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Optimal pricing policies for differentiated brands under different supply chain power structures

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Abstract: We investigate a supply chain in which a retailer is supplied by two manufacturers with differentiated brands, a good brand and an average brand. The customers in the market are segmented based on value and brand preference, namely the customer acceptance of the average brand and the customer surplus for each brand. Both horizontal competition (between the two competing manufacturers) and vertical competition (between the manufacturers and the retailer) are considered through an exploration of different power structure combinations. Multiple-stage game models are developed to examine the impact of different power structures on the pricing decisions and the profits of the manufacturers and the retailer. We find that intensified competition between the two manufacturers hurts the manufacturers and benefits the retailer. No dominance among supply chain members (the two manufacturers and the retailer) leads to the highest profit for the entire supply chain. We also find that for the two competing manufacturers, being first to announce the pricing decision results in lower profit – the second to announce benefits from knowing the rival's price. This explains why rivals prefer not to reveal decisions on prices, bid rates, and contracts, as this information represents bargaining power. The impact of customer acceptance of the average brand is also analyzed.

Keywords: Pricing; Customer's value; Brand preference; Power structure; Game theory

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1. Introduction

Many retailers sell multiple brands of a single type of product (Krishna, 1992; Baltas, 2004; Teng *et al*., 2007). It is common to see several brands of similar goods on the shelf, such as cigarettes supplied by Marlboro, Kent, and 555, or soda produced by Coca-Cola and Pepsi. Furthermore, most people prefer shopping in large malls, supermarkets, and big-box stores offering a variety of brands for many products like Macy's, Wal-Mart, and Carrefour, rather than in direct-sale stores of particular brands or exclusive shops, for more choices of goods. Unpopular brands, however, may not only take up shelf space but also increase purchasing costs and tie up available funds if they cannot be sold quickly. Therefore, it is critical for a distributor or a retailer who sells several similar goods in different brands to decide which brands should be purchased (one brand, or some brands, or many brands) and how to set prices for them, to better meet customer demand and increase profit (Kalwani *et al*., 1990; Bucklin and Lattin, 1991; Besanko, 2005; Hall *et al.*, 2010; Chen *et al.*, 2012; Luo *et al.*, 2016). To address the above issues, we study pricing models based on the customer's valuation of the goods and brand preference in a supply chain consisting of two manufacturers and a retailer, in different power structures. The two manufacturers produce substitutable products with different brands, a good brand and an average brand. The retailer may sell either of them or both of them.

Segmenting a market based on product attributes and customer's behavior, and deciding a corresponding pricing strategy, can be an effective way to respond to demand in the market. In this paper, market segmentation based on the positioning of goods and price, considering customer preference, is discussed first. The optimal prices under competition then can be derived.

Market segmentation has been well studied (Hotelling, 1990, Vandenbosch and Weinberg, 1995). Mussa and Rosen (1978) investigated a monopoly pricing problem with a quality-differentiated class of goods. The goods are offered in an imperfect market on a take-it-or-leave-it basis, and the seller exploits the possibilities for a pricing policy to allocate customers along the quality spectrum by a process of self-selection. The optimal policy is to assign different customer types to different classes of goods. Thus, it permits partial

discrimination among consumers with various intensities of demand. Shaked and Sutton (1982) described the perfect equilibrium of a three-stage game. First, firms choose (or not) to enter an industry; second, firms choose the quality of goods; and third, firms choose their prices. Prices were found to depend on both the number of entrants and the quality of their respective goods. Moorthy (1984) developed a theory of market segmentation based on consumer self-selection, which is an extension of the third-degree price discrimination model of Pigou (1920). He used a monopoly's product line design problem as a generic example of such segmentation. Moorthy (1988) examined two identical firms competing on product quality and price. He assumed that the customer prefers the high quality product to the low quality. The quality-price equilibrium strategies of both a simutaneous-product-choice model and sequential-product-choice model were obtained. Motta (1993) developed a pricing strategy based on quality and the cusomer taste in a Bertrand duopoly model. Desai (2001) examined the problem of quality segmentation in spatial markets. He developed a model in which the market has two segments, assuming that one segment values quality more than the other. Li *et al*. (2013) examined customers' self-selection among multiple versions of an information product and clarified the inability of the linear valuation function to exactly capture the customers' valuation on information products. Optimal quality levels and prices for multiple versions were obtained for a given number of versions. Abbey *et al*. (2015) studied the optimal pricing of the new and remanufactured products using a model of consumers' preferences, based on extensive experimentation. Two distinct segments of consumers were revealed and optimal prices were examined in several scenarios. Hu *et al*. (2015) investigated optimal pricing and product decisions in a crowdfunding mechanism. They found that when the buyers are sufficiently heterogeneous in their product valuations, the creator should offer a line of products with different levels of product quality and prices.

Our work is similar to the above studies on pricing decisions based on quality and consumer behavior. We assume, however, that the customers are heterogeneous in their valuation of the product. That is, each customer has an individual "willingness-to-pay" or reservation price. It is assumed that the good brand is generally valued higher than the average brand (Martin, 1996). The market is segmented thus into three segments based on customer acceptance of the average brand and the customer surplus of each brand: average brand only,

good brand only, and both. In addition, in this paper, unlike other studies, we focus on pricing policies based on the customer's valuation and brand preference.

In practice, a firm's operating performance depends not only on its operation strategies but also on its position or bargaining power in the market. Different power structures in the vertical supply chain and horizontal business partner/competitor relationships have become important factors in decision-making and profit margin (Choi, 1991; Choi, 1996; Trivedi, 1998; Choi and Fredj, 2013; Cai, 2010; Chen *et al*., 2014; Jie and Jing, 2014; Sang, 2014; Chen and Wang, 2015). In the vertical competition between supply chain members, suppliers may be in the leading position (e.g. a stronger competitive position in the supply chain). Thus, Microsoft and Intel play a more dominant role than downstream members in their supply chains. Some retailers, however, such as Wal-Mart and Carrefour (Ertek and Griffin, 2002), may be in a relatively strong competitive position and play a more dominant role than upstream members. In many cases, supply chain members may be engaged in vertical Nash competition in a local market (see examples in Cotterill and Putsis, 2001; Zhao *et al*., 2012). Members at the same echelon of a supply chain may also compete horizontally, and this will influence their decisions in strategic actions and timing. Timing will depend on the power structures in the supply chain. For example, Coca-Cola entered the Chinese market very early, while Pepsi entered late. In practice, Pepsi had to follow the retail price of Coca-Cola in the Chinese market.

Several channel power structures have been studied in the literature. Ingene and Parry (1995) examined one manufacturer supplying multiple exclusive retailers, and focused on the channel coordination. The monopoly manufacturer needs to set a single wholesale price that can be applied to all retailers. Among the work that is very relevant to the present study, Choi (1991) studied the pricing decisions of a supply chain that consists of two manufacturers and a retailer, considering linear demand and nonlinear demand. He discussed three non-cooperative games of different power structures, namely Manufacturer-Stackelberg, Retailer-Stackelberg, and Nash games between the manufacturers and the retailer. He assumed that the two manufacturers are symmetric and play a Nash game in setting prices. Choi (1996) extended this research by examining two manufacturers supplying a product to two differentiated retailers. He assumed that each manufacturer sets his wholesale price and supplies the same product to both retailers. In addition, each manufacturer determines his wholesale price based on the

observed retail price of the competing product. He found that horizontal product differentiation helps the retailers but hurts the manufacturers. He did not consider the retailer's choice in selecting the manufacturer. Ertek and Griffin (2002) discussed the impact of the power structure in a two-stage supply chain. They developed Supplier-Stackelberg and Retailer-Stackelberg structures, in order to analyze the pricing scheme for the retailer. Raju and Zhang (2005) studied a Retailer-Stackelberg channel model and discussed the coordination mechanism for the manufacturer. They found that this type of channel structure can be coordinated by either quantity discounts or a menu of two-part tariffs. Yang and Zhou (2006) analyzed a two-echelon system with a manufacturer and two competing retailers. The Manufacturer-Stackelberg model with two retailers' competitive behaviors was discussed in their study. From the Supplier-Stackelberg, Retailer-Stackelberg, and Nash game theoretic perspectives, Cai *et al.* (2009) discussed the effect of the price discount contracts and found that the scenarios with the price discount contracts may be superior to the non-contract scenarios. Wu *et al.* (2012) analyzed competitive pricing decisions in a supply chain consisting of two retailers and a supplier. They discussed six game models including vertical competition and horizontal competition. In contrast to their paper, our study examines a supply chain consisting of two manufacturers and a retailer. Fan *et al.* (2013) analyzed a dynamic pricing and production planning problem using a one-leader-multiple-follower Stackelberg differential game with unknown demand parameters. They found that the leader outperforms the followers and each firm can improve its revenue by demand learning. Chen and Wang (2015) investigated the smart phone supply chain, which consists of a handset manufacturer and a telecom service operator. Different power structures were considered and the corresponding impacts were discussed. They showed that the smart phone supply chain would choose a bundled channel in the telecom service operator Stackelberg as well as in the manufacturer Stackelberg power structure under certain conditions, while it would select a free channel in a vertical Nash power structure.

The above studies focused mainly on the supply chain's vertical competition between supply chain members. Very limited studies have attempted to examine the impact of horizontal competition on the supply chain's decisions and performance. Furthermore, they did not consider the customer's valuation on the product as a factor in the pricing decision. A few

studies, however, have considered the customer's valuation, to capture individual preference in selecting the product for a dual-channel supply chain (for example, Chiang *et al*., 2003; Chen and Bell, 2012), but these studies only considered vertical price competition. Here, we study the pricing problem in a supply chain consisting of two manufacturers and a retailer. Both the vertical price competition (between the manufacturers and the retailer) and the horizontal competition (between two competing manufacturers) are considered under different power structures.

This paper contributes to the literature in several ways. First, our study considers that customers have heterogeneous valuations on different brands. The market is segmented based on the customer's acceptance of the brand and the customer's surplus in each brand. Second, we clearly identify the conditions under which the retailer should purchase both brands or a single brand, and the corresponding pricing strategies of the manufacturers and the retailer. Third, we consider all possible power structures for the supply chain with two manufacturers and a retailer, which captures all possible market competition scenarios that could present in practice. To the best of our knowledge, there are very limited studies that combine brand preference and customer valuation in the pricing model. Here, our research aims to fulfill this gap in the literature through addressing the following key questions:

(1) Under what conditions will the retailer sell either both brands or one of them, based on the customer's valuation and brand preference?

(2) In each power structure, how can the two manufacturers and the retailer develop pricing policies to maximize their profits when the retailer sells both brands or sells single brand?

(3) What are the impacts of power structure (including horizontal power structure and vertical power structure) on the optimal pricing policies and profits of the retailer and the manufacturers, and on the performance of the entire supply chain? What is the impact of customer acceptance of the average brand?

The rest of this paper is organized as follows. In Section 2, the model formulation and assumptions are presented, and a piecewise demand function is analyzed and derived. In Section 3, we investigate the two manufacturers' and retailer's pricing decisions and obtain equilibrium solutions from two cases under different market competition scenarios. In Section

4, we discuss the effect of customer acceptance of the average brand and the power structure on the manufacturers' and retailer's optimal pricing policies and profits, as well as on the performance of the entire supply chain. The numerical examples, which complement Section 4, are presented to provide new managerial insight in Section 5. In Section 6, we conclude our research findings and propose possible extensions of this work. All proofs are in the Appendix.

2 The model

We consider a retailer who sells two substitutable products supplied by two manufacturers: a good-brand manufacturer 1 with a high-brand value, and an average-brand manufacturer 2 with an average-brand value. The retailer sets the retail prices for both brands, while the manufacturers determine the wholesale prices independently. Manufacturer i's unit production cost is c_i, wholesale price is w_i, and the retail price is p_i for two products ($i = 1, 2$). Without loss of generality, we assume that $c_1 > c_2$.

Customers are heterogeneous in their valuation on the product, depending on customers' values, views, income or level of knowledge of the product. To describe customers' heterogeneity, we model that the customer reservation price v is uniformly distributed over [0,1] within the customer population from 0 to 1 with density of 1, which catches the individual difference in product valuation (Chiang *et al*., 2003). Considering one product that is priced at p, the customer with a net surplus $v - p \geq 0$ will buy it (Chen and Bell, 2012). From Figure 1, all the customers with valuations in the interval $[p, 1]$ will buy the product. Therefore, the demand of the product is $Q = \int_p^1 dv = 1 - p$ for $0 \leq p \leq 1$.
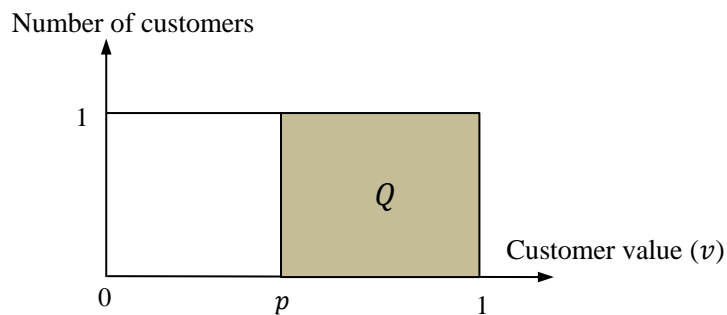

Figure 1. Distribution of customer value

Intuitively, for similar products, the good brand is always perceived by the customer to be made from a higher quality raw material and more advanced manufacturing process, and offer

better customer experience and excellent after-sales service, as compared to the average brand. In addition, an individual customer has a higher reservation price for the good brand than for the average brand (Martin, 1996). To capture this preference, we introduce a term called 'customer acceptance' of a brand. We assume that the customer perceives the good brand as perfect, and define the customer acceptance of the good brand as equal 1. Meanwhile, we introduce a parameter θ to denote customer acceptance of the average brand, where $\theta \in (0,1)$. Thus, we use v and θ to capture individual difference in valuing brands: a customer perceives the good brand to be worth v and the average brand to be worth θv, respectively. A customer with a valuation of v may purchase a good brand product if it has a nonnegative surplus $v - p_1 \geq 0$, and it may buy an average-brand product if $\theta v - p_2 \geq 0$. The customer will choose an average brand product rather than a good brand one only if $\theta v - p_2 > v - p_1$. We denote the indifferent values in whether the customer purchases (or not) the product of each brand as $v_1 = p_1$ and $v_2 = \frac{p_2}{\theta}$, respectively. The indifferent value of purchasing a good brand or an average brand is $v_{21} = \frac{p_1 - p_2}{1 - \theta}$. Through the discussion, the analysis of customer reservation price, and three indifferent values, the demand function can be modelled as follow.

Proposition 1: *The piecewise demand function of the good-brand product $D_1(p_1, p_2)$ and the average-brand product $D_2(p_1, p_2)$ can be modelled as:*

$$D_1(p_1, p_2) = \begin{cases} 1 - p_1 & 0 < \theta \leq \frac{p_2}{p_1} \\ 1 - \frac{p_1 - p_2}{1 - \theta} & \frac{p_2}{p_1} < \theta < 1 - p_1 + p_2 \\ 0 & 1 - p_1 + p_2 \leq \theta < 1 \end{cases} \qquad (1)$$

$$D_2(p_1, p_2) = \begin{cases} 0 & 0 < \theta \leq \frac{p_2}{p_1} \\ \frac{p_1 - p_2}{1 - \theta} - \frac{p_2}{\theta} & \frac{p_2}{p_1} < \theta < 1 - p_1 + p_2 \\ 1 - \frac{p_2}{\theta} & 1 - p_1 + p_2 \leq \theta < 1 \end{cases} \qquad (2)$$

Proposition 1 gives us an intuitive insight that the product demand depends on customer acceptance of the average brand θ. That is, the customer acceptance of the average brand θ lies in different ranges, namely $(0, \frac{p_2}{p_1}]$ and $[1 - p_1 + p_2, 1)$. In Corollary 1 we summarize the retailer's sales strategy for a brand.

Corollary 1: *The retailer's sales strategy for a brand is highly dependent on customer*

acceptance of the average brand θ: (a) $0 < \theta \le \underline{\theta}$, *the retailer will sell the good brand only; (b)* $\underline{\theta} < \theta < \overline{\theta}$, *the retailer will sell both brands; (c)* $\overline{\theta} \le \theta < 1$, *the retailer will sell the average brand only, where* $\underline{\theta} = \frac{p_2}{p_1}$ *and* $\overline{\theta} = 1 - p_1 + p_2$.

This Corollary implies that if the customer acceptance of the average brand is sufficiently low ($0 < \theta \le \underline{\theta}$), it is optimal for the retailer to purchase only the good brand from the manufacturer, because there is no demand for the average brand. If the customer acceptance of the average brand is sufficiently high ($\overline{\theta} \le \theta < 1$), no customer will purchase the product in the good brand, and selling the average-brand product is the optimal choice for the retailer to maximize its profit. When the customer acceptance of the average brand is moderate ($\underline{\theta} < \theta < \overline{\theta}$), the retailer's optimal sales strategy is to sell both brands. This choice requires that the retailer estimates the customer acceptance of the average brand accurately based on historical data, expertise, or industrial reports on similar brands; the retailer must be able to assess whether it should choose either both brands, or the good brand only, or the average brand only.

