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Risk control: Who cares?[*]

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Abstract

The performance of recently introduced risk-control indices is evaluated and tested with respect to a set of competing indices. Applying a method of moments methodology to these data reveals that the performance of strategies that track risk-control indices have economic and statistical significance to investors with realistic risk aversion parameter values. However, this performance varies over time and appears to be determined by macroeconomic and liquidity conditions.

Key Words: Risk control, volatility, certainty equivalent return, method of moments.

JEL Classification Codes: G53, G11, G17.

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1 Introduction

Financial crises remind investors of the perils of excessive risk taking behaviour. It is not surprising, therefore, that the post-2007 period has seen considerable growth in products that seek to highlight the virtues of limited risk exposure. One such example is the introduction of risk-control (RC) indices; e.g., the S&P Dow Jones family of RC indices.[1] Such series are based on a time-varying weighted average of a (high return/high risk) equity and (low return/low risk) cash index, with the weights determined by the expected level of equity return volatility. During periods of high expected volatility, a low equity weight is used in order to limit exposure to the risky equity market. By contrast, low expected volatility levels coincide with a high equity weight. It is the desirability of strategies based on tracking these RC indices (henceforth *cash/equity (RC) strategies*) that we examine in the current paper.

RC indices have received considerable attention in the popular financial press. For instance, the *Buttonwood* article in the Economist magazine (November 1st to 7th, 2014, p. 80) highlights the virtues of these indices (and the tracking thereof) to pension fund managers who require high returns in order to pay the pledged benefits. As cash pays small returns, these managers invest in equity with higher returns.[2] However, this leaves them exposed to bear markets such as those observed in 2001/02 and 2008/09. Therefore, for such managers the "*holy grail would be a combination of equity-like returns with reduced volatility*" (Economist, 2014). As cash/equity (RC) strategies may fulfil this need, their performance is the subject of the current paper.

The cash/equity (RC) strategies studied in the current paper are examples of a wider class of strategies referred to as dynamic strategies. Such strategies are characterised by time-varying positions in assets such that the utility of investors is maximised. Expectations of the moments associated with the returns to the assets are key inputs within this context. Early approaches

[1] Other providers tend to use similar names for their RC series. For instance, Russell Investments use the title: Russell Volatility Control Index Series.

[2] Cash is typically represented by a money market instrument.

focus on the first moment (*market timing strategies*); see, e.g., Ferson and Harvey (1993), Pesaran and Timmermann (1995), and Kandel and Stambaugh (1996). More recently, the focus has shifted to higher moments.[3] In particular, *volatility timing strategies* are demonstrated to have considerable economic value; see, e.g., West et al. (1993), Fleming et al. (2001, 2003), Marquering and Verbeek (2004), Chiriac and Voev (2011), and Taylor (2014).[4] Given this success, market practitioners have introduced their own simplified versions of volatility timing strategies, viz. cash/equity (RC) strategies. It therefore seems sensible to assess whether these particular strategies also have economic value. This represents the first contribution of the paper.

The second contribution concerns the method used to assess the economic value of the cash/equity (RC) strategies. A number of approaches have been used to assess the economic value of trading strategies within an asset allocation setting. We build on perhaps the most commonly used measure of economic value, viz. certainty equivalent return (CER); see Campbell and Thompson (2008), Welch and Goyal (2008), and Cenesizoglu and Timmermann (2012) for recent applications. Unlike previous applications of this technique, we estimate the CER within a method of moments (MM) framework.[5] There are two benefits to using the proposed framework. First, it is possible to estimate the differences in CER values across competing strategies within a single estimation procedure. Second, we are able to apply a conventional MM-based statistical test of the significance of the CER.

The MM-based CER framework is essentially a collection of pairwise comparisons, with the difference in CER values constructed for a particular strategy with respect to a competing strategy. In our case the strategies are: cash only, equity only, and a cash/equity strategy

[3]This switch reflects the widely accepted view that accurate forecasts of expected returns are particularly difficult to obtain (Merton, 1980).

[4]See Cenesizoglu and Timmermann (2008) and Jondeau and Rockinger (2012) for empirical evidence regarding the economic value of forecasting the return distribution within an asset allocation setting.

[5]The CER is defined as the guaranteed return that an investor would accept to be indifferent to taking on a risky strategy.

with a time-varying wealth allocation, viz. the cash/equity (RC) strategy. The latter strategy differentiates itself by delivering a broadly constant conditional volatility. Given that a number of different strategies are considered, it is useful to construct an overall measure of desirability based on a combination of various pairwise comparisons.

The CER framework makes use of unconditional expectations of utility over a particular sample period. Consequently, it is traditionally applied to low frequency data observed over relatively long sample periods. However, the availability of intraday data means that these expectations can be applied over much shorter sample periods; that is, a sample period of one day. In doing this, one is able to generate a daily frequency series of CER values, and to examine the determinants of these values via use of conventional regression analysis. This investigation of dynamic performance (cf. static performance) is undertaken in the second part of the paper and represents the third contribution of the paper.

A number of findings are presented. First, the cash/equity (RC) strategy delivers significant benefits (both statistical and economic) over competing strategies. These benefits are highly dependent on the risk preferences of the user, with the cash/equity (RC) strategy beating the equity strategy for highly risk-averse investors, and beating the cash strategy for risk-neutral and mildly risk-averse investors. Second, the cash/equity (RC) strategy is dominant for investors with risk preferences that coincide with those of the typical investor. However, this result is highly sensitive to transaction costs. For investors who face transaction costs in excess of 10 basis points, the cash/equity (RC) strategy is not generally preferred. Finally, strategy performance is dynamic in nature and highly dependent on the sample period used. Prior to 2006, the cash/equity (RC) strategy offers little benefit to investors. By contrast, the post-2006 period is characterised by strong performance. This finding is argued to be consistent with the inverse risk-return relationship and flight to safety behaviour observed in the post-2006 period (Ghysels et al., 2013, Baele et al., 2014, and Adrian et al., 2015).

The remainder of this paper is organised as follows. Section 2 describes the methodologies used. Section 3 contains the application. Section 4 concludes.

2 Methodologies

This section contains descriptions of the methods used to construct the RC indices, and the MM-based framework used to evaluate the economic value of competing strategies.

2.1 Constructing and tracking the RC index

The RC index consists of an underlying equity index and a hypothetical cash index (that is, a money market instrument index). These two items represent the risky and non-risky (almost risk-free) components of the index. Specifically, the RC index value at time t is given by

$$Y_t = Y_{t-1}\big(1 + L_t R_t^e + (1 - L_t)R_t^c\big), \quad t = 1, 2, \ldots, T, \tag{1}$$

where R_t^e and R_t^c denote the simple daily returns to the equity and cash indices between day $t-1$ and day t; and $L_t = f(Z^*, Z_{t-d})$ denotes the *leverage factor*, which is a function of the target volatility level Z^* (henceforth the *RC level*) and the realised volatility level Z_{t-d} observed on day $t-d$.[6] The function commonly employed is $f(Z^*, Z_{t-d}) = \min(L_{\max}, Z^*/Z_{t-d})$, where $L_{\max}$ is the maximum leverage level permitted and is set equal to 1.5. Furthermore, we assume that the delay parameter d is equal to two. These parameter values correspond to those employed in the S&P Dow Jones family of RC indices.

Realised volatility, Z_t, is based on the exponentially-weighted volatility methodology adopted

[6] The underlying equity index is often an existing equity index such as the S&P 500 index, while the cash index is constructed as follows:

$$X_t^c = X_{t-1}^c(1 + I_t D_t/360), \tag{2}$$

where I_t is the interest rate associated with the money market instrument (usually an overnight interest rate) on day t, and D_t is the number of calender days between $t-1$ and t.

by the majority of S&P-based RC indices. In particular,

$$Z_t = \max(Z_{1,t}, Z_{2,t}). \tag{3}$$

where $Z_{1,t}$ and $Z_{2,t}$ are the short and long-term volatility measures, respectively. These are given by $Z_{i,t} = \sqrt{252 \times V_{i,t}}$, with the variance $V_{i,t}$ adopting the following exponential weighting:

$$V_{i,t} = \alpha_i V_{i,t-1} + (1 - \alpha_i)(\ln(1 + R_t^e))^2. \tag{4}$$

Here the short and long-term α_i values correspond to those used in the S&P-based RC indices, that is, $\alpha_1 = 0.94$ and $\alpha_2 = 0.97$.[7] Three distinct strategies are considered based on the above representation. These are the cash strategy (achieved by restricting L_t to equal zero for all t); the equity strategy (achieved by restricting L_t to equal one for all t); and the cash/equity (RC) strategy (achieved by using the above L_t values such that the conditional volatility equals the RC level Z^*).

The RC indices can be augmented to include transaction costs.[8] Specifically, returns to strategy k net of transaction costs are given by

$$R_{k,t}^* = (1 + R_{k,t})(1 - C_t) - 1, \tag{5}$$

where $R_{k,t}$ is the return to strategy k ignoring transaction costs. The total costs of trading are given by

$$C_t = \kappa_1 |L_t - L_{t-1}^e| + \kappa_2 |1 - L_t - L_{t-1}^c|, \tag{6}$$

where κ_1 and κ_2 denote the proportional transaction costs associated with trading (tracking)

[7]The values of $V_{1,0}$ and $V_{2,0}$ are determined using the methodology described in the S&P Dow Jones Index Mathematics Methodology document downloaded from `http://www.spindices.com/documents/index-policies/methodology-index-math.pdf`.

