Effective Block Sparse Representation Algorithm for DOA Estimation with Unknown Mutual Coupling

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***Abstract*—Unknown mutual coupling effect can degrade the performance of a direction of arrival (DOA) estimation method. In this letter, a new method is proposed for uniform linear arrays (ULAs) to tackle this problem. Considering the sparse representation exploiting the inherent structure of the received data, the effective block sparse representation and the convex optimization problem is constructed using the steering vector parameterizing method. The proposed solution based on the l_1-SVD (singular value decomposition) can exploit the information provided by the whole array and the Toeplitz structure of the mutual coupling matrix (MCM) in the ULA. Simulation results are provided to demonstrate its performance with unknown mutual coupling in comparison with some existing methods.**

***Index Terms*—Direction of arrival (DOA), block sparse representation, mutual coupling, l_1-SVD.**

I. Introduction

MULTIPLE-INPUT Multiple-Output (MIMO) technique is more attractive for increasing spectral and energy efficiency in the wireless and mobile communications [1]. Meanwhile, the MIMO system has more degrees of freedom and high spatial resolution than other systems in case of the direction of arrival (DOA) estimation [2–4]. There is an issue that must be considered in the MIMO system which the array size has been given, increasing the number of antennas will lead to the decrease of the array element spacing, and then resulting in a stronger mutual coupling effect between the antenna elements.

Mutual coupling can cause severe performance degradation for those conventional direction finding methods [5, 6]. Therefore, various array calibration techniques have been proposed [7–11]. For a uniform linear array (ULA), the coupling between neighboring elements is almost the same along the array, so the number of parameters can be reduced, and the mutual coupling matrix (MCM) can be modelled as a banded symmetric Toeplitz matrix [7]. And the method in [8] used auxiliary arrays, exploiting the banded symmetric Toeplitz matrix model for the mutual coupling effect, based on the ESPRIT algorithm. The special structure of the MCM of a ULA was also employed to parameterize the steering vector for joint estimation of DOAs and MCM in [9]. With the help of the auxiliary elements, the effect of mutual coupling can be eliminated and the MUSIC and ESPRIT method can be utilized directly to the angle estimation in bistatic MIMO radar [10, 11].

Recently, sparse signal representation based methods have been proposed to tackle spectrum estimation and array processing problems [12–16, 18], outperforming many traditional direction finding algorithms. To solve the more general source localization problems, the l_1-SVD method was derived in [12], which can be used to tackle a wide variety of practical signal processing problems. An efficient direction finding method based on the separable sparse representation is derived in [13], where it utilizes a separable structure for spatial observation matrix to reduce the complexity. And a perturbed sparse Bayesian learning-based algorithm is proposed to solve the DOA estimation for off-grid signals in [15], which is a more general case in practice. By using the sparse signal reconstruction of monostatic MIMO array measurements with an overcomplete basis, the SVD of the received data matrix can be penalties based on the l_1-norm [16]. In [17, 18], the sparse signal reconstruction based method is considered for DOA estimation with a coprime array, the over-complete representation is formulated for convex optimization problem design by reconstructing the virtual uniform linear subarray covariance matrix. In addition, the application of sparse reconstruction can be devoted to the solution of the mutual coupling problem. For example, it was applied in [14] to compensate for the mutual coupling effect with the help of a group of auxiliary sensors in a ULA.

In this paper, we propose a new block sparse signal representation based DOA estimation method in the presence of unknown mutual coupling effect and no auxiliary array elements are required in the process. By constructing a new over-complete block matrix based on the inherent structure of the steering vector with mutual coupling, we can make full use of the received data of the whole array and eliminate the unknown mutual coupling effect. The resultant sparse optimization problem for DOA estimation is transformed to a convex optimization and then solved using the l_1–SVD

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method.

Notation: $[\cdot]^T$ represents the matrix and vector transpose; $diag[\cdot]$ stands for the diagonalization operation of matrix blocks; $||\cdot||_p$ denotes the p-norm of a matrix; $[\cdot]_{M\times N}$ indicates a matrix of M rows and N columns; the zero vector or zero matrix is denoted by $\mathbf{0}$.

