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A Fuzzy Interval Valued Fusion Technique for Multi-Modal 3D Face Recognition

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Abstract—This paper proposes a fuzzy interval valued multi-criteria decision making (MCDM) technique that aggregates information from multi-modal feature sets during decision making in a 3D face recognition system. In this paper, an interval valued fuzzy TOPSIS technique is applied to a 3D face recognition system that is benchmarked against a set of databases. Such a system is shown to be useful in decision making when the choice of alternatives of the feature sets is combinatorial and complex.

Keywords—fuzzy interval, MCDM, TOPSIS, 3D Face Recognition, Range Data, Disparity Maps.

I. INTRODUCTION

The field of 3D Face Recognition (3DFR) as a biometric is advancing quite rapidly and feature extraction techniques in particular are gaining importance as they play a key role in dictating the performance of a recognition system. Previous work on multi-modal features for 3D face recognition by the first author relates to fusing information at the feature extraction level[1]. The fusion occurs through a linear combination of feature vectors. The results were shown to be promising and indicated that higher order features were useful in reducing the equal error rate (EER) and increasing the recognition rates.

The concept of multi-criteria decision making (MCDM) aims at ranking decision alternatives that comprise of a number of criteria involving groups or teams as decision makers (experts). Thus MCDM involves the selection of the best, from a set of alternatives, each of which is evaluated against multiple criteria. MCDM is particularly suitable for handling preference based aggregation and has the ability to handle both qualitative and quantitative criteria. MCDM techniques include ELECTRE (Elimination et Choice Translating Reality)[2], SAW(Simple Adaptive Weighting)[3], TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [4], AHP (Analytical Hierarchy Process)[4], ANP (Analytic Network Process)[5] and SMART (Simple Multi Attribute Rating Technique) [7] to name a few. Whilst there are several MCDM techniques in use, they lack the ability to consider preferences among alternatives. TOPSIS benefits from a logical process of evaluation of distance measures from ideal positive and negative solutions [8] which suits pattern recognition problems very well.

Fuzzy set theory provides a means to making decisions under uncertainty in determining and defining the decision making criteria. Further, interval valued fuzzy set theory aids in the modelling of fuzzy linguistic variables to resolve conflicts involving multiple decision makers in deciding the method of defining linguistic variables based on fuzzy sets. MCDM based decision making has recently been applied in the development of face recognition systems involving multiple criteria with varying weights of importance. In [9], interval type-2 fuzzy logic and fuzzy integrals are used for feature extraction in the training data followed by a relevance measure used in decision making for face recognition application. In [10], Sugeno Measures and Integrals are defined by type-2 fuzzy logic and applied for face recognition to deal with uncertainty during fusion from multiple sources of information. In this paper, an interval-valued fuzzy TOPSIS method proposed in [4] is adapted for the 3D face recognition application. A set of multi-modal feature sets are treated as decision makers and performance parameters as criteria to be optimised for a set of 3D face databases that are treated as the alternatives. By applying the MCDM approach of aggregating the performance measures against different databases provides a ranking mechanism based on a comprehensive analysis of the feature modalities used for recognition. The ratings of the criteria and the importance weight of the criteria as determined a priori by the decision makers are modelled by triangular fuzzy membership functions. This technique defines the ratings and weights of criteria as interval-valued fuzzy numbers.

The rest of the paper is organised as follows: Section II introduces TOPSIS formally as an MCDM approach and provides an algorithmic system design using TOPSIS. Section III sets up the interval valued fuzzy set formalisation for TOPSIS. It also details out the algorithmic steps necessary for fuzzy TOPSIS. Section IV provides details of the 3DFR system considered and its performance characteristics are identified for the application of TOPSIS. In Section V, interval valued fuzzy TOPSIS is applied to 3DFR and the results are analysed. Section VI provides a conclusion from the results and further work.

II. MULTI-CRITERIA DECISION MAKING (MCDM) AND TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is an MCDM technique used in multi-objective problems involving group decision making through an aggregation process that identifies solutions from a finite set of alternatives. Each criterion is associated with performance ratings and importance measures based on voting. In fuzzy MCDM problems, performance ratings of criteria and their importance measures are treated as formal fuzzy numbers. As discussed in Section I, determining precisely such fuzzy numbers is difficult, and hence the need to describe them using interval values fuzzy numbers. This section defines the related MCDM terminology and the generic TOPSIS algorithm.