In addition, we assume that both the manufacturers and the retailer are rational and self-interested, that is, each of them aims to maximize its own profit. The model framework can be described as in Figure 2.
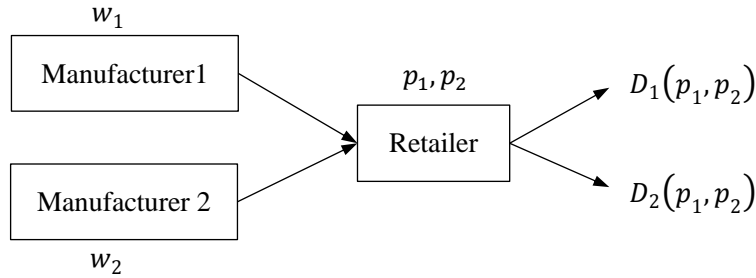

Figure 2. The model framework

Let subscripts m and r represent the manufacturers and the retailer, respectively. The manufacturers' profit functions are:

$$\pi_{m1}(w_1) = (w_1 - c_1)D_1(p_1, p_2) \tag{3}$$

$$\pi_{m2}(w_2) = (w_2 - c_2)D_2(p_1, p_2) \tag{4}$$

The retailer's profit function is:

$$\pi_r(p_1, p_2) = (p_1 - w_1)D_1(p_1, p_2) + (p_2 - w_2)D_2(p_1, p_2) \tag{5}$$

3. Equilibrium

In this section, we will examine the pricing decisions of the two manufacturers and the retailer. Consider the heterogeneous customer acceptance of the average brand θ. Two cases are discussed in this section: Case 1: retailer sells both brands for $\underline{\theta} < \theta < \overline{\theta}$, and Case 2: retailer sells single brand only for $0 < \theta \leq \underline{\theta}$ or $\overline{\theta} \leq \theta < 1$.

3.1 Case 1: retailer sells both brands

When $\underline{\theta} < \theta < \overline{\theta}$, selling both brands is the retailer's optimal policy. To capture the decisions of the two manufacturers and the retailer in the supply chain under different power structures, we model both the horizontal competition (between the two manufacturers with different brands) and the vertical competition (between the manufacturers and the retailer) as either a Stackelberg game or a Nash game, as summarized in Table 1.

Table 1. Seven game models with different power structures

Vertical competition	Horizontal competition		
	Nash (N)	Good-brand Stackelberg (G)	Average-brand Stackelberg (A)
Manufacturer Stackelberg (MS)	MNS	MGS	MAS
Retailer Stackelberg (RS)	RNS	RGS	RAS
Vertical Nash (VN)	VNN	-	-

Table 1 shows that there are seven game models with different power structures. We use k to represent a model type, where $k \in \{MNS, MGS, MAS, VNN, RNS, RGS, RAS\}$. We now discuss the decision sequence of each horizontal game under each vertical competition game.

I. Manufacturer Stackelberg (MS) model

In the case of the MS model, the manufacturers are Stackelberg leaders while the retailer is the follower. The two manufacturers may have different channel powers. So we now analyse the two manufacturers' and the retailer's pricing decisions in the MS model with balanced power and with imbalanced power.

MS model with balanced power between manufacturers (*MNS*)

In the case of the MS model with balanced power between manufacturers ($k = MNS$), the decision sequence of the manufacturers and the retailer is as follows. In the first-stage game,

the two manufacturers simultaneously announce the wholesale prices to the retailer, anticipating the retailer's prices for the products in the two brands. In the second-stage game, given the manufacturers' wholesale prices, the retailer decides the retail prices for the two brands.

MS model with good-brand manufacturer as Stackelberg leader (*MGS*)

In the case of the MS model with the good-brand manufacturer as Stackelberg leader ($k = MGS$), the decision sequence of the manufacturers and the retailer becomes: in the first-stage game, the good-brand manufacturer announces the wholesale price, anticipating the retail prices of the two brands and the wholesale price of the average brand; in the second-stage game, given the wholesale price of the good brand and anticipating the retail prices of the two brands, the average-brand manufacturer chooses its wholesale price; in the third-stage game, the retailer decides its optimal retail prices, given the two manufacturers' wholesale prices.

MS model with the average-brand manufacturer as Stackelberg leader (*MAS*)

In the case of the MS model with the average-brand manufacturer as Stackelberg leader ($k = MAS$), the decision sequence is as follows. In the first-stage game, the manufacturer with the average brand announces its wholesale price, anticipating the wholesale price of the good brand and the retail prices of the two brands. In the second-stage game, given the wholesale price of the average brand and anticipating the retail prices of the two brands, the good-brand manufacturer decides its wholesale price. In the third-stage game, the retailer chooses its optimal retail prices for both brands, given the two manufacturers' wholesale prices.

II. Vertical Nash (VN) Model

Under the vertical Nash model, the manufacturers and the retailer make their pricing decisions simultaneously ($k = VNN$). The decision sequence is: the two manufacturers decide their wholesale prices simultaneously to maximize their profits, anticipating the retailer's margin profits, while the retailer decides its retail prices for the two brands to maximize its profit, anticipating the manufacturers' wholesale prices.

III. Retailer Stackelberg (RS) model

In the case of the RS model, the retailer will be the Stackelberg leader while the two manufacturers are the followers in deciding prices. As with the MS model, we now analyse the retailer's brand choice in the RS model with balanced and imbalanced power. As in the

manufacturer Stackelberg leader models, we present the decision sequence for each game model.

RS model with balanced power between the manufacturers (RNS)

In the case of the RS game model with manufacturers' balanced power ($k = RNS$), the decision sequence of the manufacturers and the retailer is as follows: in the first-stage game, the retailer announces the retail prices of the two brands to the two corresponding manufacturers, anticipating the manufacturers' wholesale prices; in the second-stage game, the manufacturers decide the wholesale prices simultaneously, anticipating the retailer's margin profits for the two brands.

RS model with the good-brand manufacturer as Stackelberg leader (RGS)

In the case of the RS model with the good-brand manufacturer as Stackelberg leader ($k = RGS$), the decision sequence is: in the first-stage game, anticipating the wholesale prices of both manufacturers, the retailer announces retail prices for the two brands; in the second-stage game, anticipating the wholesale price of the average brand, margin profit of the good brand, and given the retail price of the average brand, the good-brand manufacturer chooses its wholesale price. In the third-stage game, the manufacturer with the average brand decides its optimal wholesale price given the wholesale price and the retail price of the good brand, and anticipating the margin profit of the average brand.

RS model with the average-brand manufacturer as Stackelberg leader (RAS)

In the case of the RS model with the average-brand manufacturer as Stackelberg leader ($k = RAS$), the decision sequence for the manufacturers and the retailer is similar to the case of $k = RGS$, except that the average-brand manufacturer acts in the second-stage game and the good-brand manufacturer in the third.

3.2 Case 2: retailer sells a single brand only

When $0 < \theta \leq \underline{\theta}$ or $\overline{\theta} \leq \theta < 1$, it is optimal for the retailer to sell either the good-brand product only or the average-brand product only, respectively. However, in this case, the upper and bottom lines keep the two manufacturers and the common retailer remaining a three players game, and the solutions give zero demand for good-brand product and average-brand product respectively.

3.3 Equilibrium solutions

To obtain equilibrium, we start by resolving the last-stage game and move back to the first-stage game for all seven game models (see Table 1). In all seven cases, the optimal solutions to the retailer's retail prices (p_1^k, p_2^k) and the manufacturers' wholesale prices (w_1^k, w_2^k) can be summarized in Proposition 2. Further, we can decide the boundary values $(\underline{\theta}^k, \overline{\theta}^k)$ for each game model, where $\underline{\theta}^k = \frac{p_2^k}{p_1^k}$ and $\overline{\theta}^k = 1 - p_1^k + p_2^k$.

Proposition 2: *For $\underline{\theta}^k < \theta < \overline{\theta}^k$, $0 < \theta \leq \underline{\theta}^k$ and $\overline{\theta}^k \leq \theta < 1$, there exists a unique optimal solution to the retailer's retail prices (p_1^k, p_2^k) and to the manufacturers' wholesale prices (w_1^k, w_2^k) respectively, which are summarized in Table 2 and 3 (Page 14). The boundary values for each game model are summarized in Table 4 (Page 15).*

From Table 4, we see that the boundary values $\underline{\theta}^k$ and $\overline{\theta}^k$ in each game model are dependent only on the unit production costs c_1 and c_2 of the two brands. This suggests that the retailer can make the brand selection decision according to Corollary 1, when it can obtain information on the unit production costs of the two manufacturers (c_1, c_2) and estimate customer acceptance of the average brand based on historical data, expertise, or industrial reports. This will not only meet the needs of specific consumer groups, but also reduce the costs of stocking and save shelf space.

Table 2. Equilibrium solutions for $\underline{\theta}^k < \theta < \overline{\theta}^k$

Game models	p_1^k	p_2^k	w_1^k	w_2^k
MNS	$1 - \dfrac{2(1-c_1)+(\theta-c_2)}{2(4-\theta)}$	$\theta - \dfrac{\theta(1-c_1)+2(\theta-c_2)}{2(4-\theta)}$	$1 - \dfrac{2(1-c_1)+(\theta-c_2)}{4-\theta}$	$\theta - \dfrac{\theta(1-c_1)+2(\theta-c_2)}{4-\theta}$
MGS	$1 - \dfrac{(2-\theta)(1-c_1)+(\theta-c_2)}{4(2-\theta)}$	$\theta - \dfrac{\theta(2-\theta)(1-c_1)+(4-\theta)(\theta-c_2)}{8(2-\theta)}$	$1 - \dfrac{(2-\theta)(1-c_1)+(\theta-c_2)}{2(2-\theta)}$	$\theta - \dfrac{\theta(2-\theta)(1-c_1)+(4-\theta)(\theta-c_2)}{4(2-\theta)}$
MAS	$1 - \dfrac{(4-\theta)(1-c_1)+(2-\theta)(\theta-c_2)}{8(2-\theta)}$	$\theta - \dfrac{\theta(1-c_1)+(2-\theta)(\theta-c_2)}{4(2-\theta)}$	$1 - \dfrac{(4-\theta)(1-c_1)+(2-\theta)(\theta-c_2)}{4(2-\theta)}$	$\theta - \dfrac{\theta(1-c_1)+(2-\theta)(\theta-c_2)}{2(2-\theta)}$
VNN	$1 - \dfrac{3(1-c_1)+(\theta-c_2)}{9-\theta}$	$\theta - \dfrac{\theta(1-c_1)+3(\theta-c_2)}{9-\theta}$	$1 - \dfrac{2[3(1-c_1)+(\theta-c_2)]}{9-\theta}$	$\theta - \dfrac{2[\theta(1-c_1)+3(\theta-c_2)]}{9-\theta}$
RNS	$1 - \dfrac{2(1-c_1)+(\theta-c_2)}{2(4-\theta)}$	$\theta - \dfrac{\theta(1-c_1)+2(\theta-c_2)}{2(4-\theta)}$	$1 - \dfrac{2(1-c_1)+(\theta-c_2)}{2(4-\theta)}$	$\theta - \dfrac{\theta(1-c_1)+2(\theta-c_2)}{2(4-\theta)}$
RGS	$1 - \dfrac{2(1-\theta)(1-c_1)+(\theta-c_2)}{2(4-3\theta)}$	$\theta - \dfrac{\theta(1-\theta)(1-c_1)+(2-\theta)(\theta-c_2)}{2(4-3\theta)}$	$1 - \dfrac{(6-5\theta)(1-c_1)+(\theta-c_2)}{2(4-3\theta)}$	$\theta - \dfrac{\theta(1-\theta)(1-c_1)+2(3-2\theta)(\theta-c_2)}{2(4-3\theta)}$
RAS	$1 - \dfrac{(2-\theta)(1-c_1)+(1-\theta)(\theta-c_2)}{2(4-3\theta)}$	$\theta - \dfrac{\theta(1-c_1)+2(1-\theta)(\theta-c_2)}{2(4-3\theta)}$	$1 - \dfrac{2(3-2\theta)(1-c_1)+(1-\theta)(\theta-c_2)}{2(4-3\theta)}$	$\theta - \dfrac{\theta(1-c_1)+(6-5\theta)(\theta-c_2)}{2(4-3\theta)}$

Table 3. Equilibrium solutions, demands and profits for $0 < \theta \le \underline{\theta}^k$ and $\overline{\theta}^k \le \theta < 1$

Game models	$0 < \theta \le \underline{\theta}^k$					$\overline{\theta}^k \le \theta < 1$				
	p_1^k	w_1^k	D_1^k	π_{m1}^k	π_r^k	p_2^k	w_2^k	D_2^k	π_{m2}^k	π_r^k
MNS	$1 - \dfrac{1-c_1}{2(2-\theta)}$	$1 - \dfrac{1-c_1}{2-\theta}$	$\dfrac{1-c_1}{2(2-\theta)}$	$\dfrac{(1-\theta)(1-c_1)^2}{2(2-\theta)^2}$	$\dfrac{(1-c_1)^2}{4(2-\theta)^2}$	$\theta - \dfrac{\theta-c_2}{2(2-\theta)}$	$\theta - \dfrac{\theta-c_2}{2-\theta}$	$\dfrac{\theta-c_2}{2\theta(2-\theta)}$	$\dfrac{(1-\theta)(\theta-c_2)^2}{2(2-\theta)^2\theta}$	$\dfrac{(\theta-c_2)^2}{4(2-\theta)^2\theta}$
MGS	$1 - \dfrac{\theta-c_2}{2\theta}$	$\dfrac{c_2}{\theta}$	$\dfrac{\theta-c_2}{2\theta}$	$\dfrac{(\theta-c_2)(c_2-\theta c_1)}{2\theta^2}$	$\dfrac{(\theta-c_2)^2}{4\theta^2}$	$\theta - \dfrac{\theta-c_2}{2(2-\theta)}$	$\theta - \dfrac{\theta-c_2}{2-\theta}$	$\dfrac{\theta-c_2}{2\theta(2-\theta)}$	$\dfrac{(1-\theta)(\theta-c_2)^2}{2(2-\theta)^2\theta}$	$\dfrac{(\theta-c_2)^2}{4(2-\theta)^2\theta}$
MAS	$1 - \dfrac{1-c_1}{2(2-\theta)}$	$1 - \dfrac{1-c_1}{2-\theta}$	$\dfrac{1-c_1}{2(2-\theta)}$	$\dfrac{(1-\theta)(1-c_1)^2}{2(2-\theta)^2}$	$\dfrac{(1-c_1)^2}{4(2-\theta)^2}$	$\theta - \dfrac{1-c_1}{2}$	$c_1 - 1 + \theta$	$\dfrac{1-c_1}{2\theta}$	$\dfrac{(1-c_1)(c_1-1+\theta-c_2)}{2\theta}$	$\dfrac{(1-c_1)^2}{4\theta}$
VNN	$1 - \dfrac{1-c_1}{3-\theta}$	$1 - \dfrac{2(1-c_1)}{3-\theta}$	$\dfrac{1-c_1}{3-\theta}$	$\dfrac{(1-\theta)(1-c_1)^2}{(3-\theta)^2}$	$\dfrac{(1-c_1)^2}{(3-\theta)^2}$	$\theta - \dfrac{\theta-c_2}{3-\theta}$	$\theta - \dfrac{2(\theta-c_2)}{3-\theta}$	$\dfrac{\theta-c_2}{(3-\theta)\theta}$	$\dfrac{(1-\theta)(\theta-c_2)^2}{(3-\theta)^2\theta}$	$\dfrac{(\theta-c_2)^2}{(3-\theta)^2\theta}$
RNS	$1 - \dfrac{1-c_1}{2(2-\theta)}$	$\dfrac{1-\theta+3c_1-\theta c_1}{2(2-\theta)}$	$\dfrac{1-c_1}{2(2-\theta)}$	$\dfrac{(1-\theta)(1-c_1)^2}{4(2-\theta)^2}$	$\dfrac{(1-c_1)^2}{4(2-\theta)}$	$\theta - \dfrac{\theta-c_2}{2(2-\theta)}$	$\dfrac{\theta-\theta^2+3c_2-\theta c_2}{2(2-\theta)}$	$\dfrac{\theta-c_2}{2\theta(2-\theta)}$	$\dfrac{(1-\theta)(\theta-c_2)^2}{4(2-\theta)^2\theta}$	$\dfrac{(\theta-c_2)^2}{4(2-\theta)\theta}$
RGS	$1 - \dfrac{1-c_1}{4}$	$1 - \dfrac{3(1-c_1)}{4}$	$\dfrac{1-c_1}{4}$	$\dfrac{(1-c_1)^2}{16}$	$\dfrac{(1-c_1)^2}{8}$	$\theta - \dfrac{\theta-c_2}{2(2-\theta)}$	$\dfrac{\theta-\theta^2+3c_2-\theta c_2}{2(2-\theta)}$	$\dfrac{\theta-c_2}{2\theta(2-\theta)}$	$\dfrac{(1-\theta)(\theta-c_2)^2}{4(2-\theta)^2\theta}$	$\dfrac{(\theta-c_2)^2}{4(2-\theta)\theta}$
RAS	$1 - \dfrac{1-c_1}{2(2-\theta)}$	$\dfrac{1-\theta+3c_1-\theta c_1}{2(2-\theta)}$	$\dfrac{1-c_1}{2(2-\theta)}$	$\dfrac{(1-\theta)(1-c_1)^2}{4(2-\theta)^2}$	$\dfrac{(1-c_1)^2}{4(2-\theta)}$	$\theta - \dfrac{\theta-c_2}{4}$	$\theta - \dfrac{3(\theta-c_2)}{4}$	$\dfrac{\theta-c_2}{4\theta}$	$\dfrac{(\theta-c_2)^2}{16\theta}$	$\dfrac{(\theta-c_2)^2}{8\theta}$

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$$\text{Table 4. Boundary values } \underline{\theta}^k \text{ and } \overline{\theta}^k$$

Game models	$\underline{\theta}^k$	$\overline{\theta}^k$
MNS	$\dfrac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$	$1-\dfrac{c_1-c_2}{2-c_1}$
MGS	$\dfrac{2+2c_1+3c_2-\sqrt{(2+2c_1+3c_2)^2-16(2+c_1)c_2}}{2(2+c_1)}$	$1-\dfrac{c_1-c_2}{2-c_1}$
MAS	$\dfrac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$	$\dfrac{5-3c_1+c_2-\sqrt{(3-3c_1+c_2)^2+4(c_1-c_2)}}{2}$
VNN	$\dfrac{1+2c_1+c_2-\sqrt{(1+2c_1+c_2)^2-12c_2}}{2}$	$1-\dfrac{2c_1-2c_2}{3-c_1}$
RNS	$\dfrac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$	$1-\dfrac{c_1-c_2}{2-c_1}$
RGS	$\dfrac{2c_2}{1+c_1}$	$1-\dfrac{c_1-c_2}{2-c_1}$
RAS	$\dfrac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$	$\min[1,2-2c_1+c_2]$

4 Impact of customer acceptance of the average brand and the power structure

In this section, we discuss the impact of customer acceptance of the average brand and horizontal and vertical power structures in the decisions and performance of the supply chain.