II. PROBLEM FORMULATION

Consider a ULA with M sensors with N far-field narrowband impinging signals $s_n(t)$, $n = 1, 2, \cdots, N$, where t is the sample index, with $t = 1, 2, \cdots, T$. Firstly, we formulate the received data model for an ideal array without mutual coupling as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

In (1), $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$ denotes the M received antenna signals, and the array steering matrix $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_N)]$ with $\mathbf{a}(\theta_n) = [1, \beta(\theta_n), \cdots, \beta(\theta_n)^{M-1}]^T$ denoting the nth signals ideal steering vector, where $\beta(\theta_n) = \exp(-j2\pi\lambda^{-1}d\sin\theta_n)$, and θ_n denotes the angle of arrival of the nth source, d is the adjacent sensor spacing, and λ is the signal wavelength. $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$ is the source signal vector, and $\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$ is the independent and identically distributed additive white Gaussian noise vector with zero mean and covariance matrix $\sigma^2\mathbf{I}$, where σ^2 denotes the power of noise and $\mathbf{I}$ is the identity matrix.

In practice, we have to consider the mutual coupling effect between closely spaced antennas. In this case, the received data model can be modified as

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{C}$ is the MCM. For ULAs, the MCM can be modelled as a banded symmetric Toeplitz matrix

$$\mathbf{C} = \begin{bmatrix} 1 & c_1 & \dots & c_{P-1} & & & & & \\ c_1 & 1 & c_1 & \cdots & c_{P-1} & & \mathbf{0} & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} & & \\ & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} \\ & \mathbf{0} & & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\ & & & & c_{P-1} & \cdots & c_1 & 1 & c_1 \\ & & & & & c_{P-1} & \cdots & c_1 & 1 \end{bmatrix}_{M\times M} \tag{3}$$

In (3), c_p is a complex number and denotes the mutual coupling coefficient between the mth and the $(m+p)$th sensor with $p = 0, 1, \cdots, P-1$, $m = 1, 2, \cdots, M$. P gives the maximum distance between antennas over which the mutual coupling effect cannot be ignored, which means that for the mth sensor, it is affected by electromagnetic coupling coming from the $(m-P+1)$th, $\cdots$, $(m-1)$th, $(m+1)$th, $\cdots$, $(m+P-1)$th sensors. For multiple snapshots, we can define $\hat{\mathbf{X}} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(T)] \in \mathbb{C}^{M\times T}$and also define $\mathbf{S}$ and $\mathbf{N}$ in a similar way. Then, we have

$$\hat{\mathbf{X}} = \mathbf{C}\mathbf{A}\mathbf{S} + \mathbf{N} \tag{4}$$

III. THE PROPOSED METHOD

In this section, the proposed sparse representation method for DOA estimation based on parameterization of the steering vector and l_1-SVD will be introduced.

A. Parameterization of the Steering Vector with Mutual Coupling

According to the data model in Section II, the ideal steering vector $\mathbf{a}(\theta)$ is distorted by the effect of mutual coupling in practice, and it should be modified as

$$\tilde{\mathbf{a}}(\theta) = \mathbf{C}\mathbf{a}(\theta) \tag{5}$$

According to (3), we can rewrite equation (5) as

$$\tilde{\mathbf{a}}(\theta) = \mathrm{H}(\theta)\mathbf{\Lambda}(\theta)\mathbf{a}(\theta) \tag{6}$$

where

$$\mathrm{H}(\theta) = \sum_{l=1-P}^{P-1} c_{|l|}\beta(\theta)^l \tag{7}$$

$$\mathbf{\Lambda}(\theta)=diag[\mu_1, \cdots, \mu_{P-1}, 1, \cdots, 1, \nu_1, \cdots, \nu_{P-1}]_{M\times M} \tag{8}$$

and for $k = 1, 2, \cdots, P-1$,

$$\mu_k = \frac{\mathrm{H}(\theta) - \sum\limits_{l=k}^{P-1} c_l\beta(\theta)^{-l}}{\mathrm{H}(\theta)}, \nu_k = \frac{\mathrm{H}(\theta) - \sum\limits_{l=P-k}^{P-1} c_l\beta(\theta)^{l}}{\mathrm{H}(\theta)} \tag{9}$$

