Task design for working mathematically

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This article will discuss the ways in which teachers talk about the strategies they use to support their students working mathematically. From a comparison of these strategies a set of principles for task design for working mathematically will be developed. I did the opening keynote at the 2016 MANSW conference, *Igniting the fire*. The focus of that keynote and the focus of this article is how teachers establish a culture of students thinking mathematically in their classrooms. I think of this as the way the teachers fan the flames of creativity and imagination in their students, sharing ways of working that support the students in carrying the torch of enjoyment of mathematics into their future lives.

When Becky Lovelock introduced me before the keynote, she reminded me of the words on a postcard that had been pinned to a noticeboard in my office when she was completing a one-year, post-degree qualification to become a mathematics teacher and I was her personal tutor:

I KNOW
YOU *BELIEVE* YOU
UNDERSTAND WHAT YOU
THINK I SAID,
BUT I AM *NOT* SURE,
YOU REALISE, THAT
WHAT YOU *HEARD,*
IS *NOT* WHAT I *MEANT*

Communication has always been tricky for me. I was born in the north of England where the Wars of the Roses saw the Houses of Lancaster and Yorkshire, their emblems the red and white roses respectively, do battle. I was born in Yorkshire and brought up in Lancashire. I am both the white and the red rose, even the accents are different. When I was at junior school in Blackpool, Lancashire, I spoke with the strong Yorkshire accent of my parents. I remember a teacher telling me that it is not 'book', 'cook', 'look' (pronounced 'buck', 'cuck', 'luck') but 'book', 'cook', 'look' (pronounced with the 'oo' as in 'boo'). There's a song that I heard as a child called 'Will ye go lassie go' that is a Scottish folk song. In this song, there is the phrase 'bloomin' heather' describing the beauty of the heather in flower. In Yorkshire dialect, there is the phrase 'bloomin' 'eck', probably from 'bloody hell' but does not sound so bad. I could not understand what the heather had done wrong to the singer. Singing the hymn, 'There is a green hill far away, without a city wall', at school, the meaning of the words confused me. Why would a green hill have a city wall anyway?

When I first saw the words on the postcard (reproduced above) in my late teens or early twenties, I felt like someone else understood the world that I live in. What would the act of teaching look like if we accepted that 'telling' students, giving explanations in classrooms, could lead to multiple confusions given their cultural and linguistic backgrounds and connections made to their own experiences? My way of coping as a young teacher was always to start a lesson with something that we could talk about, an image or a problem and, through discussion, open up those different views to give more avenues to explore.

Teaching activities

In what follows, I will share three activities from the keynote, and, for each classroom activity, ask, 'What motivates the three teachers to offer their chosen activities?' I will conclude by offering some principles for task design that encourage students in working mathematically.

Teacher 1

A long time ago, I interviewed a group of teachers individually, shortly after their first lesson of the year with a class that they had not taught before. The focus of the interviews was to uncover what motivated the teachers' way of teaching mathematics. One of the teachers used 'story' as the basis of his teaching and told the following one in the first lesson:

> I had a dream last night and in that dream this is what I heard. You must build a tower and from the top of the tower [it should look] sort of like a plus sign, from the side it should look like two staircases meeting. We haven't decided yet how big the tower should be but when we decide, you must be able to build it and organise the building of it.

In this first lesson, the ways of working mathematically that the students will use for the year are set up. Have a go at the task and then, try to do it again in a different way. The following quotation from the interview describes how the teacher envisions his mathematics teaching:

> By the end of two day's work we were going to have posters of this and I wanted the posters to be different and I wanted people to have things to look at which would be new for them and interesting, and I wanted different people to have different problems that they would be solving, partly so they would have to rely on their own thinking. And just to show that a huge range of possibilities can come out of story anyway. There is not one right answer, there are lots of answers which are valid to various degrees.

I then asked them in groups, again individually, to write down the story, giving them five minutes to do that, and then in their groups to decide what kind of questions or concerns or worries the architect has.

The teacher is already working with the students on there being more than one way of doing mathematics for any given problem and communication is encouraged through group work and sharing their work as a class.

Teacher 2

Watch the images generated up to 35 (Figure 1 is a still image of the first 35) from the web-address, http://www.datapointed.net/visualizations/math/factorization/animated-diagrams/ created by Brent Yorgey.

Ask yourself what you would draw for the number 36. Do you think there is only one answer? Alistair Bissell wrote an article in Mathematics Teaching 253, *Dancing factors: Using images to provoke discussion* in which he writes about his motivation for using the images to focus on a syllabus topic of factors, multiples and primes, with a group where many students use repeated addition for multiplication:

If there was only one right answer as to how you might represent a number with dots, it would create lots of ways for my students to get things wrong. The aim in working on these images was to create more ways for the students to be successful.

