

# Representative elementary volume (REV) of cementitious materials from three-dimensional pore structure analysis

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## Abstract

The representative elementary volume (REV) is a fundamental property of a material, but no direct measurements exist for cementitious materials. In this paper, the REV of cement pastes with supplementary cementitious materials (GGBS, PFA, SF) was determined by analysing the three-dimensional pore structure ( $> 0.2 \mu\text{m}$ ) using laser scanning confocal microscopy (LSCM). The effect of axial distortion inherent to LSCM on 3D pore structure was also investigated. A range of 3D pore parameters was measured using skeletonisation, maximal ball and random walker algorithms. Results show that axial distortion has insignificant effects on most parameters except Euler connectivity, average pore and throat volumes and directional diffusion tortuosities. Most pore parameters become independent of sampling volume at  $\approx 60^3 \mu\text{m}^3$  except diffusion tortuosities and formation factor. The REV for porosity calculated based on a statistical approach at eight realisations and 5% relative error was found to be  $\approx 100^3 \mu\text{m}^3$ .

**Keywords:** *Microstructure (B); Image analysis (B); Transport properties (C); Cement paste (D); 3D pore structure*

## 1 Introduction

Representative elementary volume (REV), also known as representative volume element (RVE), is an important parameter for understanding and modelling the properties of multi-scale composite materials such as cement-based materials. The REV is the smallest volume over which a measurement or simulation can be carried out to produce a result that is representative of the macroscopic property. This is important because it is often difficult/impractical to experimentally capture or computationally generate composite materials at full length scales. Several definitions of REV exist, but there are two common requirements [1]: (1) the REV must be of the right size to contain sufficient microstructural features to depict the macroscopic property representatively, and (2) the REV is determined for a specified property and it is essentially independent of the sampling position within the material. Bear [2] presented the concept of REV graphically as shown in Figure 1. The fluctuations in the property of interest (e.g. porosity) reduce with increasing sampling volume and the volume at which fluctuations become insignificant is taken as the REV. For an inhomogeneous medium however, the property may gradually change again as the sampling volume increases further.

The REV depends on the length scale of the features of interest. Cement-based materials contain features ranging from nanoscale gel pores and hydrates to microscale capillary pores and millimetre-sized air voids and aggregate particles. Determining the REV at the concrete scale is relatively straightforward because one could simply carry out measurements on samples of varying sizes. However, it is much more challenging to do this at the scale of capillary pores. Yet, this is important because of its relevance to mass transport processes. The REV at the capillary pore scale is generally considered to be  $100^3 \mu\text{m}^3$ . However, this value was derived from numerical modelling of computer generated 3D pore structures [3-5]. For example, Zhang *et al.* [4] adopted a numerical-statistical approach to determine the REV based on finite element simulated diffusion of tritiated water through 3D models of cement pastes (w/c 0.30 to 0.60) generated with HYMOSTRUC3D. Later, Ukrainczyk and Koenders [5] found that the REV of computer generated 3D pore structures is highly dependent on the employed numerical resolution, boundary conditions, initial particle size distribution of anhydrous cement particles and degree of hydration.

To the best of our knowledge, the REV for cementitious materials has never been measured experimentally at the capillary pore scale. This could be partly due to lack of suitable experimental techniques to characterise the 3D characteristics of pore structure at sufficiently high resolution. Recently, a new 3D imaging approach which combines laser scanning confocal microscopy (LSCM) with serial sectioning [6] has enabled 3D reconstruction of pore structure at submicron spatial resolution. The method involves stitching of sequential confocal stacks based on phase correlation

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46 and so is able to image large volumes without resolution loss. Therefore, the method not only lends itself well for 3D  
47 pore characterisation, but also opens up possibilities for determining the REV of cement-based materials.

48 A number of issues need to be addressed to achieve this. For example, accurate segmentation of the pore structure from  
49 LSCM images is a prerequisite for successful analyses, but this is particularly challenging due to the complex  
50 boundaries between pores and solid hydration products. Uneven brightness that may occur along the depth of the  
51 reconstructed image further complicates the segmentation process. Fredrich [7] segmented the pore structure of Berea  
52 stone from 3D LSCM images using the local minima between solid and pore peaks in the image histogram as a  
53 threshold. Oh and Lindquist [8] have further developed a kriging-based method that determines the threshold based on  
54 minimum variance estimation within a pre-assigned threshold range, which also lies between the solid and pore peaks.  
55 However, the determination is not straightforward and is usually user-specified. Both methods are not applicable to this  
56 study because LSCM images of cement-based materials do not exhibit bimodal distribution in the histogram. Other  
57 approaches such as iterative k-means clustering also requires an a priori threshold to be estimated [9].

58 Furthermore, the process for quantitative analysis of the 3D pore structure based on images reconstructed by such a  
59 method has not been established to date. It is also well recognised that 3D LSCM images suffer from distortion in the  
60 optical (axial) axis because of two main reasons [10, 11]. First, the resolution is inherently anisotropic along the optical  
61 axis due to the elongation of point spread function (PSF). Second, mismatch of refractive indices between the  
62 immersion medium and the sample or within the sample itself can lead to severe spherical aberrations. Such distortions  
63 can produce misleading results when volumetric measurements are made. In biological imaging, the axial distortion  
64 caused by refractive index mismatch can be corrected empirically by inserting fluorescent microspheres of known  
65 geometries into the specimen to measure the elongation [12, 13]. However, this approach is unsuitable for hardened  
66 cement-based materials. While it may be possible to place 'micro-standards' in the fresh mix, they will either end up in  
67 the pore space or be engulfed by hydration products and therefore cannot be distinguished in hardened cement paste.  
68 Depending on the size of the standards, the resulting microstructure may also be altered owing to the formation of new  
69 interfaces. Moreover, these standards are required in large quantities and dispersed within the system to ensure a  
70 homogeneous correction.

71 The main aim of this paper is to: (1) study the effects of axial distortion in LSCM images on 3D pore parameters, and  
72 (2) estimate the REV for different pore parameters and cementitious systems. A method for pore segmentation and  
73 protocol for charactering the 3D pore structure was first developed. Then, the methods were validated on measurements  
74 of ideal 3D model structures. Finally, the methods were applied to quantify over twenty 3D pore structure of real  
75 cementitious samples in order to study the effects of axial distortion and to determine the REV. A range of blended  
76 cement pastes containing CEM I, silica fume, pulverised fuel ash and ground granulated blastfurnace slag cured to 7  
77 and 90 days were tested.

78

## 79 **2 Experimental**

### 80 **2.1 Materials and sample preparation**

81 Four cement pastes containing CEM I and CEM I blended with silica fume (SF), pulverised fuel ash (PFA) or ground  
82 granulated blastfurnace slag (GGBS) were prepared and cured for 7 and 90 days to produce samples with a range of  
83 microstructure. Mix proportions are shown in Table 1. The oxide compositions and properties of the cementitious  
84 materials are given in Table 2. The Bogue composition of CEM I was 53.1% C<sub>3</sub>S, 19.1% C<sub>2</sub>S, 10.8% C<sub>3</sub>A and 7.2%  
85 C<sub>4</sub>AF. The fineness and specific gravity of the CEM I were 291 m<sup>2</sup>/kg and 3.06 respectively.

86 All wet-mixing was done in a Hobart mixer for 4 min. PFA and GGBS were dry-mixed with CEM I for 1 min before  
87 water was added. For the mix with SF, a polycarboxylate-based superplasticiser was added to the water at 0.4 wt. %  
88 binder and pre-mixed with SF for 1 min to disperse agglomerated particles. All mixes were cast in steel moulds of 100  
89 mm diameter × 25 mm height and compacted in two layers using a vibrating table. Immediately after casting, the  
90 samples were covered with plastic sheets and wet hessian to prevent loss of moisture, and left to harden at 20°C. After  
91 24 hours, the samples were demoulded and cured in a fog room at 100% RH and 20°C for 7 and 90 days.

92 For each mix and curing age, four replicate discs were prepared; one for LSCM imaging and three for mass transport  
93 measurements, the latter will be reported in a separate publication. A slight amount of bleeding was observed after  
94 casting therefore two additional disc samples were prepared per mix to measure bleed water. The bleed water on the  
95 sample surface was collected by pipette and measured periodically in accordance with BS EN 480-4:2005 [14]. The  
96 corrected free w/b ratios for each mix are given in Table 1.

97

### 98 **2.2 Samples for microscopy**

99 After curing, a block (40×20×8 mm<sup>3</sup>) was extracted from the centre of each disc (Figure 2) using a diamond saw  
100 (Logitech GTS1) for imaging. The blocks were then placed in sealed environmental chambers containing saturated

101 potassium dichromate ( $K_2Cr_2O_7$ ) for conditioning at 55% RH, 20°C. The chambers were equipped with motorised fans  
102 to generate circulating air and soda lime to minimise carbonation. The blocks were conditioned until mass loss was no  
103 more than 0.01%/day. This typically took around 90 days.

104 Following conditioning, the blocks were fully impregnated with fluorescein-doped epoxy following the method  
105 described in Wong and Buenfeld [15]. The epoxy (Struers EpoFix, refractive index 1.578) was doped with fluorescein  
106 (C.I. Solvent Yellow 43) at 0.05 wt. %, then mixed with hardener at 25:3 mass ratio and thinned with toluene at 5% wt.  
107 Earlier work [6] found that a 0.05 wt. % dye concentration produced maximum fluorescence intensity under the  
108 imaging conditions described in Section 2.3.

109 The blocks were placed under vacuum (-1 bar) for an hour to remove air and then submerged in fluorescein-doped  
110 epoxy without breaking the vacuum. Vacuum was then released to force the epoxy into the blocks. Immediately after  
111 that, the blocks were pressurised with compressed air at 2.5 bars for 2 hours to complete the impregnation. Finally, the  
112 blocks were ground and polished using successively finer abrasive grit sizes of 30, 18, 14, 9, 6, 3, 1 and 0.25  $\mu m$  until a  
113 flat and highly reflective surface was achieved.

114

### 115 2.3 LSCM and 3D pore reconstruction

116 3D images of the pore structure were reconstructed using the method described in Yio *et al.* [6]. This combines  
117 fluorescence LSCM with serial sectioning to produce a series of overlapping 3D confocal Z-stacks, which are then  
118 aligned and stitched based on phase correlation. A Leica TCS SP5 microscope equipped with HCX PL APO 40 $\times$  (NA  
119 1.25) oil immersion objective was used for imaging. The pinhole aperture size was set at 0.3 Airy unit. A 488 nm argon  
120 laser at 15% intensity was applied to induce fluorescence. At these settings, the theoretical spatial XY and Z resolutions  
121 were 0.156 and 0.534  $\mu m$  respectively. These were calculated according to the Rayleigh criterion (Pawley, 1995) from  
122 the numerical aperture (NA) of the objective lens, refractive index of immersion oil (1.518), laser excitation wavelength  
123 and pinhole aperture (see [6] for details).

124 The emission band was set to range from 500 to 600 nm to ensure that all emitted fluorescence was captured. A zoom  
125 factor of 1.8 $\times$  was applied to give a field of view of 215  $\times$  215  $\mu m^2$  and images were digitised to 2048  $\times$  2048 pixels.  
126 The final voxel size was 0.105  $\times$  0.105  $\times$  0.1  $\mu m^3$ . Based on the Nyquist theorem, the smallest pore that can be resolved  
127 in the XY and Z direction is  $\approx$  0.242  $\mu m$  (2.3 $\times$  voxel width) and  $\approx$  0.534  $\mu m$  (2.3 $\times$  voxel depth) respectively. A 2 $\times$  line  
128 averaging, 400 Hz scan speed and a 3D median-filter with 1  $\times$  1  $\times$  2 voxel radii were applied to ensure good signal-to-  
129 noise ratio.

130 Two spots  $\approx$  5 mm apart (namely A and B) near the centre of each block were imaged (Figure 2). Serial sectioning was  
131 done by grinding with 15  $\mu m$  diamond on a Struers LaboPol-5 machine at low applied force (7N) and rotation (50 rpm).  
132 Depending on the sample mix and age, grinding time was varied between 1 to 4 s per direction to remove an average of  
133 1.83 to 3.57  $\mu m$  thick material per step. The sample was imaged after each grinding step to capture a 3D Z-stack of  $\approx$  10  
134  $\mu m$  thick. Well-focused images with high signal-to-noise ratio from each stack were selected, aligned using image  
135 registration with StackReg [16] and stitched using Pairwise stitching [17]. The process of sectioning, imaging,  
136 alignment and stitching was repeated until the total thickness of the reconstructed image ranged from 108 to 150  $\mu m$   
137 (see Table 3). This required stitching of 40 to 62 stacks. The average overlapping regions between stacks ranged from  
138 34.3 to 47.9% and the average correlation coefficients (R) ranged from 0.87 to 0.93, indicating good accuracy of the  
139 reconstruction process for all samples. The final area of the reconstructed image ( $\approx$  190  $\times$  190  $\mu m^2$ ) was smaller than  
140 the field of view (215  $\times$  215  $\mu m^2$ ) due to the loss of a small region around the edges during alignment (Table 3). Full  
141 details of the image acquisition process are given in Yio *et al.* [6].

142

### 143 2.4 Pore segmentation

144 A typical LSCM image of hardened cement paste (P0.45 7d) and its brightness histogram are shown in Figure 3. The  
145 same area imaged with backscattered electron (BSE) microscopy is also shown. Similar features can be seen in both  
146 images, giving confidence in the LSCM technique. However, slight differences are inevitable due to the fact that the  
147 images were not captured at the exact same plane. The LSCM image was obtained slightly beneath the sample surface  
148 for even and optimum brightness, while the BSE image is of the sample surface.

