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Dynamical systems methods for evaluating aircraft ground manoeuvres

Bernd Krauskopf, Etienne B. Coetzee, Mark H. Lowenberg,
Simon A. Neild and Sanjiv Sharma

Abstract Evaluating the ground-based manoeuvrability of large aircraft is time consuming and costly if explored though simulations with industry-developed complex models of ground dynamics. We argue here that this type of dynamics can be investigated efficiently and with considerable precision by applying dynamical systems techniques. As an example, we consider the lateral loads experienced by an Airbus A380 when it turns off a runway.

1 Introduction

Aircraft are designed to fly and, hence, are not optimised for ground operations in the way cars or other ground vehicles may be. Nevertheless, a passenger aircraft needs to operate fast, reliably and safely on the ground in order to ensure its overall commercial success. The standard approach employed by aircraft manufacturers has been to conduct massive and expensive numerical simulations of industry-tested and parametrised models for aspects of aircraft motion to evaluate the ground performance of aircraft as part of their design, evaluation and certification.

An alternative approach is to use dynamical systems techniques that allow one to follow solutions, detecting stability changes and bifurcations, as parameters are varied. We conducted a number of systematic case studies to demonstrate that aircraft ground dynamics can be investigated without the need for expensive brute-force numerical simulations; these projects include:

- the development of a fully parameterized model of a mid-size aircraft and its use to evaluate aircraft ground turning with the goal of providing insight into safe operation limits under different conditions. The main motivation for this work was to evaluate the suitability of the existing Federal Aviation Regulations for lateral loads experienced during turning manoeuvres [8].

Bernd Krauskopf, Department of Mathematics, University of Auckland, Auckland 1142, New Zealand, e-mail: b.krauskopf@auckland.ac.nz · Etienne B. Coetzee, Airbus Operations Limited, Filton, Bristol, BS99 7AR, UK, e-mail: etienne.coetzee@airbus.com · Mark H. Lowenberg, Faculty of Engineering, University of Bristol, Bristol BS8 1TR, UK, e-mail: m.lowenberg@bristol.ac.uk · Simon A. Neild, Faculty of Engineering, University of Bristol, Bristol BS8 1TR, UK, e-mail: simon.neild@bristol.ac.uk · Sanjiv Sharma, Airbus Operations Limited, Filton, Bristol, BS99 7AR, UK, e-mail: sanjiv.sharma@airbus.com

- research on ground handling of aircraft with more than three sets of wheels, in particular, the Airbus A380 model. This work considered low-, medium- and high-speed ground manoeuvres of an A380 in comparison with an A320 [2, 3, 4].
- the development of a mathematical model of an aircraft nose landing gear, which features torsional and lateral bending modes that are coupled through the tyre dynamics. A bifurcation analysis in terms of the forward velocity and vertical force on the landing gear was used to identify regions of (unwanted) stable torsional and stable lateral shimmy oscillations [11, 12]. Subsequent work also considered shimmy oscillations in main landing gears [7] and landing gear-fuselage interactions [10].
- beyond aircraft ground dynamics, configurations and forces of landing gears during retraction and extension were considered, as well as the evaluation of control laws during flight; see the overview paper [9] for details and further references.

A common feature of the underlying mathematical models is that they contain considerable nonlinearities, for example, due to geometric constraints, the tyre-ground interface and aereodynamics forces. Therefore, their behaviour needs to be evaluated using a combination of analytical and numerical techniques; in particular, it is possible to determine the dependence of the observed behaviour on the different parameters, such as velocity and loading of the aircraft, with the numerical technique of continuation. Specifically, we developed the Dynamical Systems Toolbox (DST) [1], which incorporates the capabilities of the well-known continuation software package AUTO [5, 6] into MATLAB. The DST enables the convenient coupling of user-developed models to numerical continuation, thus, making dynamical systems available for use within an industrial setting. The DST has been incorporated in the Airbus Methods and Tools portfolio as a supported tool for the evaluation of proposed works and new designs.

Taking a specific example, we consider the problem of determining the loads on the aircraft and on the tyres that arise during ground manoeuvres. For safety reasons, these loads need to remain below agreed limits under all operational scenarios as stipulated by the relevant regulations. Hence, extensive analysis is required to cover all of the different ground menoeuvres the aircraft may perform. An important case is that of exiting a runway, which we consider here for the largest commercial airliner in operation — the Airbus A380 with a maximum ramp weight (MRW) of 577,000 kg.

