Cosmic string network evolution in arbitrary Friedmann-Lemaître models

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We use the modified ''one-scale'' model by Martins and Shellard to investigate the evolution of a GUT long cosmic string network in general Friedmann-Lemaître models (with and without a cosmological constant). Four representative cosmological models are used to show that in general there is no scaling solution. [S0556-2821(98)03602-9]

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I. INTRODUCTION

Cosmic strings might be responsible for structure formation in the universe. To understand how cosmic strings could influence the cosmic medium we need to know how the string network evolves. In recent years the theory was applied mainly to the Einstein–de Sitter model (with $\lambda_0=0$ and $\Omega_0=1$), in which the network reaches a scaling behavior (see [1,2] for a review). However, there are observations which suggest that the matter density in the universe might be less than the critical density or that the cosmological constant is nonzero [3–5]. Therefore, an investigation of the evolution of the string network in open models or in those with a cosmological constant is necessary.

The first investigation of defects in open models was done by Spergel [6]. He argued that in such models a slight departure from the scaling behavior is expected and this might have interesting consequences for structure formation [7]. Other investigations of structure formation with cosmic strings assume the scaling solution in open models *a priori* [8]. The behavior of string networks in open models was recently discussed in some detail by Martins [9] using the modified ''one-scale'' model by Martins and Shellard [10]. In this paper we use this model to extend the analysis by Martins and allow for a nonvanishing cosmological constant. We investigate the behavior in four models: a flat model with a cosmological constant and $\Omega_0<1$, a closed model with a loitering epoch [11], and for comparison we include the Einstein–de Sitter model and an open model. The cosmological parameters for the models can be found in Table I and the behavior of the scale factor is shown in Fig. 1.

II. NETWORK EVOLUTION

In the modified ''one-scale'' model the energy of a string (or the typical length scale L, defined by $\rho_\infty=\mu/L^2$) and the rms velocity of the string $v_{\rm rms}$ are treated as independent quantities, which describe the statistical properties of the network. The equations of motion of the string network can be found in [9]. The reader is referred to that paper or to [10] for more information and discussions on the modified ''one-scale'' model. The loop chopping efficiency $\tilde{c}$, a phenomenological parameter which is included in order to describe the loop production, is chosen as in [9] for the different epochs. We know the values of $\tilde{c}$ in the radiation- ($\tilde{c}_r=0.24$) and matter-dominated ($\tilde{c}_m=0.17$) epochs [10], but not in the curvature- and vacuum-dominated epochs. However, the scaling properties of the string network do not crucially depend on the parameter $\tilde{c}$ [10,9]. Therefore, we assume a smooth transition between $\tilde{c}_r$ and $\tilde{c}_m$ between the radiation- and matter-dominated epochs and $\tilde{c}$ remains constant ($=\tilde{c}_m$) in the subsequent epochs. For the calculations we assume $G\mu=10^{-6}$, corresponding to the grand unified theory (GUT) energy scale.

Our results are presented in Figs. 2–4. They are consistent with the results from Martins and Shellard [10] and Martins [9] for the Einstein–de Sitter model and for the open model. In the vacuum dominated epoch, the scale factor grows as $R\propto\exp(Ht)$ ($H=\dot{R}/R=$const). One finds

$$L\propto\exp(Ht)\propto R. \tag{2.1}$$

The cosmic strings become frozen in the cosmic expansion. Therefore cosmic strings become dominant over matter but could not influence the expansion of the universe (because $\rho_{vac}=\Lambda/8\pi G$ is constant). The values of $\log(R_0/R_{eq})$ for the different models are summarized in Table I.

In Fig. 3 we plot the number of loops produced per Hubble volume and Hubble time. Because we assume $\tilde{c}_{vac}=\tilde{c}_{curv}=\tilde{c}_m$, we *overestimate* this number in the vacuum epoch. In this epoch the strings become frozen in the

TABLE I. The four representative cosmological models. K is the curvature parameter, Ω_0 is the matter density parameter, λ_0 is the cosmological term, H_0 is the Hubble parameter (in km s^{-1} Mpc^{-1}). R_0 and R_{eq} are the scale factors at the present time and at matter-radiation equality, respectively.

Model	K	Ω_0	λ_0	H_0	$\log(R_0/R_{eq})$
1	+1	0.014	1.08	90	2.66
2	0	1.0	0.0	60	4.16
3	-1	0.1	0.0	60	3.16
4	0	0.1	0.9	60	3.16

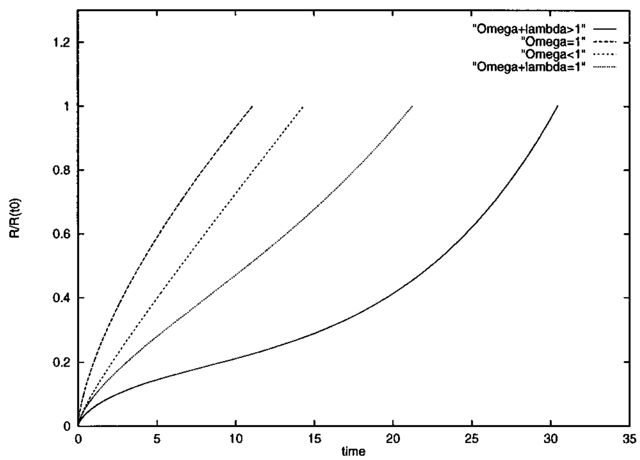

FIG. 1. The scale factor $(R(t)/R_0)$ as a function of time (in units of 10^9 years) for the four representative models.

