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# Numerical Simulation of Earthquake-Induced Liquefactions Considering the Principal Stress Rotation

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## Abstract

Dynamic loadings such as earthquake loadings can generate considerable principal stress rotation (PSR) in the saturated soil. The PSR without changes of principal stress magnitudes can generate additional excess pore water pressures and plastic strains, thus accelerating liquefaction in undrained conditions. This paper simulates a centrifuge model test using the fully coupled finite element method considering the PSR. The impact of PSR under the earthquake loading is taken into account by using an elastoplastic soil model developed on the basis of a kinematic hardening soil model with the bounding surface concept. The soil model considers the PSR by treating the stress rate generating the PSR independently. The capability of this soil model is verified by comparing the numerical predictions and experimental results. It also indicates that the PSR impact can not be ignored in predictions of soil liquefaction.

**KEYWORDS:** elastoplastic model; principal stress rotation; liquefaction; earthquake loading; non-coaxiality

## Introduction

The soil behavior under earthquake loadings is one of major research areas in both numerical simulations and experimental studies. The loading conditions under earthquakes are quite diverse and complex, but they share a common characteristic in which the soil is subjected to considerable principal stress rotation (PSR). It is important to consider the PSR impact in many types of geotechnical engineering studies under dynamic loadings. Ishihara & Towhata (1983) found that the PSR can generate plastic deformations and the non-coaxiality even without a change of principal stress magnitudes. The PSR can also generate excess pore water pressures and plastic strains in undrained conditions. Similar phenomenon is also found

by Ishihara & Yamazaki (1984), Bhatia et al. (1985), Miura et al. (1986), Gutierrez et al. (1991), etc. It is well established that the additional excess pore water pressure and plastic deformation caused by the PSR from the dynamic loading can accelerate undrained soil liquefaction. Ignoring the PSR impact may lead to unsafe designs.

At present, numerous researches have been carried out to investigate the soil behavior under earthquake loadings. One of the most famous researches is the VELACS project (Verification of Liquefaction Analysis using Centrifuge Studies). It includes a variety of centrifuge model tests and the corresponding numerical simulations in many universities and research institutes (Arulanandan & Scott, 1993). However, Arulanandan et al. (1995) claims that the predicted results from these numerical simulations have great variations and errors which may result from different soil models used by different researchers. They also state that the predicted results are largely affected by the computer codes used and it seems that the program with fully coupled governing equations performs the best among all the results. Although several researchers have implemented their soil models into these numerical simulations subsequently (Andrianopoulos et al., 2010; Sadeghian & Namin, 2013; Pak et al., 2014), there are few of them considering the PSR effect.

This paper aims to take into account the impact of PSR on the liquefaction in numerical simulations of earthquake loadings by using a well established PSR model and a fully coupled finite element program DYSAC2 (Muraleetharan et al., 1994; 1997). This model is developed on the basis of a kinematic hardening model with the bounding surface and critical state concept. The PSR soil model considers the PSR effect by treating the stress rate generating the PSR independently. The model has been validated in single element studies with different types of sand, such as Nevada sand (Yang et al., 2014), Toyoura sand (Yang & Yu, 2013), Leighton Buzzard sand (Wang et al., 2016), etc. All the results demonstrate that this model can properly simulate the PSR effects in single element studies. The focus of the paper is on the investigation of PSR impacts on boundary value problems under earthquake loadings. Firstly, the original base model and the modified PSR model will be introduced. Secondly, these two models will be tested in a single element numerical simulation, compared with experimental results with the PSR. Finally, they will be implemented into FEM software to simulate VELACS centrifuge model tests. The Model No 3 of the VELACS project is chosen to be simulated in this investigation and the comparison will be made between the original base model, the modified PSR model, and the experimental results.