4.1 Impact of the customer acceptance of the average brand θ on pricing decisions

Corollary 1 and Proposition 2 show that when $\underline{\theta}^k < \theta < \overline{\theta}^k$, the retailer sells both brands. From Table 2, we can obtain the following proposition.

Proposition 3: *For any game model $k \in \{MNS, MGS, MAS, VNN, RNS, RGS, RAS\}$, when $\theta \in (\underline{\theta}^k, \overline{\theta}^k)$, then $w_1^k > w_2^k$ and $p_1^k > p_2^k$.*

Proposition 3 indicates that in all the power structures we have discussed above, when the retailer sells both good and average brands, the optimal wholesale prices and optimal retail prices of the good brand are higher than that of the average brand. Since the customer perceives the good brand as higher value, the retailer thus can charge a higher retail price, which leaves a room for the good-brand manufacturer to charge a higher wholesale price. On the other hand, due to low customer acceptance of the average-brand product, the retailer should set a lower retail price to attract more lower-value customers, and this leads to a lower wholesale price from the average-brand manufacturer. In addition, from the proof in the

Appendix, we find that this Proposition still holds when $c_1 = c_2$. If and only if the customer acceptance of the average brand approaches one, the retail and wholesale prices of the good brand and the average brand will merge, which implies that when the customer becomes brand insensitive, the manufacturers and the retailer will not need to differentiate their prices, when $c_1 = c_2$.

4.2 Impact of horizontal power structure

Define the profit of the entire supply chain and the total sales volume as $\pi_s = \pi_{m1} + \pi_{m2} + \pi_r$ and $D = D_1 + D_2$, respectively. Furthermore, we denote the proportions of good-brand and average-brand product sold as $\alpha_1 = D_1/D$ and $\alpha_2 = D_2/D$, respectively. The impact of the horizontal power structure can be summarized in Propositions 4 (for the MS game) and 5 (for the RS game). Since $\underline{\theta}^{MGS} < \underline{\theta}^{MAS} = \underline{\theta}^{MNS}$ and $\overline{\theta}^{MNS} = \overline{\theta}^{MGS} < \overline{\theta}^{MAS}$ through graphical analysis for $0 < c_2 < c_1 < 1$, the result in Proposition 4 is given when the retailer sells both brands in all MNS, MGS, and MAS game models.

Proposition 4: *In MS model, if the customer acceptance of the average brand $\theta \in (\underline{\theta}^{MNS}, \overline{\theta}^{MNS})$, the following properties hold:*

(a) $w_1^{MNS} < w_1^{MGS}$ and $w_1^{MNS} < w_1^{MAS}$; $w_2^{MNS} < w_2^{MGS}$ and $w_2^{MNS} < w_2^{MAS}$.

(b) $p_1^{MNS} < p_1^{MGS}$ and $p_1^{MNS} < p_1^{MAS}$; $p_2^{MNS} < p_2^{MGS}$ and $p_2^{MNS} < p_2^{MAS}$.

(c) $D^{MNS} > D^{MGS}$ and $D^{MNS} > D^{MAS}$; $\alpha_1^{MGS} < \alpha_1^{MNS} < \alpha_1^{MAS}$ and $\alpha_2^{MGS} > \alpha_2^{MNS} > \alpha_2^{MAS}$.

(d) $\pi_{m1}^{MGS} > \pi_{m1}^{MNS}$ and $\pi_{m2}^{MAS} > \pi_{m2}^{MNS}$. When $\theta \in (\underline{\theta}^{MNS}, \theta_1]$, $\pi_r^{MGS} \leq \pi_r^{MAS}$ and

$\pi_s^{MGS} \leq \pi_s^{MAS}$; *when* $\theta \in (\theta_1, \overline{\theta}^{MNS})$, $\pi_r^{MGS} > \pi_r^{MAS}$ *and* $\pi_s^{MGS} > \pi_s^{MAS}$, *where*

$\theta_1 = \frac{(1-c_1)^2 + 2c_2 + (1-c_1)\sqrt{(1-c_1)^2 + 4c_2}}{2}$.

Parts (a) and (b) in Proposition 4 show that the imbalanced power between the two manufacturers leads to a higher optimal wholesale price as well as a higher retail price. This implies that the power imbalance between the two manufacturers facilitates a higher margin for both manufacturers because it softens the price competition between the two manufacturers. As a result, both manufacturers gain more profit although one may sell fewer products if it acts as the leader between the two manufacturers. The power imbalance between

the two manufacturers, however, results in reduced total demand for the retailer since the retailer has to increase prices for both brands, and may lose some lower-value customers. Further analysis also shows that if the gap in unit production cost between the two manufacturers is larger, the optimal wholesale prices and retail price of the MAS model are always higher than that of the MGS model. This implies that if the weaker manufacturer acts as the leader, both manufacturers gain more profit margins. Part (c) also shows that if the competing manufacturer acts as the leader, the other manufacturer can sell more products to the retailer. In part (d), it is interesting to see that when customer acceptance of the average brand is relatively low ($\theta \in (\underline{\theta}^{MNS}, \theta_1]$), both the retailer and the entire supply chain gain more profits if the average-brand manufacturer is the leader and the good-brand manufacturer is the follower. When customer acceptance of the average brand is relatively high ($\theta \in (\theta_1, \overline{\theta}^{MNS})$), both the retailer and the entire supply chain gain more profit if the good-brand manufacturer is the leader and the average-brand manufacturer is the follower. Proposition 4 provides decision-makers important enlightenments to make accurate management decisions. Using Microsoft and Apple as an example. Surface Pro 4 and iPad Pro are similar portable personal computers manufactured by Microsoft and Apple, respectively. It is hard to say which company dominates the other as a whole, but these two competitive electronic products were launched at different times, which may bring a competitive edge to the first. That is, the timing of product coming into the market and the initial price will have a significant effect on product sales volume and profit gain. Therefore, our analysis offers some oligopolies important decision support on when and how to make pricing policies under different horizontal power structure. The retailer, such as retail giant Amazon in the United States, SUNING and JD Mall in China, may face a variety of competition scenarios between upstream enterprises and different consumer preference in product. Our findings offer suggestions on how to price Surface Pro and iPad Pro to improve retail profit under each scenarios. In addition, it is also interesting that when the customer acceptance of average brand is relatively low, the retailer would like the average brand manufacturer to move first to push out a new product, because this can bring the retailer more profit.

As for the RS model, since $\underline{\theta}^{RGS} < \underline{\theta}^{RAS} = \underline{\theta}^{RNS}$ and $\overline{\theta}^{RNS} = \overline{\theta}^{RGS} < \overline{\theta}^{RAS}$ through

graphical analysis for $0 < c_2 < c_1 < 1$, the result in Proposition 5 is given when the retailer sells both brands in the RNS, RGS, and RAS game models.

Proposition 5: *In the RS model, if the customer acceptance of the average brand $\theta \in$ ($\underline{\theta}^{RNS}, \overline{\theta}^{RNS}$), the following properties hold:*

(a) $w_1^{RNS} < w_1^{RGS}$ and $w_1^{RNS} < w_1^{RAS}$. $w_2^{RNS} < w_2^{RGS}$ and $w_2^{RNS} < w_2^{RAS}$.

(b) $p_1^{RNS} < p_1^{RGS}$ and $p_1^{RNS} < p_1^{RAS}$. $p_2^{RNS} < p_2^{RGS}$ and $p_2^{RNS} < p_2^{RAS}$.

(c) $D^{RNS} > D^{RGS}$ and $D^{RNS} > D^{RAS}$. $\alpha_1^{RGS} < \alpha_1^{RNS} < \alpha_1^{RAS}$ and $\alpha_2^{RGS} > \alpha_2^{RNS} > \alpha_2^{RAS}$.

(d) $\pi_{m1}^{RNS} > \pi_{m1}^{RGS}$ and $\pi_{m1}^{RAS} > \pi_{m1}^{RGS}$. $\pi_{m2}^{RNS} > \pi_{m2}^{RAS}$ and $\pi_{m2}^{RGS} > \pi_{m2}^{RAS}$. When $\theta \in$ ($\underline{\theta}^{RNS}, \theta_1$], $\pi_r^{RNS} > \pi_r^{RAS} \geq \pi_r^{RGS}$ and $\pi_s^{RAS} \geq \pi_s^{RGS}$; When $\theta \in (\theta_1, \overline{\theta}^{RNS})$, $\pi_r^{RNS} > \pi_r^{RGS} > \pi_r^{RAS}$ and $\pi_s^{RAS} < \pi_s^{RGS}$, where $\theta_1 = \frac{(1-c_1)^2 + 2c_2 + (1-c_1)\sqrt{(1-c_1)^2 + 4c_2}}{2}$.

The results of (a), (b), and (c) in Proposition 5 are similar to those in Proposition 4 (MS game). However, these conclusions can explain some typical cases in the retail industry. Take Wal-Mart and Carrefour as an example, these two giant retailers are always in a strong competitive position thus having more power than their suppliers. Given the marginal profits, Wal-Mart and Carrefour would like their upstream suppliers to be in relatively fierce competition, which results in a low wholesale price and triggers high product demand. Most noteworthy is that conclusion part (d) confirms the benefits of fierce competition between upstream suppliers, the retailer, e, g. Wal-Mart and Carrefour, will gain more profits in these scenarios. And both upstream suppliers and the retailer can be better off if the upstream suppliers have equal power. The most interesting in part (d) is that when the retailer is the Stackelberg leader in the supply chain, the manufacturer that acts as the follower in competition with the other manufacturer will be more profitable. Based on this conclusion, neither manufacturers wishes to be first to announce its wholesale price. This can be explained by that the price information is regarded as trade secret. Once it leaks, it will have an extremely serious impact on its own financial benefit. From (a) and (c), we see that when the two manufacturers have equal power, their low wholesale prices allow the retailer to attract more customers, and more customers will choose their brands, as compared to cases in which one manufacturer acts as the leader in competition with the other. The factor of demand

attraction dominates the factor of higher wholesale price, so both manufacturers are more profitable. When the two manufacturers are in competition, being a follower allows the manufacturer to attract more customers, and, as a result, enhance its profit. When the customer acceptance of the average brand is lower, the system-wide profit and the retailer's profit can be higher if the average brand manufacturer has more power, and vice versa.

4.3 Impact of vertical power structure

4.3.1 Retailer sells single brand

When the retailer sells the good-brand product only, if $0 < \theta \le \underline{\theta}^k$, the profit of the entire supply chain is $\pi_{s1} = \pi_{m1} + \pi_r$; when the retailer sells the average-brand product only, if $\overline{\theta}^k \le \theta < 1$, the profit of the entire supply chain is $\pi_{s2} = \pi_{m2} + \pi_r$. With Table 3, the following proposition indicates the impact of vertical power structure for cases in which the retailer sells one brand only.

Proposition 6: *(a) When the retailer sells the good-brand product only，namely* $\theta \in (0, \underline{\theta}^k]$, *we have* $w_1^{MNS} > w_1^{VNN} > w_1^{RNS}$, $p_1^{MNS} = p_1^{RNS} > p_1^{VNN}$, $D_1^{MNS} = D_1^{RNS} < D_1^{VNN}$, $\pi_{m1}^{MNS} > \pi_{m1}^{VNN} > \pi_{m1}^{RNS}$, $\pi_r^{MNS} < \pi_r^{VNN} < \pi_r^{RNS}$ *and* $\pi_{s1}^{MNS} = \pi_{s1}^{RNS} < \pi_{s1}^{VNN}$.

(b) When the retailer sells the average-brand product only, namely $\theta \in [\overline{\theta}^k, 1)$, *then* $w_2^{MNS} > w_2^{VNN} > w_2^{RNS}$, $p_2^{MNS} = p_2^{RNS} > p_2^{VNN}$, $D_2^{MNS} = D_2^{RNS} < D_2^{VNN}$, $\pi_{m2}^{MNS} > \pi_{m2}^{VNN} > \pi_{m2}^{RNS}$, $\pi_r^{MNS} < \pi_r^{VNN} < \pi_r^{RNS}$ *and* $\pi_{s2}^{MNS} = \pi_{s2}^{RNS} < \pi_{s2}^{VNN}$.

If the customer acceptance of the average brand is lower ($\theta \in (0, \underline{\theta}^k]$) or higher ($\theta \in [\overline{\theta}^k, 1)$), results in Proposition 6 are consistent with studies in the literature (for example, Choi, 1991; Chen and Wang, 2015; Chen *et al.*, 2016). Both the retailer and the manufacturer will gain more profit when they have more market power in the supply chain. The entire supply chain, as well as the customer, however, will benefit from higher profits and lower retail prices when there is no dominant channel member. Thus this conclusion explains why the individual firm wants more power and the whole supply chain system wants to stay in a balanced competition environment. From the perspective of individual manufacturer or retailer, more power over its supply chain counter parties will enable it to capture more profit. Therefore, for the manufacturer they can gain more power by expanding production scale,

technical upgrade and product differentiation. And for the retailer, he can focus on product marketing and extending distribution channel to enhance his power. In contrast, from the perspective of the entire supply chain, it will gain more profits if there is a more balanced power relationship vertically. Therefore, on the one hand, it is important for individual manufacturer or retailer to seek solutions in enhancing their market and supply chain power in order to acquire more economic benefits. On the other hand, strategically, it is crucial for industry leaders to create a more power balanced supply chain environment that promotes fair and effective competition to improve its supply chain competitiveness, which is significant in chain to chain competition.

4.3.2 Retailer sells both brands

When the retailer sells both brands and the two manufacturers have equal market power, since $\underline{\theta}^{MNS} = \underline{\theta}^{RNS} < \underline{\theta}^{VNN}$ and $\overline{\theta}^{VNN} < \overline{\theta}^{MNS} = \overline{\theta}^{RNS}$ through graphical analysis for $0 < c_2 < c_1 < 1$, the following proposition can be obtained.

Proposition 7: *In the MNS, RNS, and VNN game models, if the customer acceptance of the average brand $\theta \in (\underline{\theta}^{VNN}, \overline{\theta}^{VNN})$, the following properties hold:*

(a) $w_1^{MNS} > w_1^{VNN}$ and $w_1^{MNS} > w_1^{RNS}$; $w_2^{MNS} > w_2^{VNN}$ and $w_2^{MNS} > w_2^{RNS}$.

(b) $p_1^{MNS} = p_1^{RNS} > p_1^{VNN}$ and $p_2^{MNS} = p_2^{RNS} > p_2^{VNN}$.

(c) $D^{MNS} = D^{RNS} < D^{VNN}$. When $\theta \in (\underline{\theta}^{VNN}, \theta_1]$, $\alpha_1^{MNS} = \alpha_1^{RNS} \leq \alpha_1^{VNN}$ and $\alpha_2^{MNS} = \alpha_2^{RNS} \geq \alpha_2^{VNN}$; when $\theta \in (\theta_1, \overline{\theta}^{VNN})$, $\alpha_1^{MNS} = \alpha_1^{RNS} > \alpha_1^{VNN}$ and $\alpha_2^{MNS} = \alpha_2^{RNS} < \alpha_2^{VNN}$, where $\theta_1 = \frac{(1-c_1)^2 + 2c_2 + (1-c_1)\sqrt{(1-c_1)^2 + 4c_2}}{2}$.

(d) $\pi_{m1}^{MNS} > \pi_{m1}^{RNS}$ and $\pi_{m2}^{MNS} > \pi_{m2}^{RNS}$; $\pi_s^{MNS} = \pi_s^{RNS}$.

Part (a) illustrates that when a manufacturer has more power than the retailer, it will set a higher wholesale price. The imbalanced vertical power, however, will result in the same retail prices, which are higher than that in a balanced vertical power structure. Part (c) shows that when the retailer and the manufacturers have equal power, the total number of products sold is higher than the case in which the retailer and the manufacturers have imbalanced power, which can attract more low-value customers. Facing relatively low customer acceptance of the

average-brand product, however, the retailer tends to increase the order proportion of the good-brand product and decrease the order proportion of the average-brand product in the VNN model, and vice versa. Part (d) shows that the two manufacturers will benefit from the Stackelberg leader model (MNS model) as compared with the RNS model. The profit of the entire supply chain is the same whether the manufacturers or the retailer is the leader in the supply chain. Therefore, combining with Proposition 7, no matter single brand or multiple brands, for each member in the supply chain, they all want to gain more power to enhance their financial benefits. However, the whole supply chain system would like to be in a balanced power environment to optimize the chain-wide performance.

In summary, as mentioned in Section 1, imbalanced power relationships among supply chain members are very common. The seven power structures discussed in this section can be observed in the retailing and manufacturing industries. Our findings provide guidance for decision-making on pricing policies based on customer value and brand differentiation within different power structures.

5 Numerical examples

To illustrate the main results and obtain additional insight into the differences among the power structures, we use a numerical example to show the impact of power structure and customer acceptance of the average brand when the retailer sells both brands for $\underline{\theta}^k < \theta < \overline{\theta}^k$. We set $c_1 = 0.6$ and $c_2 = 0.25$, which gives $\underline{\theta}^{VNN} = 0.359$ and $\overline{\theta}^{VNN} = 0.708$ (from Table 4). Notice that maximum profits can be obtained and compared in closed-form solutions, as wholesale prices and retail prices have been presented in Table 2.

Figures 3 and 4 show the manufacturers' maximal profit for game model k, where $k \in \{MNS, MGS, MAS, VNN, RNS, RGS, RAS\}$. In general, an increase in the customer acceptance of the average brand drives the profit of the average-brand manufacturer up, and reduces the profit of the good-brand manufacturer. In the horizontal power structure, in the MS model, intensified competition between manufacturers will hurt both. Thus, in the MS model, the horizontal Nash game generates the worst financial performance. In the RS model, however, the financial performance in the horizontal Nash game is neither the worst nor the best.