Since $\mathbf{\Lambda}(\theta)$ is a diagonal matrix and $\mathbf{a}(\theta)$ is a column vector, (6) can be expressed by

$$\tilde{\mathbf{a}}(\theta) = \mathrm{H}(\theta)\mathbf{J}(\theta)\mathbf{v}(\theta) \tag{10}$$

where

$$\mathbf{J}(\theta) = \begin{bmatrix} 1 & & & \\ & \beta(\theta) & & \\ & & \ddots & & \mathbf{0} \\ & & & \beta(\theta)^{P-1} & \\ & & & \vdots & \\ & \mathbf{0} & & \beta(\theta)^{M-P} & \\ & & & & \ddots \\ & & & & & \beta(\theta)^{M-1} \end{bmatrix}_{M\times(2P-1)} \tag{11}$$

$$\mathbf{v}(\theta) = [\mu_1, \cdots, \mu_{P-1}, 1, \nu_1, \cdots, \nu_{P-1}]^T \tag{12}$$

Hence, (2) can be changed to

$$\mathbf{x}(t) = [\tilde{\mathbf{a}}(\theta_1), \tilde{\mathbf{a}}(\theta_2), \cdots, \tilde{\mathbf{a}}(\theta_N)]\mathbf{s}(t) + \mathbf{n}(t) \tag{13}$$

From (10), we can see that $\mathrm{H}(\theta)$ is a scalar parameter related to the mutual coupling coefficients and DOAs. It may take a zero value for some very specific cases. However, in general, it is not zero-valued and we assume $\mathrm{H}(\theta) \neq 0$, $\theta \in [-90^o, 90^o]$ in the following discussion. Then, (13) can be further changed to

$$\mathbf{x}(t) = \mathbf{A}_{\mathrm{J}}\mathbf{\Gamma}\mathbf{s}(t) + \mathbf{n}(t) \tag{14}$$

where

$$\mathbf{A}_{\mathrm{J}} = [\mathbf{J}(\theta_1), \mathbf{J}(\theta_2), \cdots, \mathbf{J}(\theta_N)] \tag{15}$$

$$\mathbf{\Gamma} = \begin{bmatrix} \mathrm{H}(\theta_1)\mathbf{v}(\theta_1) & & & \\ & \mathrm{H}(\theta_2)\mathbf{v}(\theta_2) & & 0 \\ & & \ddots & \\ 0 & & & \mathrm{H}(\theta_N)\mathbf{v}(\theta_N) \end{bmatrix}_{N(2P-1)\times N} \tag{16}$$

Now, we can consider the distinct block columns of the matrix $\mathbf{A}_\mathrm{J}$, i.e. $\mathbf{J}(\theta_n) \in \mathbb{C}^{M\times Q}$, as a new steering vector behaving like $\mathbf{a}(\theta_n)$, $n = 1, 2, \cdots, N$, $Q = 2P-1$, thus, $\mathbf{A}_\mathrm{J}$ becomes the new manifold matrix of the array with mutual coupling. $\mathbf{\Gamma}$ is a block diagonal matrix.

B. Block Sparsity Representatiom Using the l_1-SVD Method

For the case of sparse reconstruction in direction finding without mutual coupling, we first construct an over-complete representation $\tilde{\mathbf{A}} = [\mathbf{a}(\theta_1'), \mathbf{a}(\theta_2'), \cdots, \mathbf{a}(\theta_G')] \in \mathbb{C}^{M\times G}$ to find the sparsest spectrum of the signal vector $\tilde{\mathbf{s}} \in \mathbb{C}^{G\times 1}$ to satisfy $\mathbf{x} = \tilde{\mathbf{A}}\tilde{\mathbf{s}}$ with respect to all possible DOAs $\Theta = \{\theta_g', g = 1, 2, \cdots, G\}$, where the ith row of $\tilde{\mathbf{s}}$ is nonzero and equal to $s_n(t)$ if the DOA of signal n is θ_i' , G is the number of all possible DOAs and the set Θ constitutes the sampling grid. The formulation of the problem with additive white Gaussian noise is given as follows