## **The Original Soil Model**

### **Model Formulations**

A well-established soil model with the bounding surface concept and kinematic

hardening is chosen as the base model. It employs the back-stress ratio as the hardening parameter and the state parameter to represent influences of different confining stresses and void ratios on sand behaviors. It also adopts the critical state concept and the principle of phase transformation line. However, it does not give special consideration of the PSR effect. This model will be briefly introduced, and more details about this model can be found in Manzari & Dafalias (1997) and Dafalias & Manzari (2004). It should be noted that this study is focused on the impact of PSR, and the simplified version of the above mentioned models is employed to better present the PSR impact. For example, the fabric impact in Dafalias & Manzari (2004) is not considered, which can improve simulations otherwise.

The yield function of the model is defined as:

$$f = [(\mathbf{s} - p\boldsymbol{\alpha}) : (\mathbf{s} - p\boldsymbol{\alpha})]^{1/2} - \sqrt{2/3} pm = 0 \quad (1)$$

where  $\mathbf{s}$  is the deviatoric stress tensor.  $p$  and  $\boldsymbol{\alpha}$  are the confining pressure and back-stress ratio tensor, respectively.  $\boldsymbol{\alpha}$  represents the center of yield surface in the stress ratio space while  $m$  is the radius of yield surface.  $m$  is assumed to be a small constant, indicating no isotropic hardening. The normal to the yield surface is defined as:

$$\mathbf{l} = \frac{\partial f}{\partial \boldsymbol{\sigma}} = \mathbf{n} - \frac{1}{3} (\mathbf{n}:\mathbf{r}) \mathbf{I}; \quad \mathbf{n} = \frac{\mathbf{r}-\boldsymbol{\alpha}}{\sqrt{2/3} m} \quad (2)$$

where  $\mathbf{I}$  is the isotropic tensor and  $\mathbf{n}$  represents the normal to the yield surface on the deviatoric plane.  $\mathbf{r}$  represents the stress ratio, and is equal to  $\mathbf{s} / p$ . The elastic deviatoric strain rate  $d\boldsymbol{\varepsilon}^e$  and volumetric strain rate  $d\varepsilon_v^e$  are defined as:

$$d\boldsymbol{\varepsilon}^e = d\mathbf{s} / 2G \quad (3)$$

$$d\varepsilon_v^e = dp / K \quad (4)$$

where  $G$  and  $K$  are the elastic shear module and bulk module, respectively, which are expressed as:

$$G = G_0 p_{at} [(2.97 - e)^2 / (1 + e)] (p / p_{at})^{1/2} \quad (5)$$

$$K = 2(1 + \nu)G / 3(1 - 2\nu) \quad (6)$$

where  $G_0$  is a constant,  $p_{at}$  is the atmospheric pressure,  $e$  is the void ratio, and  $\nu$  is the Poisson's ratio. The plastic strain rate  $d\boldsymbol{\varepsilon}^p$  is defined as:

$$d\boldsymbol{\varepsilon}^p = \langle L \rangle \mathbf{R} \quad (7)$$

$$L = \frac{1}{K_p} \left( \frac{\partial f}{\partial \boldsymbol{\sigma}} \right) : d\boldsymbol{\sigma} \quad (8)$$

$$\mathbf{R} = \mathbf{n} + \frac{1}{3} D \mathbf{I} \quad (9)$$

where  $L$  represents the loading index, and  $\mathbf{R}$  is the normal to the potential surface, indicating the direction of the plastic strain rate.  $K_p$  is the plastic modulus, and  $D$  is the dilatancy ratio and they are defined as:

$$K_p = \frac{2}{3} p \left[ G_0 h_0 (1 - c_h e) \left( \frac{p}{p_{at}} \right)^{-1/2} \right] \left[ \frac{\mathbf{b} : \mathbf{n}}{|\mathbf{(\alpha - \alpha_{in}) : \mathbf{n}}|} \right] \quad (10)$$

$$D = A_d \mathbf{d} : \mathbf{n} \quad (11)$$

where  $\mathbf{b}$  and  $\mathbf{d}$  are the distances between the current back-stress ratio tensor and bounding and dilatancy back-stress ratio tensors, respectively.  $h_0$ ,  $c_h$  and  $A_d$  are the model parameters.  $\alpha_{in}$  is the initial value of  $\alpha$  at the start of a new loading process and is updated when the denominator becomes negative. In some extreme cases, for example, when the void ratio is very large,  $K_p$  can become negative. In that case, care should be exercised to prevent  $K_p$  from becoming zero.

