Undrained Cavity Contraction Analysis for Prediction of Soil Behaviour around Tunnels

Pin-Qiang Mo[1] and Hai-Sui Yu[2]

[1] Assistant Research Scientist, State Key Laboratory for GeoMechanics and Deep Underground Engineering, China University of Mining & Technology, Xuzhou, 221116, China. E-mail: pinqiang.mo@cumt.edu.cn

[2] Professor, The Marjorie and Arnold Ziff Building, University of Leeds, Leeds, LS2 9JT, UK. E-mail: PVC.int@leeds.ac.uk

ABSTRACT

Cavity contraction method has been used for decades for the design of tunneling and prediction of ground settlement, by modelling the cavity unloading process from in-situ stress state. Analytical solutions of undrained cavity contraction in a unified state parameter model for clay and sand (CASM) are developed in this paper to predict the soil behaviour around tunnels. The overall behaviour of clay and sand under both drained and undrained loading conditions could be properly captured by CASM, and the large-strain and effective stress analyses of cavity contraction provide the distributions of stress/strain within the elastic, plastic and critical-state regions around a tunnel. The effects of ground condition and soil model parameters are investigated from the results of stress paths and cavity contraction curves. Comparisons of the ground reaction curve and the excess pore pressure are also provided between the predicted and measured behaviour of tunneling, using data of centrifuge tunnel tests in clay.

INTRODUCTION

With the increasing demand for construction of tunnels in urban areas, it becomes more important to understand the tunneling-induced ground movements and to investigate their effects on preexisting underground structures and other services (Mair, 2008; Kolymbas, 2008). With symmetric assumption for deep tunnels, the ground movements at the tunnel heading are in a spherical scenario, while cylindrical symmetry is used for radial movements around lining, as can be seen in Fig. 1 (after Mair & Taylor, 1993; Mair, 2008). Undrained condition for clay behaviour around the heading is often applied, indicating the sufficiently fast advance of the tunnel (Mair, 2008).

Cavity expansion theory, concerning stress/displacement fields around cavities, has been developed and applied to a variety of geotechnical problems, as described in Yu (2000). By modelling the cavity unloading process from the in-situ stress state, cavity contraction method has been used for decades for the design of tunneling and the prediction of ground settlement (e.g. Hoek & Brown, 1980; Mair & Taylor, 1993). Mair & Taylor (1993) reported simple plasticity solutions for prediction of ground deformations and pore pressure changes caused by tunnelling in clay. Closed form solutions were proposed based on linear elastic-perfectly plastic solutions of cavity contraction in a Tresca material. In the past two decades, critical state solutions were increasingly proposed to account for the dependence of soil strength with deformation history (e.g. Collins & Yu, 1996; Yu & Rowe, 1999; Chen & Abousleiman, 2012). Additionally, undrained solutions of cavity expansion were recently developed using a unified state parameter model for clay and sand (CASM), which has the ability of capturing the overall behaviour of clay and sand (Mo & Yu 2016a). The undrained expansion solutions of Mo & Yu (2016a) are modified in this paper with respect to the problems of cavity contraction and tunneling.

This paper provides novel analytical solutions of undrained cavity contraction in a unified state parameter model for clay and sand (CASM) to predict the soil behaviour around tunnels. The solution aims to propose a unified approach for cavity contraction analysis in both clay and sand with two additional soil parameters (the stress-state coefficient and the spacing ratio), as well as a non-associated flow rule. Large strain analysis is adopted for both elastic and plastic regions by using the logarithmic strains. Taking account of the effect of stress history by an effective stress analysis, the predictions of stress fields and soil deformation are compared with previous analytical results and centrifuge data, with attempts to improve the prediction of the uniform convergence under the assumption of axisymmetry.

PROBLEM DEFINITION

The contraction of a spherical/cylindrical cavity with initial radius a_0 embedded in an infinite soil under undrained condition is concerned in this paper. The geometry and kinematics of cavity contraction are illustrated schematically in Fig. 2. The initial isotropic stress state is assumed with the initial ambient pore pressure u_0. The preconsolidation pressure is referred to as p'_{y0} and $R_0 = p'_{y0}/p'_0$ represents the isotropic overconsolidation ratio in terms of the mean effective stress. The specific volume keeps as a constant ($v = v_0$) during the process of contraction for undrained analysis. Note that a compression positive notation is used in this paper.