A. MCDM Terminology

A set of important terms are defined for use in MCDM:

- **Alternatives** - these relate to the available options from which ranked selections are made.
- **Criteria or Attribute** – a set of criteria or attributes that will impact the selection of the alternatives. An attribute is a property, quality or feature of alternatives being considered. These are termed as sub-criteria or sub-attribute. An alternative is
- **Weights** – provide relative importance of criteria provided by decision makers.
- **Importance of Weights-** a heuristic approach of importance of the weights.
- **Decision Makers (DMs)** – a set of experts providing weights to each criterion.
- **Decision Matrix** – a table that is used to make objective decisions from several options. DMs rate each criterion of each alternative.

An MCDM problem may be described by a decision matrix **D**. Suppose that there are m alternatives that are assessed by n attributes or criteria, then **D** is an $m \times n$ matrix. MCDM problem is typically described as a decision matrix as follows[4]:

$$\mathbf{D} = r_{ij} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix}$$

The set of alternatives is denoted by A_1, A_2, \dots, A_m and the criteria denoted by C_1, C_2, \dots, C_n and x_{ij} represent the rating of alternative A_i with respect to criteria C_j with each element x_{ij} representing the j th criteria of the i th alternative. When the ratings are described in linguistic terms, x_{ij} is replaced by r_{ij} . Similarly, the weights of criteria may also be described linguistically.

B. TOPSIS Algorithm

The general steps in TOPSIS are:

- **Step1(a) – Construct Normalised Decision Matrix** - a transform that maps various criteria dimensions into non-dimensional attributes allowing comparisons across criteria[11]. For normalisation, each column of the decision matrix is divided by the root of sum of squares of respective criterion. This is given by the equation:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}}, i = 1, 2, \dots, n; j = 1, 2, \dots, m \quad (1)$$

- **Step1 (b) – Construct Weighted Normalised Decision Matrix** - multiply columns of r_{ij} by its associated weights to obtain the weighted and normalised decision matrix:

$$VW_{ij} = \omega_j \times r_{ij}, i = 1, 2, \dots, n; j = 1, 2, \dots, m, \quad (2)$$

where $\omega_j \rightarrow$ weight of the j th criterion and

$$W = (\omega_{j1}, \omega_{j2}, \dots, \omega_{jm}) \mid \sum_{j=1}^m \omega_j = 1.$$

- **Step2 – Determine Ideal and Negative Ideal Solution** – a set of maximum values for each criterion is ideal solution. Similarly, a set of minimum values for each criterion is negative ideal solution. These respectively are calculated as follows:

$$A^* = \{v_1^*, v_2^*, \dots, v_m^*\} = (\cup_j v_{ij} \mid j \in \Omega_b), (\cap_j v_{ij} \mid j \in \Omega_c) \quad (3)$$

$$A^- = \{v_1^-, v_2^-, \dots, v_m^-\} = (\cup_j v_{ij} \mid j \in \Omega_c), (\cap_j v_{ij} \mid j \in \Omega_b) \quad (4)$$

where Ω_c and Ω_b are the maximisation (profit category) and minimisation (cost category) criteria respectively[11].

- **Step4 – Determine separation from ideal solutions** – determine separation from ideal solution and closeness to negative ideal solution respectively:

$$S_i^+ = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^*)^2} \quad (5)$$

$$S_i^- = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^-)^2} \quad (6)$$

- **Step5 – Determine relative closeness to ideal solution.**

The relative closeness is defined as follows:

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, i = 1, 2, \dots, m, 0 \leq RC_i \leq 1. \quad (7)$$

- **Step6 – Rank alternatives based on relative closeness to ideal solutions** – Rank in decreasing order.

III. INTERVAL VALUED FUZZY AND TOPSIS

In this Section, we consider the mathematical approach presented in [12] for modelling uncertainty through the use of fuzzy sets.