2 Ground loads of an A380 during turning

The Airbus A380 has been in operation since 2007. Modeling and simulation have been an important part of the design and evaluation of this aircraft. In particular, a validated SimMechanics model of the A380 is available for the purpose of studying its behaviour on the ground. A schematic is shown in Fig. 1, where the aircraft is subject to forces, such as the engine thrust and steering, which influence the internal states describing, for example, the attitude of the aircraft and the forces at the

Fig. 1 Top-level SimMechanics model of an A380. Reproduced from [4].

individual tyres. This A380 model was developed by Airbus for real-time studies, but has now also been coupled directly to the Dynamical Systems Toolbox within Matlab. As a result, there is confidence in the validity of the results obtained from the bifurcation analysis.

To support its weight, the A380 has five landing gears with a total of 22 wheels: a dual-wheel nose landing gear (NLG), left and right wing landing gears (WLG) with four wheels each, and left and right body landing gears (BLG) with six wheels each. The naming convention for the 22 wheels is shown in Fig. 2(a). From a regulatory and operational perspective it is important to ensure that the loads at each of the wheels do not exceed their safety margins. Already for commercial aircraft of small to medium size with three landing gears it is a considerable task to determine the forces at the tyres during actual manoeuvres. For the A380 the relevant forces need to be determined at all 22 wheels, taking into account different possibilities for the weight distribution among the landing gears which, as there are more than three, are statically-redundant. As was mentioned in the introduction, loads on landing gears and tyres during different ground manoeuvres are traditionally analysed by performing a large number of numerical simulations of the model.

We first consider the lateral load (the sideways load perpendicular to the direction of travel) acting on the A380 at the centre of gravity (CG); more details and a comparison with results for an A320 can be found in [4]. The bifurcation analysis with the DST is an efficient method for determining this load over a large range of velocity V_n at the NLG versus the steering angle δ that the pilot sets. The results can be represented conveniently in the (δ, V_n)-plane, as is shown Fig. 2(b) for the extreme case of MRW, the largest possible weight of the aircraft, with a CG position as far back or aft as possible. For evaluation purposes Fig. 2(b) shows a large range of V_n of up to 35 m/s for any setting of the steering angle δ. The labelled contour lines are curves of equal lateral load factor *for the aircraft*, defined as the lateral (or sideways) force divided by the vertical (or downward) force at the CG, from 0.05 to 0.2. The contour line for 0.133 is shown thicker because it delimits the design envelope that determines the maximum permissible velocities during runway exits.

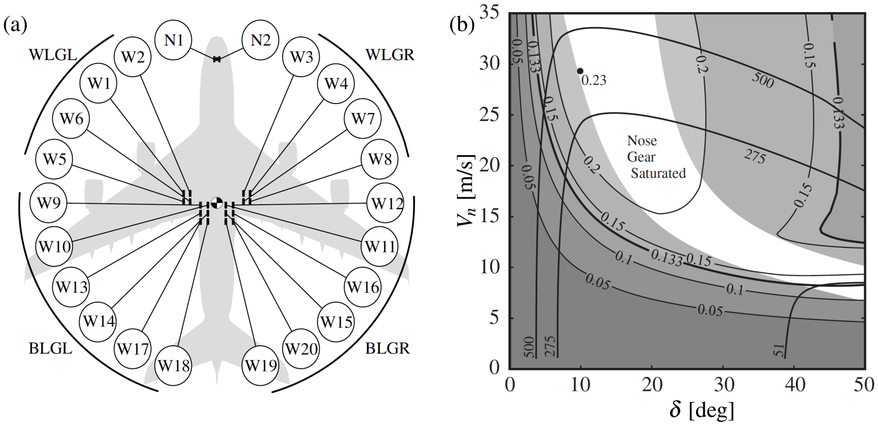

Fig. 2 (a) Wheel numbering definition for the A380; (b) contour lines in the (δ, V_n)-plane of the lateral aircraft load factor at the CG for the A380 at maximum ramp weight with aft CG position. Reproduced from [4].