149 Note that bright green pixels in the LSCM image are pores while dark pixels are solids. Unreacted cement grains (AH)  
150 are dense, thick and non-porous, so they do not fluoresce and appear black. However, hydration products such as  
151 calcium silicate hydrates (C-S-H) are less dense and have an amorphous nano-porous structure. The C-S-H may appear  
152 brighter due to a slight detectable subsurface fluorescence occurring within the optical section (Z-stack). Pores  
153 approaching the resolution limit or smaller than the voxel size will also show intermediate brightness levels due to  
154 diffraction and mixing of signals from several phases. As such, the brightness histogram of LSCM shows a peak

155 representing solids, with a broad shoulder at the higher end of the grey scale with no distinct peak for pores due to  
 156 overlapping grey levels between some phases.

157 All these create uncertainties when selecting the threshold to segment pores from solids. Realistically, a perfect  
 158 segmentation is not possible, but an objective and repeatable approach applied to all samples is needed. Due to the lack  
 159 of a local minimum in the histogram, simple thresholding based on peaks and valleys, or the overflow method [18, 19]  
 160 do not work well. Various approaches commonly applied to fluorescent images were tested (e.g. Otsu's method [20],  
 161 IsoData [21], maximum entropy [22] and triangle method [23]). It was found that the moment-preserving method by  
 162 Tsai [24] was able to yield the most satisfactory results, based on visual comparison between the original and  
 163 segmented LSCM and backscattered electron images. The method is deterministic and does not require iterations. It  
 164 computes the threshold by retaining the first three moments of the original image in the binarised image. The first  
 165 moment is the mean grey value whereas the second and the third moments describe the variance and skewness of the  
 166 image histogram respectively [25]. It is also worth noting that this method has been applied successfully to 3D  
 167 fluorescence LSCM images of a silica monolith's skeleton [26] with pores of similar sizes to cement paste. A brief  
 168 description of the method is given in Appendix I.

169 Prior to segmentation, contrast limited adaptive histogram equalisation (CLAHE) [27] was applied to each image slice  
 170 to enhance the contrast of very fine pores. A block of 23 pixels (10× the resolvable pore size) was used to define the  
 171 local region for histogram equalisation. The number of histogram bins used was 256 and the maximum slope to restrict  
 172 maximum contrast change was set to 1.5. All operations were slice-wise and performed using Fiji (v.1.51d) [28].  
 173 Following pore segmentation, 3D morphological dilation and erosion with a kernel diameter of 3 voxels was applied to  
 174 the entire image to fill in small holes, remove noise and smooth edges of the pore structure. Given that the segmentation  
 175 process was slice-wise, a second 3D median filter with radii of  $1 \times 1 \times 2$  was applied to eliminate discontinuities along  
 176 edges of the segmented pores in the axial direction.

177

## 178 2.5 Quantitative 3D pore structure analysis

179 Over twenty 3D pore parameters, summarised in Table 4, were measured. These include: a) global parameters such as  
 180 porosity, specific surface area and pore size distribution; and b) topological parameters that describe connectivity and  
 181 degree of convolution (e.g. tortuosity). Three methods were used. The first was BoneJ [29] that consists of a set of  
 182 commands including medial-axis skeletonisation [30] which finds the skeleton running through the entire pore  
 183 structure. The second method was the modified maximal ball algorithm [31] which extracts the pore network by fitting  
 184 spheres of variable sizes into the pore space and ranking them accordingly to determine the largest spheres which form  
 185 the largest pores connected by smaller spheres which form the pore throats. The third method consisted of two  
 186 Mathematica® programmes [32] which run cluster labelling to detect connected pore voxels and random walk  
 187 simulations. Details of these can be found in their respective references.

188 The skeleton tortuosity determined using the medial axis-thinning algorithm is a direct measure of the crookedness of  
 189 the pore structure. It is calculated as [33]:

$$190 \quad \tau_s = \frac{\sum_{i=1}^j L_e}{\sum_{i=1}^j L_a} \quad \text{Eq. (1)}$$

191 Where  $i$  is the index of skeleton branch,  $j$  is the total number of skeleton branches,  $L_e$  and  $L_a$  are the actual and  
 192 Euclidean lengths of each skeleton branch respectively. Another measure of the pore geometrical complexity is the  
 193 diffusion tortuosity, which is based on the ease of self-diffusion of random walkers defined as follows:

$$194 \quad \tau_D = \frac{D_f}{D(t)} = \frac{a_1^2}{d(r(\tau_t)^2)} \quad \text{as } t \text{ and } \tau_t \rightarrow \infty \quad \text{Eq. (2)}$$

195 Where  $D_f$  and  $D(t)$  are the self-diffusivity ( $\text{m}^2/\text{s}$ ) of random walkers in free space and pore space as a function of time ( $t$ )  
 196 respectively,  $a_1$  is the dimension of a cubic voxel (m) and  $r(\tau_t)^2$  is the mean square displacement of the walkers as a  
 197 function of a unit time ( $\tau_t$ ). By assigning a value of 1 to  $a_1$ , the mean square displacement becomes dimensionless and so  
 198 is the diffusion tortuosity. Unless otherwise stated, a total of 20,000 random walkers and  $1 \times 10^7$  time steps ( $\tau_t$ ) were used  
 199 in the simulations so that the walkers experienced the full tortuosity of the pore structure. The time derivatives of the  
 200 mean square displacements were calculated from the fitted slope of the mean square displacement vs. lattice walk time  
 201 curve. The slopes were fitted for  $\tau_t > 2 \times 10^6$  and  $\tau_t > 4 \times 10^6$  for 7 and 90-day samples respectively. This was to avoid

202 unrestricted diffusion of the walkers during the early stages of simulation where most walkers rarely collided with the  
203 pore walls. By breaking down the scalar mean square displacements into the X, Y and Z axes, directional diffusion  
204 tortuosities can also be calculated.

205 Another relevant parameter is the formation factor ( $F$ ), which is calculated based on the Nernst-Einstein relationship as  
206 [34-36]:

$$207 \quad F = \frac{\sigma_o}{\sigma} = \frac{D_o}{D} = \frac{\Phi_p}{\tau_D^2} \quad \text{Eq. (3)}$$

208 Where  $\sigma_o$  and  $\sigma$  are the electrical conductivities (S/m) of pore solution and saturated material respectively,  $D_o$  and  $D$  are  
209 the free and intrinsic diffusivities ( $\text{m}^2/\text{s}$ ) of the bulk system respectively,  $\Phi_p$  is the accessible porosity and  $\tau_D$  is the  
210 scalar diffusion tortuosity. There is no general consensus as to whether or not the diffusion tortuosity should be squared.  
211 Indeed, several versions of the above relationship exist where tortuosity is unsquared or square rooted, as summarised  
212 by [37]. However, it was observed that Eq. 3 gave better predictions of transport properties (to be published).

213 Two workstations were used to run the algorithms: one is equipped with an Intel® Xeon™ CPU E5-1650 0 at 3.2 GHz  
214 processor with 32 GB RAM running on 64-bit Windows 7 Enterprise and the other is equipped with an Intel® Core™  
215 i7-4770 CPU at 3.4 GHz processor with 16 GB RAM running on 64-bit Windows 8 Home.

216

## 217 2.6 Axial distortion

218 A systematic analysis was performed to investigate the effect of axial elongation in 3D LSCM images on the measured  
219 pore parameters (Table 4). A sub-volume with an area of  $100 \mu\text{m}^2$  was cropped from the main 3D images of P0.45 7d.  
220 The voxel depth was first factored by 1.05 to correct for the mismatch of refractive indices between the immersion oil  
221 and epoxy in the pore structure [6]. Following that, the sub-volume was segmented using the method described in  
222 Section 2.4. A compression factor of 0.25, 0.50, 0.75 and 1.00 was then applied to the voxel depth to create a series of  
223 compressed image volumes with total thicknesses of 35, 70, 105 and  $140 \mu\text{m}$  respectively. These image volumes were  
224 further re-sliced prior to 3D median filtering to yield isotropic voxels of  $0.105 \mu\text{m}^3$  for analysis with the methods  
225 presented in Section 2.5. All other samples were analysed for the extreme compression factor of 0.25 and 1.00.

226 Having completed the above, a suitable correction factor for elongation across all samples was determined using  
227 spherical PFA particles in P0.45 PFA 7d and 90d as benchmarks. The principle is analogous to that of microspheres but  
228 in this case, the exact sizes of PFA particles are unknown. PFA particles are intrinsically perfect spheres and hence their  
229 aspect ratios in the XY and Z directions are approximately equal to 1. This enabled a correction factor to be determined  
230 by measuring the aspect ratios of many PFA particles as a function of reducing voxel depth without knowing their exact  
231 sizes.

232 In total, 80 PFA particles were randomly selected per sample (40 for each curing age) for measurements. The voxel  
233 depth was gradually compressed at intervals of 0.05. At each compression factor, the aspect ratios of all 80 particles  
234 were measured in the orthogonal views (XZ and YZ planes). Due to complexity of the microstructure, it was difficult to  
235 use a tracing tool to delineate particle boundaries for measurement. Instead, the 'Oval selections' tool in Fiji was used  
236 to fit ellipses to the PFA particles and the aspect ratios were recorded. The compression factor that gave the average  
237 aspect ratio closest to 1 was adopted as the correction factor.

238

## 239 2.7 Determination of representative elementary volume (REV)

240 The REV for different cementitious systems were determined using a statistical approach. Four sub-volumes of  $100^3$   
241  $\mu\text{m}^3$  were cropped from the imaged spots and segmented. Following that, smaller sampling volumes of  $20^3$ ,  $40^3$ ,  $60^3$  and  
242  $80^3 \mu\text{m}^3$  were extracted from their centres (Figure 4). Thus, a total of 20 sampling volumes (10 each for spots A and B)  
243 were analysed per sample. The  $100^3 \mu\text{m}^3$  sampling volumes were selected diagonally to each other and so they overlap  
244 slightly (no more than  $\approx 7\%$ ). In some cases, the total thickness of the image volume was less than  $100 \mu\text{m}$  after being  
245 corrected for axial distortion (see Table 3 & Section 3.4). The voxel depth was also resliced to give isotropic voxels  
246 ( $0.105^3 \mu\text{m}^3$ ). A total of 160 sub-volumes were analysed for the entire study.

247 The statistical approach enables the REV to be defined for a given property, number of realisations (or number of image  
248 volumes analysed,  $n$ ) and a chosen accuracy of the estimate (indicated by relative error,  $\epsilon$ ). For example, the REV ( $\text{m}^3$ )  
249 based on porosity can be calculated using Eq. (4) and Eq. (5) [38, 39]:

$$250 \quad V_{REV} = \frac{4(1 - \Phi_T)A_3}{n\Phi_T\epsilon^2} \quad \text{Eq. (4)}$$

251 Where  $\Phi_T$  is the ‘true’ total porosity (taken as the mean porosity of the main image volume from spots A and B  $\approx 190 \times$   
252  $190 \times 100 \mu\text{m}^3$ ) and  $A_3$  is related to the variance of the porosities ( $D_p^2(V_i)$ ) measured from  $n$  realisations for each  
253 volume size ( $V_i, \text{m}^3$ ), and was determined by fitting Eq. (5) to measured data:

$$254 \quad D_p^2(V_i) = \frac{\Phi_T(1 - \Phi_T)A_3}{V_i} \quad \text{Eq. (5)}$$

255

### 256 **3 Results**

#### 257 **3.1 Pore segmentation**

258 Figure 5 shows example images (XY plane) of a relatively porous (P0.45 PFA 7d) and relatively dense (P0.45 SF 90d)  
259 system segmented using the proposed method described in Section 2.4. Generally, both systems are well-segmented.  
260 Features with defined boundaries such as the PFA and anhydrous cement (AH) particles are visible. Pores as small as  $\approx$   
261  $0.2 \mu\text{m}$  are also segmented as shown by arrows in Figure 5c and d. This demonstrates the ability of the applied  
262 methodology (Section 2.4) to segment very fine features. Several hollow shell pores or ‘Hadley’ grains are also visible  
263 [40]. However, low contrast features such as the shells of some ‘Hadley’ grains (marked by red boxes in Figure 5a and  
264 b) and some pores nestled in solids (marked by red boxes in Figure 5c and d) may not be adequately segmented.

265 Figure 6 shows the pore structure of P0.45 PFA 7d and P0.45 SF 90d in three-dimensional views. A small volume of  
266 interest ( $30^3 \mu\text{m}^3$ ) was cropped from Figure 5 as an example to highlight the pore topology. Note that although the  
267 segmentation was performed slice-wise in the XY plane, pore edges in the Z plane appear smooth with no apparent  
268 discontinuities. A compression factor of 0.725 was applied to these images (see Sections 3.2 and 3.3). The largest  
269 connected pore and its corresponding skeleton determined by mapped-labelling and medial axis thinning algorithms  
270 show that P0.45 PFA 7d is more percolated than P0.45 SF 90d. The pore networks extracted by the maximal ball  
271 algorithm also indicate that the pores and throats of P0.45 PFA 7d are much larger than those of P0.45 SF 90d,  
272 consistent with our expectation.

273

#### 274 **3.2 Validation on ideal 3D pore structures**

275 The algorithms described in Section 2.5 and Table 4 were tested on nine 3D pore models constructed from spheres and  
276 cylinders of known sizes to represent pores and throats respectively. This is essentially to check that the algorithms are  
277 correct before applying them to actual 3D pore images for determining REV. The ideal pore models were generated  
278 using 3D Draw Shape and Line [41] in Fiji. 3D dilation and erosion were applied as in Section 2.4. Three pore  
279 configurations were considered: (i) pores attached, (ii) pores linked by short throats and (iii) pores linked by long  
280 throats. Details of the nine pore models are given in Table 5 and Figure 7. The measured total volume, surface area,  
281 pore and throat sizes, throat length, skeleton length and distance between pore to pore centres were compared with  
282 theoretical values in Figure 8. Note that the theoretical values were calculated by assuming that the sphere and throat  
283 surfaces were smooth and continuous. Overall, there is a good agreement between measured and calculated values.