### Calibration and Model Simulations of Laboratory Experiments

The sand used in Model No 3 test of VELACS is Nevada sand which has a specific gravity of 2.67. Its maximum and minimum void ratios are 0.887 and 0.511, respectively. All the model parameters in both the original model and the modified model are calibrated by a series of triaxial, torsional and rotational tests for Nevada sand from Chen & Kutter (2009). While the triaxial tests do not have the PSR, the latter two tests have the PSR. The stress paths of the torsional and rotational tests are illustrated in Figure 1. The set of model parameters listed in Table 1 are used for both the single element and finite element simulations. The critical state parameters  $e_0$ ,  $\lambda_c$ ,  $\xi$  and  $M$  are determined from the quantities at the end of triaxial tests.  $c$  is determined by comparing the critical state stress ratios at triaxial compression and triaxial extension.  $m$  for the yield surface is assumed to be  $M/100$ . Parameters  $n^b$  and  $n^d$  are determined by using the approach in Li & Dafalias (2000). The parameters  $h_0$ ,  $c_h$  and  $A_0$  can be found by trial and error in curve fitting.

Some typical results are shown in Figures 2 to 5. Figure 2 shows the predicted results of the drained triaxial tests, and they generally fit the test results very well. Figures 3 and 4 show the predictions of torsional shear tests under different initial conditions. In Figure 3, it can be seen that the effective confining pressure  $p'$  is reduced to about 75 kPa, at which the  $q$ - $p'$  stress path shows the butterfly shape and  $p'$  stops reducing, and the final  $p'$  is much larger than the test result. Meanwhile, as the shear stress continues changing, no dramatic shear strain is observed, which is significantly different from the lab results. Figure 4 shows similar predictions to those in Figure 3. Figure 5 shows the predictions of the rotational test. Its simulation is similar to that in the torsional test, and there is a limited reduction of effective confining pressure and small strains, indicating no occurrence of liquefaction.

Predictions of these tests indicate that the original model is able to predict sand responses without the PSR, but is not capable of considering the PSR impact on liquefaction. This is because the model is not able to simulate the considerable volumetric reduction from the PSR.

Especially at a large stress ratio close to the phase transformation line, the model usually gives very small volumetric reduction or even volumetric expansion above the phase transformation line. As a result, it constrains the reduction of effective confining pressure near the phase transformation line under the PSR. Yang & Yu (2013) gives detailed discussions on this deficiency. To better simulate the responses under the PSR, a new model needs to be developed based on the original model, in order to properly take into account the PSR impact.

### The PSR Modified Soil Model

Detailed description of the modified model can be found in Yang & Yu (2013), and a brief introduction is given here. In the modified model, the plastic strain rate is split into the monotonic strain rate  $d\boldsymbol{\varepsilon}_m^p$  and the PSR induced strain rate  $d\boldsymbol{\varepsilon}_r^p$ , where the subscript m and r represent monotonic and PSR loading hereinafter, respectively. It should be noted that the ‘monotonic’ is used to be distinguished from the PSR stress rate, and does not represent real monotonic loading. The evolution of hardening parameter is not affected by this separate treatment. Therefore, the plastic strain rate can be expressed as:

$$d\boldsymbol{\varepsilon}_m^p = \langle L_m \rangle \mathbf{R}_m = \frac{1}{K_{pm}} \left( \frac{\partial f}{\partial \boldsymbol{\sigma}} d\boldsymbol{\sigma}_m \right) \mathbf{R}_m \quad (12)$$

$$d\boldsymbol{\varepsilon}_r^p = \langle L_r \rangle \mathbf{R}_r = \frac{1}{K_{pr}} \left( \frac{\partial f}{\partial \boldsymbol{\sigma}} d\boldsymbol{\sigma}_r \right) \mathbf{R}_r \quad (13)$$

