The effect of shear and lap arrangement on reinforcement lap strength

Marianna Micallef, Robert Vollum*

Department of Civil and Environmental Engineering, Imperial College London, London, SW7 2AZ, UK

* Corresponding author, r.vollum@imperial.ac.uk

Abstract

The paper is concerned with the design of tension laps in reinforced concrete structures. The most recent design recommendations for laps are found in fib Model Code 2010 which is likely to influence the next revision of EN-1992. This is of concern to UK industry since laps designed to MC2010 can be significantly longer than laps designed to EN-1992 which UK designers already consider excessive compared with previous UK code requirements. Unlike the previous UK code, BS8110, EN-1992 requires adjacent laps to be offset by 0.3 of the lap length which complicates reinforcement detailing. The paper describes an experimental programme which was undertaken to assess the influence on lap performance of increasing lap length beyond that required for bar yield, shear and staggering of laps. The influence of shear was assessed by comparing the performance of laps of the same length positioned in zones of uniform and varying bending moment. Reinforcement strains were monitored and detailed measurements of crack development and crack widths were obtained with digital image correlation. Results show that very long laps are inefficient with the central half contributing little to force transfer between bars. Shear was found to have no significant influence on lap strength while lapping only 50% of bars at a section increased forces in the lapped bars leading to premature bond failure. Test results are compared with EN-1992 predictions, which are shown to be conservative for the tested laps.

Keywords

Reinforced concrete, bond, tension laps, shear, staggering of laps, Eurocode 2

Highlights

- 18 tests of reinforced concrete beams with tension laps
- Very long laps are inefficient with central half contributing little to lap strength
- Shear had no significant influence on lap strength
- Lapping 50% of bars reduces lap strength compared with 100% laps

1. Introduction

The paper is concerned with the behaviour of tension laps between reinforcement bars in concrete structures. The research was motivated by concern that design lap lengths have increased progressively with time despite no evidence of lap failure in existing structures. For example, fib Model Code 2010 (MC2010) [1] requires longer laps than EN-1992 [2], which in turn requires longer laps than the superseded UK code BS8110 [3]. Already, UK designers find that the reinforcement detailing requirements of EN-1992 complicate construction and increase project costs compared with previous UK practice. Consequently, it is concerning that Cairns and Eligehausen [4] consider the current EN-1992 design lap provisions to be unsafe. Results are presented of an experimental programme which was designed to assess the influence on lap performance of: 1) increasing lap length beyond that required for bar yield, 2) shear and 3) staggering of laps.

1.1 Strength of lapped bars

The design provisions for reinforcement laps in MC2010 [1] are based on the recommendations of fib Bulletin 72 [5], which assesses the mean strength f_{stm} of lapped or anchored bars of length l_b and bar diameter ∅ as:

$$f_{stm} = 54\left(\frac{f_{cm}}{25}\right)^{0.25}\left(\frac{25}{\emptyset}\right)^{0.2}\left(\frac{l_b}{\emptyset}\right)^{0.55}\left[\left(\frac{c_{min}}{\emptyset}\right)^{0.25}\left(\frac{c_{max}}{c_{min}}\right)^{0.1} + k_m K_{tr}\right] \quad (1)$$

where f_{cm} is the mean concrete strength, c_{min} and c_{max} are the minimum and maximum of the concrete covers and half the clear bar spacing, and $k_m K_{tr}$ accounts for confinement from transverse reinforcement. The coefficients in Equation 1 are derived from curve fitting the fib tension splice test database [6].

The term in square brackets of Equation 1 equals 1.0 for the least favourable confinement conditions allowed by fib Bulletin 72. For these conditions, fib Bulletin 72 defines the characteristic lap or anchorage strength as 76% of the mean strength given by Equation 1. The design lap length is 1.5 times the characteristic lap length due to the introduction of the partial factor for concrete $\gamma_c = 1.5$. When the term in square brackets of Equation 1 equals 1.0, the resulting design lap length for reinforcement yield is 2.67 times the length l_b required by Equation 1 to develop $f_{stm} = f_{yd} = 435\ MPa$, where f_{yd} is the design reinforcement yield strength. The design bond strength in MC2010

[1] is derived using this approach, with some simplifications [5], assuming reinforcement yields. The resulting design lap lengths for reinforcement yield are typically around three times the length l_b required by Equation 1 to develop $f_{stm} = f_{yd} = 435\ MPa$.

1.2 Interaction of shear and bond

It is good practice to locate laps near points of contraflexure where shear forces can be significant. The influence of shear on lap strength is uncertain with contradictory claims in the literature. Most laboratory tested laps are situated in regions of constant bending moment to ease interpretation of results. Experimental work on anchorage lengths and laps with moment gradient has been carried out at the University of Texas at Austin [7-12]. Ferguson and Krishnaswamy [9] proposed that lap lengths should be reduced in zones of varying moment zone compared with constant moment zones. However, Orangun et al. [13] reviewed test results by others and suggested that moment gradient has little or no effect on bar stresses at lap failure. Jirsa and Breen [11] and Zekany et al. [12] tested 24 beams with shear spans of 1016 mm, 1346 mm and 2032 mm and concluded [11, 12] that increases in shear had negligible effect on bond strength. Different conclusions were reached by Reynolds and Beeby [14] who tested six beams with laps in varying moment zones. They concluded that bond strength is greatly increased by the presence of stirrups when: 1) laps are situated in zones of high shear, 2) diagonal cracking has occurred and 3) the transverse steel is highly stressed. fib Bulletin 72 [5] argues that Reynolds and Beeby’s findings [14] cannot be used in design since diagonal shear cracking cannot be relied upon.

1.3 Effect of staggering laps on lap strength

It is considered good practice to stagger laps [15] even though staggering complicates detailing and is labour intensive. EN-1992 [2] requires adjacent laps to be offset by 0.3 times the design lap length. Additionally, the design lap length is increased by 50%, if more than 50% of the bars are lapped within a section of length 1.3l, where l is the lap length. MC2010 does not require adjacent laps to be offset but allows a 30% reduction to the design lap length if no more than 33% of bars are lapped at a section. Ferguson and Briceno [16] found staggering laps to be beneficial in three point bending (3PB) tests conducted on three specimens with one pair of lapped bars and one continuous bar, and one specimen with two pairs of staggered laps. They [16] suggest a 20% reduction in lap length in situations of 50% staggering. Cairns [17] tested 17 beams in four point bending (4PB) with laps positioned in the constant moment region. The proportion of bars lapped at the same section was 33%, 50% or 100% in three, six and eight specimens respectively. Unlike Ferguson and Briceno [16], Cairns [17] did not find staggering to increase lap strength. He suggested that if allowance is made for the increase in clear distance between adjacent lapped bars, staggering decreases lap strength because lapped bars attract more force than continuous bars owing to the increased stiffness of the former. All the laps tested by Cairns [17] were shorter than required to achieve full bar strength.

2. Research significance

The research systematically investigates the effect of increasing lap length on lap strength, ductility and failure mode, in zones of uniform and varying moment. The influence of staggering laps is also investigated. Tested laps are between bars of the same as well as mixed diameter. The presented experimental results include reinforcement strains and detailed measurements of crack development and crack widths obtained with digital image correlation (DIC).

3. Description of laboratory tests

3.1 Aim

The tests investigated the benefit of: 1) providing laps longer than required to develop bar yield, 2) placing laps in zones of varying moment, and 3) staggering laps which is required in EN-1992 but not BS8110 or MC2010. Lap lengths were designed as “short”, “long” or “very long”, with the length of “long” laps just sufficient to develop bar yield according to Equation 1.

3.2 Specimen description and methodology

The test programme consisted of 18 specimens (Series A to E) measuring 450 mm wide by 250 mm deep and 4250 mm long, each having laps in the tension face. Table 1 provides details of the tested specimens and in the footnote explains the assigned specimen designation. Laps in Table 1 are classified as “short”, “long” and “very long” as defined above. The tested reinforcement arrangements are shown in cross section, elevation, and plan in Figure 1. Cover to top and bottom longitudinal reinforcement was 30 mm. All specimens were reinforced with two 12 mm compression bars and 10 mm links spaced at 200 mm centres in the lapped region. Series A to D were tested in 4PB with laps positioned in the constant moment region and Series E in 3PB with laps positioned in the longer shear span. Series A and B were reinforced with three 25 mm diameter bars lapped with bars of 25 mm (series A) or 16 mm (series B) and laps of varying lengths. Series A included a control specimen with three continuous bars. Reinforcement in series E, which investigated the influence of shear on bond strength, was detailed similarly to Series B for direct comparison and included “short”, “long” and “very long” laps. In series A, B and E, all the laps were positioned at the same section with no staggering. The influence of staggering laps between laps of mixed and the same bar diameter was investigated in series C (20 mm and 16 mm bars) and D (20 mm bars) respectively. The bar arrangements tested in series C and D were: 1) four laps at a section (100% lap), 2) four staggered laps (50 % lap), denoted “(s)”, and 3) two adjacent middle laps at a section with two continuous 20 mm diameter edge bars (50 % stagger), denoted “(c)”. In specimen 4P-16/20s-500, with four staggered laps, the clear distance between the ends of staggered laps was 30% of the lap length which is the minimum allowed by EN-1992 for tension laps.