For cavity expansion/contraction problems, the quasi-static equilibrium equation can be written as:

$$\sigma_\theta - \sigma_r = \frac{r}{m}\frac{\partial \sigma_r}{\partial r} \tag{1}$$

where the parameter 'm' is used to integrate both cylindrical ($m = 1$) and spherical ($m = 2$) scenarios; σ_r and σ_θ are the total radial and tangential stresses, and r is the radius of the material element (r_0 indicates the initial position before cavity contraction). Excess pore pressure Δu is calculated as $u - u_0$. According to Collins & Yu (1996), the mean and deviatoric effective stresses (p'; q) for cavity contraction problems can be defined as follows:

$$p' = \frac{\sigma'_r + m \cdot \sigma'_\theta}{1+m} \quad ; \quad q = \sigma'_r - \sigma'_\theta \tag{2}$$

Similarly, the volumetric and shear strains (δ; γ) are expressed as:

$$\delta = \varepsilon_r + m \cdot \varepsilon_\theta = 0 \quad ; \quad \gamma = \varepsilon_r - \varepsilon_\theta \tag{3}$$

It is assumed that strains can be decomposed additively into elastic and plastic components while yielding occurs, and superscripts 'e' and 'p' are used to distinguish the elastic and plastic components of the total strains. To accommodate the effect of large deformation in cavity contraction process, large strain analysis is adopted for both elastic and plastic regions by using logarithmic strains:

$$\varepsilon_r = -\ln\left(\frac{d\,r}{d\,r_0}\right) \quad ; \quad \varepsilon_\theta = -\ln\left(\frac{r}{r_0}\right) \tag{4}$$

The state parameter ξ was defined by Been & Jefferies (1985), representing the difference of specific volume between the current and critical states at the same mean effective stress (see Fig. 3a):

$$\xi = \nu + \lambda \, \ln p' - \Gamma \tag{5}$$

It has shown to be an important parameter to describe the behaviour of granular material over a wide range of stresses and densities (Been & Jefferies, 1985; Sladen et al., 1985; Sladen & Oswell, 1989). In addition, it is also established that the state parameter can be used to determine the soil responses for both clay and sand (Yu, 1998).

With the benefits of the concept of state parameter, Yu (1998) proposed a unified state parameter model for clay and sand, which is referred to as CASM. It is a simple constitutive model with two additional material constants introduced to the standard Cam-clay model, whereas the overall behaviour of clay and sand can be satisfactorily modelled by CASM under both drained and undrained loading conditions. The state boundary surface of CASM (Fig. 3b) is described as:

$$\left(\frac{\eta}{M}\right)^n = 1 - \frac{\xi}{\xi_R} = -\frac{ln(p'/p'_y)}{ln\, r^*} \tag{6}$$

where $\eta = -q/p'$ is known as stress ratio (note that the negative symbol indicates the negative deviatoric stress during process of contraction); n is the stress-state coefficient; $\xi_R = (\lambda - \kappa) \, \ln r^*$, is the reference state parameter; and r^* is the spacing ratio, defined as p'_y/p'_x (see Fig. 3a). In addition, a non-associated flow rule based on the Rowe's stress-dilatancy relation is adopted here to better describing the deformation of sands and other granular media:

$$\frac{\dot{\delta}^p}{\dot{\gamma}^p} = -\frac{9\,(M-\eta)}{9+3\,M-2\,M\,\eta} \times \frac{m}{m+1} \tag{7}$$