Figures 3 and 4 also show that in either the MS or the RS model, for the two manufacturers, the one who makes decisions after his rival will be more profitable. This is inconsistent with the statement of first-mover advantage, but it is also very reasonable to explain some practical phenomena. For instance, a firm's operation decisions, including choice of action and timing of implementation are always regarded as trade secrets, which are very important and should not be leaked. Thinking about that two firms compete in an open bidding. Before the biding, each firm's bid price is an important trade secret. Once it is leaked, other rivals will know and make responses. Thus less benefit can be gained than that when the bid price is not leaked. Therefore, our conclusions confirm that due to low profit levels, it is not wise to leak firms' decisions to their rivals in business competition. In the vertical power structure, the manufacturer with more power gains more profit. Manufacturers are better off when they are leaders and the retailer is the follower.
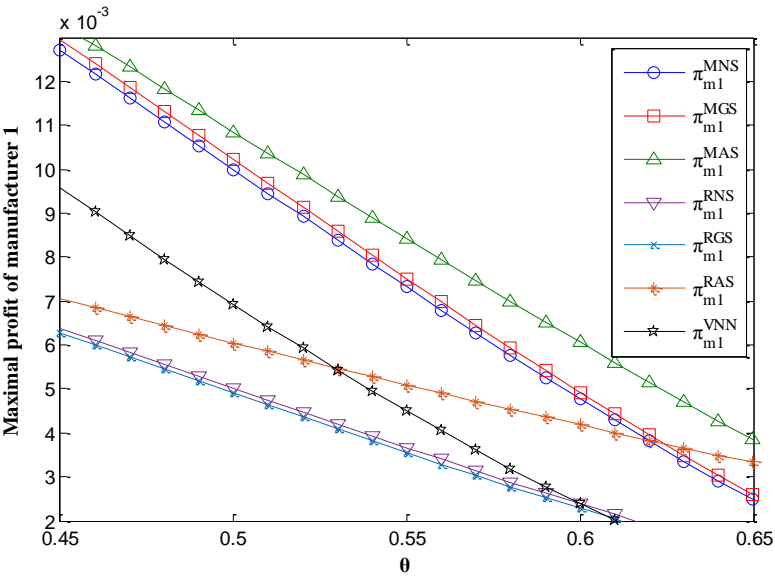

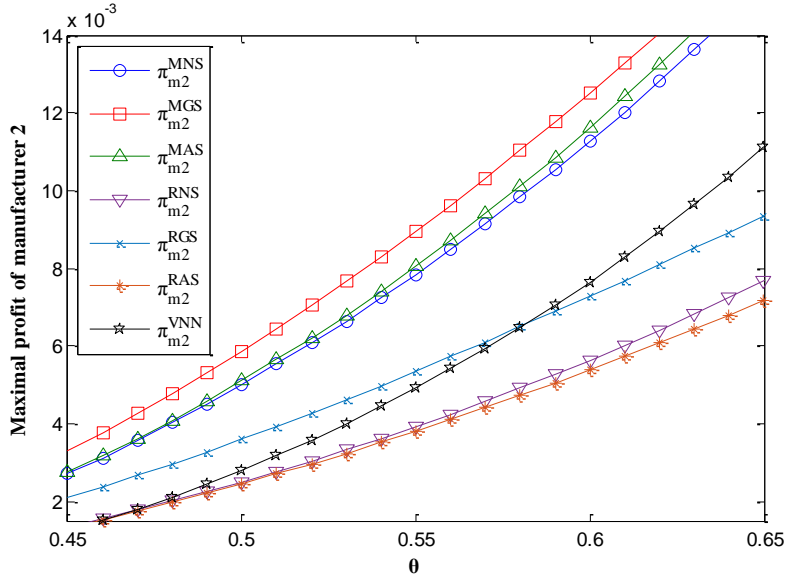**Figure 3. Maximal profits of manufacturer 1**

Figure 4. Maximal profits of manufacturer 2

Figure 5 shows the maximal profit of the retailer. As the customer acceptance of the average brand increases, the retailer is more profitable. In terms of the horizontal power structure of the two manufacturers, Figure 5 indicates that as in the RS model, the retailer will always gain the most profit if the two manufacturers have equal power in the MS model. In addition, the retailer is more profitable when it is a Stackelberg leader and the manufacturers are followers in the supply chain.
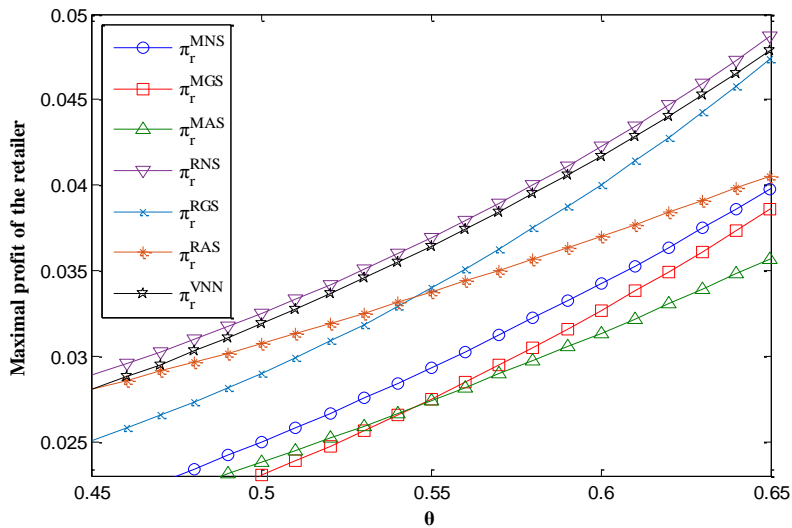

Figure 5. Maximal profit of the retailer

Figure 6 shows the chain-wide profit. The supply chain is more profitable as customer acceptance of the average brand increases and is the most profitable in the VNN model (Nash

equilibrium between the manufacturers and the retailer). That is, a supply chain in which all the members have balanced power and are in perfect competition will result in the highest chain-wide profit.
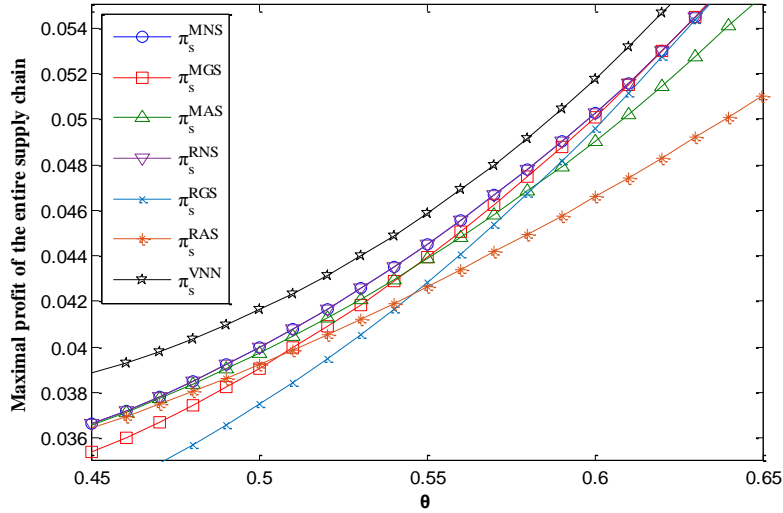

Figure 6. Maximal profit of the entire supply chain

6 Conclusions

This paper considers a supply chain with two manufacturers and one retailer. The two manufacturers are differentiated in product brand. Customers are heterogeneous in their valuation of the products of the two different brands. Based on the customer's utility function, we segment the markets and derive demand functions for the good-brand product as well as the average-brand product. We develop models with different power structures that capture the horizontal competition between the two manufacturers, as well as the vertical competition between the manufacturers and the retailer. We identify the conditions under which the retailer should sell both brands, and when it should sell either the good-brand only or the average-brand only. We find that the retailer's brand selection depends on the value of customer acceptance of the average brand. This study suggests several interesting observations and implications.

Observation 1. Given the supply chain power structure, with information on the unit production costs of the two manufacturers (c_1, c_2), the retailer can find the lower bound and the upper bound of the customer acceptance of the average brand ($\underline{\theta}^k, \overline{\theta}^k$). If the retailer can

estimate customer acceptance of the average brand based on historical data, expertise, or industrial reports on similar product brands, it can be in the position to assess whether it should choose both brands, the good brand only, or the average brand only (based on Corollary 1). When the retailer decides to sell both brands, the wholesale price and the retail price of the good brand are always higher than those of average brand, for all power structures (based on Proposition 3). This result is not affected by the unit production cost and will hold even when the unit production costs of the good brand and the average brand are equal.

Observation 2. The horizontal power structures have significant impact on the manufacturers' and retailer's pricing decisions and profits, as well as on the chain-wide profit. In either a manufacturers Stackelberg supply chain (where the retailer is the follower) or a retailer Stackelberg supply chain (where the manufacturers are followers), intensified competition between the good-brand manufacturer and the average-brand manufacturer (when they have equal power) will lead to lower wholesale prices, lower retail prices, and more sales for the retailer than the case in which the two manufacturers have imbalanced power. The retailer orders a larger proportion of product from the weaker manufacturer. When the manufacturers are leaders, they are more profitable in conditions of imbalanced power, due to softened wholesale price competition between the manufacturers as well as retail price competition between the two brands. When customer acceptance of the average brand is relatively lower, the retailer and the chain-wide are more profitable when the average-brand manufacturer is the leader. When customer acceptance of the average brand is relatively higher, the retailer and the entire chain are more profitable when the good-brand manufacturer is the leader (based on Proposition 4). When the retailer is leader, the manufacturers and the retailer are more profitable when there is a balanced power relationship between the manufacturers, due to the intensified wholesale price competition between the manufacturers and retail price competition between the two brands, which can enhance the overall demand of the retailer (based on Proposition 5). Whether either the retailer or the manufacturers are Stackelberg leader in the supply chain, the manufacturer who acts as the follower and makes decisions after his rival will be more profitable (based on Proposition 5, Figures 3 and 4). It is possible that in a game of perfect information, no one wants to move first, which would result in a lower profit level as decisions are exposed to the rival. Therefore, in practice, the

wholesale price or bid rate is always considered a trade secret between rivals, and leaks should be avoided.

Observation 3. Our findings also show the effect of vertical power structures. When the retailer sells both brands, we see that the more powerful firm always makes higher profits, but for the entire supply chain, a balanced power relationship among the two manufacturers and retailer is the best strategy (based on Proposition 7). Therefore, perfect competition helps improve the performance of the entire system through setting low prices to attract more customers.

This study provides a general analytical framework for pricing policies based on customer value and brand preference in a supply chain structure with two competing manufacturers and a retailer. We analyze the retailer's brand selection behavior with different power structures, and discuss the effect of power structures and customer acceptance of the average brand. Our study provides manufacturers and retailers with decision support that can help them develop accurate pricing strategies to improve their profits in various market positions.

As in other models used in the literature, the present model is also based on some assumptions. For example, our model assumes that a retailer sells two substitutable products with different brands purchased from different manufacturers. One meaningful extension of this work would be to consider two or multiple retailers who sell both brands, in which the chain-to-chain competition can be studied. Another extension is to consider stochastic demand based on customer value theory, to examine the impact of demand uncertainty on pricing decisions and channel structure.

Acknowledgements

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Appendix

Proof of Proposition 1:

From Section 2, we have $v_1 = p_1$, $v_2 = \frac{p_2}{\theta}$ and $v_{21} = \frac{p_1 - p_2}{1 - \theta}$. There are two conditions:

Condition 1: if $v_1 > v_2$ or $p_1 > \frac{p_2}{\theta}$, we can derive $\frac{p_1 - p_2}{1 - \theta} > p_1$ (or equivalently, $v_{21} > v_1$).

(1) If $1 > v_{21} > v_1 > v_2$ (or equivalently, $p_2 + 1 - \theta > p_1 > \frac{p_2}{\theta}$), namely $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$, then the customer whose reservation price v is in the range $[0, v_2]$ will buy neither brand, while the customer will purchase the average brand product for v in the range $[v_2, v_{21}]$ and the good brand product for v in the range $[v_{21}, 1]$. Therefore, the demands for the good brand product and for the average brand product are $D_1(p_1, p_2) = \int_{v_{21}}^{1} dv = 1 - v_{21} = 1 - \frac{p_1 - p_2}{1 - \theta}$ and $D_2(p_1, p_2) = \int_{v_2}^{v_{21}} dv = v_{21} - v_2 = \frac{p_1 - p_2}{1 - \theta} - \frac{p_2}{\theta}$, respectively.

(2) If $v_{21} \geq 1 > v_1 > v_2$, it is equivalent to $\theta \geq 1 - p_1 + p_2$. In this case, no customer will purchase the good brand product, and the customer will only purchase the average brand product if v is in the range $[v_2, 1]$. Therefore, the demands of the good brand product and the average brand product are $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = \int_{v_2}^{1} dv = 1 - v_2 = 1 - \frac{p_2}{\theta}$.

Condition 2: If $v_1 \leq v_2$ or $p_1 \leq \frac{p_2}{\theta}$, suggesting $\frac{p_1 - p_2}{1 - \theta} \leq p_1$, which is equivalent to $v_{21} \leq v_1$, then we have $v_{21} \leq v_1 \leq v_2 < 1$ or $\theta \leq \frac{p_2}{p_1}$. This implies that no customer will purchase the average brand product and the customer will only purchase the good brand product if v is in the range $[v_1, 1]$. Therefore, demands of the good brand product and the average brand product are $D_1(p_1, p_2) = \int_{v_1}^{1} dv = 1 - v_1 = 1 - p_1$ and $D_2(p_1, p_2) = 0$, respectively. This completes the proof.

Proof of Corollary 1:

With demand functions in Proposition 1, we define $\underline{\theta} = \frac{p_2}{p_1}$ and $\overline{\theta} = 1 - p_1 + p_2$. When $0 < \theta \leq \underline{\theta}$, $D_1(p_1, p_2) > 0$ and $D_2(p_1, p_2) = 0$; when $\underline{\theta} < \theta < \overline{\theta}$, $D_1(p_1, p_2) > 0$ and $D_2(p_1, p_2) > 0$; and when $\overline{\theta} \leq \theta < 1$, $D_1(p_1, p_2) > 0$ and $D_2(p_1, p_2) = 0$. This completes the proof.

Proof of Proposition 2:

MNS **model:**

Step one:

Given w_1 and w_2, considering three players game, $\pi_r(p_1, p_2) = (p_1 - w_1)(1 - \frac{p_1 - p_2}{1-\theta}) + (p_2 -$

$w_2)(\frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta})$, we get $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = \frac{-1+\theta+2p_1-2p_2-w_1+w_2}{-1+\theta}$, $\frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = -\frac{2\theta p_1-2p_2-\theta w_1+w_2}{(-1+\theta)\theta}$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} =$

$-\frac{2}{1-\theta} < 0$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} = -\frac{2}{1-\theta} - \frac{2}{\theta}$, and $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} = \frac{2}{1-\theta}$. Then $\begin{vmatrix} \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} \end{vmatrix} =$

$\frac{4}{(1-\theta)\theta} > 0$. Therefore, $\pi_r(p_1, p_2)$ is joint concave in p_1 and p_2. Let $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = \frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = 0$, we

obtain $p_1 = \frac{1+w_1}{2}$ and $p_2 = \frac{\theta+w_2}{2}$. The square $\{(p_1, p_2) | 0 \le p_1 \le 1 \cap 0 \le p_2 \le 1\}$ is divided into three

regions, because of the customer acceptance of the average brand and the retail prices. Namely, $B =$

$\{(p_1, p_2) | \frac{p_2}{p_1} < \theta < 1 - p_1 + p_2 \cap 0 \le p_1 \le 1 \cap 0 \le p_2 \le 1\}$, the demand is positive for both the good

and average brand product; $G = \{(p_1, p_2) | 0 < \theta \le \frac{p_2}{p_1} \cap 0 \le p_1 \le 1 \cap 0 \le p_2 \le 1\}$, the demand is

positive for the good-brand product only; and $A = \{(p_1, p_2) | 1 - p_1 + p_2 \le \theta < 1 \cap 0 \le p_1 \le 1 \cap 0 \le$

$p_2 \le 1\}$, the demand is positive for the average-brand product only. Thus, three cases are taken into

account.

Step two:

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Replacing p_1 and p_2, we can get $\frac{w_2}{w_1} < \theta < 1 - w_1 + w_2$, which makes three players game and positive

demand for both products. Substituting p_1 and p_2 into (3) and (4), we get $\pi_{m1}(w_1) = (w_1 - c_1)\left[1 - \right.$

$\left. \frac{\frac{1}{2}(1+w_1)-\frac{1}{2}(\theta+w_2)}{1-\theta} \right]$ and $\pi_{m2}(w_2) = (w_2 - c_2)\left[\frac{\frac{1}{2}(1+w_1)-\frac{1}{2}(\theta+w_2)}{1-\theta} - \frac{\theta+w_2}{2\theta} \right]$. Then we get $\frac{d\pi_{m1}(w_1)}{dw_1} =$

$\frac{-1+\theta-c_1+2w_1-w_2}{2(-1+\theta)}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{1}{1-\theta} < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in w_1. Similarly, we get

$\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{c_2+\theta w_1-2w_2}{2(1-\theta)\theta}$ and $\frac{d^2\pi_{m2}(w_2)}{dw_2^2} = -\frac{1}{1-\theta} - \frac{1}{\theta} < 0$. Therefore, $\pi_{m2}(w_2)$ is concave in w_2. Let

$\frac{d\pi_{m1}(w_1)}{dw_1} = \frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get $w_1^{MNS} = 1 - \frac{2(1-c_1)+(\theta-c_2)}{4-\theta}$ and $w_2^{MNS} = \theta - \frac{\theta(1-c_1)+2(\theta-c_2)}{4-\theta}$. Replacing

w_1 and w_2 with w_1^{MNS} and w_2^{MNS} into p_1 and p_2, then we have $p_1^{MNS} = 1 - \frac{2(1-c_1)+(\theta-c_2)}{2(4-\theta)}$ and

$p_2^{MNS} = \theta - \frac{\theta(1-c_1)+2(\theta-c_2)}{2(4-\theta)}$. Because of $\underline{\theta}^{MNS} = \frac{p_2^{MNS}}{p_1^{MNS}}$ and $\overline{\theta}^{MNS} = 1 - p_1^{MNS} + p_2^{MNS}$, we can get

$\underline{\theta}^{MNS} = \frac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$ and $\overline{\theta}^{MNS} = 1 - \frac{c_1-c_2}{2-c_1}$.