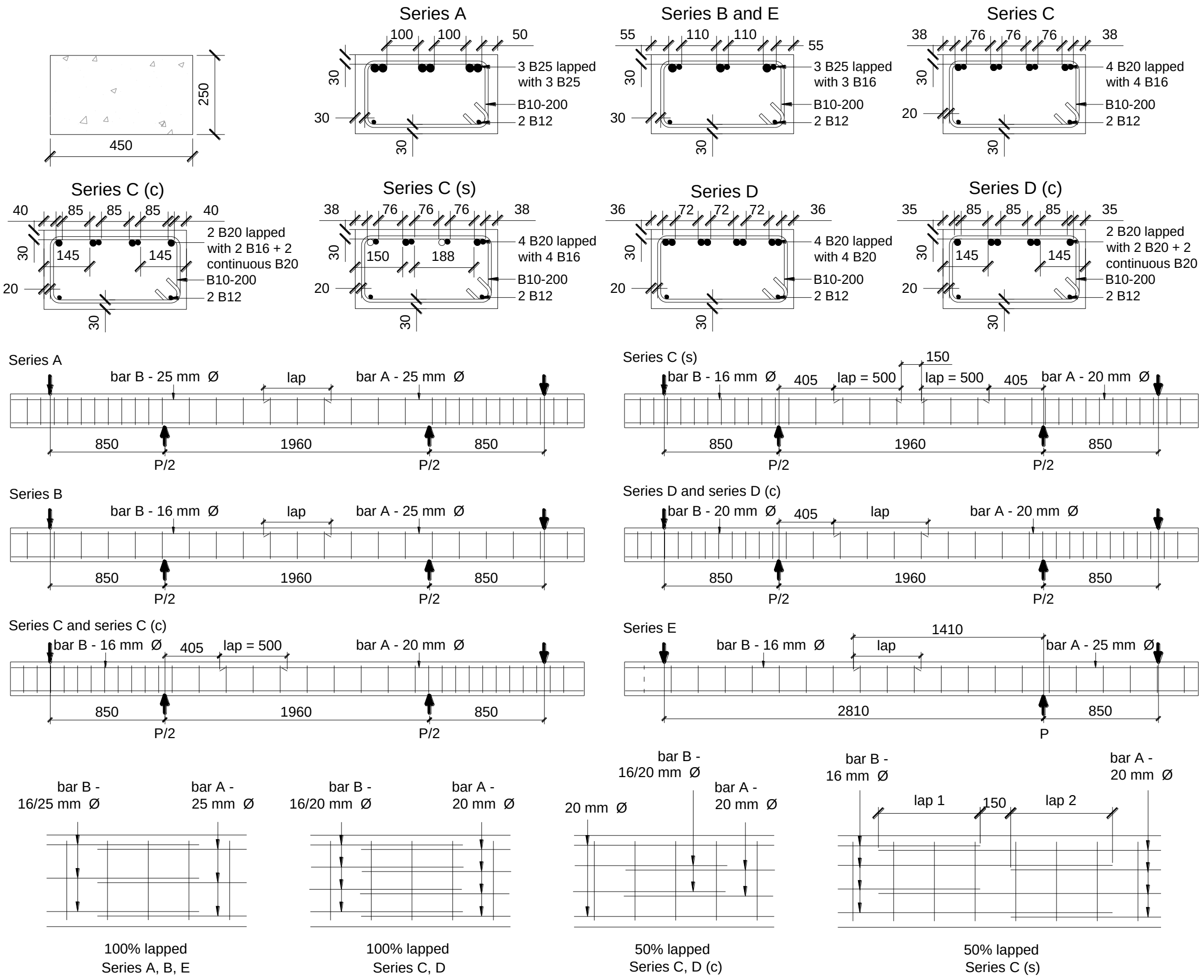


Figure 1: Reinforcement and loading arrangements

Specimens were cast in three batches (denoted “cast 1” to “cast 3” in Tables 1 and 2), from ready-mix concrete with target strength of C25/30 and medium workability (slump S3 to BS EN 206-1 [18]). The maximum aggregate size was 20 mm. Lapped tension bars were positioned at the bottom of the cast for good bond conditions. After casting, specimens were covered with a waterproof tarpaulin, demoulded after 3 to 5 days, and daily wetted over a period of two to three weeks. Tables 1 and 2 show concrete cylinder strengths on the day of testing and at 28 days respectively. Reinforcement was specified as grade 500B to BS 4449 [19]. Measured reinforcement properties are presented in Table 3. All the flexural reinforcement used in the tests had a well-defined long yield plateau which may have increased ductility in laps that failed in bond subsequent to reinforcement yield since the increase in reinforcement force after yield was minimal for strain less than around 0.02.

Specimens were loaded in 4PB or 3PB as depicted in Figure 1 with load increased monotonically to failure over around half an hour. Slabs were tested with the tension face on top to enable DIC to be used to monitor cracking from above. Vertical displacements were monitored with LVDTs. Reinforcement strains were monitored with surface mounted YFLA-5-1L strain gauges positioned at regular intervals along laps.

Throughout the paper, bars starting from the left and right beam ends, respectively, are denoted “bar B” and “bar A” as shown in the elevations of Figure 1. In laps with mixed bar diameters, “bar A” had the greatest diameter. In specimens B to E, the lap was most stressed in bar B at the end of bar A where the moment of resistance utilisation within the lap was greatest.

4. Experimental results

4.1 General behaviour and mode of failure

Measured maximum loads, maximum reinforcement stresses at lap ends and failure modes, are presented for each specimen in Table 1. Stresses were derived from section analysis as well as measured strains as subsequently discussed. Load-displacement responses are shown for series B (4PB) and E (3PB) in Figure 2a and for series C and D in Figure 2b. Loads in Figure 2 are total applied loads excluding self-weight while displacements are measured at the load points. The self-weight of specimens was around 12 kN. In the case of 4PB, average displacements at the two loading points are shown. Results for series A, which are comparable, are presented elsewhere [20].

Table 1. Specimen details and test results

series	test ID[#]	type[1]/ cast	lap length l_b (mm) (% lap)	f_{cm} [2] (N/mm²)	P_{test} [3] (kN) [failure mode[4]]	M_{test} (kNm) [lap end]	Stress at lap end (N/mm²) from: Eq 1	Eq 10	S.A. [5]	meas avg at P_{test}	meas max [7]
A	4P-25-C	- /1	n/a	25.9	352 [y]	146.4[8]	n/a	n/a	583	n/a	n/a
	4P-25/25-500	S/1	500 (100%)	26.1	238 [b]	96.7	348	292	370	406	410[m,e]
	4P-25/25-1000	L/1	1000 (100%)	25.7	344 [y, b]	142.1	507	577	558	558	596[e]
	4P-25/25-1750	VL/1	1750 (100%)	26.1	353 [y, c]	146.6	693	1023	582	565	599[e]
B	4P-16/25-275	S/1	275 (100%)	26.0	132 [b]	51.5	410	280	443	314	337[m]
	4P-16/25-350	S/2	350 (100%)	28.0	159 [b]	62.9	477	377	542	515	570[m]
	4P-16/25-500	L/2	500 (100%)	28.0	188 [y, b]	74.8	581	539	629	572	587[m]
	4P-16/25-1000	VL/2	1000 (100%)	28.0	195 [y,c]	78.3	850	1077	661	580	606[e]
C	4P-16/20-500	L/3	500 (100%)	30.4	223 [y,c]	90.5	554	574	577	548	548
	4P-16/20c-500	L/3	500 (50%)	30.6	266 [y,b]]	109.2	686	618	554	564[6]	573
	4P-16/20s-500	L/3	500 (50%)	30.7	220 [y,c]	89.2	663	620	567	555[6]	556
D	4P-20/20-700	L/3	700 (100%)	30.5	306 [y,c]	126.2	546	605	543	539	539[m,e]
	4P-20/20c-700	L/3	700 (50%)	30.6	296 [y,b]	121.7	672	650	521	582[6]	590
	4P-20/20-1050	VL/3	1050 (100%)	30.7	309 [y,c]	127.0	683	913	546	570	576[e]
E	3P-16/25-275	S/1	275 (100%)	26.2	168 [b]	50.3	411	281	434	469	488[m]
	3P-16/25-350	S/2	350 (100%)	28.1	191 [b]	57.6	478	378	496	514	570[m]
	3P-16/25-500	L/2	500 (100%)	28.1	232 [y,b]	70.5	581	540	591	572	585[m]
	3P-16/25-1000	VL/2	1000 (100%)	28.0	236 [y,s]	72.1	850	1077	606	572	574[m]

[#] Test specimen designation is as follows: #1P - #2 / #3 - #4

#1P = type of test: three point bending ("3P") or four point bending ("4P")

#2 = diameter of lapped bar B

#3 = diameter of lapped bar A ("c" denotes two edge continuous bars, and "s" denotes staggered bars)

#4 = lap length ("C" denotes continuous unlapped bars in control specimen)

[1] S = short, L = long and VL = very long

[2] Average measured compressive cylinder strength (cured in air)