Note that the relationship between the volumetric and shear strains in this paper and the conventional definitions is given by: $\dot{\delta}^p/\dot{\gamma}^p = \dot{\varepsilon}_p^p/\dot{\varepsilon}_q^p \times \frac{m}{m+1}$. The plastic potential can then be obtained by the integration of the stress-dilatancy relation (Eq. 7), and the hardening law is adopted based on a typical isotropic volumetric plastic strain hardening, as shown to be:

$$\dot{p}'_y = \frac{\nu\, p'_y}{\lambda-\kappa} \, \dot{\delta}^p \tag{8}$$

PLASTICITY SOLUTIONS

Plasticity solutions are presented in this section, for a cavity contracted from a_0 to a

until the soil around the cavity reaches the critical state (i.e. soil medium is deformed to include elastic, plastic and critical-state regions). 'c' is the radius of the elastic-plastic boundary, and c_{cs} is the radius where critical-state region initially commences. Thus, for $r > c$, soil is in elastic region; whereas for $c_{cs} < r < c$, soil is in plastic region, and critical-state zone is for soil at $a < r < c_{cs}$ (see Fig. 2b). Note that the contraction solutions are modified based on the cavity expansion solutions by Mo & Yu (2016a). Although some formulations can be found in Mo & Yu (2016b), the detailed derivations and solutions for cavity contraction are provided in this section.

Solution in Elastic Region

Soil volume within an arbitrary radius (r) can be assumed as constant with respect to undrained condition, and this relation leads to the following expression:

$$r_0{}^{m+1} - r^{m+1} = a_0{}^{m+1} - a^{m+1} = T \tag{9}$$

'T' keeps constant at a certain contraction instant and represents the volumetric change at an arbitrary radius. To describe the stress-strain relationship in elastic region, the elastic strain rates are given as follows:

$$\dot{\delta^e} = \frac{1}{K}\,\dot{p'} \quad ; \quad \dot{\gamma^e} = \frac{1}{2\,G}\,\dot{q} \tag{10}$$

where K is the elastic bulk modulus, which equals to $\frac{\nu\,p'}{\kappa}$; G is the elastic shear modulus, which is determined by $\frac{(1+m)\,(1-2\,\mu)\,\nu\,p'}{2\,[1+(m-1)\,\mu]\,\kappa}$, and μ denotes Poisson's ratio. In elastic region, elastic volumetric strain rate equals total volumetric strain rate ($\dot{\delta} = \dot{\delta^e} = 0$); thus the mean stress rate is zero based on Eq. (10), i.e. $p' = p'_0$. When the radial and tangential stresses are written as: $\sigma'_r = p'_0 + \Delta\sigma'_r$; $\sigma'_\theta = p'_0 + \Delta\sigma'_\theta$, the cumulative changes of effective stresses have the following relationship: $\Delta\sigma'_r = -m\,\Delta\sigma'_\theta$. Thus $\Delta\sigma'_\theta$ can then be derived as a function of radius r:

$$\Delta\sigma'_\theta = 2\,G_0\,\varepsilon_\theta = 2\,G_0\,\ln\left(\frac{r_0}{r}\right) = \frac{2\,G_0}{m+1}\,\ln\left(\frac{r^{m+1}+T}{r^{m+1}}\right) = A(r) \tag{11}$$

where G_0 represents the constant shear modulus in elastic region. With the aid of equilibrium Eq. (1), the incremental form of radial total stress can be obtained as:

$$\partial\,\sigma_r = \frac{m\,(m+1)}{r}\,A(r)\,\partial\,r = 2\,G_0\,m\,\frac{\ln\left(\frac{r^{m+1}+T}{r^{m+1}}\right)}{r}\,\partial\,r \tag{12}$$

The integration of Eq. (12) from r to $r = \infty$ leads to:

$$\sigma_r - p_0 = 2\, G_0\, m \int \frac{ln\left(\frac{r^{m+1}+T}{r^{m+1}}\right)}{r}\, \partial\, r \tag{13}$$

and the integration can be written as a series function:

$$\int \frac{ln\left(\frac{r^{m+1}+T}{r^{m+1}}\right)}{r}\, \partial\, r = \frac{1}{m+1} \sum_{k=1}^{\infty} \frac{\left(-T/r^{m+1}\right)^k}{k^2} = B(r) \tag{14}$$

Therefore, the distributions of stresses and strains in elastic zone are formulated as follows:

$$\begin{aligned} &\sigma_r' = p_0' - m\, A(r) \;\;;\;\; \sigma_\theta' = p_0' + A(r) \;\;; \\ &\varepsilon_r = -\frac{m}{2\, G_0} \times A(r) \;\;;\;\; \varepsilon_\theta = \frac{1}{2\, G_0} \times A(r) \;\;; \\ &\Delta u = 2\, G_0\, m\, B(r) + m\, A(r) \end{aligned} \tag{15}$$

where $A(r)$ and $B(r)$ can be determined by Equations (11) and (14).