By refusing to confirm or not, I was setting the expectation that it is the responsibility of the students to determine how many there are. This created a need for students to share their reasoning and to be convincing in what they say. The expectation is that students need to justify their comments, be convincing, and emphasised that the reasoning is valued more than the answer.

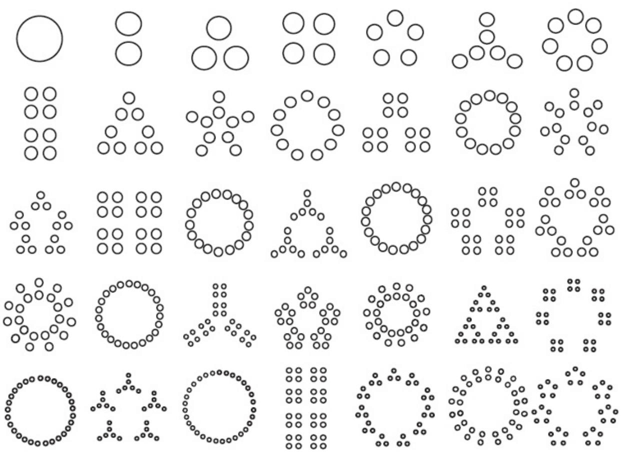


Figure 1: The first 35 images from the website

By quietly drawing interpretations of students' contributions on the board and then waiting, without judgement, I was encouraging the students to offer alternative ways of seeing. The more students that contribute their way of thinking, the more investment there is in the discussion and the more motivation the class has to resolve any issues arising:

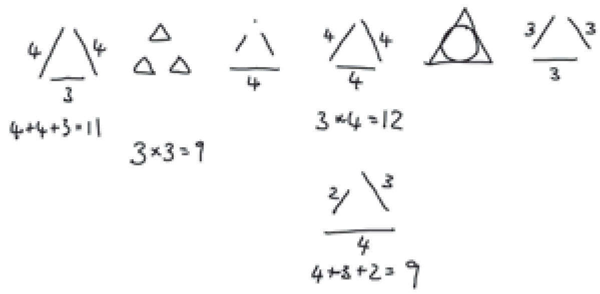


Figure 2: Drawings of students' interpretations

Here working mathematically is also evidenced through communication, with justifying part of creating a need for reasoning. Again, there is a focus on more than one right answer.

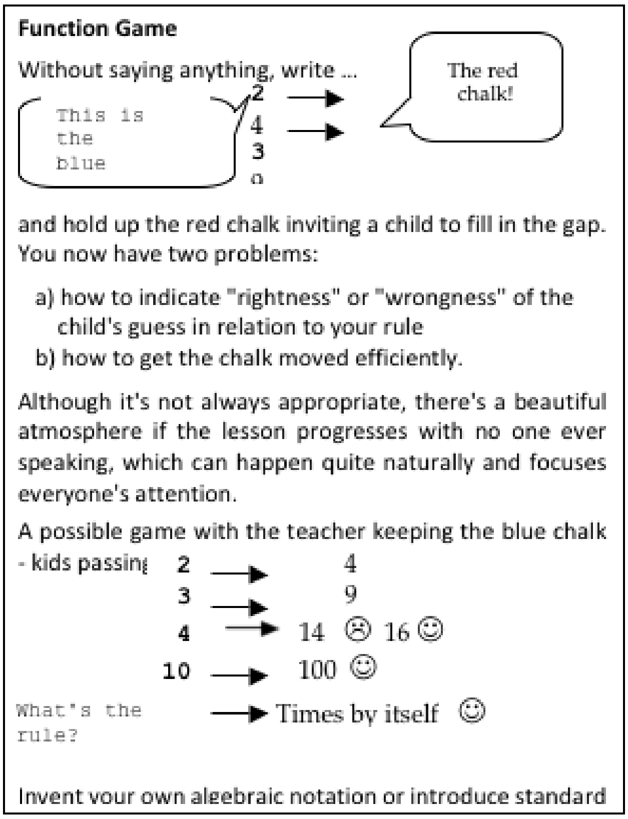
Teacher 3

I have worked with teacher 3, Alf Coles, since 1995, when he was at the end of his first year of teaching. The first ten years of that work is documented in a book, *Hearing silence: Steps to teaching mathematics* (Brown and Coles, 2008). When we were finding what to work on together in his classroom, before we worked together, he told two anecdotes of times that had fitted with his image of what he would like his teaching to be like, but so often was not:

First: During an A-Level lesson on partial fractions I was going through an example on the board, trying to prompt suggestions for what I should write. Some discussion ensued amongst the students, which ended in disagreement about what the next line should be. I said I would not write anything until there was a unanimous opinion. This started further talk and a resolution amongst themselves of the disagreement. I then continued with the rule of waiting for agreement before writing the next line on the board.