[3] Loads include self-weight

[4] Failure modes – [b] bond failure, [y] reinforcement yield, [c] flexural compression subsequent to reinforcement yield, and [s] shear failure subsequent to reinforcement yield

[5] S.A. depicts section analysis

[6] Weighted average of stresses in lapped and continuous bars

[7] Superscripts e for edge and m for middle depict location of bar in which maximum measured stress occurred

[8] Maximum moment in beam

Table 2. Concrete properties at 28 days

cast	E_c (kN/mm^2)	$f_{cu,air}$ (N/mm^2)	$f_{cu,water}$ (N/mm^2)	$f_{cyl,air}$ (N/mm^2)	$f_{cyl,water}$ (N/mm^2)	$f_{ct,air}$ (N/mm^2)*	$f_{ct,water}$ (N/mm^2)*
1	30.0	-	32.7	25.6	24.2	2.3	2.3
2	33.9	41.5	31.0	28.0	25.5	3.1	2.9
3	-	36.2	35.9	30.7	30.9	2.5	2.5

* Taken as 0.9 times the average measured splitting tensile strength

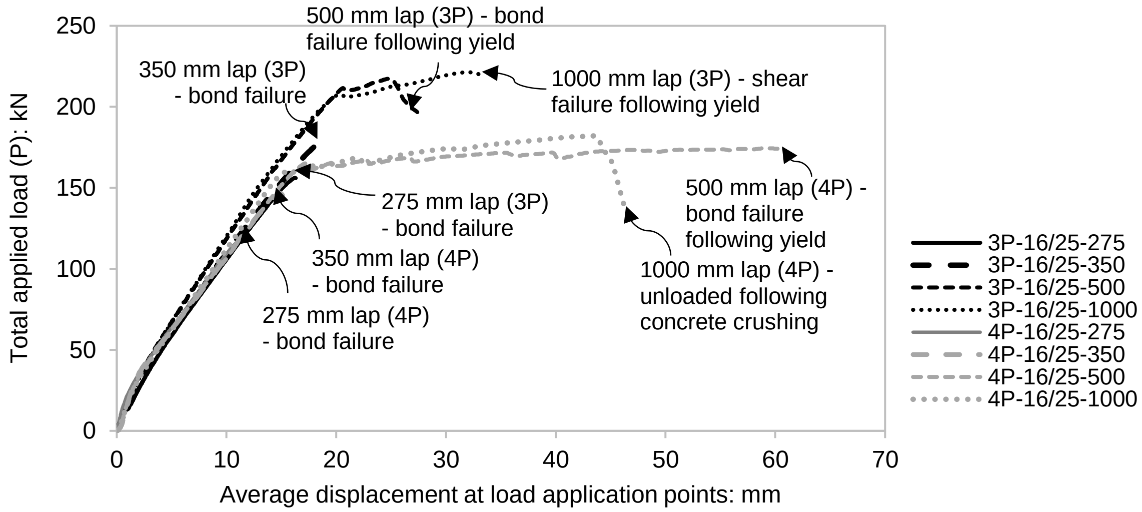
Table 3. Reinforcement properties

series A, B and E				series C and D			
Ø (mm)	E_s (kN/mm^2)	f_y (N/mm^2)	f_u (N/mm^2)	Ø (mm)	E_s (kN/mm^2)	f_y (N/mm^2)	f_u (N/mm^2)
10	-	509	621	10	-	520*	639
12	-	551	638	12	-	534	624
16	182	572	666	16	193	548	645
25	195	558	656	20	193	539	641

* Rounded stress-strain curve without a yield plateau

All “short” laps failed suddenly in bond without warning, prior to yield, as predicted by Equation 1 with strength increasing with lap length as expected. The influence of shear can be seen by comparing lap strengths, calculated with section analysis (see Table 1), of corresponding specimens in series B (4PB) and series E (3PB). The comparison shows that if anything, shear marginally reduced lap strength rather than increasing strength as found by Reynolds and Beeby [14]. However, the ratios of the failure loads of the 275 mm and 350 mm long laps to that of the 500 mm long lap, which yielded, are almost the same for each series (70% and 84% for series B and 72% and 82% for series E). Consequently, it is concluded that shear had no significant influence on the strength of the tested laps. Reinforcement yielded at the most highly stressed critical lap end, or ends, in specimens with “long” and “very long” laps. All these specimens underwent considerable plastic deformation and cracking before failure as indicated by the load displacement responses in Figure 2. Specimens with “long” laps in series A (with 25 mm bars), B and E (with 16 mm and 25 mm bars) failed in bond following yield, irrespective of the presence or absence of shear along the lap length. Similarly, both specimens with continuous bars (c) and adjacent pairs of “long” laps in series C and D failed suddenly in bond following reinforcement yield. However, specimens with four adjacent “long” laps in series C (16 mm and 20 mm bars) and D (20 mm bars), failed in flexure subsequent to reinforcement yield. Test 4P-16/20s-500, with four staggered laps, was stopped when the actuators ran out of stroke following considerable plastic deformation. Specimens with “very long” laps in the constant moment region failed in flexure. Specimen 3P-16/25-1000, with “very long” laps, failed in shear, within the shorter shear span, subsequent to reinforcement yield.

a)



b)

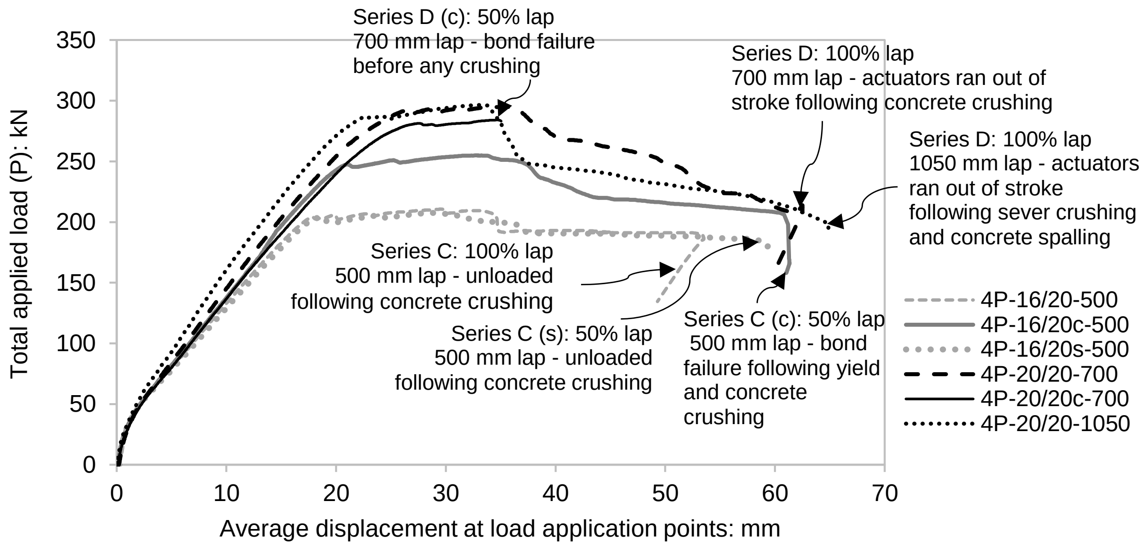


Figure 2: Load-displacement plots for a) series B and E tests, and b) series C and D tests

4.2 Reinforcement strains and bond stresses

Reinforcement strains in lapped bars are greatest at the loaded end of lapped bars and reduce to zero at the unloaded end. The following trends were observed in series A to D loaded in 4PB with three or four bars lapped at the same section within the constant moment region. Strains were similar in all bars at ends of laps as predicted assuming plane sections remain plane. Contrary to the predictions of Cairns [17] this was also true for specimens with 50% and staggered laps. Strain varied almost linearly along “short” laps above around 50% of the failure load. In “long” laps, the slope of the strain distribution was initially greatest near bar ends and almost horizontal over the central half of the lap. The strain distribution became progressively more linear with increasing load and by first yield was almost linear corresponding to uniform bond stress as assumed in EN-1992. In “very long” laps, the shape of the strain distribution at first yield was similar to that in “long” laps at low loads. Subsequent to reinforcement yield, the strain gradient increased slightly in the central half of “very long” laps, but even so the central half was relatively ineffective contributing only 26% of the maximum transferred force in specimen 4P-25/25-1750.

Figures 3a to 3c show average strains, along laps, in the edge and middle B bars (see Figure 1) for series C, D and E respectively. Strains are shown at first yield or failure if it occurred sooner. In Figure 3a, strains are shown for the left of the staggered laps in 4P-16/20s-500 (see Figure 1). Within the central part of the lap, strains are slightly larger in 4P-16/20s-500 than in 4P-16/20-500 with four bars lapped at the same section. Similarly, Figure 3b shows that the strain at the centre of the lap in 4P-20/20c-700, with continuous edge bars (50% lapped), is greater than in 4P-20/20-700 with 100% laps. The increase in strain in the specimens with partial laps is due to the reduced total area of reinforcement within the lapped section compared with 100% laps. In series E, with laps positioned in the shear span, the most highly stressed section for lapped bars was in bar B (16 mm diameter) at the end of bar A. Strain distributions along the B bars were similar to those observed for laps in constant moment regions (i.e. linear in “short” laps, linear close to yield in “long” laps, and relatively uniform in the central region of “very long” laps) as shown in Figure 3c. Strains are shown just before bond failure in 3P-16/25-275 and 3P-16/25-350 and at first yield for longer laps.