Case G: $0 < \theta \leq \frac{p_2}{p_1}$.

Replacing p_1 and p_2, we can get $0 < \theta \leq \frac{w_2}{w_1}$. Specially, $p_2 = \theta p_1$ or $w_2 = \theta w_1$ makes three players

game and positive demand for good-brand product only. Substituting p_1 and p_2 into Eq. (3), we can get

$\pi_{m1}(w_1) = (w_1 - c_1)\left[1 - \frac{\frac{1}{2}(1+w_1)-\frac{1}{2}(\theta+w_2)}{1-\theta}\right]$. Then $\frac{d\pi_{m1}(w_1)}{dw_1} = \frac{-1+\theta-c_1+2w_1-w_2}{2(-1+\theta)}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{1}{1-\theta} <$

0. Therefore, $\pi_{m1}(w_1)$ is concave in w_1. Let $\frac{d\pi_{m1}(w_1)}{dw_1} = 0$, we get $w_1 = \frac{1-\theta+c_1+w_2}{2}$. Therefore, combine

$w_1 = \frac{1-\theta+c_1+w_2}{2}$ and $w_2 = \theta w_1$, we have $w_1^{MNS} = 1 - \frac{1-c_1}{2-\theta}$ and $w_2^{MNS} = \left(1 - \frac{1-c_1}{2-\theta}\right)\theta$. Replacing w_1

and w_2 with w_1^{MNS} and w_2^{MNS} into p_1 and p_2, then we have $p_1^{MNS} = 1 - \frac{1-c_1}{2(2-\theta)}$ and $p_2^{MNS} =$

$\left[1 - \frac{1-c_1}{2(2-\theta)}\right]\theta$.

Case A: $1 - p_1 + p_2 \leq \theta < 1$.

Replacing p_1 and p_2, we can get $1 - w_1 + w_2 \leq \theta < 1$. Specially, $1 - p_1 + p_2 = \theta$ or $1 - w_1 + w_2 =$

θ makes three players game and positive demand for average-brand product only. Substituting p_1 and p_2

into Eq. (4), we can get $\pi_{m2}(w_2) = (w_2 - c_2)\left[\frac{\frac{1}{2}(1+w_1)-\frac{1}{2}(\theta+w_2)}{1-\theta} - \frac{\theta+w_2}{2\theta}\right]$. Then $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{c_2+\theta w_1-2w_2}{2(1-\theta)\theta}$

and $\frac{d^2\pi_{m2}(w_2)}{dw_2^2} = -\frac{1}{1-\theta} - \frac{1}{\theta} < 0$. Therefore, $\pi_{m2}(w_2)$ is concave in w_2 Let $\frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get $w_2 =$

$\frac{c_2+\theta w_1}{2}$. Therefore, combine $w_2 = \frac{c_2+\theta w_1}{2}$ and $w_1 = 1 + w_2 - \theta$, we have $w_1^{MNS} = 1 - \frac{\theta-c_2}{2-\theta}$ and

$w_2^{MNS} = \theta - \frac{\theta-c_2}{2-\theta}$. Replacing w_1 and w_2 with w_1^{MNS} and w_2^{MNS} into p_1 and p_2, then we have

$p_1^{MNS} = 1 - \frac{\theta-c_2}{2(2-\theta)}$ and $p_2^{MNS} = \theta - \frac{\theta-c_2}{2(2-\theta)}$.

MGS **model:**

Step one is the same with the proof of *MNS* model.

Step two:

Given w_1, substituting p_1 and p_2, we can get $\frac{w_2}{w_1} < \theta < 1 - w_1 + w_2$, which makes three players game

and positive demand for both products. Substituting p_2 into (4), we get $\pi_{m2}(w_2) = (w_2 -$

$c_2)\left[\frac{\frac{1}{2}(1+w_1)-\frac{1}{2}(\theta+w_2)}{1-\theta} - \frac{\theta+w_2}{2\theta}\right]$. Then $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{c_2+\theta w_1-2w_2}{2(1-\theta)\theta}$ and $\frac{d^2\pi_{m2}(w_2)}{dw_2^2} = -\frac{1}{1-\theta} - \frac{1}{\theta} < 0$. Therefore,

$\pi_{m2}(w_2)$ is concave in w_2. Let $\frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get $w_2 = \frac{c_2 + \theta w_1}{2}$. Based on the customer acceptance of the average brand and the retail prices, three cases are taken into account.

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Substituting w_2, p_1 and p_2 into (3), we get $\pi_{m1}(w_1) = (-c_1 + w_1)\left[1 - \frac{\frac{1}{2}(1+w_1) - \frac{1}{2}\theta + \frac{1}{4}(-c_2 - \theta w_1)}{1-\theta}\right]$. Then

$\frac{d\pi_{m1}(w_1)}{dw_1} = \frac{-2 + 2\theta - 2c_1 + \theta c_1 - c_2 + 4w_1 - 2\theta w_1}{4(-1+\theta)}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{2-\theta}{2(1-\theta)} < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in

w_1. Let $\frac{d\pi_{m1}(w_1)}{dw_1} = 0$, then $w_1^{MGS} = 1 - \frac{(2-\theta)(1-c_1) + (\theta-c_2)}{2(2-\theta)}$. Replacing w_1^{MGS} with w_1 into w_2, p_1, and

p_2, we get $w_2^{MGS} = \theta - \frac{\theta(2-\theta)(1-c_1) + (4-\theta)(\theta-c_2)}{4(2-\theta)}$, $p_1^{MGS} = 1 - \frac{(2-\theta)(1-c_1) + (\theta-c_2)}{4(2-\theta)}$ and $p_2^{MGS} = \theta - $

$\frac{\theta(2-\theta)(1-c_1) + (4-\theta)(\theta-c_2)}{8(2-\theta)}$. Because $\underline{\theta}^{MGS} = \frac{p_2^{MGS}}{p_1^{MGS}}$ and $\overline{\theta}^{MGS} = 1 - p_1^{MGS} + p_2^{MGS}$, we get $\underline{\theta}^{MGS} = $

$\frac{2 + 2c_1 + 3c_2 - \sqrt{(2+2c_1+3c_2)^2 - 16(2+c_1)c_2}}{2(2+c_1)}$ and $\overline{\theta}^{MGS} = 1 - \frac{c_1 - c_2}{2 - c_1}$.

Case G: $0 < \theta \le \frac{p_2}{p_1}$.

Given w_1, replacing p_1 and p_2, we can get $0 < \theta \le \frac{w_2}{w_1}$. Specially, $p_2 = \theta p_1$ or $w_2 = \theta w_1$ makes

three players game and positive demand for good-brand product only. Therefore, combine $w_2 = \frac{c_2 + \theta w_1}{2}$

and $w_2 = \theta w_1$, we have $w_1^{MGS} = \frac{c_2}{\theta}$ and $w_2^{MGS} = c_2$. Replacing w_1 and w_2 with w_1^{MGS} and w_2^{MGS}

into p_1 and p_2, then we have $p_1^{MGS} = 1 - \frac{\theta - c_2}{2\theta}$ and $p_2^{MGS} = \left(1 - \frac{\theta - c_2}{2\theta}\right)\theta$.

Case A: $1 - p_1 + p_2 \le \theta < 1$.

Given w_1, replacing p_1 and p_2, we can get $1 - w_1 + w_2 \le \theta < 1$. Specially, $1 - p_1 + p_2 = \theta$ or $1 - $

$w_1 + w_2 = \theta$ makes three players game and positive demand for average-brand product only. Therefore,

combine $w_2 = \frac{c_2 + \theta w_1}{2}$ and $1 - w_1 + w_2 = \theta$, we have $w_1^{MGS} = 1 - \frac{\theta - c_2}{2 - \theta}$ and $w_2^{MGS} = \theta - \frac{\theta - c_2}{2 - \theta}$.

Replacing w_1 and w_2 with w_1^{MGS} and w_2^{MGS} into p_1 and p_2, then we have $p_1^{MGS} = 1 - \frac{\theta - c_2}{2(2-\theta)}$ and

$p_2^{MGS} = \theta - \frac{\theta - c_2}{2(2-\theta)}$.

MAS **model:**

Step one is the same with the proof of *MNS* model.

Step two:

Given w_2, substituting p_1 and p_2, we can get $\frac{w_2}{w_1} < \theta < 1 - w_1 + w_2$, which makes three players game

and positive demand for both products.

And substituting p_1 into (3), we get $\pi_{m1}(w_1) = (w_1 - c_1)\left[1 - \frac{\frac{1}{2}(1+w_1)-\frac{1}{2}(\theta+w_2)}{1-\theta}\right]$. Then $\frac{d\pi_{m1}(w_1)}{dw_1} =$

$\frac{-1+\theta-c_1+2w_1-w_2}{2(-1+\theta)}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{1}{1-\theta} < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in w_1. Let $\frac{d\pi_{m1}(w_1)}{dw_1} = 0$,

we get $w_1 = \frac{1-\theta+c_1+w_2}{2}$. Based on the customer acceptance of the average brand and the retail prices, three

cases are taken into account.

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Substituting w_1, p_1 and p_2 into (4), we get $\pi_{m2}(w_2) = (-c_2 + w_2)\left[-\frac{\theta+w_2}{2\theta} + \right.$

$\left. \frac{\frac{1}{2}(-\theta-w_2)+\frac{1}{2}+\frac{1}{4}(1-\theta+c_1+w_2)}{1-\theta}\right]$. Then $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{-\theta+\theta^2-\theta c_1-2c_2+\theta c_2+4w_2-2\theta w_2}{4(-1+\theta)\theta}$ and $\frac{d^2\pi_{m2}(w_2)}{dw_2^2} = -\frac{2-\theta}{2(1-\theta)\theta} < 0$.

Therefore, $\pi_{m2}(w_2)$ is concave in w_2. Let $\frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get $w_2^{MAS} = \theta - \frac{\theta(1-c_1)+(2-\theta)(\theta-c_2)}{2(2-\theta)}$.

Replacing w_2 with w_2^{MAS} in w_1, p_1, and p_2, we get $w_1^{MAS} = 1 - \frac{(4-\theta)(1-c_1)+(2-\theta)(\theta-c_2)}{4(2-\theta)}$, $p_1^{MAS} = 1 -$

$\frac{(4-\theta)(1-c_1)+(2-\theta)(\theta-c_2)}{8(2-\theta)}$, and $p_2^{MAS} = \theta - \frac{\theta(1-c_1)+(2-\theta)(\theta-c_2)}{4(2-\theta)}$. Because $\underline{\theta}^{MAS} = \frac{p_2^{MAS}}{p_1^{MAS}}$ and $\overline{\theta}^{MAS} = 1 -$

$p_1^{MAS} + p_2^{MAS}$, we get $\underline{\theta}^{MAS} = \frac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$ and $\overline{\theta}^{MAS} = \frac{5-3c_1+c_2-\sqrt{(3-3c_1+c_2)^2+4(c_1-c_2)}}{2}$.

Case G: $0 < \theta \leq \frac{p_2}{p_1}$.

Given w_2, replacing p_1 and p_2, we can get $0 < \frac{w_2}{w_1} \leq \theta$. Specially, $p_2 = \theta p_1$ or $w_2 = \theta w_1$ makes

three players game and positive demand for good-brand product only.

Therefore, combine $w_1 = \frac{1-\theta+c_1+w_2}{2}$ and $w_2 = \theta w_1$, we have $w_1^{MAS} = 1 - \frac{1-c_1}{2-\theta}$ and $w_2^{MAS} = [1 -$

$\frac{1-c_1}{2-\theta}]\theta$. Replacing w_1 and w_2 with w_1^{MAS} and w_2^{MAS} into p_1 and p_2, then we have $p_1^{MAS} = 1 - \frac{1-c_1}{2(2-\theta)}$

and $p_2^{MAS} = \left[1 - \frac{1-c_1}{2(2-\theta)}\right]\theta$.

Case A: $1 - p_1 + p_2 \leq \theta < 1$.

Given w_2, replacing p_1 and p_2, we can get $1 - w_1 + w_2 \leq \theta < 1$. Specially, $1 - p_1 + p_2 = \theta$ or $1 -$

$w_1 + w_2 = \theta$ makes three players game and positive demand for average-brand product only. Therefore,

combine $w_1 = \frac{1-\theta+c_1+w_2}{2}$ and $w_1 = 1 + w_2 - \theta$, we have $w_1^{MAS} = c_1$ and $w_2^{MAS} = c_1 - 1 + \theta$.

Replacing w_1 and w_2 with w_1^{MAS} and w_2^{MAS} into p_1 and p_2, then we have $p_1^{MAS} = 1 - \frac{1-c_1}{2}$ and

$$p_2^{MAS} = \theta - \frac{1-c_1}{2}.$$

VNN model:

We denote the marginal profits of the good-brand product and the average-brand product as $m_1 = p_1 - w_1$ and $m_2 = p_2 - w_2$, respectively, as in Choi (1991). Then the manufacturers' profit functions in (3) and (4) become

$$\pi_{m1}(w_1) = (w_1 - c_1)\left[1 - \frac{(m_1+w_1)-(m_2+w_2)}{1-\theta}\right] \qquad \text{(P1)}$$

$$\pi_{m2}(w_2) = (w_2 - c_2)\left[\frac{(m_1+w_1)-(m_2+w_2)}{1-\theta} - \frac{m_2+w_2}{\theta}\right] \qquad \text{(P2)}$$

From (P1), we get $\frac{d\pi_{m1}(w_1)}{dw_1} = 1 - \frac{(m_1+w_1)-(m_2+w_2)}{1-\theta} - \frac{w_1-c_1}{1-\theta}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{2}{1-\theta} < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in w_1. From (P2), we get $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{(m_1+w_1)-(m_2+w_2)}{1-\theta} - \frac{m_2+w_2}{\theta} + (-\frac{1}{1-\theta} - \frac{1}{\theta})(w_2 - c_2)$ and $\frac{d^2\pi_{m2}(w_2)}{dw_2^2} = -\frac{2}{1-\theta} - \frac{2}{\theta} < 0$. Therefore, $\pi_{m2}(w_2)$ is concave in w_2.

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Considering three players game, $\pi_r(p_1,p_2) = (p_1 - w_1)(1 - \frac{p_1-p_2}{1-\theta}) + (p_2 - w_2)(\frac{p_1-p_2}{1-\theta} - \frac{p_2}{\theta})$, we get

$\frac{\partial\pi_r(p_1,p_2)}{\partial p_1} = \frac{-1+\theta+2p_1-2p_2-w_1+w_2}{-1+\theta}$, $\frac{\partial\pi_r(p_1,p_2)}{\partial p_2} = -\frac{2\theta p_1-2p_2-\theta w_1+w_2}{(-1+\theta)\theta}$, $\frac{\partial^2\pi_r(p_1,p_2)}{\partial p_1^2} = -\frac{2}{1-\theta} < 0$, $\frac{\partial^2\pi_r(p_1,p_2)}{\partial p_2^2} = -\frac{2}{1-\theta} - \frac{2}{\theta}$, and $\frac{\partial^2\pi_r(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2\pi_r(p_1,p_2)}{\partial p_2 \partial p_1} = \frac{2}{1-\theta}$. Then $\begin{vmatrix} \frac{\partial^2\pi_r(p_1,p_2)}{\partial p_1^2} & \frac{\partial^2\pi_r(p_1,p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2\pi_r(p_1,p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2\pi_r(p_1,p_2)}{\partial p_2^2} \end{vmatrix} = \frac{4}{(1-\theta)\theta} > 0$. Therefore, $\pi_r(p_1,p_2)$ is joint concave in p_1 and p_2. Let $\frac{\partial\pi_r(p_1,p_2)}{\partial p_1} = \frac{\partial\pi_r(p_1,p_2)}{\partial p_2} = \frac{d\pi_{m1}(w_1)}{dw_1} = \frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get

$w_1^{VNN} = 1 - \frac{2[3(1-c_1)+(\theta-c_2)]}{9-\theta}$, $w_2^{VNN} = \theta - \frac{2[\theta(1-c_1)+3(\theta-c_2)]}{9-\theta}$, $p_1^{VNN} = 1 - \frac{3(1-c_1)+(\theta-c_2)}{9-\theta}$, and $p_2^{VNN} = \theta - \frac{\theta(1-c_1)+3(\theta-c_2)}{9-\theta}$. Because $\underline{\theta}^{VNN} = \frac{p_2^{VNN}}{p_1^{VNN}}$ and $\overline{\theta}^{VNN} = 1 - p_1^{VNN} + p_2^{VNN}$, we can get $\underline{\theta}^{VNN} = \frac{1+2c_1+c_2-\sqrt{(1+2c_1+c_2)^2-12c_2}}{2}$ and $\overline{\theta}^{VNN} = 1 - \frac{2c_1-2c_2}{3-c_1}$.

Case G: $0 < \theta \le \frac{p_2}{p_1}$.

Specially, $p_2 = \theta p_1$ makes three players game and positive demand for good-brand product only, hence $\pi_r(p_1,p_2) = \pi_r(p_1) = (p_1 - w_1)(1 - p_1)$, we get $\frac{d\pi_r(p_1)}{dp_1} = 1 - 2p_1 + w_1$, $\frac{d^2\pi_r(p_1)}{dp_1^2} = -2 < 0$. Therefore, $\pi_r(p_1)$ is concave in p_1. Combine $\frac{d\pi_{m1}(w_1)}{dw_1} = \frac{d\pi_{m2}(w_2)}{dw_2} = \frac{d\pi_r(p_1)}{dp_1} = 0$ and $p_2 = \theta p_1$, we get $w_1^{VNN} =$

$1 - \frac{2(1-c_1)}{3-\theta}$, $w_2^{VNN} = c_2$, $p_1^{VNN} = 1 - \frac{1-c_1}{3-\theta}$, and $p_2^{VNN} = \left(1 - \frac{1-c_1}{3-\theta}\right)\theta$.