Second: Doing significant figures I wrote up a list of numbers and got the class to round them to the nearest hundred or tenth, [...] Keeping silent, I wrote, next to their answers, how many significant figures they had used in their rounding. Different explanations for what I was doing were quickly formed and a discussion followed about what significant figures were.

After these anecdotes had been related, there was a pause until Alf, energetically, said, 'It's silence, isn't it, it's silence!' This statement set up Alf's first steps in developing his teaching after we started to work together and led to the title of our book. The first activity I shared with Alf had been, in turn, shown me by the person who was my mathematics education tutor during my own one-year course becoming a mathematics teacher. It illustrates a favourite quotation of Dick Tahta, my tutor, 'In order to listen, it is necessary to keep silence…', (Father Gratry, 1944, p. 518). When I had seen Dick teach a class where he and the students were silent, using the activity that follows, I soon tried the idea out myself. What that experience taught me as the students kept asking me why I was not saying anything, is that Dick had not been doing what I was doing! It is worth sticking with trying the activity however until you can get it working. Here is the activity as it is written up in *Addendum to Cockcroft* (Brown and Waddingham, 1982, search on the title, it is freely available on the UK STEM site):



Function Game

Without saying anything, write …

This is the blue

2 → 4 →

3

The red chalk!

and hold up the red chalk inviting a child to fill in the gap. You now have two problems:

a) how to indicate "rightness" or "wrongness" of the child's guess in relation to your rule

b) how to get the chalk moved efficiently.

Although it's not always appropriate, there's a beautiful atmosphere if the lesson progresses with no one ever speaking, which can happen quite naturally and focuses everyone's attention.

A possible game with the teacher keeping the blue chalk - kids passing

2 → 4

3 → 9

4 → 14 ☹ 16 ☺

10 → 100 ☺

What's the rule? → Times by itself ☺

Invent your own algebraic notation or introduce standard

Figure 3: Function game described in *Addendum to Cockcroft*

Alf had set homework to find the rules for some linear functions and to have a go at finding the rule for the function:

$1 \rightarrow 6$

$2 \rightarrow 12$

$3 \rightarrow 20 \qquad N \rightarrow$

$4 \rightarrow 30$

$5 \rightarrow 42$

What different strategies can you use to find the rule? It might seem like there is only one right answer but there are many methods that students use to find a generalisation and the forms of the 'rules' can differ leading to exploration of equivalence. For instance, any patterns in the factors of the 'answers' that can be related to the starting numbers? What if you take a factor of two out of the answers? What's left if you take out n squared? All these strategies, and more, were used by the members of the MANSW conference in the plenary lecture.

The game is flexible in that it can be used with classes and also with individuals, for instance, if you have a student having problems with square-rooting, without speaking, start a function game on squaring with them filling in the 'answers'. At some point, when they are fluent, place a number on the right-hand side and reverse the arrow.

As Alf and I continued to work together we developed other strategies to develop a classroom culture in which it was the students doing the mathematical work and we were organising the space of the classroom. One of these strategies is the use of 'common boards'. These are spaces to share student work, as in the monuments, and also spaces where data to support problem solving can be shared so that patterns can be observed.

Surprise can be harnessed in support of student motivation in a classroom. A three-digit number, where the hundreds digit is larger than the unit digit is chosen by each member of the class who works individually. The number is reversed and subtracted, say 764 – 467. The subtraction can be done using any method. The answer, 297 in this case, is reversed and added to the answer, 297 + 792. Share with your neighbours and continue exploring. This is an example of a problem that is self-generating and self-checking although you will find different questions from students to explore, including, what would happen with 4 and more digits. What follows are two examples of 'common boards' generated from working with this problem with a class of 11 and 12 year old students:

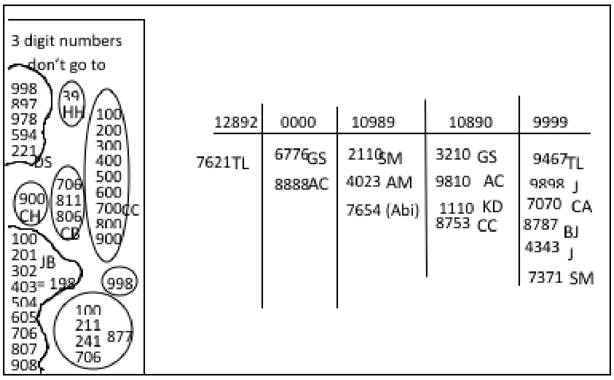
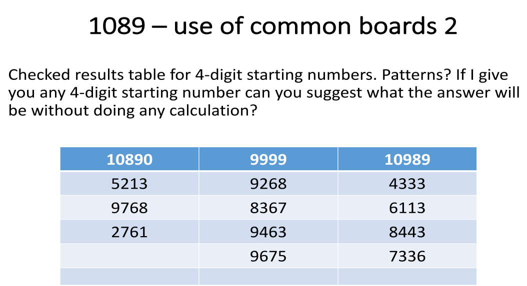


Figure 4: Common board for 4-digit starting numbers



1089 – use of common boards 2

Checked results table for 4-digit starting numbers. Patterns? If I give you any 4-digit starting number can you suggest what the answer will be without doing any calculation?

10890	9999	10989
5213	9268	4333
9768	8367	6113
2761	9463	8443
	9675	7336

Figure 5: Common board with checked answers

As the students work on the 4-digit problem, a working table (see Figure 4) is formed on a 'common board', where they place their starting number in one column of the table and, if they have a different 'answer', this is placed at the top of a new column. The students also write their initials next to the starting number they have added. The students are encouraged to check each other's contributions and, if they find a different answer, to go and talk this through with the person who wrote it in the table. In this way, an accurate set of results can be collected on a second 'common board', allowing everyone to be able to look for patterns and make conjectures. There is a useful challenge to say that at some point in the lesson you will say a 4-digit number and expect the students to be able to say which column that number will go in without having to do any calculations.

Where do you think that the following starter could lead? What's important is that you listen to what the students say.

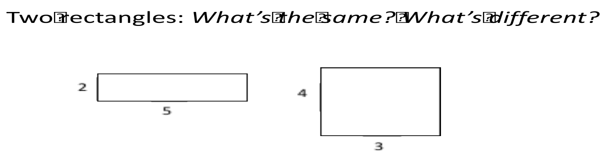


Figure 6: Comparing two examples

Working mathematically

Working mathematically has five components in the Australian National Curriculum: communicating; problem solving; reasoning; understanding; and fluency. Fluency is the only one of these components that does not end in 'ing'. For me, fluency is about do*ing* the mathematics since, at the end of all our teaching we are focused on the student learning. A question presents itself, what is learning? I have asked many different groups that question and here is a collection of answers that have been given: knowing; growing; an act of participation; absorbing; getting better at; asking questions; acquiring a skill; doing differently; satisfying curiosity; a journey; and keeping on asking questions. Many of the words and phrases used in response are active 'ing's. In the teachers' lessons, their students are being active: sharing, reasoning, justifying, convincing, storying, making connections, making meaning and throughout, being creative. The activities allow the practice that leads to fluency to be carried out whilst being mathematical. So, what can be said about task design for working mathematically, so that students are learning?

Task design for a contingent classroom

Firstly, some assumptions. Anyone we are working with, students or adults, can: make distinctions (so we can ask them what's the same, what's different?); use their power of imagery (so we can use complex images); generalise (learners can see patterns and connections and abstract). You can read more about the powers of students in a book by Caleb Gattegno, *What we owe children*. These three teachers ask their students to be active in their lessons and that means asking what they see, the patterns that they notice, the monuments that they can build.

For a conference on Task Design (ICMI Study 22), Alf Coles and I wrote about the following set of principles that can be used when designing activities for anyone to learn mathematics so that they become fluent:

- considering at least two contrasting examples (where possible, images) and collecting responses;
- asking students to comment on what is the same or different about contrasting examples and/or to pose questions;
- introducing language and notation arising from student distinctions;
- starting with a closed activity (which may involve teaching a new skill);
- having a challenge prepared in case no questions are forthcoming;
- creating opportunities for the teacher to teach further new skills and for students to practise skills in different contexts;
- opportunities for students to spot patterns, make conjectures and work on proving them, hence involving generalising and algebra.

For us, working mathematically is not separate from learning new mathematics and applying mathematical skills fluently over time. Therefore, the students are becoming mathematicians, because working mathematically is what they do.

The teachers described in this article are not the same, they have different strengths, however, they do engage their students actively in the activities they offer. Use your experiences outside the mathematics classroom, such as 'the questions an architect would ask' or 'the power of story', reading to your own young children, to support the developing culture of your classroom. The principles might be the same but how you are able to design activities is dependent on your interests.