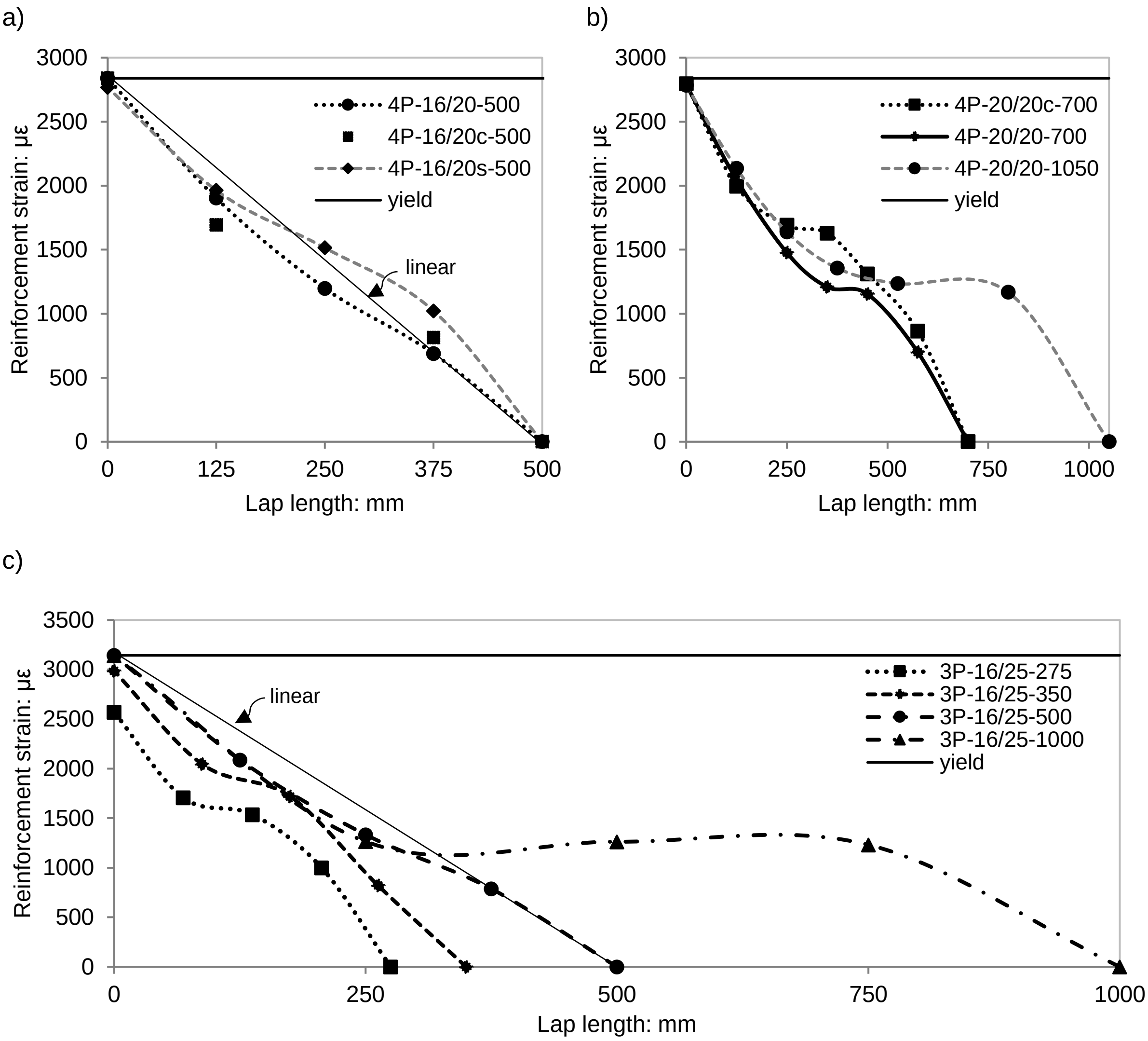


Figure 3: Strain in B bars (average of middle and edge lap) at maximum load for "short" laps and at yield for "long" and "very long" laps in series a) C, b) D and c) E

Figures 4a and 4b provide further confirmation that strains, and hence forces, are greater in staggered laps than 100% laps. The figures compare average reinforcement strains, along the lap, in bars B of 4P-20/20-700 and 4P-20/20c-700 for a range of loads shown relative to that at first yield of bar B (241 kN for 4P-20/20c-700 and 275 kN for 4P-20/20-700). Corresponding strains at the lap centreline, shown numerically in brackets, are greatest in 4P-20/20c-700, with continuous edge bars, at all loads.

The gradient of the strain plots in Figures 3 and 4 is a measure average bond stress between adjacent strain gauges. The average bond stress, $f_{b,av}$ along a lapped bar of diameter Ø is given by:

$$f_{b,av} = (F_1 - F_2)/\pi\emptyset l_b \quad (2)$$

where F_1 and F_2 are the bar forces at points 1 and 2 spaced at l_b.

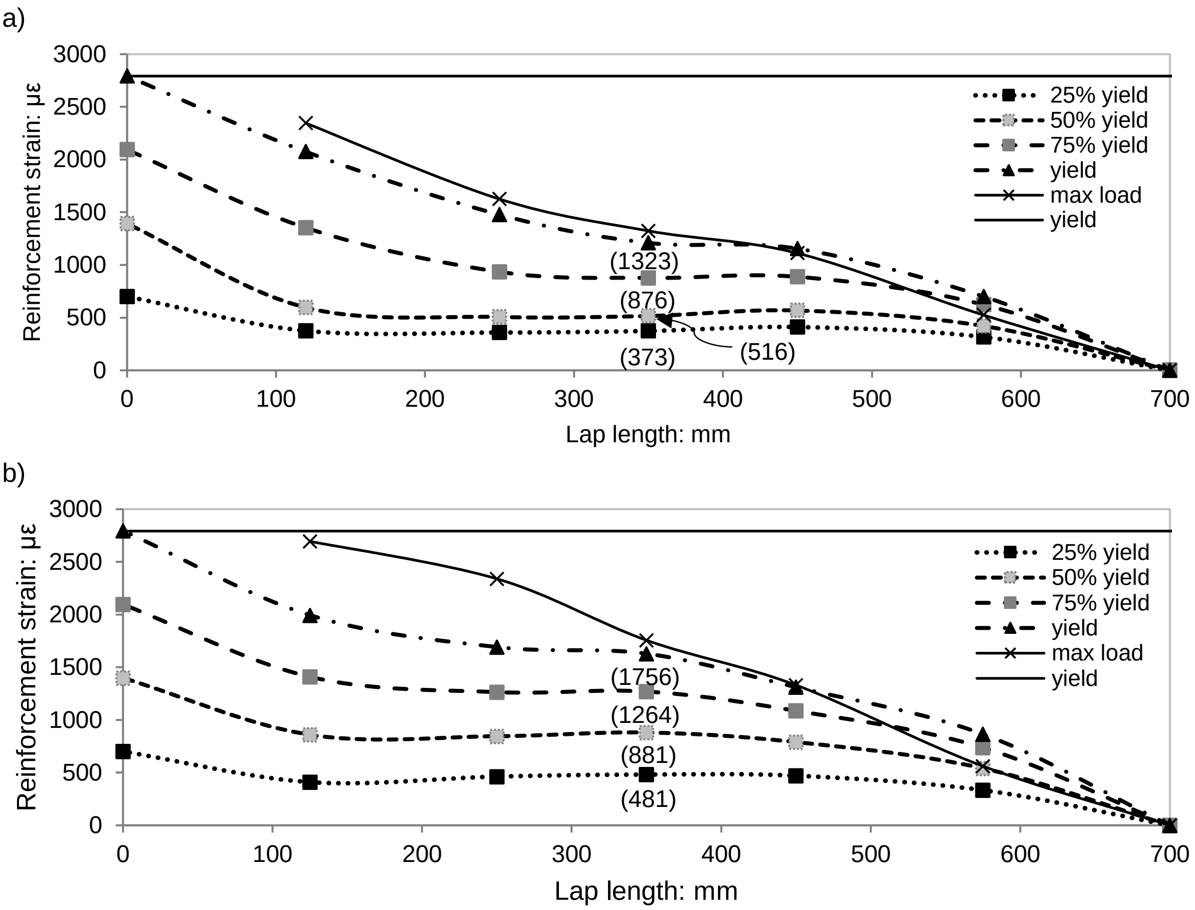


Figure 4: Strain distribution along lap (bar B) in a) 4P-20/20-700, and b) 4P-20/20c-700

Figures 5a to 5d show the average bond stress over 250 mm at the loaded end of edge (e) and middle (m) laps for all the tested specimens. Bond stresses are plotted against reinforcement stress which was derived from strain at the loaded end of the lap. Bond stresses in Figures 5b to 5d are shown for bars B. Figures 5a to 5c suggest that bond stresses at lap ends are almost independent of lap length and similar for edge and middle laps until near failure. Furthermore, comparison of Figures 5b and 5c for identical laps in zones of constant and linearly varying moment, respectively, shows no significant difference between the two. More scatter in average bond stresses is observed for series C and D, possibly due to staggering of the laps.