Case A: $1 - p_1 + p_2 \leq \theta < 1$.

Specially, $1 - p_1 + p_2 = \theta$ makes three players game and positive demand for average-brand product only, hence $\pi_r(p_1, p_2) = \pi_r(p_2) = (p_2 - w_2)(1 - \frac{p_2}{\theta})$, we get $\frac{d\pi_r(p_1)}{dp_1} = 1 - \frac{2p_2}{\theta} + w_2$, $\frac{d^2\pi_r(p_1)}{dp_1^2} = -\frac{2}{\theta} < 0$. Therefore, $\pi_r(p_2)$ is concave in p_2. Combine $\frac{d\pi_{m1}(w_1)}{dw_1} = \frac{d\pi_{m2}(w_2)}{dw_2} = \frac{d\pi_r(p_2)}{dp_2} = 0$ and $1 - p_1 + p_2 = \theta$, we get $w_1^{VNN} = c_1$, $w_2^{VNN} = \theta - \frac{2(\theta - c_2)}{3-\theta}$, $p_1^{VNN} = 1 - \frac{\theta - c_2}{3-\theta}$, and $p_2^{VNN} = \theta - \frac{\theta - c_2}{3-\theta}$.

RNS **model:**

Step one:

Given p_1 and p_2, considering three players game, from (P1), we get $\frac{d\pi_{m1}(w_1)}{dw_1} = 1 - \frac{(m_1 + w_1) - (m_2 + w_2)}{1-\theta} - \frac{w_1 - c_1}{1-\theta}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{2}{1-\theta} < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in w_1. From (P2), we get $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{(m_1 + w_1) - (m_2 + w_2)}{1-\theta} - \frac{m_2 + w_2}{\theta} + (-\frac{1}{1-\theta} - \frac{1}{\theta})(w_2 - c_2)$ and $\frac{d^2\pi_{m2}(,w_2)}{dw_2^2} = -\frac{2}{1-\theta} - \frac{2}{\theta} < 0$. Therefore, $\pi_{m2}(w_2)$ is concave in w_2. Let $\frac{d\pi_{m1}(w_1)}{dw_1} = \frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get $w_1 = 1 - \theta + c_1 - p_1 + p_2$ and $w_2 = c_2 - p_2 + \theta p_1$.

Step two:

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Substituting w_1 and w_2 into (5), we get $\pi_r(p_1, p_2) = (1 - \frac{p_1 - p_2}{1-\theta})(-1 + \theta - c_1 + 2p_1 - p_2) + (-c_2 - \theta p_1 + 2p_2)(\frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta})$. Then $\frac{\partial \pi_r(p_1, p_2)}{\partial p_1} = \frac{-3 + 3\theta - c_1 + c_2 + 4p_1 + 2\theta p_1 - 6p_2}{-1+\theta}$, $\frac{\partial \pi_r(p_1, p_2)}{\partial p_2} = \frac{-2\theta + 2\theta^2 - \theta c_1 + c_2 + 6\theta p_1 - 4p_2 - 2\theta p_2}{(1-\theta)\theta}$, $\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1^2} = -\frac{4}{1-\theta} - \frac{2\theta}{1-\theta} < 0$, $\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2^2} = 4(-\frac{1}{1-\theta} - \frac{1}{\theta}) - \frac{2}{1-\theta}$, and $\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2 \partial p_1} = \frac{5}{1-\theta} - (-\frac{1}{1-\theta} - \frac{1}{\theta})\theta$. Then $\begin{vmatrix} \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1^2} & \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2^2} \end{vmatrix} = \frac{4(4-\theta)}{(1-\theta)\theta} > 0$. Therefore, $\pi_r(p_1, p_2)$ is joint concave in p_1 and p_2. Let $\frac{\partial \pi_r(p_1, p_2)}{\partial p_1} = \frac{\partial \pi_r(p_1, p_2)}{\partial p_2} = 0$, we get $p_1^{RNS} = 1 - \frac{2(1-c_1) + (\theta - c_2)}{2(4-\theta)}$ and $p_2^{RNS} = \theta - \frac{\theta(1-c_1) + 2(\theta - c_2)}{2(4-\theta)}$. Replacing p_1 and p_2 with p_1^{RNS} and p_2^{RNS} in w_1 and w_2, we get $w_1^{RNS} = 1 - \frac{2(1-c_1) + (\theta - c_2)}{2(4-\theta)}$ and $w_2^{RNS} = \theta - \frac{\theta(1-c_1) + 2(\theta - c_2)}{2(4-\theta)}$. Because $\underline{\theta}^{RNS} = \frac{p_2^{RNS}}{p_1^{RNS}}$ and

$\overline{\theta}^{RNS} = 1 - p_1^{RNS} + p_2^{RNS}$, we get $\underline{\theta}^{RNS} = \frac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$ and $\overline{\theta}^{RNS} = 1 - \frac{c_1-c_2}{2-c_1}$.

Case G: $0 < \theta \leq \frac{p_2}{p_1}$.

Substituting w_1 and w_2 into (5), and $p_2 = \theta p_1$ makes three players game and positive demand for good-brand product only, we get $\pi_r(p_1, p_2) = \pi_r(p_1) = (1 - p_1)(-1 + \theta - c_1 + 2p_1 - \theta p_1)$. Then $\frac{d\pi_r(p_1)}{dp_1} = 3 - 2\theta + c_1 - 4p_1 + 2\theta p_1$ and $\frac{d^2\pi_r(p_1)}{dp_1^2} = -4 + 2\theta < 0$. Therefore, $\pi_r(p_1)$ is concave in p_1.

Let $\frac{d\pi_r(p_1)}{dp_1} = 0$, we get $p_1^{RNS} = 1 - \frac{1-c_1}{2(2-\theta)}$. Replacing p_1 with p_1^{RNS} in p_2, w_1 and w_2, we get

$p_2^{RNS} = \left[1 - \frac{1-c_1}{2(2-\theta)}\right]\theta$, $w_1^{RNS} = \frac{1-\theta+3c_1-\theta c_1}{2(2-\theta)}$ and $w_2^{RNS} = c_2$.

Case A: $1 - p_1 + p_2 \leq \theta < 1$.

Substituting w_1 and w_2 into (5), and $p_1 = 1 + p_2 - \theta$ makes three players game and positive demand for average-brand product only, we get $\pi_r(p_1, p_2) = \pi_r(p_2) = \frac{(\theta-p_2)(-\theta+\theta^2-c_2+2p_2-\theta p_2)}{\theta}$. Then $\frac{d\pi_r(p_2)}{dp_2} = -\frac{-3\theta+2\theta^2-c_2+4p_2-2\theta p_2}{\theta}$ and $\frac{d^2\pi_r(p_2)}{dp_1^2} = -\frac{2(2-\theta)}{\theta} < 0$. Therefore, $\pi_r(p_2)$ is concave in p_2. Let $\frac{d\pi_r(p_2)}{dp_2} = 0$, we get $p_2^{RNS} = \theta - \frac{\theta-c_2}{2(2-\theta)}$. Replacing p_2 with p_2^{RNS} in p_1, w_1 and w_2, we get $p_1^{RNS} = 1 - \frac{\theta-c_2}{2(2-\theta)}$, $w_1^{RNS} = c_1$ and $w_2^{RNS} = \frac{\theta-\theta^2+3c_2-\theta c_2}{2(2-\theta)}$.

RGS **model**

Step one:

Given p_1 and p_2, considering three players game, from (P2), we get $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{(m_1+w_1)-(m_2+w_2)}{1-\theta} - \frac{m_2+w_2}{\theta} + (-\frac{1}{1-\theta} - \frac{1}{\theta})(w_2 - c_2)$ and $\frac{d^2\pi_{m2}(,w_2)}{dw_2^2} = -\frac{2}{1-\theta} - \frac{2}{\theta} < 0$. Therefore, $\pi_{m2}(w_2)$ is concave in w_2.

Let $\frac{d\pi_{m2}(w_2)}{dw_2} = 0$, we get $w_2 = c_2 - p_2 + \theta p_1$. Substituting w_2 into (P1) and letting $p_i = w_i + m_i$, we get $\pi_{m1}(w_1) = (w_1 - c_1)\left[1 - \frac{(m_1+w_1)-(c_2-w_2+\theta m_1+\theta w_1)}{1-\theta}\right]$. Then $\frac{d\pi_{m1}(w_1)}{dw_1} = 1 - \frac{(m_1+w_1)-(c_2-w_2+\theta m_1+\theta w_1)}{1-\theta} - w_1 + c_1$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -2 < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in w_1.

Let $\frac{d\pi_{m1}(w_1)}{dw_1} = 0$ and replace $m_i + w_i$ with p_i, we get $w_1 = 1 + c_1 - \frac{p_1-p_2}{1-\theta}$.

Step two:

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Substituting w_1 and w_2 into (5), we get $\pi_r(p_1, p_2) = \left[p_1 - \left(1 + c_1 - \frac{p_1 - p_2}{1-\theta}\right)\right]\left(1 - \frac{p_1 - p_2}{1-\theta}\right) + [p_2 - $

$(c_2 - p_2 + \theta p_1)]\left(\frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}\right)$. Then $\frac{\partial \pi_r(p_1, p_2)}{\partial p_1} = \frac{3 - 4\theta + \theta^2 + c_1 - \theta c_1 - c_2 + \theta c_2 - 4p_1 + 2\theta^2 p_1 + 6p_2 - 4\theta p_2}{(1-\theta)^2}$, $\frac{\partial \pi_r(p_1, p_2)}{\partial p_2} = $

$\frac{-2\theta + 2\theta^2 - \theta c_1 + \theta^2 c_1 + c_2 - \theta c_2 + 6\theta p_1 - 4\theta^2 p_1 - 4p_2 + 2\theta p_2}{(1-\theta)^2 \theta}$, $\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1^2} = -\frac{2(2-\theta^2)}{(1-\theta)^2} < 0$, $\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2^2} = -\frac{2(2-\theta)}{(1-\theta)^2 \theta}$, and

$\frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2 \partial p_1} = \frac{2(3-2\theta)}{(1-\theta)^2}$. Then $\begin{vmatrix} \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1^2} & \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2^2} \end{vmatrix} = \frac{4(4-3\theta)}{(1-\theta)^2 \theta} > 0$. Therefore, $\pi_r(p_1, p_2)$ is

joint concave in p_1 and p_2. Let $\frac{\partial \pi_r(p_1, p_2)}{\partial p_1} = \frac{\partial \pi_r(p_1, p_2)}{\partial p_2} = 0$, we get $p_1^{RGS} = 1 - \frac{2(1-\theta)(1-c_1) + (\theta - c_2)}{2(4-3\theta)}$ and

$p_2^{RGS} = \theta - \frac{\theta(1-\theta)(1-c_1) + (2-\theta)(\theta - c_2)}{2(4-3\theta)}$. Replacing p_1 and p_2 with p_1^{RGS} and p_2^{RGS} in w_1 and w_2, we get

$w_1^{RGS} = 1 - \frac{(6-5\theta)(1-c_1) + (\theta - c_2)}{2(4-3\theta)}$ and $w_2^{RGS} = \theta - \frac{\theta(1-\theta)(1-c_1) + 2(3-2\theta)(\theta - c_2)}{2(4-3\theta)}$. Because $\underline{\theta}^{RGS} = \frac{p_2^{RGS}}{p_1^{RGS}}$ and

$\overline{\theta}^{RGS} = 1 - p_1^{RGS} + p_2^{RGS}$, we can get $\underline{\theta}^{RGS} = \frac{2c_2}{1+c_1}$ and $\overline{\theta}^{RGS} = 1 - \frac{c_1 - c_2}{2 - c_1}$.

Case G: $0 < \theta \le \frac{p_2}{p_1}$.

Substituting w_1 and w_2 into (5), and $p_2 = \theta p_1$ makes three players game and positive demand for

good-brand product only, we get $\pi_r(p_1, p_2) = \pi_r(p_1) = (2p_1 - 1 - c_1)(p_1 - 1)$. Then $\frac{d\pi_r(p_1)}{dp_1} = 3 + $

$c_1 - 4p_1$ and $\frac{d^2 \pi_r(p_1)}{dp_1^2} = -4 < 0$. Therefore, $\pi_r(p_1)$ is concave in p_1. Let $\frac{d\pi_r(p_1)}{dp_1} = 0$, we get $p_1^{RGS1} = $

$1 - \frac{1-c_1}{4}$. Replacing p_1 with p_1^{RGS} in p_2, w_1 and w_2, we get $p_2^{RGS} = \left(1 - \frac{1-c_1}{4}\right)\theta$, $w_1^{RGS} = 1 - \frac{3(1-c_1)}{4}$

and $w_2^{RGS} = c_2$.

Case A: $1 - p_1 + p_2 \le \theta < 1$.

Substituting w_1 and w_2 into (5), and $p_1 = 1 + p_2 - \theta$ makes three players game and positive demand

for average-brand product only, we get $\pi_r(p_1, p_2) = \pi_r(p_2) = \frac{(\theta - p_2)(-\theta + \theta^2 - c_2 + 2p_2 - \theta p_2)}{\theta}$. Then $\frac{d\pi_r(p_2)}{dp_2} = $

$-\frac{-3\theta + 2\theta^2 - c_2 + 4p_2 - 2\theta p_2}{\theta}$ and $\frac{d^2 \pi_r(p_2)}{dp_1^2} = -\frac{4-2\theta}{\theta} < 0$. Therefore, $\pi_r(p_2)$ is concave in p_2. Let $\frac{d\pi_r(p_2)}{dp_2} = 0$,

we get $p_2^{RGS2} = \theta - \frac{\theta - c_2}{2(2-\theta)}$. Replacing p_2 with p_2^{RGS} in p_1, w_1 and w_2, we get $p_1^{RGS} = 1 - \frac{\theta - c_2}{2(2-\theta)}$,

$w_1^{RGS} = c_1$ and $w_2^{RGS} = \frac{\theta - \theta^2 + 3c_2 - \theta c_2}{2(2-\theta)}$.

***RAS* model:**

Step one:

Given p_1 and p_2, considering three players game, from (P1), we get $\frac{d\pi_{m1}(w_1)}{dw_1} = 1 - \frac{(m_1+w_1)-(m_2+w_2)}{1-\theta} -$

$\frac{w_1-c_1}{1-\theta}$ and $\frac{d^2\pi_{m1}(w_1)}{dw_1^2} = -\frac{2}{1-\theta} < 0$. Therefore, $\pi_{m1}(w_1)$ is concave in w_1. Let $\frac{d\pi_{m1}(w_1)}{dw_1} = 0$ and replace

$m_i + w_i$ with p_i, we get $w_1 = 1 - \theta + c_1 - p_1 + p_2$.

Substituting w_1 into (P2) and let $p_i = w_i + m_i$, we get $\pi_{m2}(w_2) = (w_2 - $

$c_2)\left[\frac{(1-\theta+c_1+m_2-w_1+w_2)-(m_2+w_2)}{1-\theta} - \frac{m_2+w_2}{\theta}\right]$. Then $\frac{d\pi_{m2}(w_2)}{dw_2} = \frac{(1-\theta+c_1+m_2-w_1+w_2)-(m_2+w_2)}{1-\theta} - \frac{m_2+w_2}{\theta} - $

$\frac{w_2-c_2}{\theta}$ and $\frac{d^2\pi_{m2}(w_2)}{dw_2^2} = -\frac{2}{\theta} < 0$. Therefore, $\pi_{m2}(w_2)$ is concave in w_2. Let $\frac{d\pi_{m2}(w_2)}{dw_2} = 0$ and replace

$m_i + w_i$ with p_i, we get $w_2 = c_2 + \frac{\theta p_1 - p_2}{1-\theta}$.

Step two:

Case B: $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$.

Substituting w_1 and w_2 into (5), we get $\pi_r(p_1, p_2) = [p_1 - (1 - \theta + c_1 - p_1 + p_2)]\left(1 - \frac{p_1-p_2}{1-\theta}\right) +$

$\left[p_2 - \left(c_2 + \frac{\theta p_1 - p_2}{1-\theta}\right)\right]\left(\frac{p_1-p_2}{1-\theta} - \frac{p_2}{\theta}\right)$. Then

$\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = \frac{3-6\theta+3\theta^2+c_1-\theta c_1-c_2+\theta c_2-4p_1+2\theta p_1+6p_2-4\theta p_2}{(1-\theta)^2}$, $\frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = $

$-\frac{2\theta-4\theta^2+2\theta^3+\theta c_1-\theta^2 c_1-c_2+\theta c_2-6\theta p_1+4\theta^2 p_1+4p_2-2\theta^2 p_2}{(1-\theta)^2\theta}$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} = -\frac{2(2-\theta)}{(1-\theta)^2} < 0$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} = -\frac{2(2-\theta^2)}{(1-\theta)^2\theta}$,

and $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} = \frac{2(3-2\theta)}{(1-\theta)^2}$. Then $\begin{vmatrix} \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} \end{vmatrix} = \frac{4(4-3\theta)}{(1-\theta)^2\theta} > 0$. Therefore,

$\pi_r(p_1, p_2)$ is joint concave in p_1 and p_2. Let $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = 0$ and $\frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = 0$, we get $p_1^{RAS} = 1 -$

$\frac{(2-\theta)(1-c_1)+(1-\theta)(\theta-c_2)}{2(4-3\theta)}$ and $p_2^{RAS} = \theta - \frac{\theta(1-c_1)+2(1-\theta)(\theta-c_2)}{2(4-3\theta)}$. Replacing p_1 and p_2 with p_1^{RAS} and p_2^{RAS}

in w_1 and w_2, we get $w_1^{RAS} = 1 - \frac{2(3-2\theta)(1-c_1)+(1-\theta)(\theta-c_2)}{2(4-3\theta)}$ and $w_2^{RAS} = \theta - \frac{\theta(1-c_1)+(6-5\theta)(\theta-c_2)}{2(4-3\theta)}$.