284 All three methods (BoneJ, cluster labelling and maximal ball) gave the same total pore volumes, but slightly higher than  
285 theoretical values by 5.07% on average. This is due to the fact that the pore models are voxelised rather than having  
286 continuous smooth surfaces assumed in calculations. The measured total surface areas were on average higher than  
287 calculated values by 1.62% with BoneJ (marching cube algorithm) and 50.8% with cluster labelling. This is because  
288 triangular isosurface meshes used in BoneJ gave a better representation of curved surfaces compared to discretised  
289 voxels assumed in cluster labelling. The average pore sizes were slightly underestimated (-2.07%) by the maximal ball  
290 algorithm because the upper-limit radius of maximal ball is defined as the Euclidean distance from the centre voxel to  
291 the nearest grain voxel [31]. However, the average throat radius and throat lengths were overestimated slightly (8.28%  
292 and 15.9% respectively) because the throats in “pores attached” models (a, b, c) were enlarged during erosion/dilation  
293 due to close proximity between spheres (2 voxels). The total pore centre-to-centre lengths were accurately measured by  
294 maximal ball (1.52% mean error) while the total skeleton length obtained by medial axis thinning (BoneJ) had a slightly  
295 larger mean error of 5.21%. This was because the skeletonised branches were not straight within the spheres.  
296 Nevertheless, the topologies of all pore structures were well-preserved in the skeleton image as seen in Figure 7.

297

#### 298 **3.3 Effect of axial distortion**

299 Figure 9 shows the XZ plane of a sampling volume of P0.45 7d subjected to different axial compression factors. In can  
300 be seen that the pores appear stretched in the vertical direction when no axial compression was applied (1.00) and  
301 appear compressed when the applied factor was 0.25. Visually, an axial compression between 0.50 to 0.75 give the least  
302 distorted image. A more detailed quantitative assessment of this will be given in Section 3.4.

303 Figure 10 shows the effect of axial compression on measured pore parameters (Table 4). Note that the results are  
304 normalised to that of no compression. It can be seen that many pore parameters are insensitive to axial compression.  
305 These include total porosity, accessible (largest connected) porosity, percolation connectivity and skeleton tortuosity.  
306 Other parameters such as specific surface, shape factor, coordination number, pore and throat radii showed only a slight  
307 fluctuation. These parameters remain relatively constant because the pore and total sample volumes changed at equal  
308 proportion with axial compression. Similarly, the lengths and Euclidean distances of pore skeleton branches reduced at  
309 the same magnitude with increasing compression.

310 As expected, the average pore size, skeleton branch length, volumes and lengths of pores and throats decreased with  
311 increasing compression. This is because of the reduced dimension (shortened pore length) in the Z direction. Euler  
312 connectivity increased with compression because the contribution of the pore structure to the Euler characteristic of the  
313 entire image increased (see Table 4 and Odgaard and Gundersen, 1993 for details). The directional diffusion tortuosities  
314 were also significantly affected by compression (Figure 10c). This is because directional diffusion tortuosities were  
315 calculated as  $(1/3) / \text{fitted slope}$  of random walks in the X, Y and Z directions. Compression causes the cumulative  
316 mean square displacement of walkers (after a given simulation time) to reduce in the Z direction and increase in the XY  
317 direction. This led to increased Z tortuosity and reduced XY tortuosity.

318 Figure 10d presents the fitted slopes to mean square displacement vs. lattice walk time curves for calculating directional  
319 diffusion tortuosities. The larger the fitted slope, the lower the diffusion tortuosity. It is clear from the figure that with  
320 no compression, the contribution from the Z direction outweighs those from the XY directions, indicating an elongation  
321 of the pore structure in the original image. Axial compression decreases the contribution from Z direction and increases  
322 those from XY. Given that scalar diffusion tortuosity is equal to the sum contributions from all three directions, it  
323 remained relatively stable with increasing compression (Figure 10c). Similarly, the formation factor is insensitive to  
324 compression because it is calculated from scalar diffusion tortuosity and accessible porosity (Eq. 3).

325 The effect of axial distortion on all other samples was investigated for compression factors of 0.25 and 1.00. The  
326 porosities of these samples ranged from 14.2% to 26.4%. The results, summarised in Appendix II, are consistent with  
327 the findings from P0.45 7d above that many pore parameters are relatively insensitive to axial distortion. The  
328 parameters most affected are Euler connectivity, pore and throat volumes, and directional tortuosities.

329

### 330 3.4 Proposed correction factor for axial distortion

331 Figure 11 shows the XY and XZ planes of a PFA particle from P0.45 PFA 90d subjected to axial compression. The  
332 uncompressed XY plane clearly shows that the particle is a perfect sphere of diameter  $\approx 3.7 \mu\text{m}$ . However, in the XZ  
333 plane, the particle appears prolate and changes progressively to spherical then oblate when the axial compression factor  
334 increased to 0.5. The optimum compression factor that restored the particle to a sphere was approximately 0.7 (marked  
335 by a red box).

336 Figure 12a shows the average aspect ratios of 80 PFA particles from P0.45 PFA at 7 and 90 days measured as a  
337 function of increasing axial compression. The error bars represent the 95% confidence interval of the average based on  
338 Student's  $t$  distribution. The particle diameters determined from the minor axis of fitted ellipse without axial  
339 compression ranged from 9 to  $276 \mu\text{m}$ , thus a wide range of feature size was sampled. Note that this range of particle  
340 sizes are relatively large in comparison to the Z-resolution or the PSF. Thus, very fine features close to the Z resolution  
341 or PSF would remain elongated. The results show that the smallest aspect ratio was achieved with axial compression  $\approx$   
342 0.75 to 0.80 at 7 days, and  $\approx 0.65$  to 0.70 at 90 days. This suggests that the axial distortion was slightly more severe at  
343 90 days. Taking all data into consideration, the optimum compression was between 0.70 and 0.75 (Figure 12a).

344 Another approach to estimate the optimum correction factor was to measure the amount of compression required to give  
345 the smallest aspect ratio for every PFA particle. The results are presented as a frequency distribution histogram in  
346 Figure 12b. A spread of values is observed for both ages, indicating heterogeneity of the axial distortion. Nevertheless,  
347 the histograms show a peak at compression factors of 0.70 and 0.75, where most particles (55% for 7-day and 60% for  
348 90-day) were restored to spheres. Given that the optimum compression factors are so close, it is proposed that an  
349 average value of 0.725 is adopted as the correction factor for all samples.

350

### 351 3.5 Representative elementary volume

352 The effect of increasing sampling volume on the measured pore parameters (Table 4) is shown in Figures 13 to 15. The  
353 total time steps ( $\tau_i$ ) used for random walk simulations were 2, 4, 6, 8 and  $10 \times 10^6$  for sampling volumes of  $20^3$ ,  $40^3$ ,  $60^3$ ,  
354  $80^3$  and  $100^3 \mu\text{m}^3$  respectively while the total number of random walkers was kept at 20,000. Each data point is an  
355 average of four measurements (two from each imaging spot A & B) and normalised to that of the  $100^3 \mu\text{m}^3$ , acting as a  
356 benchmark. The error bars mark the maximum and minimum values. Note that the scale used on the Y-axes varies  
357 between plots and the X-axis represents the linear dimension of the image cube (= volume<sup>1/3</sup>).

358 The results show large scatter for all pore parameters when measurements were made on small image volumes ( $20^3$  to  
359  $40^3 \mu\text{m}^3$ ). There is also huge variability in the measured values and in some cases, the largest percolated pore structure  
360 was found to span the image volume in one or two directions only, instead of all three (X, Y and Z) directions.  
361 However, the degree of scatter for all pore parameters reduced significantly with increasing sampling volume. Most  
362 parameters became relatively constant (within  $\pm 10\%$  of the benchmark values) at  $60^3 \mu\text{m}^3$  except for the scalar  
363 diffusion tortuosity and formation factor (Figure 15).

364 The REV for porosity was calculated for all systems using Eq. (4) for a range of relative errors ( $\varepsilon = 1.0, 2.5, 5.0, 7.5$  and  
365  $10.0\%$ ). The 'true' porosity  $\Phi_T$  was taken as the mean total porosity of the main image volume ( $\approx 190 \times 190 \times 100 \mu\text{m}^3$ ) of  
366 spots A and B. The integral range  $A_3$  was determined from Eq. (5) by curve fitting the variance of measured porosities  
367 to image volume as shown in Figure 16a. The best fit  $R^2$  values for all curves were  $> 0.93$ , except for P0.45 GGBS 90d  
368 whose variances were relatively small and showed poor linearity with image volume. Overall,  $A_3$  increased with  
369 increase in porosity variance. As such,  $A_3$  is an indicator of pore structure heterogeneity. This can also be seen in Figure  
370 16b showing a positive correlation between  $A_3$  and length of REV for all systems ( $n = 8$  and  $\varepsilon = 5.0\%$ ).

371 The calculated REVs are presented in Figure 17. As expected, the size of the REV decreases with increasing number of  
372 realisations and relative error. This is because the measured pore parameters become more statistically representative if  
373 a greater number of images is analysed and if the tolerable error in the measurement is higher. There is no discernible  
374 effect of curing age on the REV. For a single realisation ( $n = 1$ ) and relative error of  $1\%$ , the REVs ranged from  $354^3$   
375  $\mu\text{m}^3$  (P0.45 GGBS 7d) to  $626^3 \mu\text{m}^3$  (P0.45 SF 90d), excluding P0.45 GGBS 90d whose  $A_3$  value was uncertain.  
376 However, as  $n$  increased to 8 and  $\varepsilon$  increased to  $5\%$ , nearly all REVs fell below  $100^3 \mu\text{m}^3$  except for P0.45 PFA 90d  
377 ( $101^3 \mu\text{m}^3$ ), P0.45 SF 7d ( $107^3 \mu\text{m}^3$ ) and P0.45 SF 90d ( $107^3 \mu\text{m}^3$ ). To achieve a REV of  $100^3 \mu\text{m}^3$  and maintaining the  
378 relative error at  $1\%$ , the number of realisations required ranged from 44 (P0.45 GGBS 7d) to 248 (P0.45 SF 90d),  
379 excluding P0.45 GGBS 90d. However, the requirement for such a high number of image volumes is not currently  
380 practical for 3D imaging.

381

## 382 4 Discussion

383 The proposed pore segmentation method may produce slight under- or over-segmentation with very low contrast  
384 features. The CLAHE operation enhances the local contrast of these features, but noise may also be enhanced and this is  
385 an inevitable side effect. The segmentation process may be affected by artefacts such as scratches and fall-out particles  
386 which occur during the sectioning process. Nevertheless, such artefacts are localised on the polished surface and will  
387 not affect overall results. This is because surface artefacts are removed once the Z-stacks are overlapped to reconstruct  
388 the 3D image. Finally, it should be recognised that a perfect segmentation method does not exist, but the approach  
389 proposed in this paper provides an objective means to pore segmentation.

390 Given that only pores connected to the sample surface and interconnected within the sample can be filled with  
391 fluorescein epoxy, the largest remaining pore following mapped-labelling or cluster labelling processes is effectively  
392 interconnected. Hence, no further segmentation such as watershed is needed to split the pore into sub-clusters. Clearly,  
393 pores which are smaller than the spatial resolutions of LSCM ( $\approx 0.2 \mu\text{m}$  in XY and  $\approx 0.5 \mu\text{m}$  in Z direction) may not be  
394 resolved properly. However, these pore sizes are already much finer than those resolvable by conventional X-ray  $\mu\text{CT}$ .

395 It has recently been shown by [42] that the critical pore entry diameter measured with mercury intrusion porosimetry  
396 (MIP) for 28-day white cement pastes cured underwater occurred at a few tens of nanometres. Such pores are obviously  
397 much smaller than the pores considered in this study. However, the very different appearance of pore structure obtained  
398 from MIP and other imaging techniques such as LSCM may not be as inconsistent as it seems. As evident from LSCM  
399 (Figure 3), large pores may in fact be interconnected through much smaller pores within the C-S-H, termed as  
400 'interhydrate' pores [43]. These are probably within regions of hydrates that exhibit intermediate brightness levels in  
401 the LSCM images. However, it should be emphasised that direct comparison between MIP and LSCM is not  
402 straightforward, as these techniques are different in terms of working principle, sample preparation (pre-drying  
403 techniques) and data interpretation.

404 Different cementitious systems tend to exhibit slightly different levels of axial distortion in LSCM images. Therefore,  
405 the proposed axial correction factor of 0.725 is an approximation. Ideally, a specific correction factor for each system  
406 should be determined. As mentioned in the Introduction, it is impractical to use micro-standards for this purpose. An  
407 alternative approach is to use random walker to find the optimal compression factor that gives equal directional  
408 diffusion tortuosities by trial-and-error. However, such a method is retrospective and is valid only if the pore structure is  
409 perfectly isotropic. Moreover, the axial distortion within a system is spatially variable depending on local porosity,  
410 phase density and refractive index. Nevertheless, as shown in Section 3.3, the major pore parameters such as porosity,  
411 specific surface, average pore and throat sizes, percolation connectivity and scalar diffusion tortuosity are not  
412 significantly affected by axial distortion, and therefore knowing the exact correction factor is not critical. The measured  
413 REV is not influenced by the choice of correction factor.

414 The determination of REV relied on averaging of four replicates and the results show this is adequate (small error bars).  
415 The obtained REV's based on the change in measured pore properties with increasing image volume for CEM I systems  
416 were between  $60^3$  to  $100^3 \mu\text{m}^3$ . These are generally in line with those determined from computer-generated 3D pore  
417 structure of pastes, e.g. Garboczi and Bentz [3] and Zhang *et al.* [4], with pre-defined water/cement ratio and cement  
418 particle size distribution and composition. Determination of REV from such models is also based on a statistical  
419 approach, where the fluctuations in the property of interest are quantified for a number of sampling volumes with  
420 increasing volume size until the property reaches a desired accuracy. Nevertheless, the sampling volumes are  
421 individually generated in modelling whereas those from experiments (e.g. Rolland du Roscoat *et al.* [39] and Mendoza  
422 *et al.* [44]) are usually sampled within the main domain at increasing length due to impracticalities with obtaining large  
423 number of datasets, as was the case in this study.