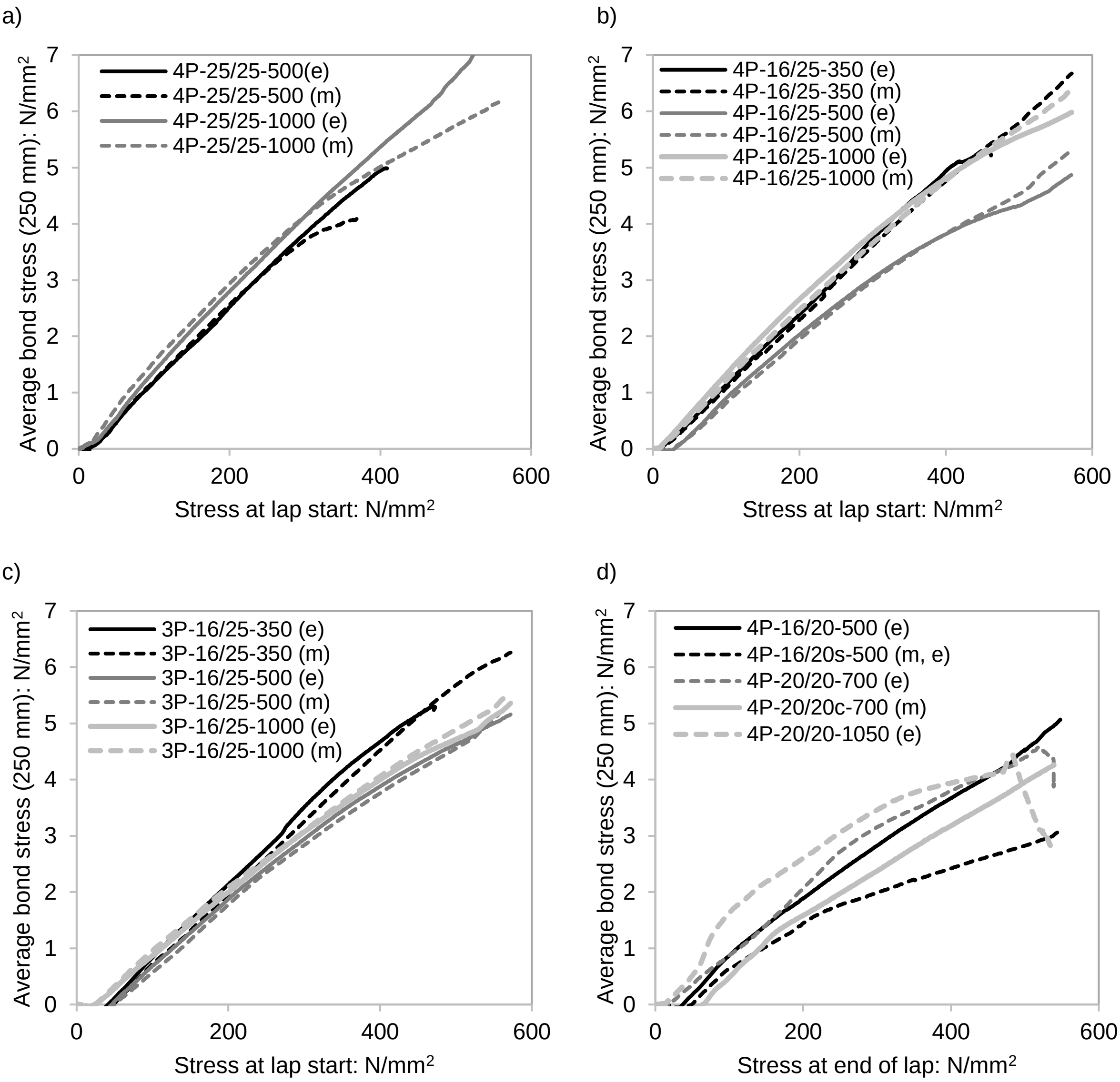


Figure 5: Average bond stresses over 250 mm from lap start in series a) A, b) B, c) E, and d) C and D

4.3 Forces in lapped bars

Bar forces within laps were calculated from reinforcement strains using the steel properties shown in Table 3. In each specimen with 100% laps within the constant moment section, the total force in lapped bars was almost constant along the lap as expected.

Figure 6 shows the total force resisted by single pairs of lapped edge bars in Series E specimens loaded in 3PB. Forces are shown at various proportions of the load at which bar B first yielded according to strain measurements as well as maximum load for 3P-16/25-1000. The forces vary almost linearly in proportion with the applied moment as expected.

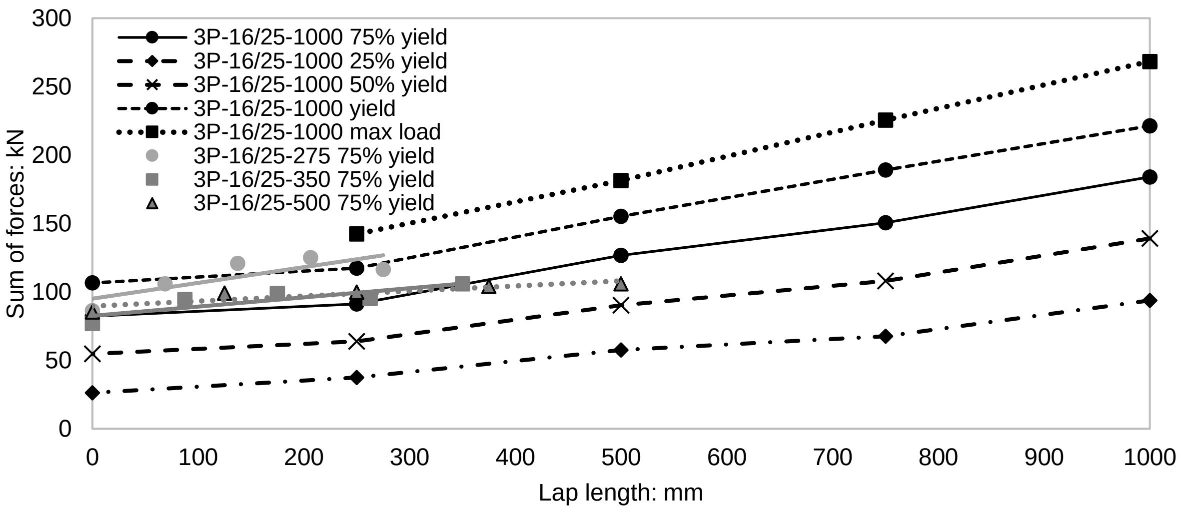


Figure 6: Sum of forces in pair of lapped bars for series E specimens

Figures 7a to 7c, respectively, show the distribution of force along reinforcement bars within the lapped length of specimens 4P-20/20-700, 4P-20/20c-700 and 4P-16/20s-500 of Series D. Forces are shown at 25%, 50% and 100% of the load at first yield (i.e. 275 kN, 295 kN, and 204 kN in Figures 7a, 7b and 7c respectively) as a percentage of the average total bar force at each lap end. The encircled plotted points in Figures 7a to 7c are estimated assuming that the total force in all the bars is constant at any section along the lap, as observed in other tests. Figure 7a shows bar forces for an edge (e) and middle (m) lap in 4P-20/20-700, which is typical of the tested specimens with 100% laps in constant moment regions. At lap ends, forces are similar in loaded bars of edge and middle laps. At the lap centreline, forces are similar in all bars with the sum of bar forces almost equal to the average total force at lap ends. This is typical for specimens with all bars lapped at a single section (100% lapped). Figure 7b shows the corresponding force distribution for 4P-20/20c-700, in which the edge bars were continuous throughout the lapped zone. At both ends of the lap, forces are similar in continuous bars and lapped bars. However, at the lap centreline, the force in the continuous bars at 100% yield is only 40% of the total compared with 50% for edge bars in 4P-20/20-700 with 100% laps. Consequently, the force resisted by each of pair of lapped bars is 20% greater in 4P-20/20c-700 than 4P-20/20-700, with 100% laps, compared with 33% greater calculated on the basis of reinforcement area neglecting slip.