A. TOPSIS Formalisation

When there are measurement uncertainties, exact modelling is made difficult and precise definitions of fuzzy membership functions that model such uncertainty is not possible[11]. In such cases, Type-2 fuzzy sets that handle linguistic uncertainties better by modelling vagueness and unreliability of information may be used. A Type-1 fuzzy system is represented by a fuzzy membership function as shown in Fig.1. A Type-2 system is derived by blurring the Type-1 membership function to the left and right as shown in Fig.2 and is defined as follows [11][13]:

$$\tilde{A} = \{((x, u), \mu_A(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$$

where $0 \leq \mu_A(x, u) \leq 1, J_x \subseteq [0,1]$ represents the primary membership of x and $\mu_A(x, u)$ is a type-1 fuzzy set called the secondary set that defines the possibilities for the primary membership. Uncertainty is defined by a region called the footprint of uncertainty (FOU) as depicted by the blurred regions in Fig.2. The FOU can be described in terms of upper and lower membership functions are as follows:

$$\left. \begin{aligned} A &= \{((x, [\mu_A^L(x), \mu_A^U(x)]), \\ \mu_A^L, \mu_A^U &: \rightarrow [0,1] \quad \forall x \in X, \quad \mu_A^L < \mu_A^U \\ \bar{\mu}_A(x) &= [\mu_A^L(x), \mu_A^U(x)] \\ \therefore A &= \{((x, \bar{\mu}_A(x))\}, x \in [-\alpha, \alpha] \end{aligned} \right\}$$

where $\mu_A^L(x), \mu_A^U(x)$ form the upper and lower bounds respectively for $\bar{\mu}_A(x)$.

(8)

A fuzzy logic system that has at least one of its sets to be of type-2 is defined as type-2 fuzzy system. Its IF-THEN rules will contain type-2 antecedent or consequent sets. In this paper, we are concerned about the inference mechanism at the point of decision making (output stage).

B. Distance Measures

As with the pattern recognition domain, a similarity measure based on minimum distance from an ideal solution and furthest distance from an ideal negative solution determines the choice of the alternative. A fuzzy approach to TOPSIS in applications of fuzzy group decision making using fuzzy triangular membership functions and determining closeness of such numbers using fuzzy interval arithmetic is adapted from [8]. This approach uses an interval valued fuzzy TOPSIS for multi-criteria decision making wherein criteria values and their weights are treated as linguistic terms and described using interval valued fuzzy numbers. The technique is adapted for fuzzy aggregation of multi-feature based recognition parameters as criteria.

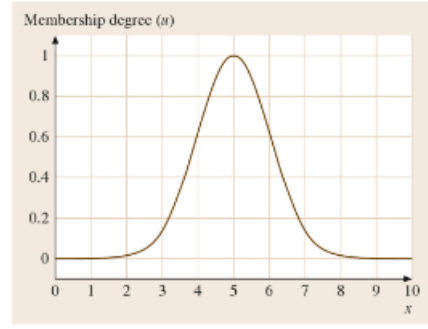


Fig. 1 Type-1 Fuzzy System [2]

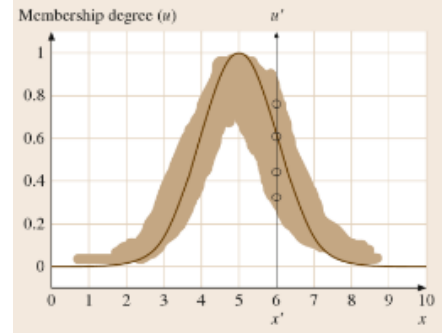


Fig. 2. Type-2 Fuzzy System [2]

Given two interval valued fuzzy numbers $N_x = [N_x^-, N_x^+]$ and $M_y = [M_y^-, M_y^+]$, the following definitions from[14] are considered:

- If $\circ \in (+, -, \times, \div)$, then $N_x \circ M_y = [N_x^- \circ M_y^-, N_x^+ \circ M_y^+]$
- The normalised Euclidean distance between N_x and M_y is given by

$$d(\tilde{N}, \tilde{M}) = \sqrt{\frac{1}{6} \sum_{i=1}^3 [(N_x^- - M_y^-)^2 + (N_x^+ - M_y^+)^2]} \quad (9)$$