[8]Proportional transaction costs are well studied in the literature (see, e.g., Constantinides, 1986).

the equity and cash indices, respectively. Here L^e_{t-1} and L^c_{t-1} are the respective leverage factors on day $t-1$ *after* rebalancing due to changes in the values of the equity and cash components of the strategy, that is, $L^e_t = L_t(1+R^e_t)/(1+R_{k,t})$ and $L^c_t = (1-L_t)(1+R^c_t)/(1+R_{k,t})$.

2.2 Measuring and testing strategy performance

The CER represents the minimum risk-free return that an investor is willing to accept in preference to a risky asset/portfolio return. Specifically, it is measured by c in the following equation:

$$u(c) = \mathrm{E}[u(R_t)], \tag{7}$$

where R_t is the return to the risky asset (or portfolio), $u(.)$ represents the utility function of the investor, and $\mathrm{E}[.]$ is the unconditional expectation operator.

Using this concept, the objective is to compare the performance of a particular strategy (say strategy 0), with a set of N competing strategies. Thus we seek the solution to the following equations:

$$u(c_0) = \mathrm{E}[u(R_{0,t})], \tag{8a}$$

$$u(c_0 - \delta_1) = \mathrm{E}[u(R_{1,t})], \tag{8b}$$

$$\vdots \tag{8c}$$

$$u(c_0 - \delta_N) = \mathrm{E}[u(R_{N,t})], \tag{8d}$$

where c_0 is the CER associated with strategy 0, and $\delta_1, \ldots, \delta_N$ are the (risk-adjusted) costs/benefits (henceforth *differential CER values*) to using strategy 0 with respect to strategies 1 to N.

The above can be written more compactly via the introduction of suitable notation. Let the vector-valued function be defined as $h(\mathbf{R}_t, \boldsymbol{\theta}) = u(c_0 - \boldsymbol{\delta}) - u(\mathbf{R}_t)$, where $\boldsymbol{\delta} = (0, \delta_1, \ldots, \delta_N)^\top$, $\boldsymbol{\theta} = (c_0, \delta_1, \ldots, \delta_N)^\top$ and $\mathbf{R}_t = (R_{0,t}, R_{1,t}, \ldots, R_{N,t})^\top$. It follows that the system can be written as the following set of $N+1$ moment conditions:

$$g(\boldsymbol{\theta}) = \mathrm{E}[h(\mathbf{R}_t, \boldsymbol{\theta})] = 0. \tag{9}$$

As the number of moment conditions equals the number of unknown parameters then the system is exactly identified. It follows that one can use the MM estimator (cf. the generalised method of moments estimator). Replacing population expectations with sample averages, the MM estimator is given by the solution to the following set of $N+1$ sample moment conditions:

$$g_T(\boldsymbol{\theta}) = \mathrm{E}_T[h(\mathbf{R}_t, \boldsymbol{\theta})] = 0. \tag{10}$$

That is, the MM estimator $\widehat{\boldsymbol{\theta}}$ is given by the solution to $g_T(\widehat{\boldsymbol{\theta}}) = 0$.

Standard results demonstrate that this MM estimator is consistent and asymptotically normal. Specifically, we have

$$\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \overset{a}{\sim} N(0, \widehat{\boldsymbol{\Lambda}}(\boldsymbol{\theta}_0)), \tag{11}$$

where

$$\widehat{\boldsymbol{\Lambda}}(\boldsymbol{\theta}_0) = \big(\widehat{\mathbf{A}}(\boldsymbol{\theta}_0)^\top\big)^{-1}\widehat{\boldsymbol{\Omega}}(\boldsymbol{\theta}_0)\big(\widehat{\mathbf{A}}(\boldsymbol{\theta}_0)\big)^{-1}, \tag{12a}$$

$$\widehat{\mathbf{A}}(\boldsymbol{\theta}_0) = \mathrm{E}_T\left[\frac{\partial h(\mathbf{R}_t, \boldsymbol{\theta}_0)}{\partial \boldsymbol{\theta}_0}\right], \tag{12b}$$

$$\widehat{\boldsymbol{\Omega}}(\boldsymbol{\theta}_0) = \mathrm{E}_T\left[h(\mathbf{R}_t, \boldsymbol{\theta}_0)h(\mathbf{R}_t, \boldsymbol{\theta}_0)^\top\right]. \tag{12c}$$

This result enables statistical inference to be applied to relative strategy performance. Thus not only do we have a method of calculating the economic value of an investment strategy (and hence its economic significance), we also have a way of testing its statistical significance.

It is possible to analytically solve the above sample moment conditions under a range of utility function assumptions. Given its widespread use, we assume that investor utility is given by the following power utility function:

$$u(\mathbf{R}_t) = \frac{(1+\mathbf{R}_t)^{1-\gamma} - 1}{1-\gamma}, \quad \gamma \geq 0, \tag{13}$$

where γ is the relative risk aversion parameter. Then, when $\gamma \neq 1$ we have the following solution:

$$\widehat{\boldsymbol{\theta}} = \begin{bmatrix} \widehat{c}_0 \\ \widehat{\delta}_1 \\ \vdots \\ \widehat{\delta}_N \end{bmatrix} = \begin{bmatrix} ((1-\gamma)\mathrm{E}_T[u(R_{0,t})] + 1)^{1/(1-\gamma)} - 1 \\ ((1-\gamma)\mathrm{E}_T[u(R_{0,t})] + 1)^{1/(1-\gamma)} - ((1-\gamma)\mathrm{E}_T[u(R_{1,t})] + 1)^{1/(1-\gamma)} \\ \vdots \\ ((1-\gamma)\mathrm{E}_T[u(R_{0,t})] + 1)^{1/(1-\gamma)} - ((1-\gamma)\mathrm{E}_T[u(R_{N,t})] + 1)^{1/(1-\gamma)} \end{bmatrix}. \tag{14a}$$

And when $\gamma = 1$ we have

$$\widehat{\boldsymbol{\theta}} = \begin{bmatrix} \widehat{c}_0 \\ \widehat{\delta}_1 \\ \vdots \\ \widehat{\delta}_N \end{bmatrix} = \begin{bmatrix} \exp(\mathrm{E}_T[u(R_{0,t})]) - 1 \\ \exp(\mathrm{E}_T[u(R_{0,t})]) - \exp(\mathrm{E}_T[u(R_{1,t})]) \\ \vdots \\ \exp(\mathrm{E}_T[u(R_{0,t})]) - \exp(\mathrm{E}_T[u(R_{N,t})]) \end{bmatrix}. \tag{14b}$$

Moreover, inference is achieved by noting that if we wish to test the null that $\boldsymbol{\theta}_0$ equals zero then the $\mathbf{A}(0)$ matrix in (12b) is given by

$$\widehat{\mathbf{A}}(0) = \begin{bmatrix} 1 & \mathbf{1}_{1\times N} \\ \mathbf{0}_{N\times 1} & \mathbf{I}_N \end{bmatrix}, \tag{15}$$

where $\mathbf{1}_{1\times N}^{\top}$ is an $(N \times 1)$ vector of ones, $\mathbf{0}_{N\times 1}$ is an $(N \times 1)$ vector of zeros, and $\mathbf{I}_N$ is an N-dimension identity matrix. The $\widehat{\boldsymbol{\Omega}}(0)$ matrix is obtained using sample means of the cross product terms.

The above framework delivers measures of the relative economic values (differential CER values) of strategy k in comparison to competing strategies. Moreover, it enables users to test the null hypothesis that these relative economic values take a particular value (most obviously zero). We can go further and use the p-values from these tests to construct a desirability index. The idea behind the desirability index is simple. Consider a strategy that dominates all competing strategies. Under the null hypothesis that the differential CER value is less than or equal to zero (against the alternative that it is greater than zero) for each comparison, this strategy will be associated with a set of low p-values. Taking the product of the p-value complements will deliver a value close to one. For this reason our desirability index is given by this product, with successful strategies having a value close to one and unsuccessful strategies having a value close to zero. Moreover, the 'dominant' strategy has the highest desirability amongst its competitors.[9]

3 Application

An application of the above methods to data is provided in this section.

3.1 Data

Official RC index data were obtained from the S&P Dow Jones Indices website `http://us.spindices.com/index-family/strategy/risk-control`, and cover the period from January 1, 2006 to December 31, 2014. The series considered are the S&P 500 RC 5%, 7%, 7.5%, 10%, 12%, and 15% total return indices.[10]

[9] An alternative criterion is also considered that is more selective. It selects a 'dominant' strategy only if it has the highest desirability index *and* has a desirability index greater than one half. It follows that this criterion only provides a winner if it is sufficiently dominant. Results associated with this criterion deliver similar results and are available on request.

[10] The percentage refers to the (annualised) RC level.

To obtain a longer span of data and to permit examination of the parameters governing the RC indices, synthetic RC data covering the period January 1, 1956 to December 31, 2014 are constructed.[11] These series are based on the techniques used to construct the official RC series as outlined in the S&P Dow Jones Index Mathematics Methodology document. The S&P 500 equity index (inclusive of dividends) and overnight dollar LIBOR rates (2006 to 2014 sample only) or federal funds rates (1956 to 2014 sample only) are used to construct these indices. These data were obtained from the CRSP US Stock database accessed via the Wharton Research Data Services (WRDS) interface; the Federal Reserve Economic Data (FRED) website `http://research.stlouisfed.org/fred2/`, and the Federal Reserve website `http://www.federalreserve.gov/releases/h15/data.htm`, respectively. All data are measured at the daily frequency.