For soil at elastic-plastic boundary ($r = c$), the stress state is on the initial yield surface (i.e. $p' = p_0'$; $q = q|_{r=c}$; $p_y' = p_{y0}'$). From the yield surface function (Eq. 6) for initial yielding, the deviatoric stress ($q|_{r=c}$) is derived as:

$$q|_{r=c} = -\left(\frac{ln\, R_0}{ln\, r^*}\right)^{\frac{1}{n}} M\, p_0' \tag{16}$$

On the other hand, the deviatoric stress can also be obtained from the distributions in elastic region (Eq. 15):

$$q|_{r=c} = -(m+1)\, A(c) = -2\, G_0\;\, ln\left(\frac{c^{m+1}+T}{c^{m+1}}\right) \tag{17}$$

Combining Equations (16) and (17) leads to the expressions of the elastic-plastic boundary radius and its original position before contraction:

$$c = \left\{\frac{-T}{1-exp\left[\left(\frac{\ln R_0}{\ln r^*}\right)^{\frac{1}{n}} \frac{M\, p_0'}{2\, G_0}\right]}\right\}^{\frac{1}{m+1}} \quad ; \quad c_0 = \left(c^{m+1} - T\right)^{\frac{1}{m+1}} \tag{18}$$

Cavity contraction starts with elastic responses, and further contraction may lead to yielding of soil around cavity. T_{yield} can be obtained from Eq. (18) for $c = a$, which is used to indicate the plastic stage when $T > T_{yield}$.

Solution in Plastic Region

When soil is in plastic region ($c_{cs} < r < c$), the elastic moduli (K and G) are not constant but functions of mean effective stress p'; and the undrained condition gives: $\delta^p = -\delta^e$. Following the integrations from $r = c$ to r, the elastic and plastic volumetric strains (Eq. 19) are derived with the aid of the elastic modulus (Eq. 10) and the hardening relation (Eq. 8), respectively:

$$\begin{aligned} \delta^e &= \int d\,\delta^e = \int_{p_0'}^{p'} \frac{\kappa}{\nu}\frac{1}{p'}\,d\,p' = \frac{\kappa}{\nu}\ln\left(\frac{p'}{p_0'}\right) \\ \delta^p &= \int d\,\delta^p = \int_{p_{y0}'}^{p_y'} \frac{\lambda-\kappa}{\nu}\frac{1}{p_y'}\,d\,p_y' = \frac{\lambda-\kappa}{\nu}\ln\left(\frac{p_y'}{p_{y0}'}\right) \end{aligned} \tag{19}$$

Substitute into Eq. (6) leads to:

$$\left(\frac{\eta}{M}\right)^n = A_1 + A_2 \times ln\, p' \tag{20}$$

where

$$A_1 = \frac{ln\,R_0 + \Lambda^{-1}\ \ln p_0'}{\ln r^*} \quad ; \quad A_2 = -\frac{\Lambda^{-1}}{\ln r^*} \quad ; \quad \Lambda = \frac{\lambda-\kappa}{\lambda} \tag{21}$$

Additionally, the differential forms of q and $\ln p_y'$ are expressed as follows:

$$\begin{aligned} d\,q &= -M \times \left\{[A_1 + A_2 \times ln\,p']^{\frac{1}{n}} + \frac{A_2}{n}\,[A_1 + A_2 \times ln\,p']^{\frac{1}{n}-1}\right\} d\,p' \\ d\ \ln p_y' &= \frac{\kappa}{\kappa-\lambda}\ d\ \ln p_y' = \frac{\kappa}{\kappa-\lambda}\,\frac{n}{A_2\,M^n}\,\eta^{n-1}\,d\,\eta \end{aligned} \tag{22}$$