$$\mathbf{x} = \tilde{\mathbf{A}}\tilde{\mathbf{s}} + \mathbf{n} \tag{17}$$

An ideal measure of sparsity is the l_0-norm constraint, but it is a difficult and intractable combinatorial optimization problem. According to [12], we use the l_1-norm minimization principle to relax the constraint, so the DOA estimation problem can be formulated as

$$min\, ||\tilde{\mathbf{s}}||_1,\ subject\ to\ \ ||\mathbf{x} - \tilde{\mathbf{A}}\tilde{\mathbf{s}}||_2^2 \le \xi^2 \tag{18}$$

Now, let us consider the case with mutual coupling. With (2) and (17), we can modify (18) as

$$min\, ||\tilde{\mathbf{s}}||_1,\ subject\ to\, ||\mathbf{x} - \mathbf{C}\tilde{\mathbf{A}}\tilde{\mathbf{s}}||_2^2 \le \xi^2 \tag{19}$$

The above representation is no longer a convex optimization problem due to the unknown mutual coupling parameter. In order to reconstruct the signal spectrum from (19), we need to construct a new over-complete matrix $\bar{\mathbf{A}}_\mathrm{J}$ in terms of a sampling grid of all potential source locations as follows

$$\bar{\mathbf{A}}_\mathrm{J} = [\mathbf{J}(\theta_1'), \mathbf{J}(\theta_2'), \cdots, \mathbf{J}(\theta_G')] \tag{20}$$

where each $M\times Q$ block matrix $\mathbf{J}(\theta_g')$ has the same structure as $\mathbf{J}(\theta)$. Meanwhile, because of the matrix in (14), the structure of the sparse signal vector is modified as below

$$\bar{\mathbf{s}} = \mathbf{\Gamma}'\tilde{\mathbf{s}} \tag{21}$$

where $\mathbf{\Gamma}' = diag[\mathrm{H}(\theta_1')\mathbf{v}(\theta_1'), \mathrm{H}(\theta_2')\mathbf{v}(\theta_2'), \cdots, \mathrm{H}(\theta_G')\mathbf{v}(\theta_G')] \in \mathbb{C}^{GQ\times G}$ is a block diagonal matrix, and the $(Qi-Q+1)$th to (Qi)th rows of $\bar{\mathbf{s}}$ are of a nonzero value if the ith row of $\tilde{\mathbf{s}}$ is nonzero and $\mathrm{H}(\theta_i') \neq 0$. So the $GQ\times 1$ signal vector $\bar{\mathbf{s}}$ has only a few nonzero blocks, each consisting of certain Q consecutive rows, i.e., $\bar{\mathbf{s}}$ has a block-based sparse spatial spectrum. Considering T samples of the received signal, we have

$$\hat{\mathbf{X}} = \bar{\mathbf{A}}_\mathrm{J}\bar{\mathbf{S}} + \mathbf{N} \tag{22}$$

where $\hat{\mathbf{X}} \in \mathbb{C}^{M\times T}$, $\bar{\mathbf{A}}_\mathrm{J} \in \mathbb{C}^{M\times GQ}$, and $\bar{\mathbf{S}} = [\bar{\mathbf{s}}(1), \bar{\mathbf{s}}(2), \cdots, \bar{\mathbf{s}}(T)] \in {}^{GQ\times T}$.

As a result, we can apply the l_2-norm for all samples and the problem can be again transformed into a convex optimization problem, as formulated below

$$min\, ||\hat{\mathbf{s}}^{l_2}||_1,\ subject\ to\, ||\hat{\mathbf{X}} - \bar{\mathbf{A}}_\mathrm{J}\bar{\mathbf{S}}||_2^2 \le \xi^2 \tag{23}$$

where $\hat{\mathbf{s}}^{l_2} = [s_1^{l_2}, s_2^{l_2}, \cdots, s_G^{l_2}]^T$, and $s_g^{l_2} = ||[s_g(1), s_g(2), \cdots, s_g(T)]||_2$. It is worth noting that $s_g(t)$ corresponds to the $(Qg-Q+1)$th to (Qg)th rows of in the tth snapshot.