424 One may argue that the observed convergence in the measured property (Figs. 13, 14 & 15) was down to the  
425 increasingly large occupation of the sampling volumes within the entire domain. It should, however, be noted that the  
426 occupation of the sampling volumes ( $20^3$ ,  $40^3$ ,  $60^3$  and  $80^3 \mu\text{m}^3$ ) within the  $100^3 \mu\text{m}^3$  image volume was only 51.2% at  
427 most. To further support our findings, the average porosity measured from each sampling volume is normalised to the  
428 porosity measured from the main image volume ( $\approx 190 \times 190 \times 100 \mu\text{m}^3$ ). Figure 18 plots the normalised porosity against  
429 the size ratio of sampling volume to main image volume. Results show that the porosities become relatively constant  
430 and fall within  $\pm 0.1$  when the sampling volume is at only  $\approx 6$  to 30% of the main image volume. This corresponds to a  
431 sampling volume of  $60^3$  to  $100^3 \mu\text{m}^3$ , which confirms the findings from Section 3.5.

432 Given that a range of binder type, curing age and pore structure was covered in this study, the findings obtained should  
433 be relevant to other systems. However, it is important to note that these results were obtained from cement pastes, based  
434 on measurements made at two locations separated by a 5 mm distance. The REV's determined are likely a lower limit  
435 (see Figure 1) [2] since larger scale spatial variation may exist, but not be accounted for. Concretes and mortars are  
436 inherently even more heterogeneous than cement pastes because of the presence of aggregate particles, interfacial  
437 transition zone (ITZ) and defects such as microcracking and segregation. Therefore, pore structure variation occurs over  
438 larger length scales and the REV of these materials will be greater than for cement pastes. However, as explained in the  
439 Introduction, determining the REV at concrete scale is less complicated because one can simply conduct tests on  
440 samples of varying sizes. For example, 3D numerical modelling of mortars and concretes containing different aggregate  
441 particle shapes with ITZ show that a numerical sample size of at least  $2.5 \times$  the largest aggregate particle is needed to  
442 obtain representative simulations of diffusivity [45]. Experimental studies also showed that concrete samples with  
443 thickness of  $10 \times$  the largest aggregate size is needed to obtain consistent permeability measurements due to drying-  
444 induced microcracking [46].

445

## 446 5 Conclusions

447 In the present work, the three-dimensional pore structure of a range of cementitious systems containing CEM I blended  
448 with silica fume, pulverised fuel ash or ground granulated blastfurnace slag cured to 7 and 90 days was imaged with an  
449 approach combining laser scanning confocal microscopy with serial sectioning and image reconstruction. The 3D pore  
450 structure ( $> 0.2 \mu\text{m}$ ) was quantified to investigate the effects of axial distortion and to estimate the representative  
451 elementary volume (REV) of these systems. A total of 160 three-dimensional images were analysed. For each image,  
452 over twenty 3D pore parameters including total porosity, specific surface area, connectivity, skeleton tortuosity,  
453 diffusion tortuosity, formation factor, pore/throat radius, length, volume, shape factor and coordination number were  
454 quantified using BoneJ, maximal ball, cluster labelling and random walker algorithms. The main findings are  
455 summarised as follows:

- 456 a. The transition of brightness from the solid to the pore phases in LSCM images of cement-based materials spans  
457 across the entire grey scale due to the varying brightness of the solid phase. This complicates pore segmentation.  
458 The proposed pore segmentation method which combines the Moments method with CLAHE to enhance the local  
459 contrast of microstructures is able to segment pores as small as  $\approx 0.2 \mu\text{m}$  with good accuracy.
- 460 b. Different cementitious systems tend to exhibit slightly different extents of axial distortion in LSCM images.  
461 Nevertheless, the axial distortion was found to have very minor effects on most of the measured pore parameters  
462 including total porosity, specific surface area, percolation connectivity, average pore and throat radii, and scalar  
463 diffusion tortuosity. A generic correction factor of 0.725 was proposed based on the measured aspect ratios of 80  
464 PFA particles as a function of increasing axial compression.
- 465 c. Size of REV depends on the number of volumes sampled and averaged, the target % relative error and degree of  
466 variability (heterogeneity) of the pore structure. Based on the average results of four replicates, most pore  
467 parameters were found to be independent of the image volume size at  $60^3 \mu\text{m}^3$  except for diffusion tortuosity and  
468 formation factor. The REV's for porosity calculated based on eight realisations and a relative error of 5% for

469 different systems were all found to be  $\leq 100^3 \mu\text{m}^3$  except for P0.45 PFA 90d ( $101^3 \mu\text{m}^3$ ), P0.45 SF 7d ( $107^3 \mu\text{m}^3$ )  
470 and P0.45 SF 90d ( $107^3 \mu\text{m}^3$ ).

471

## 472 **Acknowledgements**

473 M.H.N. Yio would like to acknowledge the Dixon scholarship awarded by the Department of Civil and Environmental  
474 Engineering, Imperial College London. We thank Mr. Andrew Morris and Mr. Wenjie Zhou for their assistance with  
475 sample preparation and data collection.

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570

571

572 **Appendix I**

573

574 The Moments method proposed by Tsai [24] considers the grey-level image as a blurred version of the binarised image.  
575 It computes the threshold of an image by retaining the first three moments of the original image in the binarised image.  
576 The first three moments of an image are calculated as:

577 
$$m_i = \frac{1}{N} \sum_j N_j (z_j)^i = \sum_j p_j (z_j)^i \quad \text{with } i = 1, 2, 3 \quad \text{Eq. (I.1)}$$

578 Where  $N$  is the total number of pixels in the image,  $N_j$  is the total number of pixels with grey value  $z_j$ , and  $p_j = N_j / N$ .  
579 For bi-level thresholding, the pixels of the image are grouped into two classes, below- and above-threshold pixels. The  
580 moments of the binarised image  $m_i'$  are expressed as:

581 
$$m_i' = p_0 z_0^i + p_1 z_1^i \quad \text{Eq. (I.2)}$$

582 Where  $p_0$  and  $p_1$  are the fractions and  $z_0$  and  $z_1$  are the replacement grey values representative of the below- and above-  
583 threshold pixels respectively. By keeping the moments unchanged,  $m_i = m_i'$  and by solving Eq. (I.2) for  $m_i$ ,  $p_0$  and  $p_1$ ,  $z_0^i$   
584 and  $z_1^i$  can be determined. The threshold is taken as the grey value which corresponds to the  $p_0$ -tile of the histogram.

585

## Appendix II

**Table II.1: Effect of axial compression (at 0.25) on measured pore parameters for all samples. Results are normalised to that of no compression.**

Sample ID	Age (days)	BoneJ							Maximal ball							
		Total porosity	Accessible porosity	Percolation connectivity	Euler connectivity	Mesh specific surface area	Skeleton tortuosity	Avg. pore size	Avg. pore shape factor	Avg. pore connection number	Avg. pore volume	Avg. pore radius	Avg. throat shape factor	Avg. throat length	Avg. throat radius	Avg. throat volume
P0.45	7	1.00	1.00	1.00	2.05	1.26	1.00	0.79	0.91	1.00	0.50	1.02	1.00	0.78	0.96	0.46
	90	0.99	0.99	1.00	2.45	1.33	1.01	0.75	0.96	0.97	0.40	0.86	1.00	0.74	0.81	0.39
P0.45 SF	7	0.99	0.99	1.00	2.19	1.26	1.01	0.80	0.93	0.96	0.48	0.92	1.00	0.74	0.86	0.48
	90	0.99	0.98	0.99	2.22	1.25	1.02	0.83	0.91	0.87	0.41	0.91	1.00	0.67	0.85	0.43
P0.45 PFA	7	1.00	1.00	1.00	2.86	1.43	1.01	0.65	0.95	1.10	0.41	0.87	1.00	0.83	0.81	0.41
	90	0.99	0.99	1.00	2.07	1.30	1.01	0.77	0.93	0.91	0.44	0.94	1.01	0.76	0.91	0.44
P0.45 GGBS	7	1.00	0.99	1.00	1.91	1.29	1.01	0.74	0.92	0.98	0.48	1.02	1.00	0.81	0.96	0.50
	90	0.99	0.99	0.99	2.09	1.28	1.01	0.78	0.95	0.92	0.42	0.90	1.00	0.75	0.85	0.41
<b>Average</b>		0.99	0.99	1.00	2.23	1.30	1.01	0.76	0.93	0.97	0.44	0.93	1.00	0.76	0.88	0.44

Sample ID	Age (days)	Cluster labelling and random walker						
		Voxel S <sub>p</sub> A	Voxel S <sub>p</sub> B	Diffusion tortuosity X	Diffusion tortuosity Y	Diffusion tortuosity Z	Scalar diffusion tortuosity	Formation factor
P0.45	7	1.28	1.29	0.52	0.49	2.97	1.12	1.13
	90	1.35	1.36	0.54	0.66	3.94	1.13	1.14
P0.45 SF	7	1.29	1.30	0.56	0.57	3.24	1.23	1.24
	90	1.29	1.30	0.58	0.49	3.66	1.54	1.56
P0.45 PFA	7	1.44	1.44	0.60	0.58	3.24	1.12	1.13
	90	1.33	1.33	0.75	0.48	10.97	1.10	1.11
P0.45 GGBS	7	1.31	1.32	0.48	0.47	3.25	1.18	1.19
	90	1.31	1.32	0.58	0.65	3.92	1.67	1.70
<b>Average</b>		1.32	1.33	0.58	0.55	4.40	1.26	1.27

## Representative elementary volume (REV) of cementitious materials from three-dimensional pore structure analysis

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 UK

**Table 1: Mix proportions.**

Sample ID	CEM I (kg/m <sup>3</sup> )	SCM (wt.% total binder)	SF (kg/m <sup>3</sup> )	PFA (kg/m <sup>3</sup> )	GGBS (kg/m <sup>3</sup> )	Water (kg/m <sup>3</sup> )	Total w/b	Free w/b*
P0.45	1290	-	-	-	-	581	0.45	0.426
P0.45 SF	1158	9	115	-	-	573	0.45	0.445
P0.45 PFA	965	23	-	288	-	564	0.45	0.434
P0.45 GGBS	505	60	-	-	758	568	0.45	0.415

\* After correcting for water lost to bleeding

**Table 2: Oxide compositions and properties of cementitious materials used.**

Binder	Oxide composition (wt.%)									LOI (%)	Laser granulometry d <sub>50</sub> (μm)
	CaO	SiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>2</sub> O <sub>3</sub>	MgO	Na <sub>2</sub> O <sub>(eq)</sub>	K <sub>2</sub> O	SO <sub>3</sub>	Cl <sup>-</sup>		
CEM I	63.4	20.8	5.4	2.4	1.5	0.3	0.7	2.9	<0.1	2.10	N/A
SF	0.2	98.6	0.3	0.0	0.1	0.2	-	0.1	-	N/A	0.25
PFA	0.1	72.2	24.3	0.4	0.1	0.3	-	0.1	-	N/A	7.00
GGBS	40.8	36.5	11.6	1.4	7.5	0.5	-	2.1	-	-0.99	8.00

**Table 3: Details of the 3D pore reconstruction.**

Sample ID	XY field of view (μm <sup>2</sup> )		Total number of stacks		Total reconstructed thickness (μm)		Average overlapping region (%)		Average R	
	A	B	A	B	A	B	A	B	A	B
	P0.45 7d	190 × 190	161 × 186	52	62	117	137	42.0	42.1	0.93
P0.45 90d	190 × 190	190 × 190	44	40	143	130	37.4	38.4	0.92	0.93
P0.45 SF 7d	190 × 190	190 × 136	43	40	142	141	35.1	35.6	0.88	0.88
P0.45 SF 90d	190 × 190	188 × 188	47	47	144	148	39.0	39.4	0.90	0.90
P0.45 PFA 7d	190 × 190	190 × 190	62	56	146	138	43.1	42.9	0.89	0.89
P0.45 PFA 90d	190 × 190	190 × 190	43	42	147	150	34.3	36.2	0.89	0.90
P0.45 GGBS 7d	190 × 190	187 × 187	59	58	108	140	47.9	39.3	0.92	0.92
P0.45 GGBS 90d	190 × 129	180 × 180	43	44	141	145	37.8	37.9	0.88	0.87

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**Table 4: Quantification of 3D pore parameters using different methods.**

Method	Software	Algorithm	Parameters
BoneJ (Doube et al., 2010)	Fiji (v.1.51d)	Voxel counting	Total segmented porosity ( $\Phi$ ) = total volume of pore / total volume of image $\times$ 100%
		Mapped labelling	<ul style="list-style-type: none"> <li>• Accessible porosity (<math>\Phi_p</math>) = volume of largest percolated pore in X, Y and Z / total volume of image <math>\times</math> 100%</li> <li>• Percolation connectivity (<math>X</math>) = accessible porosity / total porosity</li> </ul>
		Euler characteristic	Euler connectivity = $1 - \Delta\chi$ , where $\Delta\chi$ represents the contribution from largest connected pore (Odgaard and Gundersen, 1993)
		Marching cube	Mesh specific surface area ( $S_p$ ) = surface area of pore mesh / volume of largest connected pore ( $m^{-1}$ )
		3D medial surface axis thinning	Skeleton tortuosity of largest connected pore ( $\tau_s$ ) (see Eq. (1) in text)
Modified maximal ball (Dong and Blunt, 2009)	Windows Command Prompt	Modified maximal ball	<ul style="list-style-type: none"> <li>• Total segmented porosity (<math>\Phi</math>) as in BoneJ;</li> <li>• Pore and throat radii: inscribed radii of the largest spheres in pores and throats respectively (m);</li> <li>• Throat length: see Figure 5 in Dong and Blunt (2009) (m);</li> <li>• Pore connection (coordination) number: number of pores linked to each pore defined;</li> <li>• Pore and throat volume: number of voxels associated with each pore or throat block defined (<math>m^3</math>);</li> <li>• Pore and throat shape factor = volume*length / surface area<sup>2</sup> of each pore or throat block defined</li> </ul>
			<ul style="list-style-type: none"> <li>• Total segmented porosity (<math>\Phi</math>), accessible porosity (<math>\Phi_p</math>) and percolation connectivity (<math>X</math>) as in BoneJ;</li> <li>• Voxel specific surface area (<math>S_p</math>) = surface area of voxels of largest connected pore without (A) and with (B) considering pore faces on the edges / volume of largest connected pore (<math>m^{-1}</math>)</li> </ul>
Mathematica programmes (Nakashima and Kamiya, 2007)	Mathematica® (v.10.4) (Wolfram Research, Champaign, Illinois)	Cluster labelling	<ul style="list-style-type: none"> <li>• Total segmented porosity (<math>\Phi</math>), accessible porosity (<math>\Phi_p</math>) and percolation connectivity (<math>X</math>) as in BoneJ;</li> <li>• Voxel specific surface area (<math>S_p</math>) = surface area of voxels of largest connected pore without (A) and with (B) considering pore faces on the edges / volume of largest connected pore (<math>m^{-1}</math>)</li> </ul>
		Random walker	<ul style="list-style-type: none"> <li>• Scalar (<math>\tau_D</math>) and directional (<math>\tau_d</math>) diffusion tortuosity of largest connected pore (see Eq. (2) in text);</li> <li>• Formation factor (F) of largest connected pore = <math>\tau_D^2 / \Phi_p</math></li> </ul>

Table 5: Details of 3D pore models used for validation.