C. Proposed Interval-valued Fuzzy TOPSIS Formalisation

The fuzzy decision matrix r_{ij} and weights ω_j are assumed to be triangular fuzzy numbers (TFN) defined generally by

$$\tilde{x} = \{(x_1, x_2, x_3), (x'_1, x'_2, x'_3)\} [(x_1, x'_1); x_2; (x_3, x'_3)]$$

whose average values are defined as follows:

$$\left. \begin{aligned} \tilde{x}_{ij} &= \frac{1}{K} [\tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \dots + \tilde{x}_{ij}^K] \\ \tilde{w}_{ij} &= \frac{1}{K} [\tilde{w}_{ij}^1 + \tilde{w}_{ij}^2 + \dots + \tilde{w}_{ij}^K] \end{aligned} \right\} \quad (10)$$

IV. 3D FACE RECOGNITION AND FUZZY INTEGRAL

In this Section, we consider a case study for MCDM in the field of Biometrics namely a 3D Face Recognition (3DFR) proposed in [1] in deciding the best set of features and their combinations based on their recognition performance.

A. 3DFR System Description

The overview of the system in [1] from an MCDM perspective is shown in Fig.3. For clarity, the pre-processing steps in 3DFR have been omitted as the main focus relates to the feature set representation for an MCDM problem solving. The 3DFR consists of the following key stages:

Stage1 – Feature Extraction: The feature set is constructed starting with a fundamental unary feature set. The unary features indicate the angle at which the images are sampled during feature extraction. Basic variations in intersecting planar angles with an image include 0° , 45° , 90° and 135° and the corresponding features are treated as uni-modal features represented as θ_i . Additional feature sets are organised into three multi-modal categories θ_i depending on the level of the linear combination of unary features. Each category has further sub-categories with permutation ${}^n C_r$ as a result of a linear combination of the fundamental uni-modal feature vector. Thus, an N -ary set of multi-modal features are derived as follows:

- a) Unary features, $n = 1$, $\theta_1 \in {}^1 C_4 = \{0,45,90,135\}$
- b) Binary features, $n = 2$, $\theta_2 \in {}^2 C_4 = \{0+45, 0+90, 45+90, 0+135, 45+135, 90+135\}$
- c) Ternary features, $n = 3$, $\theta_3 \in {}^3 C_4 = \{0+45+90, 0+45+135, 0+90+135, 45+90+135\}$
- d) Quadruple features, $n = 4$: $\theta_4 \in {}^4 C_4 = \{0+45+90+135\}$

Stage 2 – Model Construction/Modelling: Two models are constructed namely, an *average* morphed image model and an *individual* image based model. In the former case, only a single model exists for a subject (individual) and in the latter, the number of models are as many samples as available for each subject. The average models are formed by morphing or averaging the samples for a subject thus arriving at a single model/subject. For the individual model, each sample is treated as a separate model for the same subject. This leads to multiple models for an individual subject. Stages 1 and 2 satisfy transitivity relation.

Stage 3- Database for Benchmarking: For the MCDM example, we consider 2 main databases details of which can be found in [1]. Further categorisation of these databases is provided in TABLE I.

Stage 4- Classification and Matching: The classical Fischer’s Linear Discriminant Analysis (FLDA) is applied