When the number of data samples is large $(T > K)$, the computational complexity of the above optimization process will be very high. To reduce the complexity and also the sensitivity to noise, we can apply singular value decomposition (SVD) to the received data matrix $\hat{\mathbf{X}}$ to reduce its dimension. Denote the SVD of $\hat{\mathbf{X}}$ by $\hat{\mathbf{X}} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}$, and we further have

$$\hat{\mathbf{X}} = \mathbf{U}_S\mathbf{\Lambda}_S\mathbf{V}_S^H + \mathbf{U}_N\mathbf{\Lambda}_N\mathbf{V}_N^H \tag{24}$$

where $\mathbf{\Lambda}_S$ and $\mathbf{\Lambda}_N$ are diagonal matrices whose diagonal entries correspond to the N largest singular values and the remaining $M-N$ singular values, respectively. The unitary matrices $\mathbf{U}_S$ and $\mathbf{V}_S$ correspond to the signal subspace, while the unitary matrices $\mathbf{U}_N$ and $\mathbf{V}_N$ correspond to the noise subspace. Together we have $\mathbf{U} = [\mathbf{U}_S\,\mathbf{U}_N]$, $\mathbf{V} = [\mathbf{V}_S\,\mathbf{V}_N]^T$, and $\mathbf{\Lambda} = diag[\mathbf{\Lambda}_S\,\mathbf{\Lambda}_N]$. Then $\hat{\mathbf{X}}$. $\hat{\mathbf{X}}$ can be reduced to

$$\hat{\mathbf{X}}_R = \mathbf{C}\mathbf{A}\mathbf{S}_R + \mathbf{N}_R \tag{25}$$

where $\hat{\mathbf{X}}_R = \hat{\mathbf{X}}\mathbf{V}_S$, $\mathbf{S}_R = \mathbf{S}\mathbf{V}_S$, and $\mathbf{N}_R = \mathbf{N}\mathbf{V}_S$. Then, in a similar way, we can define $\bar{\mathbf{S}}_R = \bar{\mathbf{S}}\mathbf{V}_S = [\hat{\mathbf{s}}_R(1), \hat{\mathbf{s}}_R(2), \cdots, \hat{\mathbf{s}}_R(N)]$, $\hat{\mathbf{s}}_R^{l_2} = [\hat{s}_1^{l_2}, \hat{s}_2^{l_2}, \cdots, \hat{s}_G^{l_2}]$, and $\hat{s}_g^{l_2} = ||\bar{\mathbf{S}}_R\left((Qg-Q+1):Qg,\ :\right)||_2$, and arrive at the following formulation with a much reduced dimension

$$min\, ||\hat{\mathbf{s}}_R^{l_2}||_1,\ subject\ to\, ||\hat{\mathbf{X}}_R - \bar{\mathbf{A}}_\mathrm{J}\bar{\mathbf{S}}_R||_2^2 \le \xi^2 \tag{26}$$

According to the knowledge of the distribution, we can apply the l_1-SVD method and the upper value of $||\mathbf{N}_R||_2$ with a 99% confidence interval to select the regularization parameter ξ as described in [12]. As (26) shows, we have applied the parameterized steering vector operation to the manifold matrix of the array with mutual coupling. Thus, the spatial spectrum of $\bar{\mathbf{S}}_R$ is block sparse, which is related to the constructed over-complete matrix. And the computational complexity of solving (26) through the second-order cone programming is $O((NGQ)^3)$. So we employ the recursive grid refinement procedure [14] to reduce the calculation time.

Note that in our discussion, we have ignored the case of $\mathrm{H}(\theta)$= 0. This may happen for some specific combination of angle and coupling coefficient values, which means the array will not be able to receive the signal correctly for those directions and as a result, the proposed method will fail. However, $\mathrm{H}(\theta)$ is a continuous function for given coupling coefficients, so the chance of $\mathrm{H}(\theta)$= 0 has a measure of zero and we can say in general the proposed solution is valid and effective as demonstrated by the following simulation results.