The second part of the analysis makes use of intraday data. In particular, five-minute frequency S&P 500 index values and all trades in the S&P 500 index futures and SPDR S&P 500 ETF Trust markets traded on the Chicago Mercantile Exchange (CME) and the New York Stock Exchange (NYSE), respectively, are obtained from *TickData, Inc.* These data are collected over the periods starting in April 21, 1982 (futures data), February 1, 1983 (index data) and February 1, 1993 (ETF data) and ending in December 31, 2014, and cover the daytime trading periods only.[12]

In addition, we make use of measures of market conditions that in turn are functions of commonly used series. In particular, daily frequency Chicago Board Options Exchange (CBOE) Volatility Index (VIX) data are obtained from the CBOE website `https://www.cboe.com/micro/vix/`, and the Aruoba, Diebold and Scotti (2009) daily frequency real time measure of business conditions (henceforth the ADS index) are obtained from the Federal Reserve Bank of Philadelphia website `https://www.philadelphiafed.org/research-and-data/`. Both of

[11]This is the maximum span possible given the availability of a suitable equity and cash index.
[12]The trading times are restricted to Monday to Friday, 9.00am to 3.00pm (CT).

these series cover the period, January 1, 1990 to December 31, 2014.

3.2 Static performance analysis

The analysis begins by investigating the performance of the cash/equity (RC) strategy using the MM methodology described in section 2.

3.2.1 Summary statistics

Summary statistics associated with the official and synthetic cash/equity (RC), cash, and equity strategies are provided in Table 1. To enable comparison of the official and synthetic datasets, the common sample period January 1, 2006 to December 31, 2014 is used. We confine results to the RC levels associated with the official RC index, that is, 5%, 7%, 7.5%, 10%, 12%, and 15%. Moreover, statistics associated with the synthetic cash/equity (RC) strategy based on the sample period January 1, 1956 to December 31, 2014 are provided.

Insert Table 1 here

The results accord to prior expectations. First, as the RC level increases, the mean return increases. Moreover, the reward to risk to the cash/equity (RC) strategy appears greater than that associated with the equity strategy as indicated by the Sharpe ratios. This implies that altering the leverage factor not only maintains a constant volatility level, but also generates greater returns. Second, the performance of the official cash/equity (RC) strategy is very similar to its synthetic counterpart. For instance, the mean annualised returns to the official and synthetic cash/equity (RC) strategies with a 15% RC level are 10.233% and 10.244%, respectively. Thus the construction of the synthetic RC index seems valid. Third, the cash strategy delivers the lowest mean and volatility. Moreover, the choice of daily interest rate is likely to affect performance, with the mean federal funds rate lower than the mean overnight dollar LIBOR rate.[13] Fourth, the performance of the strategies change over the longer 1956 to 2014 sample period. However, relative strategy performance remains broadly unchanged.

The results in Table 1 also indicate that the cash/equity (RC) strategy returns are negatively skewed. Moreover, they are more negatively skewed than the equity strategy returns in the post-2006 period only. It is during this period that the cash/equity (RC) strategy enjoys a noticeably higher Sharpe ratio in comparison to the equity strategy. This observation raises the possibility that this superior performance (in the post-2006 period) may simply be compensation for taking on additional skewness risk; see Amaya et al. (2015) for empirical evidence of a negative relationship between individual equity returns and skewness.

To gain further insight into the performance of the strategies, Figure 1 contains a plot of the cumulative returns to each $1 invested in two cash/equity (RC) strategies (with 5% and 15% RC levels), and the equity and cash strategies over the 2006 to 2014 sample period. Two additional plots contain cumulative returns to the official and synthetic cash/equity (RC) strategies. The first plot provides clear evidence that the cash/equity (RC) strategies are able to avoid the large losses to equity during the 2008/09 financial crisis period by switching to cash. The second and third plots confirm that there are almost no differences between the performance of the official and synthetic cash/equity (RC) strategies.

Insert Figure 1 here

The time series return plots in Figure 1 also highlight the differences between the strategies. In particular, the equity strategy returns exhibit time-varying levels of volatility, while the cash/equity (RC) strategy returns are characterised by (near) constant conditional volatility levels. Thus the cash/equity (RC) strategy is able to avoid episodes of high volatility by shifting out of equity (cf. the equity strategy).

[13] The official and synthetic RC indices are based on the S&P 500 equity index and overnight dollar LIBOR interest rates. However, over the 1956 to 2014 sample, it is necessary to use federal funds rates as overnight dollar LIBOR interest rates are not available over this period.

Figure 1 also contains plots of daily bid-ask spreads and the leverage factor levels associated with the two cash/equity (RC) strategies (with 5% and 15% RC levels). The bid-ask spreads are constructed using the method proposed by Roll (1984) applied to all trades in the S&P 500 index futures contract with shortest maturity. The bid-ask spreads exhibit considerable variability of the period, with a peak coinciding with the high volatility episode observed during late 2008. Moreover these higher bid-ask spread levels appear to occur when the cash/equity (RC) strategies are most active as evinced by the large swings in leverage factor levels at this time. Taken together this evidence suggests that the superior performance of the cash/equity (RC) strategies during the post-2006 period may not occur if transaction costs are taken into account. This conjecture will be examined later in the paper.

3.2.2 The costs/benefits of RC

The use of synthetic cash/equity (RC) strategies enables examination of performance over a longer period – an important feature given the likely users of these strategies such as pension fund managers. To this end, the results in Table 2 provide the differential CER values and an indication of the (one-sided) significance of the null hypothesis of zero costs to using the cash/equity (RC) strategy with RC levels of 5%, 7.5%, 10%, 12.5%, and 15%. In addition, we consider an RC level that delivers the same returns to the cash/equity (RC) strategy as the equity strategy (henceforth this is referred to as the *equity-calibrated RC level*). These are calculated using the power utility function with risk aversion parameter values given by $\gamma = 0, 1, 2, 4, 8, 16, 32$, and data observed from January 1, 1956 to December 31, 2014.

Insert Table 2 here

The results highlight a number of features of the data. First, for risk-neutral investors, the equity strategy is the best for RC levels less than 15%. For instance, the cash/equity (RC) strategy with an RC level of 10% delivers annualised differential CER values (returns under risk-neutrality) of −1.7% with respect to the equity strategy and 3.9% with respect to the cash strategy. The last of these is significantly different from zero (at the 1% level). Second, as the risk aversion parameter value increases, the performance of the cash/equity (RC) strategy improves with respect to (more risky) equity (becoming significant when $\gamma \geq 8$) and worsens with respect to (less risky) cash (becoming insignificant when $\gamma \geq 8$). Importantly, this improvement with respect to equity occurs when the equity-calibrated RC level is used – implying that the cash/equity (RC) strategy is capable of delivering equity-like returns with lower variance. Finally, the RC level alters these costs/benefits in predictable ways. In particular, as the RC level is increased, the cash/equity (RC) strategy becomes less cash-like and more equity-like. This affects the differential CER values and, to a slightly lesser extent, the significance of these values.

To gain additional insight into relative performance, the first row in Figure 2 provides plots of the desirability of the cash/equity (RC) strategies (each with a different RC level) against the cash and equity strategy benchmarks.[14] As risk aversion increases, the desirability decreases with respect to cash, and increases with respect to equity (for modest risk aversion levels only). Taking the product of these individual desirability values gives an overall picture of the desirability of the cash/equity (RC) strategies – see panel (c). This plot shows that there are differences over the cash/equity (RC) strategies, with low (high) RC level strategies more desirable for more (less) risk-averse investors. It is also noticeable that the desirability index exhibits an inverted U-shaped pattern over risk aversion space, reaching a peak level of around 0.9. This result ultimately reflects the tradeoff between the cash/equity (RC) strategy dominating cash at low risk aversion levels, and equity at high risk aversion levels.

[14]This particular desirability measure is given by the complement of the p-value associated with the null hypothesis that the cash/equity (RC) strategy performance is inferior or equal to the benchmark strategy performance.

Insert Figure 2 here

3.2.3 Constant transaction cost effects

The analysis thus far has assumed zero transaction costs. This assumption is now relaxed and the overall desirability of each strategy (with respect to all other strategies) is calculated. The results in Table 3 provide the risk aversion regions in which a particular strategy is dominant (henceforth referred to as the *dominant risk aversion region*), and are based on the following constant proportional transaction costs: $\kappa_1 = \kappa_2 = 0, 0.0005, 0.001, 0.0025, 0.005$. These costs are lower than the estimated values associated with trading individual shares within the S&P 500 index; see Hasbrouck (2009) for estimates of around 0.01 (100 basis points). However, they cover the most likely costs incurred when tracking the S&P 500 index using closely related securities. For instance, the mean daily bid-ask spread (based on the Roll estimator applied to S&P 500 index futures transaction data over the period 2006 to 2014) equals 0.0001465 (that is, 1.465 basis points); see Hasbrouck (2004) for similar evidence, and Poterba and Shoven (2002) who provide an estimate of 0.00096 (9.6 basis points) based on ETF data. Thus the middle value of 0.001 represents the transaction cost likely to be incurred when using ETF data.

Insert Table 3 here

The results confirm those in previous tables that equity is the dominant strategy for low risk aversion investors, cash dominates for high risk aversion investors, and the cash/equity (RC) strategy dominates for intermediate investors. Importantly, for low transaction costs the cash/equity (RC) strategy dominates for risk aversion regions that coincide with risk aversion levels observed in previous studies; see, e.g., Aït-Sahalia and Lo (2000) and Bliss and Panigirtzoglou (2004) for empirical evidence that the risk aversion levels of the typical investor under power utility range from 3 to 12. For instance, the cash/equity (RC) strategy with an RC level of 10%, and transaction costs of 10 basis points, dominates for $\gamma \in [4, 6]$.

The success of the cash/equity (RC) strategy erodes as transaction costs increase. Indeed for transaction costs of 25 basis points and higher, this success largely disappears.[15] Instead it is the cash and equity strategies that dominate. This change in performance is due to the lower turnover levels associated with the cash and equity strategies.