Together with the boundary condition: $\gamma^e|_{r=c} = \frac{-(m+1)}{2\,G_0}\,A(c)$ based on Eq. (15), the elastic deviatoric strain (γ^e) in plastic region is obtained through the integration:

$$\begin{aligned} \int d\,\gamma^e = \gamma^e - \gamma^e|_{r=c} &= \frac{[1+(m-1)\,\mu]\,\kappa}{(1+m)\,(1-2\,\mu)\,\nu}\int_{q|_{r=c}}^{q}\frac{1}{p'}\,d\,q \\ &= -\frac{[1+(m-1)\,\mu]\,\kappa\,M}{(1+m)\,(1-2\,\mu)\,\nu}\left\{\frac{n}{(1+n)A_2}[A_1 + A_2 \times ln\,p']^{\frac{1}{n}+1} + [A_1 + A_2 \times ln\,p']^{\frac{1}{n}}\right. \\ &\qquad \left. - \frac{n}{(1+n)A_2}[A_1 + A_2 \times ln\,p_0']^{\frac{1}{n}+1} - [A_1 + A_2 \times ln\,p_0']^{\frac{1}{n}}\right\} \end{aligned} \tag{23}$$

Accordingly, the integration of plastic deviatoric strain (γ^p) is derived based on the stress-dilatancy relation (Eq. 7):

$$\begin{aligned} \gamma^p &= -\int_{ln\,p_{y0}'}^{ln\,p_y'} \frac{(9+3\,M-2\,M\,\eta)\,(\lambda-\kappa)\,(m+1)}{9\,\nu\,(M-\eta)\,m}\,d\,ln\,p_y' \\ &= \frac{\kappa\,n\,(m+1)}{9\,\nu\,A_2\,M^n\,m}\left\{\frac{2\,M}{n}[\eta^n - {\eta_c}^n] + (9 + 3\,M - 2\,M^2)\int_{\eta_c}^{\eta}\frac{\eta^{n-1}}{M-\eta}\,d\,\eta\right\} \end{aligned} \tag{24}$$

where $\eta_c = -q|_{r=c}/p_0'$, and the integration form can also be written as series functions:

$$\int \frac{\eta^{n-1}}{M-\eta}\, d\,\eta = \begin{cases} 0 & (\eta_c = M) \\ \frac{\eta^n}{M} \sum_{k=0}^{\infty} \left[\frac{1}{n+k} \times \left(\frac{\eta}{M}\right)^k\right] & (\eta_c < M) \\ \sum_{k=0}^{\infty} \left[-M^k \frac{\eta^{n-1-k}}{n-1-k}\right] & (\eta_c > M) \end{cases} \tag{25}$$

For associated flow rule of standard Cam-clay model, the stress-dilatancy relation can be rewritten as: $\dot{\delta^p}/\dot{\gamma^p} = (M-\eta) \times \frac{m}{m+1}$, hence the plastic deviatoric strain in Eq. (24) needs to be replaced by:

$$\gamma^p = \frac{\kappa\, n\, (m+1)}{v\, A_2\, M^n\, m} \int_{\eta_c}^{\eta} \frac{\eta^{n-1}}{M-\eta}\, d\,\eta \tag{26}$$

Combining Equations (3), (19), (23), and (24) leads to the distribution of radial and tangential strains. However, to obtain the total stresses and the excess pore water pressure, a numerical integration is required based on the equilibrium Eq. (1):

$$\int \partial\sigma_r = -m \int \frac{q}{r}\, d\, r \tag{27}$$

Solution for soil in critical-state region

When the cavity is contracted further after plastic stage, critical-state region commences from the cavity wall. The boundary of the critical state soil is referred as to c_{cs}, and the critical-state region is for soil where $a < r < c_{cs}$. In critical-state region, the deviatoric and mean effective stresses remain constants, and expressions can be given as:

$$\begin{aligned} p'_{cs} &= \left(\frac{R_0}{r^*}\right)^{\Lambda} p'_0 = exp\left[\frac{\Gamma - v}{\lambda}\right] \\ q_{cs} &= -p'_{cs} \times M \\ p'_{y,cs} &= p'_{cs} \times r^* = \left(\frac{R_0}{r^*}\right)^{\Lambda} r^*\, p'_0 \end{aligned} \tag{28}$$