Such contours can be followed directly in the parameters V_n and δ, without the need for simulations. Similarly, one can determine directly the boundary of the white region in the centre where the NLG saturates; this means that the NLG tyres cannot produce enough lateral force to hold the aircraft on the desired turn. It can be seen that the majority of this region is above the allowable aircraft load factor of 0.133. We now consider the allowable manoeuvre speeds to show that the small nose gear saturation region that lies below the allowable aircraft load factor contour, in the bottom right corner of Fig. 2(b), is not actually entered in practice. Taking constant radius turns, of 51m, 275m and 500m, the velocity-steering angle relationships are plotted and labelled. The constant turn with a radius of 51 m is in the lower right corner of Fig. 2(b). While this curve enters the white region, it does so only for velocities V_n well above the recommended velocity of 4 m/s for this turn.

We now consider the loads at the individual tyres of the four main landing gears during a 90° turn with a radius of 51 m. During this manoeuvre the steering angle δ of the NLG is ramped up from zero to about 32° and then brought back to zero at the end of the turn. The aircraft velocity reduces considerably when the steering angle is increased — the nose tyres then effectively act as brakes. This is why a velocity controller is used to adjust the thrust of the engines so that a constant velocity of 4 m/s is maintained during the entire 90° turn. Continuation analysis can be used to determine how the forces at each of the 20 main gear wheels build up and reach a maximun during this runway exit manoeuvre.

Figure 3 shows the resulting maximum loads, again in terms of load factors, but this time *for each tyre* rather than for the aircraft. Distinguished here are the steady-state load factor, which is the one determined by numerical continuation, and the dynamic load factor, which is calculated from simulations. First of all, the maximum steady-state values are larger and hence the critical ones, except that they are are about 10%–20% smaller for W17–W20 (which are the tyres at the back axles of the

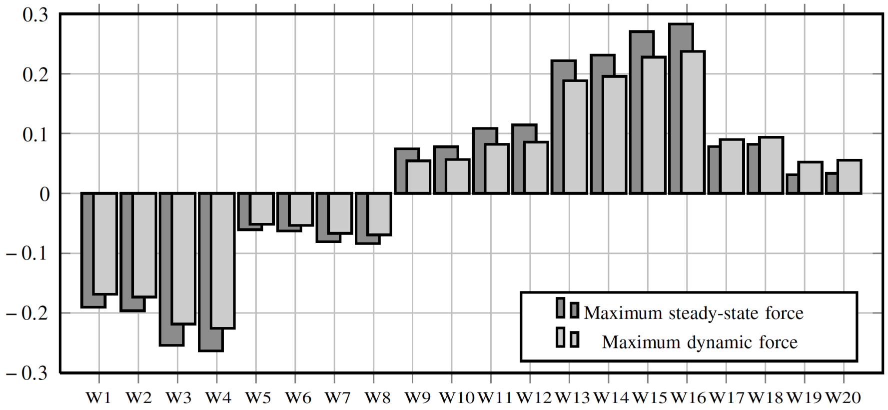

Fig. 3 Steady-state and dynamic tyre load factors on main gear tyres for the A380 when a radius of 51 m and a velocity of 4 m/s is maintained at the nose gear. Reproduced from [4].

inner BLGs, which is actually steered). A more detailed analysis (not reported here) suggests that this is due to the straightening out at the end of the turn, and we argue that numerical continuation is indeed a valid tool for investigating the maximum tyre loads during turns. Figure 3 illustrates that there are considerable differences in the magnitudes of the loads at the different tyres during this ground manoevre. Tyres W1–W8 of the outer wing landing gears (see Fig. 2(a)) have negative load factors (the force is against the direction of the turn). Tyres W9–W20 of the inner body landing gears, on the other hand, experience positive loads, with the largest load occurring at tyres W13–W16 of the middle axle.

3 Conclusions

The introduction of dynamical systems methods, which are implemented in the Dynamical Systems Toolbox, into the industrial practice provides Airbus with new capability for the evaluation of aircraft ground performance and the potential of considerable savings in time and costs. Airbus in the UK have described its impact as an approximately 80% reduction in time and associated costs for ground maneuverability analysis; this estimate is made by comparing the time taken to conduct global assessments using bifurcation and other dynamical systems techniques versus the time to obtain exhaustively a large number of point solutions.

As an example of this new capability, we presented a study of the loads at the centre of gravity and at individual tyres that the 22 wheel A380 experiences when making a runway turn. These numerical continuation results were obtained within the Dynamical Systems Toolbox under Matlab for an industry developed and validated model of the aircraft. This type of detailed knowledge of forces and their build-up during different ground manoeuvres may be used to inform design decisions and operational procedures.

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