It is assumed that  $K_{pm} = K_p$  and  $\mathbf{R}_m = \mathbf{R}$  (equation 9 & 10) because the original model is for the non-PSR loading. The direction of PSR strain rate  $\mathbf{R}_r$  can be expressed as:

$$\mathbf{R}_r = \mathbf{n}_r + \frac{1}{3} D_r \mathbf{I} \quad (14)$$

where  $\mathbf{n}_r$  is the direction of deviatoric plastic strain rate and can be approximated as  $\mathbf{n}$  for simplicity.  $D_r$  is the dilatancy ratio for the PSR loading rate, it can be derived from the postulate of the PSR dilatancy ratio of Gutierrez et al (1991) on the basis of work and energy dissipation. In this model, it can be expressed as:

$$D_r = A_r (1 - \alpha/\alpha_\theta^b) \alpha \quad (15)$$

where  $A_r$  is a constant for the impact of PSR on the dilatancy.  $\alpha/\alpha_\theta^b$  can be approximated as the cosine of the angle between the principal stress and plastic strain rate. Compared with equation (15), equation (11) for the flow rule in the original model can predict dilatancy at a large stress ratio, such as when the stress ratio is above the phase transformation line, because it can not distinguish the PSR stress rate from the total stress rate. Equation (15) indicates that the PSR at a relatively high stress ratio can still generate substantial volumetric reduction, such as near the phase transformation line. The plastic modulus  $K_{pr}$  for PSR loading rate is defined as:

$$\mathbf{K}_{pr} = \frac{2}{3} p \left[ \mathbf{G}_0 h_{0r} (1 - c_h e) \left( \frac{p}{p_{at}} \right)^{-1/2} \right] \left( \frac{|\mathbf{b} : \mathbf{n}|}{|(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \mathbf{n}|} \right)^{\zeta_r} \quad (16)$$

where  $h_{0r}$  and  $\zeta_r$  are new model parameters associated with the PSR. In order to make  $\mathbf{K}_{pr}$  more sensitive to the stress ratio,  $\zeta_r$  is usually larger than unity.

To complete the model, the definition of PSR loading rate  $d\boldsymbol{\sigma}_r$  is required. To determine  $d\boldsymbol{\sigma}_r$  in general stress space, it is first considered in the space with only x and y directions denoted as  $\boldsymbol{\alpha}$ . The physical meaning of  $d\boldsymbol{\sigma}_r$  compared with  $d\boldsymbol{\sigma}$  is illustrated in Figure 6, in which  $d\boldsymbol{\sigma}$  is split into  $d\boldsymbol{\sigma}_r$  and  $d\boldsymbol{\sigma}_m$ .  $d\boldsymbol{\sigma}_m$  is along the direction of the current stress vector, and  $d\boldsymbol{\sigma}_r$  can be obtained from  $d\boldsymbol{\sigma} - d\boldsymbol{\sigma}_m$ . Their relationship can be expressed as  $d\boldsymbol{\sigma}_r^\alpha = \mathbf{N}_r^\alpha d\boldsymbol{\sigma}$ , and can be written in matrix form as:

$$\begin{pmatrix} d\sigma_{rx}^\alpha \\ d\sigma_{ry}^\alpha \\ d\sigma_{rxy}^\alpha \end{pmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{(\sigma_x - \sigma_y)^2}{8t_J^\alpha} & -\frac{1}{2} + \frac{(\sigma_x - \sigma_y)^2}{8t_J^\alpha} & -\frac{(\sigma_x - \sigma_y)\sigma_{xy}}{2t_J^\alpha} \\ -\frac{1}{2} + \frac{(\sigma_x - \sigma_y)^2}{8t_J^\alpha} & \frac{1}{2} - \frac{(\sigma_x - \sigma_y)^2}{8t_J^\alpha} & \frac{(\sigma_x - \sigma_y)\sigma_{xy}}{2t_J^\alpha} \\ -\frac{(\sigma_x - \sigma_y)\sigma_{xy}}{4t_J^\alpha} & \frac{(\sigma_x - \sigma_y)\sigma_{xy}}{4t_J^\alpha} & 1 - \frac{\sigma_{xy}^2}{t_J^\alpha} \end{bmatrix} \begin{pmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_{xy} \end{pmatrix} \quad (17)$$