Pore configurations	Model	Number of pore	Number of throat	Pore radius (voxel)	Pore centre to pore centre length (voxel)	Throat radius (voxel)
Pore attached	a	3	3	32	66	
	b	4	6			
	c	8	12			
Pore linked by short throats	d	3	3	24	84	8
	e	4	6			
	f	8	12			
Pore linked by long throats	g	3	3	16	104	
	h	4	6			
	i	8	12			

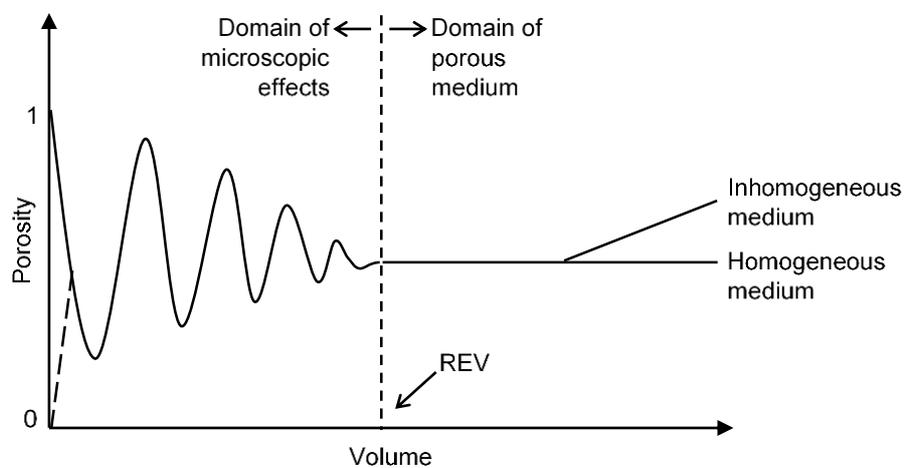


Figure 1: Change in property of interest (e.g. porosity) as a function of sample volume to define REV (after Bear [2]).

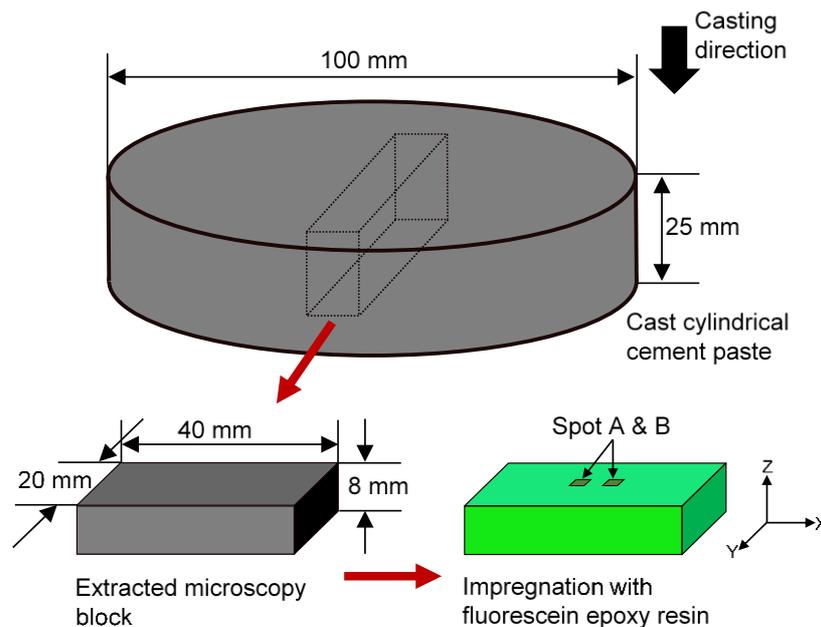


Figure 2: Preparation of epoxy impregnated block for 3D LSCM from cast cylindrical sample (not to scale).

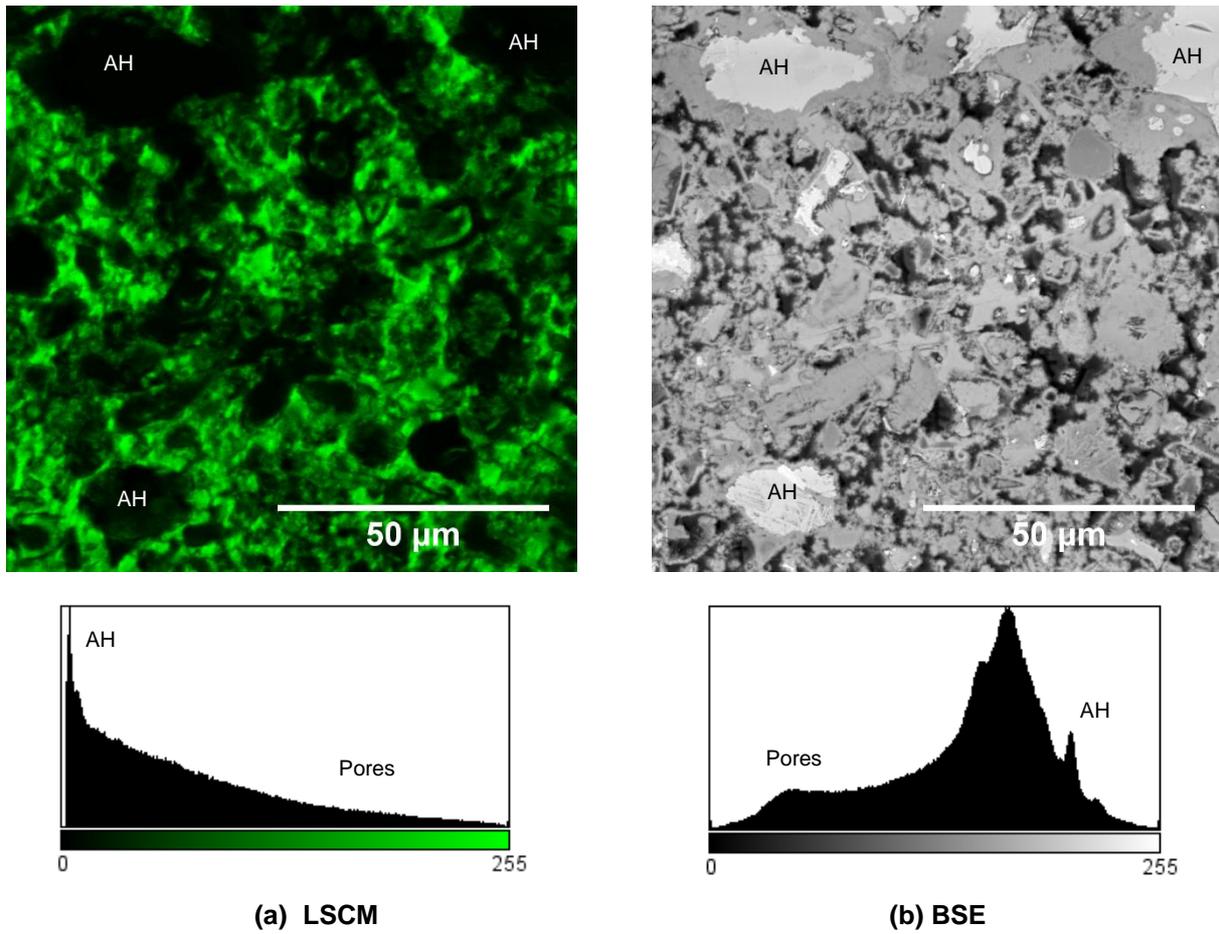
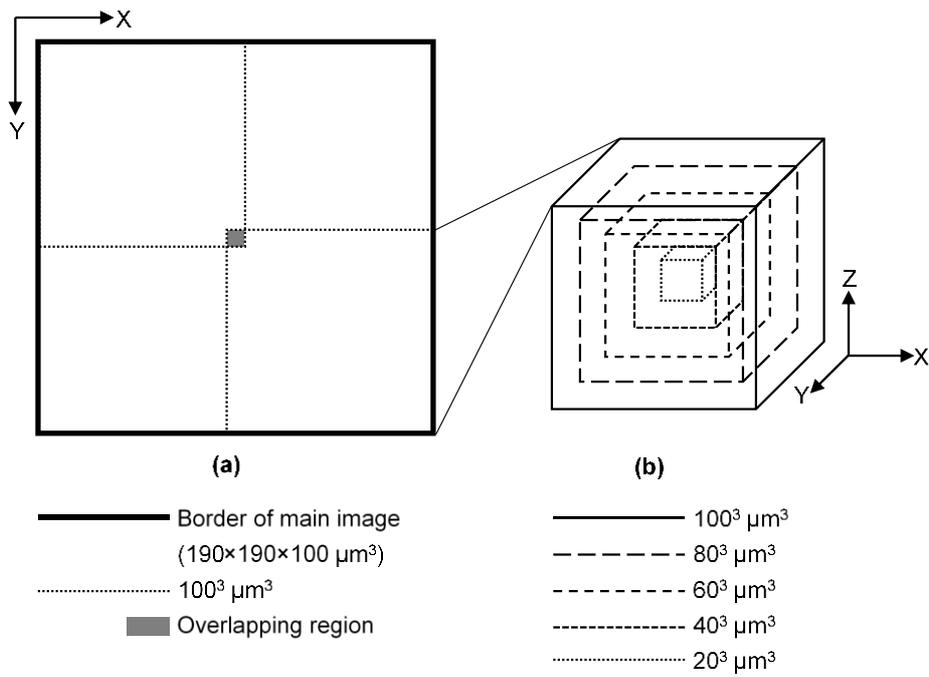
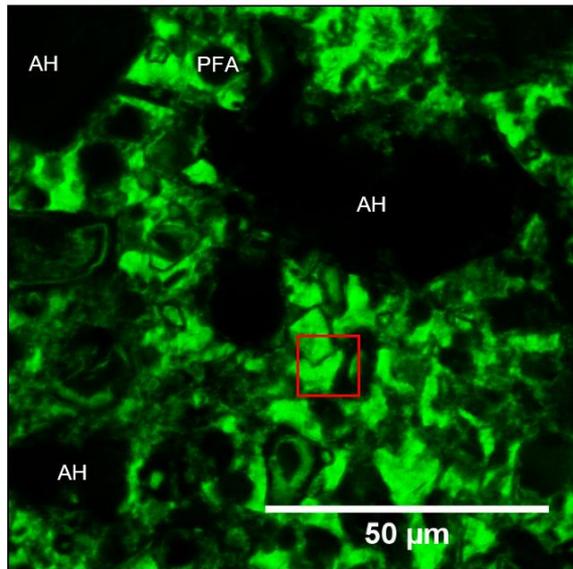


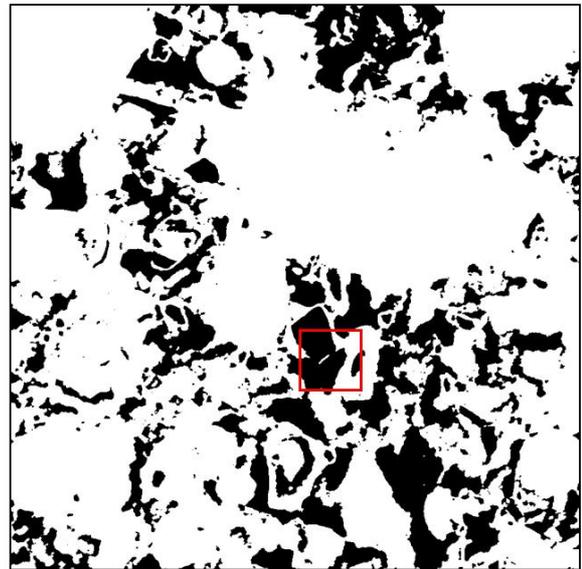
Figure 3: Comparison between LSCM and BSE images of hardened cement paste impregnated with fluorescein-doped epoxy captured at the same area. Sample is P0.45 7d.