Because $\underline{\theta}^{RAS} = \frac{p_2^{RAS}}{p_1^{RAS}}$ and $\overline{\theta}^{RAS} = 1 - p_1^{RAS} + p_2^{RAS}$, we get $\underline{\theta}^{RAS} = \frac{1+c_1+c_2-\sqrt{(1+c_1+c_2)^2-8c_2}}{2}$ and

$\overline{\theta}^{RAS} = \min[1, 2 - 2c_1 + c_2]$.

Case G: $0 < \theta \le \frac{p_2}{p_1}$.

Substituting w_1 and w_2 into (5), and $p_2 = \theta p_1$ makes three players game and positive demand for

good-brand product only, we get $\pi_r(p_1, p_2) = \pi_r(p_1) = \frac{(-1+p_1)(1-\theta+c_1-2p_1+\theta p_1)}{2(-2+\theta)(-1+\theta)\theta}$. Then $\frac{d\pi_r(p_1)}{dp_1} = $

$\frac{3-2\theta+c_1-4p_1+2\theta p_1}{2(-2+\theta)(-1+\theta)\theta}$ and $\frac{d^2\pi_r(p_1)}{dp_1^2} = \frac{1}{(-1+\theta)\theta} < 0$. Therefore, $\pi_r(p_1)$ is concave in p_1. Let $\frac{d\pi_r(p_1)}{dp_1} = 0$, we

get $p_1^{RAS} = 1 - \frac{1-c_1}{2(2-\theta)}$. Replacing p_1 with p_1^{RAS} in p_2, w_1 and w_2, we get $p_2^{RAS} = \left[1 - \frac{1-c_1}{2(2-\theta)}\right]\theta$,

$w_1^{RAS} = \frac{1-\theta+3c_1-\theta c_1}{2(2-\theta)}$ and $w_2^{RAS} = c_2$.

Case A: $1 - p_1 + p_2 \leq \theta < 1$.

Substituting w_1 and w_2 into (5), and $p_1 = 1 + p_2 - \theta$ makes three players game and positive demand

for average-brand product only, we get $\pi_r(p_1, p_2) = \pi_r(p_2) = -\frac{(\theta+c_2-2p_2)(\theta-p_2)}{2(-2+\theta)(-1+\theta)\theta^2}$. Then $\frac{d\pi_r(p_2)}{dp_2} =$

$\frac{3\theta+c_2-4p_2}{2(-2+\theta)(-1+\theta)\theta^2}$ and $\frac{d^2\pi_r(p_2)}{dp_1^2} = -\frac{2}{(2-\theta)(1-\theta)\theta^2} < 0$. Therefore, $\pi_r(p_2)$ is concave in p_2. Let $\frac{d\pi_r(p_2)}{dp_2} = 0$,

we get $p_2^{RAS} = \theta - \frac{\theta-c_2}{4}$. Replacing p_2 with p_2^{RAS} in p_1, w_1 and w_2, we get $p_1^{RAS} = 1 - \frac{1-c_2}{4}$, $w_1^{RAS} =$

c_1 and $w_2^{RAS} = \theta - \frac{3(\theta-c_2)}{4}$. This completes the proof.

Proof of Proposition 3:

From Proposition 2, we get $w_1^{MNS} - w_2^{MNS} = \frac{2-3\theta+\theta^2+2c_1-\theta c_1-c_2}{4-\theta}$ and $p_1^{MNS} - p_2^{MNS} =$

$\frac{6-8\theta+2\theta^2+2c_1-\theta c_1-c_2}{2(4-\theta)}$. For $\underline{\theta}^{MNS} < \theta < \overline{\theta}^{MNS}$ and $1 > c_1 > c_2$, we get $2 - 3\theta + \theta^2 + 2c_1 - \theta c_1 - c_2 >$

0 and $6 - 8\theta + 2\theta^2 + 2c_1 - \theta c_1 - c_2 > 0$. Therefore, $\frac{2-3\theta+\theta^2+2c_1-\theta c_1-c_2}{4-\theta} > 0$ and

$\frac{6-8\theta+2\theta^2+2c_1-\theta c_1-c_2}{2(4-\theta)} > 0$. That is, $w_1^{MNS} > w_2^{MNS}$ and $p_1^{MNS} > p_2^{MNS}$.

From Proposition 2, we get $w_1^{MGS} - w_2^{MGS} = \frac{2-2\theta+2c_1-\theta c_1-c_2}{4}$ and $p_1^{MGS} - p_2^{MGS} = \frac{6-6\theta+2c_1-\theta c_1-c_2}{8}$. For

$\underline{\theta}^{MGS} < \theta < \overline{\theta}^{MGS}$ and $1 > c_1 > c_2$, we get $2 - 2\theta + 2c_1 - \theta c_1 - c_2 > 0$ and $6 - 6\theta + 2c_1 - \theta c_1 -$

$c_2 > 0$. Therefore, $\frac{2-2\theta+2c_1-\theta c_1-c_2}{4} > 0$ and $\frac{6-6\theta+2c_1-\theta c_1-c_2}{8} > 0$. That is, $w_1^{MGS} > w_2^{MGS}$ and $p_1^{MGS} >$

p_2^{MGS}.

From Proposition 2, we get $w_1^{MAS} - w_2^{MAS} = \frac{4-7\theta+3\theta^2+4c_1-3\theta c_1-2c_2+\theta c_2}{4(2-\theta)}$ and $p_1^{MAS} - p_2^{MAS} =$

$\frac{12-19\theta+7\theta^2+4c_1-3\theta c_1-2c_2+\theta c_2}{8(2-\theta)}$. For $\underline{\theta}^{MAS} < \theta < \overline{\theta}^{MAS}$ and $1 > c_1 > c_2$, we get $4 - 7\theta + 3\theta^2 + 4c_1 -$

$3\theta c_1 - 2c_2 + \theta c_2 > 0$ and $12 - 19\theta + 7\theta^2 + 4c_1 - 3\theta c_1 - 2c_2 + \theta c_2 > 0$. Therefore,

$\frac{4-7\theta+3\theta^2+4c_1-3\theta c_1-2c_2+\theta c_2}{4(2-\theta)} > 0$ and $\frac{12-19\theta+7\theta^2+4c_1-3\theta c_1-2c_2+\theta c_2}{8(2-\theta)} > 0$. That is, $w_1^{MAS} > w_2^{MAS}$ and

$p_1^{MAS} > p_2^{MAS}$.

From Proposition 2, we get $w_1^{VNN} - w_2^{VNN} = \frac{3-4\theta+\theta^2+6c_1-2\theta c_1-4c_2}{9-\theta}$ and $p_1^{VNN} - p_2^{VNN} =$

$\frac{6-7\theta+\theta^2+3c_1-\theta c_1-2c_2}{9-\theta}$. Because $\underline{\theta}^{VNN} < \theta < \overline{\theta}^{VNN}$ and $1 > c_1 > c_2$, we get $3 - 4\theta + \theta^2 + 6c_1 - 2\theta c_1 -$

$4c_2 > 0$ and $6 - 7\theta + \theta^2 + 3c_1 - \theta c_1 - 2c_2 > 0$. Therefore, $\frac{3-4\theta+\theta^2+6c_1-2\theta c_1-4c_2}{9-\theta} > 0$ and

$\frac{6-7\theta+\theta^2+3c_1-\theta c_1-2c_2}{9-\theta} > 0$. That is, $w_1^{VNN} > w_2^{VNN}$ and $p_1^{VNN} > p_2^{VNN}$.

From Proposition 2, we get $w_1^{RNS} - w_2^{RNS} = \frac{2-3\theta+\theta^2+6c_1-2\theta c_1-5c_2+\theta c_2}{2(4-\theta)}$ and $p_1^{RNS} - p_2^{RNS} =$

$\frac{6-8\theta+2\theta^2+2c_1-\theta c_1-c_2}{2(4-\theta)}$. For $\underline{\theta}^{RNS} < \theta < \overline{\theta}^{RNS}$ and $1 > c_1 > c_2$, we get $2 - 3\theta + \theta^2 + 6c_1 - 2\theta c_1 -$

$5c_2 + \theta c_2 > 0$ and $6 - 8\theta + 2\theta^2 + 2c_1 - \theta c_1 - c_2 > 0$. Therefore, $\frac{2-3\theta+\theta^2+6c_1-2\theta c_1-5c_2+\theta c_2}{2(4-\theta)} > 0$ and

$\frac{6-8\theta+2\theta^2+2c_1-\theta c_1-c_2}{2(4-\theta)} > 0$. That is, $w_1^{RNS} > w_2^{RNS}$ and $p_1^{RNS} > p_2^{RNS}$.

From Proposition 2, we get $w_1^{RGS} - w_2^{RGS} = \frac{2-3\theta+\theta^2+6c_1-6\theta c_1+\theta^2 c_1-5c_2+4\theta c_2}{2(4-3\theta)}$ and $p_1^{RGS} - p_2^{RGS} =$

$\frac{(1-\theta)(6-4\theta+2c_1-\theta c_1-c_2)}{2(4-3\theta)}$. For $\underline{\theta}^{RGS} < \theta < \overline{\theta}^{RGS}$ and $1 > c_1 > c_2$, we get $2 - 3\theta + \theta^2 + 6c_1 - 6\theta c_1 +$

$\theta^2 c_1 - 5c_2 + 4\theta c_2 > 0$ and $6 - 4\theta + 2c_1 - \theta c_1 - c_2 > 0$. Therefore, $\frac{2-3\theta+\theta^2+6c_1-6\theta c_1+\theta^2 c_1-5c_2+4\theta c_2}{2(4-3\theta)} >$

0 and $\frac{(1-\theta)(6-4\theta+2c_1-\theta c_1-c_2)}{2(4-3\theta)} > 0$. That is, $w_1^{RGS} > w_2^{RGS}$ and $p_1^{RGS} > p_2^{RGS}$.

From Proposition 2, we get $w_1^{RAS} - w_2^{RAS} = \frac{2-4\theta+2\theta^2+6c_1-5\theta c_1-5c_2+4\theta c_2}{2(4-3\theta)}$ and $p_1^{RAS} - p_2^{RAS} =$

$\frac{(1-\theta)(6-5\theta+2c_1-c_2)}{2(4-3\theta)}$. For $\underline{\theta}^{RAS} < \theta < \overline{\theta}^{RAS}$ and $1 > c_1 > c_2$, we get $2 - 4\theta + 2\theta^2 + 6c_1 - 5\theta c_1 - 5c_2 +$

$4\theta c_2 > 0$ and $6 - 5\theta + 2c_1 - c_2 > 0$. Therefore, $\frac{2-4\theta+2\theta^2+6c_1-5\theta c_1-5c_2+4\theta c_2}{2(4-3\theta)} > 0$ and

$\frac{(1-\theta)(6-5\theta+2c_1-c_2)}{2(4-3\theta)} > 0$. That is, $w_1^{RAS} > w_2^{RAS}$ and $p_1^{RAS} > p_2^{RAS}$. This completes the proof.

Proof of Proposition 4:

(a) $w_1^{MNS} - w_1^{MGS} = -\frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)}{2(-4+\theta)(-2+\theta)}$, $w_1^{MNS} - w_1^{MAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{4(-4+\theta)(-2+\theta)}$, $w_2^{MNS} - w_2^{MGS} =$

$-\frac{\theta^2(2-2\theta-2c_1+\theta c_1+c_2)}{4(-4+\theta)(-2+\theta)}$ and $w_2^{MNS} - w_2^{MAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{2(-4+\theta)(-2+\theta)}$. For any $\theta \in [\underline{\theta}^{MNS}, \overline{\theta}^{MNS}]$ and $0 <$

$c_2 < c_1 < 1$, we obtain $2 - 2\theta - 2c_1 + \theta c_1 + c_2 > 0$ and $-\theta + \theta^2 - \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore,

$w_1^{MNS} < w_1^{MGS}$, $w_1^{MNS} < w_1^{MAS}$, $w_2^{MNS} < w_2^{MGS}$ and $w_2^{MNS} < w_2^{MAS}$.

(b) $p_1^{MNS} - p_1^{MGS} = -\frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)}{4(-4+\theta)(-2+\theta)}$, $p_1^{MNS} - p_1^{MAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{8(-4+\theta)(-2+\theta)}$ $p_2^{MNS} - p_2^{MGS} =$

$-\frac{\theta^2(2-2\theta-2c_1+\theta c_1+c_2)}{8(-4+\theta)(-2+\theta)}$ and $p_2^{MNS} - p_2^{MAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{4(-4+\theta)(-2+\theta)}$. Because $2 - 2\theta - 2c_1 + \theta c_1 + c_2 > 0$

and $-\theta + \theta^2 - \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore, $p_1^{MNS} < p_1^{MGS}$, $p_1^{MNS} < p_1^{MAS}$, $p_2^{MNS} < p_2^{MGS}$ and $p_2^{MNS} < p_2^{MAS}$.

(c) (i) $D^{MNS} - D^{MGS} = \frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)}{8(-4+\theta)(-2+\theta)}$ and $D^{MNS} - D^{MAS} = -\frac{-\theta+\theta^2-\theta c_1+2c_2-\theta c_2}{4(-4+\theta)(-2+\theta)}$. Because $2 - 2\theta - 2c_1 + \theta c_1 + c_2 > 0$ and $-\theta + \theta^2 - \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore, $D^{MNS} > D^{MGS}$ and $D^{MNS} > D^{MAS}$.

(ii) $\alpha_1^{MNS} - \alpha_1^{MGS} = \frac{\theta^2(\theta-c_2)(2-2\theta-2c_1+\theta c_1+c_2)}{(-1+\theta)(-3\theta+\theta c_1+2c_2)(6\theta-2\theta^2-2\theta c_1+\theta^2 c_1-4c_2+\theta c_2)}$ and $\alpha_1^{MAS} - \alpha_1^{MNS} = -\frac{\theta^2(-1+c_1)(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{2(-1+\theta)(-3\theta+\theta c_1+2c_2)(-3\theta+\theta^2+\theta c_1+2c_2-\theta c_2)}$. For any $\theta \in [\underline{\theta}^{MNS}, \overline{\theta}^{MNS}]$ and $0 < c_2 < c_1 < 1$, we obtain $-3\theta + \theta c_1 + 2c_2 < 0$, $6\theta - 2\theta^2 - 2\theta c_1 + \theta^2 c_1 - 4c_2 + \theta c_2 > 0$ and $-3\theta + \theta^2 + \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore, $\alpha_1^{MGS} < \alpha_1^{MNS} < \alpha_1^{MAS}$. Because $\alpha_2 = 1 - \alpha_1$, $\alpha_2^{MGS} > \alpha_2^{MNS} > \alpha_2^{MAS}$.

(d) (i) $\pi_{m1}^{MGS} - \pi_{m1}^{MNS} = \frac{\theta^2(2-2\theta-2c_1+\theta c_1+c_2)^2}{16(-4+\theta)^2(-2+\theta)(-1+\theta)} > 0$ and $\pi_{m2}^{MAS} - \pi_{m2}^{MNS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)^2}{16(-4+\theta)^2(-2+\theta)(-1+\theta)} > 0$.

(ii) $\pi_r^{MGS} - \pi_r^{MAS} = -\frac{(-8+3\theta)(-\theta+\theta^2+2\theta c_1-\theta c_1^2-2\theta c_2+c_2^2)}{64(-2+\theta)^2}$ and $\pi_s^{MGS} - \pi_s^{MAS} = -\frac{(-8+5\theta)(-\theta+\theta^2+2\theta c_1-\theta c_1^2-2\theta c_2+c_2^2)}{64(-2+\theta)^2}$. Here $\theta_1 = \frac{1-2c_1+c_1^2+2c_2+(1-c_1)\sqrt{1-2c_1+c_1^2+4c_2}}{2}$ where $\theta_1 \in [\underline{\theta}^{MNS}, \overline{\theta}^{MNS}]$, which makes $\pi_r^{MGS} - \pi_r^{MAS} = 0$ and $\pi_s^{MGS} - \pi_s^{MAS} = 0$. That is, when $\theta \in (\underline{\theta}^{MNS}, \theta_1]$, $\pi_r^{MGS} \leq \pi_r^{MAS}$ and $\pi_s^{MGS} \leq \pi_s^{MAS}$; when $\theta \in (\theta_1, \overline{\theta}^{MNS})$, $\pi_r^{MGS} > \pi_r^{MAS}$ and $\pi_s^{MGS} > \pi_s^{MAS}$. This completes the proof.

Proof of Proposition 5:

(a) $w_1^{RNS} - w_1^{RGS} = -\frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)}{(-4+\theta)(-4+3\theta)}$, $w_1^{RNS} - w_1^{RAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{2(-4+\theta)(-4+3\theta)}$, $w_2^{RNS} - w_2^{RGS} = -\frac{\theta^2(2-2\theta-2c_1+\theta c_1+c_2)}{2(-4+\theta)(-4+3\theta)}$ and $w_2^{RNS} - w_2^{RAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{(-4+\theta)(-4+3\theta)}$. For any $\theta \in [\underline{\theta}^{RNS}, \overline{\theta}^{RNS}]$ and $0 < c_2 < c_1 < 1$, we obtain $2 - 2\theta - 2c_1 + \theta c_1 + c_2 > 0$ and $-\theta + \theta^2 - \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore, $w_1^{RNS} < w_1^{RGS}$, $w_1^{RNS} < w_1^{RAS}$, $w_2^{RNS} < w_2^{RGS}$ and $w_2^{RNS} < w_2^{RAS}$.

(b) $p_1^{RNS} - p_1^{RGS} = -\frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)}{(-4+\theta)(-4+3\theta)}$, $p_1^{RNS} - p_1^{RAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{2(-4+\theta)(-4+3\theta)}$, $p_2^{RNS} - p_2^{RGS} = -\frac{\theta^2(2-2\theta-2c_1+\theta c_1+c_2)}{2(-4+\theta)(-4+3\theta)}$ and $p_2^{RNS} - p_2^{RAS} = \frac{\theta(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{(-4+\theta)(-4+3\theta)}$. Because $2 - 2\theta - 2c_1 + \theta c_1 + c_2 > 0$ and $-\theta + \theta^2 - \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore, $p_1^{RNS} < p_1^{RGS}$, $p_1^{RNS} < p_1^{RAS}$, $p_2^{RNS} < p_2^{RGS}$ and $p_2^{RNS} < p_2^{RAS}$.