3.2.4 Sample effects

The recent introduction of RC indices by various providers may be due to a number of reasons. One possibility is that it simply reflects developments in the modeling and forecasting of volatility. Alternatively, the financial crisis has reminded investors of the downside risk associated with their equity positions. There is however a third, more cynical, rationale. It may be the case that cash/equity (RC) strategies have performed well empirically *only* over recent years. To examine this conjecture we re-estimate the dominant risk aversion regions using two sample periods: January 1, 1956 to December 31, 2005 and January 1, 2006 to December 31, 2014. The results associated with the cash/equity (RC) strategy with various RC levels are provided in Table 4 (pre-2006 period) and Table 5 (post-2006 period).

Insert Tables 4 & 5 here

The results provide clear evidence of the superior performance of the cash/equity (RC) strategy during the recent sample period only. For instance, under all (non-zero) transaction costs, the cash/equity (RC) strategy with the equity-calibrated RC level never dominates in the first sub-period. By contrast, in the second sub-period, this cash/equity (RC) strategy remains dominant over a wider range of risk aversion levels and up to and including transaction costs of 25 basis points.

[15] The impact of transaction costs has a large impact on the cash/equity (RC) strategy because of the frequent updating of the leverage factor values as new information becomes available. It follows that these costs can be mitigated by updating less often – though the mitigation effects are minor. Results associated with lower frequency updating are available on request.

The plots in Figure 2 provide additional proof of the superior performance of the cash/equity (RC) strategy over the latter sub-period. In particular, the second row (pre-2006 data) and third row (post-2006 data) of this figure provide plots of the desirability of the cash/equity (RC) strategy with various RC levels in comparison to the other strategies. A key factor driving the difference in overall desirability over the two samples is the superior performance of the cash/equity (RC) strategy with the equity-calibrated RC level over the equity strategy. In panels (e) and (h) of Figure 2, we see that as risk aversion levels increase, this cash/equity (RC) strategy is more desirable than the equity strategy, with this desirability stronger in the post-2006 period.

It is therefore natural to ask why the cash/equity (RC) strategy with the equity-calibrated RC level performs so well in the post-2006 period. Recall that the returns to the equity strategy exhibit time-varying levels of volatility, while the cash/equity (RC) strategy returns do not. It follows that there are periods when the equity strategy is exposed to high volatility levels, whereas the cash/equity (RC) strategy is exposed to a constant level of volatility. If the high volatility levels deliver high returns (and these more than compensate for the low returns delivered during the low volatility periods) then the performance of the equity strategy would be superior to the cash/equity (RC) strategy.

The contrary evidence to this prediction observed in the post-2006 sample is consistent with the empirical results concerning the break down in the positive and linear risk-return relationship around the financial crisis documented in Ghysels et al. (2013) and Adrian et al. (2015), and with the argument that this period is characterised by flight to safety behaviour (Baele et al., 2014). The latter behaviour is characterised by periods of market stress (high equity return volatility) and low equity returns as investors switch to safer assets. Under such conditions users of the equity strategy will experience high volatility and low returns leading to inferior performance.

3.2.5 Time-varying transaction cost effects

The evidence is Figure 1 shows that proportional transaction costs vary over time. Moreover, it is quite possible that they are at their highest level precisely when the cash/equity (RC) strategies are most active. The product of these effects may result in very high *total* transaction costs (see (6)), rendering the cash/equity (RC) strategies inferior to competing strategies. To examine this conjecture we recalculate the dominance regions based on trading inclusive of time-varying proportional transaction costs, denoted κ_t, as given the Roll bid-ask spread estimator applied to S&P 500 index futures transaction data. Specifically, we re-estimate the dominant risk aversion regions over the sample periods: April 21, 1982 to December 31, 2015, and the associated pre and post-2006 sub-periods.[16] Moreover, as actual transaction costs are likely to be higher than the estimated bid-ask spreads (because of broker commissions, or use of ETF securities with higher transaction costs) we multiply the estimated bid-ask spreads by one of the following: $\tau = 0, 1, 10, 25, 50$. This amounts to replacing κ_1 and κ_2 in (6) with $\tau\kappa_t$. The results associated with the cash/equity (RC) strategy (using the equity-calibrated RC level) are provided in Table 6.

Insert Table 6 here

The results demonstrate that when proportional transaction costs are given by the estimated bid-ask spread (that is, $\tau = 1$), the cash/equity (RC) strategy has a dominant risk aversion region from 1 to 7 (full sample), 1 to 7 (pre-2006 sample) and 0 to 8 (post-2006 sample). Moreover, as τ increases, differences in the results over the samples emerge. In particular, the post-2006 sample results indicate that the cash/equity (RC) strategy is able to dominate the other strategies even over high τ values. For instance, during this period, this strategy has a dominant risk aversion region from 1 to 7 even when proportional transaction costs are ten

[16] The start of this sample period represents the earliest date on which S&P 500 index futures data are available.

times higher than the estimated bid-ask spreads (that is, $\tau = 10$). By contrast, during the pre-2006 period, this strategy is not dominant over any risk aversion region when $\tau > 1$. Thus, this evidence does not support the conjecture that cash/equity (RC) relative performance in the post-2006 period is driven by the insufficient account of high transaction costs observed during this period.

3.3 Dynamic performance analysis

The sample effects documented above can be further examined by considering relative performance at each point in time. To this end, we develop a new methodology based on the use of intraday data.

3.3.1 Measuring performance

The above analysis assesses average performance over the entire sample period; see use of the unconditional expectation in the definition of the CER in equation (7). However, as the cash/equity (RC) strategy is conditional in nature it may offers benefits not captured by the unconditional CER approach. To address this shortcoming we augment the analysis by developing a CER measure that is allowed to vary continuously over the sample. This is achieved via use of intraday data (cf. the interday data used previously).

The new CER measure is based on repeated calculation of expectations based on intraday data, where each day t has unit length with the full grid of all observation points given by $G = \{t_1, \ldots, t_M\}$. Specifically, we replace the full sample averages in (10) with daily sample averages (based on intraday data), such that the MM estimator of the daily CER measure is given by the solution to the following set of $N + 1$ sample moment conditions:

$$g_M(\boldsymbol{\theta}_t) = \mathrm{E}_M[h(\mathbf{R}_{t_m}, \boldsymbol{\theta}_t)] = 0. \tag{16}$$

where $\boldsymbol{\theta}_t = (c_{0,t}, \delta_{1,t}, \ldots, \delta_{N,t})^\top$ and $\mathbf{R}_{t_m} = (R_{0,t_m}, R_{1,t_m}, \ldots, R_{N,t_m})^\top$. Here $\mathrm{E}_M[.]$ represents the sample average of intraday data observed each day. The vector-valued function is defined as $h(\mathbf{R}_{t_m}, \boldsymbol{\theta}_t) = u(c_{0,t} - \boldsymbol{\delta}_t) - u(\mathbf{R}_{t_m})$. In turn, we assume that investor utility is represented by the following second-order approximation to the instantaneous power utility function (*felicity function*):

$$u(\mathbf{R}_{t_m}) = \mathbf{R}_{t_m} - \frac{\gamma}{2}\mathbf{R}_{t_m}^2, \tag{17}$$

where γ is the relative risk aversion parameter.[17]

In our particular application $\mathbf{R}_{t_m}$ consists of intraday (five-minute frequency) returns to the cash/equity (RC) strategy, the equity only strategy ($R^e_{t_m}$), and the cash only strategy ($R^c_{t_m}$). If we assume that cash delivers zero returns then the above felicity function can be expressed as

$$u(\mathbf{R}_{t_m}) = \begin{bmatrix} L_t R^e_{t_m} - \frac{\gamma}{2} L_t^2 (R^e_{t_m})^2 \\ R^e_{t_m} - \frac{\gamma}{2}(R^e_{t_m})^2 \\ 0 \end{bmatrix}, \tag{18}$$

where L_t is the daily leverage factor value used in the cash/equity (RC) strategy.[18] Substituting this expression into the sample moment conditions given by (16) and rearranging leads to the following differential CER values:

$$\widehat{\delta}_{1,t} = -\sqrt{(1 - 2\gamma L_t M_{1,t} + \gamma^2 L_t^2 M_{2,t})/\gamma^2} + \sqrt{(1 - 2\gamma M_{1,t} + \gamma^2 M_{2,t})/\gamma^2}, \tag{19a}$$

$$\widehat{\delta}_{2,t} = -\sqrt{(1 - 2\gamma L_t M_{1,t} + \gamma^2 L_t^2 M_{2,t})/\gamma^2} + 1/\gamma. \tag{19b}$$

where $M_{1,t} = \mathrm{E}_M[R_{1,t_m}]$ and $M_{2,t} = \mathrm{E}_M[R^2_{1,t_m}]$ are the first and second daily realised moments of intraday equity returns.

[17]The analysis can easily be extended to higher order approximations. The virtue of using the second-order approximation is that it enables derivation of compact formulae relating to relative performance.

[18]The assumption of zero returns to cash enables derivation of compact formulae.

The differential CER values of the cash/equity (RC) strategy with the equity-calibrated RC level in comparison to the equity strategy are provided in Figure 3. The risk aversion parameters used correspond to assuming that investors are risk-neutral ($\gamma = 0$) or highly risk-averse ($\gamma = 32$). These values are based on the use of intraday data observed over the period January 1, 1990 to December 31, 2014.[19]

Insert Figure 3 here

The kernel-smoothed plots in Figure 3 demonstrate that the relative performance of the cash/equity (RC) strategy varies considerably over time. Moreover, for risk-averse investors ($\gamma = 32$), this strategy appears to enjoy two distinct episodes of superior performance. These coincide with the high volatility periods observed in the aftermath of the 2001 US recession and in the 2008/09 financial crisis. During these episodes, the utility to the equity strategy falls as there is insufficient reward to taking on the extra risk (that is, returns do not increase with risk during these periods). The net effect is an increase in the relative performance of the cash/equity (RC) strategy during these periods. This result in consistent with flight to safety behaviour.