RESULTS AND DISCUSSION

Comparisons with Results of Solutions by Yu & Rowe (1999)

In this section, the results of soil behaviour around deep tunnels are presented by using the provided plasticity solutions of cavity contraction in undrained condition, as also shown in Mo & Yu (2016b). As the yield criterion of the original Cam-clay model can be recovered from CASM by selecting the material constants: $n = 1.0$ and $r^* = 2.7183$, the validation of the solutions is carried out by comparing the results of original Cam-clay model with the results of solutions by Yu & Rowe (1999). The values of the critical state parameters, chosen to be relevant for London clay, are identical to Yu & Rowe (1999). It needs to be noted that the ambient pore pressure is not included in the results of total stresses (i.e. $\sigma = \sigma' + \Delta u$).

Figures 4 and 5 present the results of soil behaviour around tunnels using cylindrical and spherical scenarios, with the overconsolidation ratio of $R_0 = 1.001$. The final contraction for both cylindrical and spherical tests is $a_0/a = 1.95$ and 1.12, respectively. Subplots (a) show the cavity pressure and the excess pore pressure at the cavity wall during unloading. The obtained ground reaction curves caused by the tunneling are usually referred to as the convergence-confinement graphs (Panet & Guenot, 1982). The decreasing relationship between the support pressure and tunnel deformation is provided by the curve of σ_r. Negative excess pore pressure is predicted after an increasing stage at the initial contraction. Subplots (b) show the distributions of soil displacement (U), which is normalized by the cavity radius (a). The results are found to be comparable with data from Yu & Rowe (1999) when using non-associated flow rule, while identical results are shown for tests using associated flow rule.

Parametric study of cavity contraction

After validation of the proposed plasticity solutions by original Cam-clay model, parametric study is carried out in this section to investigate the variation of stress and deformation distributions with overconsolidation ratio (R_0) and soil parameters. The reference soil parameters are selected to simulate London clay ($\Gamma = 2.759$, $\lambda = 0.161$, $\kappa = 0.062$, $\mu = 0.3$, $n = 2.0$, $r^* = 3.0$, $\phi_{tx} = 22.75°$), as suggested by Yu (1998). The friction constant M is determined by: $M = \frac{2(m+1)\sin\phi_{cs}}{(m+1)-(m-1)\sin\phi_{cs}}$, where the critical state friction angle ϕ_{cs} can be assumed based on the triaxial critical state friction: $\phi_{cs} = \phi_{tx}$ for spherical scenario and $\phi_{cs} = 1.125\ \phi_{tx}$ for cylindrical scenario.

Fig. 6 presents the strain distributions around both spherical and cylindrical contracted cavities for $a_0/a = 2$. It can be seen that contraction results in negative radial strain and positive tangential strain; spherical scenario has larger radial strain and smaller tangential strain when comparing with cylindrical scenario. Radial deformation for both spherical and cylindrical contraction is shown in Fig. 7. In addition, the variation of strain distributions or radial deformation with soil parameters and overconsolidation ratio is not obvious due to the kinematics of undrained cavity contraction.

Fig. 8 shows the stress paths in normalised $p' - q$ space for $a_0/a = 1$ to 2. Two spherical tests are for overconsolidation ratio $R_0 = 1.001$ and 10, with the initial specific volume ν_0 as 2.0. After initial yielding, plastic region is generated around the cavity, and the stress path is gradually approaching the critical state line. The undrained plasticity solutions provide the exact stress paths after yielding. Both ultimate normalized mean and deviatoric effective stresses decrease with overconsolidation ratio. It should be noted that the stress paths for spherical scenario (Fig. 9) overlaps with cylindrical scenario in normalised $p' - q$ space.