(c) (i) $D^{RNS} - D^{RGS} = \frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)}{2(-4+\theta)(-4+3\theta)}$ and $D^{RNS} - D^{RAS} = -\frac{-\theta+\theta^2-\theta c_1+2c_2-\theta c_2}{4(-4+\theta)(-4+3\theta)}$. Because $2-2\theta-$

$2c_1 + \theta c_1 + c_2 > 0$ and $-\theta + \theta^2 - \theta c_1 + 2c_2 - \theta c_2 < 0$. Therefore, $D^{RNS} > D^{RGS}$ and $D^{RNS} > D^{RAS}$.

(ii) $\alpha_1^{RNS} - \alpha_1^{RGS} = \frac{\theta^2(\theta-c_2)(2-2\theta-2c_1+\theta c_1+c_2)}{(-1+\theta)(-3\theta+\theta c_1+2c_2)(3\theta-2\theta^2-\theta c_1+\theta^2 c_1-2c_2+\theta c_2)}$ and $\alpha_1^{RAS} - \alpha_1^{RNS} =$

$-\frac{\theta^2(-1+c_1)(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)}{2(-1+\theta)(-3\theta+\theta c_1+2c_2)(-3\theta+\theta^2+\theta c_1+2c_2-\theta c_2)}$. For any $\theta \in [\underline{\theta}^{RNS}, \overline{\theta}^{RNS}]$ and $0 < c_2 < c_1 < 1$, we obtain

$-3\theta + \theta c_1 + 2c_2 < 0, 3\theta - 2\theta^2 - \theta c_1 + \theta^2 c_1 - 2c_2 + \theta c_2 > 0$ and $-3\theta + \theta^2 + \theta c_1 + 2c_2 - \theta c_2 < 0$.

Therefore, $\alpha_1^{RGS} < \alpha_1^{RNS} < \alpha_1^{RAS}$. Because $\alpha_2 = 1 - \alpha_1$, $\alpha_2^{RGS} > \alpha_2^{RNS} > \alpha_2^{RAS}$.

(d) $\pi_{m1}^{RNS} - \pi_{m1}^{RGS} = -\frac{\theta^3(2-2\theta-2c_1+\theta c_1+c_2)^2}{4(-4+\theta)^2(-1+\theta)(-4+3\theta)^2} > 0$, $\pi_{m1}^{RAS} - \pi_{m1}^{RGS} = -\frac{\theta(-\theta+\theta^2+\theta c_1^2+2c_2-2\theta c_2-2c_1 c_2+c_2^2)}{4(-4+3\theta)^2}$,

$\pi_{m2}^{RNS} - \pi_{m2}^{RAS} = -\frac{\theta^2(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)^2}{4(-4+\theta)^2(-1+\theta)(-4+3\theta)^2} > 0$ and $\pi_{m2}^{RGS} - \pi_{m2}^{RAS} = -\frac{\theta(-\theta+\theta^2+\theta c_1^2+2c_2-2\theta c_2-2c_1 c_2+c_2^2)}{4(-4+3\theta)^2}$. For

any $\theta \in [\underline{\theta}^{RNS}, \overline{\theta}^{RNS}]$ and $0 < c_2 < c_1 < 1$, we obtain $-\theta + \theta^2 + \theta c_1^2 + 2c_2 - 2\theta c_2 - 2c_1 c_2 + c_2^2 < 0$.

That is, $\pi_{m1}^{RAS} - \pi_{m1}^{RGS} > 0$ and $\pi_{m2}^{RGS} - \pi_{m2}^{RAS} > 0$. Therefore, $\pi_{m1}^{RNS} > \pi_{m1}^{RGS}$, $\pi_{m1}^{RAS} > \pi_{m1}^{RGS}$, $\pi_{m2}^{RNS} > \pi_{m2}^{RAS}$

and $\pi_{m2}^{RGS} > \pi_{m2}^{RAS}$. (ii) $\pi_r^{RNS} - \pi_r^{RGS} = -\frac{\theta(2-2\theta-2c_1+\theta c_1+c_2)^2}{4(-4+\theta)(-1+\theta)(-4+3\theta)} > 0$, $\pi_r^{RNS} - \pi_r^{RAS} =$

$-\frac{(-\theta+\theta^2-\theta c_1+2c_2-\theta c_2)^2}{4(-4+\theta)(-1+\theta)(-4+3\theta)} > 0$, $\pi_r^{RAS} - \pi_r^{RGS} = \frac{-\theta+\theta^2+2\theta c_1-\theta c_1^2-2\theta c_2+c_2^2}{4(-4+3\theta)}$ and $\pi_s^{RAS} - \pi_s^{RGS} =$

$\frac{-\theta+\theta^2+2\theta c_1-\theta c_1^2-2\theta c_2+c_2^2}{4(-4+3\theta)}$. Here $\theta_1 = \frac{1-2c_1+c_1^2+2c_2+(1-c_1)\sqrt{1-2c_1+c_1^2+4c_2}}{2}$ where $\theta_1 \in [\underline{\theta}^{RNS}, \overline{\theta}^{RNS}]$, which

makes $\pi_r^{RGS} - \pi_r^{RAS} = 0$ and $\pi_s^{RGS} - \pi_s^{RAS} = 0$. That is, When $\theta \in (\underline{\theta}^{RNS}, \theta_1]$, $\pi_r^{RNS} > \pi_r^{RAS} \geq \pi_r^{RGS}$

and $\pi_s^{RAS} \geq \pi_s^{RGS}$; When $\theta \in (\theta_1, \overline{\theta}^{RNS})$, $\pi_r^{RNS} > \pi_r^{RGS} > \pi_r^{RAS}$ and $\pi_s^{RAS} < \pi_s^{RGS}$. This completes the

proof.

Proof of Proposition 6:

(a) For $\theta \in (0, \underline{\theta}^k]$, from Table 3, we get $w_1^{MNS} - w_1^{VNN} = \frac{(1-\theta)(1-c_1)}{(3-\theta)(2-\theta)} > 0$, $w_1^{MNS} - w_1^{RNS} =$

$\frac{(1-\theta)(1-c_1)}{2(2-\theta)} > 0$ and $w_1^{VNN} - w_1^{RNS} = \frac{(1-\theta)^2(1-c_1)}{2(3-\theta)(2-\theta)} > 0$; $p_1^{MNS} - p_1^{RNS} = 0$ and $p_1^{MNS} - p_1^{VNN} =$

$\frac{(1-\theta)(1-c_1)}{2(3-\theta)(2-\theta)} > 0$; $D_1^{MNS} - D_1^{RNS} = 0$ and $D_1^{MNS} - D_1^{VNN} = -\frac{(1-\theta)(1-c_1)}{2(3-\theta)(2-\theta)} < 0$; $\pi_{m1}^{MNS} - \pi_{m1}^{VNN} =$

$\frac{(1-\theta)(1+2\theta-\theta^2)(1-c_1)^2}{2(3-\theta)^2(2-\theta)^2} > 0$, $\pi_{m1}^{MNS} - \pi_{m1}^{RNS} = \frac{(1-\theta)(1-c_1)^2}{4(2-\theta)^2} > 0$ and $\pi_{m1}^{VNN} - \pi_{m1}^{RNS} = \frac{(1-\theta)^2(7-3\theta)(1-c_1)^2}{4(3-\theta)^2(2-\theta)^2} > 0$;

$\pi_r^{MNS} - \pi_r^{VNN} = -\frac{(1-\theta)(7-3\theta)(1-c_1)^2}{4(3-\theta)^2(2-\theta)^2} < 0$, $\pi_r^{MNS} - \pi_r^{RNS} = -\frac{(1-\theta)(1-c_1)^2}{4(2-\theta)^2} < 0$ and $\pi_r^{VNN} - \pi_r^{RNS} =$

$-\frac{(1-\theta)^2(1-c_1)^2}{4(3-\theta)^2(2-\theta)} < 0$. That is, $w_2^{MNS} > w_2^{VNN} > w_2^{RNS}$, $p_2^{MNS} = p_2^{RNS} > p_2^{VNN}$, $D_2^{MNS} = D_2^{RNS} < D_2^{VNN}$,

$\pi_{m2}^{MNS} > \pi_{m2}^{VNN} > \pi_{m2}^{RNS}$ and $\pi_r^{MNS} < \pi_r^{VNN} < \pi_r^{RNS}$. $\pi_{s1}^{MNS} = \pi_{m1}^{MNS} + \pi_r^{MNS} = \frac{(3-2\theta)(1-c_1)^2}{4(2-\theta)^2}$, $\pi_{s1}^{VNN} =$

$\pi_{m1}^{VNN} + \pi_r^{VNN} = \frac{(2-\theta)(1-c_1)^2}{(3-\theta)^2}$ and $\pi_{s1}^{RNS} = \pi_{m1}^{RNS} + \pi_r^{RNS} = \frac{(3-2\theta)(1-c_1)^2}{4(2-\theta)^2}$, so $\pi_{s1}^{MNS} = \pi_{s1}^{RNS}$ and $\pi_{s1}^{MNS} -$

$\pi_{s1}^{VNN} = -\frac{(1-\theta)^2(5-2\theta)(1-c_1)^2}{4(3-\theta)^2(2-\theta)^2} < 0$. That is, $\pi_{s1}^{MNS} = \pi_{s1}^{RNS} < \pi_{s1}^{VNN}$.

(b) For $\theta \in [\overline{\theta}^k, 1)$, from Table 3, $w_2^{MNS} - w_2^{VNN} = \frac{(1-\theta)(\theta-c_2)}{2(3-\theta)(2-\theta)} > 0$, $w_2^{MNS} - w_2^{RNS} = \frac{(1-\theta)(\theta-c_2)}{2(2-\theta)} > 0$

and $w_2^{VNN} - w_2^{RNS} = \frac{(1-\theta)^2(\theta-c_2)}{2(3-\theta)(2-\theta)} > 0$; $p_2^{MNS} - p_2^{RNS} = 0$ and $p_2^{MNS} - p_2^{VNN} = \frac{(1-\theta)(\theta-c_2)}{2(3-\theta)(2-\theta)} > 0$;

$D_2^{MNS} - D_2^{RNS} = 0$ and $D_2^{MNS} - D_2^{VNN} = -\frac{(1-\theta)(\theta-c_2)}{2(3-\theta)(2-\theta)\theta} < 0$; $\pi_{m2}^{MNS} - \pi_{m2}^{VNN} = \frac{(1-\theta)(1+2\theta-\theta^2)(\theta-c_2)^2}{2(3-\theta)^2(2-\theta)^2\theta} > 0$,

$\pi_{m2}^{MNS} - \pi_{m2}^{RNS} = \frac{(1-\theta)(\theta-c_2)^2}{4(2-\theta)^2\theta} > 0$ and $\pi_{m2}^{VNN} - \pi_{m2}^{RNS} = \frac{(1-\theta)^2(7-3\theta)(\theta-c_2)^2}{4(3-\theta)^2(2-\theta)^2\theta} > 0$; $\pi_r^{MNS} - \pi_r^{VNN} =$

$-\frac{(1-\theta)(7-3\theta)(\theta-c_2)^2}{4(3-\theta)^2(2-\theta)^2\theta} < 0$, $\pi_r^{MNS} - \pi_r^{RNS} = -\frac{(1-\theta)(\theta-c_2)^2}{4(2-\theta)^2\theta} < 0$ and $\pi_r^{VNN} - \pi_r^{RNS} = -\frac{(1-\theta)^2(\theta-c_2)^2}{4(3-\theta)^2(2-\theta)\theta} < 0$.

That is, $w_2^{MNS} > w_2^{VNN} > w_2^{RNS}$, $p_2^{MNS} = p_2^{RNS} > p_2^{VNN}$, $D_2^{MNS} = D_2^{RNS} < D_2^{VNN}$, $\pi_{m2}^{MNS} > \pi_{m2}^{VNN} > \pi_{m2}^{RNS}$

and $\pi_r^{MNS} < \pi_r^{VNN} < \pi_r^{RNS}$. $\pi_{s2}^{MNS} = \pi_{m2}^{MNS} + \pi_r^{MNS} = \frac{(3-2\theta)(\theta-c_2)^2}{4(2-\theta)^2\theta}$, $\pi_{s2}^{VNN} = \pi_{m2}^{VNN} + \pi_d^{VNN} =$

$\frac{(2-\theta)(\theta-c_2)^2}{(3-\theta)^2\theta}$ and $\pi_{s2}^{RNS} = \pi_{m2}^{RNS} + \pi_r^{RNS} = \frac{(3-2\theta)(\theta-c_2)^2}{4(2-\theta)^2\theta}$, so $\pi_{s2}^{MNS} = \pi_{s2}^{RNS}$ and $\pi_{s2}^{MNS} - \pi_{s2}^{VNN} =$

$-\frac{(1-\theta)^2(5-2\theta)(\theta-c_2)^2}{4(3-\theta)^2(2-\theta)^2\theta} < 0$. That is, $\pi_{s2}^{MNS} = \pi_{s2}^{RNS} < \pi_{s2}^{VNN}$. This completes the proof.

Proof of Proposition 7:

(a) $w_1^{MNS} - w_1^{VNN} = \frac{6-5\theta-\theta^2-6c_1+4\theta c_1+c_2+\theta c_2}{(9-\theta)(4-\theta)}$, $w_1^{MNS} - w_1^{RNS} = \frac{2-2\theta-2c_1+\theta c_1+c_2}{2(4-\theta)}$, $w_2^{MNS} - w_2^{RNS} =$

$\frac{\theta-\theta^2+\theta c_1-2c_2+\theta c_2}{2(4-\theta)}$ and $w_2^{MNS} - w_2^{VNN} = \frac{5\theta-5\theta^2+\theta c_1+\theta^2 c_1-6c_2+4\theta c_2}{(9-\theta)(4-\theta)}$. For any $\theta \in [\underline{\theta}^{VNN}, \overline{\theta}^{VNN}]$ and $0 <$

$c_2 < c_1 < 1$, we obtain $6 - 5\theta - \theta^2 - 6c_1 + 4\theta c_1 + c_2 + \theta c_2 > 0$, $2 - 2\theta - 2c_1 + \theta c_1 + c_2 > 0$, $\theta -$

$\theta^2 + \theta c_1 - 2c_2 + \theta c_2 > 0$ and $5\theta - 5\theta^2 + \theta c_1 + \theta^2 c_1 - 6c_2 + 4\theta c_2 > 0$. That is, $w_1^{MNS} > w_1^{VNN}$,

$w_1^{MNS} > w_1^{RNS}$, $w_2^{MNS} > w_2^{rn}$ and $w_2^{MNS} > w_2^{VNN}$.

(b) $p_1^{MNS} - p_1^{RNS} = 0$, $p_1^{MNS} - p_1^{VNN} = \frac{6-5\theta-\theta^2-6c_1+4\theta c_1+c_2+\theta c_2}{2(9-\theta)(4-\theta)}$, $p_2^{MNS} - p_2^{RNS} = 0$ and $p_2^{MNS} -$

$p_2^{VNN} = \frac{5\theta-5\theta^2+\theta c_1+\theta^2 c_1-6c_2+4\theta c_2}{2(9-\theta)(4-\theta)}$. For any $\theta \in [\underline{\theta}^{MNS}, \overline{\theta}^{MNS}]$ and $0 < c_2 < c_1 < 1$, we obtain $6 -$

$5\theta - \theta^2 - 6c_1 + 4\theta c_1 + c_2 + \theta c_2 > 0$ and . $5\theta - 5\theta^2 + \theta c_1 + \theta^2 c_1 - 6c_2 + 4\theta c_2 > 0$ That is, $p_1^{MNS} =$

$p_1^{RNS} > p_1^{VNN}$ and $p_2^{MNS} = p_2^{rn} > p_2^{VNN}$.

(c) (i) $D^{MNS} - D^{RNS} = 0$ and $D^{RNS} - D^{VNN} = -\frac{5\theta-5\theta^2+\theta c_1+\theta^2 c_1-6c_2+4\theta c_2}{2(-9+\theta)(-4+\theta)\theta} < 0$. That is, $D^{MNS} =$

$D^{RNS} < D^{VNN}$.

(ii) $\alpha_1^{MNS} - \alpha_1^{RNS} = 0$ and $\alpha_1^{RNS} - \alpha_1^{VNN} = -\frac{\theta(-\theta+\theta^2+2\theta c_1-\theta c_1^2-2\theta c_2+c_2^2)}{(-1+\theta)(-3\theta+\theta c_1+2c_2)(-4\theta+\theta c_1+3c_2)}$. Here $\theta_1 =$

$\frac{1-2c_1+c_1^2+2c_2+(1-c_1)\sqrt{1-2c_1+c_1^2+4c_2}}{2}$ where $\theta_1 \in [\underline{\theta}^{MNS}, \overline{\theta}^{MNS}]$, which makes $\alpha_1^{RNS} - \alpha_1^{VNN} = 0$. That is,

when $\theta \in (\underline{\theta}^{MNS}, \theta_1]$, $\alpha_1^{MNS} = \alpha_1^{RNS} \leq \alpha_1^{VNN}$; when $\theta \in (\theta_1, \overline{\theta}^{MNS})$, $\alpha_1^{MNS} = \alpha_1^{RNS} > \alpha_1^{VNN}$. Because

$\alpha_2 = 1 - \alpha_1$, when $\theta \in (\underline{\theta}^{MNS}, \theta_1]$, $\alpha_2^{MNS} = \alpha_2^{RNS} \geq \alpha_2^{VNN}$; when $\theta \in (\theta_1, \overline{\theta}^{MNS})$, $\alpha_2^{MNS} = \alpha_2^{RNS} <$

α_2^{VNN}. This completes the proof.