The above results assume that the non-cash strategies track the S&P 500 index. As tracking the index is likely to be costly it could be argued that they are unrealistic. To address this issue, the analysis is repeated using closely related traded assets, namely, S&P 500 index futures contracts and the SPDR S&P 500 ETF Trust. The results presented in panels (b) and (c) of Figure 3 provide evidence that is similar to that given in panel (a). Thus, for investors who trade futures and ETF markets there appears to be similar dynamic variation in performance.

[19]This restricted sample period reflects the availability of the exogenous variables used in the subsequent analysis.

3.3.2 The determinants of performance

The next stage of the analysis examines the determinants of the relative performance of the cash/equity (RC) strategy with the equity-calibrated RC level. Five daily frequency exogenous variables are considered: lagged realised variance (RV), lagged variance risk premium (VRF), lagged flight to safety (FTS), lagged bid-ask spread (BAS), and lagged business conditions (ADS) variables.[20] The relationship between relative performance and these exogenous variables is investigated using the following time series regression:

$$\widehat{\delta}_{i,t} = \alpha_{i,0}+\alpha_{i,1}\mathrm{RV}_{t-1}+\alpha_{i,2}\mathrm{VRP}_{t-1}+\alpha_{i,3}\mathrm{FTS}_{t-1}+\alpha_{i,4}\mathrm{BAS}_{t-1}+\alpha_{i,5}\mathrm{ADS}_{t-1}+\epsilon_{i,t}, \quad t = 1, 2, \ldots, T, \tag{20}$$

for $i = 1, 2$. The RV variable is constructed by summing squared five minute frequency S&P 500 index returns, the VRP variable is constructed by taking the difference between the square of the VIX index and expectations of RV, the FTS variable represents the FTS probability based on a threshold model applied to the ordinal FTS index used in Baele et al. (2014), and the BAS variable is constructed using the method proposed by Roll (1984) applied to all trades in the S&P 500 index futures contract with shortest maturity.[21]

Use of the RV variable is motivated by the results presented above that appear to show that performance is affected by market volatility. The remaining variables attempt to measure risks that are not captured by the second-order felicity function used in the analysis. In particular, VRP has been interpreted as a measure of the representative agent's risk aversion (see, e.g., Rosenberg and Engle, 2002, Bakshi and Madan, 2006, and Bollerslev et al., 2008), or macroeconomic uncertainty risk (see, e.g., Bollerslev et al., 2009, and Drechsler and Yaron, 2008). The FTS variable captures FTS episodes whereby a switch from equity to bonds occurs during peri-

[20] The use of lagged variables establishes the direction of causality from the variables to relative performance.

[21] The heterogeneous autoregressive (HAR) model introduced by Corsi (2009) is used to generate forecasts of future realised variance. See Bekaert and Hoerova (2014) for a similar approach to measuring the VRP.

ods of market stress (see, e.g., Baele et al., 2014, for evidence of the relationship between FTS and market liquidity). The BAS variable attempts to capture liquidity constraints that occur during periods of financial market turmoil (Nagel, 2012). Finally, the ADS business conditions variable attempts to capture other forms of risk that covary with the macroeconomy.

The results in panel (a) of Table 7 contain the ordinary least squares (OLS) standardised coefficients associated with the above regression for the cash/equity (RC) strategy with the equity-calibrated RC level against the equity benchmark strategy where $\gamma = 0, 1, 2, 4, 8, 16, 32$. These results are based on using data observed over the period January 1, 1990 to December 31, 2014.[22]

Insert Table 7 here

The results reveal a number of important features. First, as aversion to risk increases, all exogenous variables appear to have increasing explanatory power. For instance, the adjusted R^2 monotonically increases from 1.6% (when $\gamma = 0$) to 14.8% (when $\gamma = 32$). Second, all five exogenous variables appear to have a role in predicting future cash/equity (RC) performance, with RV having the greatest economic significance (as evinced by the standardised OLS coefficients). Moreover, for risk-averse investors the majority of the effects are consistent with their theoretical (risk) interpretations such that RV, FTS and BAS have positive and significant coefficients, while ADS has a negative and significant coefficient.[23] Similar results hold when using futures and ETF data; see panels (b) and (c) in Table 7. Therefore, the relative performance of the cash/equity (RC) strategy appears to be determined by risk factors relating to macroeconomic and liquidity conditions.

It is also of interest to consider the extent to which the exogenous variables are able to

[22]The sample period represents the longest span of available exogenous variable data.

[23]A negative and significant VRP coefficient is observed under risk neutrality ($\gamma = 0$). This result is consistent with the positive relationship between equity returns and VRP found in the literature; see, e.g., Bollerslev et al. (2009).

explain the large increases in differential CER values particularly in the aftermath of the 2001 US recession and during the 2008/09 financial crisis. To this end, we plot the differential CER values after adjusting for the effects of the exogenous variables (achieved by plotting the residuals from (20)). The results in panels (d), (e) and (f) in Figure 3 show that for the index, futures and ETF market uses, the large increases are considerably reduced with flatter profiles apparent. Thus, we are able to rationalise the performance peaks via the effects of the exogenous variables.

3.3.3 Equilibrium risk preferences

The above framework assumes that investor risk aversion is constant over time. However, a number of theoretical asset pricing models have been developed that require the representative agent to have time-varying risk aversion in order to explain features such as stock return predictability or the equity premium (see, e.g., Campbell and Cochrane, 1999). This assumption can be investigated by calculating the risk aversion levels that drive differential CER values to zero at all points in time. As we have a differential CER value for each benchmark then it follows that we have two 'equilibrium' γ levels (at each point in time). The above formulae for $\delta_{1,t}$ and $\delta_{2,t}$ are equated to zero and solved for γ to give the following equilibrium γ values:

$$\widehat{\gamma}^*_{1,t} = 2M_{1,t}/(1 + L_t)M_{2,t}, \tag{21a}$$

$$\widehat{\gamma}^*_{2,t} = 2M_{1,t}/L_t M_{2,t}. \tag{21b}$$

Here $\widehat{\gamma}^*_{1,t}$ and $\widehat{\gamma}^*_{2,t}$ correspond to equilibrium γ values when using the equity and cash strategy benchmarks, respectively. If $M_{1,t}$ is positive (negative) then $\widehat{\gamma}^*_{2,t}$ is greater (less) than $\widehat{\gamma}^*_{1,t}$.

Kernel-smoothed plots of the equilibrium γ values are provided in Figure 4. Only $\widehat{\gamma}^*_{1,t}$ values are presented as we confine attention to the cash/equity (RC) strategy with the equity-calibrated RC level. These are based on using index, futures and ETF data observed over the period January 1, 1990 (February 1, 1993 when using ETF data) to December 31, 2014.

Insert Figure 4 here

The plots in Figure 4 show that there is considerable variation in risk attitudes over time. During the early 2000s the bear market was sufficiently intense ($M_{1,t} < 0$) that all equilibrium γ values become negative. Since this period (and particularly after the 2008/09 financial crisis) investors appear to have become increasingly averse to risk. To examine whether this finding is due to the use of a particular felicity function, we repeat the analysis using a fourth-order felicity function.[24] The results confirm that there has indeed been an increase in risk aversion levels over recent years – a finding consistent with the survey based evidence of Guiso et al. (2013). Thus it is quite possible that if investors have a utility function with time-varying risk aversion then cash/equity (RC) strategies may provide little economic value (that is, deliver differential CER values close to zero).

4 Conclusion

Cash/equity (RC) strategies provide a safer investment during periods of high volatility than a naked equity position, yet permit exposure to the equity market during calm periods. The net result is that they provide investors with a middle ground in terms of risk and return. For reasonable risk aversion parameter values and transaction costs, such strategies dominate cash only and equity only strategies. However, this dominance decays as transaction costs increase and appears to be highly sensitive to the sample period used. Additional analysis reveals that

[24]This function is given by

$$u(\mathbf{R}_{t_m}) = \mathbf{R}_{t_m} - \frac{\gamma}{2}\mathbf{R}_{t_m}^2 + \frac{\gamma(1+\gamma)}{6}\mathbf{R}_{t_m}^3 - \frac{\gamma(\gamma+1)(\gamma+2)}{24}\mathbf{R}_{t_m}^4,$$

where previous notation is maintained.

risk factors based on realised volatility, bid-ask spreads, flight to safety effects and business conditions contribute to the observed performance of cash/equity (RC) strategies.

The results of this paper have important implications for investors and the designers of financial products. On the demand side, fund managers should be wary of tracking RC indices unless they can trade at exceptionally low transaction costs. Moreover, they should confine tracking to periods in which the benefits have economic value adjusted for risks based on macroeconomic and liquidity conditions. On the supply side, for providers of indices it may be fruitful to invest more effort into improving the quality of RC indices via use of more accurate volatility forecasting models. Models based on the use of realised volatility that incorporate options-based implied volatility measures such as the VIX index may prove fruitful.