The distributions of effective stresses (σ'_r, σ'_θ) and excess pore pressure (Δu) are presented in Fig. 10(a, b, c) respectively for both spherical and cylindrical scenarios. Stresses are normalised by undrained shear strength (s_u, defined as 0.5 $M\ exp[(\Gamma - \upsilon)/\lambda]$, based on the Mohr circle of effective stresses at failure), and the radial coordinate is normalised by cavity radius a. Critical state regions can be found in Fig. 9(a, b), where effective stresses keep constant. Blue circular symbols represent the elastic-plastic boundary (c) for tests with $R_0 = 10$, while c/a is larger than 15 for tests with $R_0 = 1$. At critical state, normalized effective stresses are independent of overconsolidation ratio. The elastic-plastic boundary is shown to decrease with overconsolidation ratio, and cylindrical cavity contraction has larger size of plastic region compared with spherical scenario. Negative excess pore pressure is generated during undrained cavity contraction, as shown in Fig. 9(c). Δu increases with overconsolidation ratio, and spherical scenario has larger value of negative excess pore pressure compared with cylindrical scenario. Fig. 9(d) shows the cavity contraction-pressure curves with different scenarios and overconsolidation ratio. Cavity pressure decreases with contraction, and negative cavity pressure might occur caused by the excess pore pressure.

Parametric study was also carried out to investigate the effects of two additional soil parameters of CASM (n and r^*), as presented in Figures 10 ~ 13. The stress-state coefficient n, varying from 1.0 to 2.5, has small influence on the distributions of normalized radial effective stress, for both spherical and cylindrical scenarios (see Fig. 10a and Fig. 11a). Constant value was also found at $r/a = 5.5$ for spherical contraction and $r/a = 9.5$ for cylindrical contraction. However, both normalized tangential effective stress and negative excess pore pressure increase with the stress-state coefficient. In addition, positive excess pore pressure appears in plastic region for soil with small value of stress-state coefficient. Compared with cylindrical scenario, higher negative excess pore pressure was found for spherical contraction.

The effects of spacing ratio r^*, varying from 2.0 to 5.0, have been shown in Fig. 12 for spherical scenario and Fig. 13 for cylindrical scenario. The increases of normalized effective stress with spacing ratio are obvious, relative to the effects of stress-state coefficient (see Fig. 12a,b and Fig. 13a,b). Conversely, negative excess pore pressure decreases with the spacing ratio, and positive excess pore pressure appears for soil with large value of spacing ratio. Due to the constant normalised effective stresses at critical state region, cavity contraction-pressure curves increase with spacing ratio, resulting from the effects on excess pore pressure.

Comparisons with Results of Centrifuge Tests by Mair (1979)

The proposed analytical solutions are related to soil behaviour around tunnels, with comparisons to centrifuge results by Mair (1979). Fig. 14(a) presents the prediction of tunnel crown displacement for the selected centrifuge test 2DP with cover to diameter ratio: $H/D = 1.67$. The tunnel test in clay can be assumed to be undrained condition. According to Mair (1979) and Yu & Rowe (1999), soil properties are chosen as: $\Gamma = 3.92$, $\lambda = 0.3$, $\kappa = 0.05$, $M = 0.8$, $\mu = 0.3$, $s_u = 26kPa$. The ground reaction curve indicates the crown displacement with reducing the tunnel support pressure. The crown displacement shows comparable results with the previous analytical results (Yu & Rowe, 1999) and the centrifuge data (Mair, 1979).

Fig. 14(b) shows the prediction of the distribution of excess pore pressure around a tunnel in soft clay. According to Mair & Taylor (1993), the equivalent stability ratio is defined as: $N = (p_0 - \sigma_{li})/s_u$, where σ_{li} represents the support pressure on the lining. Comparing with the centrifuge data for three different unloading stages of the tunnel test ($N = 2.4, 3.3, 4.2$), the excess pore pressure is generally well predicted in the plastic region. Additionally, the proposed analytical solution provides the variation of the excess pore pressure and the plastic region with the soil properties and overconsolidation ratio, as well as the equivalent stability ratio, which was the only influence factor reported by Mair & Taylor (1993).