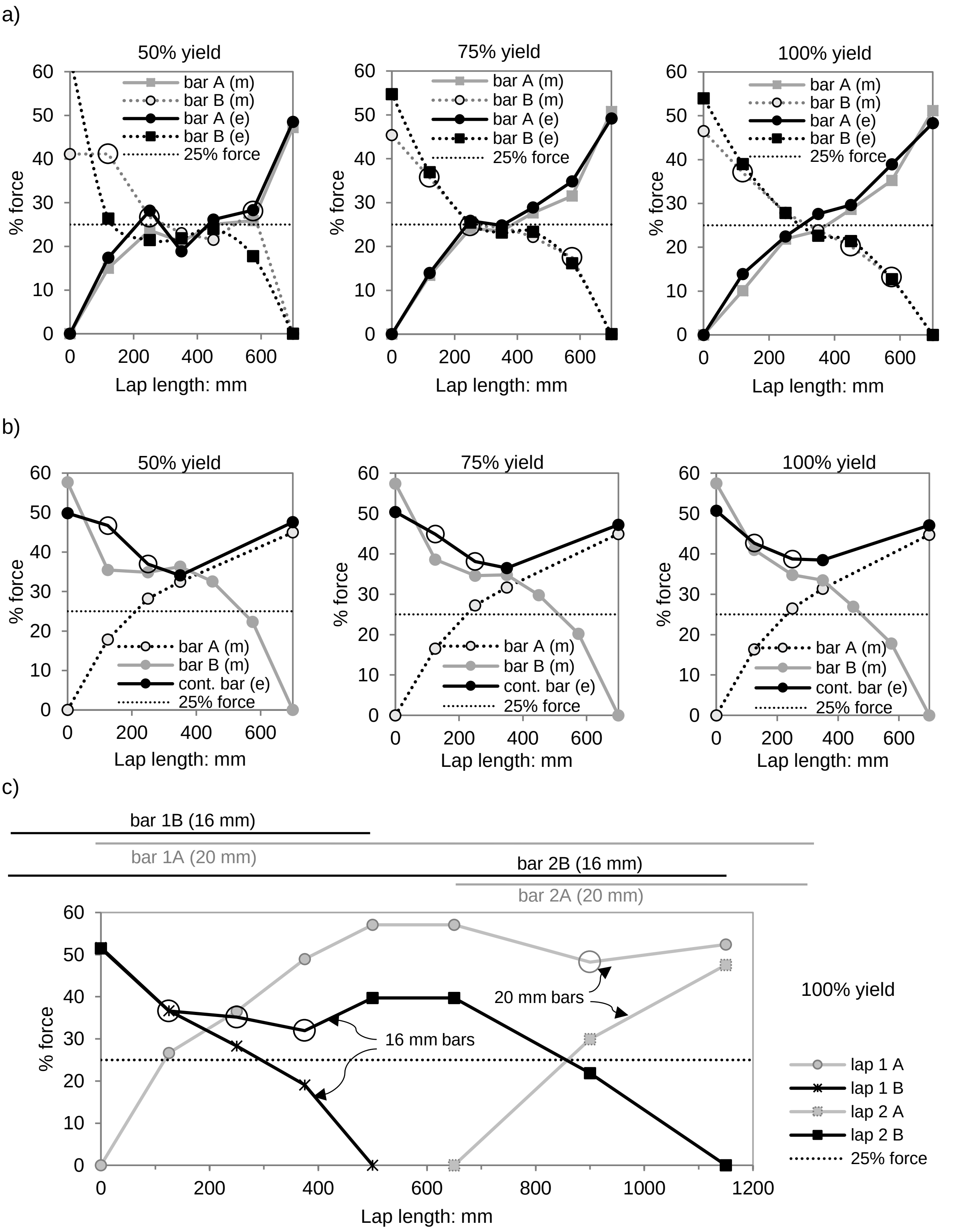


Figure 7: Force distribution in a) 4P-20/20-700, b) 4P-20/20c-700, and c) 4P-16/20s-500

Similarly, Figure 7c shows the distribution of force between bars in the edge (lap 1) and middle (lap 2) laps of 4P-16/20s-500 with staggered laps. At the left hand end of bar A, forces are equal in bars 1B and 2B which both have diameters of 16 mm. At the right hand end of the first lap, the ratio of forces carried by bar 1A, with diameter 20 mm, and bar 2B is in proportion to their areas (i.e. 60:40). At the right hand end of the second lap, forces are almost equal in bars 1A and bar 2A with diameters of 20 mm.

Figure 7c also shows that where laps are staggered, the share of force carried by the continuous bar (e.g. bar 2B in lap 1 and bar 1A in lap 2) reduces towards the centreline of laps due to the greater stiffness of the lapped bars. Consequently, staggering laps increases the force in lapped bars at the lap centreline even though the total force transferred between bars along the lap appears unchanged. The effect of this is to increase the maximum bond stress in beams with staggered laps. Consequently, lap failure occurred, subsequent to bar yield, in 4P-16/20c-500 and 4P-20/20c-700, with continuous bars, but not 4P-16/20-500 and 4P-20/20-700 with 100% laps. The increased force in staggered laps was identified by Cairns [17] on the basis of compatibility considerations and simplified finite element analysis. However, the reduction in strain within the continuous bar along the lap is at odds with Cairns [17] assumption of uniform strain.

4.4 Crack formation and crack widths

Initial cracking within the lapped zone was across the slab width at lap ends. Subsequently further transverse cracking developed over and midway between stirrups accompanied by longitudinal cracking, which initiated over longitudinal bars at lap ends. With increasing load, the longitudinal cracks progressively extended from lap ends and equalled half the lap length immediately before bond failure. For specimens of the same series, longitudinal crack lengths, measured from the lap end to the lap centre, are almost independent of lap length at any given load. For example, the maximum length of longitudinal cracks in 4P-16/25-350 was 175 mm immediately before failure at 159 kN when splitting developed over the complete lap length. At the same load, the maximum length of longitudinal cracks in both 4P-16/25-500 and 4P-16/25-1000 was also around 175 mm. Similarly, the maximum length of longitudinal cracks in of 4P-16/25-500 and 4P-16/25-1000 was 250 mm at the failure load of 4P-16/25-500 which failed in bond due to longitudinal splitting along the complete lap length of 500 mm.

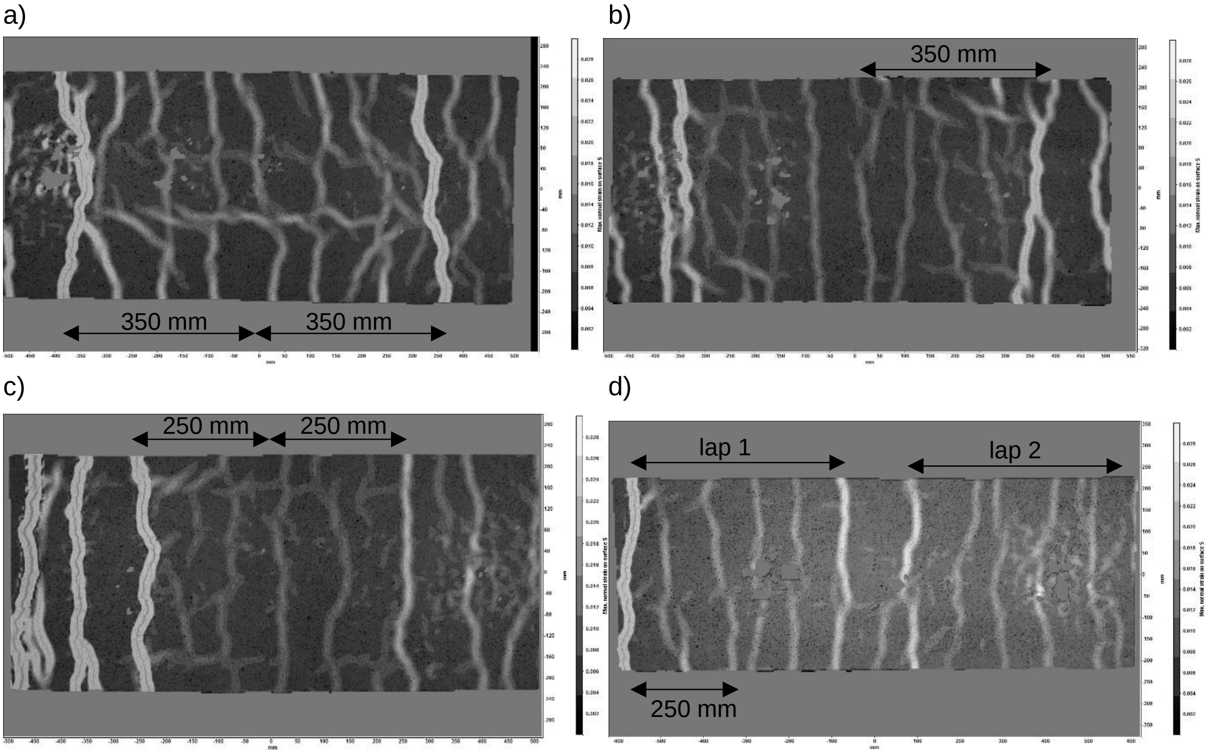


Figure 8: Crack patterns near failure load in a) 4P-20/20c-700, b) 4P-20/20-700, c) 4P-16/20-500, and d) 4P-16/20s-500

This phenomena is illustrated in Figure 8 of which 8a and 8b compare crack patterns just before bond failure in 4P-20/20c-700 (50% lapped) and at the same load of 295 kN in 4P-20/20-700 (100% lapped) which failed in flexure. In both specimens, the maximum length of the longitudinal cracks, measured from lap ends towards lap centres, is 350 mm. Figures 8c and 8d, respectively, show cracks in 4P-16/20-500 (100% lapped) and 4P-16/20s-500, with staggered laps, at peak load. Both specimens failed in flexure at almost the same load. The length of longitudinal cracks in 4P-16/20-500 is 250 mm as in 4P-16/20C-500 at failure when the lap was similarly stressed. Comparison of Figures 8c and 8d, shows that longitudinal cracks in 4P-16/20s-500 are intermittent and shorter than in 4P-16/20-500 with four laps. Hence, there was a clear difference in longitudinal crack formation (and lap strength) between specimens with continuous bars (which failed in bond) and 4P-16/20s-500 with staggered laps, even though the percentage of lapped bars was 50% in each case.