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Table 1 – Summary statistics (official and synthetic data)

		Statistic				
Strategy	RC Level	MN	VL	SK	KT	SR
Panel A: 2006 to 2014						
Cash/Equity	5.0%	4.415	5.035	−0.619	6.082	0.531
(official RC)	7.0%	5.960	6.962	−0.548	5.554	0.606
	7.5%	6.259	7.460	−0.548	5.549	0.605
	10.0%	7.308	10.068	−0.621	6.099	0.553
	12.0%	8.511	12.042	−0.580	5.706	0.562
	15.0%	10.233	14.786	−0.519	5.262	0.574
Cash/Equity	5.0%	4.629	5.123	−0.435	7.235	0.607
(synthetic RC)	7.0%	5.740	7.125	−0.564	6.347	0.592
	7.5%	6.018	7.628	−0.578	6.278	0.590
	10.0%	7.395	10.143	−0.616	6.145	0.579
	12.0%	8.574	12.092	−0.575	5.688	0.583
	15.0%	10.244	14.791	−0.521	5.255	0.590
Equity		9.950	21.136	−0.080	13.432	0.399
Cash (Fed Funds)		1.432	0.162	2.677	10.824	0.000
Cash (1D LIBOR)		1.518	0.165	2.605	10.366	0.000
Panel B: 1956 to 2014						
Cash/Equity	5.0%	7.155	5.071	−0.484	8.567	0.374
(synthetic RC)	7.0%	7.937	7.092	−0.477	8.523	0.377
	7.5%	8.136	7.596	−0.476	8.525	0.379
	10.0%	9.132	10.080	−0.473	8.626	0.384
	12.0%	9.940	11.973	−0.462	8.758	0.391
	15.0%	11.014	14.430	−0.468	9.303	0.399
Equity		10.846	15.618	−0.557	22.134	0.358
Cash (Fed Funds)		5.261	0.334	2.593	13.144	0.000

Notes: This table contains the mean (MN), volatility (VL), skewness (SK), kurtosis (KT) and Sharpe ratio (SR) (with MN and VL given in annual percentage terms) associated with the cash, equity, and official and synthetic cash/equity (RC) strategies. Strategies are based on official RC indices that employ the S&P 500 equity index and overnight (1D) USD LIBOR interest rates, or on strategies based on synthetic RC indices that employ the S&P 500 equity index and federal funds rates. The first panel uses data observed from January 1, 2006 to December 31, 2014, and second uses data observed from January 1, 1956 to December 31, 2014.

Table 2 – The costs/benefits of cash/equity (RC) strategies

	Risk Aversion Parameter (γ)						
Benchmark Strategy	0	1	2	4	8	16	32
Panel A: 5.0% RC level							
Equity	−0.037	−0.026	−0.015	0.007	0.054**	0.162**	0.756*
Cash	0.019**	0.018**	0.016**	0.014*	0.009	−0.002	−0.023
Panel B: 7.5% RC level							
Equity	−0.027	−0.018	−0.008	0.011	0.051**	0.145**	0.711
Cash	0.029**	0.026**	0.023*	0.017*	0.006	−0.018	−0.067
Panel C: 10.0% RC level							
Equity	−0.017	−0.010	−0.003	0.012	0.043**	0.119**	0.643
Cash	0.039**	0.034**	0.029*	0.018	−0.002	−0.044	−0.135
Panel D: 12.5% RC level							
Equity	−0.007	−0.002	0.002	0.012	0.032**	0.085**	0.553
Cash	0.049**	0.041**	0.033*	0.018	−0.014	−0.078	−0.226
Panel E: 15.0% RC level							
Equity	0.002	0.003	0.005	0.009	0.018*	0.047*	0.439
Cash	0.058**	0.047**	0.037*	0.016	−0.027	−0.117	−0.340
Panel F: Equity-calibrated RC level							
Equity	0.000	0.002	0.005	0.010	0.021*	0.056**	0.466
Cash	0.056**	0.046**	0.036*	0.016	−0.024	−0.108	−0.312

Notes: This table contains differential CER estimates (given in annualised terms) associated with cash/equity (RC) strategies (with various RC levels) against various benchmark strategies. All strategies assume that investors have power utility, and are based on data observed over the period January 1, 1956 to December 31, 2014. Rejections of the null that the differential CER estimates are less than or equal to zero are given by * (5% significance) and ** (1% significance).

Table 3 – Dominant risk aversion regions (constant transaction costs, full sample)

	Transaction Cost Level (in basis points)									
	0		5		10		25		50	
Strategy	L	U	L	U	L	U	L	U	L	U
Panel A: 5.0% RC level										
Equity	0	3	0	3	0	4	0	4	0	4
Cash/Equity (RC)	4	14	4	12	4	10	5	5		
Cash	15	C	13	C	11	C	6	C	5	C
Panel B: 7.5% RC level										
Equity	0	2	0	2	0	3	0	4	0	4
Cash/Equity (RC)	3	9	3	8	4	7				
Cash	10	C	9	C	8	C	5	C	5	C
Panel C: 10.0% RC level										
Equity	0	2	0	2	0	3	0	4	0	4
Cash/Equity (RC)	3	7	3	6	4	6				
Cash	8	C	7	C	7	C	5	C	5	C
Panel D: 12.5% RC level										
Equity	0	1	0	2	0	3	0	4	0	4
Cash/Equity (RC)	2	6	3	5	4	5				
Cash	7	C	6	C	6	C	5	C	5	C
Panel E: 15.0% RC level										
Equity			0	1	0	3	0	4	0	4
Cash/Equity (RC)	0	5	2	5	4	4				
Cash	6	C	6	C	5	C	5	C	5	C
Panel F: Equity-calibrated RC level										
Equity			0	1	0	3	0	4	0	4
Cash/Equity (RC)	0	5	2	5	4	4				
Cash	6	C	6	C	5	C	5	C	5	C

Notes: This table contains the dominant risk aversion regions (lower and upper points, L and U) for the cash/equity (RC) strategy (with various RC levels), the cash strategy and the equity strategy. Various constant proportional transaction costs are assumed. The constant C represents an unspecified number greater than 100. All strategies assume that investors have power utility, and are based on data observed over the period January 1, 1956 to December 31, 2014.

Table 4 – Dominant risk aversion regions (constant transaction costs, pre-2006 sample)

	Transaction Cost Level (in basis points)									
	0		5		10		25		50	
Strategy	L	U	L	U	L	U	L	U	L	U
Panel A: 5.0% RC level										
Equity	0	3	0	3	0	3	0	4	0	4
Cash/Equity (RC)	4	13	4	11	4	9				
Cash	14	C	12	C	10	C	5	C	5	C
Panel B: 7.5% RC level										
Equity	0	3	0	3	0	3	0	4	0	4
Cash/Equity (RC)	4	9	4	7	4	6				
Cash	10	C	8	C	7	C	5	C	5	C
Panel C: 10.0% RC level										
Equity	0	2	0	3	0	4	0	4	0	4
Cash/Equity (RC)	3	7	4	6	5	5				
Cash	8	C	7	C	6	C	5	C	5	C
Panel D: 12.5% RC level										
Equity	0	2	0	3	0	4	0	4	0	4
Cash/Equity (RC)	3	6	4	5						
Cash	7	C	6	C	5	C	5	C	5	C
Panel E: 15.0% RC level										
Equity			0	4	0	4	0	4	0	4
Cash/Equity (RC)	0	5								
Cash	6	C	5	C	5	C	5	C	5	C
Panel F: Equity-calibrated RC level										
Equity			0	4	0	4	0	4	0	4
Cash/Equity (RC)	0	5								
Cash	6	C	5	C	5	C	5	C	5	C

Notes: This table contains the dominant risk aversion regions (lower and upper points, L and U) for the cash/equity (RC) strategy (with various RC levels), the cash strategy and the equity strategy. Various constant proportional transaction costs are assumed. The constant C represents an unspecified number greater than 100. All strategies assume that investors have power utility, and are based on data observed over the period January 1, 1956 to December 31, 2005.

Table 5 – Dominant risk aversion regions (constant transaction costs, post-2006 sample)

	Transaction Cost Level (in basis points)									
	0		5		10		25		50	
Strategy	L	U	L	U	L	U	L	U	L	U
Panel A: 5.0% RC level										
Equity	0	2	0	2	0	2	0	2	0	3
Cash/Equity (RC)	3	23	3	22	3	20	3	15	4	8
Cash	24	C	23	C	21	C	16	C	9	C
Panel B: 7.5% RC level										
Equity	0	1	0	1	0	1	0	2	0	3
Cash/Equity (RC)	2	15	2	14	2	13	3	10	4	5
Cash	16	C	15	C	14	C	11	C	6	C
Panel C: 10.0% RC level										
Equity	0	1	0	1	0	1	0	2	0	3
Cash/Equity (RC)	2	11	2	10	2	9	3	7		
Cash	12	C	11	C	10	C	8	C	4	C
Panel D: 12.5% RC level										
Equity	0	0	0	0	0	1	0	2	0	3
Cash/Equity (RC)	1	9	1	8	2	8	3	6		
Cash	10	C	9	C	9	C	7	C	4	C
Panel E: 15.0% RC level										
Equity			0	0	0	0	0	1	0	3
Cash/Equity (RC)	0	8	1	7	1	7	2	6		
Cash	9	C	8	C	8	C	7	C	4	C
Panel F: Equity-calibrated RC level										
Equity			0	0	0	0	0	1	0	3
Cash/Equity (RC)	0	8	1	7	1	7	2	6		
Cash	9	C	8	C	8	C	7	C	4	C

Notes: This table contains the dominant risk aversion regions (lower and upper points, L and U) for the cash/equity (RC) strategy (with various RC levels), the cash strategy and the equity strategy. Various constant proportional transaction costs are assumed. The constant C represents an unspecified number greater than 100. All strategies assume that investors have power utility, and are based on data observed over the period January 1, 2006 to December 31, 2014.