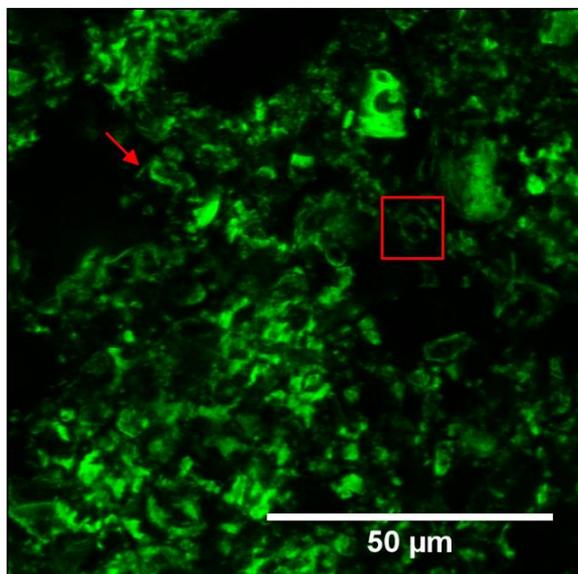
**Figure 4: Cropping of sampling volumes from the main 3D image for REV analysis: (a) XY plane of the main 3D image; and (b) 3D views of cropped sampling volumes. Diagram not to scale.**



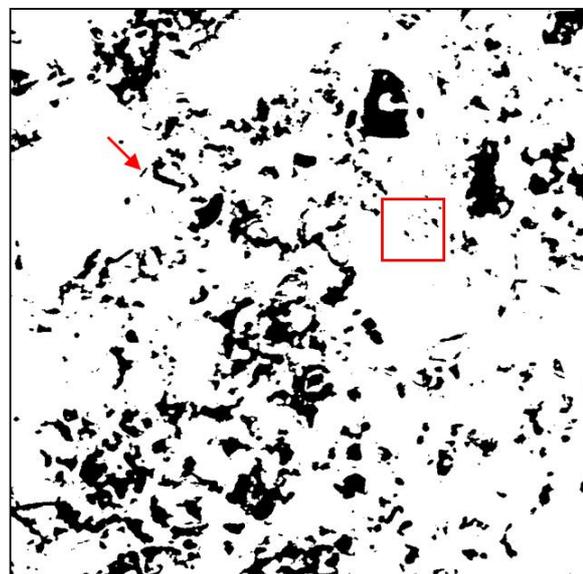
(a) P0.45 PFA 7d



(b) Pores segmented from (a); porosity = 24.4%

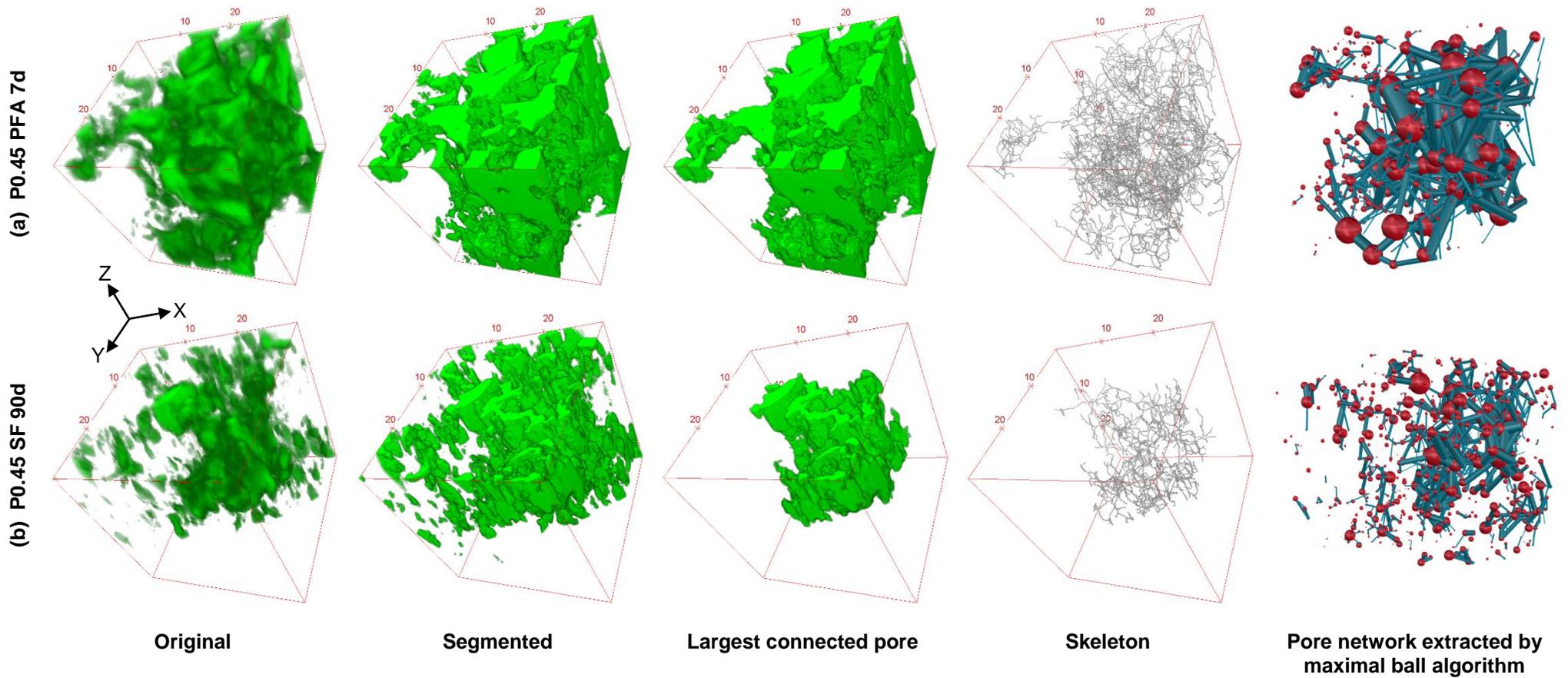


(c) P0.45 SF 90d



(d) Pores segmented from (c); porosity = 16.9%

Figure 5: Example pore segmentation demonstrated on (a, b) P0.45 PFA 7d; and (c, d) P0.45 SF 90d.



**Figure 6: Example 3D views of the pore structure of (a) P0.45 PFA 7d; and (b) P0.45 SF 90d. All image volumes are  $30^3 \mu\text{m}^3$ . Images were generated using Fiji except for the pore networks, which were visualised using Rhinoceros 5 (Robert McNeel & Associates, Seattle). In the pore networks, green cylinders represent pore throats while red spheres represent ‘ancestor’ pores.**

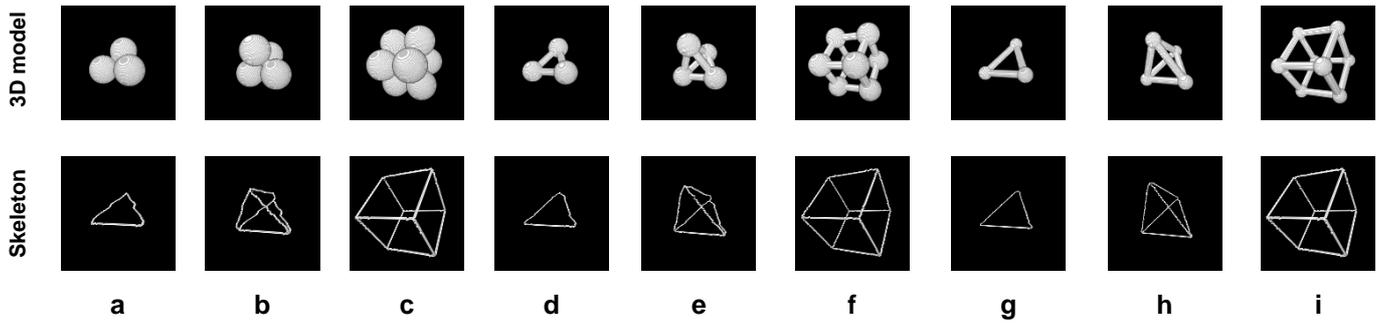


Figure 7: 3D pore models and their respective skeletons computed by the medial-axis thinning algorithm for validation.

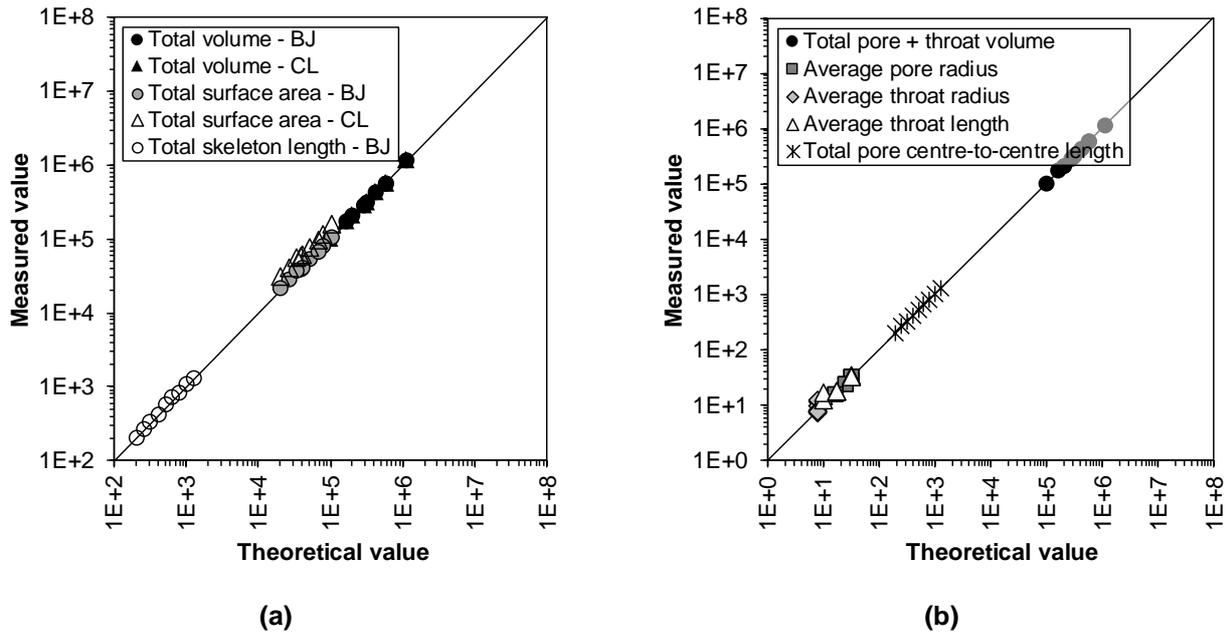


Figure 8: Comparison between measured pore parameters and theoretical values for the 3D pore models shown in Fig. 7. Measurements were made using (a) BoneJ (BJ) and cluster labelling (CL); (b) maximal ball algorithms.

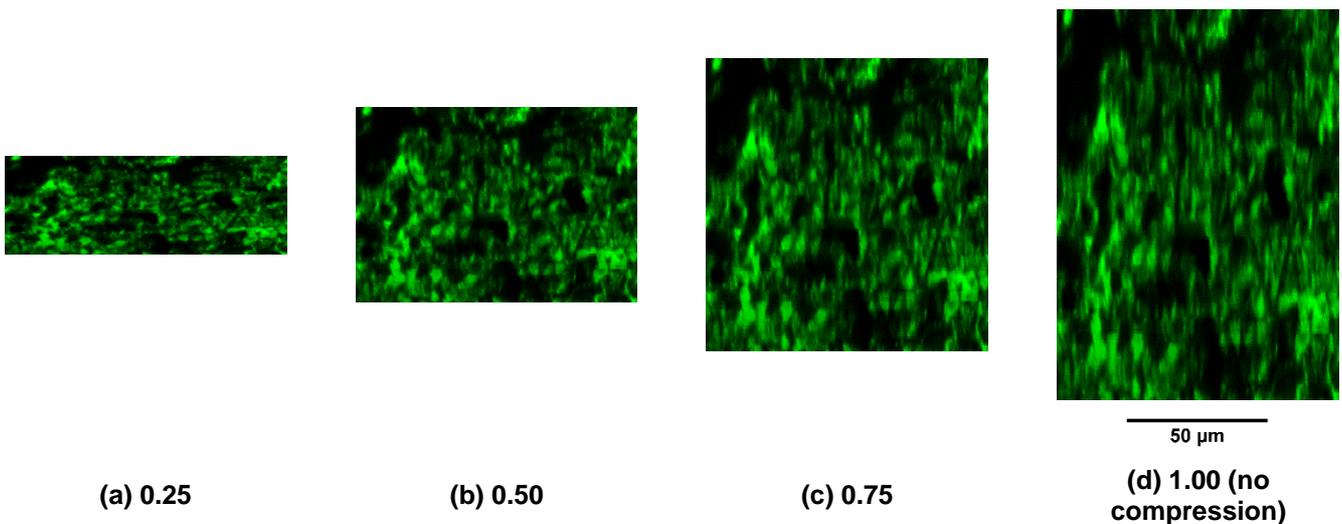
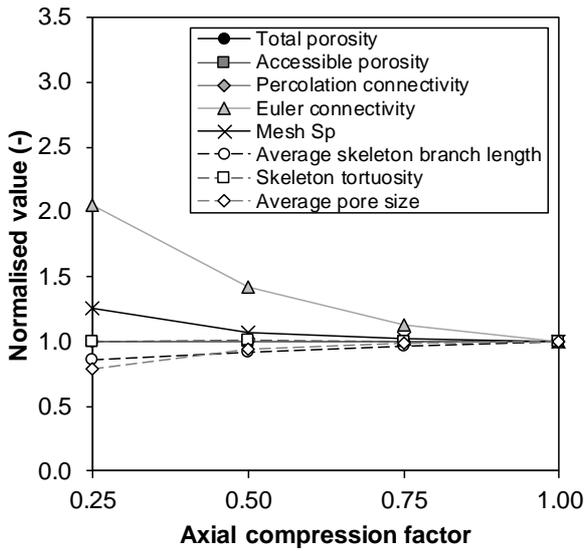
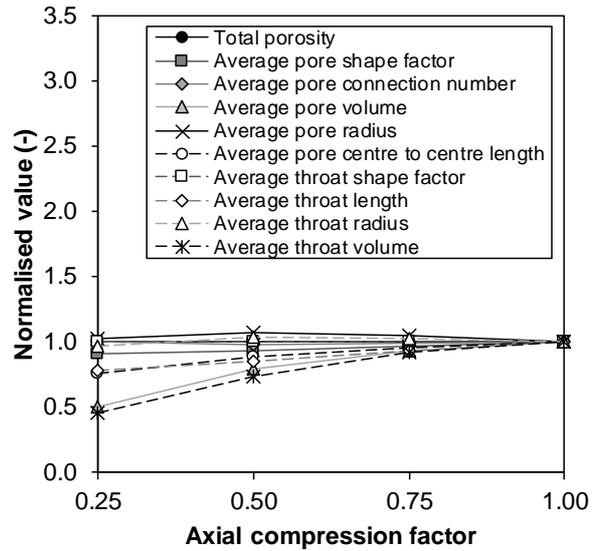


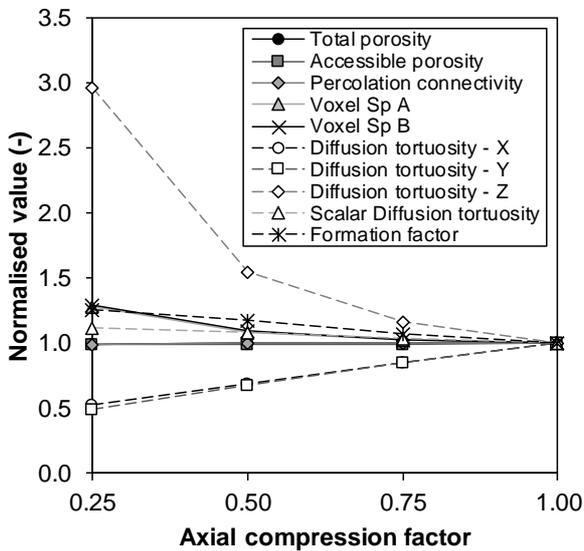
Figure 9: Applying axial compression to correct optical distortion in the Z axis of LSCM images. Sample is P0.45 7d.