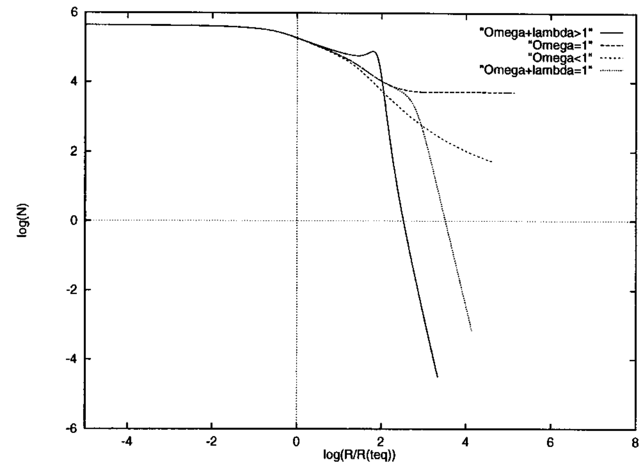

FIG. 3. The logarithm of the number N of loops produced per Hubble volume and Hubble time as a function of $\log(R/R_{eq})$.

cosmic expansion and therefore the probability of loop production decreases. As a result, the parameter $\widetilde{c}$ should also decrease. In the case of the loitering epoch in the closed model, the number is *underestimated*. The velocity of the strings increases (see Fig. 3) and so the probability of loop production. Therefore, the parameter $\widetilde{c}$ should increase in the loitering epoch. In contrast with open models or to the Einstein–de Sitter case, the energy loss (due to loop production) of the string network at the present epoch is not as efficient as in models with a cosmological constant.

One would expect that the string network approaches scaling in the radiation-dominated epoch, after a transition the network approaches scaling in the matter-dominated epoch and than the network becomes frozen in the cosmic expansion in the vacuum-dominated epoch. However, from Fig. 2 one can see that *the scaling in the matter-dominated epoch is not reached by the network*. The reason for this is that in the Einstein–de Sitter model the transition between the scaling behavior in the radiation- and the matter-dominated epoch is much longer than previously estimated [10]. In the other models the matter-dominated epoch is too short for the network to reach scaling. The same holds for the open model, where the universe becomes curvature dominated [9].

In Fig. 4 we plot the transition from the matter to the vacuum epoch for the models with a cosmological constant. In the case of the closed model there is a loitering epoch between the matter-dominated and the vacuum-dominated epochs. The transition between these two regimes is a very slow process.

We point out that the results do not depend strongly on the values of χ in the ansatz for k, which is a parameter related to the small scale structure of the strings [10].

The main difference between the network evolution in open models and in models with a cosmological constant is that the energy loss of the network at the present epoch is larger in open models. This has impact on the gravitational wave background and/or high energy particle fluxes from cosmic strings. It will also affect the structure formation theory with cosmic strings.

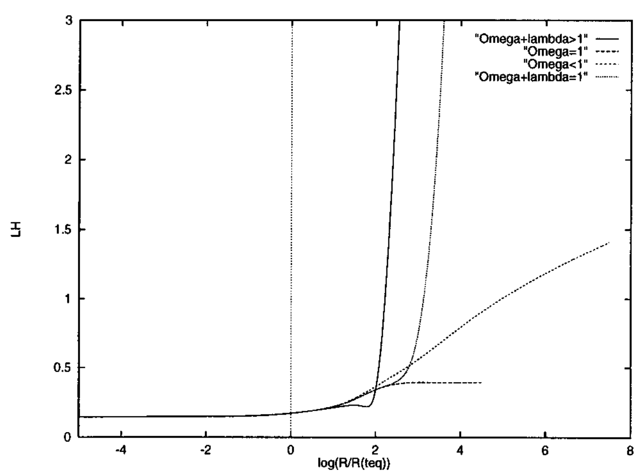

FIG. 2. Transition to the matter-dominated epoch. We plot LH as a function of $\log(R/R_{eq})$, L is the typical length scale of the string network, H is the Hubble parameter, and R_{eq} is the scale factor at radiation-matter equality.

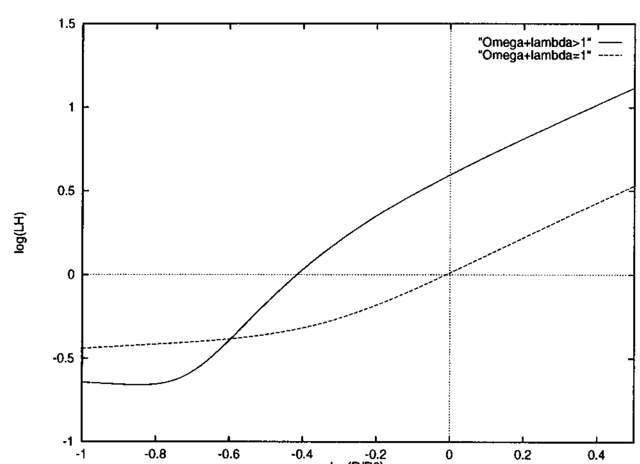

FIG. 4. The transition to the vacuum-dominated epoch in the models with cosmological constant. We plot $\log(LH)$ as a function of $\log(R/R_0)$.

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