Table 6 – Dominant risk aversion regions (time-varying transaction costs)

	Transaction Cost Multiple (τ)									
	0		1		10		25		50	
Strategy	L	U	L	U	L	U	L	U	L	U
Panel A: Full sample										
Equity			0	0	0	2	0	5	0	5
Cash/Equity (RC)	0	7	1	7	3	6				
Cash	8	C	8	C	7	C	6	C	6	C
Panel B: Pre-2006 sample										
Equity			0	0	0	5	0	6	0	6
Cash/Equity (RC)	0	7	1	7						
Cash	8	C	8	C	6	C	7	C	7	C
Panel C: Post-2006 sample										
Equity					0	0	0	2	0	3
Cash/Equity (RC)	0	8	0	8	1	7	3	5		
Cash	9	C	9	C	8	C	6	C	4	C

Notes: This table contains the dominant risk aversion regions (lower and upper points, L and U) for the cash/equity (RC) strategy (using the equity-calibrated RC level), the cash strategy and the equity strategy. Proportional transaction costs are given by a multiple (τ) times the estimated daily bid-ask spread. The constant C represents an unspecified number greater than 100. All strategies assume that investors have power utility, and are based on data observed over the periods: April 21, 1982 to December 31, 2014 (panel (a)), April 21, 1982 to December 31, 2005 (panel (b)), and January 1, 2006 to December 31, 2014 (panel (c)).

Table 7 – The determinants of cash/equity (RC) strategy performance

	Risk Aversion Parameter (γ)						
Regression Coefficient	0	1	2	4	8	16	32
Panel A: Index-based performance determinants							
RV	−0.007	0.003	0.014	0.035*	0.075**	0.151**	0.268**
VRP	−0.149**	−0.145**	−0.141**	−0.133**	−0.115**	−0.081**	−0.019
FTS	0.067**	0.068**	0.068**	0.068**	0.067**	0.066**	0.060**
BAS	0.051**	0.052**	0.054**	0.058**	0.065**	0.076**	0.092**
ADS	−0.021	−0.024	−0.026	−0.030*	−0.039**	−0.054**	−0.077**
Adjusted R^2	0.016**	0.015**	0.015**	0.016**	0.023**	0.054**	0.148**
Panel B: Futures-based performance determinants							
RV	−0.004	0.008	0.019	0.042*	0.086**	0.167**	0.288**
VRP	−0.150**	−0.145**	−0.140**	−0.130**	−0.110**	−0.071**	−0.005
FTS	0.068**	0.069**	0.069**	0.070**	0.070**	0.070**	0.066**
BAS	0.048**	0.049**	0.051**	0.054**	0.059**	0.068**	0.078**
ADS	−0.023	−0.026	−0.028	−0.032*	−0.041**	−0.056**	−0.077**
Adjusted R^2	0.016**	0.015**	0.015**	0.017**	0.025**	0.060**	0.163**
Panel C: ETF-based performance determinants							
RV	−0.008	0.002	0.013	0.034	0.074**	0.147**	0.255**
VRP	−0.156**	−0.150**	−0.144**	−0.132**	−0.108**	−0.061**	0.017
FTS	0.077**	0.077**	0.077**	0.077**	0.077**	0.076**	0.069**
BAS	0.055**	0.057**	0.058**	0.062**	0.068**	0.078**	0.090**
ADS	−0.023	−0.026	−0.029	−0.035*	−0.047**	−0.067**	−0.096**
Adjusted R^2	0.018**	0.017**	0.017**	0.018**	0.026**	0.062**	0.165**

Notes: This table contains the parameters (standardised coefficients and adjusted R^2 statistics) associated with regressions of daily differential CER values (against the equity benchmark) on the lagged realised variance (RV), lagged variance risk premium (VRP), lagged flight to safety (FTS), lagged bid-ask spread (BAS), and lagged business conditions (ADS) variables. The cash/equity (RC) strategy uses the synthetic RC index with the equity-calibrated RC level. All strategies assume that investors have a second-order felicity function, and are based on S&P 500 index return data (panel (a)), S&P 500 index futures return data (panel (b)) or S&P 500 index ETF return data (panel (c)) observed over the period January 1, 1990 (February 1, 1993 for ETF data) to December 31, 2014. Rejections of the null that each coefficient (tested via a conventional t-test) or overall fit (tested via a conventional F-test) equal zero are given by * (5% significance) and ** (1% significance).

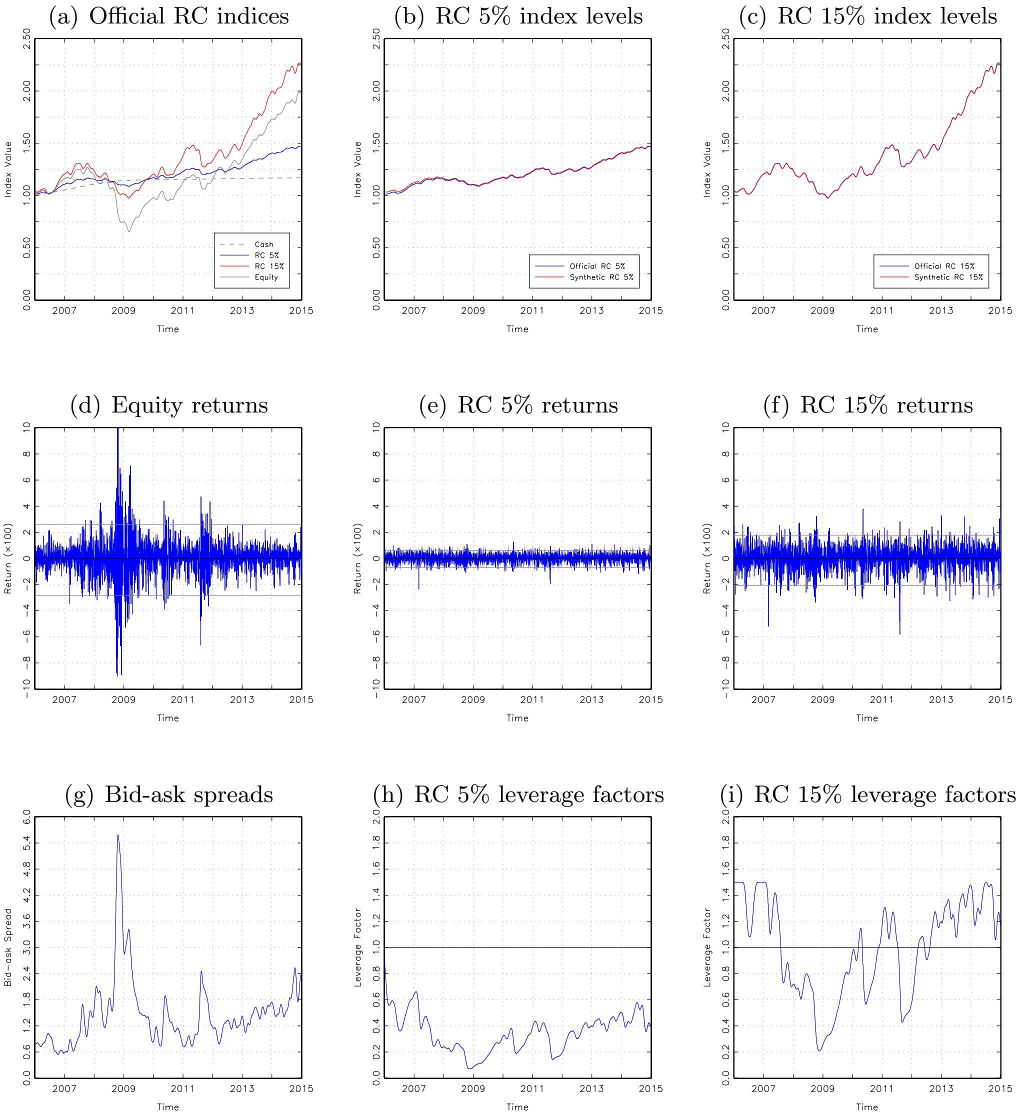

Figure 1 – Strategy performance

This figure contains time series plots of the cumulative returns to the cash, equity, and cash/equity (RC) strategies (with 5% and 15% RC levels). The cash/equity (RC) strategies in panel (a) are based on synthetic RC data, and in panels (b) and (c) are based on official and synthetic RC data. The figure also contains time series plots of the equity strategy returns (panel (d)), the cash/equity (RC) strategy returns with 5% and 15% RC levels (panels (e) and (f), respectively), bid-ask spreads (panel (g)), and the cash/equity (RC) strategy leverage factor levels with 5% and 15% RC levels (panels (h) and (i), respectively). All plots are based on data observed over the period January 1, 2006 to December 31, 2014.