Crack widths were estimated using the La Vision DaVis software by applying virtual extensometers to processed DIC images. Transverse crack widths were measured at intersections with each longitudinal tension bar and in between bars. Maximum longitudinal (depicted "l") and transverse (depicted "t") crack widths are shown in Figures 9a to 9d for Series B to E. The maximum transverse crack widths occurred at lap ends while maximum longitudinal crack widths occurred around 100 mm from lap ends. In “long” and “very long” laps, maximum transverse crack widths were almost independent of lap length (500 mm and 1000 mm laps in Figures 9a and 9e, and 700 mm and 1050 mm laps in Figure 9c). Direct comparison of maximum transverse crack widths in Figure 9b for 4P-16/20-500 (100% lapped), 4P-16/20c-500 and 4P-16/20s-500 (50% lapped) shows that staggering laps slightly reduced crack widths between 70% and 100% of yield. However, comparison of crack widths in 4P-20/20-700 (100% lapped) and 4P-20/20c-700 in Figure 9c shows no reduction in transverse crack widths due to staggering of laps. Longitudinal cracks are considerably narrower than transverse cracks at all applied loads.

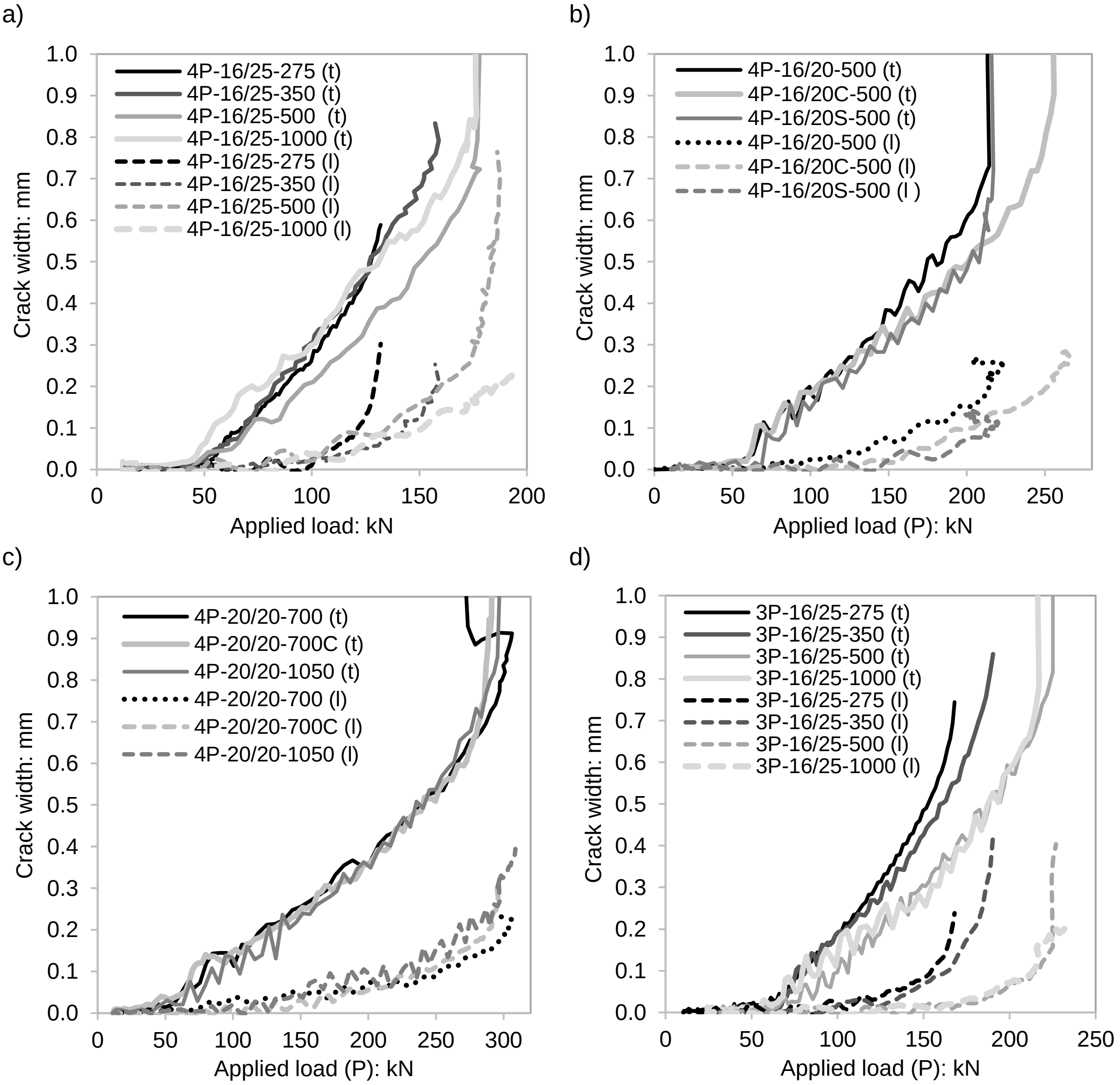


Figure 9: Crack widths in series a) B, b) C, c) D and d) E

5. Comparing test results with predictions

EN-1992 calculates the basic required anchorage length $l_{b,rqd}$ as:

$$l_{b,rqd} = (\emptyset/4)(\sigma_{sd}/f_{bd}) \tag{3}$$

in which $\emptyset$ is the bar diameter, σ_{sd} is the design stress in the reinforcement bar at the position the anchorage is measured from and f_{bd} is the design bond strength which is given by:

$$f_{bd} = 2.25\eta_1\eta_2 f_{ctd} \tag{4}$$

in which η_1 relates to bond conditions and is 1.0 for good bond, η_2 is related to bar diameter and is 1.0 for $\emptyset \leq 32$ mm and $f_{ctd} = \frac{f_{ctk}}{\gamma_c} = 0.21 f_{ck}^{2/3}/\gamma_c$ is the design concrete tensile strength where the subscript k denotes characteristic.

EN-1992 requires adjacent laps to be staggered by $0.3l_{b,d}$ where $l_{b,d}$ is the design lap length which is given by:

$$l_{b,d} = \alpha_1\alpha_2\alpha_3\alpha_5\alpha_6 l_{b,rqd} \tag{5}$$

in which the coefficients α_1 to α_5 are defined in Table 8.2 of EN-1992. For straight bars, α_1, α_3 and α_5 can conservatively be taken as 1.0. The coefficient $\alpha_2 = 1 - \frac{0.15(c_d - \emptyset)}{\emptyset} \geq 0.7;\ \leq 1.0$ in which c_d is the least of the cover and half the clear bar spacing. When more than 50% of the bars are lapped over a section of length $1.3l_{b,d}$, $\alpha_6 = 1.5$. For 50% lapping, $\alpha_6 = 1.4$.

Table 1 shows total measured failure loads, including self-weight, corresponding bending moments at the most highly stressed lap end and associated reinforcement stresses calculated with section analysis (depicted "S.A."). In the section analysis, plane sections were assumed to remain plane and the following compressive stress strain relationship from EN-1992 [2] was used for concrete:

$$\sigma_c = f_{cm}\left(\frac{k\eta - \eta^2}{1+(k-2)\eta}\right) \quad \text{for} \quad 0 \leq \varepsilon_c\ (^0/_{00}) \leq 3.5 \tag{6}$$

where:

f_{cm} is the mean concrete cylinder strength, which is taken as the measured cylinder strength for the calculations in this paper,

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \tag{7}$$

$$\varepsilon_{c1}\ (^0/_{00}) = 0.7 f_{cm}^{\ 0.31} \tag{8}$$

$$k = 1.05E_{cm}\varepsilon_{c1}/f_{cm} \tag{9}$$

ε_c is the strain in the compression zone concrete, ε_{c1} is the strain at peak compressive stress, and E_{cm} is the mean concrete elastic modulus which was taken as the 28 day measured values with cast 2 values used for analysis of cast 3.

For specimens in which significant strain hardening occurred, the flexural resistance was underestimated by section analysis with σ_c from Equation 6. For these tests, the reinforcement stress at peak load was calculated with plastic section analysis, neglecting the compression reinforcement, using a rectangular concrete stress block with stress equal to f_{cm}.

Table 1 also shows measured average (depicted “meas avg”) and maximum (depicted “meas max”) reinforcement stresses derived from strains at lap ends as well as mean lap strengths calculated with Equation 1 and EN-1992 neglecting the limit of f_y. The “meas avg” stresses are averages of stresses in edge and middle B bars (see Figure 1) at maximum load, while the “meas max” stresses are for individual bars. In beams where flexural yield occurred, the “meas max” stresses developed subsequent to peak load due to the reduction in flexural lever arm following the onset of concrete crushing in the flexural compression zone. If 4P-16/25-275 is excluded, since the measured strain appears to have been significantly reduced by tension in uncracked concrete, the mean and standard deviation of the ratio of “meas avg” and S.A. reinforcement stresses are 1.00 and 0.07, respectively. This is considered reasonable agreement.