where  $t_J^\alpha = (\sigma_x - \sigma_y)^2 / 4 + \sigma_{xy}^2$ . Similarly, in the  $\beta$  space (y, z) and  $\gamma$  space (z, x), they can be defined as  $d\boldsymbol{\sigma}_r^\beta = \mathbf{N}_r^\beta d\boldsymbol{\sigma}$  and  $d\boldsymbol{\sigma}_r^\gamma = \mathbf{N}_r^\gamma d\boldsymbol{\sigma}$ . Combining  $d\boldsymbol{\sigma}_r^\alpha$ ,  $d\boldsymbol{\sigma}_r^\beta$  and  $d\boldsymbol{\sigma}_r^\gamma$ , and letting  $d\sigma_{rx} = d\sigma_{rx}^\alpha + d\sigma_{rx}^\gamma$ ,  $d\sigma_{ry} = d\sigma_{ry}^\alpha + d\sigma_{ry}^\beta$  and  $d\sigma_{rz} = d\sigma_{rz}^\beta + d\sigma_{rz}^\gamma$ ,  $d\boldsymbol{\sigma}_r$  in the general stress space can be obtained as:

$$d\boldsymbol{\sigma}_r = \mathbf{N}_r d\boldsymbol{\sigma} \quad (18)$$

With the formulations derived above, the elastoplastic stiffness can be obtained. The total stress rate-strain rate relationship can be defined as:

$$d\boldsymbol{\sigma} = \mathbf{E}(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}^p) = \mathbf{E}(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_m^p - d\boldsymbol{\varepsilon}_r^p) \quad (19)$$

$$\mathbf{E}_{ijkl} = K\delta_{ij}\delta_{kl} + G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2/3\delta_{ij}\delta_{kl}) \quad (20)$$

where  $\mathbf{E}$  is the elastic stiffness tensor.  $\mathbf{N}_r$  is the tensor which plays the role of projecting the total stress rate onto the PSR direction, and it has the following characteristics.

$$\mathbf{E}\mathbf{N}_r = 2G\mathbf{N}_r \quad (21)$$

From mathematical manipulations and equation (21), the relationship between the stress and strain rates can be expressed as:

$$d\boldsymbol{\sigma} = \mathbf{E}^{ep} d\boldsymbol{\varepsilon} \quad (22)$$

$$\mathbf{E}^{ep} = \mathbf{E} - \mathbf{B}_1 \left[ \frac{(\mathbf{E}\mathbf{R})(\mathbf{I}\mathbf{N}_r^*)}{K_{pr} + \mathbf{I}\mathbf{N}_r^* \mathbf{R}_r} - \frac{(\mathbf{E}\mathbf{R})(\mathbf{I}\mathbf{E})}{K_{pr} + \mathbf{I}\mathbf{E}\mathbf{R}_r} \right] - \mathbf{B}_2 \left[ \frac{(\mathbf{E}\mathbf{R}_r)(\mathbf{I}\mathbf{N}_r^*)}{\mathbf{I}\mathbf{N}_r^* \mathbf{R}_r} - \frac{(\mathbf{E}\mathbf{R}_r)(\mathbf{I}\mathbf{E})}{K_p + \mathbf{I}\mathbf{E}\mathbf{R}_r} \right] \quad (23)$$

$$\mathbf{N}_r^* = 2G\mathbf{N}_r \quad (24)$$

$$B_1 = \left( \frac{\mathbf{IN}_r^* \mathbf{R}}{K_{pr} + \mathbf{IN}_r^* \mathbf{R}_r} - \frac{K_p + \mathbf{IER}}{K_{pr} + \mathbf{IER}_r} \right)^{-1} \quad (25)$$

$$B_2 = \left( \frac{K_{pr} + \mathbf{IN}_r^* \mathbf{R}_r}{\mathbf{IN}_r^* \mathbf{R}} - \frac{K_{pr} + \mathbf{IER}_r}{K_p + \mathbf{IER}} \right)^{-1} \quad (26)$$

The above formulations show that the stiffness tensor is independent of stress increments, and the stress and strain increments have a linear relationship, which indicates the easy numerical implementations. In these equations, if  $K_{pr}$  is set to be  $K_p$  and  $\mathbf{R}_r$  to be  $\mathbf{R}$ , they will be downgraded to the formulations in the classical plasticity. Three new model parameters related to the PSR are incorporated into the modified PSR model. They are  $h_{0r}$  and  $\xi_r$  for the plastic modulus, and  $A_r$  for the flow rule. All of them are independent of the monotonic loading, and can be easily calibrated through the pure rotational loading paths.