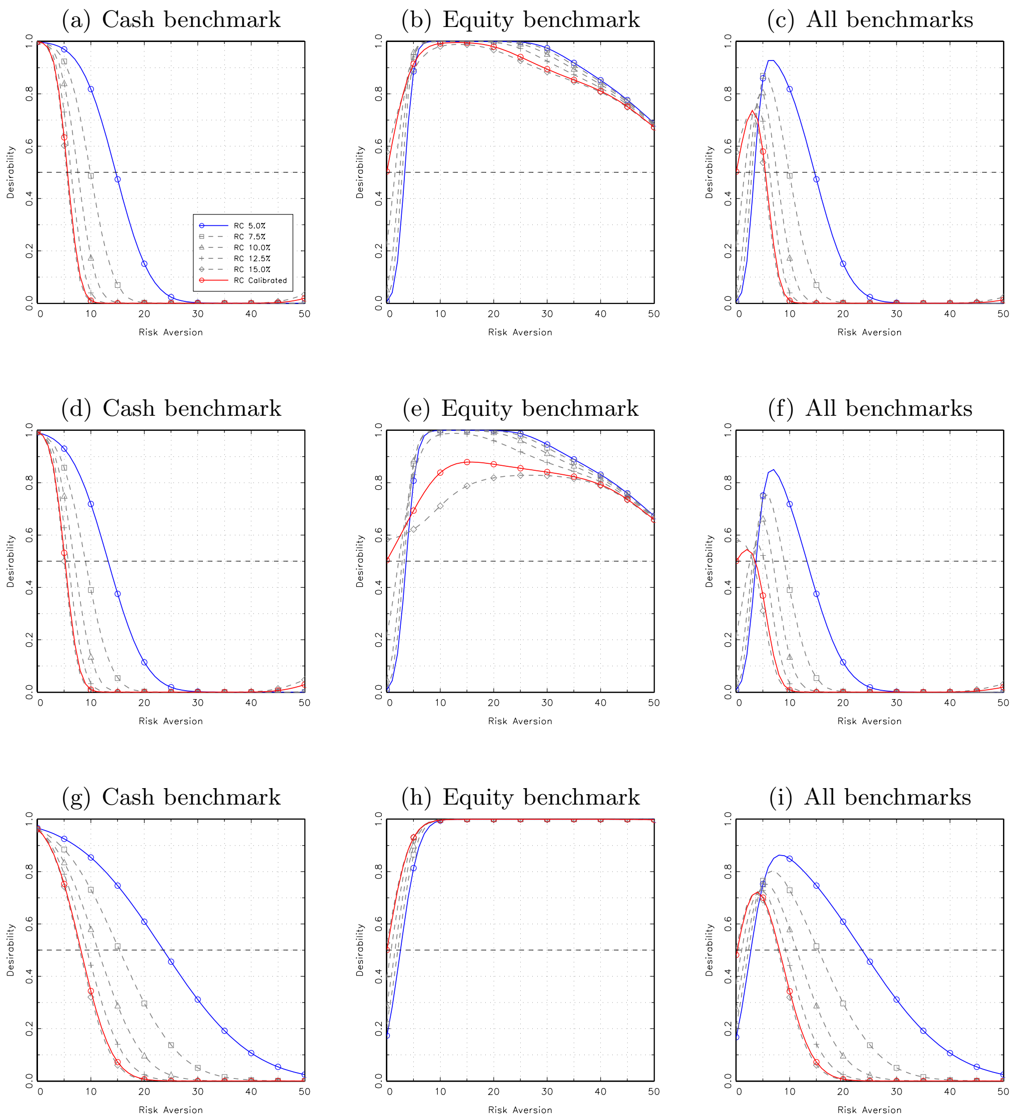

Figure 2 – Cash/equity (RC) strategy desirability
This figure contains plots of the desirability of the cash/equity (RC) strategies (with various RC levels) against the following benchmark strategies: cash, equity, and all of these strategies. These assume that investors have power utility. Panels (a) to (c) are based on data observed over the period January 1, 1956 to December 31, 2014; panels (d) to (f) are based on data observed over the period January 1, 1956 to December 31, 2005; and panels (g) to (i) are based on data observed over the period January 1, 2006 to December 31, 2014.

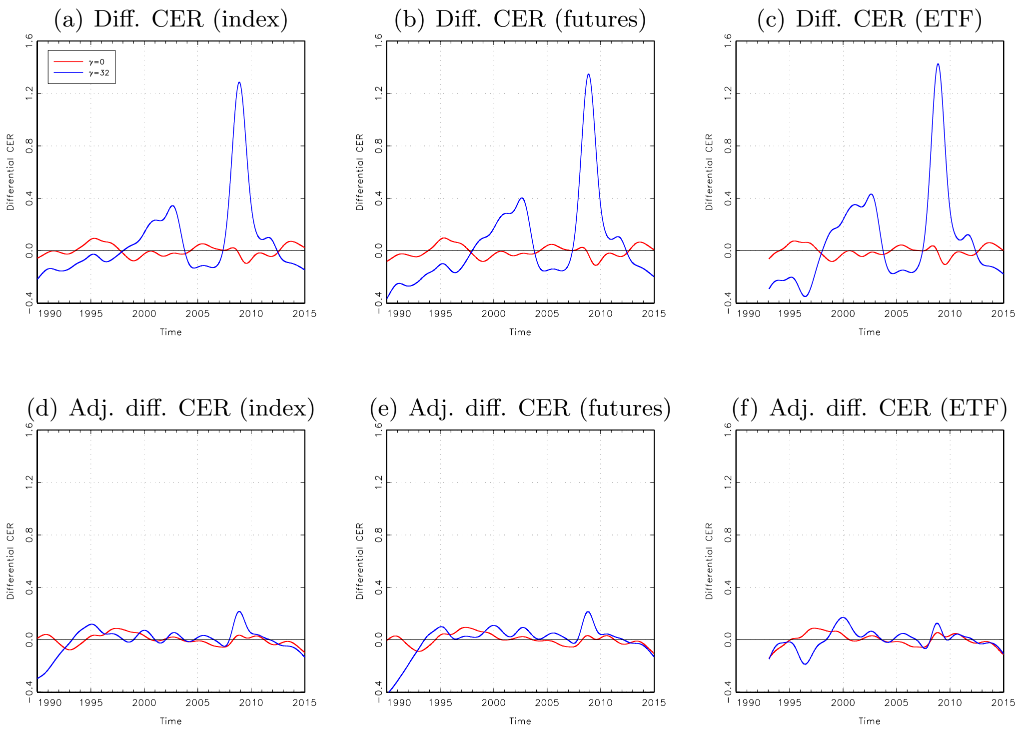

Figure 3 – Cash/equity (RC) strategy costs/benefits
This figure contains (kernel-smoothed) time series plots of daily realised differential certainty equivalent return values given in annualised terms (denoted *diff. CER*) associated with the cash/equity (RC) strategy (using the equity-calibrated RC level) against the equity strategy, and their counterparts adjusted for realised variance (RV), variance risk premium (VRP), flight to safety (FTS), bid-ask spread (BAS), and business conditions (ADS) effects (denoted *adj. diff. CER*). Higher (lower) diff. CER values indicate superior (inferior) cash/equity (RC) strategy performance. Results in panels (a) and (d) are based on index data, panels (b) and (e) are based on futures data, and panels (c) and (f) are based on ETF data. All strategies assume that investors have a second-order felicity function, and are based on index, futures or ETF data observed over the period January 1, 1990 (February 1, 1993 for ETF data) to December 31, 2014.

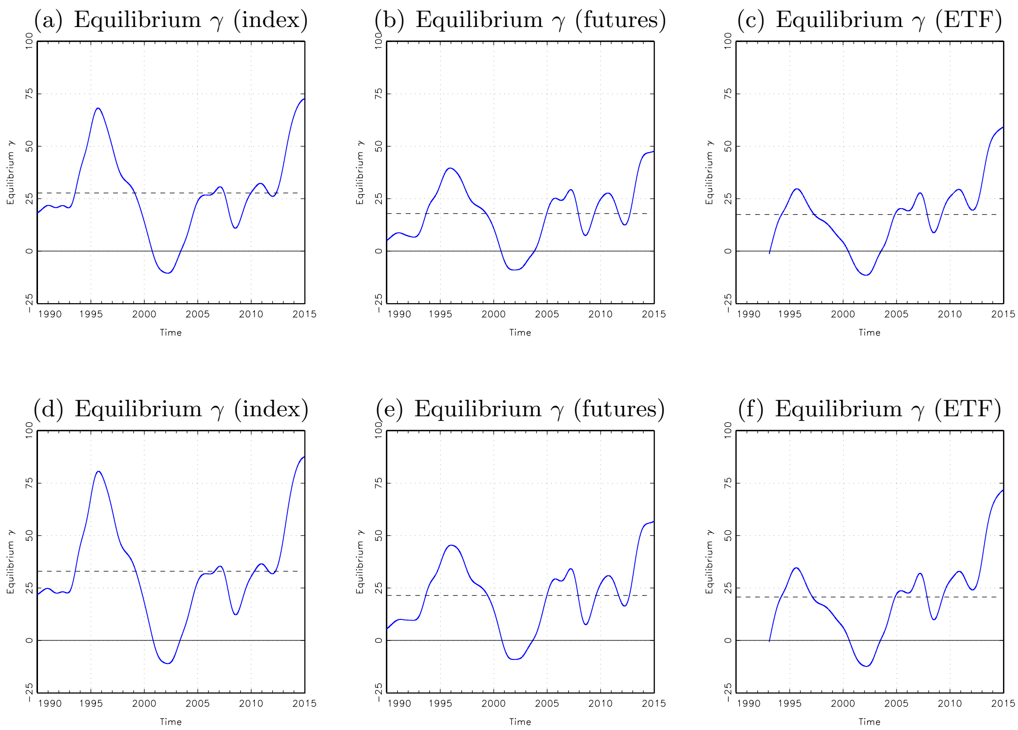

Figure 4 – Equilibrium risk preferences
This figure contains (kernel-smoothed) time series plots of the equilibrium risk preference parameter γ, such that the differential CER values associated with the cash/equity (RC) strategy (using the equity-calibrated RC level) against the equity strategy equal zero. The sample average of the equilibrium risk preference parameter is indicated by the dashed line. Results in panels (a) and (d) are based on index data, panels (b) and (e) are based on futures data, and panels (c) and (f) are based on ETF data. All strategies assume that investors have either a second-order felicity function (panels (a) to (c)) or a fourth-order felicity function (panels (d) to (f)), and are based on index, futures or ETF data observed over the period January 1, 1990 (February 1, 1993 for ETF data) to December 31, 2014.