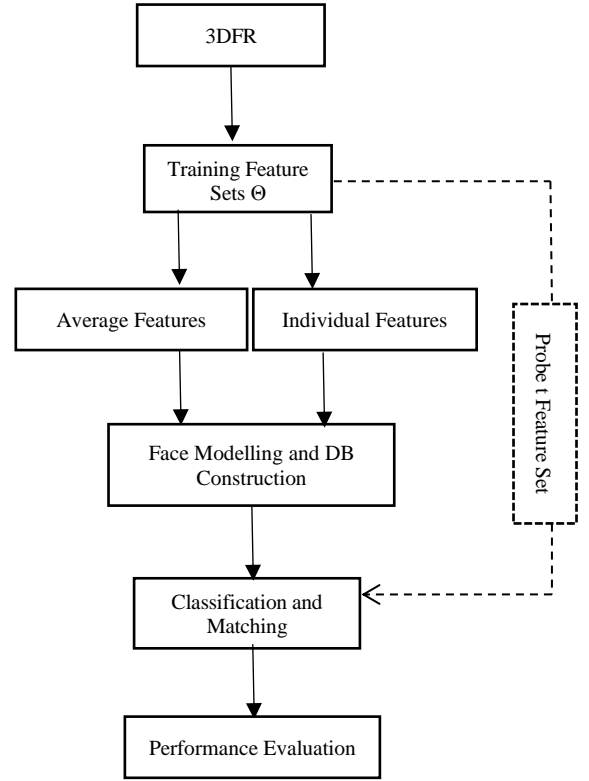


Fig. 3. 3DFR System Architecture from an MCDM Perspective

TABLE I. 3DFR DATABASE PARTITIONS FOR MCDM

	Name Student/FRVT	Model Ind/Avg	Subjects	Samples/ Subject, Total
1	7.5-Ind	Individual	100	10; 1000
2	7.5-Avg	Average	100	10; 1000
3	12.5-Ind	Individual	100	10; 1000
4	12.5-Avg	Average	100	10; 1000
5	FRVT-Ind	Individual	973	Varies; 973
5	FRVT-Avg	Average	973	Varies; 275

for query processing where a query image from the *Probe Feature Set* is matched against the model feature sets in the DB. The result is a ranked set of images based on distance measures between the query and the models. Usually, the system is designed to perform well so long as the expected result is within a rank threshold. The lower this rank value, better is the performance. For example, if a query (probe) image identifies the right subject to lie within the top few ranks, then the system is rated to be good.

Stage 5- Performance Evaluation: The result of ranking is analysed from the receiver operating characteristics (ROC) of Rank Vs Cumulative Match as shown in Fig. 4. A probe image is given rank- k when the actual subject is ranked in position k by the face recognition system. An ideal system is expected to perform at rank-1. With practical systems, the identification rate is an estimate of the probability that a subject is identified correctly at least at rank- k . In Fig. 4, a better system is one that works within a small threshold on rank and is closest to the top left corner of the graph.

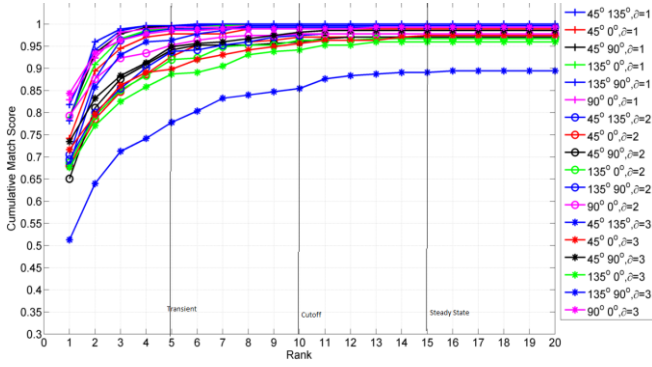


Fig. 4. RECEIVER OPERATING CHARACTERISTICS (ROC)

TABLE II. FRVT-AVG DB- RANK VS CUMULATIVE MATCH

	Criteria→ Alternatives↓	Transient $T_{\mathcal{R}}$	Cut-off T_{cutoff}	Steady State T_{SSR}
1	$\Theta \downarrow$	Score at Rank=5	Score at Rank=10	Score at Rank=17
2	H	0.99275	0.90646	1
3	V	1	1	1
4	D45	0.93478	0.98551	0.9052
5	D135	0.7971	0.87681	0.91304
6	D45H	0.99275	1	1
7	D135H	1	1	1
8	VH	1	1	1
9	VD135	0.99275	1	1
10	VD45	0.99275	1	1
11	VD45	0.99275	0.992	1
12	D45D135	0.94928	0.97826	0.99275
13	VD135H	1	1	1
14	D45D135H	1	1	1
15	VD45D135H	1	1	1

TABLE III. CRITERIA FUZZY IMPORTANCE AND INTERVAL VALUED WEIGHTS

	DM1	DM2	g	g'	h	l	l'
$T_{\mathcal{R}}$	H	H	0.75	0.77	0.80	0.88	0.91
T_{cutoff}	H	H	0.80	0.83	0.88	0.85	0.94
T_{SSR}	MH	VH	0.85	0.88	0.90	0.95	0.97

For the system considered, we define three parameters namely, the transient, cut-off and steady state responses. The **transient response** determines a rank threshold $T_{\mathcal{R}}$ at which an acceptable performance is attained. We consider $T_{\mathcal{R}} = 5$. The **cut-off** is the rank at which good performance is expected which is much higher than $T_{\mathcal{R}}$. This is denoted and assigned as $T_{cutoff} = 10$. Lastly, the **steady state response** (SSR) is the rank at which the performance reaches saturation. This is denoted and assigned as $T_{SSR} = 17$. These response points are indicated in Fig. 1. Therefore, criteria $C = \{T_{\mathcal{R}}, T_{cutoff}, T_{SSR}\}$.