The above equations indicate that the PSR stress rate can generate considerable volumetric reduction. The modified PSR model is used to simulate the above mentioned tests, shown in Figures 3-5. These figures show that the modified model can reduce the effective confining pressure further than the original model. In addition, its reduction in the last few cycles is accompanied with drastic increase of strain, indicating the occurrence of liquefaction. The simulations with the modified model are in better agreement with the test results than the original model. It should be noted that, in these torsional and rotational shear tests and simulations, the effective confining pressure can not reach zero due to the existence of shear stress. The liquefaction manifests itself by the dramatic increase of shear strains.

## Finite Element Analysis

### Problem definition

The centrifuge test of Model No.3 in VELACS project is selected to assess the ability of the modified model. This is a water saturated layer of sand deposited in a laminar box of the depth of 220 mm, shown in Figure 8. The model is divided horizontally into two sand layers which have the relative density of 40% and 70%, respectively. The laminar box is subjected to the base motion illustrated in Figure 9. The base shaking in the vertical direction is negligible and the base shaking in the horizontal direction is the major shaking. The accelerations along the height of the soil sample are measured with 7 accelerometers. 10 pore water pressure transducers are used to measure the pore water pressures. The lateral deformations and settlements are measured by 6 displacement LVDT transducers. In total, 23 transducers are used, shown in Figure 7.

To simulate the centrifuge test, the two dimensional finite element computer code DYSAC2 with the fully coupled analysis is used. This program adopts the finite element



solution of the dynamic governing equations for a saturated porous media and a three parameter time integration scheme called the Hilber-Hughes-Taylor  $\alpha$  method. A predictor/multi-corrector algorithm is also used to provide the quadratic accuracy. The details of this method are given in Muraleetharan et al. (1994; 1997). The problem is simulated in the model scale with the gravitational acceleration of 50 g. The whole box is divided into 162 elements, shown in Figure 9. No horizontal water flow is allowed on the side boundaries, and no vertical water flow is allowed on the base, which is also fixed to the ground. The nodes with the symbol 'x' in Figure 9 are tied together, which results in the same displacement among them. This is to account for the boundary conditions of the laminar box, which are rigid. Besides, the nodes in the adjacent rows on the left-hand and right-hand sides of the box are tied up with one another to give the transition between the soil elements and the rigid sides in the laminar box. The permeability coefficient of Nevada sand is  $4.6E10^{-5}$  from the study by Arulmoli et al (1992). All the quantities including the pore water pressure, stress, strain and the displacement are recorded for 30 seconds, because the liquefaction spreads over the majority of the model after 30 seconds.

#### Predicted results and comparison with the experimental data

Figure 10 shows the pore water pressure of typical locations P1, P3 and P7 in the loose sand, and those at typical locations P2, P6 and P10 in the dense sand. In the loose sand, the predicted water pressure from the modified model reaches nearly the same peak value at the same time as the experimental data, and liquefaction is reached. However, the results from the original model significantly underestimate the pore water pressures and do not reach the liquefaction. For example, in location P1, the peak pore water pressure from the original model is 29 kPa, which is 16 kPa lower than the experimental value. Generally, the results from the modified model agree better with the experimental data and reach the liquefaction, although they slightly overestimate the pore water pressure in the early stage. In the dense sand, while the modified model slightly overestimates the pore water pressure during the full stage, the original model overestimates the pore water pressure during the early stage and still underestimates the pore water pressure during the later stage, and do not bring the soil to liquefaction. Figure 11 shows the stress path of  $p'$ - $q$  at a typical location P10, predicted by using the original and modified models. It shows the decrease of the effective confining stress and the butterfly shape in the final stages. While the modified model brings  $p'$  to zero, the original model only brings  $p'$  to the lowest value of 5 kPa. It is obvious that the modified model brings the sand to liquefaction, and the original model doesn't. Figure 12 shows the path of shear stress and normal stress difference at a typical location P10 to illustrate the PSR. Although the stress path is random, it clearly indicates the continuous PSR, and the difference of predictions between the original and the modified model comes from the continuous PSR

impact.