The mean EN-1992 lap strengths in Table 1 were calculated with $\alpha_1 = \alpha_3 = \alpha_5 = 1, \alpha_6 = 1.5$ (100% lap) or 1.4 (50% lap) and $\gamma_c = 1.0$. With these assumptions, the mean lap failure stress according to EN-1992, is found by rearranging equations 3 and 5 to be:

$$\sigma_{s,m} = \frac{4}{\phi}\frac{l_b}{\alpha_2\alpha_6} f_{bm} \leq f_y \tag{10}$$

In which the mean bond strength is given by Equation 4 with $\gamma_c = 1.0$. The mean concrete tensile strength was calculated in accordance with EN-1992 as $f_{ctm} = 0.3f_{ck}^{2/3}$ in which f_{ck} was assumed for tests performed in a laboratory to be $f_{cm} - 4$ MPa [21].

The only two specimens for which Equation 1 overestimates the reinforcement stress at bond failure from section analysis are 4P-16/20c-500 and 4P-20/20c-700, with 50% laps and continuous edge bars. Lap failure in 4P-16/20c-500 and 4P-20/20c-700 resulted from the central pair of lapped bars attracting greater average force along the lap than the continuous bars. This phenomena is unaccounted for by Equation 1, which falsely predicts laps to be stronger in specimens with continuous edge bars than comparable specimens with 100% laps owing to increased bar spacing in the former.

Theoretically, only the staggered laps in 4P-16/20s-500 comply with the detailing requirements of EN-1992 since adjacent laps in other tests are not staggered by $0.3l_{b,d}$. The stirrup spacing of 200 mm also exceeds the EN-1992 limit of 150 mm for designed transverse reinforcement at laps. Despite this, Equation 10 is seen to underestimate the strength of the tested short laps but to give reasonable mean strengths for long laps which failed in bond subsequent to flexural yield. If the maximum reinforcement stress is limited to f_y, the mean of $\sigma_{S.A.}/\sigma_{s,m}$ for all the tested specimens is 1.06 and 1.16 for $\sigma_{s,m}$ from Equations 1 and 10 respectively. The corresponding standard deviations of $\sigma_{S.A.}/\sigma_{s,m}$ for Equations 1 and 10 respectively are 0.05 and 0.20. Characteristic lap strengths obtained by multiplying Equation 10 by 0.7 are all less than measured if the lap strength is limited to f_y suggesting that EN-1992 provides safe characteristic lap strengths for the tested specimens even though the specimens did not comply with the detailing rules of EN-1992 for laps.

6. Conclusions

A total of 18 specimens were tested to investigate the influence on strength and ductility of lap length and arrangement. Laps are classified as “short”, “long” and “very long” with “long” laps just able to develop bar yield according to Equation 1 of fib Bulletin 72. Some laps were tested in regions of varying moment to investigate the influence of shear on bond strength. Tested lap arrangements were 100% lapped, 50% lapped with continuous bars and 50% staggered laps. Of these, only the staggered laps comply with the detailing requirements of EN-1992 which require adjacent laps to be offset by 0.3 times the lap length. The following is a summary of the key conclusions from the study:

1. If limited to f_y, Equation 1 of fib Bulletin 72 for mean lap strength gives good predictions of the failure stress of the tested laps calculated with section analysis. Consequently, the conservatism of the MC2010 design provisions which are derived from Equation 1, relate to the adopted safety format.

2. If limited to f_y, Equation 10 for mean lap strength according to EN-1992 gives conservative predictions for all the tested laps, but underestimates the strength of short laps since it assumes bond strength to be independent of lap length.
3. Shear had no significant influence on the strength of the tested laps.
4. In specimens with 100% lapped bars at a section, “short” laps failed suddenly in bond and reinforcement yielded at critical ends of “long” laps as predicted by Equation 1. Significant plastic deformation developed in all specimens with “long” laps including those in which bond failure eventually occurred. No additional ductility resulted from increasing lap lengths beyond that required to achieve reinforcement yield.
5. Strain measurements show that the central half of the tested “very long” laps contributed little to the transmission of forces between bars.
6. Average bond stresses between strain gauges at loaded lap ends were almost independent of lap length and greatest at lap ends.
7. Within zones of uniform moment, the total force resisted by lapped bars was almost constant along laps. Furthermore, in 100% laps, forces in each pair of lapped bars were almost equal.
8. Both specimens with continuous edge bars and 50% laps failed in bond unlike directly comparable specimens with 100% laps. This occurred because the total force resisted by the lapped bars in specimens with continuous bars increased along the lap to a maximum at the lap centre with a corresponding reduction in force in the continuous bars. This effect is explained by the greater stiffness of the lapped bars compared with the continuous bars and brings into doubt the merits of staggering laps.

7. Acknowledgements

The research work disclosed in this publication is partially funded by the REACH HIGH Scholars Programme – Post-Doctoral Grants. The grant is part-financed by the European Union, Operational Programme II – Cohesion Policy 2014-2020 “Investing in human capital to create more opportunities and promote the wellbeing of society - European Social Fund”. The authors acknowledge technical and financial contribution from the Concrete Centre, in particular Mr Charles Goodchild. The help received from Mr Les Clark and Mr Bob Hewitt, laboratory technicians in the Structures Lab in the

Department of Civil and Environmental Engineering at Imperial College London, is also acknowledged.

References

1. fib, *fib Model code for concrete structures 2010*, fib, Editor. 2013, Ernst & Sohn: Lausanne, Switzerland.
2. BSI, *BS EN 1992-1-1:2004 Eurocode 2 Design of concrete structures - Part 1-1: General rules and rules for buildings*. 2011, BSI.
3. BSI, *BS 8110-1:1997 Structural use of concrete - Part 1: Code of practice for design and construction*. 2007, BSI.
4. Cairns, J. and R. Eligehausen, *Evaluation of EC2 rules for the design of tension lap joints.* The Structural Engineer, 2014. **92**(9): p. 44-52.
5. fib, *Technical report Bulletin 72 - Bond and anchorage of embedded reinforcement: background to the fib Model Code for concrete structures 2010*. 2014, fib: Lausanne, Switzerland.
6. fib Task Group 4.5 "Bond models", *Splice test database*. 2005.
7. Ferguson, P.M., *Design criteria for overhanging ends of bent caps*. 1964, The University of Texax at Austin: Center for Highway Research.
8. Ferguson, P.M., J.E. Breen, and J.N. Thompson, *Pullout tests on high strength reinforcing bars.* ACI Journal Proceedings, 1965. **62**(8).
9. Ferguson, P.M. and C.N. Krishnaswamy, *Tensile lap splices Part 2: Design recommendations for retaining wall splices and large bar splices*. 1971, Center for Highway Research: The University of Texas at Austin.
10. Ferguson, P., M. and J.N. Thompson, *Development length of high strength reinforcing bars in bond.* ACI Journal Proceedings, 1972. **59**(7).
11. Jirsa, J.O. and J.E. Breen, *Influence of casting position and shear on development and splice length - design recommendations*. 1981, Center for transportation research: The University of Texas at Austin.
12. Zekany, A.J., et al., *The influence of shear on lapped splices in reinforced concrete*. 1981, Center for transportation research bureau of engineering research: The University of Texas at Austin, Texas.
13. Orangun, C.O., J.O. Jirsa, and J.E. Breen, *The strength of anchored bars: a reevaluation of test data on development length and splices*. 1975, Center for Highway Research: The University of Texas at Austin.
14. Reynolds, G.C. and A.W. Beeby, *Bond strength of deformed bars. Bond in concrete.* 1982, London: Applied Science Publishers. 434-445.
15. IStructE and Concrete Society, *Standard method of detailing structural concrete - a manual for best practice*. 3 ed. 2006: The Institute of Structual Engineers.
16. Ferguson, P.M. and E.A. Briceno, *Tensile lap splices Part 1: Retaining wall type, varying moment zone*. 1969, Center for Highway Research: The University of Texas at Austin.
17. Cairns, J., *Staggered lap joints for tension reinforcement.* Structural Concrete, 2014. **15**(1): p. 45-54.
18. BSI, *BS EN 206-1:2000 - Concrete - Part 1: Specification, performance, production and conformity*. 2006, BSI.
19. BSI, *BS 4449:2005+A3:2016 Steel for the reinforcement of concrete - Weldable reinforcing steel - Bar coil and decoiled product - Specification*. 2016, BSI: U.K.
20. Micallef, M., R.L. Vollum, and B.A. Izzuddin, *Investigating the need for long laps in reinforced concrete elements*, in *fib Symposium 2017 - High tech concrete: where technology and engineering meet*, D.A. Hordick and M. Lukovic, Editors. 2017, Springer International Publishing AG 2018: Maastricht, The Netherlands. p. 1549-1557.
21. DIN EN 206:2014-07, *Concrete – Specification, performance, production and conformity*, German version EN 206:2013, Berlin, July 2014.