B. Sample Experimental Results and Analysis

From the ROC, we consider the performance of the 3DFR in terms of $T_{\mathcal{R}}$, T_{cutoff} and T_{SSR} . As an example, TABLE II. provides these performance measures for the various feature sets for the FRVT DB whose model is *individual*. The second column indicates multi-modality namely unary in rows 2-5, binary in rows 6-10, ternary in rows 12-14 and quadruple in row 15. For this architecture, inferences may be made on the system performance in terms of $T_{\mathcal{R}}$, T_{cutoff} and T_{SSR} :

1. A mapping of feature sets Vs Best Performers across the board are as follows:
 - a) $\Theta_1 \rightarrow \{V\}$.
 - b) $\Theta_2 \rightarrow \{D135H, H\}$.
 - c) $\Theta_3 \rightarrow \{VD135H, D45D135H\}$.
 - d) Similarly, $\Theta_4 \rightarrow \{VD45D135H\}$ performs well across the board.
2. The worst performers across the board include: $\{D135, D45D135\}$.
3. Similar analysis may be carried out for each response characteristic namely, $T_{\mathcal{R}}$, T_{cutoff} and T_{SSR} . For instance, the best $T_{\mathcal{R}}$ is obtained for $\{V, D135H, VH, VD135H\}$.

The complexity of performance analysis varies significantly based on the following factors: {number of databases \times feature modality \times model representation}. This complexity makes a final decision based on the choice of the above factors difficult as brought out in [1]. To combat this situation and arrive at an objective measure in decision using the multi-objective multi-criteria performances, the MCDM approach is opted.

V. APPLICATION OF INTERVAL-VALUED FUZZY TOPSIS IN RANKING THE PERFORMANCE OF MULTI-MODAL 3DFR SYSTEM

In this Section, we propose to apply the the fuzzy TOPSIS formalisation in Section III for the 3DFR system described in Section IV.A. Both models of *individual* and *average* are included. For brevity, we consider only one DB namely, Student_7.5. Only the unary set of features are used as DMs. The computations can easily be extended for other multi-modal features.

The following information is assumed to be provided by domain experts or by applying a heuristic approach:

Heuristic factors: Importance criteria are defined linguistically (TABLE III.). The two DB models are treated as decision makers (DM) in deciding the importance measures of the criteria (only). The ratings of criteria are defined linguistically (Table IV). TABLE V. contains the intervalued TFN linguistic descriptions of the importance criteria in TABLE IV. DMs use these linguistic descriptions to assess the alternatives. Such assessments are captured in TABLE VI.

for $C = \{T_{\mathcal{R}}, T_{cutoff}, T_{SSR}\}$. The weights of criteria are defined by

TABLE IV. FUZZY INTERVAL MAPPING FOR RATINGS

		g	g'	h	l	l'
Very Poor	VP	0	0	0	1	1.5
Poor	P	0	0.5	1	3	3.5
Moderately Poor	MP	0	1.5	3	5	5.5
Fair	F	3	3.5	5	7	7.5
Moderately Good	MG	5	5.5	7	8	9.5
Good	G	6	7.5	9	10	10
Very good	VG	9	9.5	9.5	10	10
Excellent	EX	9	9.5	10	10	10

TABLE V. FUZZY INTERVAL MAPPING FOR IMPORTANCE

		g	g'	h	l	l'
Very Low	VL	0	0	0.5	0.5	0.5
Low	L	0.4	0	0.5	0.6	0.6
Medium Low	ML	0.5	1	0.6	0.7	0.7
Medium	M	0.7	1	0.7	0.8	0.9
Medium Hig	MH	0.8	1	0.9	0.8	0.9
High	H	0.9	1	0.9	1	1
Very High	VH	1	1	1	1	1