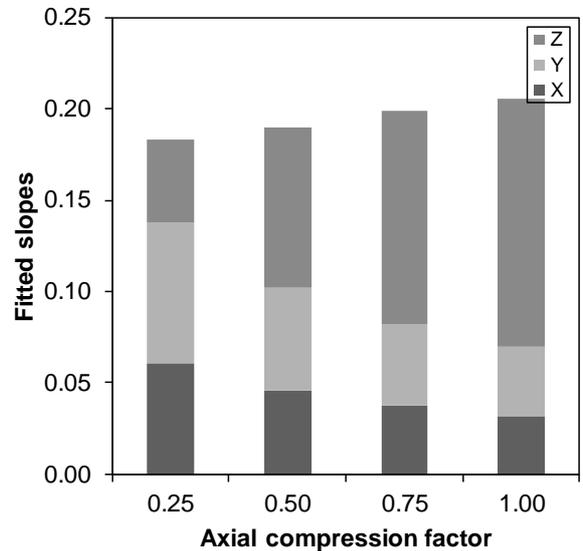
(a) BoneJ



(b) Maximal ball



(c) Cluster labelling & random walker



(d) Fitted slopes for diffusion tortuosity

Figure 10: Effect of axial compression on pore parameters quantified with (a) BoneJ; (b) maximal ball; and (c) cluster labelling and random walker algorithms. Sample is P0.45 7d. (d) Contributions from X, Y and Z directions to the scalar diffusion tortuosity.

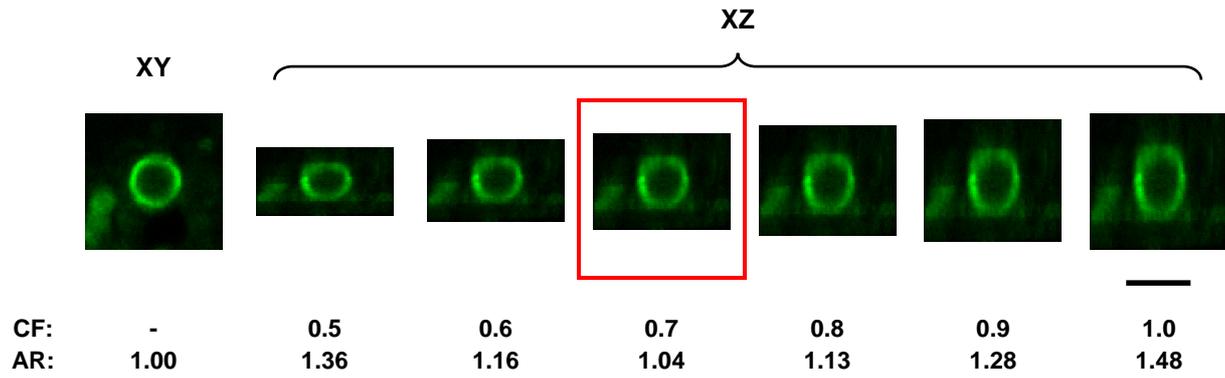


Figure 11: Determining the optimal compression factor (CF) for correcting axial distortion in the Z axis based on aspect ratio (AR) of spherical PFA particles. The compression factor giving the lowest aspect ratio is ~ 0.7. Scale bar is 5  $\mu$ m. Sample is P0.45 PFA 90d.

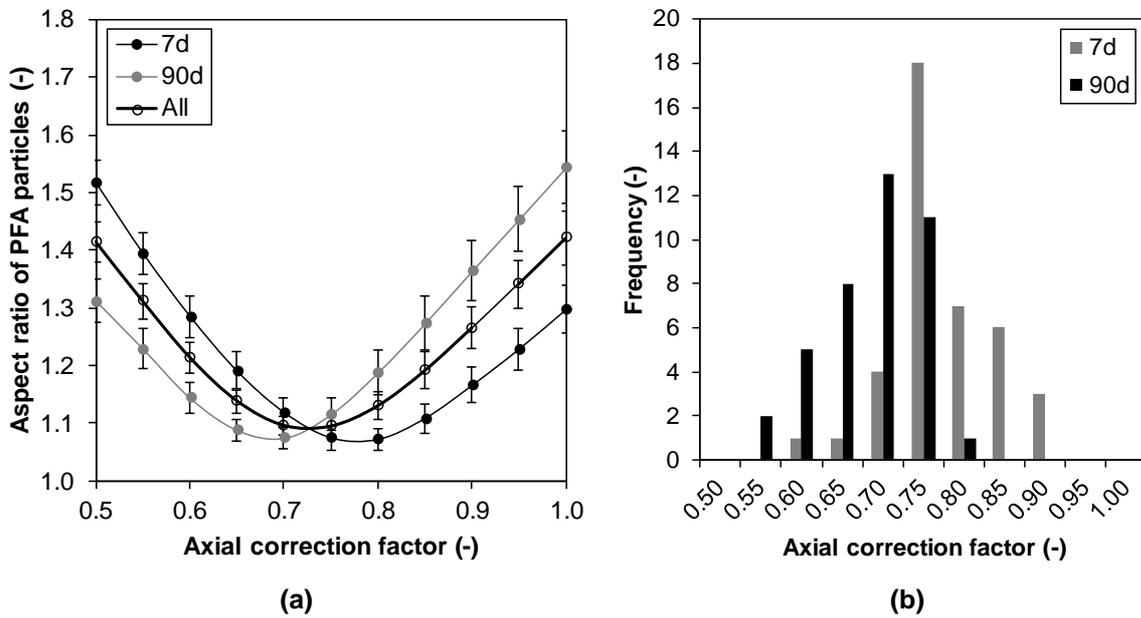


Figure 12: Results of 80 PFA particles from P0.45 PFA 7d and 90d: (a) average aspect ratio vs. axial compression factor; and (b) frequency histogram of axial compression factors giving the minimum aspect ratio of each PFA particle.

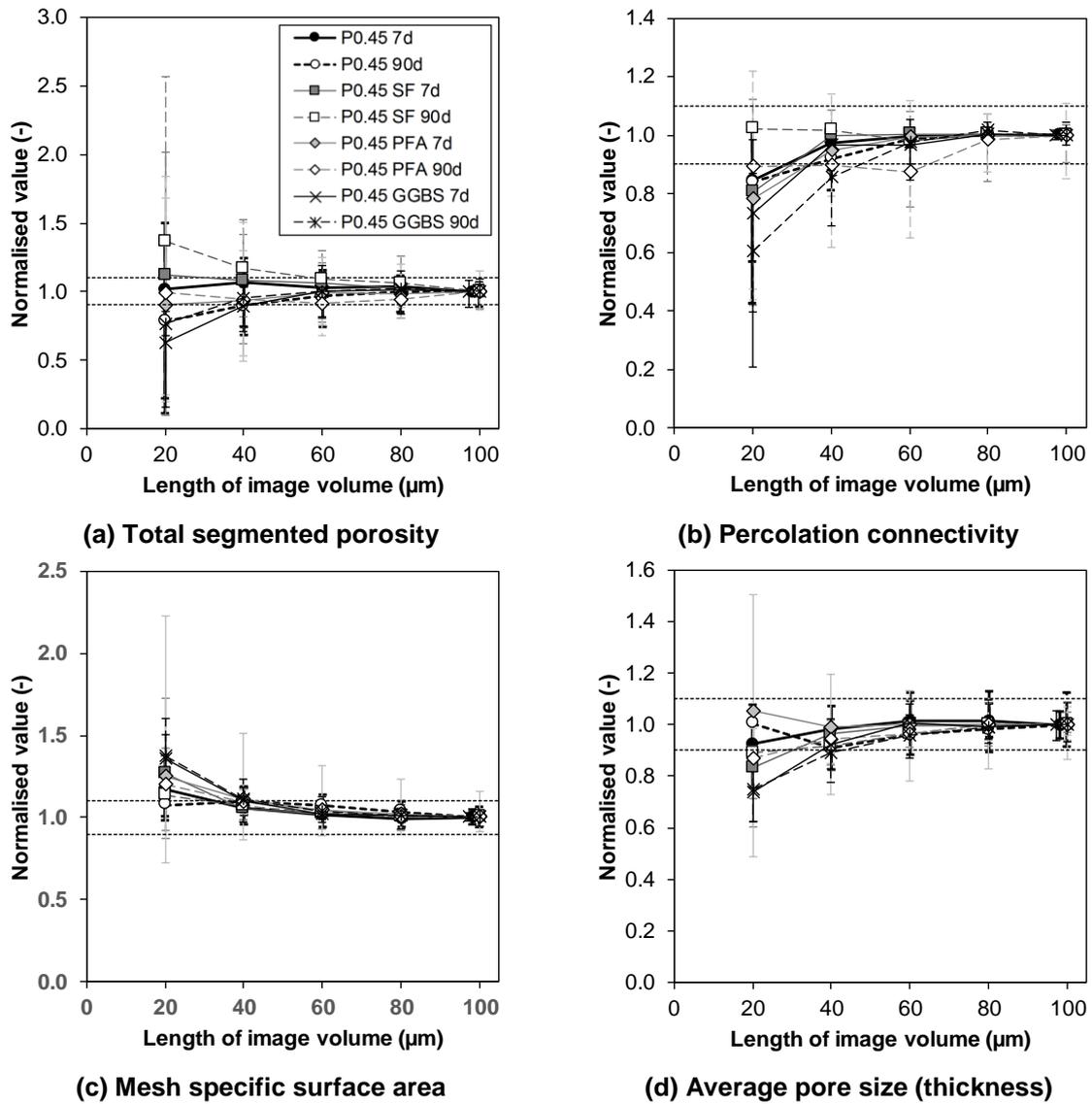
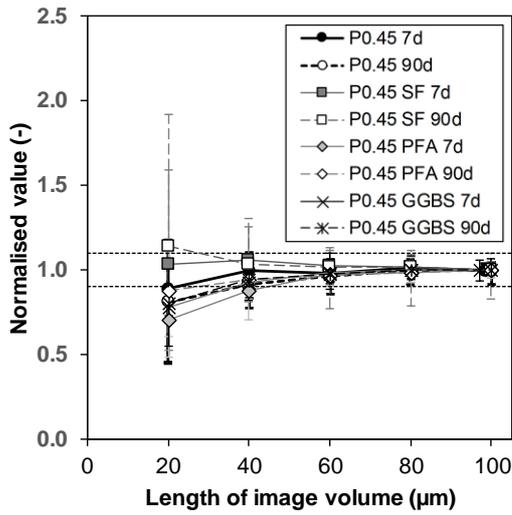
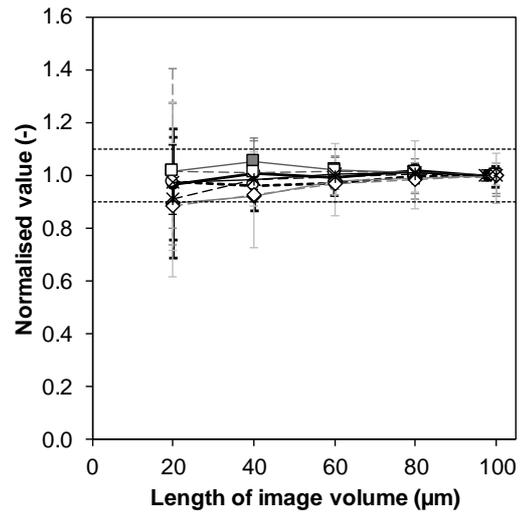


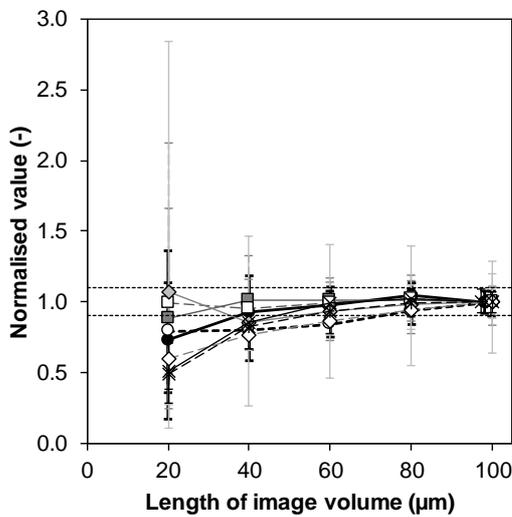
Figure 13: Change in (a) total porosity; (b) percolation connectivity; (c) mesh specific surface area; and (d) average pore size as a function of image volume size for all samples. Each data point is an average of four measurements and normalised to the value at  $100^3 \mu\text{m}^3$ . Error bars show max/min values. Horizontal dashed lines mark the 0.9 to 1.0 boundaries.