The settlements of typical locations L5 in the loose sand and L6 in the dense sand are shown in Figures 13, respectively. The results from the two models again show significant difference. Although the settlements all increase after the start of the shaking, the settlements from the original model only reaches the maximum value of 2.8 mm and 4 mm in 30 seconds at locations L5 and L6, respectively, far from the experimental results due to its failure in the prediction of liquefaction. On the other hand, the modified model brings the maximum settlements to 85 mm and 125 mm, respectively, which is reasonably closer to the experimental results.

## **Conclusion**

This paper presents application of a soil PSR model in the study of PSR impact on undrained soil behavior. The PSR model is developed on a base model with the bounding surface concept and soil critical state concept, and the PSR induced stress rate is treated separately using an independent hardening and flow rule. The PSR model and the original model are first used to study soil behavior in single element laboratory tests involving the PSR. It shows that the predictions by the PSR model can bring the soil to liquefaction, and agree better with the experimental results than the original model. It indicates the importance to independently consider the PSR in soil models. The PSR model and the original model are also used to simulate a centrifugal test of sand under earthquake loading, which leads to significant PSR. The original soil model fails to bring soil to liquefaction, and predicts very limited settlements which are much smaller than the experimental results. On the other hand, the PSR model brings soil to liquefaction, and its predictions are in reasonable agreement with experimental results. It further indicates the importance to give special treatment of PSR in soil models in boundary value problems involving the PSR.

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### **Table and Figure Captions**

Table 1: Model parameters of Nevada sand used in the single element and finite element simulations

Figure 1: Stress paths of torsional shear tests (left) and rotational shear tests (right) (a) and stress conditions (b) (Chen & Kutter, 2009)

Figure 2: Test results and model predictions of (a) stress strain behaviors and (b) volumetric strain responses for the monotonic loadings. (N70D501:  $D_r=74\%$ ,  $p=50\text{kPa}$ ; N70D1001:  $D_r=72\%$ ,  $p=100\text{kPa}$ ; N70D100C:  $D_r=85\%$ ,  $p=100\text{kPa}$ ; N70D2501:  $D_r=75\%$ ,  $p=250\text{kPa}$  (Chen & Kutter, 2009))

Figure 3: Test results and model predictions of (a)  $q-p'$  stress paths and (b) stress strain behaviors for the torsional shear tests NK138U51 (Chen & Kutter, 2009) ( $D_r=71\%$ , cell pressure= $400\text{kPa}$ ,  $K=1.38$ )

Figure 4: Test results and model predictions of (a)  $q-p'$  stress paths and (b) stress strain behaviors for the torsional shear tests NK73CU6 (Chen & Kutter, 2009)

Figure 5: Test results and model predictions of the rotational shear tests (Chen & Kutter, 2009)  
( $D_r=68\%$ , cell pressure=213kPa,  $K=0.73$ )

Figure 6: Schematic illustration of the total, monotonic, and PSR stress increments in the space of  $((\sigma_x - \sigma_y)/2, \sigma_{xy})$

Figure 7: The configuration and the location of measuring instruments for the centrifuge model test

Figure 8: Base motion of acceleration (a) Horizontal (b) Vertical

Figure 9: Elements and boundary conditions of the finite element model

Figure 10: Comparison of time history of excess pore water pressure between the predicted results and the experimental results

Figure 11: Predicted stress paths to illustrate liquefaction in location P10

Figure 12: Predicted stress path to illustrate the PSR in location P10

Figure 13. Comparison of time history of settlement between the predicted and experimental results