TABLE VI. ALLOCATION OF CRITERIA IMPORTANCE BY DMs

Applicant	$T_{\mathcal{R}}$				T_{Cutoff}				T_{SSR}			
	V	D45	D135	H	V	D45	D135	H	V	D45	D135	H
7.5-Ind	F	P	F	F	G	MGG	G	G	F	P	F	F
7.5-Avg	G	MP	G	G	VG	G	VG	VG	G	MP	G	G
12.5-Ind	G	VG	G	G	VG	VG	VG	VG	G	VG	G	G
12.5-Avg	G	VG	VG	G	VG	VG	VG	VG	G	VG	VG	G
FRGC-Ind	G	MG	MP	G	VG	G	MP	G	G	MG	MP	G
FRGC-Avg	EX	MG	MP	EX	EX	VG	MG	G	EX	MG	MP	EX

interval-valued TFN (TABLE VI.) by averaging the weights assigned by the DMs.

Profit Factor Ω_b : Since our aim is to maximise the recognition performance, we consider the profit factor and hence determine the global maxima c_j^+ of the TFNs in 0

Ideal Solutions: From Eqns. (3) and (4), we assume:
 $A^+ = [(1,1); 1.(1,1)]$; $A^- = [(0,0); 0.(0,0)]$;

A. Decision Making with Unary Features-Algorithm

Step1: Given the various linguistic descriptions and interval valued TFNs, we consider the DM assessments with linguistic descriptions in TABLE VI. These are mapped to corresponding interval valued assessments in 0 using the

interval mapping in TABLE IV. From (1) and (2), Table VII is both normalised using its global maxima and weighted using values in TABLE III. corresponding to each criterion.

Step2: From (10), the DMs assessments in Table VII are averaged based on corresponding interval points resulting in in the last block of Table VII.

Step3: From (3)-(4), ideal and negative ideal solutions are determined. The results are tabulated in TABLE VIII.

Step4: (5)-(7) are used to determine the distance from ideal solutions and the relative closeness measures. Based on minimum distance, the alternatives are ranked as shown in the last column of Table VIII. Thus the final ranking of DBs when using unary features is: $DB_3 > DB_4 > DB_2 > DB_6 > DB_5 > DB_1$.

The unary features work particularly well for the student DB with 12.5 mm lens due to its high resolution of the pair of lens used during image capture. The ranking does tally with the ground truth from the respective RoCs for the 3DFR.

VI. CONCLUSION

In this paper, the MCDM technique TOPSIS is applied for ranking a multi-modal 3DFR system that involves multi-objective, multi-criteria evidence-based decision making under uncertainty using interval valued fuzzy logic. A case study has been provided and the results are very promising. Further work relates to a detailed analysis of multi-modal features and fuzzy equivalence for classification.

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TABLE VII. INTERVAL VALUED FUZZY NORMALISED, WEIGHTED, AVERGAED DECISION MATRIX