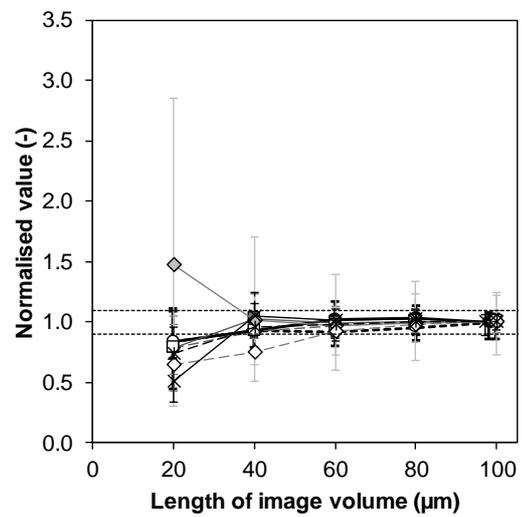
(a) Average pore connection number



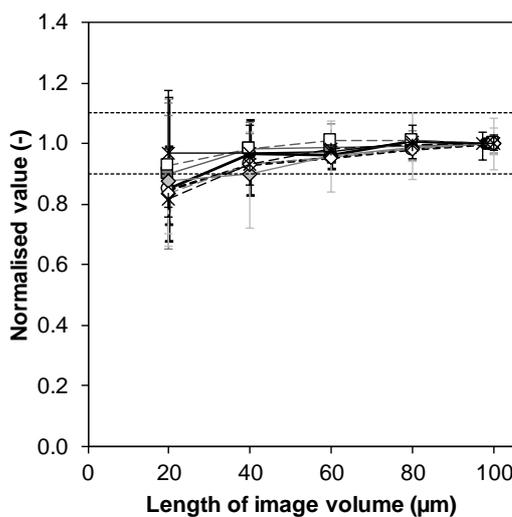
(b) Average pore radius



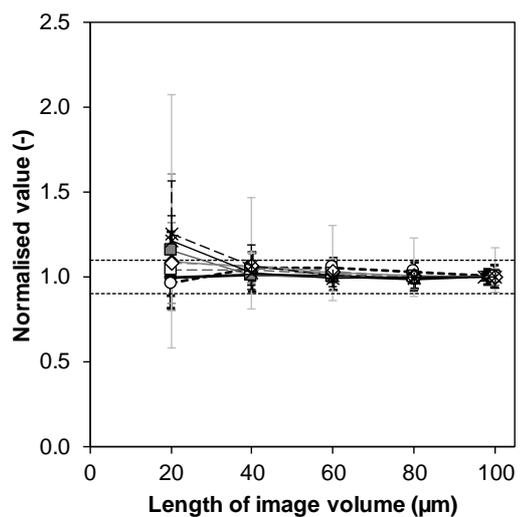
(c) Average pore volume



(d) Average throat volume



(e) Average throat length



(f) Voxel specific surface area A

Figure 14: Change in (a) average pore connection number; (b) average pore radius; (c) average pore volume; (d) average throat volume; (e) average throat radius; and (f) voxel specific surface area as a function of image volume size for all samples. Each data point is an average of four measurements and normalised to that at  $100^3 \mu\text{m}^3$ . Error bars show max/min values. Horizontal dashed lines mark the 0.9 to 1.0 boundaries.

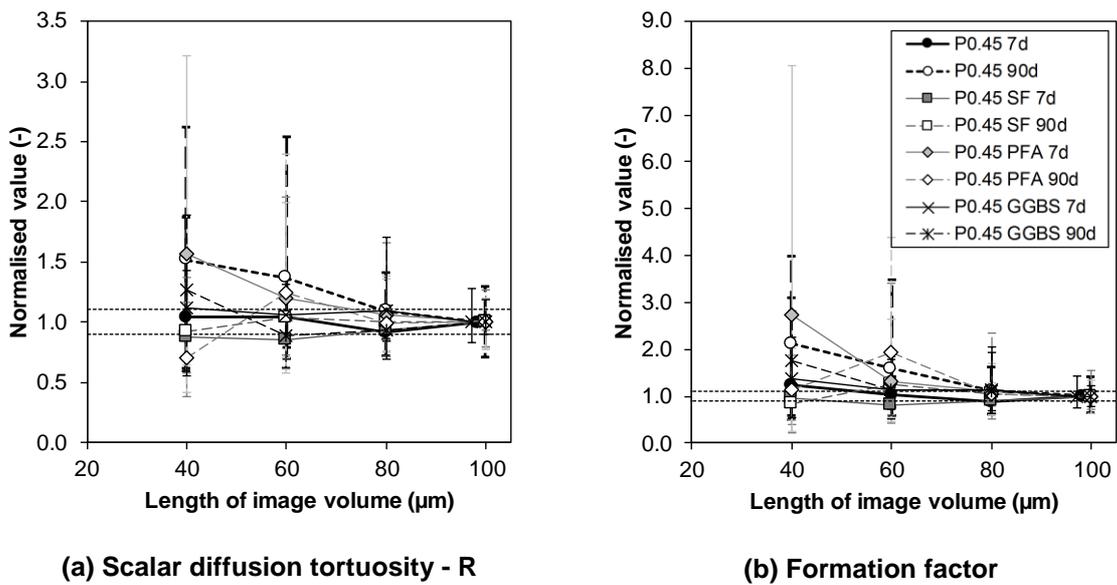


Figure 15: Change in (a) scalar diffusion tortuosity; and (b) formation factor as a function of image volume size for all samples. Each data point is an average of four measurements and normalised to that at  $100^3 \mu\text{m}^3$ . Error bars show max/min values. Horizontal dashed lines mark the 0.9 to 1.0 boundaries.

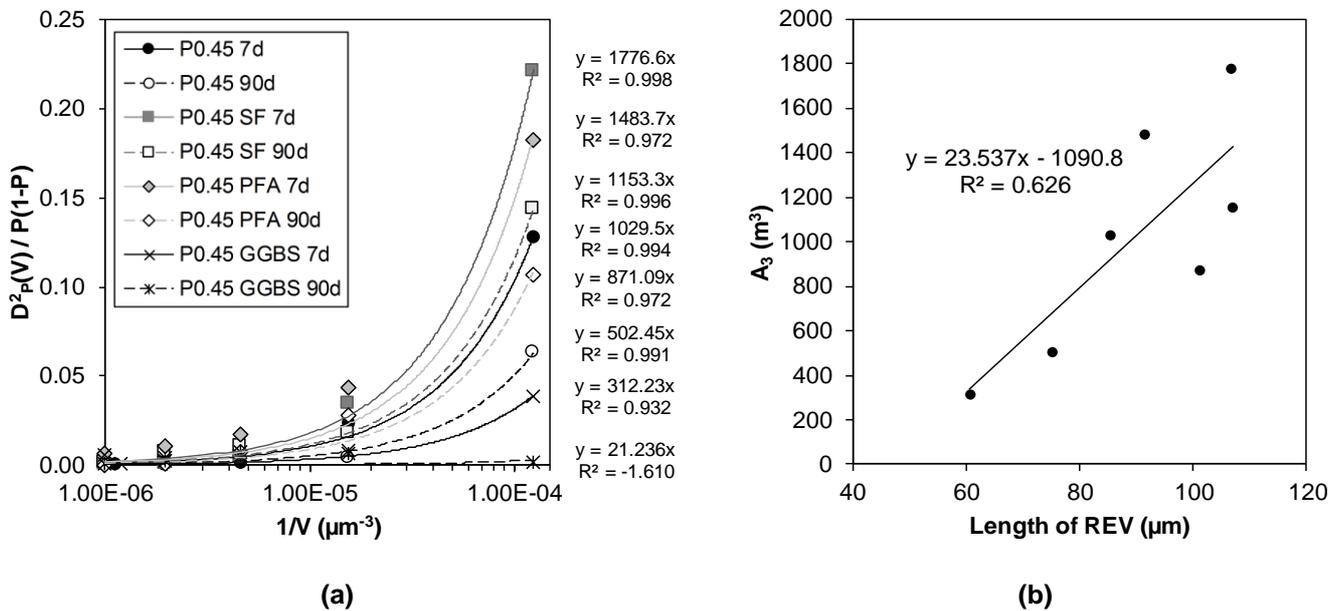


Figure 16: a) Determination of  $A_3$  by curve fitting Eq. (5) to the variances of measured porosities as a function of image volume. Note that Eq. (5) has been rearranged so that the fitted slope gives  $A_3$ . b) Relation between  $A_3$  and length of REV ( $n = 8$ ;  $\varepsilon = 5\%$ ).

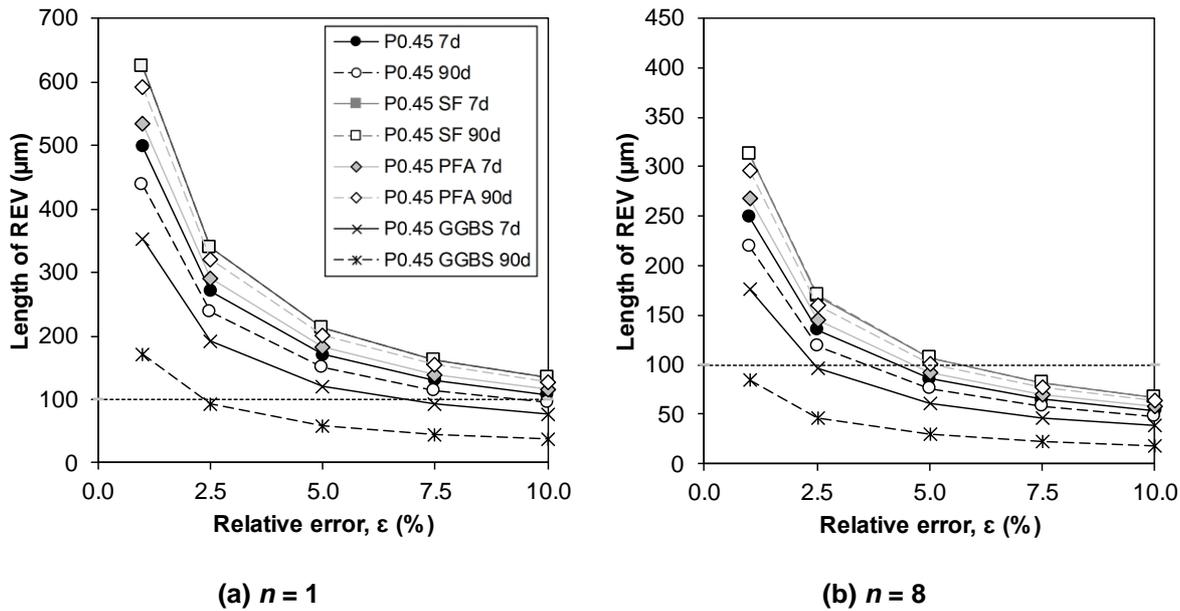


Figure 17: REV for porosity (Eq. 4) as a function of relative error  $\varepsilon$  for (a)  $n = 1$ , and (b)  $n = 8$ , where  $n$  is the number of realisations (number of image volumes analysed). Horizontal dashed lines mark the REV of  $100^3 \mu\text{m}^3$ .

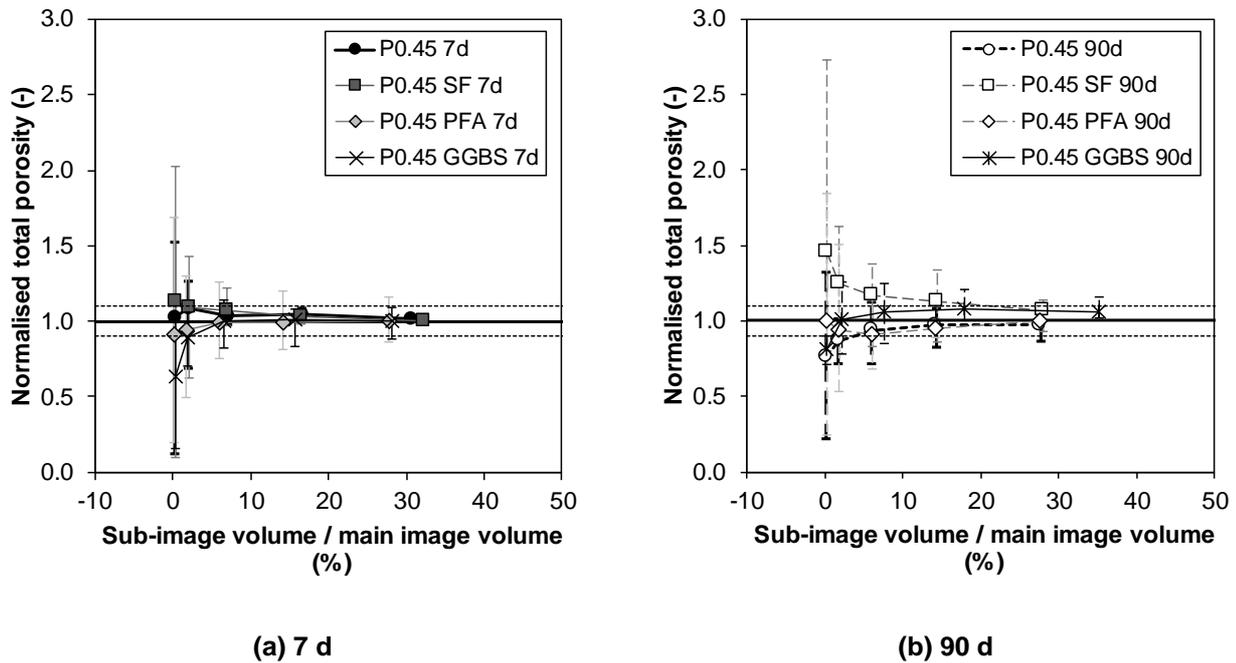


Figure 18: Total porosity measured from sampling volumes ( $20^3$ ,  $40^3$ ,  $60^3$ ,  $80^3$  and  $100^3 \mu\text{m}^3$ ) normalised to that measured from the main image volumes ( $\approx 190 \times 190 \times 100 \mu\text{m}^3$ ) plotted against ratio of sampling volume to the main image volume. Each data point is an average of four measurements. Error bars show the maximum and minimum values. Horizontal dashed lines mark the range from 0.9 to 1.0.