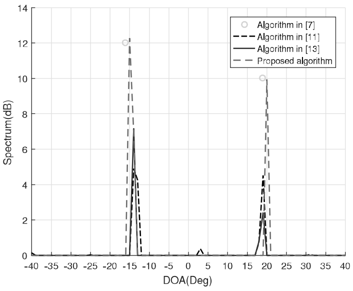

Fig. 1. Spatial spectrum obtained by the proposed algorithm in comparison with the algorithms in [8], [12] and [14].

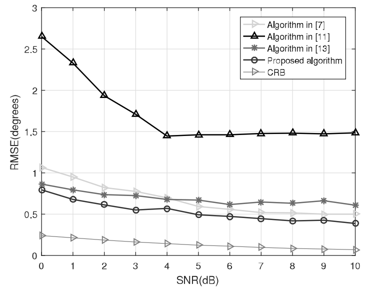

Fig. 2. RMSE of DOA versus SNRs with snapshot number T=400.

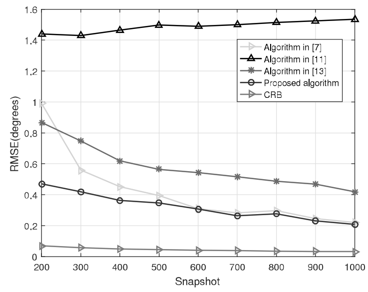

Fig. 3. RMSE of DOA versus snapshot number with SNR=20dB.

IV. SIMULATION RESULTS

In this section, simulation results are provided to show the performance of the proposed method. The number of far-field narrowband signals is $N=2$ with directions θ_1 and θ_2, respectively, the number of the ULA elements is $M=10$, and the number of nonzero mutual coupling coefficients is $P=4$. The root mean squared error (RMSE) is adopted as a performance index.

Firstly, we show the spectrum obtained by our method and the methods in [8], [12] and [14] in Fig. 1, with S-NR=5dB, snapshot number T=200, and directions $\theta_1=-15°$, and θ_2=20°. The mutual coupling coefficients are $c_1 = 0.4864-0.4776j$, $c_2 = 0.2325+0.1914j$, and $c_3 = 0.1163-0.1089j$. We can see that only our method can identify the directions of the sources correctly, while the methods in [8], [12] and [14] exhibit a large deviation from the true values. In particular, the method in [12] even led to a pseudo peak close to 5° due to the lack of consideration of mutual coupling.

Secondly, the performance of the proposed method is tested by comparing with the methods in [8], [12] and [14] at an SNR varying from 0dB to 10dB and with 400 snapshots, and directions $\theta_1=-12.1°$, and θ_2=15.9°. Fig. 2 shows the RMSE versus SNR curves obtained by averaging 400 Monte-Carlo simulations. The mutual coupling coefficients are $c_1 = 0.43301 - 0.351j$, $c_2 = 0.2618 + 0.2176j$, and $c_3 = 0.1414 - 0.1414j$. And we used the adaptive grid refinement approach to improve the measurement accuracy. As shown in Fig. 2, the proposed method has the superior resolution performance, that is because [12] suffers from lack of effective solution to the mutual coupling problem, while the method in [8] and [14] has given up the information received by $(2P-1)$ sensors located at the two ends of the ULA.

The third simulation examines the performance of our method at a snapshot number varying from 200 to 1000 with 200 Monte-Carlo experiments, and directions $\theta_1=-12.1°$, and θ_2=15.9°. The SNR is fixed at 20dB, and the mutual coupling coefficients are $c_1 = 0.5844 - 0.5476j$, $c_2 = 0.2625 + 0.1414j$, and $c_3 = 0.1163 - 0.1289j$. As shown in Fig. 3, again the proposed method has achieved the best performance.

V. CONCLUSION

In this paper, a new method based on sparse representation has been proposed to solve the DOA estimation problem in the presence of unknown mutual coupling for a ULA. The proposed algorithm can be considered as a combination of the parameterized steering vector and the l_1-SVD method, where the original non-convex problem with unknown mutual coupling parameters was transformed into a block-sparsity based convex problem by exploiting the banded symmetric Toeplitz property of the mutual coupling matrix. As shown in simulations, the proposed method has demonstrated a superior performance over existing solutions.

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