Criteria	Applica	DM1-H					DM2-V					DM3-D45					DM4-D135					Eqn.10 \tilde{x}_{ij}				
		g	g'	h	l	l'	g	g'	h	l	l'	g	g'	h	l	l'	g	g'	h	l	l'	g	g'	h	l	l'
T_R	7.5 - Ind	2.5	3.5	5	6.5	7.5	0	0.5	1	2.5	4	2.5	4	5	7	7.5	2.5	3.5	5	7	7.5	0.2	0.3	0.5	0.6	0.7
	7.5 - Avg	5.5	7.5	9	9.5	10	0	1.5	3	4.5	6	5.5	8	9	10	10	5.5	7.5	9	10	10	0.5	0.7	0.8	0.9	1.0
	12.5 - Ind	5.5	7.5	9	9.5	10	8.5	9.5	9.5	10	10	5.5	8	9	10	10	5.5	7.5	9	10	10	0.6	0.8	0.9	1.0	1.0
	12.5 - Avg	5.5	7.5	9	9.5	10	8.5	9.5	9.5	10	10	8.5	10	10	10	10	5.5	7.5	9	10	10	0.6	0.8	0.9	1.0	1.0
	FRGC - Ind	5.5	7.5	9	9.5	10	4.5	5.5	7	8	10	0	2	3	5	5.5	5.5	7.5	9	10	10	0.5	0.6	0.8	0.9	1.0
	FRGC - Avg	8.5	9.5	10	10	10	4.5	5.5	7	8	10	0	2	3	5	5.5	8.5	9.5	10	10	10	0.7	0.9	0.9	0.9	1.0
T_{Cutoff}	7.5 - Ind	5.5	7.5	9	9.5	10	4.5	5.5	7	8	10	5.5	8	9	10	10	5.5	7.5	9	10	10	0.5	0.7	0.9	0.9	1.0
	7.5 - Avg	8.5	9.5	9.5	10	10	5.5	7.5	9	9.5	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.8	0.9	0.9	1.0	1
	12.5 - Ind	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.9	1.0	1.0	1.0	1
	12.5 - Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.9	1.0	1.0	1.0	1
	FRGC - Ind	8.5	9.5	9.5	10	10	5.5	7.5	9	9.5	10	0	2	3	5	5.5	5.5	7.5	9	10	10	0.5	0.7	0.8	0.8	0.9
	FRGC - Avg	8.5	9.5	10	10	10	8.5	9.5	9.5	10	10	4.5	6	7	8	9.5	5.5	7.5	9	10	10	0.7	0.8	0.9	0.9	1.0
T_{SSR}	7.5 - Ind	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.9	1.0	1.0	1.0	1.0
	7.5 - Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.9	1.0	1.0	1.0	1
	12.5 - Ind	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.9	1.0	1.0	1.0	1
	12.5 - Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	10	10	10	10	8.5	9.5	10	10	10	0.9	1.0	1.0	1.0	1
	FRGC - Ind	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	2.5	4	5	7	7.5	5.5	7.5	9	10	10	0.6	0.8	0.8	0.9	0.9
	FRGC - Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	5.5	8	9	10	10	8.5	9.5	10	10	10	0.8	0.9	1.0	1.0	1.0

TABLE VIII. INTERVAL VALUED FUZZY NORMALISED, WEIGHTED, AVERGAED DECISION MATRIX

	Distance from A' [(1,1);1;(1,1)]										Distance from A' [(0,0);0;(0,0)]										Di1+	Di1-	Di2+	Di2-	RC1	RC2	RC	Rank										
	0.7	0.6	0.5	0.3	0.2	0.3	0.2	0.1	0.1	0	0.1	0	0	0	0	0	0	0	0	0.3									0.2	0.3	0.6	0.6	0.9	0.5	1	0.81	0.9	0.9
7.5 - Ind	0.7	0.6	0.5	0.3	0.2	0.3	0.2	0.1	0.1	0	0.1	0	0	0	0	0	0	0	0	0.3	0.2	0.3	0.6	0.6	0.9	0.5	1	0.81	0.9	0.9	1.3	1.05	1.87	2.07	0.53	0.45	0.49	1
7.5 - Avg	0.5	0.3	0.2	0.1	0	0.1	0.1	0	0	0	0.1	0	0	0	0	0.1	0.2	0	1	0.7	0.4	0.6	0.7	0.7	0.9	0.5	1	0.81	0.9	0.9	0.9	0.70	2.20	2.38	0.52	0.43	0.48	4
12.5 - Ind	0.3	0.1	0.1	0	0	0.1	0	0	0	0	0.1	0	0	0	0	0.2	0.4	1	1	0.8	0.5	0.6	0.7	0.7	0.9	0.5	1	0.73	0.9	0.9	0.8	0.57	2.34	2.50	0.52	0.43	0.47	6
12.5 - Avg	0.2	0.1	0.1	0	0	0.1	0	0	0	0	0.1	0	0	0	0	0.3	0.4	1	1	0.8	0.5	0.6	0.7	0.7	0.9	0.5	1	0.73	0.9	0.9	0.7	0.55	2.36	2.52	0.52	0.43	0.47	5
FRGC - Ind	0.5	0.3	0.2	0.1	0	0.4	0.2	0.1	0.1	0	0.2	0	0	0	0	0.1	0.2	0	0	0.6	0.2	0.3	0.4	0.5	0.7	0.3	0	0.57	0.7	0.8	1.3	1.03	1.87	2.08	0.53	0.45	0.49	2
FRGC - Avg	0.4	0.2	0.2	0.1	0	0.2	0.1	0	0	0	0.1	0	0	0	0	0.2	0.3	0	1	0.6	0.3	0.4	0.6	0.6	0.9	0.4	1	0.77	0.9	0.9	1.0	0.77	2.13	2.32	0.52	0.44	0.48	3

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