As noted by Yu & Rowe (1999), the cavity solutions tend to underpredict the observed mid-surface settlement, probably owing to the shallow tunnel test with the effect of free ground surface. As the tunneling induced deformation is a combination of three components: uniform convergence, ovalisation, and vertical translation (e.g. Verruijt & Booker, 1996; Gonzalez & Sagaseta, 2001; Pinto & Whittle, 2006), the present solution provides an approach to improve the prediction of the uniform convergence under the assumption of axisymmetry. Further study is therefore required to incorporate the effects of ovalisation and vertical translation for the prediction of soil deformation around a tunnel.

CONCLUSIONS

By modelling cavity unloading process, analytical solutions of undrained cavity contraction in a unified state parameter model for clay and sand (CASM) were proposed in this paper to predict the soil behaviour around tunnels, including stress fields and crown/ground settlements. Taking the advantages of CASM with the ability of capturing overall behaviour of clay and sand, large-strain and effective stress analyses of cavity contraction provided the distributions of stress/strain within elastic and plastic regions around tunnels. The results of soil behaviour around tunnels using cylindrical and spherical scenarios showed identical results with previous analytical solutions using original Cam-clay model. The parametric study was carried out to investigate the variation of stress and deformation distributions with overconsolidation ratio R_0 and soil parameters (i.e. stress-state coefficient n and spacing ratio r^*).

Although the variation of strain distributions or radial deformation with soil parameters and overconsolidation ratio is not obvious, both ultimate normalized mean and deviatoric effective stresses decrease with overconsolidation ratio, as well as the elastic-plastic boundary. The negative excess pore pressure, generated during undrained cavity contraction, increases with the overconsolidation ratio. The stress-state coefficient has small influence on the distributions of normalized radial effective stress, whereas both normalized tangential effective stress and negative excess pore pressure increase with the stress-state coefficient. Conversely, the increases of normalized effective stress with spacing ratio are relatively obvious; negative excess pore pressure decreases with the spacing ratio; and positive excess pore pressure appears for soil with large value of spacing ratio. Good agreement with the centrifuge data of the ground reaction curve and the excess pore pressure indicates the ability for prediction of soil behaviour around tunnels and the potential implication of cavity contraction solution to tunnel modelling.

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NOTATION

The following symbols are used in this paper:

a	=	radius of cavity;
c	=	radius of the elastic/plastic boundary;
c_{cs}	=	radius of the critical-state region boundary;
g	=	undrained gap parameter;
m	=	parameter to combine cylindrical and spherical scenarios;
n	=	stress-state coefficient for CASM;
p'	=	mean effective stress;
p'_{y0}	=	preconsolidation pressure;
q	=	deviatoric effective stress;
r	=	radial position of soil element around the cavity;
r^*	=	spacing ratio for the concept of state parameter;
s_u	=	undrained shear strength for soil;

G, G_0	=	elastic shear modulus and small-strain shear modulus of soil;
H	=	cover of tunnel (from tunnel crown to surface);
K	=	elastic bulk modulus;
N	=	equivalent stability ratio;
R_0	=	isotropic overconsolidation ratio, defined as p'_{y0}/p'_0;
T	=	parameter for volumetric change of cavity, defined as ${a_0}^{m+1} - a^{m+1}$;
U	=	radial displacement after cavity contraction;
Δu	=	excess pore pressure;
δ, γ	=	volumetric and shear strain;
$\varepsilon_r, \varepsilon_\theta$	=	radial and tangential strains;
η	=	stress ratio, defined as q/p';
ν	=	specific volume;
ϕ_{cs}	=	critical state friction angle;
σ'_r, σ_r	=	effective and total radial stresses;
$\sigma'_\theta, \sigma_\theta$	=	effective and total tangential stresses;
ξ	=	state parameter;
ξ_R	=	reference state parameter;
$M, \kappa, \lambda, \Gamma, /$	=	critical state soil parameters.

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