \frac{2n-3}{2(2n-1)}}.$$

Thus, $[q, (d, f)]$ is implementable if and only if

$$\frac{\phi n(q+c_m) - \frac{1}{2}\Delta(q) + c_m + \frac{n(2n-3)}{2n-1}c_m}{\frac{n(2n-3)}{2n-1}} \leq \frac{-\phi(q+c_m) + \frac{1}{2}\Delta(q') - c_m}{\phi + \frac{2n-3}{2(2n-1)}}. \quad (A.13)$$

(a) Note that by taking n to infinity, (A.13) coincides with (4). Hence, if (4) holds for $q = 1$ with a strict inequality, (A.13) holds for $q = 1$ for large n. Thus, $q' = 1$ for large n. However, note that
$$\frac{-\phi(q+c_m) + \frac{1}{2}\Delta(q) - c_m}{\phi + \frac{2n-3}{2(2n-1)}} \geq f_n^{\max}$$

for $q = 1$, and hence we can pick a fee $f' \geq f_n^{\max}$.

(b) If (4) fails for $q = 1$ with a strict inequality, then (A.13) fails for $q = 1$ for large n. Then, for large n, both constraints are binding for the second-best allocations. Since the two conditions, (A.13) and (9), coincide at the limit, the convergence follows. Now, suppose that (4) holds for $q = 1$ with an equality. Then both $f_n^{\max}$ and $f_n^{\min}$ converge to the same limit as f'. $\square$

Now, the benefit of forming this new link per period is then less than

$$-c_l + \frac{1}{2}|\Delta(q_n^*) - \Delta(q')| + \frac{1}{2(2n-1)}f + \frac{2n-2}{2(2n-1)}|f_n^* - f'| \equiv -c_l + T(n).$$

However, Claim 2 implies that we can choose f' such that $T(n) \to 0$ as n goes to infinity.

Finally, since the contract is given by $q = q_n^*$, $d = q_n^* + c_m$ and $f = f_n^*$, and since

$f_n^* \le f_{g_n^*}^{\max}$, we have

$$-\phi(q_n^* + c_m) + \frac{1}{2}\Delta(q_n^*) - c_m \ge \left(\frac{n-1}{2n-1} + \phi\right) f_n^*.$$

Thus, for a leaf agent to sever a link and to receive no trade, since $c_l \le 1/2\phi(q_n^* + c_m)$, the gain per period is less than

$$-\left[\frac{1}{2}\Delta(q_n^*) - c_m - \frac{n-1}{2n-1}f_n^* - c_l\right] \le 0.$$

∎